Fundamentals of Computing
Tutorial 3

1. For each of the following functions, determine whether it is one-to-one and/or onto, and briefly explain your answer. If the function is not onto, describe its range.

   (a) $succ : \mathbb{N} \to \mathbb{N}$, where $succ(n) = n + 1$.

   (b) $+ : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$, where $+(n, m) = n + m$.

2. **Definition:** The *composition* of functions $g : A \to B$ and $f : B \to C$ is the function $(f \circ g) : A \to C$ such that $(f \circ g)(a) = f\big(g(a)\big)$, for each $a \in A$. (The composition $f \circ g$ is only defined when the domain of f coincides with the codomain of g.)

   For $\mathbb{N}^+ \to \mathbb{N}^+$ functions defined by

   $$f(n) = 2n + 1 \qquad \text{and} \qquad g(n) = 3n - 1,$$

   describe the compositions

   (a) $f \circ f$,

   (b) $f \circ g$,

   (c) $g \circ f$,

   (d) $g \circ g$.

3. Let f and g be $\mathbb{N} \to \mathbb{N}$ functions defined by

   $$f(n) = \begin{cases} n - 1 & \text{if } n > 5, \\ n + 1 & \text{if } 0 \leq n \leq 5, \end{cases} \qquad \text{and} \qquad g(n) = 3n + 2\,.$$

   Describe the compositions

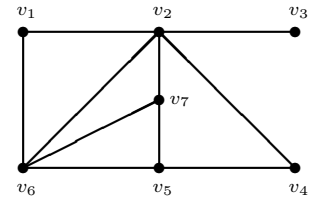
   (a) $f \circ f$,

   (b) $f \circ g$,

   (c) $g \circ f$.

4. Consider the following graph G:

   G

   (a) Find the degree of each vertex in G. Identify all isolated and pendant vertices.

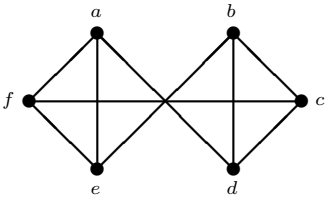
   (b) Is the sequence $(v_1, v_6, v_2, v_5, v_4)$ a path? Explain your answer.

   (c) Is the sequence $(v_6, v_7, v_5, v_6, v_2, v_1, v_6)$ a simple cycle? Explain your answer.

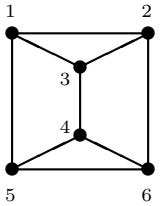
   (d) Represent G by an adjacency matrix.

5. Determine whether the following graphs are isomorphic or not, and explain your answer:

   G_1

   G_2

6. Consider the finite automaton $A = (Q, \Sigma, \delta, s, F)$, where $Q = \{s, q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$, $F = \{s, q_1, q_2\}$, and the transition function δ is given by the transition table

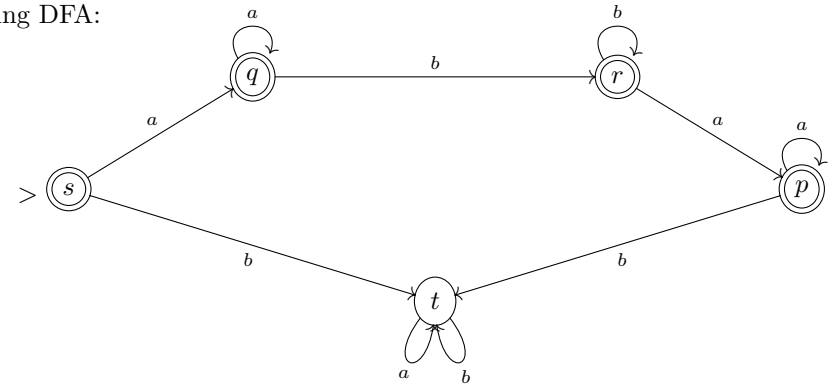
	0	1
s	s	q_1
q_1	s	q_2
q_2	s	q_3
q_3	q_3	q_3

   (a) Draw a graphical representation of A.

   (b) Does A accept the words 010, 000110100, 001111000?

   (c) Describe (in English) the language accepted by A.

7. Consider the following DFA:



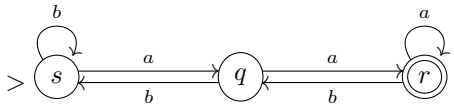
   (a) Give the formal description of the automaton, using a transition table.

   (b) Find the computation of the automaton on the input string *aaabba* and determine if the string is accepted.

   (c) Find the computations of the automaton on the input strings *aabaab* and *aabbaab* and determine if the strings are accepted.

   (d) Describe the language accepted by the automaton.

8. Consider the following DFA:



   (a) Give the formal description of the automaton, using a transition table.

   (b) Find the computations of the automaton on the input strings *aababa* and *bbaababaaa* and determine if the strings are accepted. Is it true that the automaton accepts all strings of length ≥ 2 ending with an a?

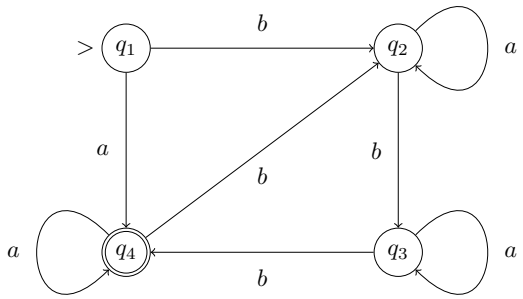
   (c) Find the computations of the automaton on the input strings *aabaab* and *bbaababb* and determine if the strings are accepted.

   (d) Bonus: describe the language accepted by the automaton.

9. What language is accepted by the following automaton:

答案：a^ja*(ba*)^k where k is divisible by 3, and j =(0,1)
or in set language:
{a^n(ba^m)^k I n>=0, m>=0, k is divisible by 3 } minus the empty set

pay attention to if it would accept an empty string



w is any word

10. What language is accepted by the automaton over the alphabet $\Sigma = \{a, b, c\}$ with the states $Q = \{s, r, t, q\}$ (where s is the initial state), the accepting states $F = \{q\}$, and the transition function δ given by the table

	a	b	c
s	t	s	s
t	t	q	s
q	r	q	q
r	r	s	q

w(abz)^k I where w doesn't contain 'ab', k is odd, and z does not contain « ab » and does not end up with a

11. Design a DFA that accepts all those strings over the alphabet $\Sigma = \{a, b, c\}$

    (a) that contain at least one occurrence of a;

    (b) in which c is always followed by a;

    (c) that do not contain a sub-word of the form abc;

    (d) that contain at least one occurrence of each of the symbols a, b and c;

    (e) that satisfy conditions (a)–(c) above.

正确答案：any string that doesn't end with a with an odd number of « ab »s