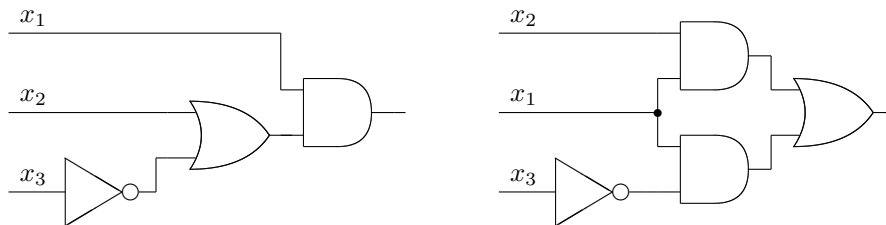


## Exercises (FoC 2010)

1. Which of these sentences are propositions? What are the truth values of those that are propositions?
  - (a) Liverpool is the capital of the UK.
  - (b)  $x + 2 = 11$ .
  - (c)  $2 + 3 = 5$ .
  - (d) Answer this question.
2. What is the negation of each of these propositions?
  - (a) Today is Thursday.
  - (b) There is no pollution in London.
  - (c)  $2 + 3 = 5$ .
  - (d) The summer in London is hot and sunny.
3. Construct a truth table for each of the following Boolean formulas.
  - (a)  $A \wedge \neg A$ .
  - (b)  $A \vee \neg A$ .
  - (c)  $(A \vee \neg B) \rightarrow B$ .
  - (d)  $(A \rightarrow B) \leftrightarrow (\neg B \rightarrow \neg A)$ .
  - (e)  $(A \rightarrow B) \rightarrow (B \rightarrow A)$ .
  - (f)  $(A \oplus B) \rightarrow (A \oplus \neg B)$ .
4. Which of the following pairs of formulas are logically equivalent?
  - (a)  $(A \rightarrow B) \wedge (A \rightarrow C)$  and  $A \rightarrow (B \wedge C)$ .
  - (b)  $(A \rightarrow B) \rightarrow C$  and  $A \rightarrow (B \rightarrow C)$ .
  - (c)  $(A \wedge B) \rightarrow C$  and  $(A \rightarrow C) \wedge (B \rightarrow C)$ .
5. Consider the Boolean functions  $f(x_1, x_2, x_3)$  and  $g(x_1, x_2, x_3)$  realised by the following digital circuits:



Construct the truth-tables for these functions and represent them as formulas over  $\{\wedge, \vee, \neg\}$ .

6. Construct a Boolean formula that realises the Boolean function given by the following truth-table:

$x_1$	$x_2$	$x_3$	$f(x_1, x_2, x_3)$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Can you simplify it? (Hint: use  $\rightarrow$  and  $\neg$ .)

7. Construct the simplest Boolean formula that realises the Boolean function given by the following truth-table:

$x_1$	$x_2$	$f(x_1, x_2)$
0	0	0
0	1	0
1	0	1
1	1	1

8. Design a Boolean circuit to input a 3-bit value and output its two's complement value (see lecture 2, page 9).
9. Consider two Boolean formulas and show that they realise the same Boolean function:

$$x_1 \wedge (x_1 \vee x_2) \quad \text{and} \quad x_1 \vee (x_1 \wedge x_2).$$

Find the simplest Boolean formula representing this Boolean function.

10. Show that the Boolean function given by the formula

$$((x_1 \rightarrow x_2) \rightarrow x_1) \rightarrow x_1$$

is equal to 1.

11. Is it possible to realise  $\neg x$  as a formula with  $\wedge$  and  $\vee$  only?
12. Given the machine 32-bit word

1011 1111 1110 0000 0000 0000 0000

find the decimal number represented by this word assuming that it is

- a two's complement integer
  - an unsigned integer
  - a single precision IEEE 754 floating-point number.
13. Represent the number -12 as a two's complement 32-bit binary number and as an IEEE 754 32-bit floating-point number. Represent -0.125 as an IEEE 754 32-bit floating-point number.
14. Draw a Boolean circuit for the logical equation

