Supplementary Problems for 201-NYC-05 Science

An open source collection of
24 CEGEP level linear algebra problems from 6 authors.

April 5, 2017

1 Matrix Inverses

1.1 [MH] Find the matrix X such that

$$\left(X^{\mathsf{T}} - 3I\right)\begin{bmatrix}1 & 3\\ 2 & 5\end{bmatrix} = \begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$$

1.2 [YL] Given $C = \begin{bmatrix}1 & 2\\ 5 & 0\end{bmatrix}$ and $D = \begin{bmatrix}-1 & 1\\ 4 & -3\end{bmatrix}$. Solve the given equations for X.

a. $CXD = 10I$

b. $C((DX)^{\mathsf{T}} - 2I)^{-1} = C$

2 Properties of Determinants and Adjoint of a Matrix

2.1 [MB] Knowing that the cofactor matrix of A is

$$\operatorname{cof}(A) = \begin{bmatrix}3 & -6 & 5\\ -4 & 3 & -5\\ 5 & -5 & 5\end{bmatrix}$$

and that the first row of A is $\begin{bmatrix}2 & 1 & -1\end{bmatrix}$

a. Find $\operatorname{adj}(A)$.

b. Find $\det(A)$.

c. Find the minors M_{12} and M_{22} of A.

d. Find A^{-1}.

2.2 [YL] Let B be a 3×3 matrix where $\det(B) = 3$. Find

$$\det\left(2B + B^2\operatorname{adj}(B)\right).$$

2.3 [MB] Consider two 4×4 matrices A and B, with $\det(A) = -2$ and $\det(B) = 3$. Find the determinant of M, knowing that $\det(2B^{\mathsf{T}}MA^{-1}B) = \det(\operatorname{adj}(A)A^2B)$.

2.4 [YL] Given

$$A = \begin{bmatrix}10 & 1 & 1 & 1 & 9 & 3 & 4 & 1 & 0 & 9\\ 0 & 9 & 1 & 1 & 4 & 9 & 2 & 7 & 7 & 9\\ 0 & 0 & 8 & 1 & 1 & 4 & 9 & 2 & 7 & 7\\ 0 & 0 & 0 & 7 & 1 & 1 & 4 & 9 & 2 & 7\\ 0 & 0 & 0 & 0 & 6 & 1 & 1 & 4 & 9 & 2\\ 0 & 0 & 0 & 0 & 0 & 5 & 1 & 1 & 4 & 9\\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 1 & 1 & 4\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 1 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

$$B = \begin{bmatrix}4 & 3 & -2 & 8\\ 2 & 3 & 1 & -4\\ -1 & 2 & -1 & 4\\ 1 & 2 & 2 & -8\end{bmatrix}, \quad C = \begin{bmatrix}4 & 3 & -2 & 8\\ 0 & 3 & 1 & -4\\ 0 & 0 & -1 & 4\\ 0 & 0 & 0 & 8\end{bmatrix}.$$

a. Given G is a 4×4 matrix show that BG is not invertible.

b. Given F is a 10×10 invertible matrix evaluate $\det\left(F^{101}\operatorname{adj}(A)(F^{-1})^{101}\right)$.

c. Evaluate $\det\left(2\operatorname{adj}(A) + 3A^{-1}\right)$, if possible.

d. Given D is a 10×10 matrix such that $A^{-1}D^2 = I$ determine $\det(D)$, if possible.

e. Evaluate $\det\left(B^{101}\operatorname{adj}(C) + BC^{2015}\right)$, if possible.

f. Given E is a square matrix and $\det\left(\frac{\det(C)}{2}E^{\mathsf{T}}A^3\right) = \pi$ find $\det(E)$, if possible.

g. Given H is a 10×10 matrix and $\det(2HA + H\operatorname{adj}(A)A^2) = 0$ show that H singular.

h. Evaluate $\det(\operatorname{adj}(A) + I)$, if possible.

i. Determine $\operatorname{adj}(\operatorname{adj}(A))$, if possible.

3 Norm and Dot Product

3.1 [SM] Determine all values of k for which the vectors are orthogonal.

