



<http://algs4.cs.princeton.edu>

## 5.2 TRIES

---

- ▶ *R-way tries*
- ▶ *ternary search tries*
- ▶ *character-based operations*

# Summary of the performance of symbol-table implementations

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Order of growth of the frequency of operations.

implementation	typical case			ordered operations	operations on keys
	search	insert	delete		
red-black BST	$\log N$	$\log N$	$\log N$	yes	<code>compareTo()</code>
hash table	$1^\dagger$	$1^\dagger$	$1^\dagger$	no	<code>equals()</code> <code>hashCode()</code>

$^\dagger$  under uniform hashing assumption

Q. Can we do better?

A. Yes, if we can avoid examining the entire key, as with string sorting.

# String symbol table basic API

---

**String symbol table.** Symbol table specialized to string keys.

```
public class StringST<Value>
```

```
    StringST()
```

*create an empty symbol table*

```
    void put(String key, Value val)
```

*put key-value pair into the symbol table*

```
    Value get(String key)
```

*return value paired with given key*

```
    void delete(String key)
```

*delete key and corresponding value*

```
    :
```

**Goal.** Faster than hashing, more flexible than BSTs.

# String symbol table implementations cost summary

implementation	character accesses (typical case)				dedup	
	search hit	search miss	insert	space (references)	moby.txt	actors.txt
red-black BST	$L + c \lg^2 N$	$c \lg^2 N$	$c \lg^2 N$	$4N$	1.40	97.4
hashing (linear probing)	$L$	$L$	$L$	$4N$ to $16N$	0.76	40.6

## Parameters

- $N$  = number of strings
- $L$  = length of string
- $R$  = radix

file	size	words	distinct
moby.txt	1.2 MB	210 K	32 K
actors.txt	82 MB	11.4 M	900 K

**Challenge.** Efficient performance for string keys.



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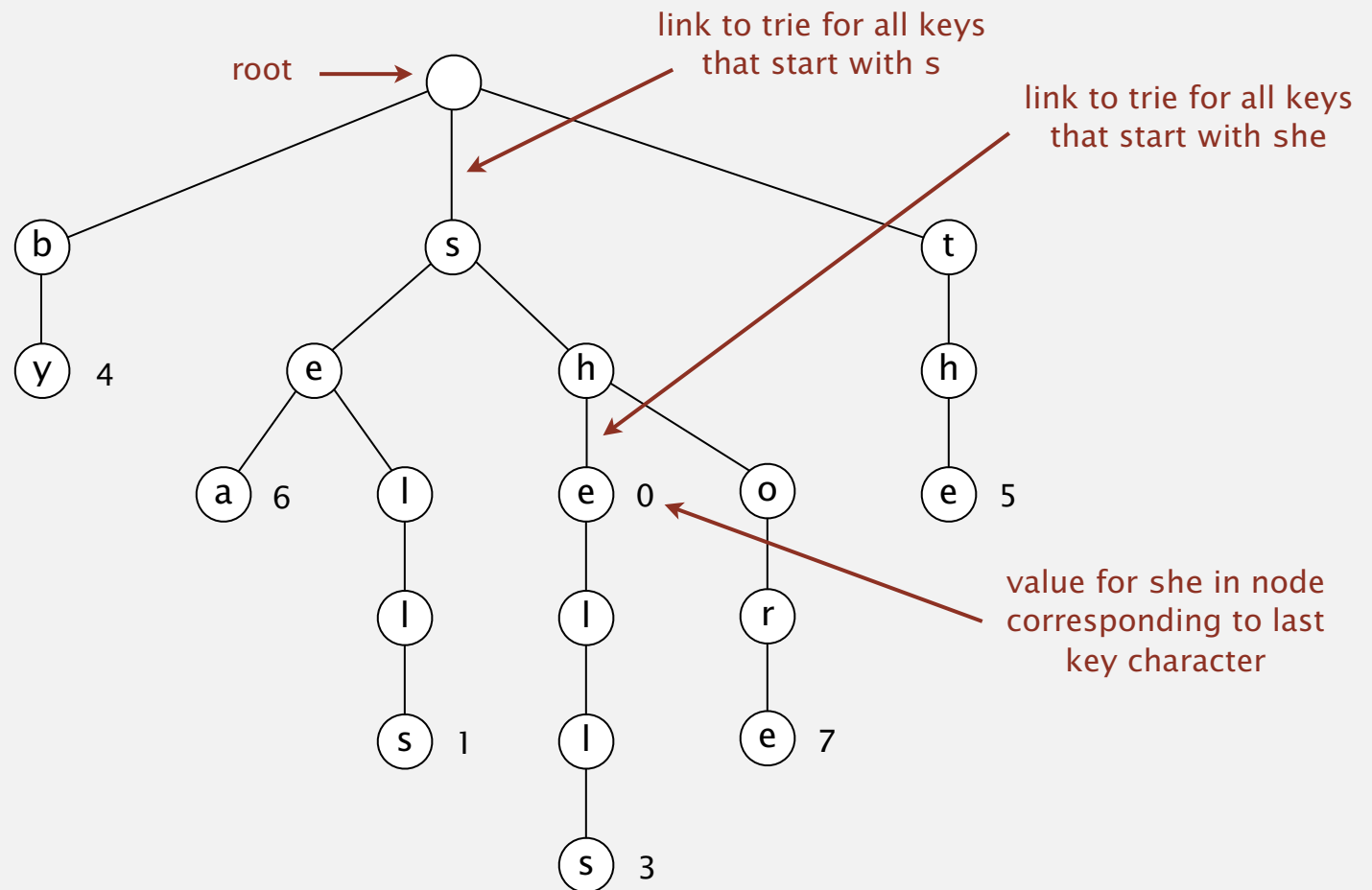
# Tries

**Tries.** [from **re**trieval, but pronounced "try"]

- Store characters in nodes (not keys).
- Each node has  $R$  children, one for each possible character.
- For now, we do not draw null links.

for now we do not  
draw null links

key	value
by	4
sea	6
sells	1
she	0
shells	3
shore	7
the	5

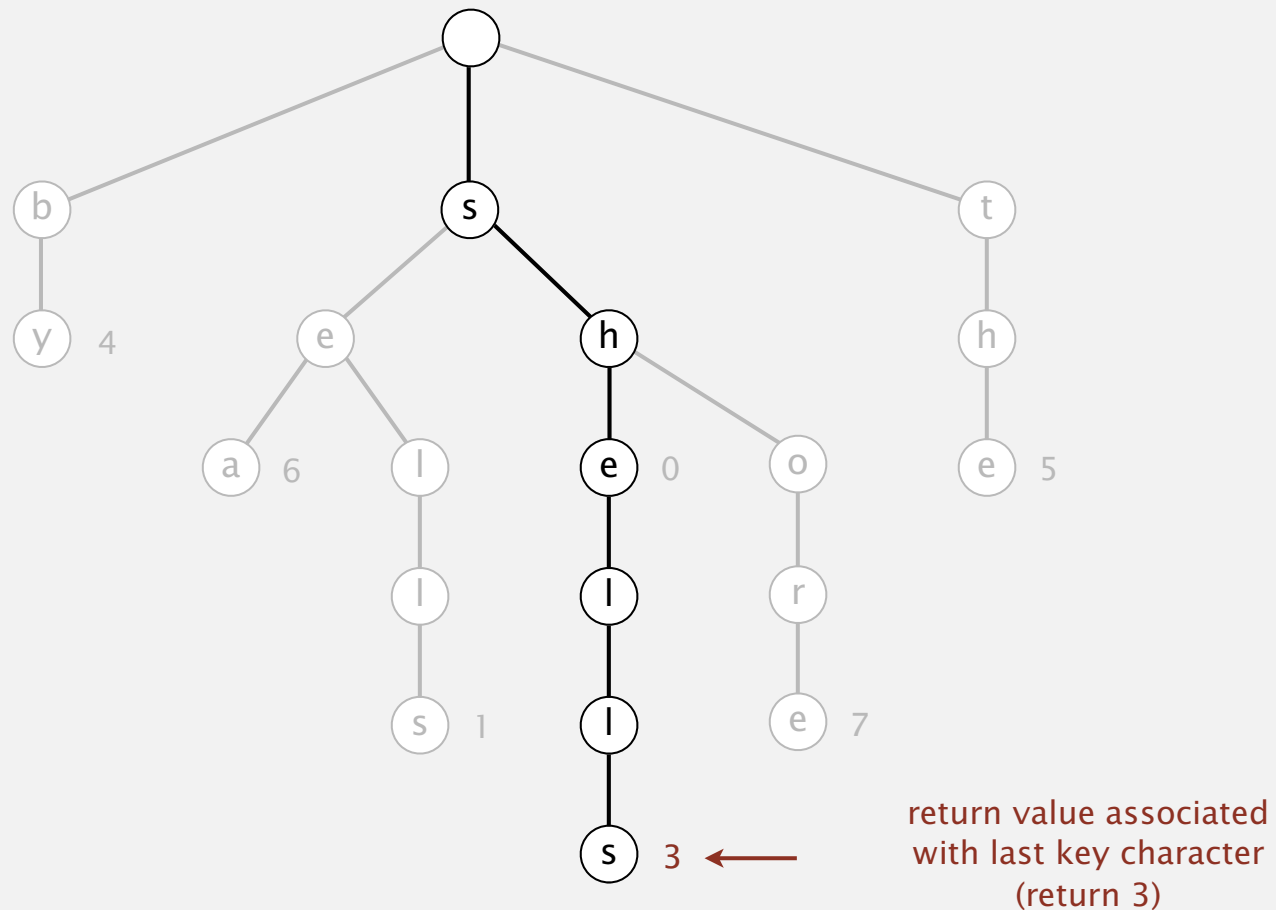


## Search in a trie

Follow links corresponding to each character in the key.

- **Search hit:** node where search ends has a non-null value.
- Search miss: reach null link or node where search ends has null value.

```
get("shells")
```

