

Proposition de Correction d'éléments de Logique 2023

Exo 1

① $\exists M \in \mathbb{N}, \forall x \in \mathbb{N}$ tel que $x \leq M$

$\exists M \in \mathbb{N}, \exists x \in \mathbb{N}$ tel que $x > M$

② $\exists M \in \mathbb{R}_+^+, \forall x \in \mathbb{R}, |f(x)| \leq M$ (dans \mathbb{C} , il n'y a pas de comparaison donc on utilise le module)

③ $\forall a, b \in \mathbb{R}$ tel que $a > b$, ~~on a~~ $\exists a, b \in \mathbb{N} \cap \mathbb{D} \neq \emptyset$

Exo 2

① Tous les repas sont mangés par des (certains) étudiants de Maths-Infos

② Certains repas sont mangés par tous les étudiants

③ Certains étudiants mangent tous les repas

Exo 3 (Relation binaire)

Sur l'ensemble $E = \{m \in \mathbb{N} : m \leq 23\}$, on considère la relation binaire suivante

$a R b$ si $a^3 - b^3$ est un multiple de 6

①

* Reflexivité

Soit $a \in E$

$a^3 - a^3 = 0$ et 0 est un multiple de 6.

D'où $a R a$. Ainsi R est réflexive.

* Symétrie

Soient $a, b \in E$ tel que $a R b$.

A-t-on $b R a$?
non

$$a R b \Rightarrow a^3 - b^3 = 6k \text{ avec } k \in \mathbb{Z}$$

$$\Rightarrow b^3 - a^3 = 6(-k), k \in \mathbb{Z}$$

$$\Rightarrow b^3 - a^3 = 6p, p = -k \in \mathbb{Z}$$

d'où bRa. Ainsi R est symétrique.

* Transitivité

Soient $a, b, c \in \mathbb{Z}$ tel que aRb, bRc

A-t-on aRc ?
oui

$$aRb \Rightarrow a^3 - b^3 = 6p, \quad p \in \mathbb{Z} \quad (1)$$

$$bRc \Rightarrow b^3 - c^3 = 6q, \quad q \in \mathbb{Z} \quad (2)$$

$$(1) + (2) \Rightarrow a^3 - c^3 = 6(p+q)$$

$$\Rightarrow a^3 - c^3 = 6r, \text{ avec } r = p+q \in \mathbb{Z}$$

d'où aRc

Ainsi R est transitive.

Bilan: R étant réflexive, symétrique et transitive

Conclusion: R est une relation d'équivalence.

② Les classes d'équivalence de 0 et 1.

$$* \bar{0} = \{x \in E \mid x \equiv 0\}$$

$$x \in \bar{0} \Leftrightarrow x \equiv 0$$

$$\Leftrightarrow x - 0 = 6h \text{ avec } h \in \mathbb{Z}$$

$$\Leftrightarrow x^3 = 6h, h \in \mathbb{Z} \text{ il y a } 6 \text{ divise } x^3.$$

Tableau de Congruence modulo 6

x	0	1	2	3	4	5
x^3	0	1	2	3	4	5

À partir de tableau de Congruence $6/x$.

$$\text{Ainsi } x = 6h, h \in \mathbb{Z}$$

$$\bar{0} = 6\mathbb{Z} = \{0, 6, 12, 18\}$$

$$* \bar{1} = \{x \in E \mid x \equiv 1\}$$

$$x \in \bar{1} \Leftrightarrow x - 1 = 6h, h \in \mathbb{Z}$$

$$\Leftrightarrow x^3 = 1 + 6h, h \in \mathbb{Z}$$

$$\Leftrightarrow x^3 = 1 \pmod{6}$$

$x \in \bar{1} \Leftrightarrow x \equiv 1[6]$ d'après le tableau de congruence

$$\Leftrightarrow x = 1 + 6k, k \in \mathbb{Z}$$

$$\text{Où } \bar{1} = 1 + 6\mathbb{Z} = \{-1, 7, 13, 19\}$$

③ Déterminons toutes les classes d'équivalences.