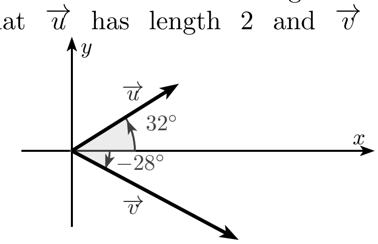
a. $(3, -1)$, $(2, k)$

b. $(3, -1)$, (k, k^2)

c. $(1, 2, 3)$, $(3, -k, k)$

d. $(1, 2, 3)$, $(k, k, -k)$

3.2 [MH] Suppose that $\vec{u}$ and $\vec{v}$ are two vectors in the xy-plane with directions as given in the diagram and such that $\vec{u}$ has length 2 and $\vec{v}$ has length 3.



a. Find $\boldsymbol{u} \cdot \boldsymbol{v}$.

b. Find $\|\boldsymbol{u} + \boldsymbol{v}\|$.

3.3 [SM] Prove the parallelogram law for the norm:

$$\|\boldsymbol{a} + \boldsymbol{b}\|^2 + \|\boldsymbol{a} - \boldsymbol{b}\|^2 = 2\|\boldsymbol{a}\|^2 + 2\|\boldsymbol{b}\|^2$$

for all vectors in $\mathbb{R}^n$.

3.4 [JH] Show that $\|\boldsymbol{u}\| = \|\boldsymbol{v}\|$ if and only if $\boldsymbol{u} + \boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$ are perpendicular. Give an example in $\mathbb{R}^2$.

4 Lines

4.1 [GHC] Write the vector and parametric equations of the lines described.

a. Passes through $P = (2, -4, 1)$, parallel to $\boldsymbol{d} = (9, 2, 5)$.

b. Passes through $P = (6, 1, 7)$, parallel to $\boldsymbol{d} = (-3, 2, 5)$.

c. Passes through $P = (2, 1, 5)$ and $Q = (7, -2, 4)$.

d. Passes through $P = (1, -2, 3)$ and $Q = (5, 5, 5)$.

e. Passes through $P = (0, 1, 2)$ and orthogonal to both $\boldsymbol{d}_1 = (2, -1, 7)$ and $\boldsymbol{d}_2 = (7, 1, 3)$.

f. Passes through $P = (5, 1, 9)$ and orthogonal to both $\boldsymbol{d}_1 = (1, 0, 1)$ and $\boldsymbol{d}_2 = (2, 0, 3)$.

g. Passes through the point of intersection and orthogonal of both lines, where $\boldsymbol{x} = (2, 1, 1) + t(5, 1, -2)$ and $\boldsymbol{x} = (-2, -1, 2) + t(3, 1, -1)$.

h. Passes through the point of intersection and orthogonal to both lines, where

$$\boldsymbol{x} = \begin{cases} x = t \\ y = -2 + 2t \\ z = 1 + t \end{cases} \quad \text{and} \quad \boldsymbol{x} = \begin{cases} x = 2 + t \\ y = 2 - t \\ z = 3 + 2t \end{cases}.$$

i. Passes through $P = (1, 1)$, parallel to $\boldsymbol{d} = (2, 3)$.

j. Passes through $P = (-2, 5)$, parallel to $\boldsymbol{d} = (0, 1)$.

4.2 [**GHC**] Determine if the described lines are the same line, parallel lines, intersecting or skew lines. If intersecting, give the point of intersection.

a. $\boldsymbol{x} = (1, 2, 1) + t(2, -1, 1)$ and $\boldsymbol{x} = (3, 3, 3) + t(-4, 2, -2)$

b. $\boldsymbol{x} = (2, 1, 1) + t(5, 1, 3)$ and $\boldsymbol{x} = (14, 5, 9) + t(1, 1, 1)$

c. $\boldsymbol{x} = (3, 4, 1) + t(2, -3, 4)$ and $\boldsymbol{x} = (-3, 3, -3) + t(3, -2, 4)$

d. $\boldsymbol{x} = (1, 1, 1) + t(3, 1, 3)$ and $\boldsymbol{x} = (7, 3, 7) + t(6, 2, 6)$

e. $$\boldsymbol{x} = \begin{cases} x = 1 + 2t \\ y = 3 - 2t \\ z = t \end{cases} \quad \text{and} \quad \boldsymbol{x} = \begin{cases} x = 3 - t \\ y = 3 + 5t \\ z = 2 + 7t \end{cases}$$