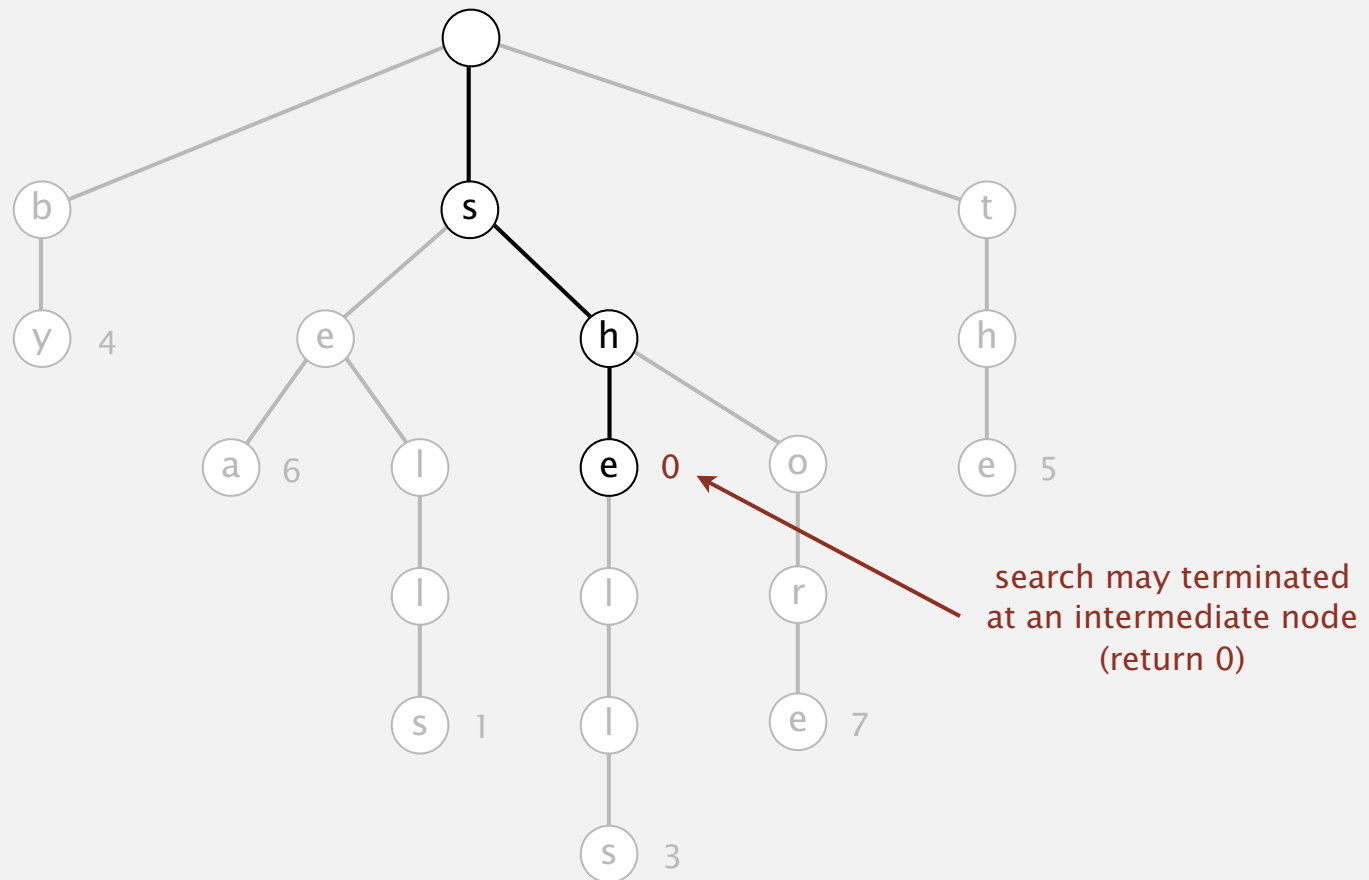


# Search in a trie

Follow links corresponding to each character in the key.

- **Search hit:** node where search ends has a non-null value.
- Search miss: reach null link or node where search ends has null value.

**get("she")**



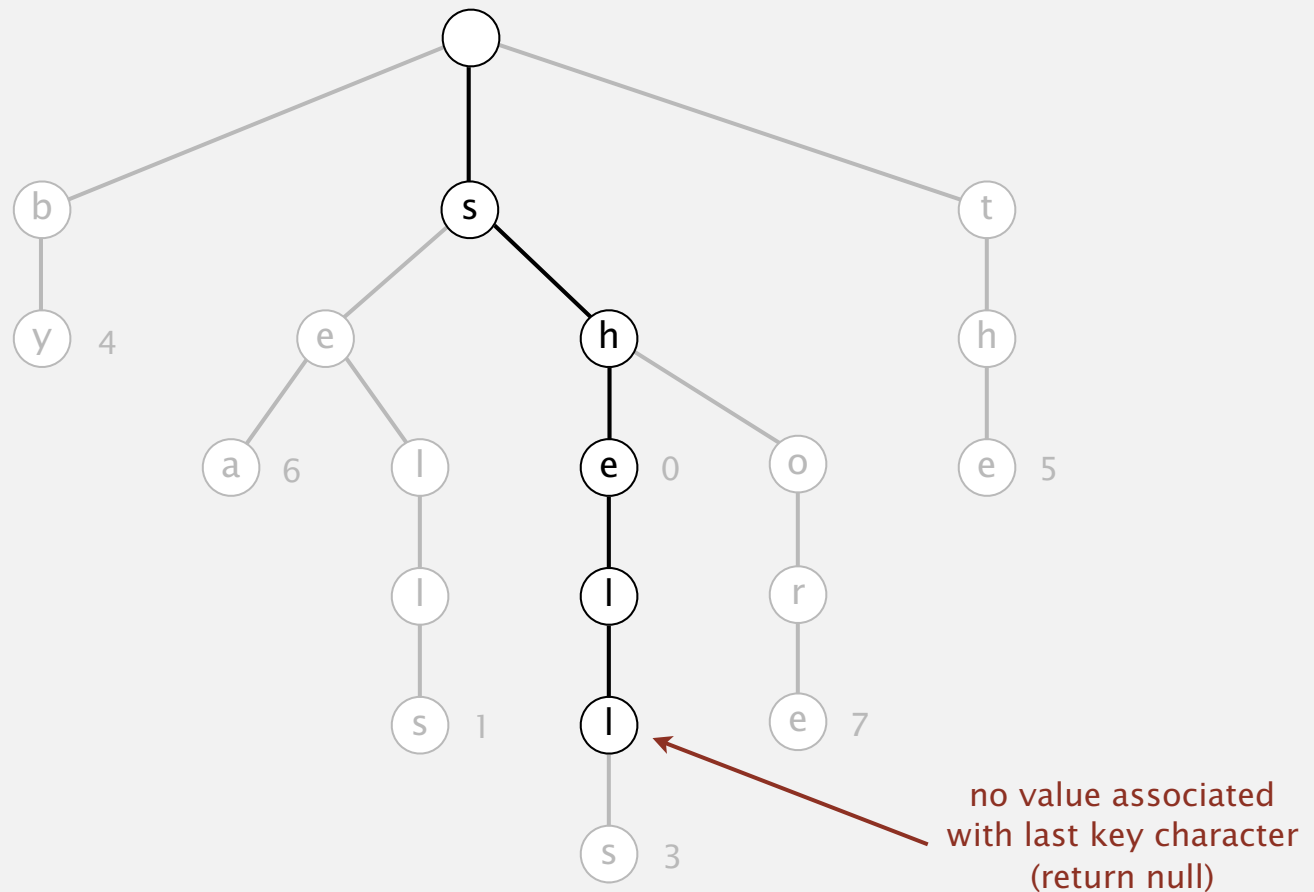


## Search in a trie

Follow links corresponding to each character in the key.

- Search hit: node where search ends has a non-null value.
- **Search miss:** reach null link or node where search ends has null value.

```
get("shell")
```

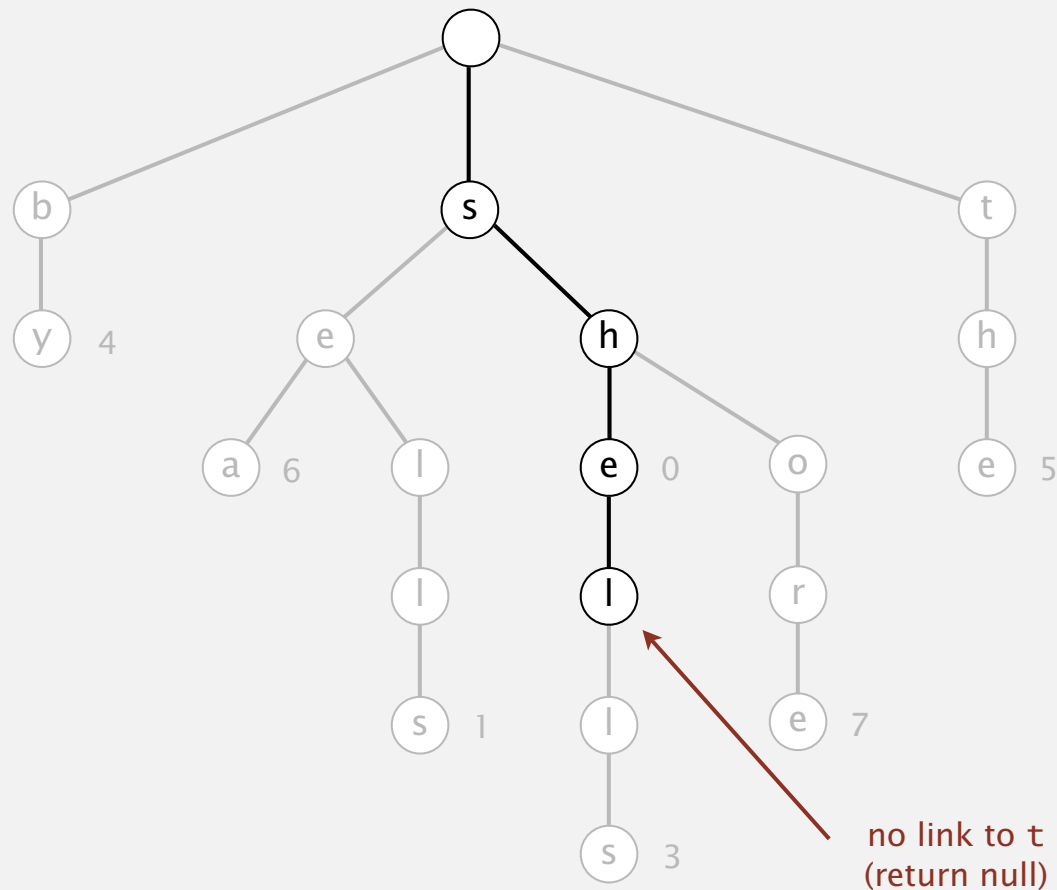


## Search in a trie

Follow links corresponding to each character in the key.

- Search hit: node where search ends has a non-null value.
- **Search miss:** reach null link or node where search ends has null value.

```
get("shelter")
```

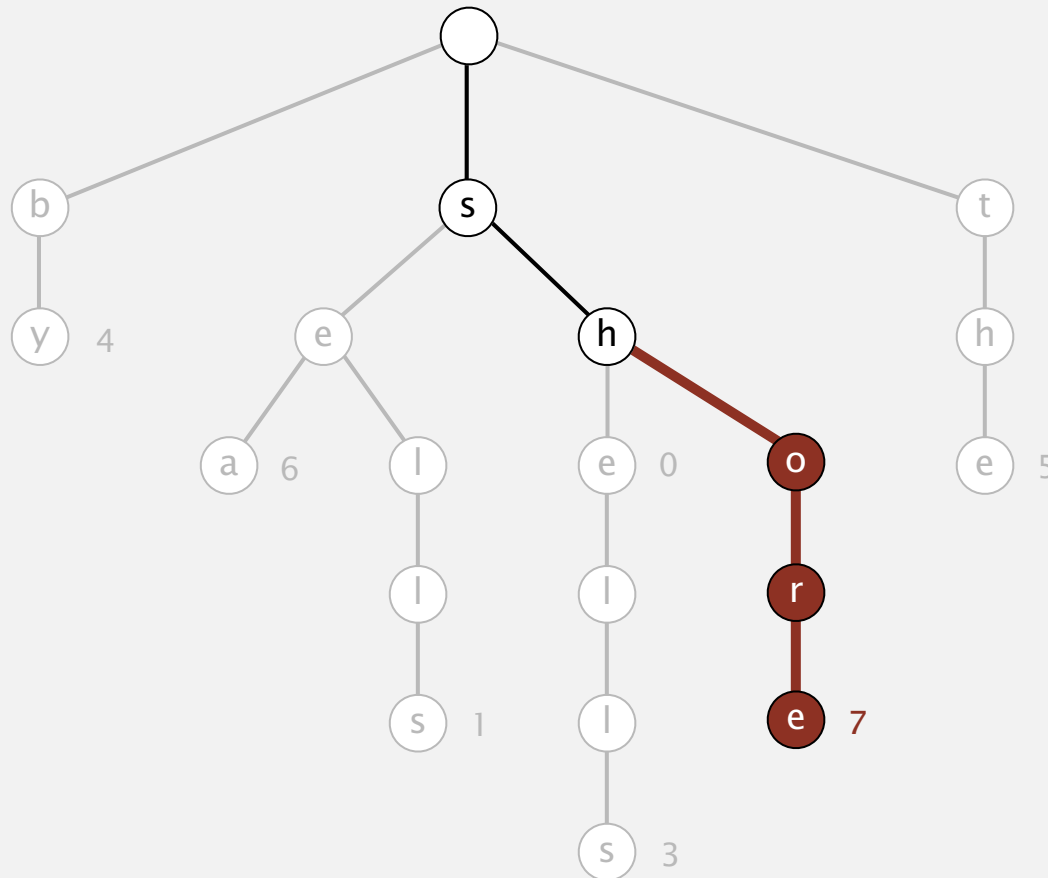


## Insertion into a trie

Follow links corresponding to each character in the key.

