

$$\begin{cases} \Delta u + k^2 u = 0, & \text{in } Y \setminus \overline{D}, \\ \Delta u + k_i^2 u = 0, & \text{in } D_i, \\ u|_+ - u|_- = 0, & \text{on } \partial D, \\ \frac{\partial u}{\partial \nu} \Big|_- - \delta \frac{\partial u}{\partial \nu} \Big|_+ = 0, & \text{on } \partial D, \\ u(x + \ell) = e^{i(\alpha + i\beta) \cdot \ell} u(x), & \text{for all } \ell \in \Lambda \end{cases}$$