ACCOMPLISH - Numerical Differentiation

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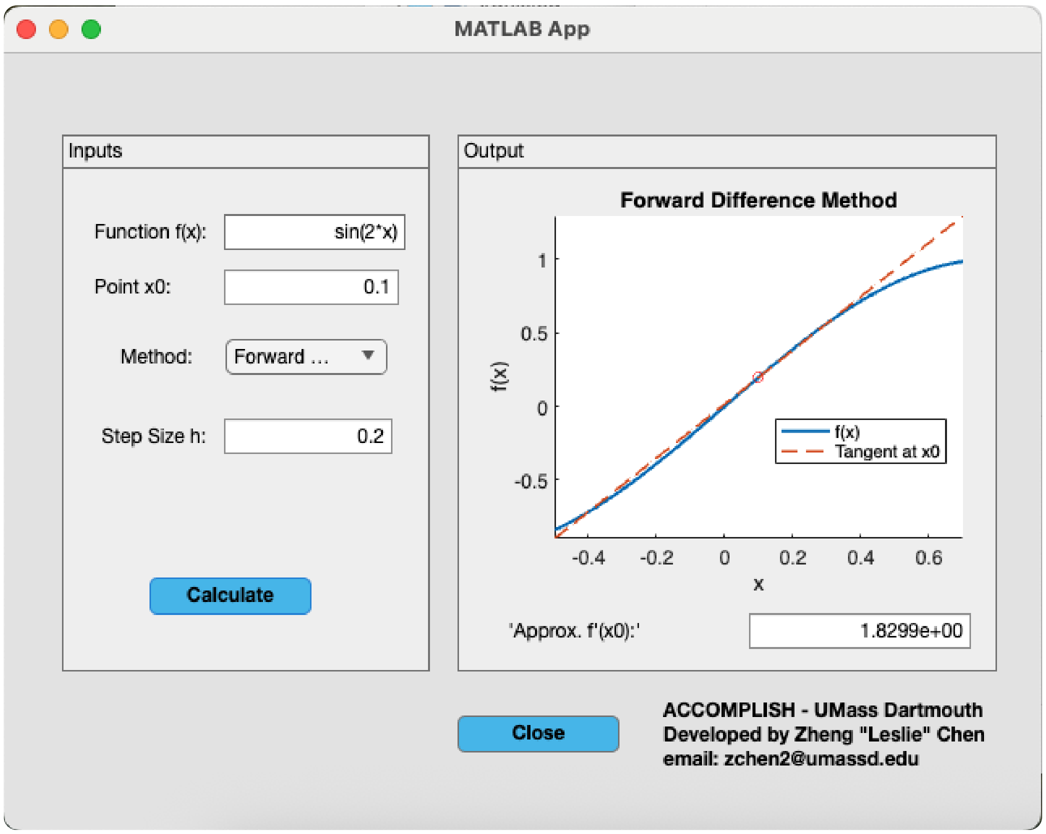
Goal

In this module, students will finish building a MATLAB App Designer application that approximates derivatives of arbitrary functions. Starting from the provided template (`Numerical_Differentiation_App.mlapp`), they will implement all finite-difference algorithms in the `numerical_differentiation.m` file.



Overview of the GUI

1. **User inputs**
   - **Function:** Enter the formula, for example `f(x) = sin(x)`
   - **Evaluation Point:** Specify the value of x_0
   - **Method:** Select a finite-difference formula from the drop-down menu
2. **App framework**
   - **Starter File:** `Numerical_Differentiation_App.mlapp`
   - **Algorithm Library:** All differentiation routines are implemented as separate functions in `numerical_differentiation.m`
   - **Instructor Version:** A complete app (with working code) is provided to the advisor only
3. **Student assignment**
   - Review the mathematical background for each finite-difference formula
   - Fill in the missing code blocks in `numerical_differentiation.m`
   - Detailed guidance and derivations will be provided in the sections that follow

