Enumeration and matroids

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Let P be a predicate such as $P(x, y)$ is decidable in polynomial time in $|x|$ and $|y| \leq Q(|x|)$ with Q a fixed polynomial.

Our problem is to find the set $A(x) = \{y \mid P(x, y)\}$ The problems of this form are the class **Enum**$\cdot\mathrm{P}$.

We want to find $A(x)$ as fast as possible.
As the process is dynamic, we want the time between each member of the set we output to be as short as possible.

Definition

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To an enumeration problem $\textsc{Enum}{\cdot}\mathcal{A}$, we associate the following characteristic decision problem :

$\textsc{AllSolution}_A$
Input: An instance x of A and a set M included in $A(x)$
Output: Accept if $A(x) - M \neq \varnothing$, reject otherwise

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If $\text{Enum}\cdot\mathcal{A} \in$ **TotalP** then $\text{AllSolution}_A \in \mathrm{P}$.

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A formula is $2\mathrm{CNF}$ if every clause has only two literals.

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- $\textsc{Enum}{\cdot}\mathit{Horn}$ is not in **TotalP** unless $\mathrm{P} = \mathrm{NP}$ thanks to the problem $\textsc{AllSolution}_{\textsc{Horn}}$
- $\textsc{Enum}{\cdot}2\mathit{CNF}$ is proven to be in **TotalP**.

Definition

A problem $\textsc{Enum}{\cdot}\mathcal{A}$ is decidable in incremental polynomial time **IncP**, if there is an algorithm which on every instance x and every integer $k \leq |A(x)|$ return the $(k+1)^{th}$ solution in time polynomial in $|x|$ and k.

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$\textsc{Enum}{\cdot}\textsc{MatroidCircuit}$ is in **IncP** as we will explain it later.

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$\textsc{Enum}\cdot\textsc{MaxIndependentSet}$ is in **DelayP** and the algorithm enumerate the solutions in lexicographic order.
In reverse lexical order, it is not **DelayP** anymore [David S. Johnson, Christos H. Papadimitriou and Mihalis Yannakakis].

Definition

A problem $\textsc{Enum}{\cdot}\mathcal{A}$ is in the class **QueryP** if there are an order on the set $A(x)$ and an algorithm which computes the i^{th} solution of $A(x)$ in time polynomial in $|x|$. If there is no i^{th} solution the algorithm must tell so in polynomial time.

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MSO queries on a term can be enumerated in linear time and the problem is also in **QueryP**.

Under widely accepted complexity hypothesis we have the following inclusions :

$$\textbf{QueryP} \subsetneq \textbf{StrongDelayP} \subsetneq \textbf{DelayP} \subseteq \textbf{IncP} \subsetneq \textbf{TotalP} \subsetneq \textbf{Enum}{\cdot}\mathrm{P}$$

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The enumeration of bipartite matching has be proven to be in **StrongDelayP** by Takeaki Uno but if it is in **QueryP** we would have $\mathrm{P} = \#\mathrm{P}$.

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The enumeration of maximal independent sets of a graph is in **DelayP** but not in **StrongDelayP** unless $\mathrm{P} = \mathrm{NP}$.

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The inclusion between **DelayP** and **IncP** is not known to be strict and it would be very satisfying to have this separation too !

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From a TFNP problem we create a problem wich has an exponential number of solution for each solution of the first one.
It is a **TotalP** problem, because the trivial algorithm is polynomial in the number of solution.
It is not **IncP** except if $\mathrm{NP} \cap \mathrm{coNP} = \mathrm{P}$.

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We want to enumerate the solutions of a FewP problem, if it is in **TotalP** then we have $\mathrm{FewP} = \mathrm{P}$.

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We look for the minimal x for this order satisfying the equation $Bx = y$.

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Question by Dimitris J. Kavvadias, Martha Sideri, Elias C. Stavropoulos in “Generating all maximal models of a Boolean expression”.

Matroids have been design to abstract the notion of dependence.

Definition

A matroid is a pair $(E, \mathcal{I})$, E is a finite set and $\mathcal{I}$ is included in the power set of E. Elements of $\mathcal{I}$ are said to be independent sets, the others are dependent sets.

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1. $\varnothing \in \mathcal{I}$
2. If $I \in \mathcal{I}$ and $I' \subseteq I$, then $I' \in \mathcal{I}$
3. If I_1 and I_2 are in $\mathcal{I}$ and $|I_1| < |I_2|$, then there is an element e of $I_2 - I_1$ such that $I_1 \cup e \in \mathcal{I}$.

The first concrete example of matroid is the vector matroid.

Let A be a matrix, the ground set E is the set of the columns and a set of columns is independent if the vectors are linearly independent.
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$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Here the set $\{1, 2, 4\}$ is independent and $\{1, 2, 3\}$ is dependent.

The second example is the cycle matroid of a graph.

