Intro/Recap of Multiple Linear Regression (MLR)

AKA Ordinary Least Squares
See LinearRegression in scikit-learn

The MLR Model

Note: I will use typical **statistics notation:**

coefficients are called βs, the dependent variable is Y, and estimates are indicated by "hats".

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \epsilon$$

Key General Issues

- What are the model assumptions? Are they valid?
- How do you estimate the parameters from data?
  - (i) cost function (true or surrogate?)
  - (ii) optimization method
- How do you evaluate your model?
  - (i) training/validation/test/scoring error
  - (ii) performance measures

Assumptions behind the MLR Model

(i) The conditional mean of Y is linear in the X_j variables.

(ii) The error term (deviations from line)

- are normally distributed
- independent from each other
- identically distributed (i.e., they have constant variance)

$$Y|X_1 \ldots X_p \sim N(\beta_0 + \beta_1 X_1 \ldots + \beta_p X_p, \sigma^2)$$

Then minimizing Mean Squared Error (MSE) on the training data yields the Maximum Likelihood Estimate (MLE) solution of the assumed *generative model.*

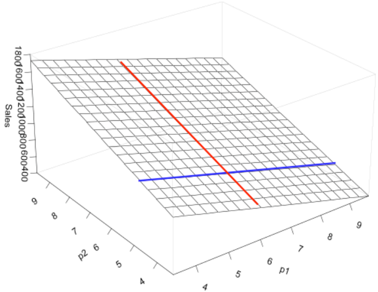
Q: What do the βs mean?

MLR On Sales Data

Consider sales of a product as predicted by price of this product (P1) and the price of a competing product (P2).

$$Sales = \beta_0 + \beta_1 P1 + \beta_2 P2 + \epsilon \text{ ; } Thus \ \beta_j = \frac{\partial E[Y|X_1, \ldots, X_p]}{\partial X_j}$$

Holding all other variables constant, β_j is the average change in Y per unit change in X_j.



Q: Will your sales go up if you reduce the price?

Least Squares

Model: $Sales_i = \beta_0 + \beta_1 P1_i + \beta_2 P2_i + \epsilon_i,\ \epsilon \sim N(0, \sigma^2)$

Regression Statistics	
Multiple R	0.99
R Square	0.99
Adjusted R Square	0.99
Standard Error	28.42
Observations	100.00

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2.00	6004047.24	3002023.62	3717.29	0.00
Residual	97.00	78335.60	807.58		
Total	99.00	6082382.84			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	115.72	8.55	13.54	0.00	98.75	132.68
p1	-97.66	2.67	-36.60	0.00	-102.95	-92.36
p2	108.80	1.41	77.20	0.00	106.00	111.60

$b_0 = \hat{\beta}_0 = 115.72,\ b_1 = \hat{\beta}_1 = -97.66,\ b_2 = \hat{\beta}_2 = 108.80,$

$s = \hat{\sigma} = 28.42$

Note that $R^2 = \text{corr}(Y, \hat{Y})^2$

Plug-in Prediction in MLR

Suppose that by using advanced corporate espionage tactics, I discover that my competitor will charge \$10 the next quarter. After some marketing analysis I decided to charge \$8. How much will I sell?

Our model is

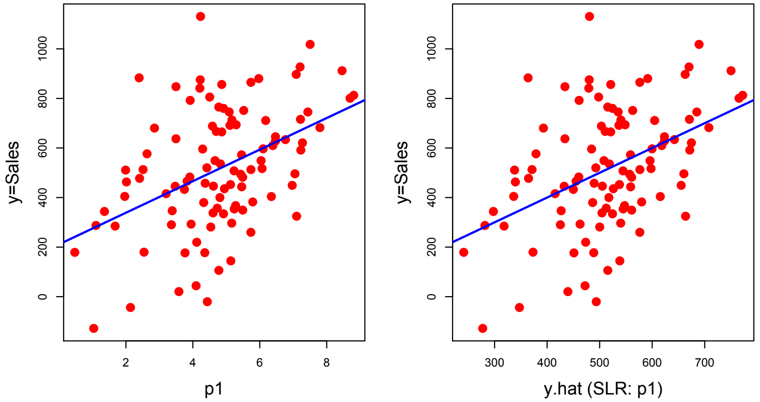
$$Sales = \beta_0 + \beta_1 P1 + \beta_2 P2 + \epsilon$$

Our estimates are $b_0 = 115$, $b_1 = -97$, $b_2 = 109$ and $s = 28$; i.e., $\epsilon \sim N(0, 28^2)$

Q: How will you estimate of sales when P1=8, P2=10 (95% confidence interval)?