- Encounter a null link: create new node.
- Encounter the last character of the key: set value in that node.

**put("shore", 7)**



# Trie construction demo

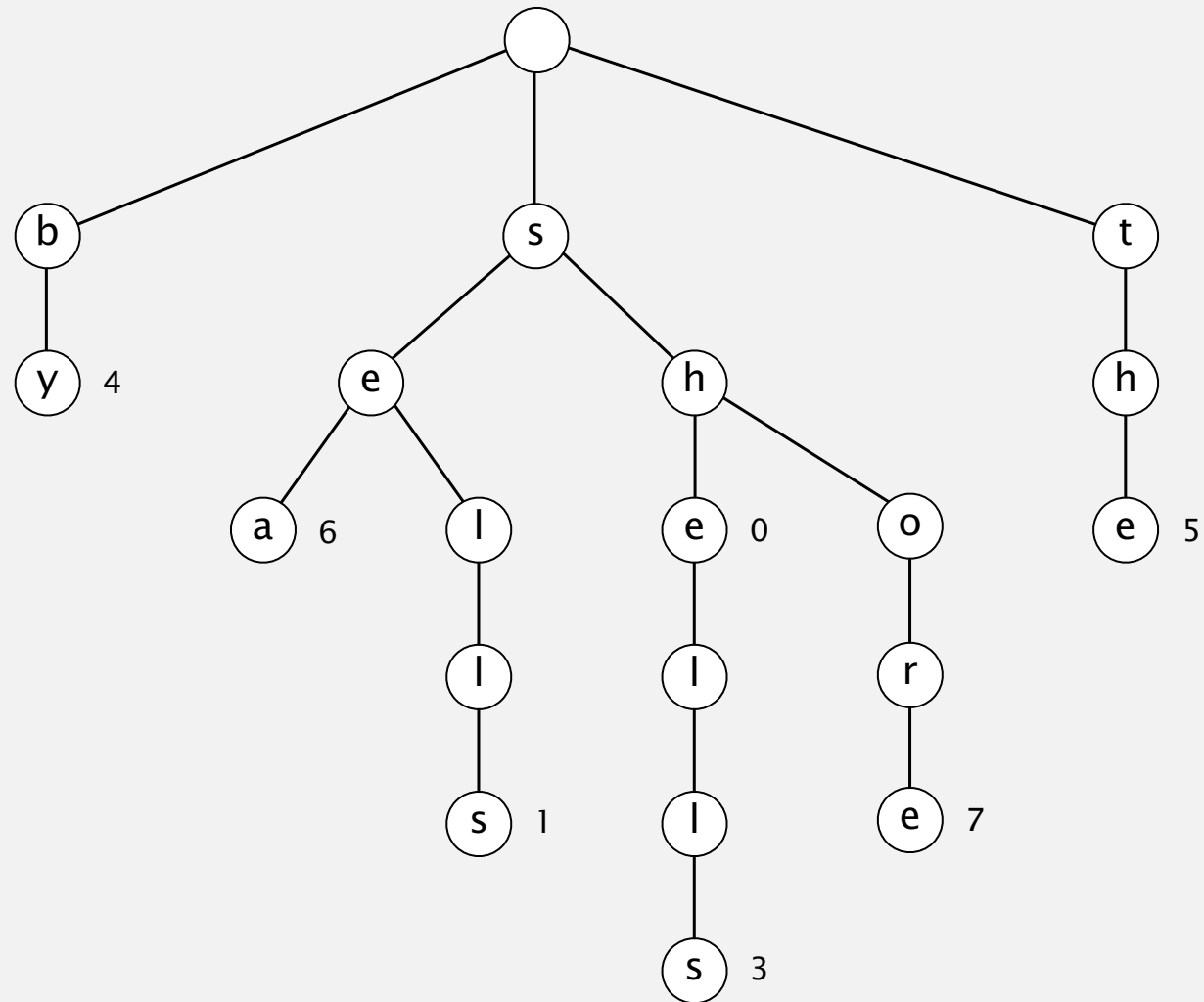
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trie



# Trie construction demo

## trie

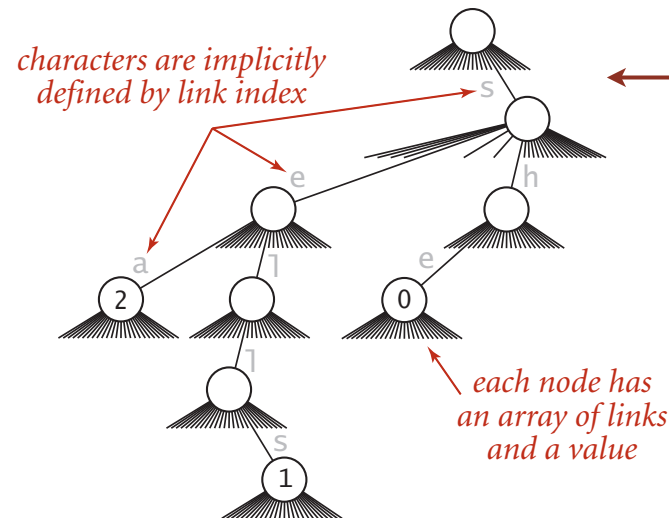
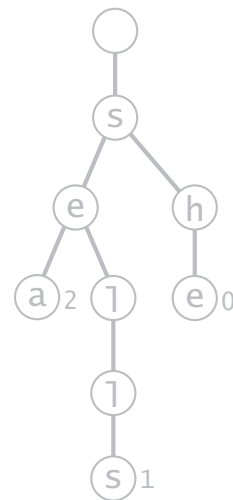


## Trie representation: Java implementation

**Node.** A value, plus references to  $R$  nodes.

```
private static class Node
{
    private Object value;
    private Node[] next = new Node[R];
}
```

use Object instead of Value since  
no generic array creation in Java



- neither keys nor characters are explicitly stored

- each node has an array of links and a value

## Trie representation

## R-way trie: Java implementation

---

```
public class TrieST<Value>
{
    private static final int R = 256;    ← extended ASCII
    private Node root = new Node();

    private static class Node
    { /* see previous slide */ }

    public void put(String key, Value val)
    { root = put(root, key, val, 0); }

    private Node put(Node x, String key, Value val, int d)
    {
        if (x == null) x = new Node();
        if (d == key.length()) { x.val = val; return x; }
        char c = key.charAt(d);
        x.next[c] = put(x.next[c], key, val, d+1);
        return x;
    }
}
```

:

## R-way trie: Java implementation (continued)

---

```
⋮  
public boolean contains(String key)  
{ return get(key) != null; }  
  
public Value get(String key)  
{  
    Node x = get(root, key, 0);  
    if (x == null) return null;  
    return (Value) x.val; ← cast needed  
}
```

```
private Node get(Node x, String key, int d)  
{  
    if (x == null) return null;  
    if (d == key.length()) return x;  
    char c = key.charAt(d);  
    return get(x.next[c], key, d+1);  
}
```

```
}
```



# Trie performance

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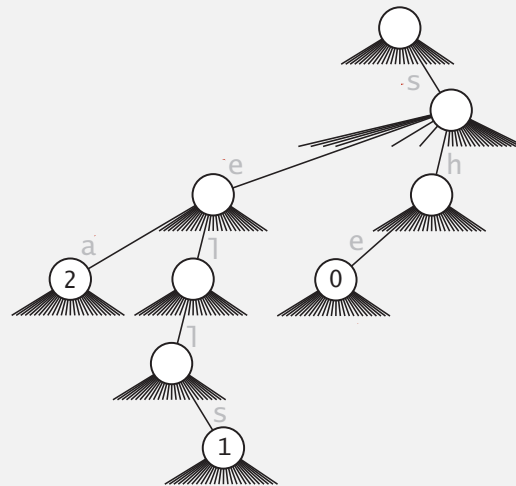
**Search hit.** Need to examine all  $L$  characters for equality.

**Search miss.**

- Could have mismatch on first character.
- Typical case: examine only a few characters (sublinear).

**Space.**  $R$  null links at each leaf.

(but sublinear space possible if many short strings share common prefixes)



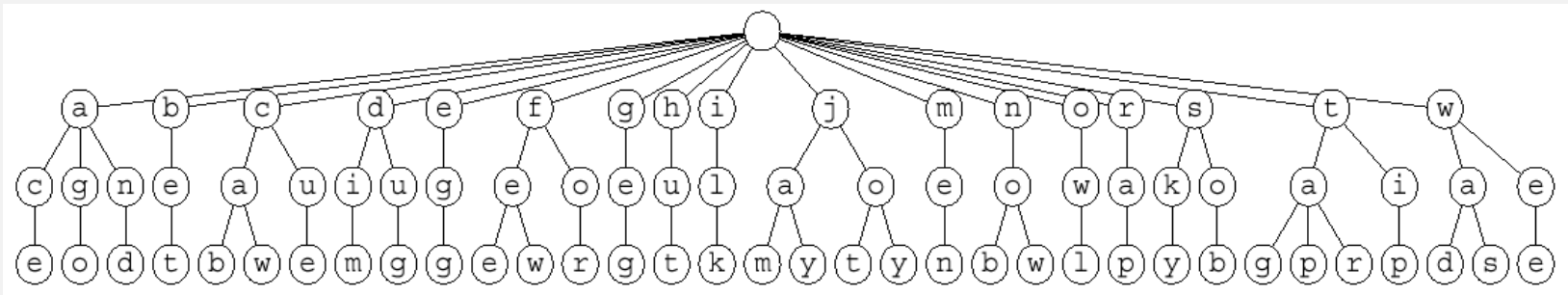
**Bottom line.** Fast search hit and even faster search miss, but wastes space.

# Popular interview question

**Goal.** Design a data structure to perform efficient spell checking.

**Solution.** Build a 26-way trie (key = word, value = bit).

English-language text



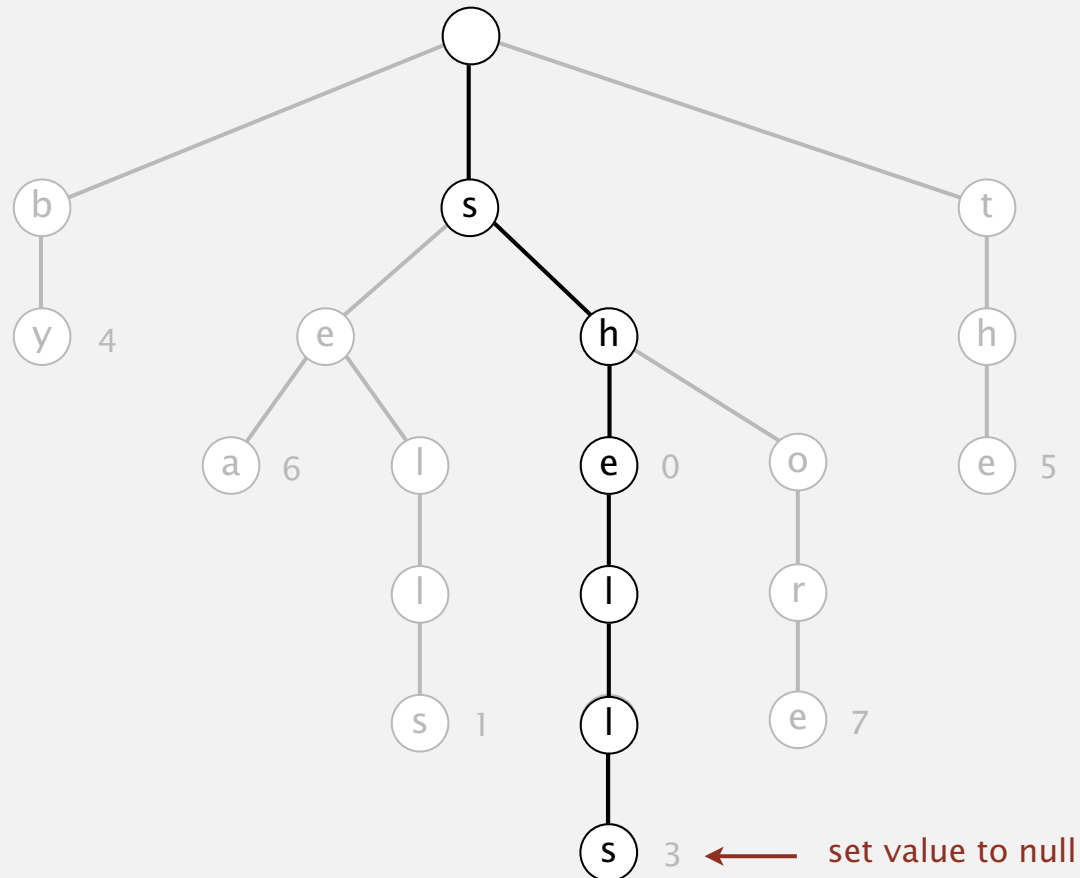
ace  
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jam  
jay  
jot  
joy  
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now  
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rap  
sky  
sob  
tag  
tap  
tar  
tip  
wad  
was  
wee

# Deletion in an R-way trie

To delete a key-value pair:

- Find the node corresponding to key and set value to null.
- If node has null value and all null links, remove that node (and recur).

