Représentation compacte d’espaces d’états

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Sécurité et Fiabilité
M2 SAR – UPMC

Outline

- **Decision Diagrams for Model-checking:**
  - **Binary Decision Diagrams :**
    - *Symbolic approach to model-checking*
  - **Saturation**
    - *A more effective fixpoint strategy*
  - **Hierarchical Set Decision Diagrams**
    - *Introduce structured descriptions*
  - **Instantiable Transition Systems**
    - *A framework to exploit SDD*

Binary Decision Diagrams → Saturation → Set Decision Diagrams → Instantiable Transition System

Reduced Ordered Binary Decision Diagrams

(RO)BDD → BDD for Model-checking → BDD extensions

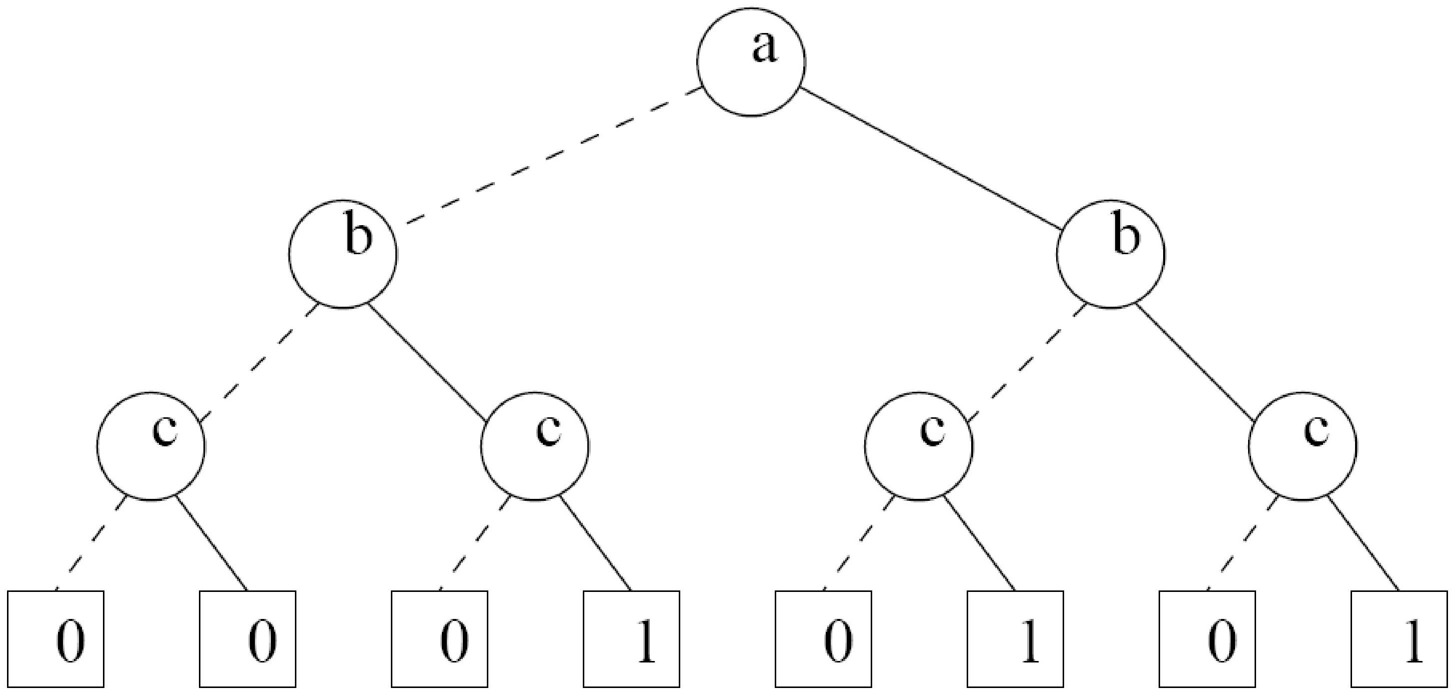
Binary Decision Diagrams

- Most cited document in computer science according to citeseer [in 2005, in 2017 down to place 61]:
  1. Graph-Based Algorithms for Boolean Function Manipulation – Bryant (1986)
  In this paper we present a new data structure for representing Boolean functions and an associated set of...
- Introduces Reduced Ordered Binary Decision Diagrams (RO)BDD
- What is a (RO)BDD ?

A compact data structure to represent boolean functions

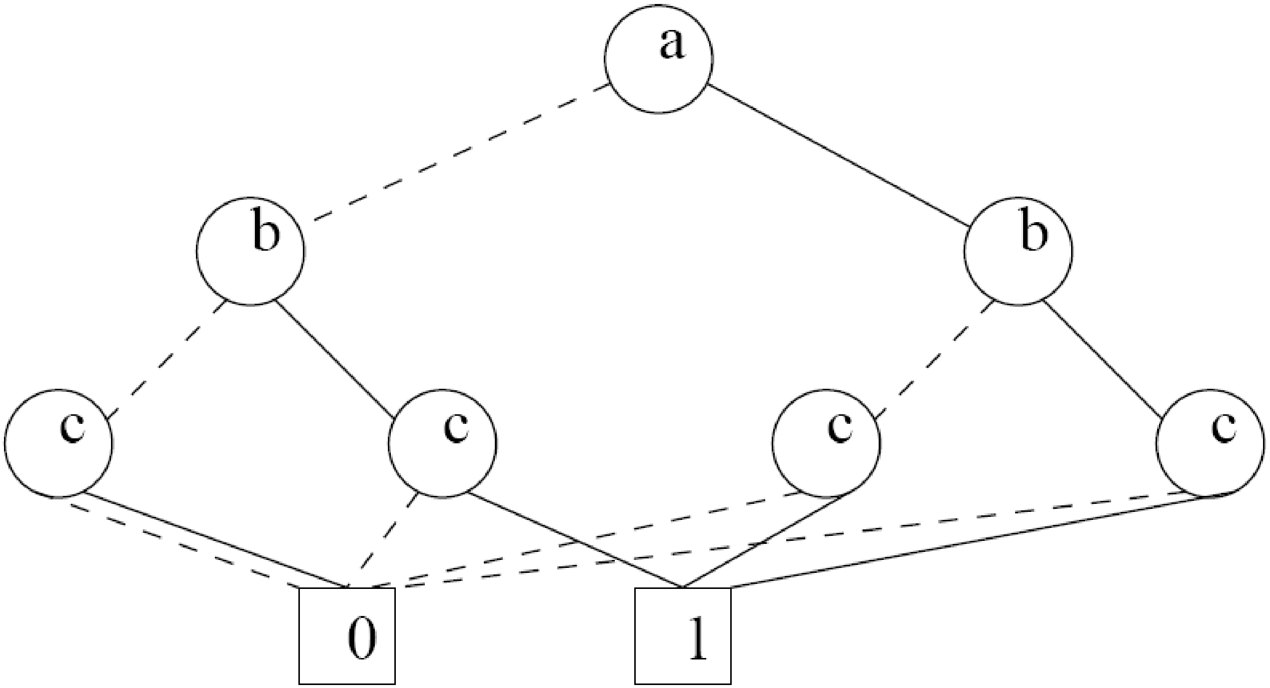
BDD : an example : f = (a OR b) AND c

- An ordered (a<b<c) binary decision diagram for f :



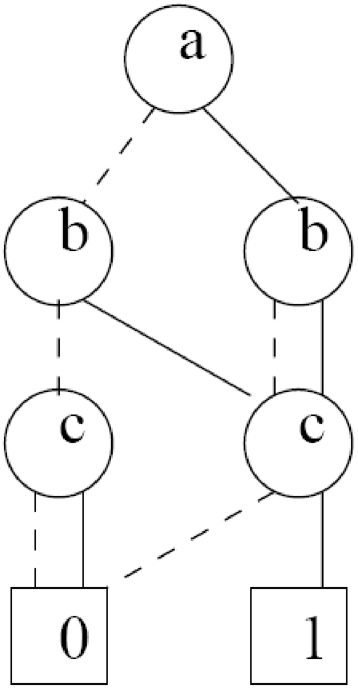
BDD : an example : f = (a OR b) AND c

- Reduction : single occurrence of terminals



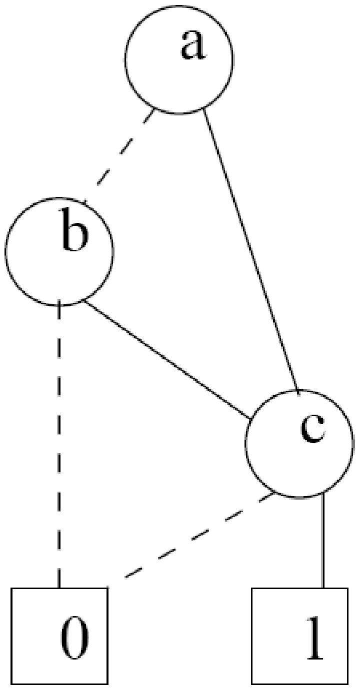
BDD : an example : f = (a OR b) AND c

- Recursively from terminals : single occurrence of any node
  - Uses a unicity table for nodes (hash table)
  - Node hash key based on : node + hash key of sons



BDD : an example : f = (a OR b) AND c

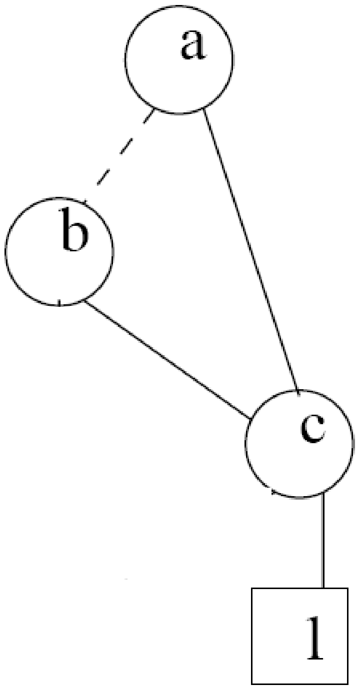
- Remove useless nodes
  - Criterion : variable value does not influence truth value of formula
  - <=> both son arcs point to same node



a	b	c	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

BDD : an example : f = (a OR b) AND c

- Zero-suppressed variant :
  - Remove paths that lead to 0 (false)
  - Represents the set of "true" values of f



a	b	c	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Properties of ROBDD

- Given a boolean expression y, or a function $f : \mathbb{B}^K \rightarrow \mathbb{B}$, there is a unique BDD encoding it (given a variable order $\mathrm{x}_K, \ldots, \mathrm{x}_1$)
- Many functions have a very compact encoding as a BDD
- The constant functions 0 and 1 are represented by the nodes $Zero$ and One, respectively
- Test whether a boolean expression is constantly true or false in $O(1)$ time, given its BDD encoding
- Test whether two boolean expressions are equivalent in $O(1)$ time, given their BDD encoding

Properties of BDD : Choice of an order

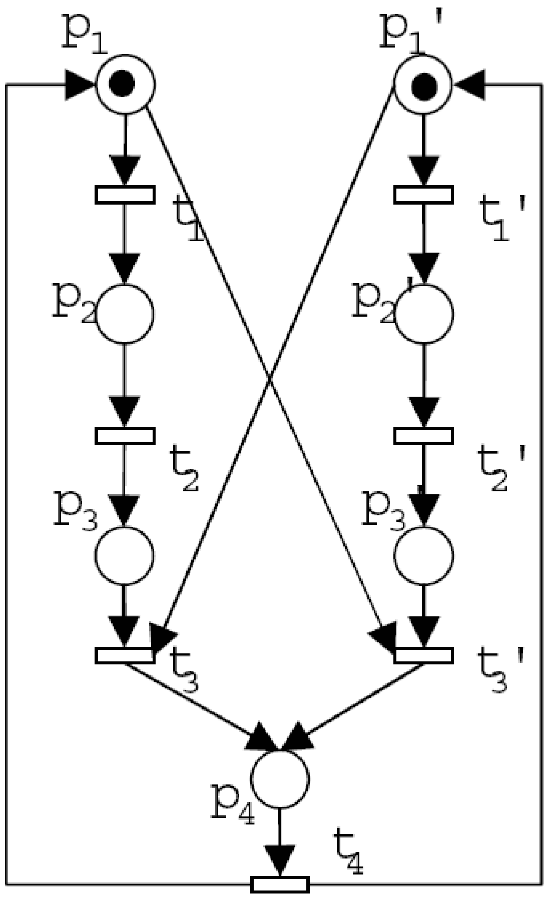
- The variable ordering affects the size of the BDD, consider $x_K \Leftrightarrow y_K \wedge \cdots \wedge x_1 \Leftrightarrow y_1$
  - with the order $(x_K, y_K, \ldots, x_1, y_1)$ $O(K)$ nodes
  - with the order $(x_K, \ldots, x_1, y_K, \ldots, y_1)$ $O(2^K)$ nodes
- The BDD encoding of some functions is large (exponential) for any order
  - the expression for bit 32 of the 64-bit result of the multiplication of two 32-bit integers
- Finding the optimal ordering is an NP-complete problem