4.3 [**SM**] For the given point and line, find by projection the point on the line that is closest to the given point, and find the distance from the point to the line.

a. point $(0, 0)$, line $\boldsymbol{x} = (1, 4) + t(-2, 2)$, $t \in \mathbb{R}$

b. point $(2, 5)$, line $\boldsymbol{x} = (3, 7) + t(1, -4)$, $t \in \mathbb{R}$

c. point $(1, 0, 1)$, line $\boldsymbol{x} = (2, 2, -1) + t(1, -2, 1)$, $t \in \mathbb{R}$

d. point $(2, 3, 2)$, line $\boldsymbol{x} = (1, 1, -1) + t(1, 4, 1)$, $t \in \mathbb{R}$

4.4 [**GHC**] Find the distance between the two lines.

a. $\boldsymbol{x} = (1, 2, 1) + t(2, -1, 1)$ and $\boldsymbol{x} = (3, 3, 3) + t(4, 2, -2)$.

b. $\boldsymbol{x} = (0, 0, 1) + t(1, 0, 0)$ and $\boldsymbol{x} = (0, 0, 3) + t(0, 1, 0)$.

4.5 [**YL**] Determine whether the two lines intersect, are parallel or are skew lines. Find a point on each line which is closest to the other line, are those points unique? Find the shortest distance between the lines.

a. $\boldsymbol{x} = (1 - 2t,\ 2 - t,\ 3 + 3t)$ and $\boldsymbol{x} = (4 + 3t,\ -1 + t,\ 2 + t)$

b. $\boldsymbol{x} = (-4,\ 5,\ -2) + t(3,\ ,-4,\ 1)$ and $\boldsymbol{x} = (-3,\ 3,\ 5) + t(4,\ -5,\ -2)$

c. $$\mathcal{L}_1 : \begin{cases} x = 3t \\ y = -1 + 2t \\ z = 1 + 2t \end{cases} \text{and } \mathcal{L}_2 : \begin{cases} x = 1 - 6t \\ y = -4t \\ z = 2 - 4t \end{cases}.$$

d. $y = 2x + 1$ and $2y - 4x + 1$

4.6 [**MB**] Consider the points $P(1,\ 0,\ 1)$, $Q(-2,\ 1,\ 2)$, and $R(3,\ 2,\ 1)$, and the vector $\boldsymbol{v} = (4,\ 0,\ -1)$ in $\mathbb{R}^3$.

a. Consider L_1, the line passing through P and Q, and L_2 the line passing through R and parallel to $\boldsymbol{v}$. Determine whether these lines are parallel, intersect, or are skew.

b. If L_1 and L_2 intersects, find their point of intersection, then find the distance between the two lines.

5 Planes

5.1 [**GHC**] Give the equation of the described plane in standard and general forms.

a. Passes through $(2, 3, 4)$ and has normal vector $\boldsymbol{n} = (3, -1, 7)$.

b. Passes through $(1, 3, 5)$ and has normal vector $\boldsymbol{n} = (0, 2, 4)$.

c. Passes through the points $(1, 2, 3)$, $(3, -1, 4)$ and $(1, 0, 1)$.

d. Passes through the points $(5, 3, 8)$, $(6, 4, 9)$ and $(3, 3, 3)$.

e. Contains the intersecting lines $\boldsymbol{x} = (2, 1, 2) + t(1, 2, 3)$ and $\boldsymbol{x} = (2, 1, 2) + t(2, 5, 4)$.

f. Contains the intersecting lines $\boldsymbol{x} = (5, 0, 3) + t(-1, 1, 1)$ and $\boldsymbol{x} = (1, 4, 7) + t(3, 0, -3)$.

g. Contains the parallel lines $\boldsymbol{x} = (1, 1, 1) + t(1, 2, 3)$ and $\boldsymbol{x} = (1, 1, 2) + t(1, 2, 3)$.