$$C = (A \wedge \neg B) \vee (A \wedge B).$$

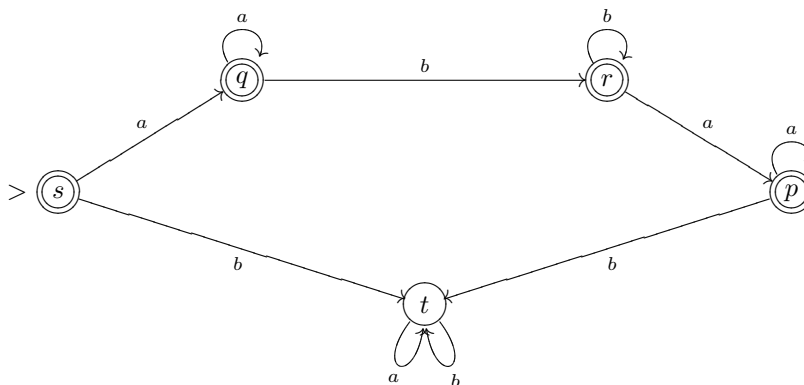
What does this circuit compute?

15. Convert the following decimal numbers to their binary equivalents:  $1.5_{10}$ ,  $1.1_{10}$ ,  $(1/3)_{10}$ .  
 Convert the following binary numbers to their decimal equivalents:  $1.1_2$ ,  $101.101_2$ .

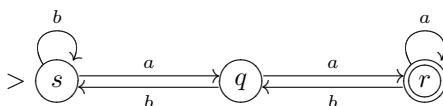
16. What decimal number does this two's complement binary number represent?

1111 1111 1111 1111 1111 1110 0000 1100

17. Consider the following DFA:

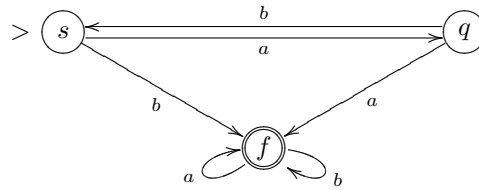


- Give the formal description of the automaton (using a transition table).
  - Find the computation of the automaton on the input string  $aaabba$  and determine if the string is accepted.
  - Find the computations of the automaton on the input strings  $aabaab$  and  $aabbaab$  and determine if the strings are accepted.
  - Describe (informally) the language accepted by the automaton.
18. Consider the following DFA:



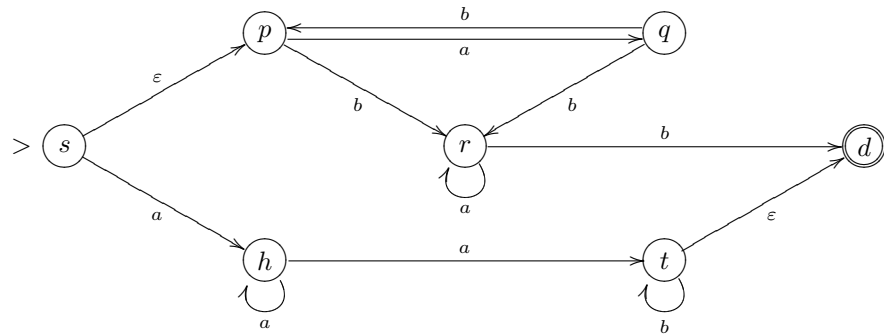
- Give the formal description of the automaton (using a transition table).
  - Find the computations of the automaton on the input strings  $aababa$  and  $bbaababaaa$  and determine if the strings are accepted. Is it true that the automaton accepts all strings of length  $\geq 2$  ending with an  $a$ ?
  - Find the computations of the automaton on the input strings  $aabaab$  and  $bbaababb$  and determine if the strings are accepted.
  - Describe the language accepted by the automaton.
19. Is there a deterministic finite automaton  $A$  accepting the empty word  $\varepsilon$ ? Is there an  $A$  over the alphabet  $\{0, 1\}$  such that  $L(A) = \{\varepsilon\}$ ? Explain your answers.
20. For each of the following languages, construct (in form of a state transition diagram) a deterministic finite automaton accepting it.
- all strings over  $\{\square, \diamond\}$  containing at least three  $\square$ s
  - all strings over  $\{\square, \diamond\}$  that begin with a  $\square$  and end with a  $\diamond$
  - all strings over  $\{0, 1\}$  except the empty string  $\varepsilon$ .

21. Consider the following finite automaton:



- Is the automaton deterministic or nondeterministic?
- Find the computation(s) of the automaton on the input strings *ababab* and *ababa*, and determine if the strings are accepted.
- Find the computation(s) of the automaton on the input strings *ababb*, *ba*, and *aa*, and determine if the strings are accepted.
- Describe (informally) the language accepted by the automaton.