**delete("shells")**

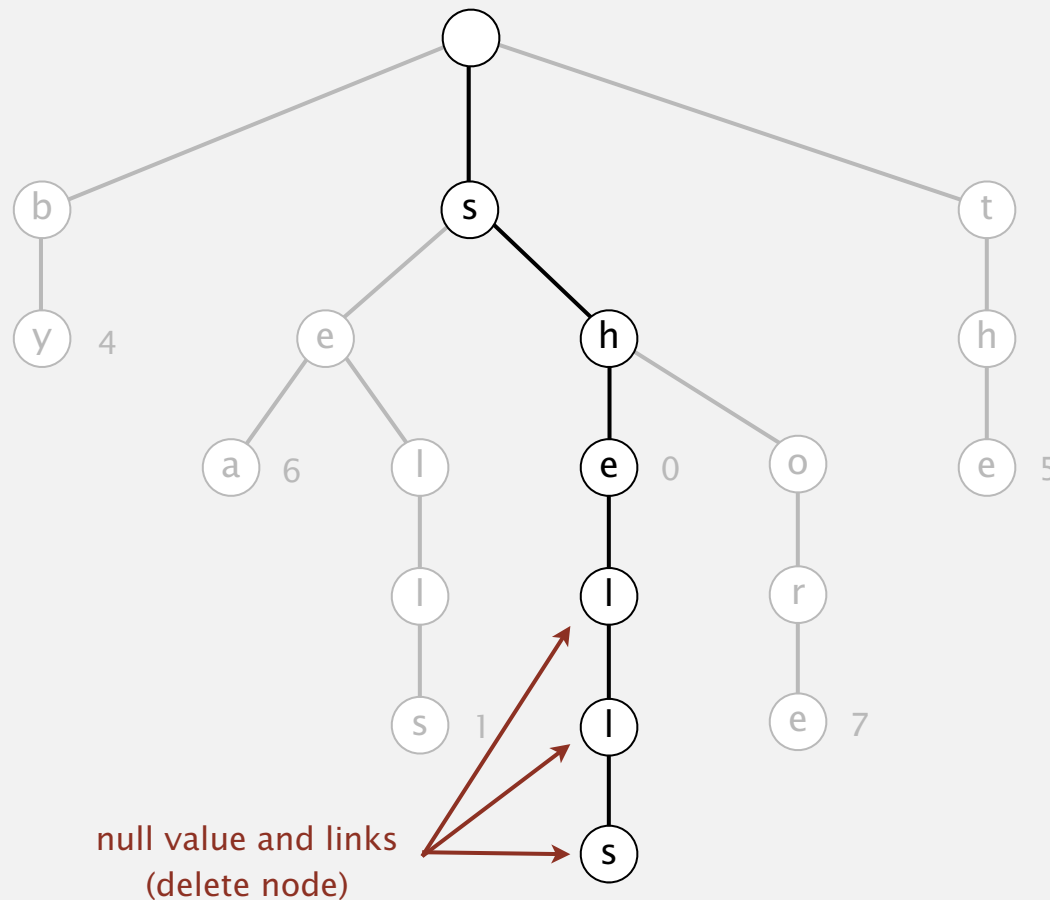


# Deletion in an R-way trie

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**delete("shells")**



# String symbol table implementations cost summary

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hashing (linear probing)	$L$	$L$	$L$	$4N$ to $16N$	0.76	40.6
R-way trie	$L$	$\log_R N$	$L$	$(R+1) N$	1.12	out of memory

## R-way trie.

- Method of choice for small  $R$ .
- Too much memory for large  $R$ .

**Challenge.** Use less memory, e.g., 65,536-way trie for Unicode!



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# Ternary search tries

---

- Store characters and values in nodes (not keys).
- Each node has 3 children: smaller (left), equal (middle), larger (right).

## Fast Algorithms for Sorting and Searching Strings

Jon L. Bentley\*

Robert Sedgewick#

### Abstract

We present theoretical algorithms for sorting and searching multikey data, and derive from them practical C implementations for applications in which keys are character strings. The sorting algorithm blends Quicksort and radix sort; it is competitive with the best known C sort codes. The searching algorithm blends tries and binary search trees; it is faster than hashing and other commonly used search methods. The basic ideas behind the algo-

that is competitive with the most efficient string sorting programs known. The second program is a symbol table implementation that is faster than hashing, which is commonly regarded as the fastest symbol table implementation. The symbol table implementation is much more space-efficient than multiway trees, and supports more advanced searches.

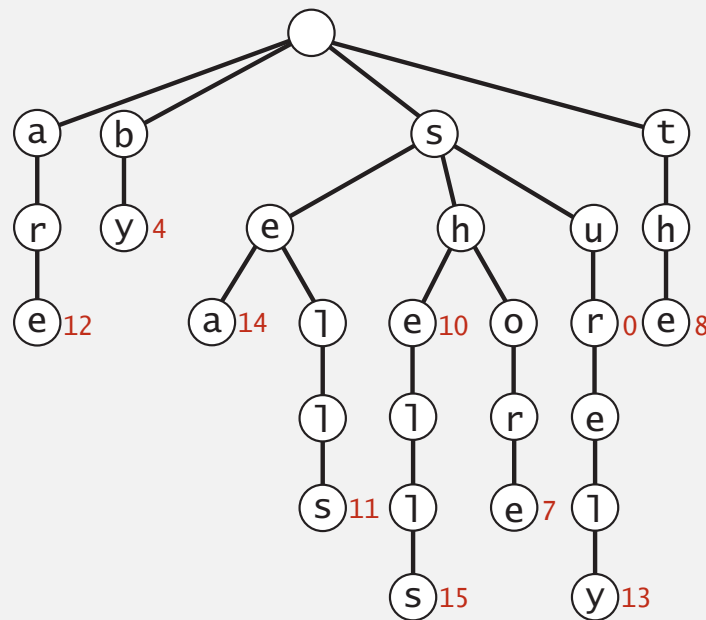
In many application programs, sorts use a Quicksort implementation based on an abstract compare operation,



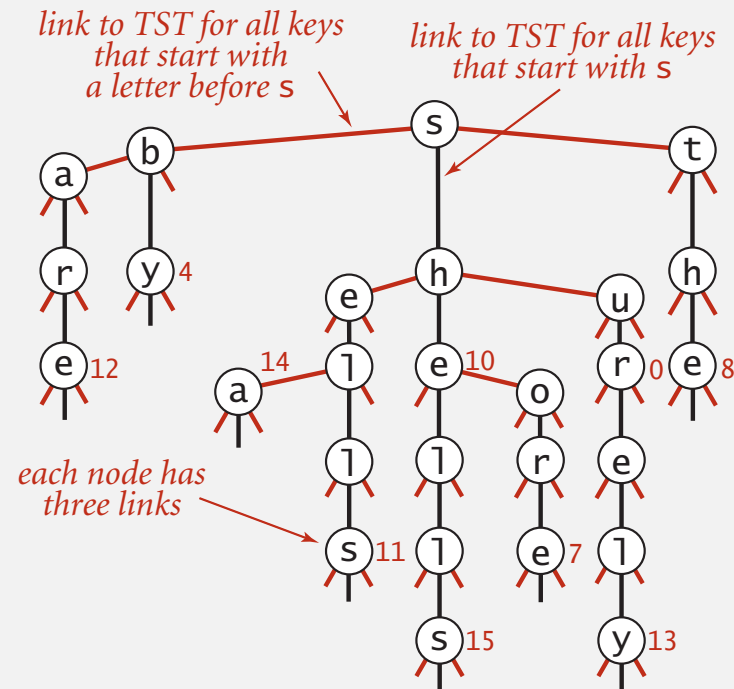


# Ternary search tries

- Store characters and values in nodes (not keys).
- Each node has **3** children: smaller (left), equal (middle), larger (right).



TST representation of a trie



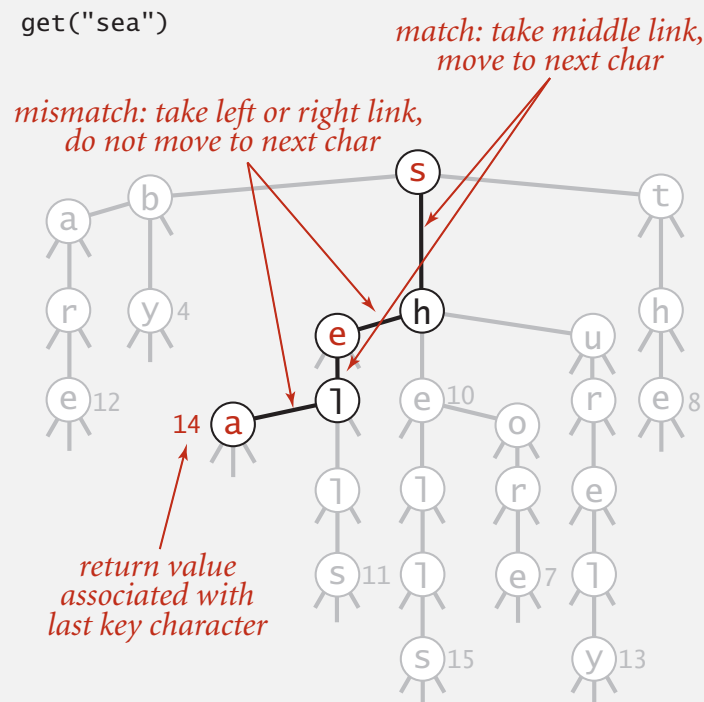
## Search in a TST

Follow links corresponding to each character in the key.

- If less, take left link; if greater, take right link.
- If equal, take the middle link and move to the next key character.