h. Contains the parallel lines $\boldsymbol{x} = (1, 1, 1) + t(4, 1, 3)$ and $\boldsymbol{x} = (2, 2, 2) + t(4, 1, 3)$.

i. Contains the point $(2, -6, 1)$ and the line

$$\boldsymbol{x} = \begin{cases} x = 2 + 5t \\ y = 2 + 2t \\ z = -1 + 2t \end{cases}$$

j. Contains the point $(5, 7, 3)$ and the line

$$\boldsymbol{x} = \begin{cases} x = t \\ y = t \\ z = t \end{cases}$$

k. Contains the point $(5, 7, 3)$ and is orthogonal to the line $\boldsymbol{x} = (4, 5, 6) + t(1, 1, 1)$.

l. Contains the point $(4, 1, 1)$ and is orthogonal to the line

$$\boldsymbol{x} = \begin{cases} x = 4 + 4t \\ y = 1 + 1t \\ z = 1 + 1t \end{cases}$$

m. Contains the point $(-4, 7, 2)$ and is parallel to the plane $3(x - 2) + 8(y + 1) - 10z = 0$.

n. Contains the point $(1,2,3)$ and is parallel to the plane $x=5$.

5.2 [**SM**] Determine the scalar equation of the plane with the given vector equation.

a. $\boldsymbol{x}=(1,4,7)+s(2,3,-1)+t(4,1,0),\ s,t\in\mathbb{R}$

b. $\boldsymbol{x}=(2,3,-1)+s(1,1,0)+t(-2,1,2),\ s,t\in\mathbb{R}$

c. $\boldsymbol{x}=(1,-1,3)+s(2,-2,1)+t(0,3,1),\ s,t\in\mathbb{R}$

5.3 [**GHC**] Find the point of intersection between the line and the plane.

a. line: $(1,2,3)+t(3,5,-1)$, plane: $3x-2y-z=4$

b. line: $(1,2,3)+t(3,5,-1)$, plane: $3x-2y-z=-4$

c. line: $(5,1,-1)+t(2,2,1)$, plane: $5x-y-z=-3$

d. line: $(4,1,0)+t(1,0,-1)$, plane: $3x+y-2z=8$

5.4 [**SM**] Given the plane $2x_1-x_2+3x_3=5$, for each of the following lines, determine if the line is parallel to the plane, orthogonal to the plane, or neither parallel nor orthogonal. If the answer is "neither", determine the angle between the direction vector of the line and the normal vector of the plane.

a. $\boldsymbol{x}=(3,0,4)+t(-1,1,1),\ t\in\mathbb{R}$

b. $\boldsymbol{x}=(1,1,2)+t(-2,1,-3),\ t\in\mathbb{R}$

c. $\boldsymbol{x}=(3,0,0)+t(1,1,2),\ t\in\mathbb{R}$

d. $\boldsymbol{x}=(-1,-1,2)+t(4,-2,6),\ t\in\mathbb{R}$

e. $\boldsymbol{x}=t(0,3,1),\ t\in\mathbb{R}$

5.5 [**SM, YL**] Use a projection (onto or perpendicular to) to find the distance from the point to the plane. And find the closest point on the plane to the given point.

a. point $(-1,-1,1)$, plane $2x_1-x_2-x_3=4$

b. point $(0,2,-1)$, plane $2x_1-x_3=5$

c. point $(2,3,1)$, plane $3x_1-x_2+4x_3=5$

d. point $(-2,3,-1)$, plane $2x_1-3x_2-5x_3=5$

5.6 [**SM**] Determine a vector equation of the line of intersection of the given planes.

a. $x+3y-z=5$ and $2x-5y+z=7$

b. $2x-3z=7$ and $y+2z=4$

5.7 [**SM**] In each case, determine whether the given pair of lines has a point of intersection; if so, determine the scalar equation of the plane containing the lines, and if not, determine the distance between the lines.

