Virtual Memory

1. A computer uses virtual memory implemented by paging. The TLB lookup takes 100 ns and the update takes 200 ns. Accessing the PT is ten times slower than accessing the TLB. The TLB-hit ratio is 0.6. How much CPU time is needed on average

   - to find out the physical address of a referenced word (assuming that the word is in main memory) and
   - to do the necessary updates to the page tables?

2. A computer uses 48-bit virtual addresses, 32-bit physical addresses, 2 KB pages and byte-level addressing. A page table entry consists of a page (frame) number and 11 extra bits (present, modified, access, etc.). Compute

   (a) the size of the virtual address space,

   (b) the size of the physical address space,

   (c) the number of pages,

   (d) the number of page frames,

   (e) the size of a page table,

   (f) the size of an inverted page table.

Solutions

1. In case of a TLB-hit, we just need 100 ns to read the PTE from the TLB.

   In case of a TLB-miss, we need TLB lookup (fail), PT lookup and TLB update: $100\ ns + 200\ ns + 1\ \mu s = 1,300\ ns$.

   Taking the weighted average we get: $0.6 \times 100 + 0.4 \times 1,300 = 60 + 520 =$ 580 (ns).

2. We need 11 bits in the adresses to specify the offset, the rest specify the page number (in case of a virtual address) or the frame number (in case of a physical address).

   (a) Since there are 2^{48} virtual addresses and every address specifies a byte-long word, the size of the virtual address space is $2^{48} = 2^8\ T = 256\ T$(bytes).

   (b) Since there are 2^{32} physical addresses and every address specifies a byte-long word, the size of the physical address space is $2^{32} = 2^2\ G = 4\ G$(bytes).

   (c) The page section of a virtual address is 37 bit long, so there are $2^{37} = 2^7\ G = 128\ G$ pages.

   (d) The frame section of a physical address is 21 bit long, so there are $2^{21} = 2^1\ M = 2\ M$ frames.

   (e) There are 128 G (the number of pages) entries in the page table and every PTE contains a frame number (21 bits) and the various extra bits (11 bits): $32 \times 128\ G = 4096\ G$(bits)$=$ 512 G(bytes).

   (f) There are 2 M (the number of frames) entries in the inverted page table and every entry contains a page number (37 bits) and the various extra bits (11 bits): $48 \times 2\ M = 96\ M$(bits)$=$ 12 M(bytes).