Model checking
and BDD

(RO)BDD → BDD for Model-checking → BDD extensions

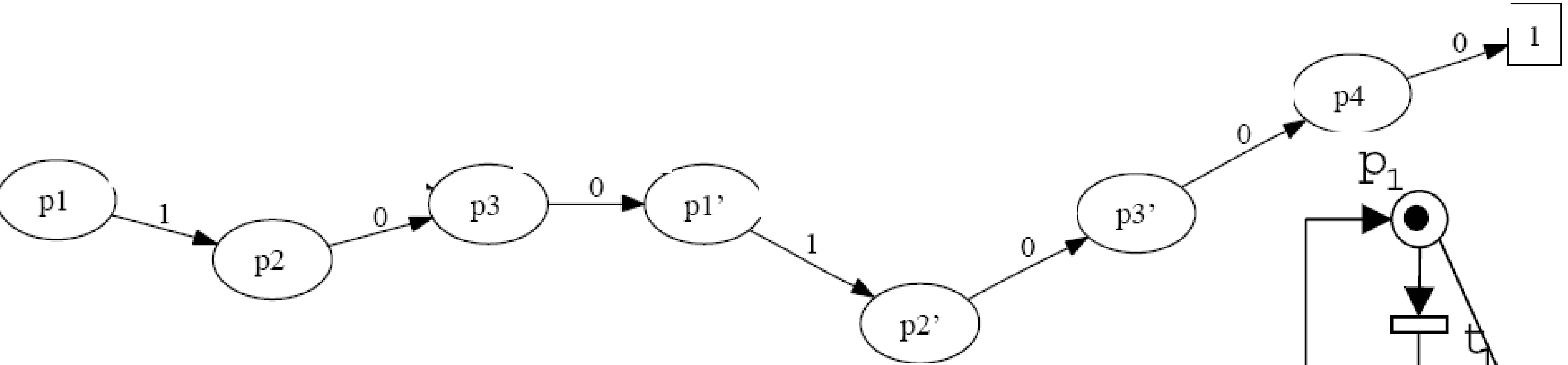
Representing a state-space using DD

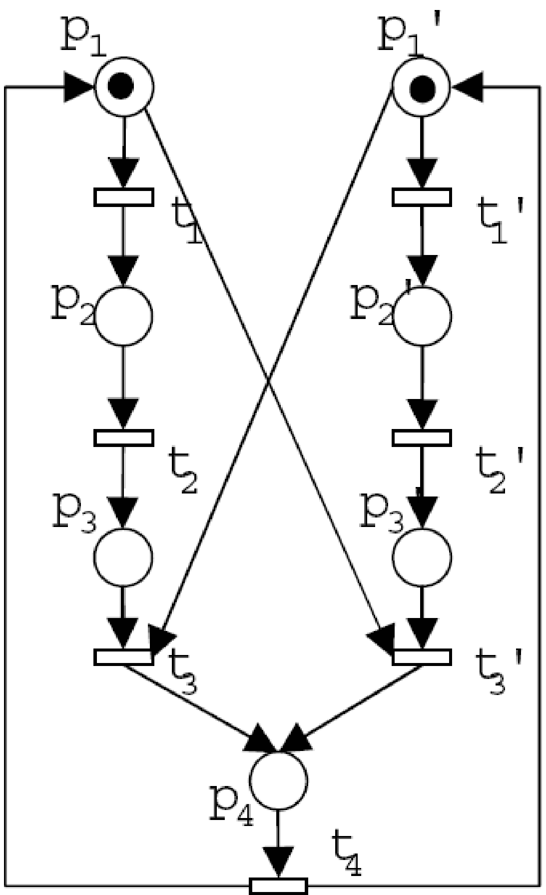
- **Principle :**
  - *A path in the structure represents a reachable state*
  - *A state S is described by the value of its state variables*
- **Example :**



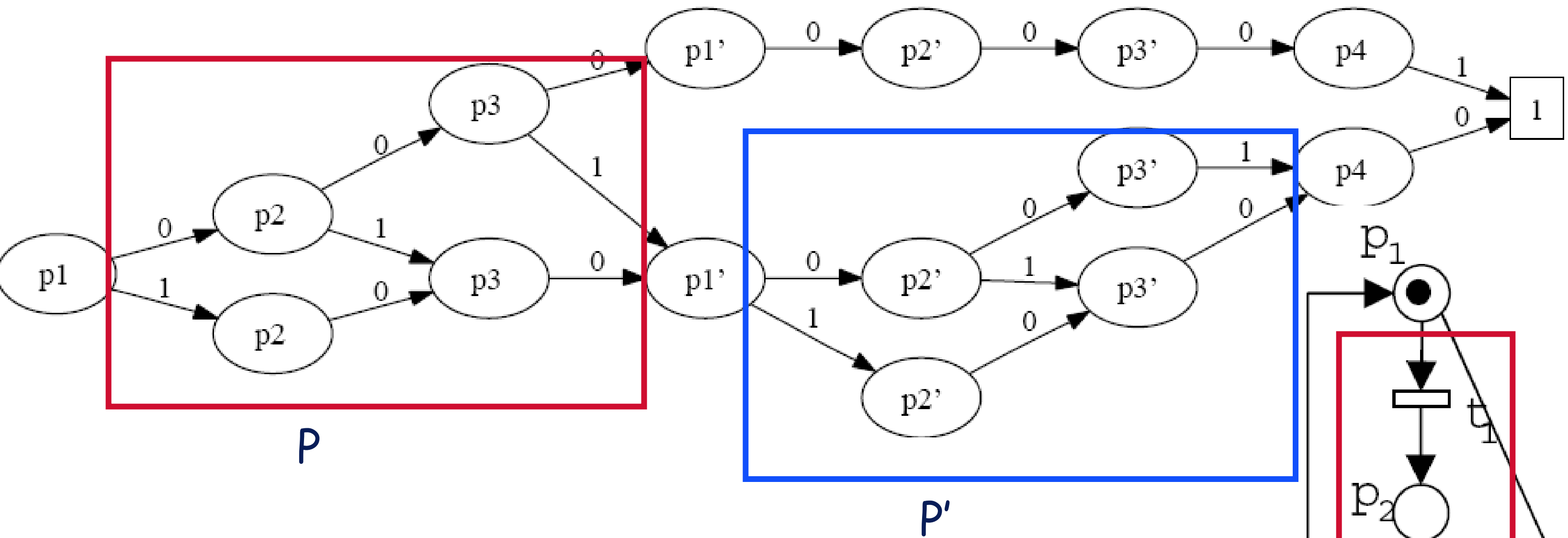
A mutual exclusion protocol for 2 process p and p'

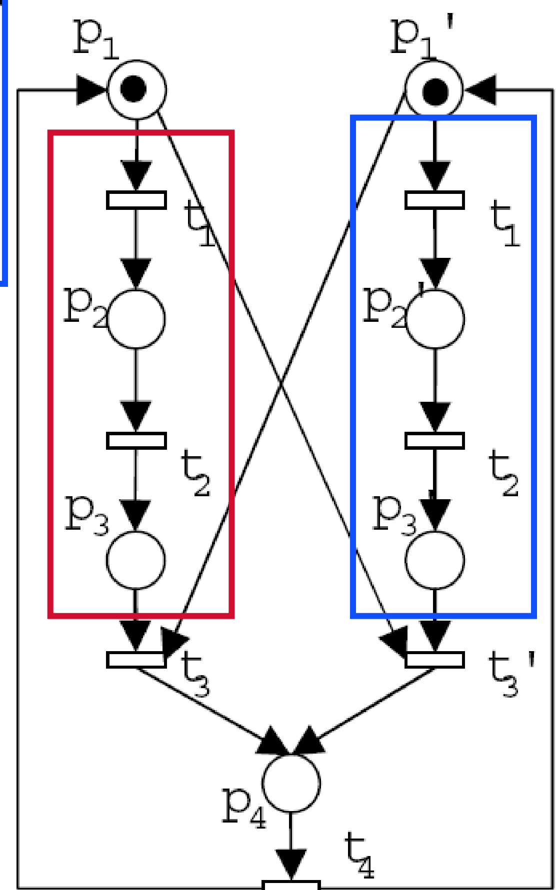
Example


p1
1
p2
0
p3
0
p1'
1
p2'
0
p3'
0
p4
0
1


p1
p1'
t1
t1'
p2
p2'
t2
t2'
p3
p3
t3
t3'
p4
t4

Example

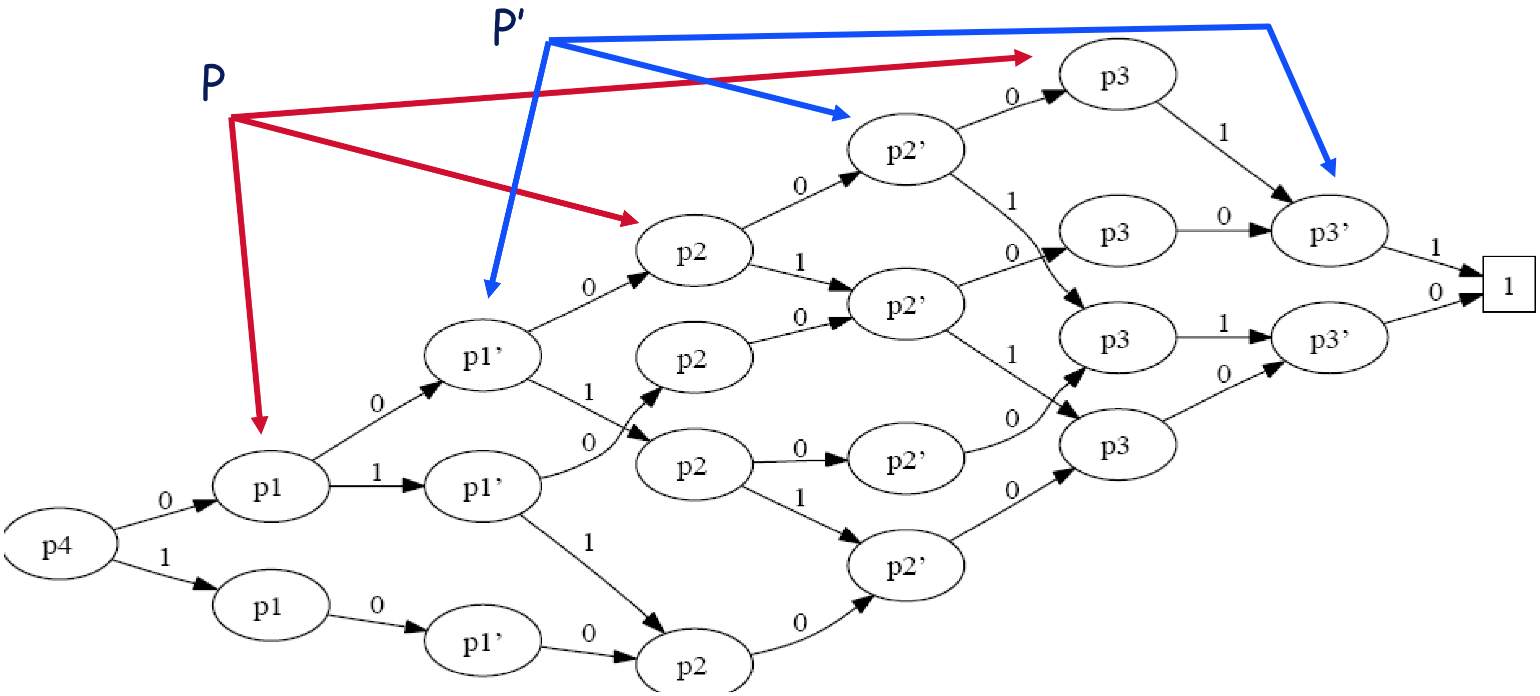


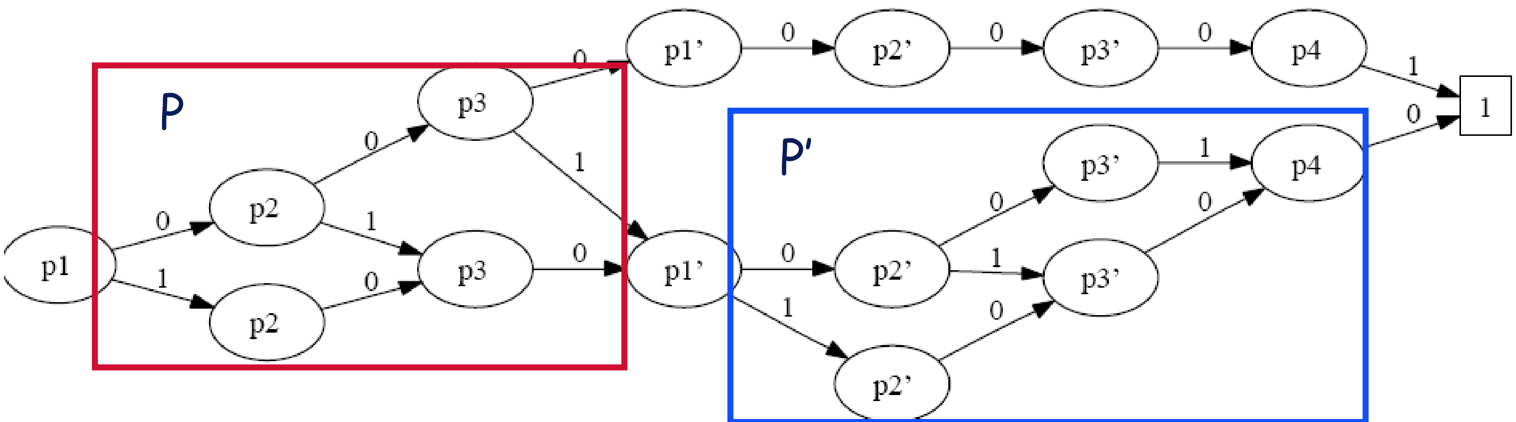


Good representation for Globally Asynchronous Locally Synchronous (GALS) systems :

- Independence of local actions (t1 and t2) is well captured

Variable Ordering issues





Growth in node size w.r.t. order

Nb Proc	Nb States	Consecutive order	Interlaced order
2	10	16	21
5	244	40	260
10	59050	80	13321
15	1.434e+07	120	589838
20	3.486e+09	160	-out of ram-

- Linear growth vs exponential growth for poor ordering clearly visible
- Linear growth of representation but exponential growth of state-space size with appropriate ordering !!
- For some problems BDD based techniques allow to go much further than explicit representation techniques

Efficiency of BDD

- Computations on BDD use a cache to limit complexity
- Cache is of the form
  - Key:<operation, operand_1,..,operand_n >-> value:result
  - Where operands and result are BDD nodes
- Example :
  - Cache : contains <union, a, b > -> c if (a U b) = c

union of BDD

- union (a,b)
  - If (a=0 or b=1) return b;
  - If (a=1 or b=0) return a;
  - If (a=b) return a;
  - If (<union,a,b> -> r in cache) return r;
  - BDD r= createBDD(
    - *0 => union(a[0],b[0]),*
    - *1=> union(a[1],b[1]))*
  - Cache.add(<union,a,b> -> r)
  - Return r;