22. Determine if the nondeterministic automaton

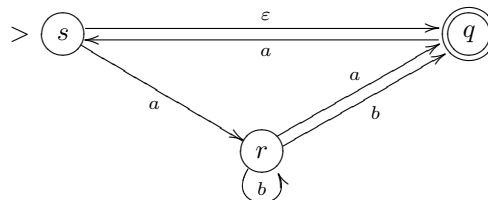


accepts the strings

- abb*
- abbbb*
- aabb*

In each case, if the string is accepted, give a corresponding computation.

- Design a (deterministic or nondeterministic) finite automaton  $A$  such that  $L(A)$  consists of all the strings of  $a$ 's and  $b$ 's that contain *bbab* as a substring.
- Transform, using the subset construction, the following nondeterministic finite automaton into an equivalent deterministic finite automaton. Then delete the unreachable states. Show your working.



- For each of the following regular expressions, find three of the shortest strings in the corresponding regular language:
  - $a^*(b \cup abb)b^*b$

(ii)  $(a \cup ab)(a^* \cup ab)^*b$

26. For each of the following regular expressions, list three of the shortest strings that are **not** in the corresponding regular language:

(i)  $a^*aabb^*$

(ii)  $(aa)^*(bba)^*(bb)^*$

(iii)  $a^*(bba)^*b^*$

27. Let  $\Sigma = \{\square, \diamond\}$ . For each of the following languages over  $\Sigma$ , find a regular expression representing it:

(i) All strings that contain exactly one  $\square$ .

(ii) All strings that begin with  $\square\square$ .

(iii) All strings that begin with  $\square\square$  and end with  $\diamond\diamond$ .

(iv) All strings that contain either the substring  $\square\square\square$  or the substring  $\diamond\diamond\diamond$ .

28. Let  $\Sigma = \{0, 1\}$ . Show that the language

$$L = \{w \mid 0110 \text{ is a suffix of } w\}$$

is regular.

29. Apply the procedure given in the lectures to convert each of the following regular languages to a finite automaton accepting it:

(i)  $L[(a \cup b)^*ab]$

(ii)  $L[((0 \cup 1)^*1)^*]$

30. Consider the following transition table of a Turing machine:

$s$	$\sqcup$	$h$	$\sqcup$
$s$	$a$	$q$	$b$
$s$	$b$	$q$	$a$
$q$	$a$	$s$	$\rightarrow$
$q$	$b$	$s$	$\rightarrow$
$q$	$\sqcup$	$q$	$\sqcup$
$s$	$\triangleright$	$s$	$\rightarrow$
$q$	$\triangleright$	$s$	$\rightarrow$

(i) Give the computation of the machine starting with the configuration  $(s, \triangleright abbabba)$ .

(ii) Give the computation of the machine starting with the configuration  $(s, \triangleright baaba)$ .

(iii) Experiment with more input words. Then describe in English what this Turing machine does.

31. Let  $\Sigma = \{a, b, \triangleright, \sqcup\}$ . Give the complete transition tables for the following Turing machines having tape alphabet  $\Sigma$ :

(i) The machine scans the tape to the right until it finds the first blank cell, then halts.

(ii) The machine scans the tape to the right until it finds the first pattern  $ab$  (i.e., an  $a$  followed immediately by a  $b$ ), then halts. If there is no such pattern, then the machines does not terminate.

32. Give the complete transition table of a Turing machine which computes the function

$$f(w) = \begin{cases} w & \text{if } |w| \text{ is even,} \\ wa & \text{if } |w| \text{ is odd,} \end{cases}$$

defined for all strings of  $a$ 's. (Note that 0 counts as even.)

33. Consider the 'left-shifting' machine described on slide 101, and assume that now not only  $a$ 's but  $b$ 's as well can be written on the tape (that is, the machine has to be able to shift any string of  $a$ 's and  $b$ 's one cell to the left). Give the transition table of this modified 'left-shifting' machine.

34. (i) Give an implementation level description in English of a Turing machine that computes the  $\mathbb{N} \rightarrow \mathbb{N}$  function

$$f(n) = \begin{cases} n & \text{if } n \geq 4 \\ 0 & \text{otherwise.} \end{cases}$$

- (ii) Give the complete transition table of this Turing machine.