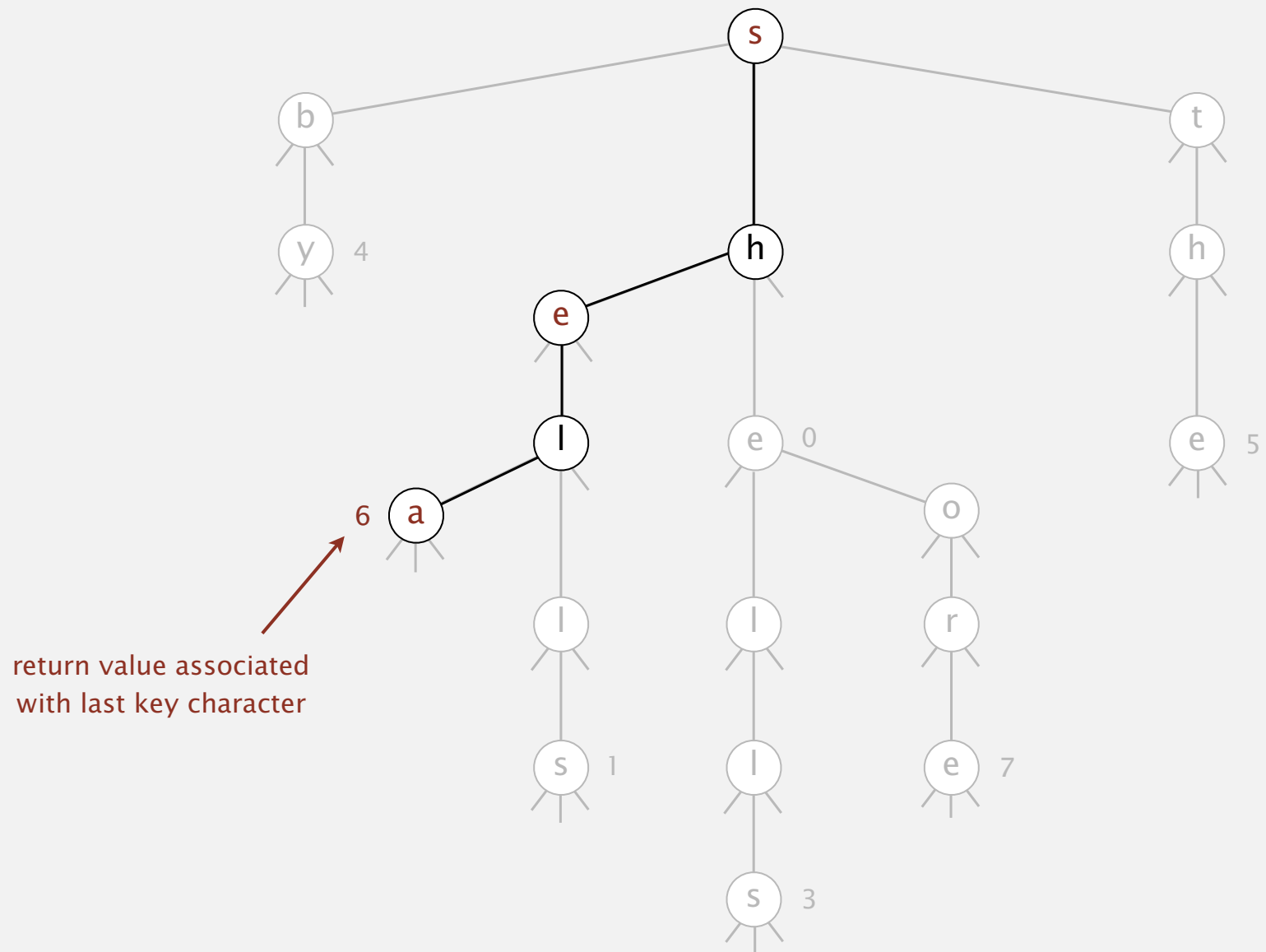
**Search hit.** Node where search ends has a non-null value.

