

### More Exercises (2017)

1. A professor of logic meets 10 of his former students—Albert, Alice, Bob, Bertha, Clifford, Connie, David, Dora, Edgar, Edith—who have become five married couples. When asked about their husbands, the ladies gave the following answers:
  - Alice: My husband is Clifford, and Bob has married Dora.
  - Bertha: My husband is Albert, and Bob has married Connie.
  - Connie: Clifford is my husband, Berthas husband is Edgar.
  - Dora: My husband is Bob, and David has married Edith.
  - Edith: Yes, David is my husband. And Alberts wife is Alice.

Additional true information coming from the men was that every lady gave one correct and one wrong answer. This was sufficient to find out the truth. Reproduce the professors argument.

2. Check the validity of the following arguments by using truth-tables.
  - Suppose  $(P \vee Q) \rightarrow R$  and  $Q$ . Therefore  $R$ .
  - Suppose  $P \rightarrow (Q \vee R)$  and  $P$ . Therefore  $R$ .
3. Formalise the following argument in propositional logic and demonstrate its validity. “If I graduate this semester, then I will have passed physics. If I do not study physics for 10 hours a week, then I will not pass physics. If I study physics for 10 hours a week, then I cannot play volleyball. Therefore, I will not graduate this semester if I play volleyball.”
4. Answer true or false for each of the following statements:
  - (a)  $\{2, 4, 5\} \subseteq \{2n \mid n \in \mathbb{N}\}$ .
  - (b)  $\{e, i, v, z\} \subseteq \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, r, s, t, u, v, x, y, z\}$ .
  - (c)  $\{2n \mid n \in \mathbb{N}\} = \{2n + 1 \mid n \text{ is a natural number}\}$ .
  - (d)  $\{1, 2, 3, 4, 5\} = \{5, 3, 2, 4, 1\}$ .
5. Describe each of the following sets by listing its elements.
  - (a)  $\{n \mid n \in \mathbb{N} \text{ and } 19 < n < 26\}$ .
  - (b)  $\{2k + 1 \mid k \text{ is an even integer between } -5 \text{ and } 3\}$ .
  - (c)  $\{x \mid x \text{ is a letter in the words EXAM COMMITTEE}\}$ .
6. Describe each of the following sets in terms of a property of its elements (that is, using the ‘description by properties’ notation).
  - (a) The set of dates in the month of July.
  - (b)  $\{1, 4, 9, 16, 25, 36, 49\}$ .
7. Show that there are sets  $A, B, C$  such that  $(A \cap B) \cup C \neq A \cap (B \cup C)$ .
8. Let  $X$  and  $Y$  be sets such that  $|X| = m$  and  $|Y| = n$  for some  $m, n \in \mathbb{N}^+$ . How many one-to-one functions are there from  $X$  to  $Y$ ?

9. Determine whether each of these sets is the power set of a set, where  $a$  and  $b$  are distinct elements:

- $\emptyset$ ;
- $\{\emptyset, \{a\}\}$ ;
- $\{\emptyset, \{a\}, \{\emptyset, a\}\}$ ;
- $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ .

10. Let  $A = \{a, b, c, d\}$  and  $B = \{y, z\}$ . Find  $A \times B$  and  $B \times A$ . Describe sets  $X$  and  $Y$  for which  $X \times Y = Y \times X$ .

11. Which of the following functions are ‘onto’?

- $f: \{a, b, c, d\} \rightarrow \{1, 2, 3\}$  defined by  $f(a) = 3$ ,  $f(b) = 2$ ,  $f(c) = 1$ , and  $f(d) = 3$ ;
- $g: \mathbb{Z} \rightarrow \mathbb{Z}$  defined by  $g(x) = x^2$ ;
- $g: \mathbb{Z} \rightarrow \mathbb{Z}$  defined by  $g(x) = x + 1$ ;
- $h: \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$  defined by  $g(x, y) = 2x - y$ ;
- $h: \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$  defined by  $g(x, y) = x^2 - 4$ .

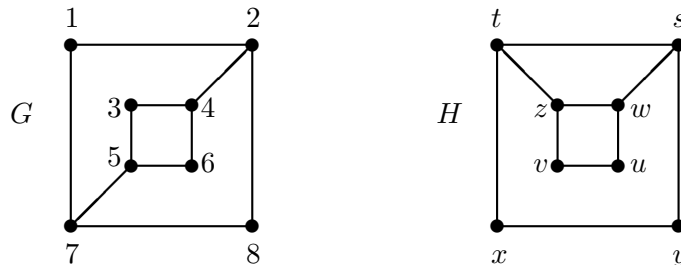
12. Determine whether each of these functions is a bijection from  $\mathbb{R}$  to  $\mathbb{R}$ :

- $f(x) = 2x + 1$ ;
- $f(x) = x^2 + 1$ ;
- $f(x) = x^3$ .

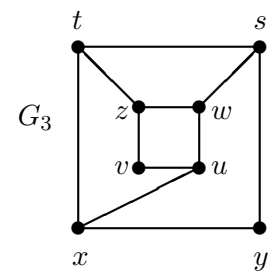
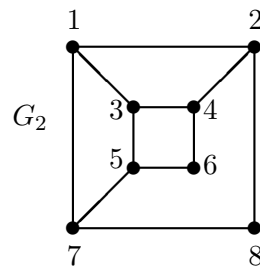
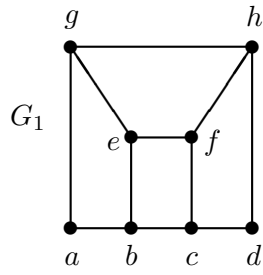
13. In each of the following cases, either draw an undirected graph having the given property, or explain why no such graph exists.

- (a) Having six vertices, each of degree 3.
- (b) Having six vertices and four edges.
- (c) Having four edges, and four vertices of degrees 1, 2, 3, 4, respectively.

14. Determine whether the graphs below are isomorphic:



15. Of the three following graphs, two are isomorphic and the third is not. Determine which pair are isomorphic and which one is the third nonisomorphic one. Explain your answers.



16. Let  $\Sigma = \{x, y\}$ . For each of the following languages over  $\Sigma$ , find a regular expression representing it:

- (a) All strings that do not contain the substring  $xx$ .
- (b) All strings that contain at least one  $y$  and do not end with  $xx$ .

17. Give context-free grammars for the following languages

- $L = \{a^n b^m \mid n \neq m\}$ .
- $L = \{a^n b^m c^k \mid n = m + k\}$ .
- $L = \{a^n b^m c^k \mid n \neq m + k\}$ .