
Collaboration Policy: You are encouraged to collaborate with up to 3 other students, but all work submitted must be your own *independently* written solution. List the computing ids of all of your collaborators in the `collabs` command at the top of the tex file. Do not share written notes, documents (including Google docs, Overleaf docs, discussion notes, PDFs), or code. Do not seek published or online solutions for any assignments. If you use any published or online resources (which may not include solutions) when completing this assignment, be sure to cite by naming the book etc. or listing a website's URL. Do not submit a solution that you are unable to explain orally to a member of the course staff. Any solutions that share similar text/code will be considered in breach of this policy. Please refer to the syllabus for a complete description of the collaboration policy.

Collaborators: list collaborators's computing IDs

Sources: Cormen, et al, Introduction to Algorithms. (*add others here*)

PROBLEM 1 *Load Balancing*

You work for a print shop with 4 printers. Each printer i has a queue with n jobs: $j_{i,1}, \dots, j_{i,n}$. Each job has a number of pages, $p(j_{i,m})$. A printer's workload $W_i = \sum_{\ell} p(j_{i,\ell})$ is the sum of all pages across jobs for for that printer. Your goal is to *equalize* the workload across all 4 printers so that they all print the same number of total pages. You may only remove jobs from the end of their queues, i.e., job $j_{i,n}$ must be removed before job $j_{i,n-1}$, and you are allowed to remove a different number of jobs from each printer. Give a **greedy algorithm** to determine the maximum equalized workload (possibly 0 pages) across all printers. Be sure to state your greedy choice property.

Solution:

Greedy choice property: Remove a job from the printer with the highest workload.

Let w_1, w_2, w_3 and w_4 represent the current workload of each printer. First, check if $w_1 = w_2 = w_3 = w_4$ and if true, return w_1 (the algorithm is finished and all workloads are equal). Else, continue. Take the printer with the highest workload and remove a job from it. While the sum of all workloads ≥ 0 , repeat the steps above.

PROBLEM 2 *Short Answer Questions.* (You don't have to explain your answers in your submission, but you should understand the reason behind your answer.)

- A. True or false? Issuing the largest coin first will always solve the *coin change problem* if only two coins are available: the penny and one larger coin. Assume the amount of change is \geq the larger coin.

Solution: True

- B. True or false? The *interval scheduling problem* is always guaranteed to have an optimal solution that contains the interval with the earliest finish time.

Solution: True

- C. Choose one: In our proof of the correctness of the greedy solution to the *interval scheduling problem*, we exchanged the interval i selected by our greedy choice with another interval that finished earlier/later than interval i . (Your answer is one of "earlier" or "later.")

Solution: Later

- D. True or false? A *feasible solution* for the *Huffman encoding problem* is any valid prefix-free code-word table T .

Solution: False

PROBLEM 3 *Optimal Substructure*

Please answer the following questions related to *Optimal Substructure*.

- A. What's the difference in how dynamic programming algorithms versus greedy algorithms use *optimal substructure*?

Solution: In dynamic programming, optimal substructure is used to create an optimal solution that contains the optimal solutions to sub-problems. At every step, every choice is evaluated recursively and the best choice is picked. In greedy algorithms, an optimal substructure allows the algorithm to make the locally best choice at every step, even if it is not the best choice in the long run.

- B. Why did we need to prove the optimal substructure for our greedy Huffman coding algorithm?

Solution: We needed to show that making the least frequent characters siblings would make the optimal tree. Once it was proven true for a smaller problem, the algorithm could also be used to solve larger problems.

PROBLEM 4 *Gradescope Submission*

Submit a version of this .tex file to Gradescope with your solutions added, along with the compiled PDF. You should only submit your .pdf and .tex files.