$$\text{On a: } \bar{0} = 6\mathbb{Z} = \{0, 6, 12, 18\}$$

$$\bar{1} = 1 + 6\mathbb{Z} = \{-1, 7, 13, 19\}$$

$$\bar{2} = \{x \in \mathbb{Z} / x \equiv 2\}$$

$$x \in \bar{2} \Leftrightarrow x^3 - 2^3 = 6h, h \in \mathbb{Z}$$

$$\Leftrightarrow x^3 - 8 = 6h, h \in \mathbb{Z}$$

$$\Leftrightarrow x^3 = 8 + 6h$$

$$\Leftrightarrow x^3 = 2 + 6(1+h), h \in \mathbb{Z}$$

$$\Leftrightarrow x^3 \equiv 2[6]$$

$\Leftrightarrow x^3 \equiv 2[6]$ d'après le tableau de congruence

$$\Leftrightarrow \text{~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~ ~~8~~ ~~9~~ ~~10~~ ~~11~~ ~~12~~ ~~13~~ ~~14~~ ~~15~~ ~~16~~ ~~17~~ ~~18~~ ~~19~~ ~~20~~ ~~21~~ ~~22~~ ~~23~~ ~~24~~ ~~25~~ ~~26~~ ~~27~~ ~~28~~ ~~29~~ ~~30~~ ~~31~~ ~~32~~ ~~33~~ ~~34~~ ~~35~~ ~~36~~ ~~37~~ ~~38~~ ~~39~~ ~~40~~ ~~41~~ ~~42~~ ~~43~~ ~~44~~ ~~45~~ ~~46~~ ~~47~~ ~~48~~ ~~49~~ ~~50~~ ~~51~~ ~~52~~ ~~53~~ ~~54~~ ~~55~~ ~~56~~ ~~57~~ ~~58~~ ~~59~~ ~~60~~ ~~61~~ ~~62~~ ~~63~~ ~~64~~ ~~65~~ ~~66~~ ~~67~~ ~~68~~ ~~69~~ ~~70~~ ~~71~~ ~~72~~ ~~73~~ ~~74~~ ~~75~~ ~~76~~ ~~77~~ ~~78~~ ~~79~~ ~~80~~ ~~81~~ ~~82~~ ~~83~~ ~~84~~ ~~85~~ ~~86~~ ~~87~~ ~~88~~ ~~89~~ ~~90~~ ~~91~~ ~~92~~ ~~93~~ ~~94~~ ~~95~~ ~~96~~ ~~97~~ ~~98~~ ~~99~~ ~~100~~ ~~101~~ ~~102~~ ~~103~~ ~~104~~ ~~105~~ ~~106~~ ~~107~~ ~~108~~ ~~109~~ ~~110~~ ~~111~~ ~~112~~ ~~113~~ ~~114~~ ~~115~~ ~~116~~ ~~117~~ ~~118~~ ~~119~~ ~~120~~ ~~121~~ ~~122~~ ~~123~~ ~~124~~ ~~125~~ ~~126~~ ~~127~~ ~~128~~ ~~129~~ ~~130~~ ~~131~~ ~~132~~ ~~133~~ ~~134~~ ~~135~~ ~~136~~ ~~137~~ ~~138~~ ~~139~~ ~~140~~ ~~141~~ ~~142~~ ~~143~~ ~~144~~ ~~145~~ ~~146~~ ~~147~~ ~~148~~ ~~149~~ ~~150~~ ~~151~~ ~~152~~ ~~153~~ ~~154~~ ~~155~~ ~~156~~ ~~157~~ ~~158~~ ~~159~~ ~~160~~ ~~161~~ ~~162~~ ~~163~~ ~~164~~ ~~165~~ ~~166~~ ~~167~~ ~~168~~ ~~169~~ ~~170~~ ~~171~~ ~~172~~ ~~173~~ ~~174~~ ~~175~~ ~~176~~ ~~177~~ ~~178~~ ~~179~~ ~~180~~ ~~181~~ ~~182~~ ~~183~~ ~~184~~ ~~185~~ ~~186~~ ~~187~~ ~~188~~ ~~189~~ ~~190~~ ~~191~~ ~~192~~ ~~193~~ ~~194~~ ~~195~~ ~~196~~ ~~197~~ ~~198~~ ~~199~~ ~~200~~ ~~201~~ ~~202~~ ~~203~~ ~~204~~ ~~205~~ ~~206~~ ~~207~~ ~~208~~ ~~209~~ ~~210~~ ~~211~~ ~~212~~ ~~213~~ ~~214~~ ~~215~~ ~~216~~ ~~217~~ ~~218~~ ~~219~~ ~~220~~ ~~221~~ ~~222~~ ~~223~~ ~~224~~ ~~225~~ ~~226~~ ~~227~~ ~~228~~ ~~229~~ ~~230~~ ~~231~~ ~~232~~ ~~233~~ ~~234~~ ~~235~~ ~~236~~ ~~237~~ ~~238~~ ~~239~~ ~~240~~ ~~241~~ ~~242~~ ~~243~~ ~~244~~ ~~245~~ ~~246~~ ~~247~~ ~~248~~ ~~249~~ ~~250~~ ~~251~~ ~~252~~ ~~253~~ ~~254~~ ~~255~~ ~~256~~ ~~257~~ ~~258~~ ~~259~~ ~~260~~ ~~261~~ ~~262~~ ~~263~~ ~~264~~ ~~265~~ ~~266~~ ~~267~~ ~~268~~ ~~269~~ ~~270~~ ~~271~~ ~~272~~ ~~273~~ ~~274~~ ~~275~~ ~~276~~ ~~277~~ ~~278~~ ~~279~~ ~~280~~ ~~281~~ ~~282~~ ~~283~~ ~~284~~ ~~285~~ ~~286~~ ~~287~~ ~~288~~ ~~289~~ ~~290~~ ~~291~~ ~~292~~ ~~293~~ ~~294~~ ~~295~~ ~~296~~ ~~297~~ ~~298~~ ~~299~~ ~~300~~ ~~301~~ ~~302~~ ~~303~~ ~~304~~ ~~305~~ ~~306~~ ~~307~~ ~~308~~ ~~309~~ ~~310~~ ~~311~~ ~~312~~ ~~313~~ ~~314~~ ~~315~~ ~~316~~ ~~317~~ ~~318~~ ~~319~~ ~~320~~ ~~321~~ ~~322~~ ~~323~~ ~~324~~ ~~325~~ ~~326~~ ~~327~~ ~~328~~ ~~329~~ ~~330~~ ~~331~~ ~~332~~ ~~333~~ ~~334~~ ~~335~~ ~~336~~ ~~337~~ ~~338~~ ~~339~~ ~~340~~ ~~341~~ ~~342~~ ~~343~~ ~~344~~ ~~345~~ ~~346~~ ~~347~~ ~~348~~ ~~349~~ ~~350~~ ~~351~~ ~~352~~ ~~353~~ ~~354~~ ~~355~~ ~~356~~ ~~357~~ ~~358~~ ~~359~~ ~~360~~ ~~361~~ ~~362~~ ~~363~~ ~~364~~ ~~365~~ ~~366~~ ~~367~~ ~~368~~ ~~369~~ ~~370~~ ~~371~~ ~~372~~ ~~373~~ ~~374~~ ~~375~~ ~~376~~ ~~377~~ ~~378~~ ~~379~~ ~~380~~ ~~381~~ ~~382~~ ~~383~~ ~~384~~ ~~385~~ ~~386~~ ~~387~~ ~~388~~ ~~389~~ ~~390~~ ~~391~~ ~~392~~ ~~393~~ ~~394~~ ~~395~~ ~~396~~ ~~397~~ ~~398~~ ~~399~~ ~~400~~ ~~401~~ ~~402~~ ~~403~~ ~~404~~ ~~405~~ ~~406~~ ~~407~~ ~~408~~ ~~409~~ ~~410~~ ~~411~~ ~~412~~ ~~413~~ ~~414~~ ~~415~~ ~~416~~ ~~417~~ ~~418~~ ~~419~~ ~~420~~ ~~421~~ ~~422~~ ~~423~~ ~~424~~ ~~425~~ ~~426~~ ~~427~~ ~~428~~ ~~429~~ ~~430~~ ~~431~~ ~~432~~ ~~433~~ ~~434~~ ~~435~~ ~~436~~ ~~437~~ ~~438~~ ~~439~~ ~~440~~ ~~441~~ ~~442~~ ~~443~~ ~~444~~ ~~445~~ ~~446~~ ~~447~~ ~~448~~ ~~449~~ ~~450~~ ~~451~~ ~~452~~ ~~453~~ ~~454~~ ~~455~~ ~~456~~ ~~457~~ ~~458~~ ~~459~~ ~~460~~ ~~461~~ ~~462~~ ~~463~~ ~~464~~ ~~465~~ ~~466~~ ~~467~~ ~~468~~ ~~469~~ ~~470~~ ~~471~~ ~~472~~ ~~473~~ ~~474~~ ~~475~~ ~~476~~ ~~477~~ ~~478~~ ~~479~~ ~~480~~ ~~481~~ ~~482~~ ~~483~~ ~~484~~ ~~485~~ ~~486~~ ~~487~~ ~~488~~ ~~489~~ ~~490~~ ~~491~~ ~~492~~ ~~493~~ ~~494~~ ~~495~~ ~~496~~ ~~497~~ ~~498~~ ~~499~~ ~~500~~ ~~501~~ ~~502~~ ~~503~~ ~~504~~ ~~505~~ ~~506~~ ~~507~~ ~~508~~ ~~509~~ ~~510~~ ~~511~~ ~~512~~ ~~513~~ ~~514~~ ~~515~~ ~~516~~ ~~517~~ ~~518~~ ~~519~~ ~~520~~ ~~521~~ ~~522~~ ~~523~~ ~~524~~ ~~525~~ ~~526~~ ~~527~~ ~~528~~ ~~529~~ ~~530~~ ~~531~~ ~~532~~ ~~533~~ ~~534~~ ~~535~~ ~~536~~ ~~537~~ ~~538~~ ~~539~~ ~~540~~ ~~541~~ ~~542~~ ~~543~~ ~~544~~ ~~545~~ ~~546~~ ~~547~~ ~~548~~ ~~549~~ ~~550~~ ~~551~~ ~~552~~ ~~553~~ ~~554~~ ~~555~~ ~~556~~ ~~557~~ ~~558~~ ~~559~~ ~~560~~ ~~561~~ ~~562~~ ~~563~~ ~~564~~ ~~565~~ ~~566~~ ~~567~~ ~~568~~ ~~569~~ ~~570~~ ~~571~~ ~~572~~ ~~573~~ ~~574~~ ~~575~~ ~~576~~ ~~577~~ ~~578~~ ~~579~~ ~~580~~ ~~581~~ ~~582~~ ~~583~~ ~~584~~ ~~585~~ ~~586~~ ~~587~~ ~~588~~ ~~589~~ ~~590~~ ~~591~~ ~~592~~ ~~593~~ ~~594~~ ~~595~~ ~~596~~ ~~597~~ ~~598~~ ~~599~~ ~~600~~ ~~601~~ ~~602~~ ~~603~~ ~~604~~ ~~605~~ ~~606~~ ~~607~~ ~~608~~ ~~609~~ ~~610~~ ~~611~~ ~~612~~ ~~613~~ ~~614~~ ~~615~~ ~~616~~ ~~617~~ ~~618~~ ~~619~~ ~~620~~ ~~621~~ ~~622~~ ~~623~~ ~~624~~ ~~625~~ ~~626~~ ~~627~~ ~~628~~ ~~629~~ ~~630~~ ~~631~~ ~~632~~ ~~633~~ ~~634~~ ~~635~~ ~~636~~ ~~637~~ ~~638~~ ~~639~~ ~~640~~ ~~641~~ ~~642~~ ~~643~~ ~~644~~ ~~645~~ ~~646~~ ~~647~~ ~~648~~ ~~649~~ ~~650~~ ~~651~~ ~~652~~ ~~653~~ ~~654~~ ~~655~~ ~~656~~ ~~657~~ ~~658~~ ~~659~~ ~~660~~ ~~661~~ ~~662~~ ~~663~~ ~~664~~ ~~665~~ ~~666~~ ~~667~~ ~~668~~ ~~669~~ ~~670~~ ~~671~~ ~~672~~ ~~673~~ ~~674~~ ~~675~~ ~~676~~ ~~677~~ ~~678~~ ~~679~~ ~~680~~ ~~681~~ ~~682~~ ~~683~~ ~~684~~ ~~685~~ ~~686~~ ~~687~~ ~~688~~ ~~689~~ ~~690~~ ~~691~~ ~~692~~ ~~693~~ ~~694~~ ~~695~~ ~~696~~ ~~697~~ ~~698~~ ~~699~~ ~~700~~ ~~701~~ ~~702~~ ~~703~~ ~~704~~ ~~705~~ ~~706~~ ~~707~~ ~~708~~ ~~709~~ ~~710~~ ~~711~~ ~~712~~ ~~713~~ ~~714~~ ~~715~~ ~~716~~ ~~717~~ ~~718~~ ~~719~~ ~~720~~ ~~721~~ ~~722~~ ~~723~~ ~~724~~ ~~725~~ ~~726~~ ~~727~~ ~~728~~ ~~729~~ ~~730~~ ~~731~~ ~~732~~ ~~733~~ ~~734~~ ~~735~~ ~~736~~ ~~737~~ ~~738~~ ~~739~~ ~~740~~ ~~741~~ ~~742~~ ~~743~~ ~~744~~ ~~745~~ ~~746~~ ~~747~~ ~~748~~ ~~749~~ ~~750~~ ~~751~~ ~~752~~ ~~753~~ ~~754~~ ~~755~~ ~~756~~ ~~757~~ ~~758~~ ~~759~~ ~~760~~ ~~761~~ ~~762~~ ~~763~~ ~~764~~ ~~765~~ ~~766~~ ~~767~~ ~~768~~ ~~769~~ ~~770~~ ~~771~~ ~~772~~ ~~773~~ ~~774~~ ~~775~~ ~~776~~ ~~777~~ ~~778~~ ~~779~~ ~~780~~ ~~781~~ ~~782~~ ~~783~~ ~~784~~ ~~785~~ ~~786~~ ~~787~~ ~~788~~ ~~789~~ ~~790~~ ~~791~~ ~~792~~ ~~793~~ ~~794~~ ~~795~~ ~~796~~ ~~797~~ ~~798~~ ~~799~~ ~~800~~ ~~801~~ ~~802~~ ~~803~~ ~~804~~ ~~805~~ ~~806~~ ~~807~~ ~~808~~ ~~809~~ ~~810~~ ~~811~~ ~~812~~ ~~813~~ ~~814~~ ~~815~~ ~~816~~ ~~817~~ ~~818~~ ~~819~~ ~~820~~ ~~821~~ ~~822~~ ~~823~~ ~~824~~ ~~825~~ ~~826~~ ~~827~~ ~~828~~ ~~829~~ ~~830~~ ~~831~~ ~~832~~ ~~833~~ ~~834~~ ~~835~~ ~~836~~ ~~837~~ ~~838~~ ~~839~~ ~~840~~ ~~841~~ ~~842~~ ~~843~~ ~~844~~ ~~845~~ ~~846~~ ~~847~~ ~~848~~ ~~849~~ ~~850~~ ~~851~~ ~~852~~ ~~853~~ ~~854~~ ~~855~~ ~~856~~ ~~857~~ ~~858~~ ~~859~~ ~~860~~ ~~861~~ ~~862~~ ~~863~~ ~~864~~ ~~865~~ ~~866~~ ~~867~~ ~~868~~ ~~869~~ ~~870~~ ~~871~~ ~~872~~ ~~873~~ ~~874~~ ~~875~~ ~~876~~ ~~877~~ ~~878~~ ~~879~~ ~~880~~ ~~881~~ ~~882~~ ~~883~~ ~~884~~ ~~885~~ ~~886~~ ~~887~~ ~~888~~ ~~889~~ ~~890~~ ~~891~~ ~~892~~ ~~893~~ ~~894~~ ~~895~~ ~~896~~ ~~897~~ ~~898~~ ~~899~~ ~~900~~ ~~901~~ ~~902~~ ~~903~~ ~~904~~ ~~905~~ ~~906~~ ~~907~~ ~~908~~ ~~909~~ ~~910~~ ~~911~~ ~~912~~ ~~913~~ ~~914~~ ~~915~~ ~~916~~ ~~917~~ ~~918~~ ~~919~~ ~~920~~ ~~921~~ ~~922~~ ~~923~~ ~~924~~ ~~925~~ ~~926~~ ~~927~~ ~~928~~ ~~929~~ ~~930~~ ~~931~~ ~~932~~ ~~933~~ ~~934~~ ~~935~~ ~~936~~ ~~937~~ ~~938~~ ~~939~~ ~~940~~ ~~941~~ ~~942~~ ~~943~~ ~~944~~ ~~945~~ ~~946~~ ~~947~~ ~~948~~ ~~949~~ ~~950~~ ~~951~~ ~~952~~ ~~953~~ ~~954~~ ~~955~~ ~~956~~ ~~957~~ ~~958~~ ~~959~~ ~~960~~ ~~961~~ ~~962~~ ~~963~~ ~~964~~ ~~965~~ ~~966~~ ~~967~~ ~~968~~ ~~969~~ ~~970~~ ~~971~~ ~~972~~ ~~973~~ ~~974~~ ~~975~~ ~~976~~ ~~977~~ ~~978~~ ~~979~~ ~~980~~ ~~981~~ ~~982~~ ~~983~~ ~~984~~ ~~985~~ ~~986~~ ~~987~~ ~~988~~ ~~989~~ ~~990~~ ~~991~~ ~~992~~ ~~993~~ ~~994~~ ~~995~~ ~~996~~ ~~997~~ ~~998~~ ~~999~~ ~~1000~~ ~~1001~~ ~~1002~~ ~~1003~~ ~~1004~~ ~~1005~~ ~~1006~~ ~~1007~~ ~~1008~~ ~~1009~~ ~~1010~~ ~~1011~~ ~~1012~~ ~~1013~~ ~~1014~~ ~~1015~~ ~~1016~~ ~~1017~~ ~~1018~~ ~~1019~~ ~~1020~~ ~~1021~~ ~~1022~~ ~~1023~~ ~~1024~~ ~~1025~~ ~~1026~~ ~~1027~~ ~~1028~~ ~~1029~~ ~~1030~~ ~~1031~~ ~~1032~~ ~~1033~~ ~~1034~~ ~~1035~~ ~~1036~~ ~~1037~~ ~~1038~~ ~~1039~~ ~~1040~~ ~~1041~~ ~~1042~~ ~~1043~~ ~~1044~~ ~~1045~~ ~~1046~~ ~~1047~~ ~~1048~~ ~~1049~~ ~~1050~~ ~~1051~~ ~~1052~~ ~~1053~~ ~~1054~~ ~~1055~~ ~~1056~~ ~~1057~~ ~~1058~~ ~~1059~~ ~~1060~~ ~~1061~~ ~~1062~~ ~~1063~~ ~~1064~~ ~~1065~~ ~~1066~~ ~~1067~~ ~~1068~~ ~~1069~~ ~~1070~~ ~~1071~~ ~~1072~~ ~~1073~~ ~~1074~~ ~~1075~~ ~~1076~~ ~~1077~~ ~~1078~~ ~~1079~~ ~~1080~~ ~~1081~~ ~~1082~~ ~~1083~~ ~~1084~~ ~~1085~~ ~~1086~~ ~~1087~~ ~~1088~~ ~~1089~~ ~~1090~~ ~~1091~~ ~~1092~~ ~~1093~~ ~~1094~~ ~~1095~~ ~~1096~~ ~~1097~~ ~~1098~~ ~~1099~~ ~~1100~~ ~~1101~~ ~~1102~~ ~~1103~~ ~~1104~~ ~~1105~~ ~~1106~~ ~~1107~~ ~~1108~~ ~~1109~~ ~~1110~~ ~~1111~~ ~~1112~~ ~~1113~~ ~~1114~~ ~~1115~~ ~~1116~~ ~~1117~~ ~~1118~~ ~~1119~~ ~~1120~~ ~~1121~~ ~~1122~~ ~~1123~~ ~~1124~~ ~~1125~~ ~~1126~~ ~~1127~~ ~~1128~~ ~~1129~~ ~~1130~~ ~~1131~~ ~~1132~~ ~~1133~~ ~~1134~~ ~~1135~~ ~~1136~~ ~~1137~~ ~~1138~~ ~~1139~~ ~~1140~~ ~~1141~~ ~~1142~~ ~~1143~~ ~~1144~~ ~~1145~~ ~~1146~~ ~~1147~~ ~~1148~~ ~~1149~~ ~~1150~~ ~~1151~~ ~~1152~~ ~~1153~~ ~~1154~~ ~~1155~~ ~~1156~~ ~~1157~~ ~~1158~~ ~~1159~~ ~~1160~~ ~~1161~~ ~~1162~~ ~~1163~~ ~~1164~~ ~~1165~~ ~~1166~~ ~~1167~~ ~~1168~~ ~~1169~~ ~~1170~~ ~~1171~~ ~~1172~~ ~~1173~~ ~~1174~~ ~~1175~~ ~~1176~~ ~~1177~~ ~~1178~~ ~~1179~~ ~~1180~~ ~~1181~~ ~~1182~~ ~~1183~~ ~~1184~~ ~~1185~~ ~~1186~~ ~~1187~~ ~~1188~~ ~~1189~~ ~~1190~~ ~~1191~~ ~~1192~~ ~~1193~~ ~~1194~~ ~~1195~~ ~~1196~~ ~~1197~~ ~~1198~~ ~~1199~~ ~~1200~~ ~~1201~~ ~~1202~~ ~~1203~~ ~~1204~~ ~~1205~~ ~~1206~~ ~~1207~~ ~~1208~~ ~~1209~~ ~~1210~~ ~~1211~~ ~~1212~~ ~~1213~~ ~~1214~~ ~~1215~~ ~~1216~~ ~~1217~~ ~~1218~~ ~~1219~~ ~~1220~~ ~~1221~~ ~~1222~~ ~~1223~~ ~~1224~~ ~~1225~~ ~~1226~~ ~~1227~~ ~~1228~~ ~~1229~~ ~~1230~~ ~~1231~~ ~~1232~~ ~~1233~~ ~~1234~~ ~~1235~~ ~~1236~~ ~~1237~~ ~~1238~~ ~~1239~~ ~~1240~~ ~~1241~~ ~~1242~~ ~~1243~~ ~~1244~~ ~~1245~~ ~~1246~~ ~~1247~~ ~~1248~~ ~~1249~~ ~~1250~~ ~~1251~~ ~~1252~~ ~~1253~~ ~~1254~~ ~~1255~~ ~~1256~~ ~~1257~~ ~~1258~~ ~~1259~~ ~~1260~~ ~~1261~~ ~~1262~~ ~~126~~$$