Complexity of BDD operations

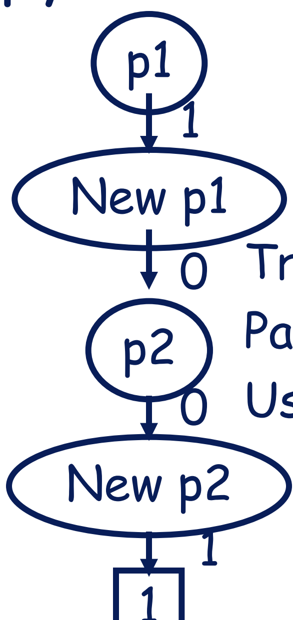
- createBDD uses a unicity table based on node structure
  - Hash on value of son nodes
- Operation cache =>
  - complexity proportional to #nodes(a) x #nodes(b) => number of nodes
  - Without it complexity would be proportional to number of PATHS in the structure
- Intersection differs from union only in terminal cases
  - If (a = 1 or b = 0) return b
  - If (a = 0 or b = 1) return a

State space computation

- **Classic algorithm is based on Breadth-first exploration BFS**
  - **Consider a system composed of k state variables (i.e. state space represented as a k-level BDD)**
  - **Transition relation represented using 2k variables**

    (i,j) = (i1,j1, ..ik,jk)

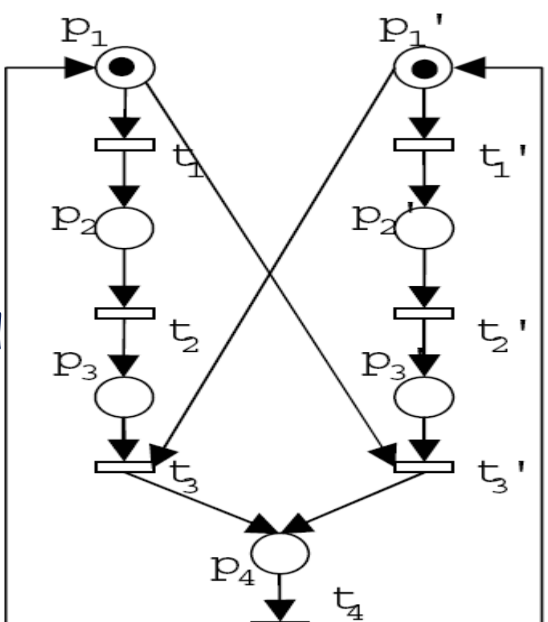
    System can go from i to j in one step
  - **A special synchronized product (relational product) operation is defined to apply such a transition to a system**



Transition BDD for Next(t1)

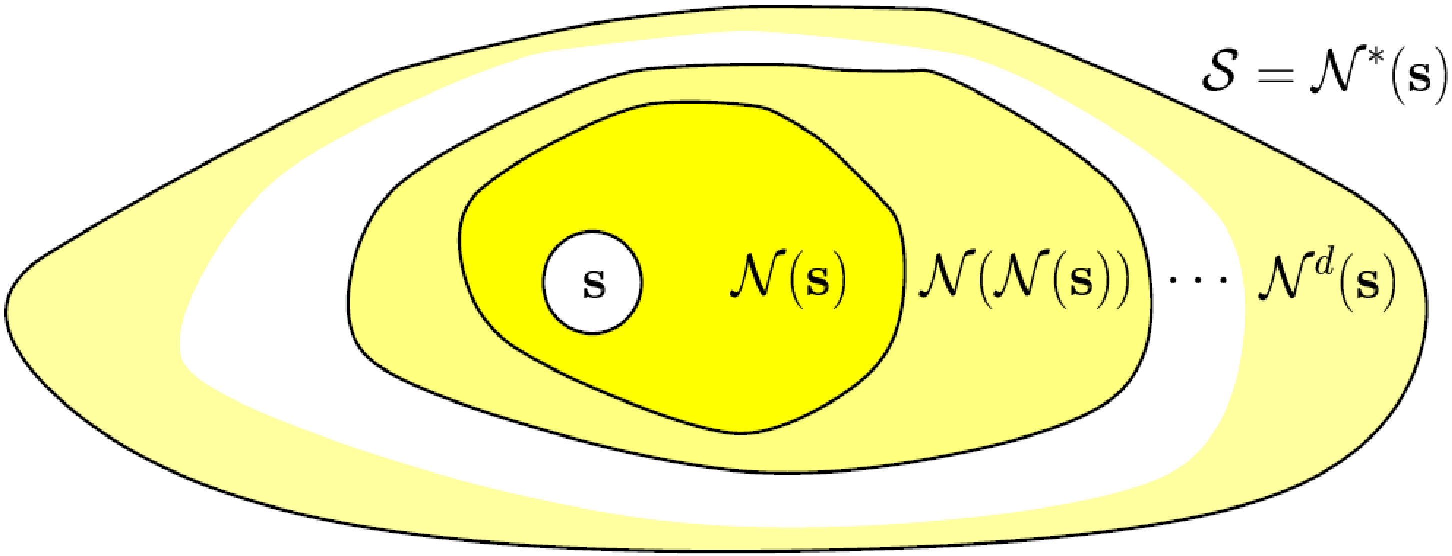
Paths to 0 terminal not represented

Useless variables removed



Symbolic state space generation: Classic fixed-point iteration

Given the *initial state* $\mathbf{s}$ and the *next-state function* $\mathcal{N}$, we obtain the *state space* $\mathcal{S}$:



The *number of iterations* equals one plus the maximum distance d of any state from $\mathbf{s}$

The *peak* BDD size is usually achieved well before reaching the *final* BDD size at step d

Slide by G. Ciardo

Combining transitions

- BDD representing transitions can be combined using union
  - Full transition relation
    - *NextAll = union (Next(t1)+..+Next(tn))*
- Algorithm for BFS(s0)
  - S := S0
  - N := 0
  - While (N != S)
    - N := S
    - S := S U NextAll(S)
  - Return S

"Symbolic Model Checking: 10 20 States and Beyond" (LICS'1990) Burch, Clarke, McMillan, Dill, Hwang

State space representation size

- BFS performs better than the "intuitive algorithm"
  - Size of representation is not directly linked to number of states manipulated
  - Re-evaluating a transition on an already reached state is likely to ctose a cache-hit thus has experimentally low cost.
- Algorithm for newBFS(s0)
  - S := S0
  - N := S0
  - While (N != 0)
    - N := NextAll(U) \ S
    - S := S U N
  - Return S

Only computes NextAll on newly reached states

Decomposing the transition relation [Roig'95]

- The idea is to cluster some transitions but keep an expression of the transition relation in parts
  - Next(i) for I in 1..nbClusters = union (Next(tj),..Next(tj+k))
  - Create one cluster for each process (requires structural information)
- Not quite BFS anymore as chainings may occur
- Solves problems experienced with the size of NextAll BDD
- **Algorithm for chainBFS(s0)**
  - S := S0
  - N := 0
  - While (N != S)
    - N := S
    - For (i in 1..nbClusters)
      - S := S U Next(i) (S)
  - Return S

Comparing the four approaches

Slide by G. Ciardo

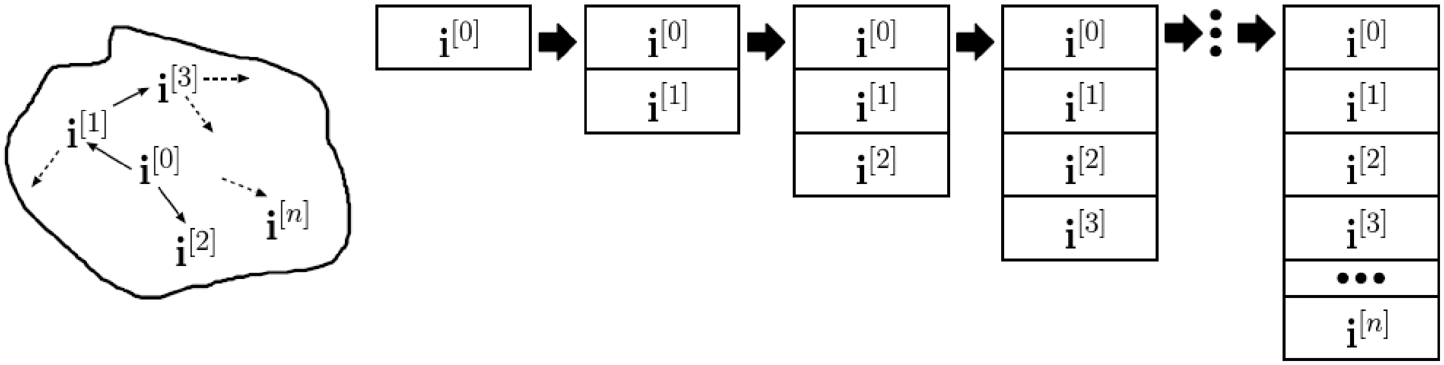
Performances using Smart

N	$\|\mathcal{S}\|$	Time (sec) New BFS	Time (sec) BFS	Time (sec) New Chain	Time (sec) Chain	Memory (MB) New BFS	Memory (MB) BFS	Memory (MB) New Chain	Memory (MB) Chain	Memory (MB) final
Dining Philosophers: $K=N$, $\|\mathcal{S}_k\|=34$ for all k										
50	2.2×10^{31}	37.6	36.8	1.3	1.3	146.8	131.6	2.2	2.2	0.0
100	5.0×10^{62}	644.1	630.4	5.4	5.3	$>$999.9	$>$999.9	8.9	8.9	0.0
1000	9.2×10^{626}	—	—	895.4	915.5	—	—	895.2	895.0	0.3
Slotted Ring Network: $K=N$, $\|\mathcal{S}_k\|=15$ for all k										
5	5.3×10^{4}	0.2	0.3	0.1	0.1	0.8	1.1	0.3	0.2	0.0
10	8.3×10^{9}	21.5	24.1	2.1	1.2	39.0	45.0	5.7	3.3	0.0
15	1.5×10^{15}	745.4	771.5	18.5	8.9	344.3	375.4	35.1	20.2	0.0
Round Robin Mutual Exclusion: $K=N+1$, $\|\mathcal{S}_k\|=10$ for all k except $\|\mathcal{S}_1\|=N+1$										
10	2.3×10^{4}	0.2	0.3	0.1	0.1	0.6	1.2	0.1	0.1	0.0
20	4.7×10^{7}	2.7	4.4	0.3	0.3	5.9	12.8	0.5	0.5	0.0
50	1.3×10^{17}	263.2	427.6	2.9	2.8	126.7	257.7	4.3	3.8	0.1
FMS: $K=19$, $\|\mathcal{S}_k\|=N+1$ for all k except $\|\mathcal{S}_{17}\|=4$, $\|\mathcal{S}_{12}\|=3$, $\|\mathcal{S}_7\|=2$										
5	2.9×10^{6}	0.7	0.7	0.1	0.1	2.6	2.2	0.4	0.2	0.0
10	2.5×10^{9}	7.0	5.8	0.5	0.3	18.2	14.7	2.3	1.3	0.0
25	8.5×10^{13}	677.2	437.9	12.9	5.1	319.7	245.3	42.7	21.2	0.1

Explicit vs. symbolic state space generation

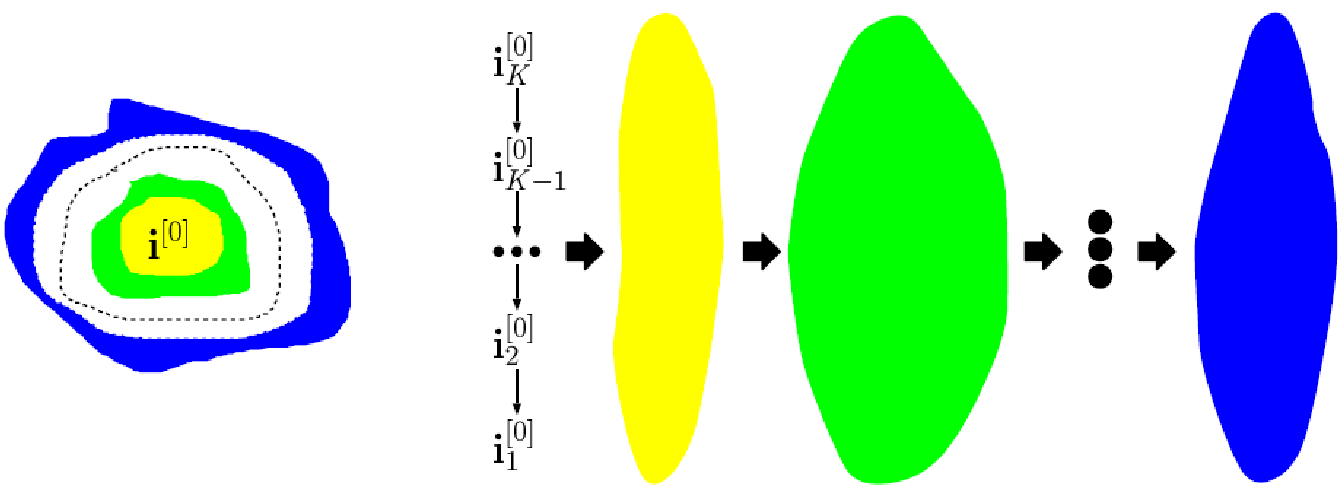
Explicit generation of the state space $\mathcal{S}$ adds *one state* at a time

- memory O(states), increases linearly, peaks at the end



Symbolic generation of the state space $\mathcal{S}$ with decision diagrams adds *sets of states* instead

- memory O(decision diagram nodes), grows and shrinks, usually peaks before the end



Using the state space representation

- The state space representation allows to easily verify safety properties
  - Can we reach a “bad” state
  - Can A and B both be true simultaneously
- State space generation is the basis for more complex temporal logic properties such as CTL
  - CTL properties can be expressed as nested fix points of the transition relation and its reverse $Next^{-1}$

Some BDD extensions

(RO)BDD → BDD for Model-checking → BDD extensions

Decision Diagrams : Widely accepted in verification tools

- SMV (US):
  - FSM/Kripke structure
  - emblematic first symbolic enabled verification tool. Now uses (NuSMV 2-Italy) library Cudd.
- Uppaal (Den-Nor):
  - Hybrid systems
  - uses Difference Bounded Matrix diagrams to represent clocks
- Prism (UK) :
  - Stochastic process algebra
  - uses Matrix DD and Multi-terminal DD for stochastic verification
- Smart (US) :
  - Stochastic Petri nets
  - uses integer valued DD, both CTL and stochastic solution engine (+saturation)
- Red (Taiwan) :
  - Timed automata
  - Specific solution for real time systems

