Learning Optics

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Lecture Outline

- **Ray Optics**
- Citations

Introduction

Picturing light as rays is useful for predicting imaging properties.

As $\lambda \to 0$, Maxwell equation's become the eikonal equation, which governs ray direction in a medium with a varying $n(\vec{r})$.

Fermat's principle is deduced from eikonal equation.

Snell's law is derived from fermat's principle.

Eikonal Equation

$$\nabla^2 \vec{E}(\vec{r},t) + \frac{[n(\vec{r})]^2\omega^2}{c^2}\vec{E}(\vec{r},t) = 0 \quad \text{(wave equation in isotropic medium)}$$

$$\vec{E}(\vec{r},t) = \vec{E}_0 e^{i[k_{vac}R(\vec{r})-\omega t]}, k_{vac} = \frac{\omega}{c}\left(\frac{rad}{m}\right) \quad \text{(trial solution)}$$

We plug trial solution in, perform laplacian, arrange terms, and make the approximation that $\frac{1}{k_{vac}} = \frac{\lambda_{vac}}{2\pi} \to 0$, we get

$$\left[\nabla R(\vec{r}) \cdot \nabla R(\vec{r}) - [n(\vec{r})]^2\right]\vec{E}_0(\vec{r}) = 0 \quad \text{(Simplified wave equation)}$$

$$\implies \nabla R(\vec{r}) \cdot \nabla R(\vec{r}) = [n(\vec{r})]^2$$

$$\implies \boxed{\nabla R(\vec{r}) = n(\vec{r})\hat{s}(\vec{r})} \quad \text{(eikonal function)}$$

If $R(\vec{r})$ (length) is real, no absorption or amplification. $R(\vec{r}) =$

Fermat's Principle

$$\nabla \times [\nabla R(\vec{r})] = \nabla \times [n(\vec{r})\hat{s}(\vec{r})] = 0 \qquad \text{(curl of eikonal function)}$$

$$\int_A \nabla \times [n(\vec{r})\hat{s}(\vec{r})] = 0 \qquad \text{(integrate over open surface A)}$$

$$\oint_C n\hat{s} \cdot d\vec{l} = 0 \qquad \text{(By Stoke's Theorem)}$$

$$\implies \int_A^B n\hat{s} \cdot d\vec{l} \text{is independent of path}$$

Notice that

$$\int_A^B n\hat{s} \cdot d\vec{l} = \min \int_A^B n d\vec{l}$$

We define **Optical Path Length** as

Paraxial Ray Theory

Propagation of rays through optical systems can be approximated as *paraxial*, nearly parallel to the axis of these systems.

Paraxial ray theory predicts stability of laser cavities, to see if ray drift away from optical axis.

Proof of Snell’s Law

Construction to prove Snell’s Law

Constrained minimization problem: Minimize $n_1 d_1 _1 + $

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- **Citations**

[1] Saleh, B. E. A., & Teich, M. C. (2019). Fundamentals of photonics (3rd ed.). Wiley. [2] Peatross, Justin, and Michael Ware. Physics of Light and Optics. 2015 ed., January 31, 2025 revision, Department of Physics, Brigham Young University. optics.byu.edu.