**Search miss.** Reach a null link or node where search ends has null value.



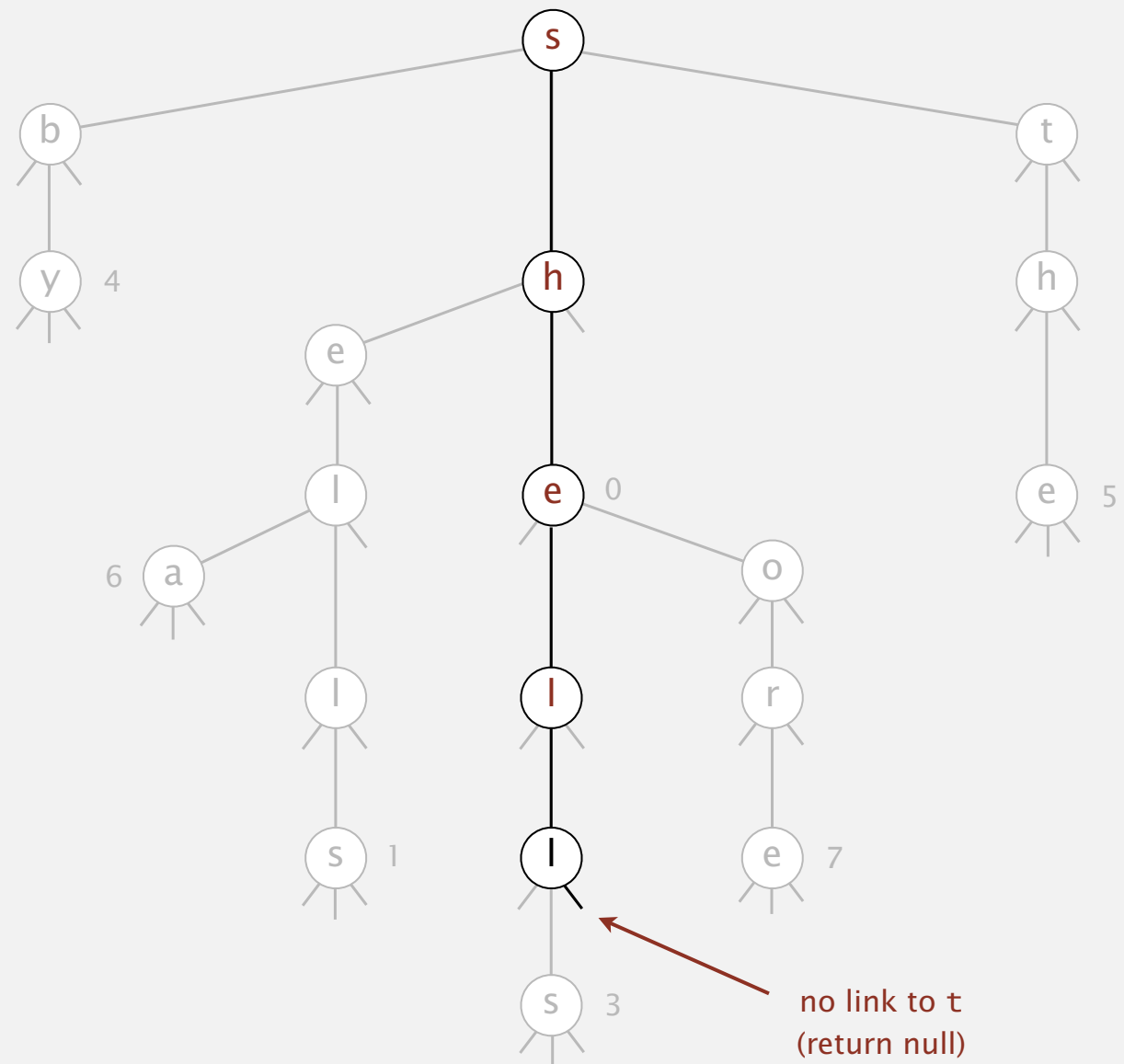
# Search hit in a TST

`get("sea")`



# Search miss in a TST

`get("shelter")`



# Ternary search trie construction demo

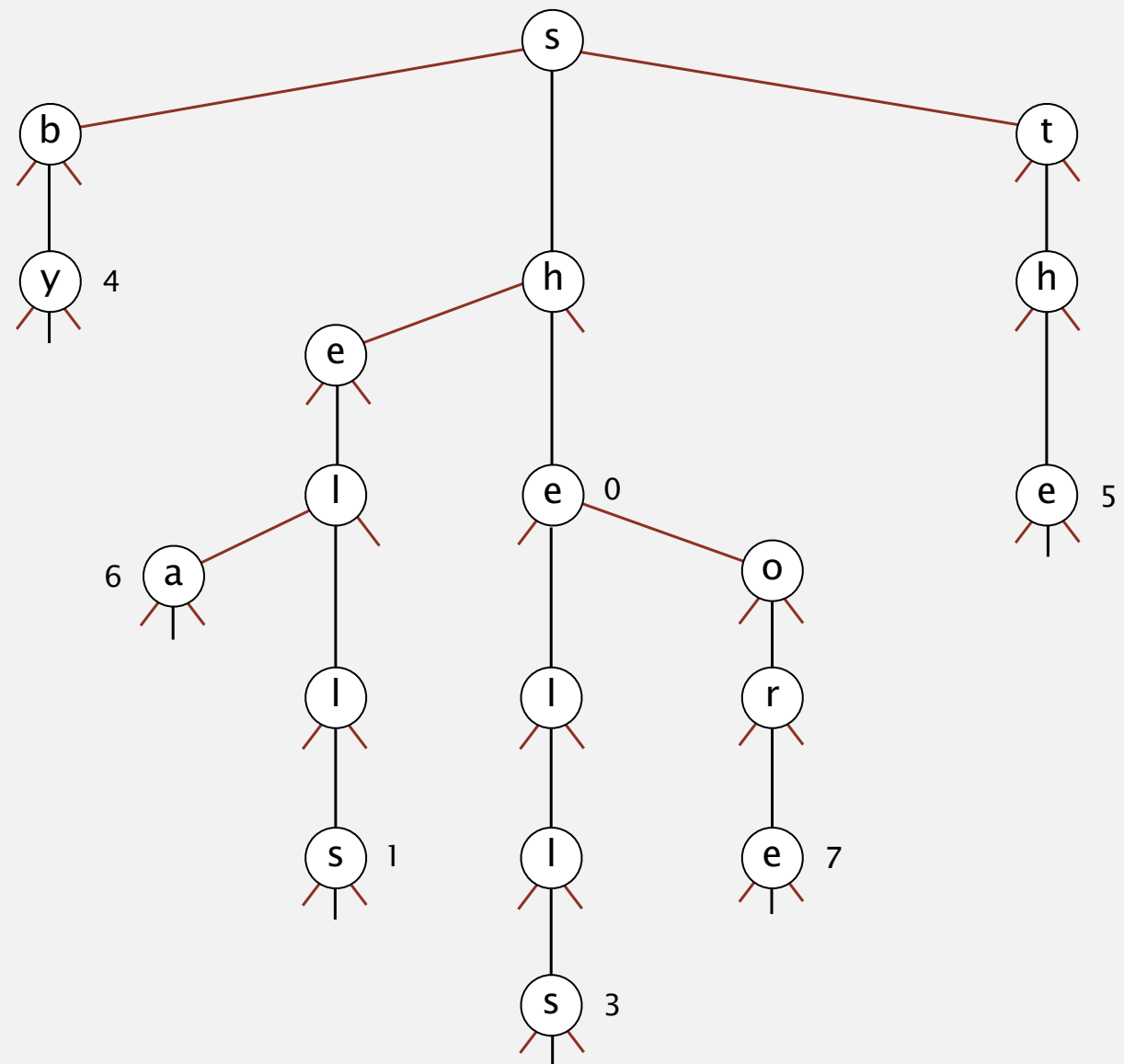
---

ternary search trie



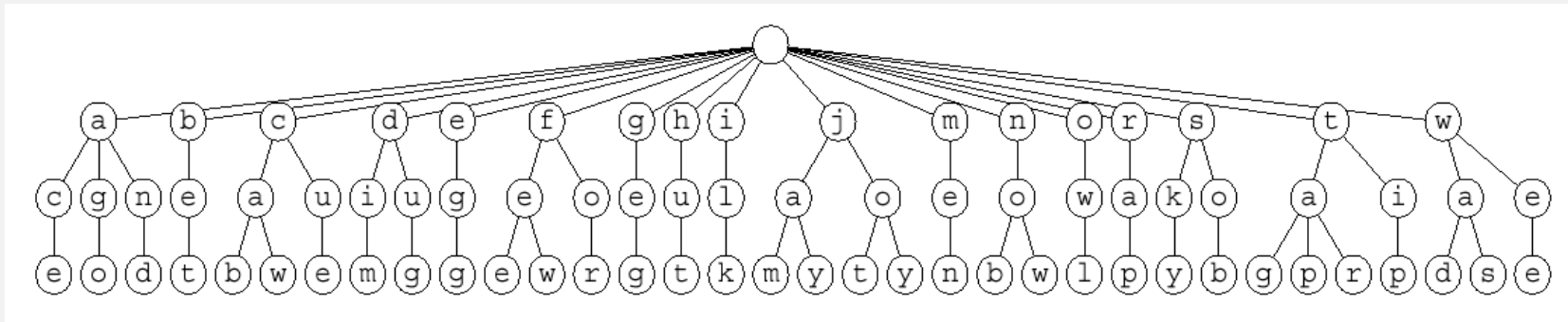
# Ternary search trie construction demo

ternary search trie



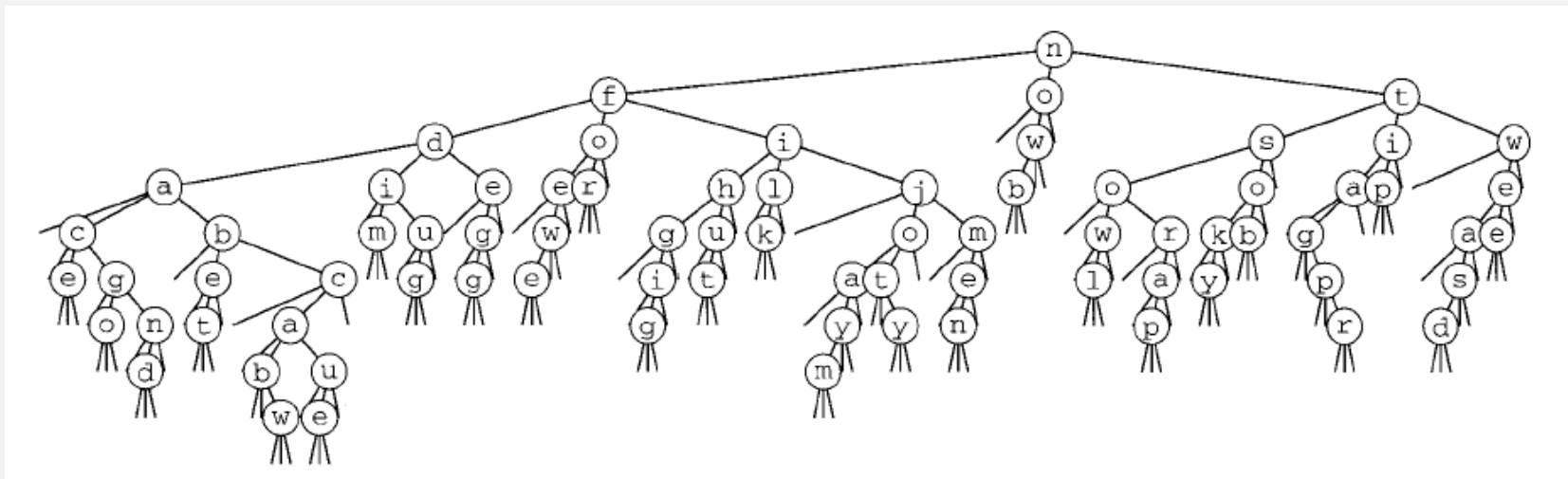
# 26-way trie vs. TST

26-way trie. 26 null links in each leaf.



26-way trie (1035 null links, not shown)

TST. 3 null links in each leaf.



TST (155 null links)

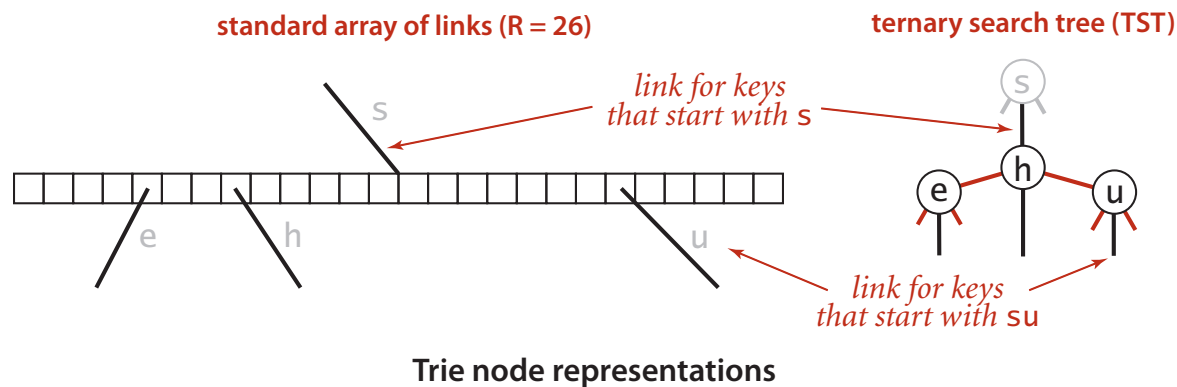
now  
for  
tip  
ilk  
dim  
tag  
jot  
sob  
nob  
sky  
hut  
ace  
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men  
egg  
few  
jay  
owl  
joy  
rap  
gig  
wee  
was  
cab  
wad  
caw  
cue  
fee  
tap  
ago  
tar  
jam  
dug  
and

# TST representation in Java

## A TST node is five fields:

- A value.
- A character  $c$ .
- A reference to a left TST.
- A reference to a middle TST.
- A reference to a right TST.

```
private class Node
{
    private Value val;
    private char c;
    private Node left, mid, right;
}
```





## TST: Java implementation

---

```
public class TST<Value>
{
    private Node root;

    private class Node
    { /* see previous slide */ }

    public void put(String key, Value val)
    { root = put(root, key, val, 0); }

    private Node put(Node x, String key, Value val, int d)
    {
        char c = key.charAt(d);
        if (x == null) { x = new Node(); x.c = c; }
        if (c < x.c) x.left = put(x.left, key, val, d);
        else if (c > x.c) x.right = put(x.right, key, val, d);
        else if (d < key.length() - 1) x.mid = put(x.mid, key, val, d+1);
        else x.val = val;
        return x;
    }
}
```

⋮

## TST: Java implementation (continued)

---

```
⋮  
public boolean contains(String key)  
{ return get(key) != null; }
```

```
public Value get(String key)  
{  
    Node x = get(root, key, 0);  
    if (x == null) return null;  
    return x.val;  
}
```

```
private Node get(Node x, String key, int d)  
{  
    if (x == null) return null;  
    char c = key.charAt(d);  
    if (c < x.c) return get(x.left, key, d);  
    else if (c > x.c) return get(x.right, key, d);  
    else if (d < key.length() - 1) return get(x.mid, key, d+1);  
    else return x;  
}
```

# String symbol table implementation cost summary

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R-way trie	$L$	$\log_R N$	$L$	$(R + 1) N$	1.12	out of memory
TST	$L + \ln N$	$\ln N$	$L + \ln N$	$4 N$	0.72	38.7

**Remark.** Can build balanced TSTs via rotations to achieve  $L + \log N$  worst-case guarantees.

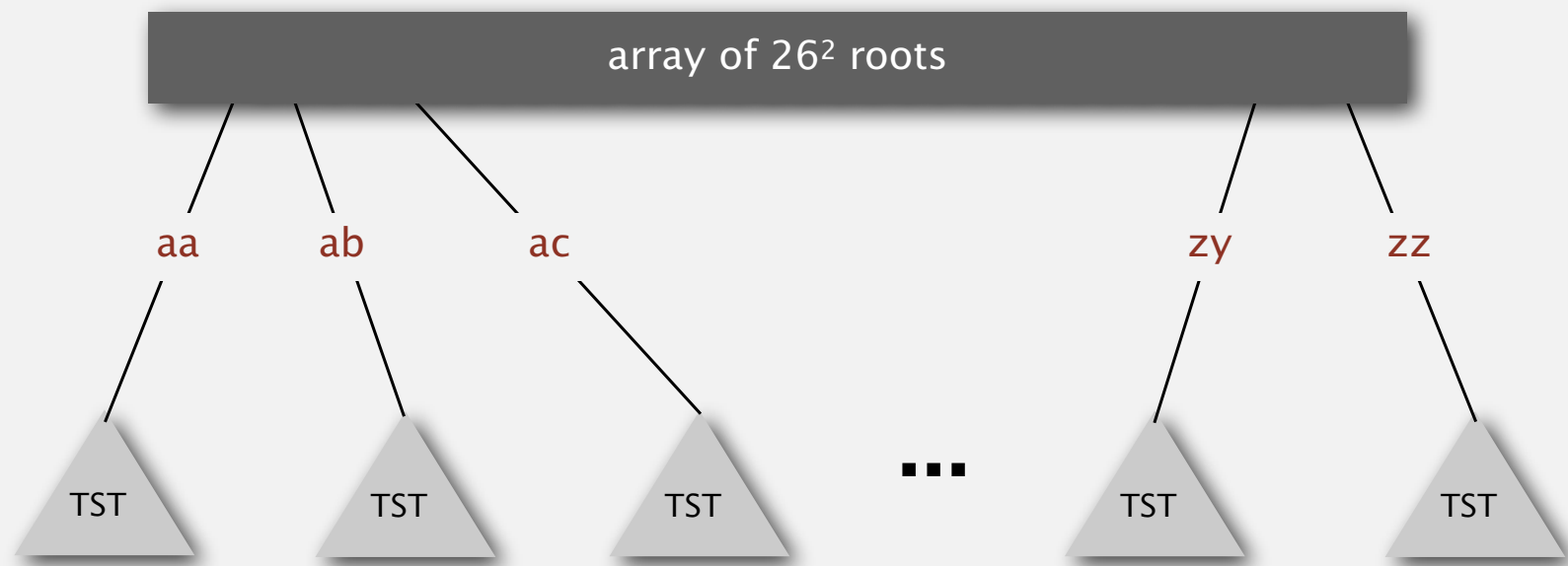
**Bottom line.** TST is as fast as hashing (for string keys), space efficient.

# TST with $R^2$ branching at root

---

## Hybrid of R-way trie and TST.

- Do  $R^2$ -way branching at root.
- Each of  $R^2$  root nodes points to a TST.



Q. What about one- and two-letter words?

# String symbol table implementation cost summary

implementation	character accesses (typical case)				dedup	
	search hit	search miss	insert	space (references)	moby.txt	actors.txt
red-black BST	$L + c \lg^2 N$	$c \lg^2 N$	$c \lg^2 N$	$4 N$	1.40	97.4
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R-way trie	$L$	$\log_R N$	$L$	$(R + 1) N$	1.12	out of memory
TST	$L + \ln N$	$\ln N$	$L + \ln N$	$4 N$	0.72	38.7
TST with $R^2$	$L + \ln N$	$\ln N$	$L + \ln N$	$4 N + R^2$	0.51	32.7

**Bottom line.** Faster than hashing for our benchmark client.

# TST vs. hashing

---

## Hashing.

- Need to examine entire key.
- Search hits and misses cost about the same.
- Performance relies on hash function.
- Does not support ordered symbol table operations.

## TSTs.

- Works only for strings (or digital keys).
- Only examines just enough key characters.
- Search miss may involve only a few characters.
- Supports ordered symbol table operations (plus others!).