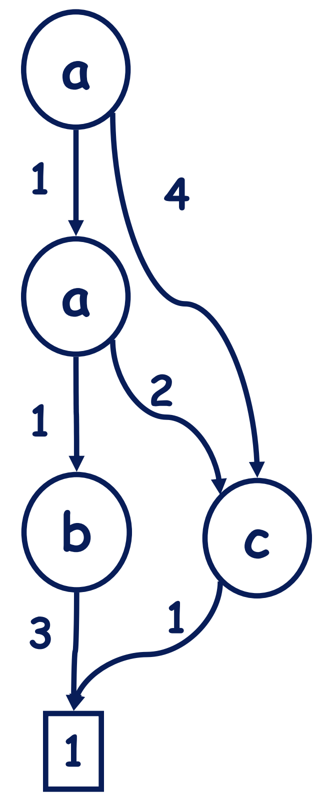
Integer valued Decision Diagrams

- Some variants:
  - Multi-way DD (Ciardo&Miner Icatpn'99)
  - Data Decision Diagrams (Couvreur et al. Icatpn'02)
- Variables may have an integer domain instead of boolean domain
  - Usually zero-suppressed, to allow arbitrary variable domains provided the actual reachable set is finite
  - Using BDD, one has to decode integer state variables into their log_2(n) bit representation
  - Problem : complexity also linked to nb arcs/node, not only number of nodes
- Data Decision Diagram
  - No variable order (handled in the union operation to handle incompatibilities)
  - Homomorphisms to define the transition relation

$$a \xrightarrow{4} c \xrightarrow{1} 1$$

$$a \xrightarrow{1} a \xrightarrow{2} c \xrightarrow{1} 1$$

$$a \xrightarrow{1} a \xrightarrow{1} b \xrightarrow{3} 1$$



Multi terminal Decision Diagrams

- Multi-terminal (MTDD [Fujita+ '97] or Algebraic DD [Bahar+ '93]):
- Instead of single terminal 1, use several terminals
- Allows to give a correspondence between a state and a characteristic it has
  - Not just presence or absence of a state
- Example 1: integer terminal
  - Terminal gives the distance (number of steps) from initial state of any state
  - union handles same path with different terminals => keep the smallest terminal
  - Useful for finding shortest witness or counter-example traces
- Example 2: Real valued terminal
  - Used in stochastic/probabilistic systems, gives the probability of being in a state
  - union handles the approximation (2 terminals x and y considered equal if |x-y| < epsilon)

The Saturation Algorithm for Decision diagrams

Binary Decision Diagrams → **Saturation** → Set Decision Diagrams → Instantiable Transition System

Transitive Closure : Fixpoint

Algorithm 1: Four variants of a transitive closure loop.

Data: $\{Hom\}\ T$: the set of transitions encoded as h_{Trans} homomorphisms
$\$\ m_0$: initial state encoded as $r(M)$ SDD
$\$\ todo$: new states to explore
$\$\ reach$: reachable states

$a)$ Explicit reachability style

```
begin
    todo := m0
    reach := m0
    while todo ≠ 0 do
        $ tmp := T(todo)
        todo := tmp \ reach
        reach := reach + tmp
end
```

$b)$ Standard symbolic BFS loop

```
begin
    todo := m0
    reach := 0
    while todo ≠ reach do
        reach := todo
        todo := todo + T(todo) ≡ (T + Id)(todo)
end
```

$c)$ Chaining loop

```
begin
    todo := m0
    reach := 0
    while todo ≠ reach do
        reach := todo
        for t ∈ T do
            todo := (t + Id)(todo)
end
```

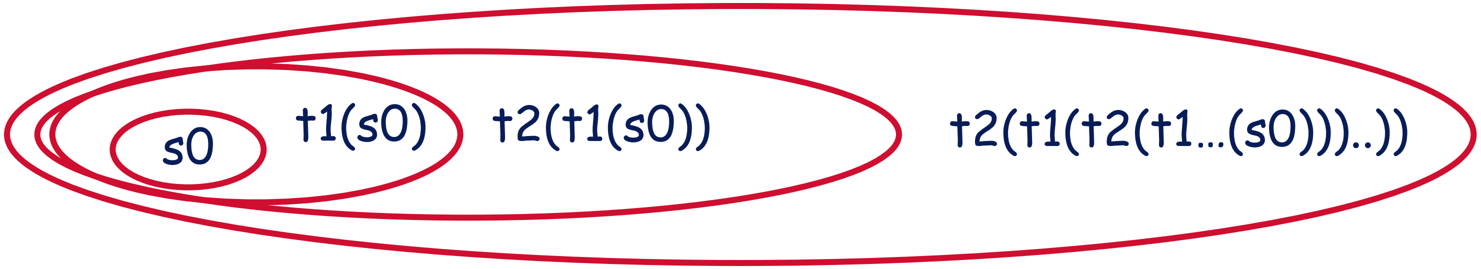
$d)$ Saturation enabled

```
begin
    reach := (T + Id)★(m0)
end
```

Saturation

- Model-checking using decision diagrams => (nested) transitive closures over the transition relation
- Optimizing complexity of this operation critical to efficiency
- [BCM'92] based on <u>BFS style iterations</u>, n iterations required where n is depth of "deepest" state

- [Roig'95] <u>Chaining</u> may converge faster, based on clusters of transitions, no longer strict BFS



- [Ciardo'01] <u>Saturation</u> is empirically 1 to 3 orders of magnitude better

Saturation vs BFS

- Saturation algorithm: [Ciardo et al. TACAS'01]
  - Fire transitions from the leaves (terminals) up to root
  - Go to ancestor of a node iff. The current node is saturated : all events that only affect this variable and variables below it have been fired until a fixpoint is reached
  - Each time a node is affected by an event, resaturate it.
- Not BFS anymore, firing order of events follows data structure
  - Huge reduction of time and space complexity
  - Good tackling of intermediate peak size effect
- However :
  - Definition of saturation algorithm is complex
  - Cannot be implemented directly with public API of DD libraries

Our contribution : Automatic saturation

Transitive Closure

- The transitive closure or *fixpoint* noted * is a unary operator
- Evaluated by $h^*(d)$:
  - repeat : d = h(d)
  - until : d == h(d)
- Evaluation may not terminate
  - depends on the homomorphism
  - if it does, evaluation described as finite composition :
    - $h^* (d) = h \circ h \circ \ldots h (d)$
  - Thus h^* is a homomorphism
- To cumulate states, use of a common construction :
  - $(h + id)^*$
- Allows to implement a leaf to root saturation strategy

Fixpoint : an example

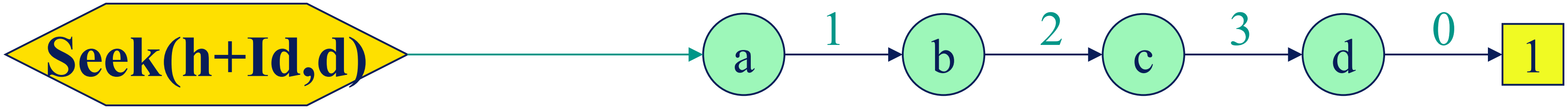
$$Seek(h,v)(e,x) = \begin{cases} h^* \circ e \xrightarrow{x} Id & \textit{if } v=e \\ e \xrightarrow{x} Seek(h,v) & \textit{otherwise} \end{cases}$$

$$Seek(h,v)(1) \quad = T$$

$$Max(z)(e,x) = \begin{cases} e \xrightarrow{x} Id & \textit{if } x<z \\ 0 & \textit{otherwise} \end{cases}$$

$$Max(z)(1) \quad = T$$

$h = Max(3) \circ Inc(d)$ // Increment *d* up to 2



Fixpoint : an example

$$\mathrm{Seek}(h,v)(e,x) = \begin{cases} h^{*} \circ e \xrightarrow{x} \mathrm{Id} & \textit{if } v=e \\ e \xrightarrow{x} \mathrm{Seek}(h,v) & \textit{otherwise} \end{cases}$$

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$$\mathrm{Max}(z)(1) \quad = \mathrm{T}$$

$h = \mathrm{Max}(3) \circ \mathrm{Inc}(d)$ // Increment *d* up to 2

a single traversal of these nodes

Fixpoint : an example

$$\mathrm{Seek}(h,v)(e,x) = \begin{cases} h^* \circ e \xrightarrow{x} \mathrm{Id} & \textit{if } v=e \\ e \xrightarrow{x} \mathrm{Seek}(h,v) & \textit{otherwise} \end{cases}$$

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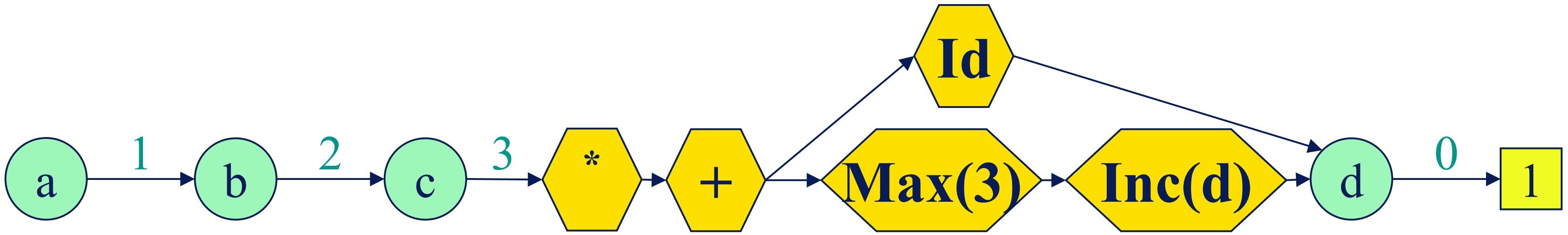

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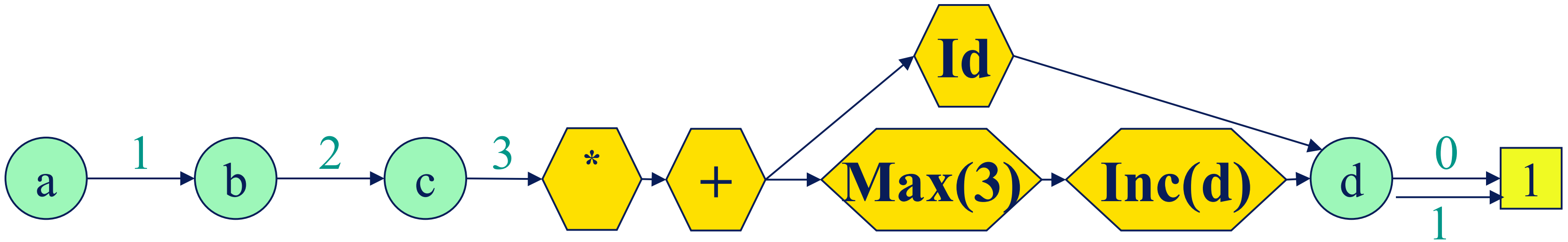


Fixpoint : an example

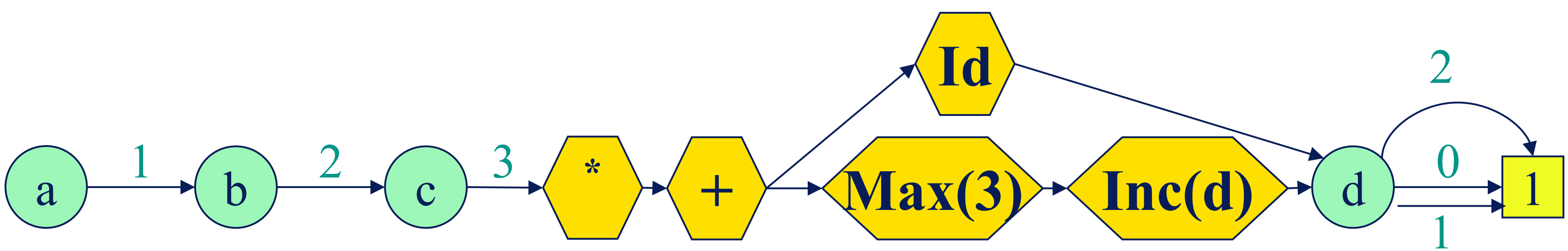
$$\mathrm{Seek}(h,v)(e,x) = \begin{cases} h^* \circ e \xrightarrow{x} \mathrm{Id} & \textit{if } v=e \\ e \xrightarrow{x} \mathrm{Seek}(h,v) & \textit{otherwise} \end{cases}$$

$$\mathrm{Seek}(h,v)(1) = T$$

$$\mathrm{Max}(z)(e,x) = \begin{cases} e \xrightarrow{x} \mathrm{Id} & \textit{if } x<z \\ 0 & \textit{otherwise} \end{cases}$$

$$\mathrm{Max}(z)(1) = T$$

$h = \mathrm{Max}(3) \circ \mathrm{Inc}(d)$ // Increment *d* up to 2



Fixpoint : an example

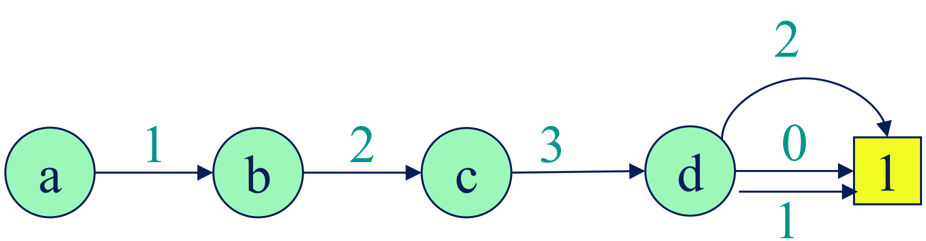
$$Seek(h,v)(e,x) = \begin{cases} h^* \circ e \xrightarrow{x} Id & \textit{if } v=e \\ e \xrightarrow{x} Seek(h,v) & \textit{otherwise} \end{cases}$$

$$Seek(h,v)(1) = T$$

$$Max(z)(e,x) = \begin{cases} e \xrightarrow{x} Id & \textit{if } x<z \\ 0 & \textit{otherwise} \end{cases}$$

