Problem Set 5

Due: 5:00pm PST, Thursday, October 1st — EE 483, Fall 2020

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1. Assume that a discrete-time signal $x[n]$ is formed by sampling the following continuous-time signal
$$x(t) = 2\sin(1000\pi t)\cos(1000\pi t)$$
with a sampling interval T.

   (a) What is the Nyquist rate for this signal in Hz.

   (b) What is the DTFT of $x[n]$ if we sample exactly at the Nyquist rate instead of slightly above it? Does this cause a problem with this signal?

   (c) Find and sketch the DTFT of $x[n]$ for each of the following sampling intervals: (i) $T = 0.0001$ seconds, (ii) $T = 0.0002$ seconds, (iii) $T = 0.0005$ seconds.

   (d) For each of the sampling intervals from part (c), find the signal $y(t)$ that would be obtained as the output of an ideal D/A converter (where the D/A converter assumes the same sampling rate as the A/D converter).

   (e) Repeat part (d), but now assume that the ideal D/A converter incorrectly assumes that the sampling rate was half of what it actually was. What happens to the signal when it is reconstructed assuming the wrong sampling rate?

2. An analog signal $x(t)$ has a Fourier transform $X(j\Omega)$ given by
$$X(j\Omega) = \begin{cases} 3 + \Omega/2, & -6 \le \Omega < 0 \\ 3 - \Omega/2, & 0 \le \Omega \le 6 \\ 0, & \text{else} \end{cases}$$
with Ω in units of radians/second. The signal is sampled with period T.

   - Sketch $X(j\Omega)$. Based on the symmetry characteristics of this Fourier spectrum, what can you infer about the characteristics of this signal in the time domain?
   - Sketch the DTFT of the sampled sequence $x[n]$ assuming (i) $T = \pi/3$; (ii) $T = \pi/6$; (iii) $T = \pi/9$; (iv) $T = \pi/12$; and (v) $T = \pi/16$.

   In all cases, make sure to label all of the key frequencies and amplitudes appearing in the plots.

3. Consider a signal $x(t)$ that is bandlimited to 20 kHz (i.e., its Fourier transform is zero for frequencies larger than 20 kHz). You are asked to bandpass filter the signal $x(t)$ to create a new signal $y(t)$. The new signal $y(t)$ should have the same spectrum as $x(t)$ for frequencies between 4 kHz and 10 kHz (these frequencies are passed perfectly through the system), while all other frequency content should be rejected (i.e., set to zero). This bandpass filtering operation is to be accomplished using a combination of ideal A/D conversion (with sampling interval T), discrete-time filtering, and ideal D/A conversion (for matched sampling interval T).

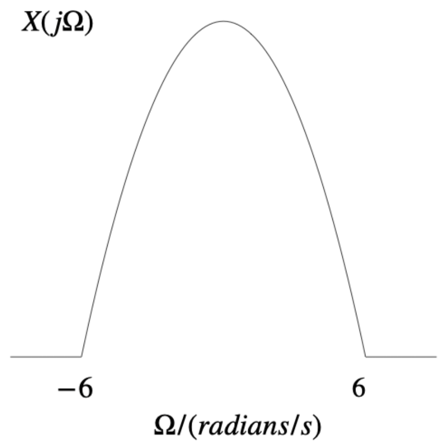
(a) What is the largest sampling interval T that will prevent aliasing at the A/D converter when the signal $x(t)$ is converted into the discrete timesignal $x[n] = x(t)|_{t=nT}$ for $n \in \mathbb{Z}$?

(b) Assume that you use a sampling rate that is twice the Nyquist rate. Design the frequency response of an ideal discrete-time filter $H(e^{j\omega})$ so that the overall system will have the desired bandpass filtering effect.

(c) It turns out that in this case, it is possible to allow some aliasing during A/D conversion and still achieve the desired bandpass filtering effect.

   i. Find the maximum value of T for which the combined system (A/D, discrete-time filter, D/A) can still perform the desired filtering action if the discrete-time filter is modified appropriately.

4. An analog signal $x(t)$ has the Fourier transformation as following:



And we sample this signal with the Nyquist rate.

(a) Compute the Nyquist rate and sketch the DTFT $X(e^{j\omega})$ of $x[n]$.

(b) Now we up-sample $x[n]$ by 2 to get a discrete signal $w[n]$. Represent its DTFT $W(e^{j\omega})$ using $X(e^{j\omega})$ and sketch it.

(c) Use $X(e^{j\omega})$ to recover $x(t)$, i.e., represent $x(t)$ using $X(e^{j\omega})$.

(d) Suppose that we get a new sequence $y[n]$ by interpolating $x[n]$ as following:

$$y[2n] = x[n], \quad y[2n+1] = x[n].$$

Represent $Y(e^{j\omega})$ using $X(e^{j\omega})$.

(e) Can you recover $x(t)$ using $Y(e^{j\omega})$? If no, explain why; If yes, represent $x(t)$ using $Y(e^{j\omega})$.

5. An analog signal $x(t)$ has a Fourier transform $X(j\Omega)$ given by

$$X(j\Omega) = \begin{cases} 3 - \Omega/2, & 3 \leq \Omega < 6 \\ 0, & \text{else} \end{cases}$$

(a) What's the Nyquist rate f_0 for this signal? Sketch the DTFT if we sample it with its Nyquist rate.

(b) What if we sample it with $f_0/2$? Sketch its DTFT. Can we recover this signal if we sample with $f_0/2$?

(c) What's the lowest sampling rate such that we can recover this signal? Sketch the DTFT for this case.