## Bottom line. TSTs are:

- Faster than hashing (especially for search misses).
- More flexible than red-black BSTs. [stay tuned]



## 5.2 TRIES

---

- ▶ *R-way tries*
- ▶ *ternary search tries*
- ▶ *character-based operations*



## 5.2 TRIES

---

- ▶ *R-way tries*
- ▶ *ternary search tries*
- ▶ *character-based operations*



# String symbol table API

---

**Character-based operations.** The string symbol table API supports several useful character-based operations.

key	value
by	4
sea	6
sells	1
she	0
shells	3
shore	7
the	5

**Prefix match.** Keys with prefix sh: she, shells, and shore.

**Wildcard match.** Keys that match .he: she and the.

**Longest prefix.** Key that is the longest prefix of shellsort: shells.

# String symbol table API

---

```
public class StringST<Value>
```

```
    StringST()
```

*create a symbol table with string keys*

```
    void put(String key, Value val)
```

*put key-value pair into the symbol table*

```
    Value get(String key)
```

*value paired with key*

```
    void delete(String key)
```

*delete key and corresponding value*

```
    :
```

```
    Iterable<String> keys()
```

*all keys*

```
    Iterable<String> keysWithPrefix(String s)
```

*keys having s as a prefix*

```
    Iterable<String> keysThatMatch(String s)
```

*keys that match s (where . is a wildcard)*

```
    String longestPrefixOf(String s)
```

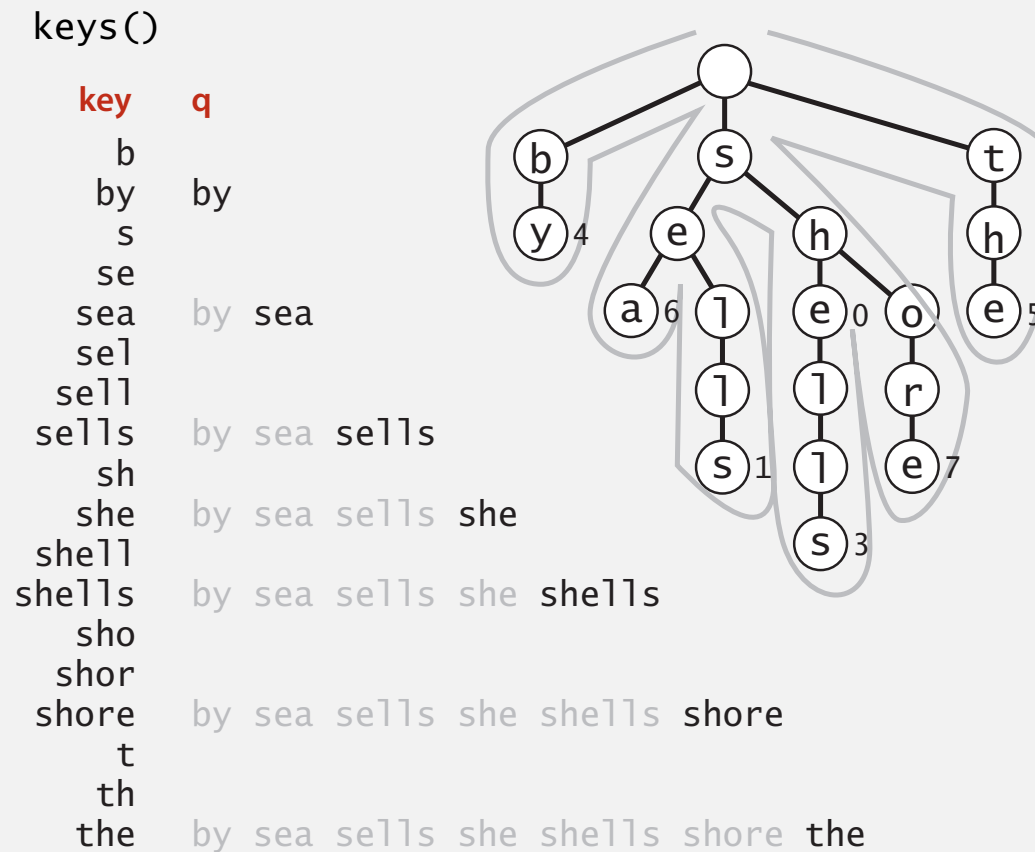
*longest key that is a prefix of s*

**Remark.** Can also add other ordered ST methods, e.g., `floor()` and `rank()`.

## Warmup: ordered iteration

To iterate through all keys in sorted order:

- Do inorder traversal of trie; add keys encountered to a queue.
- Maintain sequence of characters on path from root to node.



## Ordered iteration: Java implementation

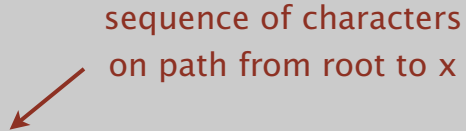
---

To iterate through all keys in sorted order:

- Do inorder traversal of trie; add keys encountered to a queue.
- Maintain sequence of characters on path from root to node.

```
public Iterable<String> keys()
{
    Queue<String> queue = new Queue<String>();
    collect(root, "", queue);
    return queue;
}

private void collect(Node x, String prefix, Queue<String> q)
{
    if (x == null) return;
    if (x.val != null) q.enqueue(prefix);
    for (char c = 0; c < R; c++)
        collect(x.next[c], prefix + c, q);
}
```



sequence of characters  
on path from root to x

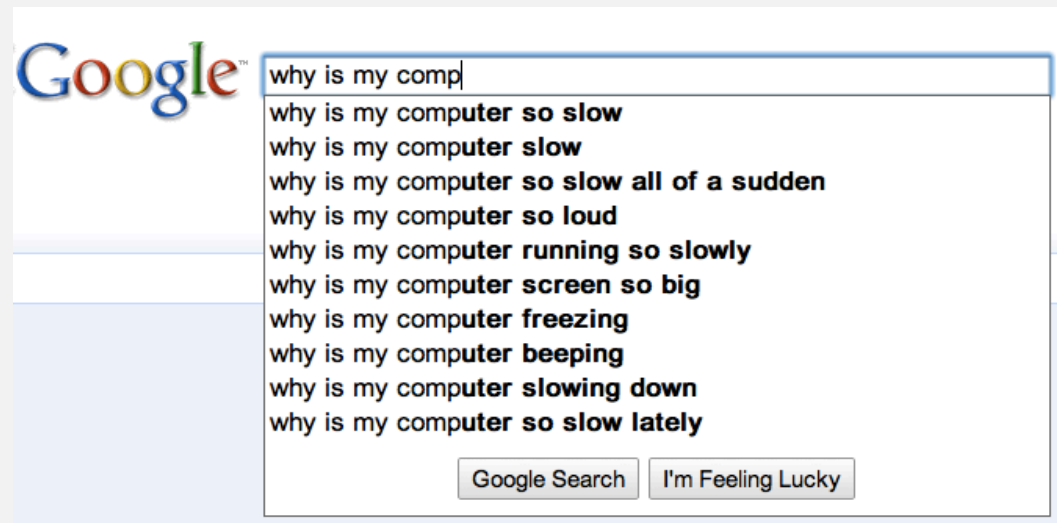
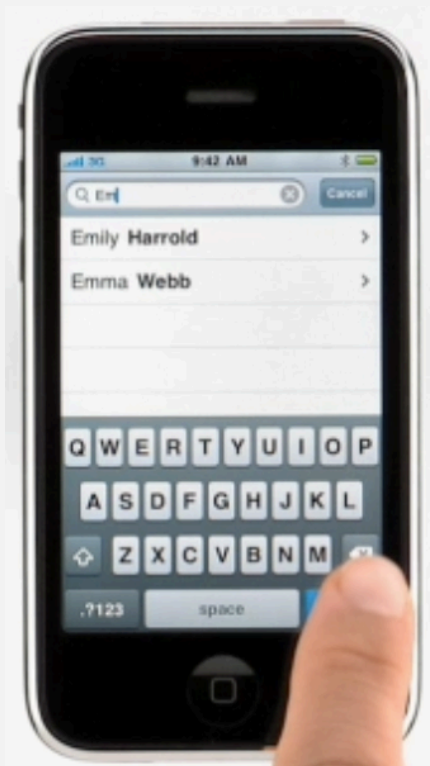
# Prefix matches

---

Find all keys in a symbol table starting with a given prefix.

**Ex.** Autocomplete in a cell phone, search bar, text editor, or shell.

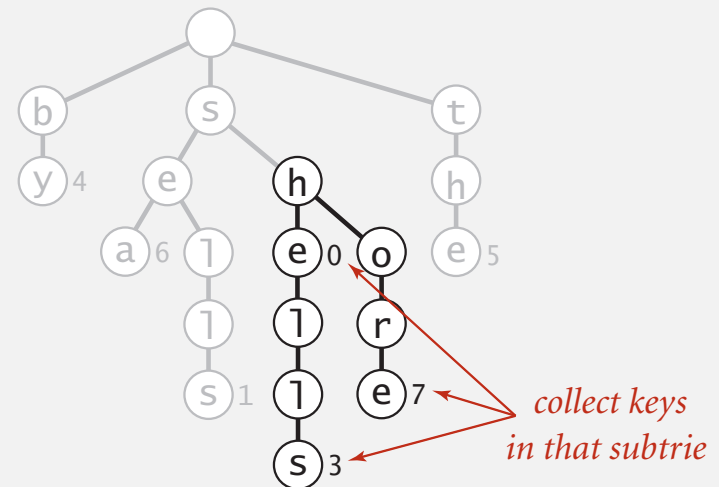
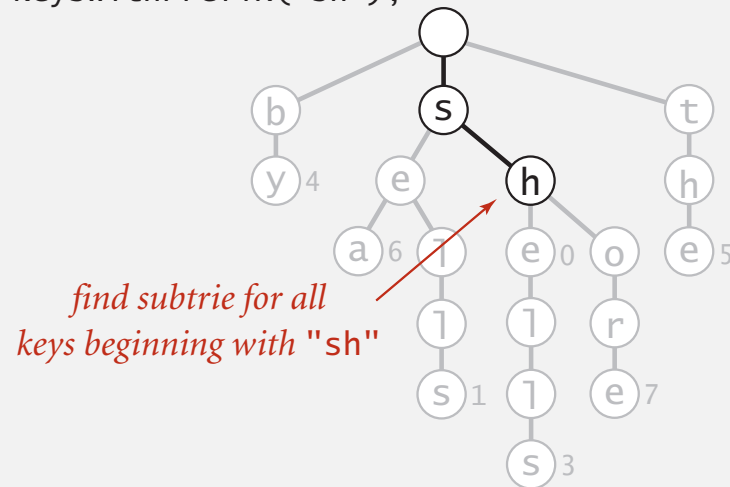
- User types characters one at a time.
- System reports all matching strings.



## Prefix matches in an R-way trie

## Find all keys in a symbol table starting with a given prefix.

```
keysWithPrefix("sh");
```



```
public Iterable<String> keysWithPrefix(String prefix)
{
    Queue<String> queue = new Queue<String>();
    Node x = get(root, prefix, 0);
    collect(x, prefix, queue);
    return queue;
}
```

root of subtree for all strings  
beginning with given prefix

key	queue
sh	
she	she
shell	
shells	she shells
shore	she shells shore

## Longest prefix

---

Find longest key in symbol table that is a prefix of query string.

**Ex.** To send packet toward destination IP address, router chooses IP address in routing table that is longest prefix match.