a. $\boldsymbol{x}=(1,3,1)+s(-2,-1,1)$ and $\boldsymbol{x}=(0,1,4)+t(3,0,1)$, $s,t\in\mathbb{R}$

b. $\boldsymbol{x}=(1,3,1)+s(-2,-1,1)$ and $\boldsymbol{x}=(0,1,7)+t(3,0,1)$, $s,t\in\mathbb{R}$

c. $\boldsymbol{x}=(2,1,4)+s(2,1,-2)$ and $\boldsymbol{x}=(-2,1,5)+t(1,3,1)$, $s,t\in\mathbb{R}$

d. $\boldsymbol{x}=(0,1,3)+s(1,-1,4)$ and $\boldsymbol{x}=(0,-1,5)+t(1,1,2)$, $s,t\in\mathbb{R}$

5.8 [**MB**] Given the following: parametric equations of two skew lines L_1 and L_2, two nonparallel planes P_1 and P_2, coordinates of the point Q, that does not lie on L_1, L_2, P_1, nor P_2.

a. Does a line that is perpendicular to L_1 and passes through Q exist? If so, is it unique? How would the equation of such a line (if it exist), be obtained with the provided information?

b. Does a line parallel to the intersection of the planes P_1 and P_2 exist? If so, is it unique? How would the equation such a line (if it exist), be obtained with the provided information?

c. Does a plane containing L_1 and L_2 exists? Is so, is it unique? How would the equation of such a plane (if it exists), be obtained with the provided information?

6 Answers to Exercises

Note that either a hint, a final answer or a complete solution is provided.

1.1 $X=\begin{bmatrix}-4 & 2\\ 1 & -4\end{bmatrix}$

1.2

a. $X=\begin{bmatrix}-8 & -2\\ -19 & -6\end{bmatrix}$

b. $X=\begin{bmatrix}9 & 3\\ 12 & 3\end{bmatrix}$

2.1

a. $\operatorname{adj}(A)=\begin{bmatrix}3 & -4 & 5\\ -6 & 3 & -5\\ 5 & -5 & 5\end{bmatrix}$

b. $\det(A)=-5$

c. $M_{12}=6$ and $M_{22}=3$

d. $A^{-1}=\frac{1}{-5}\begin{bmatrix}3 & -6 & 5\\ -4 & 3 & -5\\ 5 & -5 & 5\end{bmatrix}$

2.2 375

2.3 $\det(M)=\frac{4}{3}$

2.4

a. Hint: take the determinant of BG.

b. $(10!)^9$

c. $\frac{(2(10!)+3)^{10}}{10!}$

d. $\pm\sqrt{10!}$

e. 0

f. $\frac{2^{10}\pi}{(96)^{10}(10!)^3}$

g. Hint: Show that $\det(H)(2+\det(A))^{10}\det(A)=0$ then conclude that $\det(H)=0$.

h. $\frac{(10+10!)(9+10!)\cdots(2+10!)(1+10!)}{(10!)^{10}}$

i. Hint: Show that $\operatorname{adj}(\operatorname{adj}(A))=(\det(A))^{n-2}A$.

3.1

a. $k = 6$
b. $k = 0$ or $k = 3$
c. $k = -3$
d. any $k \in \mathbb{R}$

3.2

a. 3
b. $\sqrt{19}$

3.3 Expand the left side of the equation by using the fact that $\|\boldsymbol{v}\|^2 = \boldsymbol{v} \cdot \boldsymbol{v}$ for any vector $\boldsymbol{v}$ to get to the right side.

3.4 Where $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^n$, the vectors $\boldsymbol{u} + \boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$ are perpendicular if and only if $0 = (\boldsymbol{u} + \boldsymbol{v}) \cdot (\boldsymbol{u} - \boldsymbol{v}) = \boldsymbol{u} \cdot \boldsymbol{u} - \boldsymbol{v} \cdot \boldsymbol{v}$, which shows that those two are perpendicular if and only if $\boldsymbol{u} \cdot \boldsymbol{u} = \boldsymbol{v} \cdot \boldsymbol{v}$. That holds if and only if $\|\boldsymbol{u}\| = \|\boldsymbol{v}\|$.