MATLAB GUI Resources

- **App Designer YouTube tutorial (recommended)**
  https://youtu.be/nb0jHVXKY2w
- **App Designer documentation**
  https://www.mathworks.com/help/matlab/app-designer.html
- **Create and Run a Simple App Using App Designer tutorial**
  https://www.mathworks.com/help/matlab/creating_guis/create-a-simple-app-or-gui-using-app-designer.html
- **MATLAB App Designer product page**
  https://www.mathworks.com/products/matlab/app-designer.html

Included Differentiation Methods

Students will implement these finite-difference formulas:

- **Two-point formulas**
  - Forward-difference
  - Backward-difference
- **Three-point formulas**
- **Five-point formulas**
- **Second-derivative midpoint formula** (optional)

Next, we will provide mathematical derivations and coding hints for each method so you can seamlessly integrate them into the GUI.

Table of Contents

To approximate $f'(x_0)$:

To approximate $f''(x_0)$ (optional):

Two-point_formulas

Derivation

- **Forward-difference**: From the Taylor expansion of $f(x_0 + h)$, we have

$$f(x_0 + h) = f(x_0) + h\,f'(x_0) + O(h^2)\,.$$

  Rearranging gives

$$f'(x_0) \;\approx\; \frac{f(x_0+h)-f(x_0)}{h} + O(h)\,,$$

a first-order accurate formula.

- **Backward-difference**: Similarly, expanding $f(x_0 - h)$ yields

$$f(x_0-h) = f(x_0) - h\,f'(x_0) + O(h^2)\,,$$

so

$$f'(x_0) \;\approx\; \frac{f(x_0)-f(x_0-h)}{h} + O(h)\,,$$

also first-order accurate.

Coding hints

- Inside your `Calculate_pushbutton_Callback`, branch on `handles.Method` to distinguish forward vs. backward.
- For **forward**, you will need `f(x0 + h)` and `f(x0)`.
- For **backward**, you will need `f(x0)` and `f(x0 - h)`.
- Compute the difference of those two values and then divide by `h`.

Three-point_formulas

Derivation

- **Central-difference**: Using Taylor series for $f(x_0 \pm h)$ and subtracting gives

$$f'(x_0) \;\approx\; \frac{f(x_0+h)-f(x_0-h)}{2h} + O(h^2)\,,$$

a second-order accurate midpoint formula.

- **Endpoint (forward/backward)**: At boundaries, the three-point endpoint formula is

$$f'(x_0) \;\approx\; \frac{-3f(x_0)+4f(x_0+h)-f(x_0+2h)}{2h} + O(h^2)$$

(and its backward analogue).

Coding hints

- Branch on the three sub-methods (central, forward-endpoint, backward-endpoint).
- **Central** uses `f(x0 + h)` and `f(x0 - h)`.
- **Endpoint** uses three sample points: at `x0`, at one step away (`±h`), and at two steps away (`±2*h`).
- Combine those three function values with the weights from the derivation, then divide by `2*h`.

Five-point_formulas

Derivation

The fourth-order accurate central five-point formula comes from combining Taylor expansions at $x_0 \pm h$ and $x_0 \pm 2h$ to cancel lower-order error terms:

$$f'(x_0) \;\approx\; \frac{f(x_0-2h) \;-\; 8\,f(x_0-h) \;+\; 8\,f(x_0+h) \;-\; f(x_0+2h)}{12\,h} \;+\; O(h^4).$$

Coding hints

- This central formula requires four off-grid evaluations: `x0 ± h` and `x0 ± 2*h`, plus `x0` itself (if you verify consistency).
- Look back at the derivation to pick the four coefficients $\{1, -8, 8, -1\}$.
- Form the weighted sum of those four `f( ... )` calls and divide by `12*h`.

Second-derivative_midpoint_formula

Derivation

- From the Taylor expansions at $x_0 \pm h$, the second-derivative midpoint formula is

$$f''(x_0) \approx \frac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{h^2} + O(h^2).$$

Coding hints

- Evaluate the function at `x0 + h`, `x0`, and `x0 - h` to obtain the three sample values.
- Multiply these values by the weights $\{1, -2, 1\}$ to form the numerator of the approximation.
- Divide the weighted sum by h^2 to compute the second-derivative estimate.
- Store the result in its own variable (for example, `secondDerivative`) to keep it separate from any first-derivative outputs.