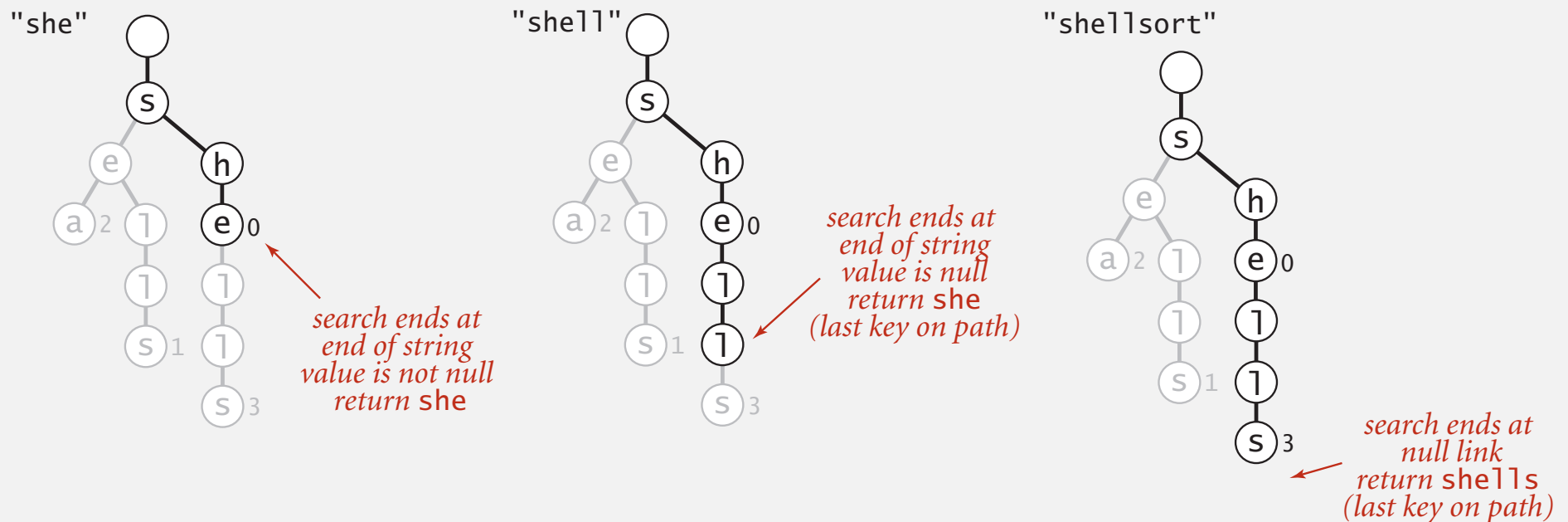
"128"	 represented as 32-bit binary number for IPv4 (instead of string)
"128.112"	
"128.112.055"	
"128.112.055.15"	
"128.112.136"	LongestPrefixOf("128.112.136.11") = "128.112.136"
"128.112.155.11"	LongestPrefixOf("128.112.100.16") = "128.112"
"128.112.155.13"	LongestPrefixOf("128.166.123.45") = "128"
"128.222"	
"128.222.136"	

**Note.** Not the same as floor: floor("128.112.100.16") = "128.112.055.15"

# Longest prefix in an R-way trie

Find longest key in symbol table that is a prefix of query string.

- Search for query string.
- Keep track of longest key encountered.



Possibilities for longestPrefixOf()



## Longest prefix in an R-way trie: Java implementation

---

Find longest key in symbol table that is a prefix of query string.

- Search for query string.
- Keep track of longest key encountered.

```
public String longestPrefixOf(String query)
{
    int length = search(root, query, 0, 0);
    return query.substring(0, length);
}

private int search(Node x, String query, int d, int length)
{
    if (x == null) return length;
    if (x.val != null) length = d;
    if (d == query.length()) return length;
    char c = query.charAt(d);
    return search(x.next[c], query, d+1, length);
}
```

# T9 texting

---

**Goal.** Type text messages on a phone keypad.

**Multi-tap input.** Enter a letter by repeatedly pressing a key until the desired letter appears.

"a much faster and more fun way to enter text"

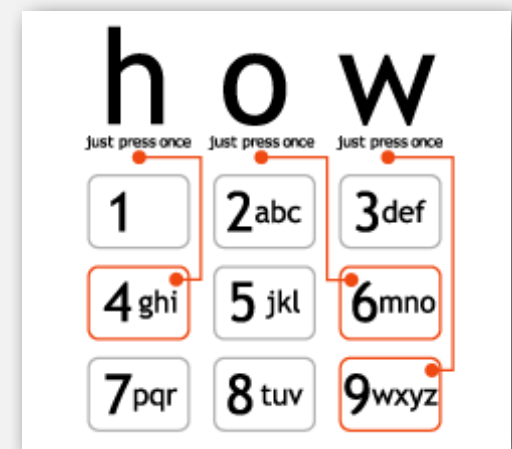
**T9 text input.**

- Find all words that correspond to given sequence of numbers.
- Press 0 to see all completion options.

**Ex.** hello

- Multi-tap: 4 4 3 3 5 5 5 5 5 5 6 6 6
- T9: 4 3 5 5 6

**Q.** How to implement?



[www.t9.com](http://www.t9.com)

# A letter to t9.com

---

To: info@t9support.com

Date: Tue, 25 Oct 2005 14:27:21 -0400 (EDT)

Dear T9 texting folks,

I enjoyed learning about the T9 text system from your webpage, and used it as an example in my data structures and algorithms class. However, one of my students noticed a bug in your phone keypad

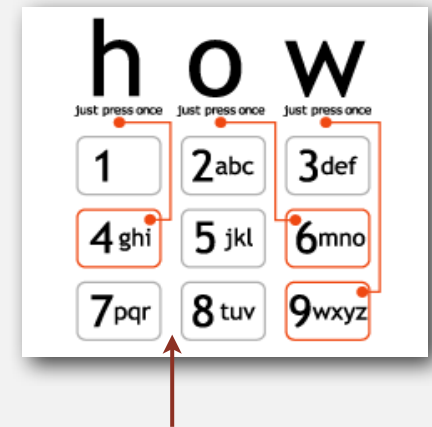
<http://www.t9.com/images/how.gif>

Somehow, it is missing the letter s. (!)

Just wanted to bring this information to your attention and thank you for your website.

Regards,

Kevin



where the @#\$% is the "s" ???

## A world without 's' ?

---

To: "'Kevin Wayne'" <wayne@CS.Princeton.EDU>

Date: Tue, 25 Oct 2005 12:44:42 -0700

Thank you Kevin.

I am glad that you find T9 o valuable for your cla. I had not noticed thi before. Thank for writing in and letting u know.

Take care,

Brooke nyder

OEM Dev upport

AOL/Tegic Communication

1000 Dexter Ave N. uite 300

eattle, WA 98109

ALL INFORMATION CONTAINED IN THIS EMAIL IS CONSIDERED  
CONFIDENTIAL AND PROPERTY OF AOL/TEGIC COMMUNICATIONS

# Patricia trie

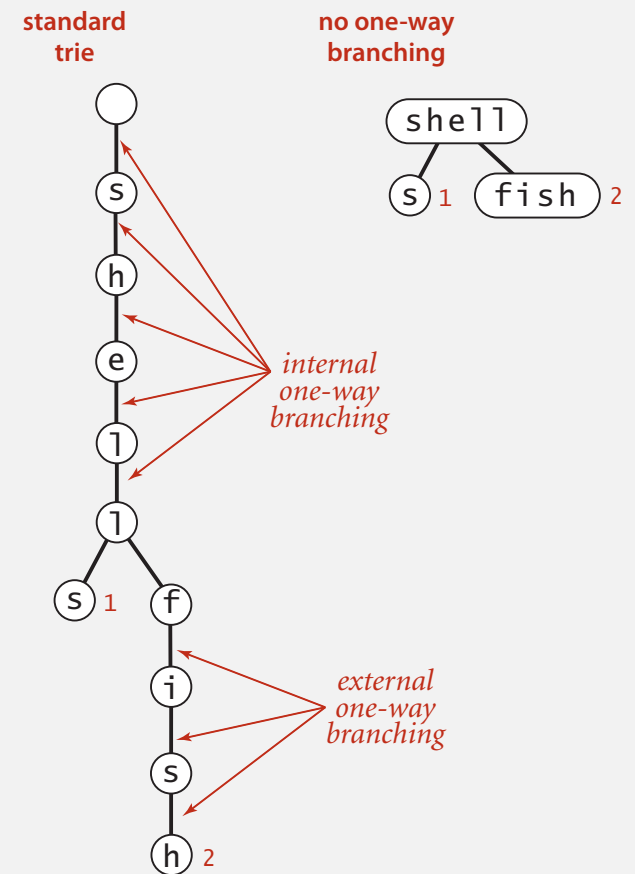
**Patricia trie.** [Practical Algorithm to Retrieve Information Coded in Alphanumeric]

- Remove one-way branching.
- Each node represents a sequence of characters.
- Implementation: one step beyond this course.

```
put("shells", 1);  
put("shellfish", 2);
```

## Applications.

- Database search.
- P2P network search.
- IP routing tables: find longest prefix match.
- Compressed quad-tree for N-body simulation.
- Efficiently storing and querying XML documents.



**Also known as:** crit-bit tree, radix tree.

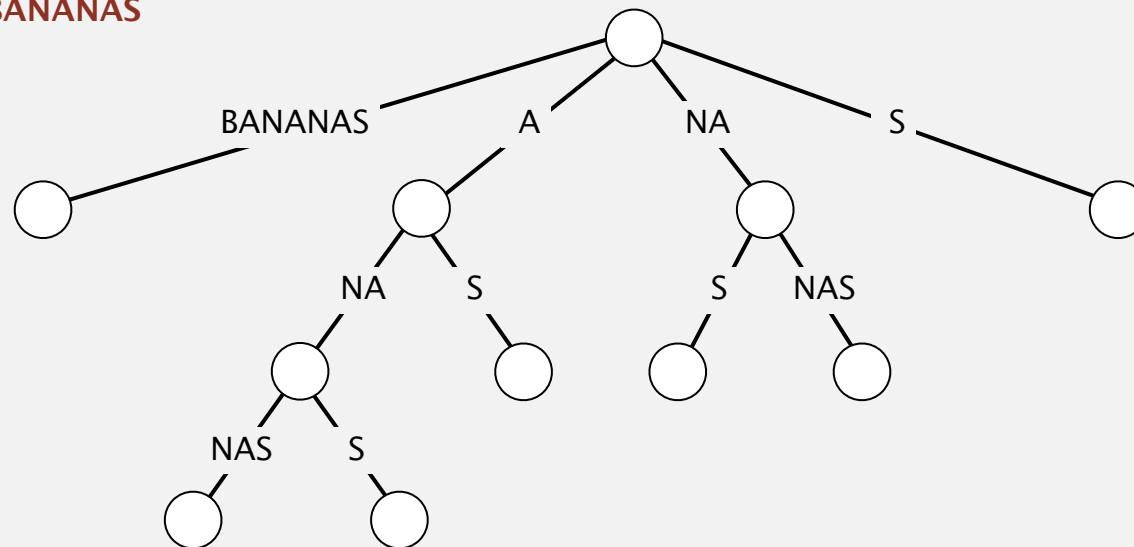
# Suffix tree

---

## Suffix tree.

- Patricia trie of suffixes of a string.
- Linear-time construction: beyond this course.

suffix tree for BANANAS



## Applications.

- Linear-time: longest repeated substring, longest common substring, longest palindromic substring, substring search, tandem repeats, ....
- Computational biology databases (BLAST, FASTA).

# String symbol tables summary

---

A success story in algorithm design and analysis.

## Red-black BST.

- Performance guarantee:  $\log N$  key compares.
- Supports ordered symbol table API.

## Hash tables.

- Performance guarantee: constant number of probes.
- Requires good hash function for key type.

## Tries. R-way, TST.

- Performance guarantee:  $\log N$  **characters** accessed.
- Supports character-based operations.

**Bottom line.** You can get at anything by examining 50-100 bits (!!!)



## 5.2 TRIES

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- ▶ *R-way tries*
- ▶ *ternary search tries*
- ▶ *character-based operations*





<http://algs4.cs.princeton.edu>

## 5.2 TRIES

---

- ▶ *R-way tries*
- ▶ *ternary search tries*
- ▶ *character-based operations*