

# Learning Optics

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# Lecture Outline

- **Ray Optics**
- Citations

# Introduction

Picturing light as rays is useful for predicting imaging properties.

As  $\lambda \rightarrow 0$ , Maxwell equation's become the **eikonal equation**, which governs ray direction in a medium with a varying  $n(\vec{r})$ .

**Fermat's principle** is deduced from eikonal equation.

**Snell's law** is derived from fermat's principle.

# The Eikonal Equation

Wave equation in isotropic medium:

$$\nabla^2 \vec{E}(\vec{r}, t) + \frac{[n(\vec{r})]^2 \omega^2}{c^2} \vec{E}(\vec{r}, t) = 0$$

Trial solution:

$$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{i[k_{vac} R(\vec{r}) - \omega t]}, k_{vac} = \frac{\omega}{c} \left( \frac{rad}{m} \right)$$

We plug trial solution in, perform laplacian, arrange terms, and make the approximation that  $\frac{1}{k_{vac}} = \frac{\lambda_{vac}}{2\pi} \rightarrow 0$ , we get the simplified wave equation

$$[\nabla R(\vec{r}) \cdot \nabla R(\vec{r}) - [n(\vec{r})]^2] \vec{E}_0(\vec{r}) = 0$$

$$\implies \nabla R(\vec{r}) \cdot \nabla R(\vec{r}) = [n(\vec{r})]^2$$



# Fermat's Principle

Curl of eikonal function is

$$\nabla \times [\nabla R(\vec{r})] = \nabla \times [n(\vec{r})\hat{s}(\vec{r})] = 0$$

Integrating over open surface A, we get

$$\int_A \nabla \times [n(\vec{r})\hat{s}(\vec{r})] = 0 \quad ()$$

Apply Stoke's Theorem, we have

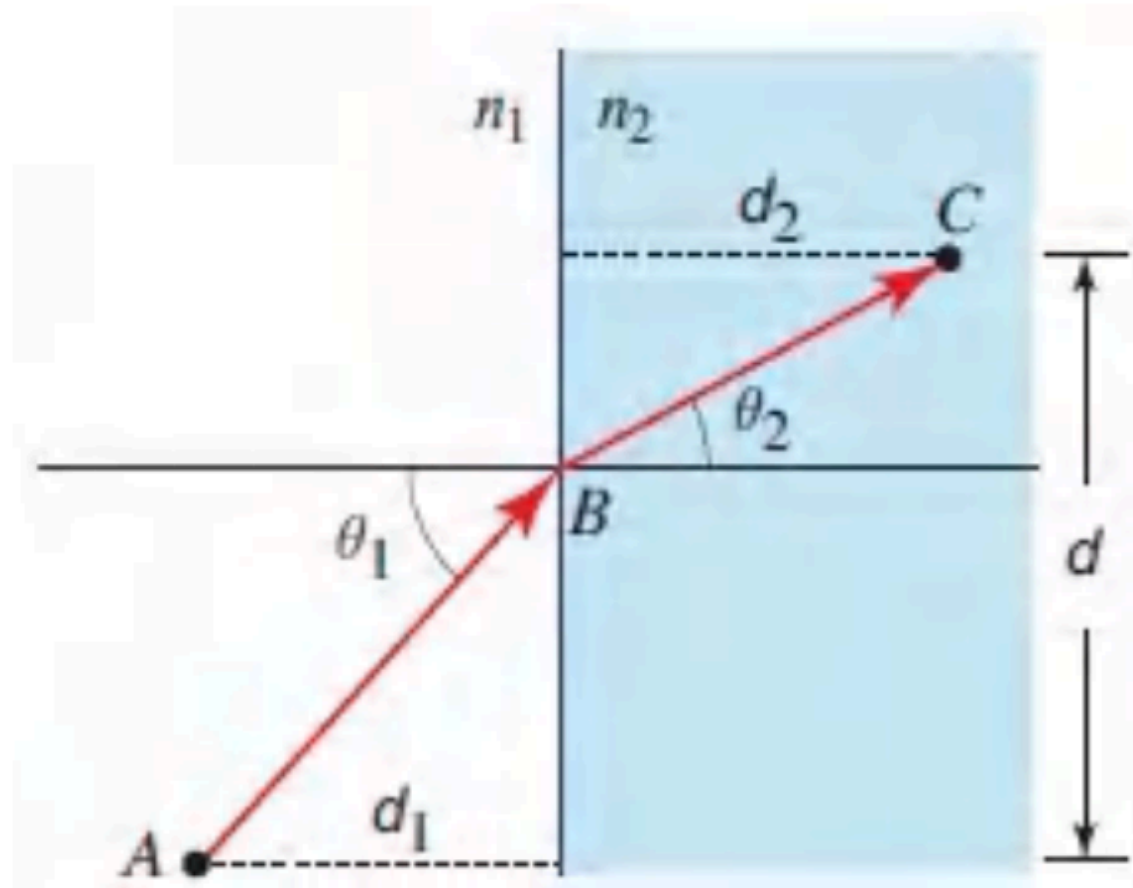
$$\oint_C n\hat{s} \cdot d\vec{l} = 0$$
$$\implies \int_A n\hat{s} \cdot d\vec{l}$$

# Paraxial Ray Theory

Propagation of rays through optical systems can be approximated as *paraxial*, nearly parallel to the axis of these systems.

**Paraxial ray** theory predicts stability of laser cavities, to see if ray drift away from optical axis.

# Proof of Snell's Law



Construction to prove Snell's Law

Constrained minimization problem: Minimize  $n_1 d_1 + n_2 d_2$

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[1] Saleh, B. E. A., & Teich, M. C. (2019). Fundamentals of photonics (3rd ed.). Wiley. [2] Peatross, Justin, and Michael Ware. Physics of Light and Optics. 2015 ed., January 31, 2025 revision, Department of Physics, Brigham Young University. [optics.byu.edu](http://optics.byu.edu).