ETH zürich

Department of Computer Science

Formal Methods and Functional Programming – Assignment 4

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Natural Deduction

a)

1. $A \vee B \to C \to A \wedge C \vee B \wedge C \equiv (A \vee B) \to (C \to ((A \wedge C) \vee (B \wedge C)))$.
2. $(A \to B \to C) \to A \wedge B \to C \equiv (A \to (B \to C)) \to ((A \wedge B) \to C)$

b)

$(A \vee B) \to (C \to ((A \wedge C) \vee (B \wedge C))) \equiv \top$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash A \vee B}\text{ axiom} \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, A \vdash A}\text{ axiom} \quad \dfrac{}{\Gamma, A \vdash C}\text{ axiom}}{\Gamma, A \vdash (A \wedge C)}\wedge\text{-I}}{\Gamma, A \vdash (A \wedge C) \vee (B \wedge C)}\vee\text{-IL} \quad \dfrac{\dfrac{\dfrac{}{\Gamma, B \vdash B}\text{ axiom} \quad \dfrac{}{\Gamma, B \vdash C}\text{ axiom}}{\Gamma, B \vdash (B \wedge C)}\wedge\text{-I}}{\Gamma, B \vdash (A \wedge C) \vee (B \wedge C)}\vee\text{-IR}}{A \vee B, C \vdash (A \wedge C) \vee (B \wedge C)}\vee\text{-E}}{A \vee B \vdash C \to ((A \wedge C) \vee (B \wedge C))}\to\text{-I}}{\vdash (A \vee B) \to (C \to ((A \wedge C) \vee (B \wedge C)))}\to\text{-I}$$

where $\Gamma := A \vee B, C$.

$(A \to (B \to C)) \to ((A \wedge B) \to C) \equiv \top$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash A \to (B \to C)}\text{ axiom} \quad \dfrac{\dfrac{}{\Gamma \vdash A \wedge B}\text{ axiom}}{\Gamma \vdash A}\wedge\text{-EL}}{\Gamma \vdash B \to C}\to\text{-E} \quad \dfrac{\dfrac{}{\Gamma \vdash A \wedge B}\text{ axiom}}{\Gamma \vdash B}\wedge\text{-ER}}{A \to (B \to C), A \wedge B \vdash C}\to\text{-E}}{A \to (B \to C) \vdash (A \wedge B) \to C}\to\text{-I}}{\vdash (A \to (B \to C)) \to ((A \wedge B) \to C)}\to\text{-I}$$

where $\Gamma := A \to (B \to C), A \wedge B$.

c)

Rules:

$$\dfrac{\Gamma \vdash A \to B \qquad \Gamma \vdash B \to A}{\Gamma \vdash A \leftrightarrow B}\leftrightarrow\text{-I}$$

$$\dfrac{\Gamma \vdash A \leftrightarrow B}{\Gamma \vdash A \to B}\leftrightarrow\text{-EL}$$

$$\dfrac{\Gamma \vdash A \leftrightarrow B}{\Gamma \vdash B \to A}\leftrightarrow\text{-ER}$$

$(A \leftrightarrow B) \to (B \leftrightarrow A) \equiv \top$:

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A \leftrightarrow B \vdash A \leftrightarrow B}\text{ axiom}}{A \leftrightarrow B \vdash B \to A}\leftrightarrow\text{-ER} \quad \dfrac{\dfrac{}{A \leftrightarrow B \vdash A \leftrightarrow B}\text{ axiom}}{A \leftrightarrow B \vdash A \to B}\leftrightarrow\text{-EL}}{A \leftrightarrow B \vdash B \leftrightarrow A}\leftrightarrow\text{-I}}{\vdash (A \leftrightarrow B) \to (B \leftrightarrow A)}\to\text{-I}$$