4.1

a. vector: $\boldsymbol{x} = (2, -4, 1) + t(9, 2, 5)$
parametric: $x = 2 + 9t$, $y = -4 + 2t$, $z = 1 + 5t$

b. vector: $\boldsymbol{x} = (6, 1, 7) + t(-3, 2, 5)$
parametric: $x = 6 - 3t$, $y = 1 + 2t$, $z = 7 + 5t$

c. Answers can vary: vector: $\boldsymbol{x} = (2, 1, 5) + t(5, -3, -1)$
parametric: $x = 2 + 5t$, $y = 1 - 3t$, $z = 5 - t$

d. Answers can vary: vector: $\boldsymbol{x} = (1, -2, 3) + t(4, 7, 2)$
parametric: $x = 1 + 4t$, $y = -2 + 7t$, $z = 3 + 2t$

e. Answers can vary; here the direction is given by $\boldsymbol{d}_1 \times \boldsymbol{d}_2$:
vector: $\boldsymbol{x} = (0, 1, 2) + t(-10, 43, 9)$
parametric: $x = -10t$, $y = 1 + 43t$, $z = 2 + 9t$

f. Answers can vary; here the direction is given by $\boldsymbol{d}_1 \times \boldsymbol{d}_2$:
vector: $\boldsymbol{x} = (5, 1, 9) + t(0, -1, 0)$
parametric: $x = 5$, $y = 1 - t$, $z = 9$

g. Answers can vary; here the direction is given by $\boldsymbol{d}_1 \times \boldsymbol{d}_2$:
vector: $\boldsymbol{x} = (7, 2, -1) + t(1, -1, 2)$
parametric: $x = 7 + t$, $y = 2 - t$, $z = -1 + 2t$

h. Answers can vary; here the direction is given by $\boldsymbol{d}_1 \times \boldsymbol{d}_2$:
vector: $\boldsymbol{x} = (2, 2, 3) + t(5, -1, -3)$
parametric: $x = 2 + 5t$, $y = 2 - t$, $z = 3 - 3t$

i. vector: $\boldsymbol{x} = (1, 1) + t(2, 3)$
parametric: $x = 1 + 2t$, $y = 1 + 3t$

j. vector: $\boldsymbol{x} = (-2, 5) + t(0, 1)$
parametric: $x = -2$, $y = 5 + t$

4.2

a. Parallel.
b. Intersecting; $(12, 3, 7)$.
c. Intersecting; $(9, -5, 13)$.
d. Skew.

4.3

a. $\left(\frac{5}{2}, \frac{5}{2}\right)$, $\frac{5}{\sqrt{2}}$
b. $\left(\frac{58}{17}, \frac{91}{17}\right)$, $\frac{6}{\sqrt{17}}$
c. $\left(\frac{17}{6}, \frac{1}{3}, -\frac{1}{6}\right)$, $\sqrt{\frac{29}{6}}$
d. $\left(\frac{5}{3}, \frac{11}{3}, -\frac{1}{3}\right)$, $\sqrt{6}$

4.4

a. $3/\sqrt{2}$
b. 2

4.5

a. Skew lines. Point on first line: $\left(\frac{5}{3},\ \frac{7}{3},\ 2\right)$, Point on second line: $\left(3,\ -\frac{4}{3},\ \frac{5}{3}\right)$, unique closest points, shortest distance between the two lines: $\sqrt{\frac{46}{3}}$.

b. The two lines intersect. $(5, -7, 1)$ is the point of intersection, hence closest to the other line. And the shortest distance is 0.

c. Parallel lines. Point on first line: $\left(\frac{21}{17},\ -\frac{3}{17},\ \frac{31}{17}\right)$, Point on second line: $(1,\ 0,\ 2)$, not unique. And the shortest distance is $\sqrt{\frac{2}{17}}$.

d. Parallel lines. Point on first line: $\left(-\frac{3}{5},\ -\frac{1}{5}\right)$, Point on second line: $\left((0,\ -\frac{1}{2}\right))$, not unique. And the shortest distance is $\frac{3}{2}\sqrt{\frac{1}{5}}$.

4.6

a. The two lines intersect.
b. The lines intersect at $(-5,\ 2,\ 3)$ and the distance is 0.