$$x \in \bar{2} \Leftrightarrow x = 2 + 6p \text{ avec } p \in \mathbb{Z}$$

$$\bar{2} = 2 + 6\mathbb{Z} = \{2, 8, 14, 20\}$$

On montre de façon analogue que :

$$\bar{3} = \{3, 9, 15, 21\}$$

$$\bar{4} = \{4, 10, 16, 22\}$$

$$\bar{5} = \{5, 11, 17, 23\}$$

Exo 4

① Exemple d'une application f de \mathbb{Z} dans \mathbb{N}^* qui soit injective.

$$f: \mathbb{Z} \longrightarrow \mathbb{N}^*$$

$$n \longmapsto \begin{cases} 2n & \text{si } n > 0 \\ -2n+1 & \text{si } n \leq 0 \end{cases}$$

f est bien définie et il n'y a pas ~~d'image~~ partage d'image donc f est injective.

Justification

Soient $a, b \in \mathbb{Z}$ tel que $f(a) = f(b)$

A-t-on $a = b$?

1^{er} Cas : Si $a > 0$ et $b > 0$

$$\text{on a } f(a) = f(b) \Rightarrow 2a = 2b$$

$\Rightarrow a = b$ dans ce cas, f est injective.

2^e Cas : Si $a < 0$ et $b < 0$

$$f(a) = f(b) \Rightarrow -2a + 1 = -2b + 1$$

$$\Rightarrow -2a = -2b$$

$$\Rightarrow -a = -b$$

$\Rightarrow a = b$ Dans ce cas f est injective.

Conclusion : Dans tous les cas, f est injective.

② Trouver une application g de \mathbb{N}^* dans \mathbb{Z} qui soit surjective

soit g cette application

$$g: \mathbb{N}^* \longrightarrow \mathbb{Z}$$
$$n \longmapsto \begin{cases} \frac{n}{2} & \text{si } n \text{ est pair} \\ -\frac{n+1}{2} & \text{si } n \text{ est impair.} \end{cases}$$

Soit $b \in \mathbb{Z}$, existe-t-il $n \in \mathbb{N}^*$ tel que $g(n) = b$?

1^{er} Cas: si $b \leq 0$

$$g(n) = b \Rightarrow \frac{-n+1}{2} = b$$

$$\Rightarrow -n+1 = 2b$$

$\Rightarrow n = -2b+1$ Dans ce cas g est surjective

2^e Cas: si $b > 0$

$$\text{on a } g(n) = b \Rightarrow \frac{n}{2} = b$$

$\Rightarrow n = 2b$ donc g est surjective

Conclusion: g est donc surjective.