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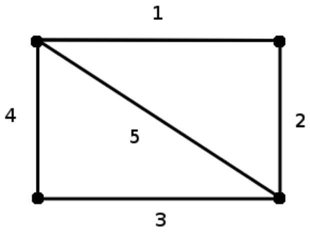

Here the set $\{1, 2, 4\}$ is independent and $\{1, 2, 3, 4\}$ or $\{1, 2, 5\}$ is dependent.

Lemma

Any cycle matroid is a representable matroid, i.e. it is isomorph to a vector matroid.

Label the vertices of a graph by $1, \ldots, n$ and the edges by $1, \ldots, m$.

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This matrix represents the former graph:

$$\mathbf{X} = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

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In a cycle matroid it is a spanning tree of the graph.

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Lemma (Circuit elimination property)

Let C_1 and C_2 two circuits of a matroid, $e \in C_1 \cap C_2$, then there is a circuit C_3 such that $C_3 \subseteq (C_1 \cup C_2) - e$.

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Remark

We can axiomatically define the matroids with a predicate base or circuit rather than the independence predicate.

We can also define concept as the rank or the span of a set, in the same way we do in linear algebra.

There is a strong link between our problem and the circuit of a vector matroid.

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Lemma

There is a bijection between the minimal solutions of the system and the circuits of M containing the vector y.

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Clearly the vector matroids verify this condition, because gaussian elimination allows to decide the independence of a set of vector in cubic time.

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Set an order on the ground set and do a depth first search on the tree of the independent sets.

Theorem

$\textsc{Enum}\cdot\textsc{MatroidBase} \in$ **StrongDelayP**.

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1. If the matroid is is defined by a whole subspace of a vector space, then it is in **DelayP**.
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3. If the matroid is the cycle matroid of a graph, it is in **DelayP**.

Definition

Let B be a basis of a matroid M and x not in B then there exists a unique circuit $C = C(B, x)$ such that $x \in C \subseteq B \cap x$. This circuit is called the fundamental circuit of x in the base B.

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Exactly as the unicity of the decomposition of a vector in a basis.

We compute a fundamental circuit in a basis B by at most $|B|$ call to the independence oracle.
Hence we compute all the fundamental circuits in a certain base in poly time.

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After gaussian elimination the matrix look like this.

$$\mathbf{A} = \left(\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{array}\right)$$

The three first columns are the base and were the columns $1, 2$ and 4.

The two other columns are defined by their fundamental circuit.

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Then C is the set of the circuits of a matroid.

If C contains all the fundamental circuits of a matroid M, then C is the set of circuits of M.

Theorem

Computing the circuits of a matroid is in **IncP**.

This result has been proven in the article "On the Complexity of Some Enumeration Problems for Matroids" by Leonid G. Khachiyan, Endre Boros, Khaled M. Elbassioni, Vladimir Gurvich and Kazuhisa Makino.

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This algorithm work in incremental polynomial time and output all the circuits of the matroid.

Definition

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Definition (connectivity)

The equivalence classes of $\sim$ are called the connected components of the matroid and the matroid is said to be connected if there is only one component.

If we look for the circuit containing a fixed element, we have to search in the connected component of this element.

Theorem

Let e be an element of a connected matroid M, and C_e be the set of circuits of M containing e. Then the circuits of M not containing e are the minimal sets of the form

$$(C_1 \cup C_2) - \bigcap \{C \in C_e | C \subseteq C_1 \cup C_2\}$$

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where C_1 and C_2 are distinct members of $\mathcal{C}_e$.

Then we have a surjection from $\mathcal{C}_e \times \mathcal{C}_e$ to $\mathcal{C}$ the set of circuit, therefore $|\mathcal{C}_e|^2 > |\mathcal{C}|$.

Theorem

Computing all the circuits through one fixed element is in **IncP***.*

Again it is a result in the article "On the Complexity of Some Enumeration Problems for Matroids".

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Thanks to the previous remark we do only a polynomial number of step in the number of solution. We have proven that the problem is in **TotalP**, but with a few improvements we have a **IncP** algorithm.

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  This problem is a generalization of the problem to find a minimal extension of a partial solution for an affine formula which is NP-complete too.
- When $k = 2$ and the matroid is binary, the decision problem is in P.
- The problem to find a cycle in a graph with k fixed edges is fixed parameter tractable in k.

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3. Enumerate the circuits of a representable matroid with k fixed elements in **IncP** or **TotalP** for at least $k = 2$
4. Find a simpler structure where the problem is in **DelayP**
5. Prove that the enumeration of the circuits of a matroid is not **DelayP**

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A theorem proven by Petr Hlinený. It's a FPT result, the dependence is not polynomial in t or in the length of the formula.

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Let $\phi(\bar{x})$ a formula of MSO_M with free variables $\vec{x}$. Given a matroid, we want to enumerate all the $\bar{x}$ which makes the formula $\phi(\bar{x})$ true. This problem is in **QueryP** *over the matroids of bounded branch-width t.*

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The problem to find the circuits of a matroid with k fixed elements is therefore easy when the matroids are of bounded branch-width.