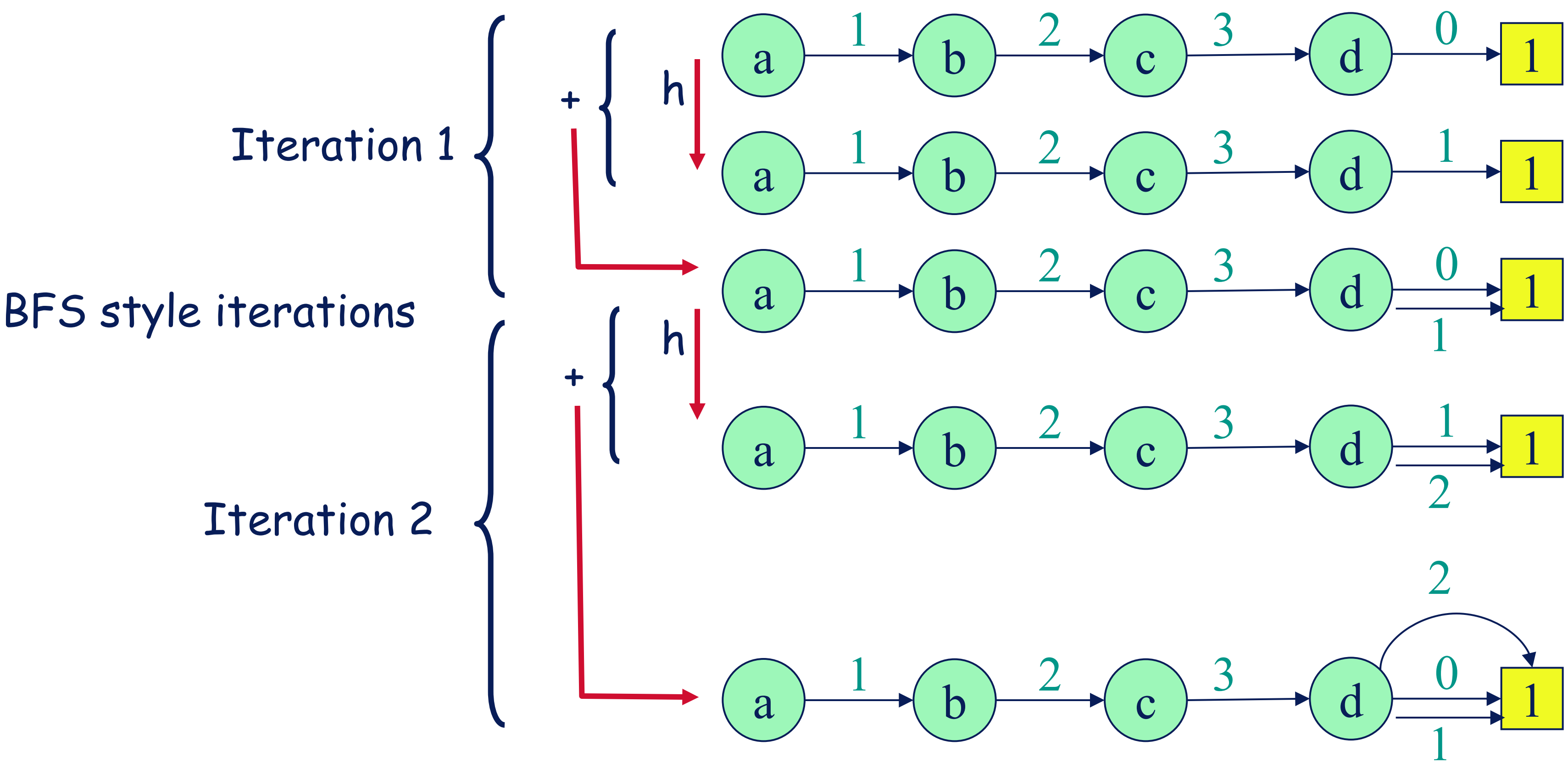
$$Max(z)(1) = T$$

$h = Max(3) \circ Inc(d)$ // Increment *d* up to 2



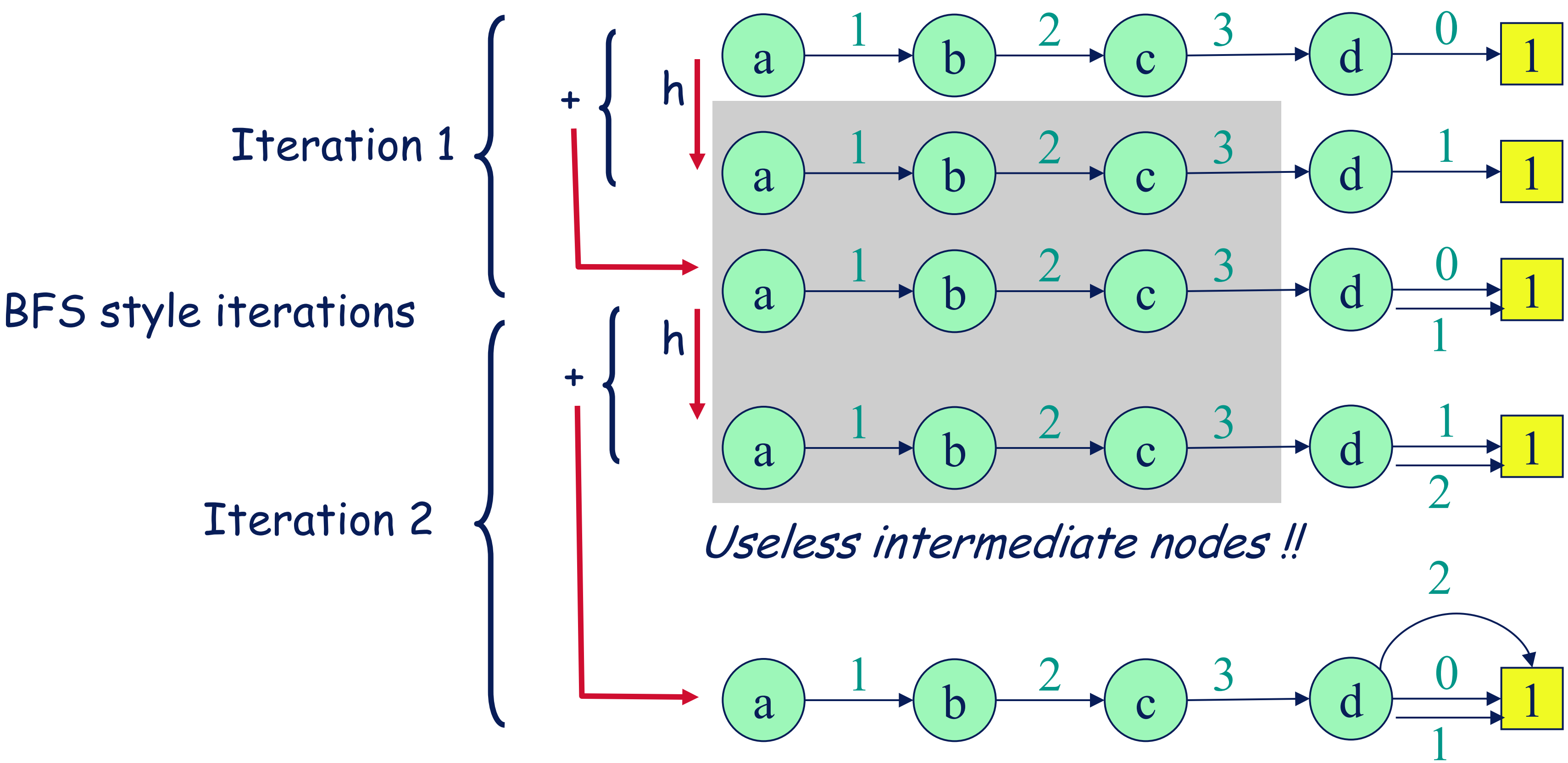
Fixpoint conclusions

- Transitive closure or fixpoint allows:
  - single traversal of the top of the tree
  - less intermediate nodes



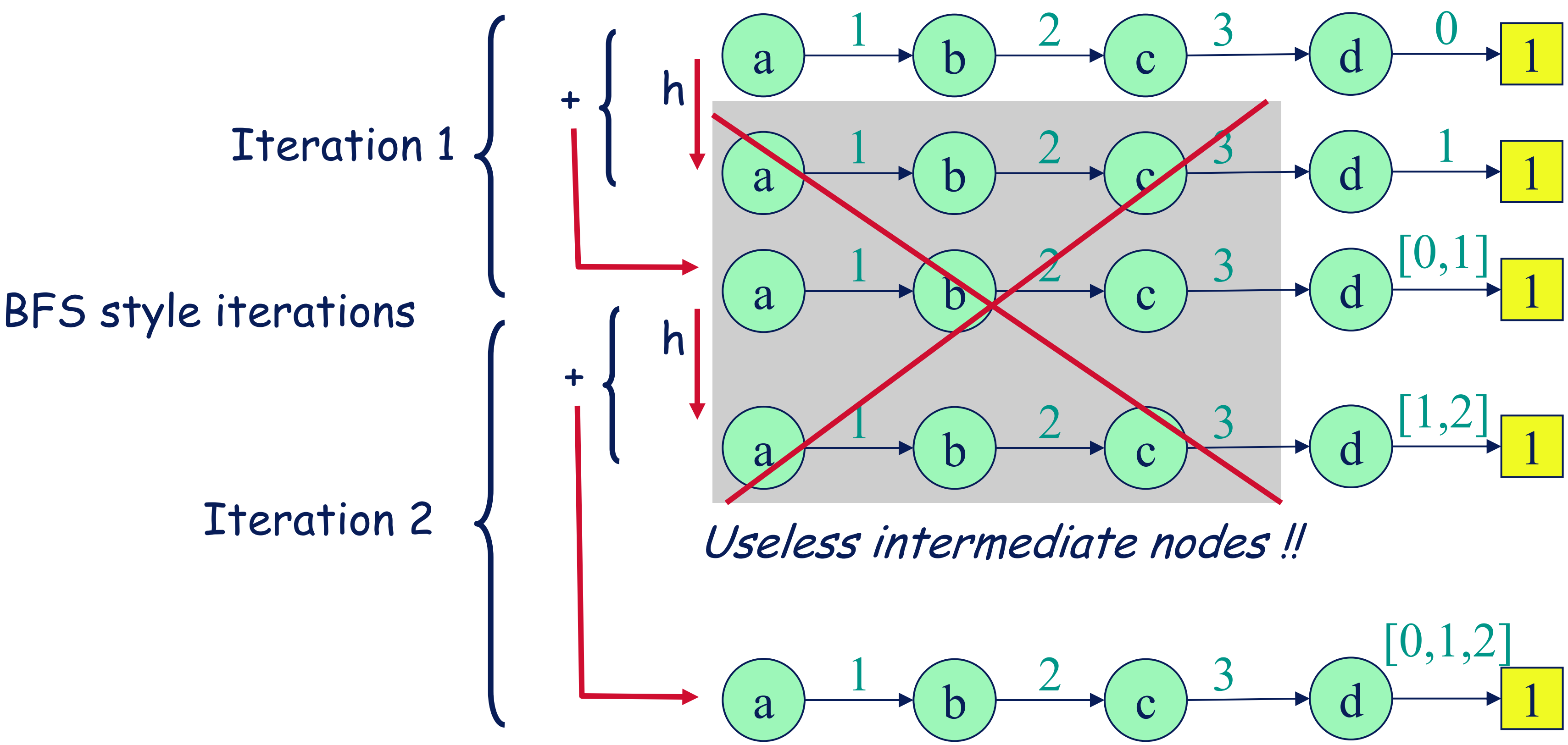
Fixpoint conclusions

- Transitive closure or fixpoint allows:
  - single traversal of the top of the tree *=> cost of + and h*
  - less intermediate nodes



Saturation effect

- Nested transitive closure or fixpoint = saturation allows:
  - single traversal of the top of the tree *=> cost of + and h*
  - less intermediate nodes



Performance measures : effect of saturation

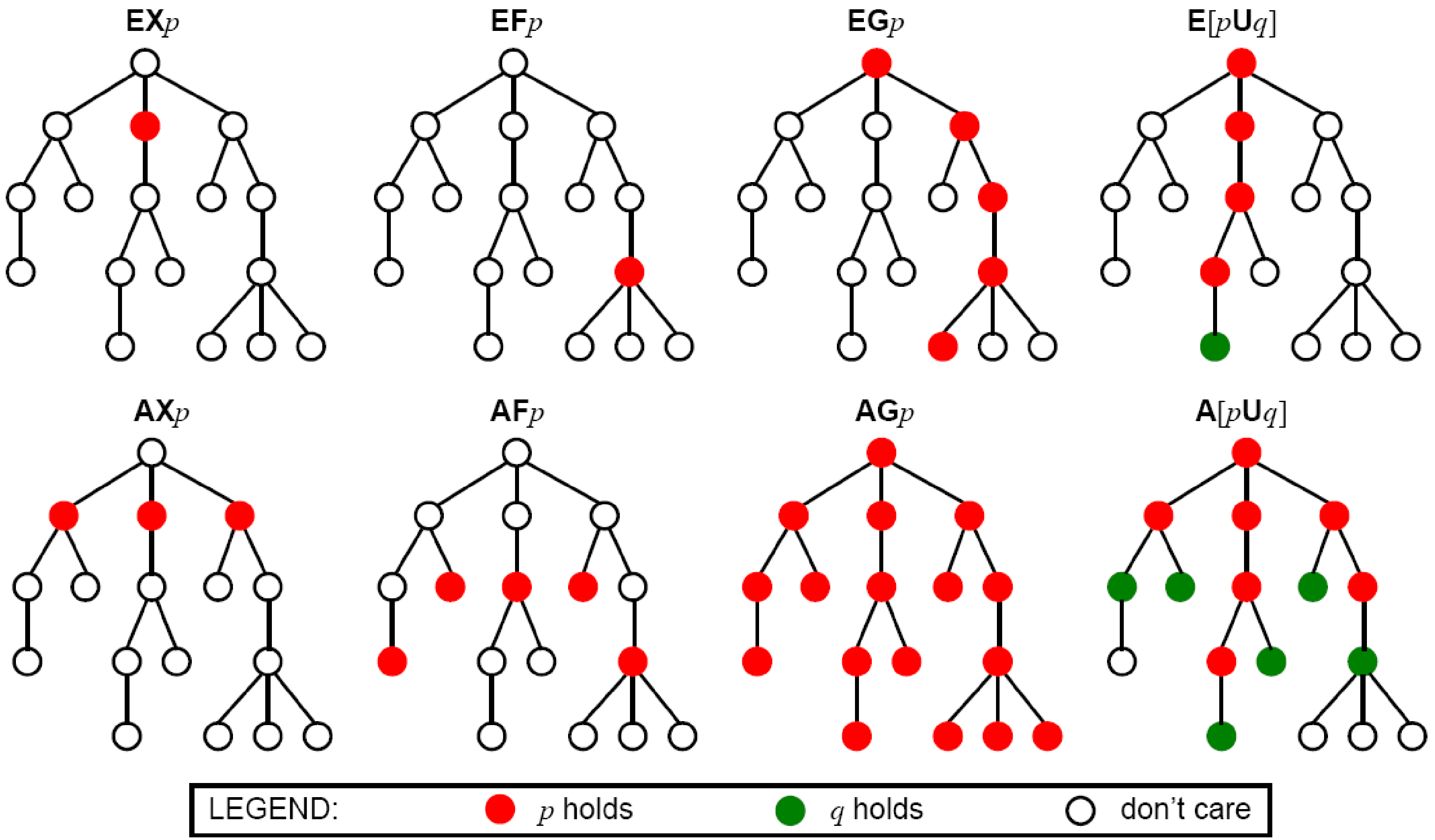
- Transitive closure allows more efficiency
  - Manual Saturation "à la Ciardo" (Tacas'01 and '03) using * operator
  - Organize events by highest variable affected

