For return on 21 January 2019 (late submission: 28 January 2019)

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1. **(6%)** (a) Construct the truth-table for the Boolean function given by the formula

$$F \;=\; (B \to A) \to \big((A \wedge B \wedge C) \vee (\neg B \wedge C)\big).$$

(b) Find a Boolean circuit with AND, OR and NOT gates only that computes the Boolean function in (a) above and contains as few gates as possible.

(c) Determine whether the following argument is logically correct:

'Suppose that the formulas F in (a), $C \to A$ and $A \to B$ are all true. Then B is true'.

Show your working.

2. **(3%)** Design a Boolean circuit (with as few gates as possible) for the Boolean function that checks whether a 3-bit unsigned binary number is less than 3 (hint: construct the truth-table for the input bits A_2, A_1, A_0).

3. **(5%)** Formalise the following argument in Boolean logic, and decide whether it is correct or not. Explain your answer.

> If Jones did not meet Smith last night then either Smith was the murderer or Jones is lying. If Smith was not the murderer then Jones did not meet Smith last night and the murder took place after midnight. If the murder took place after midnight then either Smith was the murderer or Jones is lying. Therefore, Smith was the murderer.

(Suggestion: use the propositional variable A for 'Jones did not meet Smith last night', B for 'Smith was the murderer', C for 'Jones is lying', and D for 'The murder took place after midnight'.)

4. **(6%)** Given the machine 32-bit word

1100 0001 1011 0000 0000 0000 0000 0000

find the decimal number represented by this word assuming that it is

(a) a two's complement integer;

(b) an unsigned integer;

(c) a single precision IEEE 754 floating-point number.

5. **(6%)** Find computer representations of the following numbers:

(a) -107 as a two's complement 32-bit binary number;

(b) -107 as an IEEE 754 32-bit floating-point number;

(c) -15.375 as an IEEE 754 32-bit floating-point number.

6. **(6%)** Give an example of a $\mathbb{Z} \to \mathbb{N}$ function that is

(a) one-to-one but not onto,

(b) onto but not one-to-one,

(c) both one-to-one and onto,

(d) neither one-to-one nor onto.

7. **(6%)** For each of the following functions, determine whether or not it is one-to-one and whether or not it is onto:

(a) $f\colon \mathbb{R} \to \mathbb{R}$ given by $f(x) = (x+1)/(x-2)$;

(b) $f\colon \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ given by $f(m,n) = m+n-1$;

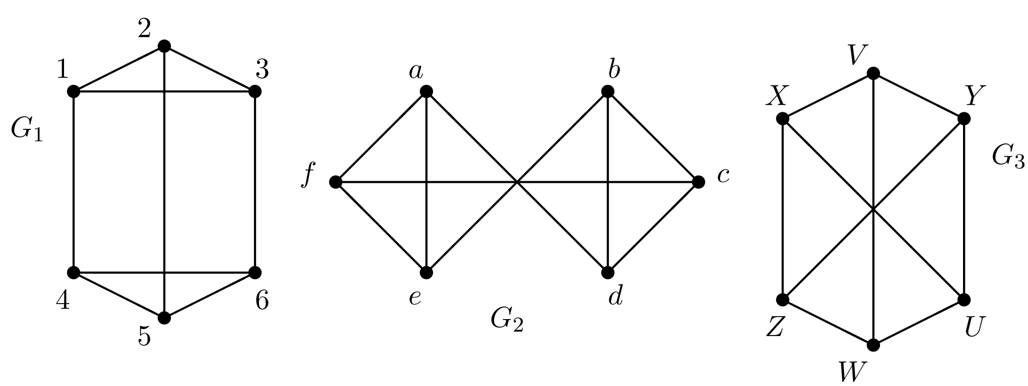
(c) $f\colon \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ given by $f(m,n) = |m| - |n| - 1$.

8. **(9%)** (a) Draw an undirected graph represented by the following adjacency matrix:

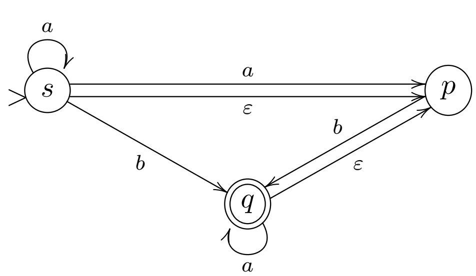
$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Is the graph simple? Explain your answer.

(b) Determine which pairs of the following graphs G_1, G_2 and G_3 are isomorphic and which are not, and explain your answer:



9. **(12%)** Consider the following NFA:



(a) Give all the computations of the automaton on the input strings bb, aa, ab, aba, and ε, and determine if the strings are accepted.

(b) Transform the automaton, using the subset construction, into an equivalent deterministic finite automaton and remove the unreachable states. Show your working.

(c) Describe the language of the automaton by means of a regular expression.

(d) Describe the language of the automaton by means of a context-free grammar.

10. **(8%)** (a) Design a DFA A such that $L(A)$ consists of all *even* binary numbers, assuming that words over $\{0, 1\}$ starting with 0 and containing more than one symbol are not proper numbers and should be rejected.

(b) Design an NFA for the language in (a) with as few states as possible.

11. **(6%)** Convert the regular language $L[(x \cup yy)^*(y \cup (xx)^*)]$ to a finite automaton accepting it. (If you do not follow the algorithm discussed in the lecture slides, explain the intuition behind your construction.)

12. **(8%)**(a) Give a context-free grammar for the language $\{a^i b^j c^k \mid i = j \text{ or } i = k,\ i, j, k \geq 0\}$ over the alphabet $\{a, b, c\}$. Show the derivation of $aabcc$ in your grammar.

(b) What is the language over the alphabet $\{a, b\}$ defined by the following context-free grammar with start variable S:

$$S \to aSa, \quad S \to bTb, \qquad T \to bT, \quad T \to b$$

Is this language regular? Give an informal explanation of your answer.

13. **(6%)** Construct a pushdown automaton for the language $\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i + j = k\}$.

14. **(5%)** Consider the following transition table of a Turing machine (with initial state s):

s	0	h	0
s	1	q	$\rightarrow$
s	$\sqcup$	s	$\sqcup$
s	$\triangleright$	s	$\rightarrow$
q	0	q	$\rightarrow$
q	1	q	$\rightarrow$
q	$\sqcup$	p	$\leftarrow$
q	$\triangleright$	q	$\rightarrow$
p	0	p	$\rightarrow$
p	1	h	0
p	$\sqcup$	h	0
p	$\triangleright$	p	$\rightarrow$

(i) Give the computations of the machine on inputs 10, 111 and 110.

(ii) Describe in English what this Turing machine does.

15. **(8%)** (a) Give an implementation level description in English of a Turing machine computing the function $f : \mathbb{N} \to \mathbb{N}$ given by $f(n) = 8 \times n + 2$.

(b) Give a transition table of your Turing machine and the computation of this machine for $n = 0$.