# Learning Optics

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# Lecture Outline

- **Ray Optics**
- Citations

# Introduction

Picturing light as rays is useful for predicting imaging properties.

As $\lambda \to 0$, Maxwell equation's become the eikonal equation, which governs ray direction in a medium with a varying $n(\vec{r})$.

Fermat's principle is deduced from eikonal equation.

Snell's law is derived from fermat's principle.

# Eikonal Equation

$$\nabla^2 \vec{E}(\vec{r},t) + \frac{[n(\vec{r})]^2\omega^2}{c^2}\vec{E}(\vec{r},t) = 0 \quad \text{(wave equation in isotropic medium)}$$

$$\vec{E}(\vec{r},t) = \vec{E}_0 e^{i[k_{vac}R(\vec{r})-\omega t]}, k_{vac} = \frac{\omega}{c}\left(\frac{rad}{m}\right) \quad \text{(trial solution)}$$

We plug trial solution in, perform laplacian, arrange terms, and make the approximation that $\frac{1}{k_{vac}} = \frac{\lambda_{vac}}{2\pi} \to 0$, we get

$$[\nabla R(\vec{r}) \cdot \nabla R(\vec{r}) - [n(\vec{r})]^2]\vec{E}_0(\vec{r}) = 0 \quad \text{(Simplified wave equation)}$$

$$\implies \nabla R(\vec{r}) \cdot \nabla R(\vec{r}) = [n(\vec{r})]^2$$

$$\implies \boxed{\nabla R(\vec{r}) = n(\vec{r})\hat{s}(\vec{r})} \qquad \text{(eikonal function)}$$

If $R(\vec{r})$ (length) is real, no absorption or amplification. $R(\vec{r}) = constant$ is one wavefront. $\nabla R(\vec{r})$ is the local direction of propagation.

# Fermat’s Principle

$$\nabla \times [\nabla R(\vec{r})] = \nabla \times [n(\vec{r})\hat{s}(\vec{r})] = 0 \qquad \text{(curl of eikonal function)}$$

$$\int_A \nabla \times [n(\vec{r})\hat{s}(\vec{r})] = 0 \qquad \text{(integrate over open surface A)}$$

$$\oint_C n\hat{s} \cdot d\vec{l} = 0 \qquad \text{(By Stoke’s Theorem)}$$

$$\implies \int_A^B n\hat{s} \cdot d\vec{l} \text{is independent of path}$$

Notice that

$$\int_A^B n\hat{s} \cdot d\vec{l} = \min \int_A^B n d\vec{l}$$

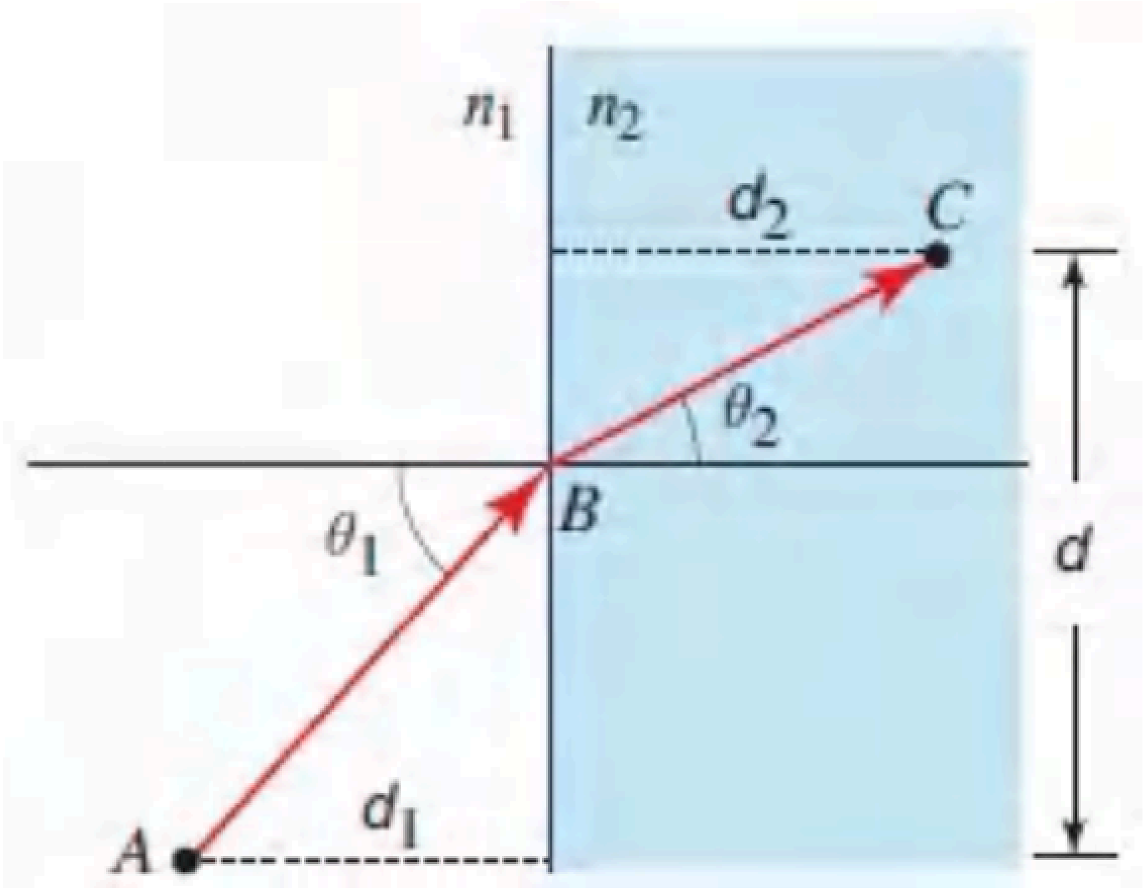
We define **Optical Path Length** as

# Paraxial Ray Theory

Propagation of rays through optical systems can be approximated as *paraxial*, nearly parallel to the axis of these systems.

Paraxial ray theory predicts stability of laser cavities, to see if ray drift away from optical axis.

# Proof of Snell's Law




Construction to prove Snell's Law

Constrained minimization problem: Minimize $n_1 d_1 _1 + $

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[1] Saleh, B. E. A., & Teich, M. C. (2019). Fundamentals of photonics (3rd ed.). Wiley. [2] Peatross, Justin, and Michael Ware. Physics of Light and Optics. 2015 ed., January 31, 2025 revision, Department of Physics, Brigham Young University. optics.byu.edu.