				PNDDD no sat		PNDDD sat	
Model	N	Nb. States	final nodes	total nodes	time (s)	total nodes	time (s)
Dining Philosophers	50	2.23e+31	1387	13123	11.6	10739	0.09
	100	4.97e+62	2787	26823	54.19	21689	0.18
	200	2.47e+125	5587	54223	234	43589	0.39
	1000	9.18e+626	27987	-	-	218789	2.1
Slotted Ring Protocol	10	8.29e+09	1281	35898	83.07	45970	0.8
	15	1.46e+15	2780	118054	595	132126	2.26
	50	1.72e+52	29401	-	-	3.58e+06	61.58
Flexible Manufacturing System	10	2.50+09	580	8604	2.06	11202	0.17
	25	8.54e+13	2545	50489	28.75	85962	1.58
	50	4.24e+17	8820	231464	240.4	490062	9.78
	80	1.58e+20	21300	-	-	1.72e+06	37.06
Kanban	10	1.01e+09	257	26862	20.47	5837	0.06
	50	1.04e+16	3217	-	-	209117	3.96
	100	1.73e+19	11417	-	-	1.32e+06	28.09
	200	3.17e+22	42817	-	-	9.23e+06	238.95

CTL Model checking

- Computation Tree Logic
  - (infinite) tree of all possible executions of a system
  - Superset of Boolean first order logic
    - *AND, OR, NOT, TRUE, FALSE*
  - Additional temporal operators
    - *G : Generally (p holds in all states)*
    - *X p: neXt (p holds in a successor state by one step)*
    - *F p: Future (p holds in a state reachable in arbitrary number of steps)*
    - *p U q: Until (p holds until q holds)*
  - Modalities : Exists, Always

CTL semantics



EX, EU, and EG are a complete set of CTL operators, since:

$$\mathsf{AX}p = \neg\mathsf{EX}\neg p \qquad \mathsf{EF}p = \mathsf{E}[true \,\mathsf{U}\, p] \qquad \mathsf{E}[p\mathsf{R}q] = \neg\mathsf{A}[\neg p\mathsf{U}\neg q]$$

$$\mathsf{AF}p = \neg\mathsf{EG}\neg p \qquad \mathsf{A}[p \,\mathsf{U}\, q] = \neg\mathsf{E}[\neg q \,\mathsf{U}\, \neg p \;\wedge\; \neg q] \wedge \neg\mathsf{EG}\neg q \qquad \mathsf{A}[p\mathsf{R}q] = \neg\mathsf{E}[\neg p\mathsf{U}\neg q]$$

$$\mathsf{AG}p = \neg\mathsf{EF}\neg p$$

Reminder Symbolic Tools

- We have
  - A compact representation for sets of states
  - A representation of transition relations, as a set of pairs of states
- A model provides (So,Next,Lab)
  - An initial state So
  - A successor relation Next, that gives the set of successors reachable in one step from a set of states
  - A labeling function, that tags individual states with the truth value of atomic propositions (e.g. x<3)

Model-checking CTL

- Start from syntactic tree of the formula
- Compute all states S(p) satisfying a formula p recursively
- Terminal cases :
  - p=True : all reachable states
  - p=False : empty set of states
  - p=Atomic predicate (e.g. x<3): all reachable states satisfying p
- Boolean cases :
  - p AND q : intersect states S(p) and S(q)
  - p OR q : union states S(p) and S(q)
  - NOT p : all reachable states set minus S(p)
- Temporal operators :
  - Reduce to EX, EU, EG using equivalences
  - Algorithms on next slides

EX p: next

- Given states satisfying p, S(p)
  - Compute predecessors of S(p)
    - *If a state is a predecessor of a state satisfying p, it satisfies neXt p*
  - Requires Pred, i.e. the invert of Next
    - *Easy in classic model, just revert pairs of states in the representation*
    - *Can be more difficult if computed dynamically : risk of unreachable states*
    - *Still possible in general by intersection with forward reachable set*

BDD based algorithm for EX(f)

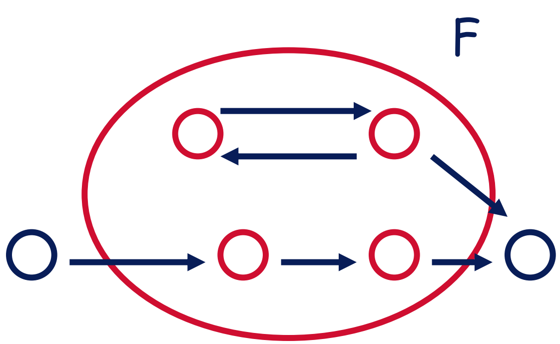
- Let F be the set of states satisfying “f”
  - F can be built by selecting states from the full state space
- EX(F)
  - S := $Next^{-1}$ (F)
  - Return S

Operator EG for EG (f)

- Let F be the set of states satisfying “f”
- EG(F)
  - S := F
  - N := 0
  - While (N != S)
    - N := S
    - S := S ∩ $Next^{-1}$ (S)
  - Return S

Initialize with states that verify f
Potentially all these states verify Gf

Remove some potential candidates state
If s verifies Gf, s verifies f
and successor verifies “f”



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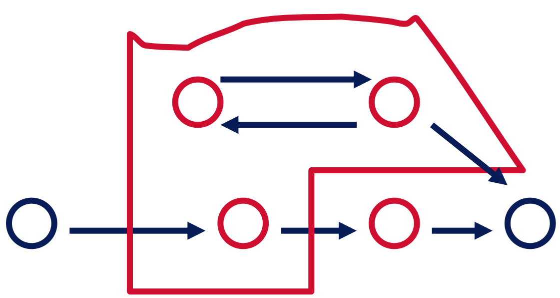

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Initialize with states that verify f
Potentially all these states verify Gf

Remove some potential candidates state
If s verifies Gf, s verifies f
and successor verifies "f"

$$\boxed{(Next^{-1} \cap Id)^* \circ F}$$

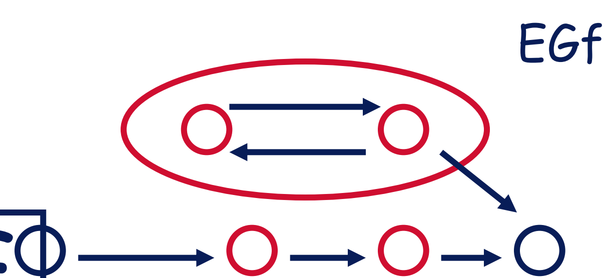
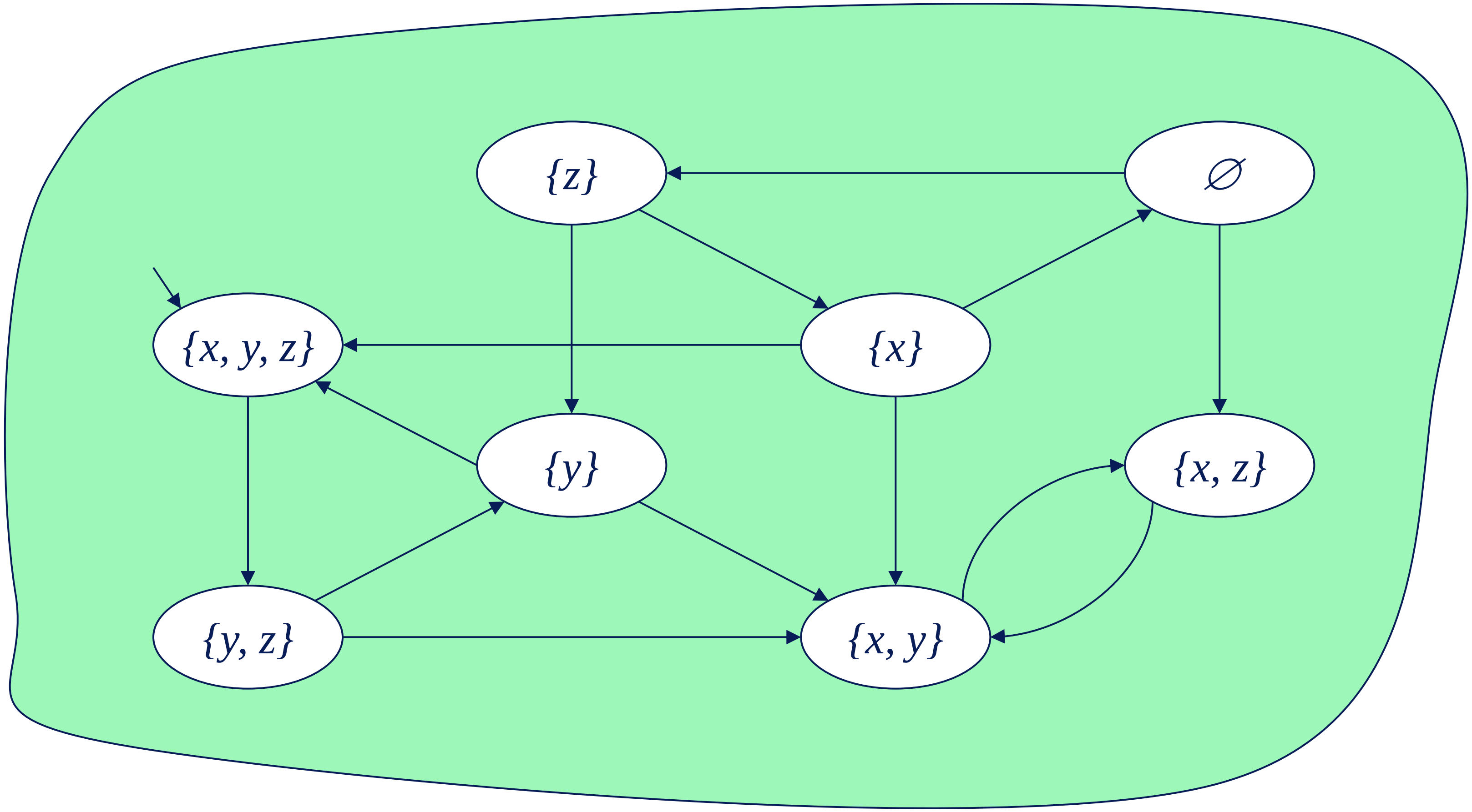
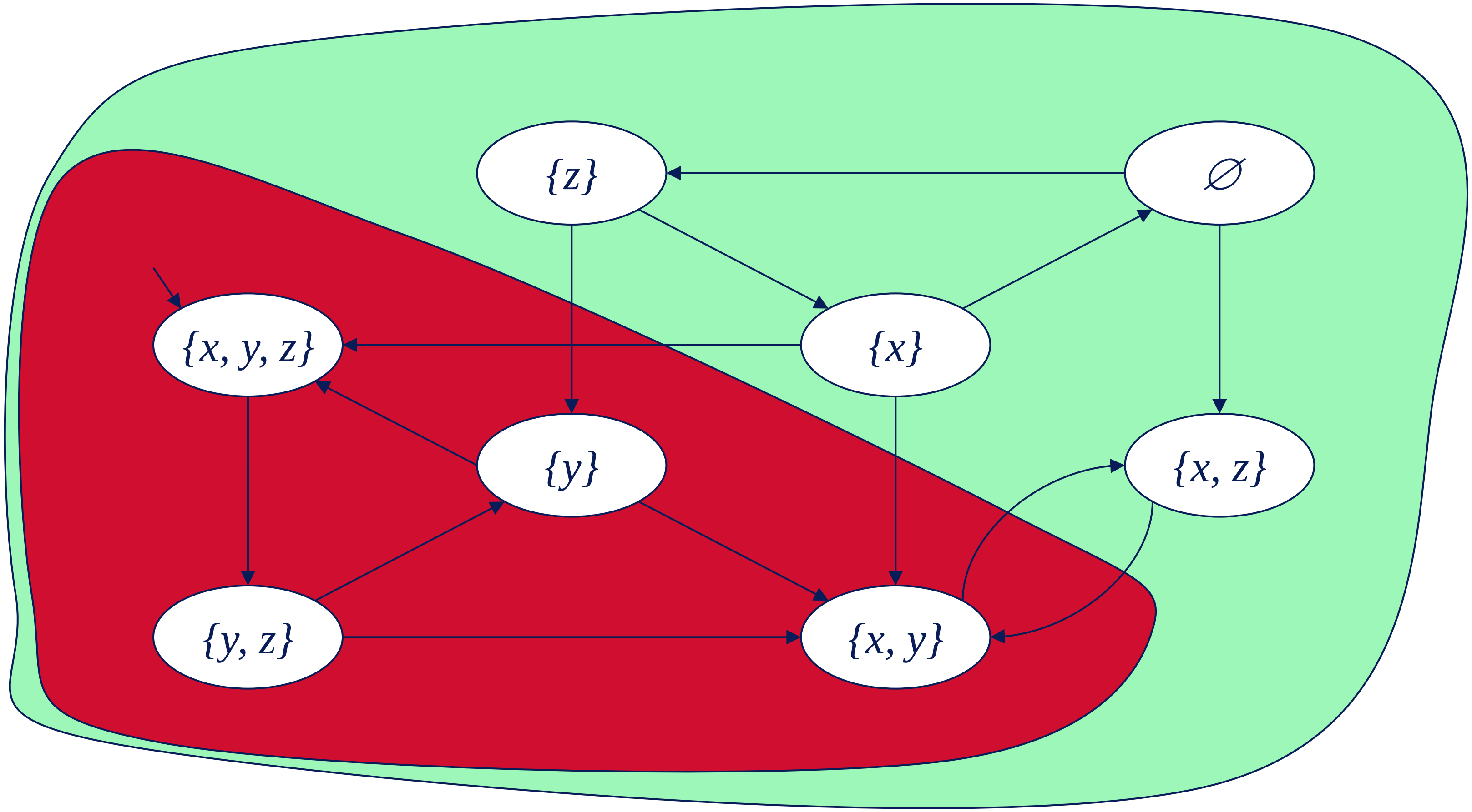


