Lung Cancer Survival Prediction with Bayesian Generalised Linear Models

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Introduction

This is an example introduction.

```
x <- 1
y <- 3
x + 2 * y
```

```
## [1] 7
```

Weibull Generalised Linear Model

Let $y \sim \text{Weibull}(\alpha, \sigma)$, so that

$$\text{Weibull}(y|\alpha, \sigma) = \frac{\alpha}{\sigma} \left(\frac{y}{\sigma}\right)^{\alpha-1} \exp\left(-\left(\frac{y}{\sigma}\right)^{\alpha}\right),$$

for $y \in [0, \infty), \alpha \in \mathbb{R}^+$, and $\sigma \in \mathbb{R}^+$.

Motivating the distribution

The Weibull distribution is often used as a more flexible and complex alternative to the semi-parametric proportional hazard Cox model for modelling time to failure events, since the hazard rate is not taken to be constant with time.

The Weibull distribution as a member of the exponential family

Now take α fixed and finite, then it can be shown that this distribution belongs to the exponential family since we can write it's probability density function

$$\text{Weibull}(y|\sigma) = \alpha y^{\alpha-1} \exp(-y^{\alpha}\sigma^{-\alpha} - \alpha \log \sigma),$$

with

$$
\begin{aligned}
b(y) &= \alpha y^{\alpha-1} \\
\eta &= \sigma^{-\alpha} \\
T(y) &= -y^{\alpha} \\
a(\eta) &= \alpha \log \sigma.
\end{aligned}
$$

Defining the link function

Looking at our sufficient statistic $\eta = \sigma^{-\alpha}$, it can be shown that

$$\sigma = \exp\left(\frac{\log \eta}{-\alpha}\right)$$

where we construct $\eta = \exp(\boldsymbol{X}\beta)$, noting that η is strictly positive. Thus we choose a log link function for our GLM such that

$$\sigma = \exp(\boldsymbol{X}\beta).$$