ETH zürich

Department of Computer Science

Formal Methods and Functional Programming – Assignment 1

Author: Yannis Huber (17-570-110)

Teaching assistant: Tobias Oberdörfer

Induction

a)

$$\frac{\Gamma \vdash P[n/0] \qquad \Gamma, \forall m \in \mathbb{N}.P[n/m] \vdash P[n/m+1]}{\Gamma, \forall m \in \mathbb{N}.P[n/m] \vdash \forall n \in \mathbb{N}.P} \text{ induction}$$

I am not sure if it is necessary to keep the induction assumption as part of the assumptions after the derivation or if it is possible to discard it, i.e. if it is sufficient to have $\Gamma \vdash \forall n \in \mathbb{N}.P$ on the bottom of the derivation rule?

b)

We want to show that $\forall n \in \mathbb{N}.\texttt{fibLouis}\ n = \texttt{fibEva}\ n$. We will first prove using induction over n, that:

$$P \equiv \forall n \in \mathbb{N}, \texttt{aux}\ n = (\texttt{fibLouis}\ n, \texttt{fibLouis}\ (n+1)).$$

holds, under the assumption that $\forall m \in \mathbb{N}.P[n/m]$ holds.

Base case: $P[n/0]$

$$\begin{aligned}
\texttt{aux}\ 0 &= (0,1) && \texttt{(aux.1)}\\
&= (\texttt{fibLouis}\ 0, 1) && \texttt{(fibLouis.1)}\\
&= (\texttt{fibLouis}\ 0, \texttt{fibLouis}\ 1) && \texttt{(fibLouis.2)}\\
&= (\texttt{fibLouis}\ 0, \texttt{fibLouis}\ (0+1)) && \texttt{(arithmetic)}
\end{aligned}$$

Q.E.D.

Step case: $\forall m \in \mathbb{N}.P[n/m] \rightarrow P[n/m+1]$

$$\begin{aligned}
\texttt{aux}\ (m+1) &= \texttt{next}\ (\texttt{aux}\ ((m+1)-1)) && \texttt{(aux.2)}\\
&= \texttt{next}\ (\texttt{aux}\ (m)) && \texttt{(arithmetic)}\\
&= \texttt{next}\ (\texttt{fibLouis}\ m, \texttt{fibLouis}\ (m+1)) && \texttt{(induction hypothesis)}\\
&= (\texttt{fibLouis}\ (m+1), \texttt{fibLouis}\ m + \texttt{fibLouis}\ (m+1)) && \texttt{(next.1)}\\
&= (\texttt{fibLouis}\ (m+1), \texttt{fibLouis}\ (m+1) + \texttt{fibLouis}\ m) && \texttt{(arithmetic)}\\
&= (\texttt{fibLouis}\ (m+1), \texttt{fibLouis}\ (m+2)) && \texttt{(fibLouis.3)}\\
&= (\texttt{fibLouis}\ (m+1), \texttt{fibLouis}\ ((m+1)+1)) && \texttt{(arithmetic)}
\end{aligned}$$

Q.E.D.

This concludes our proof that P holds for any $n \in \mathbb{N}$ and we can proceed to prove $\forall n \in \mathbb{N}.\texttt{fibLouis}\ n = \texttt{fibEva}\ n$:

$$\begin{aligned}
\forall n \in \mathbb{N}.\texttt{fibEva}\ n &= \texttt{fst}\ (\texttt{aux}\ n) && \texttt{(fibEva.1)}\\
&= \texttt{fst}\ (\texttt{fibLouis}\ n, \texttt{fibLouis}\ (n+1)) && \text{(P)}\\
&= \texttt{fibLouis}\ n && \texttt{(fst)}
\end{aligned}$$

Q.E.D.