Illustration for *EG y*



$\pi^0(S) = S$

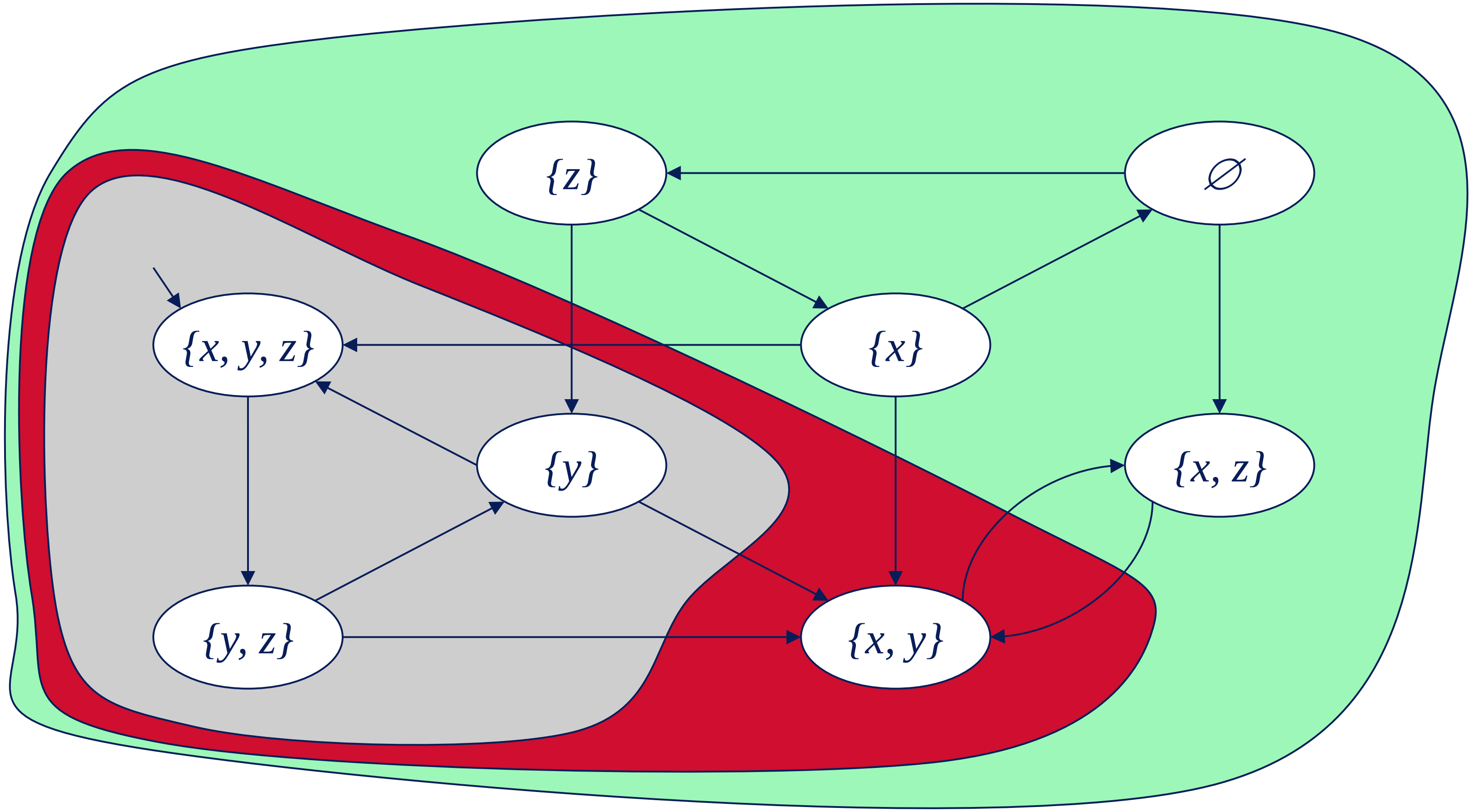
Illustration for *EG y*



$\pi^1(S) = S_K(y) \cap pre(S)$

States not satisfying *y* have been excluded

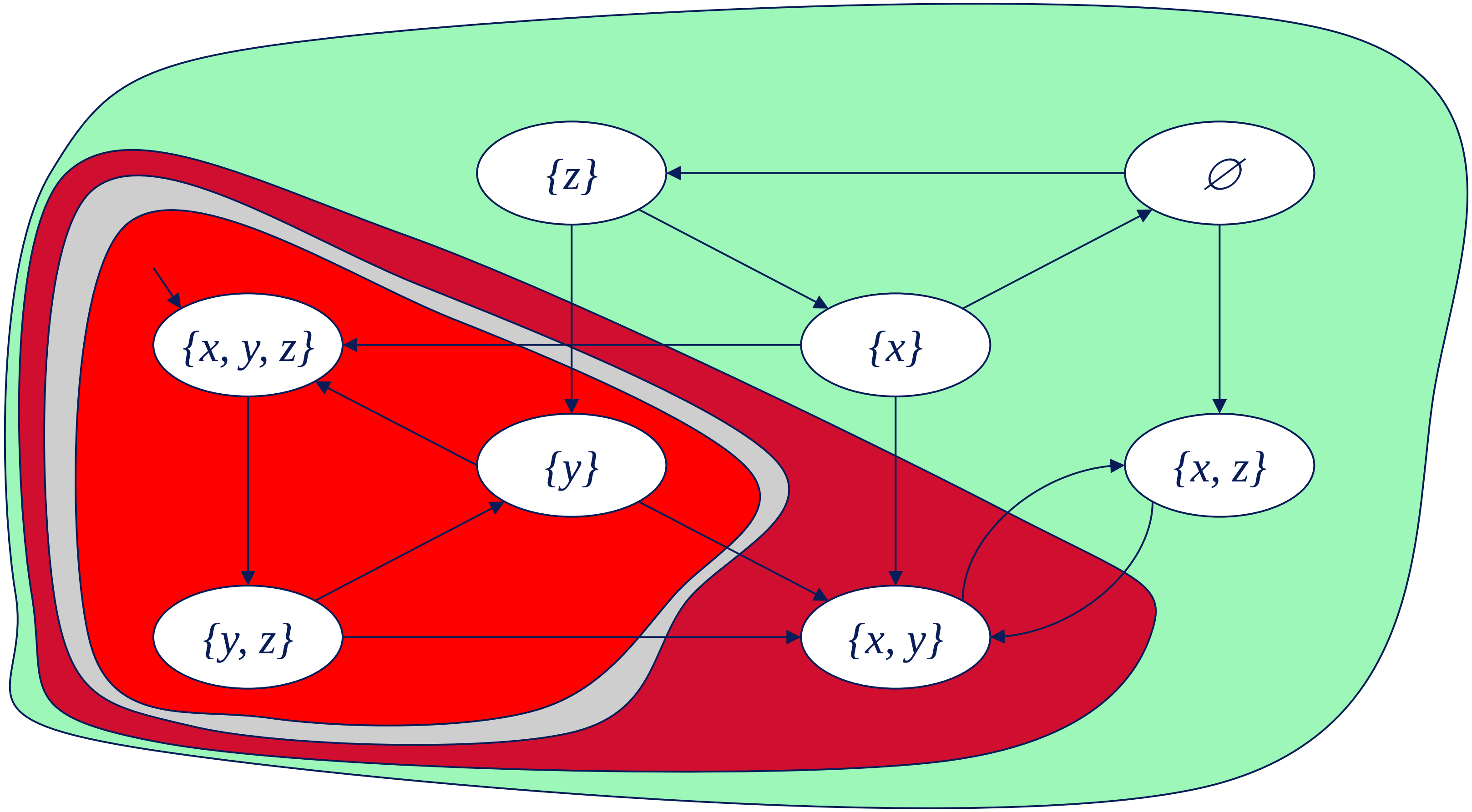
Illustration for *EG y*



$\pi^2(S) = S_K(y) \cap pre(\pi^1(S))$

States having all its successors outside π^1 have been excluded

Illustration for *EG y*



$\pi^3(S) = S_K(y) \cap pre(\pi^2(S))$

The fixed point has been reached

Operator EU for E(f U g)

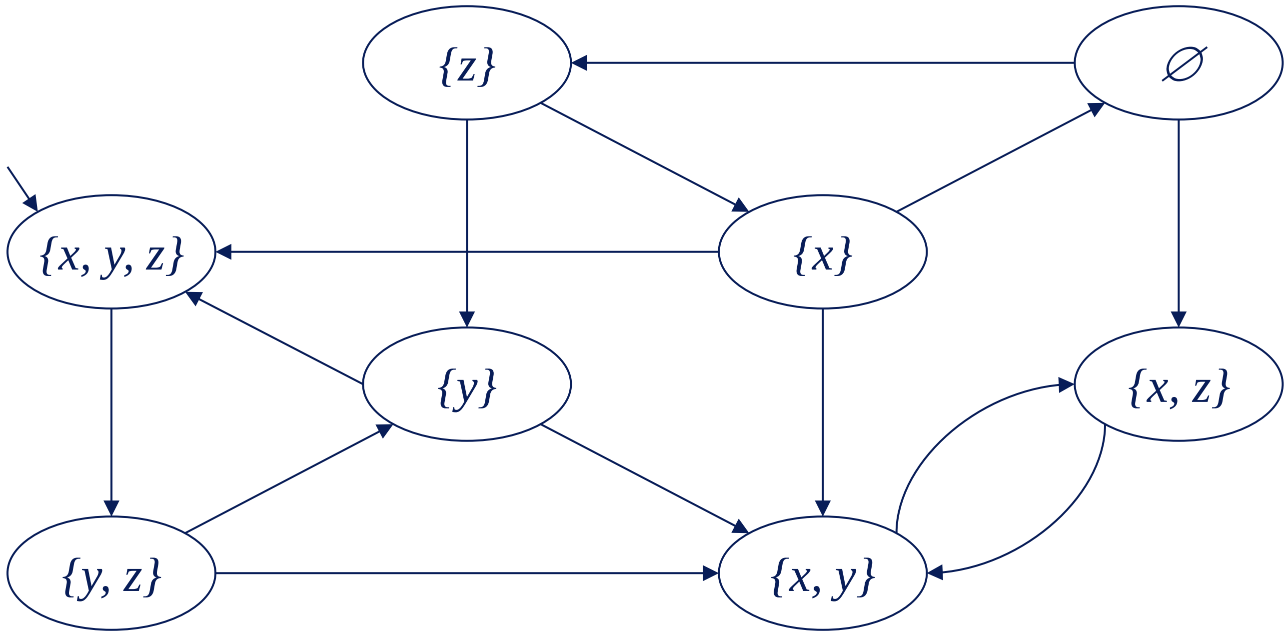
- Let F and G be the set of states satisfying "f" and "g"
- EU(F,G)
  - S := G
  - N := 0
  - While (N != S)
    - N := S
    - S := S U (F o Next^{-1} (S))
  - Return S