5.1

a. Standard form: $3(x-2) - (y-3) + 7(z-4) = 0$
general form: $3x - y + 7z = 31$

b. Standard form: $2(y-3) + 4(z-5) = 0$
general form: $2y + 4z = 26$

c. Answers may vary;
Standard form: $8(x-1) + 4(y-2) - 4(z-3) = 0$
general form: $8x + 4y - 4z = 4$

d. Answers may vary;
Standard form: $-5(x-5) + 3(y-3) + 2(z-8) = 0$
general form: $-5x + 3y + 2z = 0$

e. Answers may vary;
Standard form: $-7(x-2) + 2(y-1) + (z-2) = 0$
general form: $-7x + 2y + z = -10$

f. Answers may vary;
Standard form: $3(x-5) + 3(z-3) = 0$
general form: $3x + 3z = 24$

g. Answers may vary;
Standard form: $2(x-1) - (y-1) = 0$
general form: $2x - y = 1$

h. Answers may vary;
Standard form: $2(x-1)+(y-1)-3(z-1)=0$
general form: $2x+y-3z=0$

i. Answers may vary;
Standard form: $2(x-2)-(y+6)-4(z-1)=0$
general form: $2x-y-4z=6$

j. Answers may vary;
Standard form: $4(x-5)-2(y-7)-2(z-3)=0$
general form: $4x-2y-2z=0$

k. Answers may vary;
Standard form: $(x-5)+(y-7)+(z-3)=0$
general form: $x+y+z=15$

l. Answers may vary;
Standard form: $4(x-4)+(y-1)+(z-1)=0$
general form: $4x+y+z=18$

m. Answers may vary;
Standard form: $3(x+4)+8(y-7)-10(z-2)=0$
general form: $3x+8y-10z=24$

n. Standard form: $x-1=0$
general form: $x=1$

5.2

a. $x-4y-10z=-85$
b. $2x-2y+3z=-5$
c. $-5x-2y+6z=15$

5.3

a. No point of intersection; the plane and line are parallel.
b. The plane contains the line, so every point on the line is a "point of intersection."
c. $(-3,-7,-5)$
d. $(3,1,1)$

5.4

a. The line is parallel to the plane.
b. The line is orthogonal to the plane.
c. The line is neither parallel nor orthogonal to the plane, $\theta \approx 0.702$ radians.
d. The line is orthogonal to the plane.
e. The line is parallel to the plane.

5.5

a. $\sqrt{6}$, $(1,-2,0)$
b. $\frac{4}{\sqrt{5}}$, $\left(\frac{8}{5},2,-\frac{9}{5}\right)$
c. $\frac{2}{\sqrt{26}}$, $\left(\frac{23}{13},\frac{40}{13},\frac{9}{13}\right)$
d. $\frac{13}{\sqrt{38}}$, $\left(-\frac{25}{19},\frac{75}{38},-\frac{103}{38}\right)$

5.6

a. $\boldsymbol{x}=\left(\frac{46}{11},\frac{3}{11},0\right)+t(-2,-3,-11)$, $t\in\mathbb{R}$
b. $\boldsymbol{x}=\left(\frac{7}{2},4,0\right)+t(3,-4,2)$, $t\in\mathbb{R}$

5.7

a. Point of intersection: $(-3,1,3)$, $-x+5y+3z=17$
b. No point of intersection, $\frac{9}{\sqrt{35}}$
c. No point of intersection, $\frac{23}{3\sqrt{10}}$
d. Point of intersection $(1,0,7)$, $3x-y-z=-4$

5.8

a. Such a line exists, but it is not unique. One of such line is obtained by finding a vector $\boldsymbol{d}$ perpendicular to the direction vector of L_1, then the equation of the line is given by $\boldsymbol{x}=\boldsymbol{Q}+t\boldsymbol{d}$ where $t\in\mathbb{R}$.
b. Such a line exists and is unique. The line can be obtained by finding a vector $\boldsymbol{d}$ parallel to the intersection of L_1 and L_2, then the equation of the line is given by $\boldsymbol{x}=\boldsymbol{Q}+t\boldsymbol{d}$ where $t\in\mathbb{R}$.
c. Such a plane does not exist since the lines are skew.

References

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