

# Optics

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# Lecture Outline

- **Ray Optics**
- Citations

# Introduction

Picturing light as rays is useful for predicting imaging properties.

As  $\lambda \rightarrow 0$ , Maxwell equation's become the **eikonal equation**, which governs ray direction in a medium with a varying  $n(\vec{r})$ .

**Fermat's principle** is deduced from eikonal equation.

**Snell's law** is derived from fermat's principle.

# Eikonal Equation

$$\nabla^2 \vec{E}(\vec{r}, t) + \frac{[n(\vec{r})]^2 \omega^2}{c^2} \vec{E}(\vec{r}, t) = 0 \quad (\text{wave equation in isotropic medium})$$

$$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{i[k_{vac} R(\vec{r}) - \omega t]}, \quad k_{vac} = \frac{\omega}{c} \left( \frac{rad}{m} \right) \quad (\text{trial solution})$$

We plug trial solution in, perform laplacian, arrange terms, and make the approximation that  $\frac{1}{k_{vac}} = \frac{\lambda_{vac}}{2\pi} \rightarrow 0$ , we get

$$[\nabla R(\vec{r}) \cdot \nabla R(\vec{r}) - [n(\vec{r})]^2] \vec{E}(\vec{r}, t) = 0 \quad (\text{simplified wave equation})$$

$$\implies \nabla R(\vec{r}) \cdot \nabla R(\vec{r}) = [n(\vec{r})]^2$$

$$\implies \boxed{\nabla R(\vec{r}) = n(\vec{r}) \hat{s}(\vec{r})} \quad (\text{eikonal function})$$

If  $R(\vec{r})$  (length) is real, no absorption or amplification.  $R(\vec{r}) = \text{constant}$  is one wavefront.  $\nabla R(\vec{r})$  is the local direction of propagation.

# Fermat's Principle

$$\nabla \times [\nabla R(\vec{r})] = \nabla \times [n(\vec{r})\hat{s}(\vec{r})] \quad (\text{curl of eikonal function})$$

$$\int_A \nabla \times [n(\vec{r})\hat{s}(\vec{r})] \quad (\text{integrate over open surface A})$$

$$\oint_C n\hat{s} \cdot d\vec{l} = 0 \quad (\text{By Stoke's Theorem})$$

$$\Rightarrow \int_A^B n\hat{s} \cdot d\vec{l} \text{ is independent of path}$$

Notice that

$$\int_A^B n\hat{s} \cdot d\vec{l} = \min \int_A^B n d\vec{l}$$

We define **Optical Path Length** as



# Paraxial Ray Theory

Propagation of rays through optical systems can be approximated as *paraxial*, nearly parallel to the axis of these systems.

**Paraxial ray** theory predicts stability of laser cavities, to see if ray drift away from optical axis.

# Proof of Snell's Law

Construction to prove Snell's Law

Constrained minimization problem: Minimize  $n_1 d_{1-1} + n_2 d_{2-2}$

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[1] Saleh, B. E. A., & Teich, M. C. (2019). Fundamentals of photonics (3rd ed.). Wiley. [2] Peatross, Justin, and Michael Ware. Physics of Light and Optics. 2015 ed., January 31, 2025 revision, Department of Physics, Brigham Young University. [optics.byu.edu](http://optics.byu.edu).