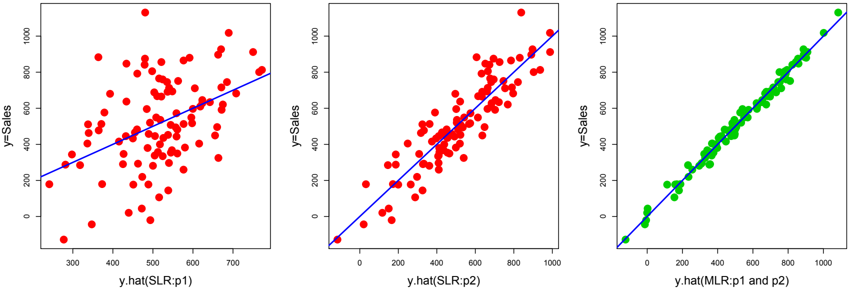
Fitted Values in MLR

With just $P1$...



- Left plot: *Sales* vs $P1$ (something odd?)
- Right plot: Sales vs. $\hat{y}$ (only $P1$ as a regressor)

Fitted Values in MLR

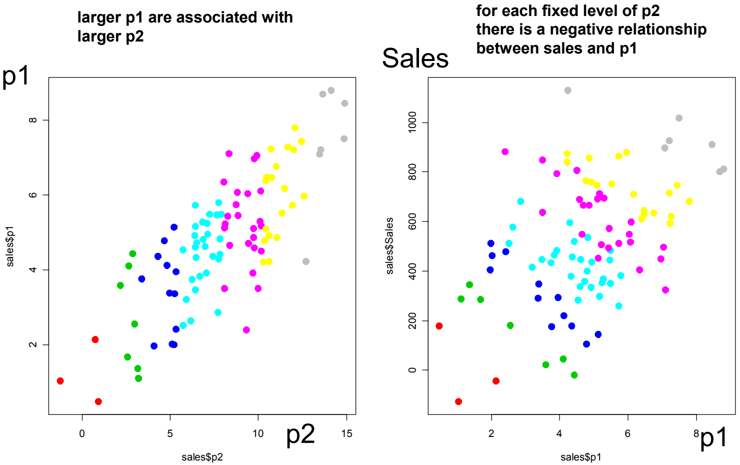


- First plot: *Sales* regressed on $P1$ alone..
- Second plot: *Sales* regressed on $P2$ alone...
- Third plot: Sales regressed on $P1$ and $P2$

Also look at residuals

Solving the Puzzle

- Let's look at a subset of points where $P1$ varies and $P2$ is held approximately constant...



Key Points to Remember

1. How dependencies between the X's affect our interpretation of a multiple regression.

   Any time a report says two variables are related and there's a suggestion of a "causal" relationship, ask yourself whether or not other variables might be the real reason for the effect.

   - Example: Why is it better to model beer vs. weight rather than beer vs. both height and weight?

2. How dependencies between the X's inflate standard errors (aka multicolinearity)

   - in MLR, the standard errors are defined by the following formula:

$$s_{b_j}^2 = \frac{s^2}{\text{(N-1)(variation in } X_j \text{ not associated with other } X\text{'s)}}$$

3. Correlation does not imply causation

4. Succinct models with the "right" predictors are superior

   - Reject predictor when the p-value is less than 0.05 (i.e. when the $|t_j| > 2$)

More Decisions

- How many X's do you have and what are they?
  - Bank Example: dummy coding and interaction effects
  - What if number of (potential) predictors is very large (p vs. n)
- Outliers in X or in Y
- Transformation of Variables (look at residuals!)
  - Non-constant residuals may suggest log transform