Initialize with states that verify g

Keep only predecessors that verify f

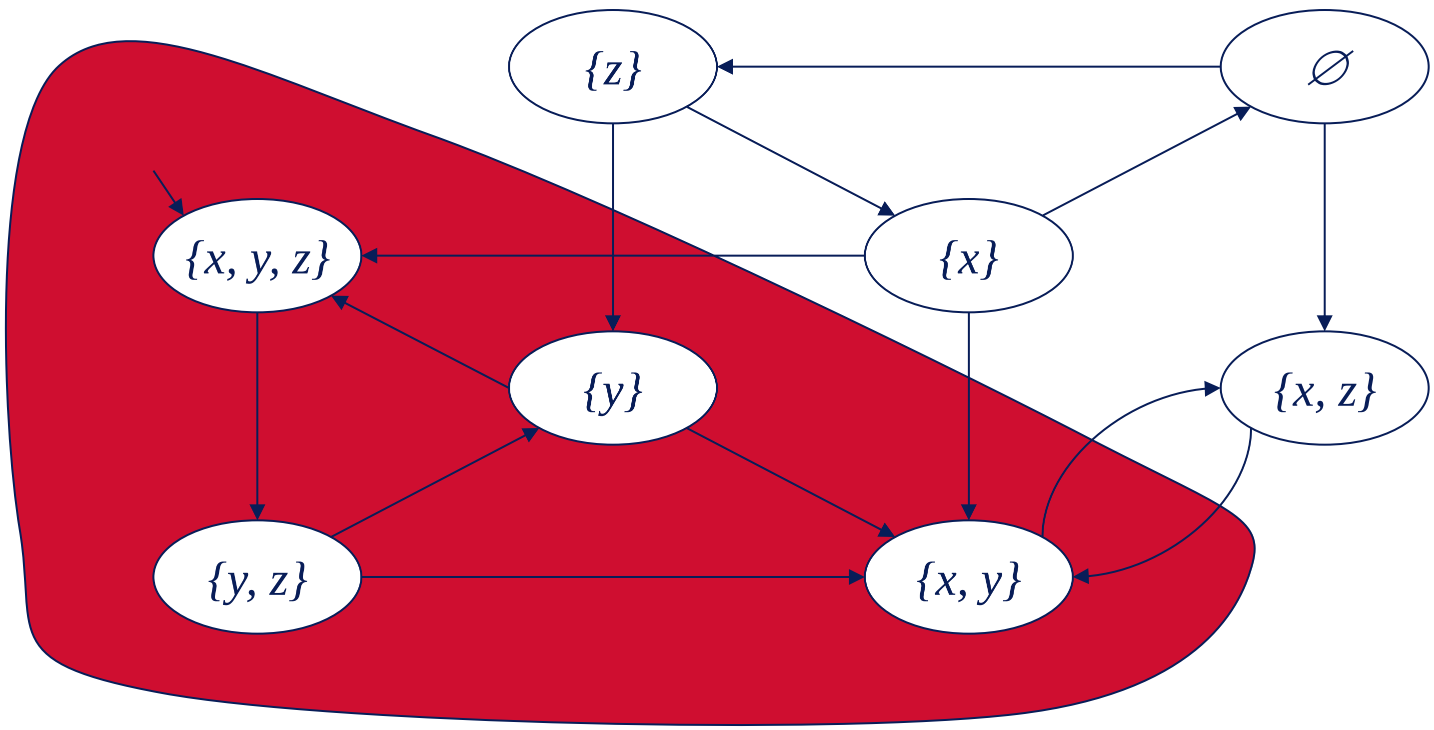
$$\boxed{(F \circ Next^{-1} + Id)^* \circ G}$$

Illustration for z *EU* y



$\xi^0(\varnothing) = \varnothing$

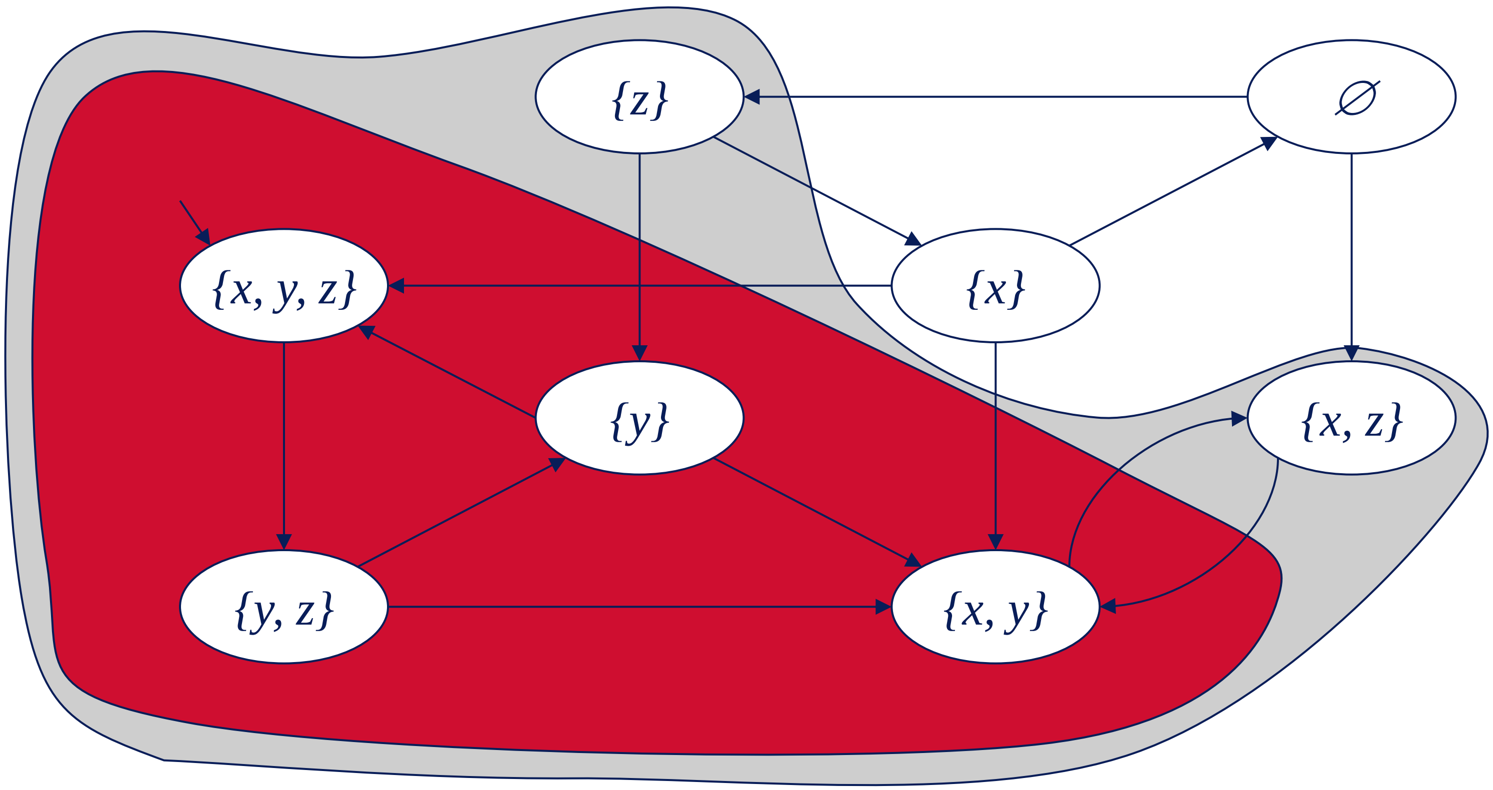
Illustration for z *EU* y



$\xi^1(\varnothing) = S_K(y) \cup (S_K(z) \cap pre(\xi^0(\varnothing)))$

States satisfying y have been added

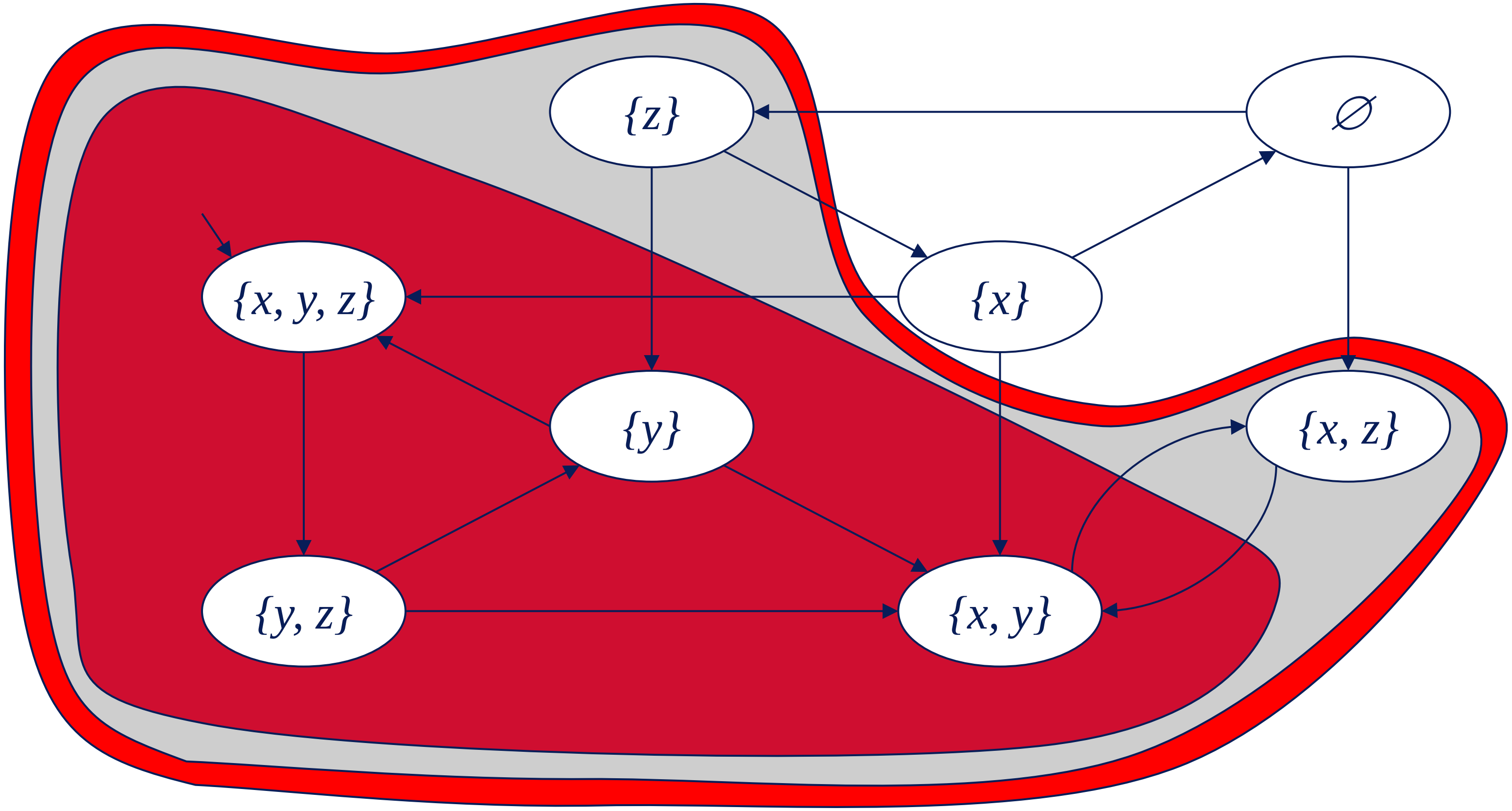
Illustration for *z* ***EU*** *y*



$\xi^2(\emptyset) = S_K(y) \cup (S_K(z) \cap pre(\xi^1(\emptyset)))$

States satisfying z and having at least a successor in ξ^1 have been added

Illustration for z *EU* y



$\xi^3(\emptyset) = S_K(y) \cup (S_K(z) \cap pre(\xi^2(\emptyset)))$

The fixed point has been reached

Conclusion on CTL

- CTL (Branching time) can specify safety properties and some liveness properties
- CTL can be efficiently implemented (linear complexity w.r.t. to the Kripke structure), provided a good management of sets of states.
- Fairness needs to augment the capability of CTL model checkers (SCC searches are needed).
- CTL fair model checkers can be used to verify CTL and also LTL formula.
- CTL does not provide a single counter example when the property does not hold. The output is the set of states that satisfy the formula (maybe huge).
  - Witness paths or counter examples can still be exhibited
- CTL model checkers cannot answer before labeling the initial state with the truth value of the formula.

A Guarded Action Language to express system semantics

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Joint work with
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Nov. 2013 – ENS Cachan
Journée AFSEC

Model Driven Development and Model-checking

- In MDD approaches
  - Build a Domain Specific Language
  - Use model transformation for specific targets
- Choosing a target formalism
  - Expressive enough to capture your semantics
  - Efficient solution engine
- We propose ITS/GAL formalism
  - Allows to express discrete state semantics
  - Symbolic model-checking

Guarded Action Language

- GAL : a « DSL to express Semantics »
  - Simple to use, easy C-like syntax
  - Straightforward Petri net style concurrent semantics
  - Integer variables and arrays + arbitrarily nested array expressions
  - Efficient symbolic solution engine
  - Subsumed by Instantiable Transition Systems (ITS), allowing hierarchical composition of GAL modules
- Meant to be a back-end target in a transformation process.
- Define your semantics in GAL.

GAL example


```
GAL system {
    // Variable declarations
    int variable = 5 ;
    array [2] tab = (1, 2) ;

    transition t1 [variable > 9] {
        tab [0] = tab [1] * tab [0] ;
        variable = variable * 5 ;
    }

    transition t2 [variable == 23] label "a" {
        tab [1] = 0 ;
    }
}
```

GAL Variables

- All variables are 32 bit integers or arrays of fixed size.
- Any variable must be initialized
  - int a = 0;
  - array [3] tab = (0,0,0);

Arithmetic and Boolean

- Terminal expressions are signed constants, parameters, variables, array access with arbitrary index expression
  - 3,-2,$MAX,x,tab[x+1],tab[tab[$MAX-x]]
- All C operators supported
  - Bitwise : &, |, ^, <<, >>, ~
  - Integer : +, -, *, /, %, **
- Boolean expressions
  - Basics : true, false, &&, ||, !
  - Comparisons of integers: ==,!=, <,<=, >,>=
- x = (y == 255) * 100;   // x is 0 or 100

GAL Statements

- <lhs=rhs> assign integer expression rhs to variable designated by lhs.
- <s1;..;sn> sequence of statements, <nop> the empty sequence
- <ite(c,t,f)> an if-then-else statement
- <for(min, max, b)> a limited form of iteration
- <abort> return the empty set (!)
- <call(a)> call a label (i.e. an arbitrary transition with label a) of « self »
- <fixpoint(b)> fixpoint statement

GAL Transitions

- Tuple : <label, guard, body>
- Fire : In any state where guard is enabled, process body statement(s) atomically
- Tau (empty) label for local transitions
- Labeled transitions are not fireable by Locals outside of call or synchronization
- Guard is a boolean expression
- Body is a statement

System Parameters

- Allow easier configuration of a model

```
GAL paramSystem ($N = 2, $K = 1) {
    int variable = $N ;
    array [2] tab = ($N + $K, $N - 1) ;
    transition t1 [variable > $N] {
        tab [$K] = tab [1] * tab [0] ;
        variable = variable * 5 ;
    }
    transition t2 [variable == $N] label "a" {
        tab [1] = 0 ;
    }
}
```

Range Type definitions & Transition parameters


```
GAL paramDef ($N=2) {
    typedef paramType = 0..$N;
    typedef paramType2 = 0..1;
    int variable = 0;

    // a transition compactly modeling ($N+1)*2
    // basic transitions
    transition trans (paramType $p1, paramType2 $p2)
              [$p1 != $p2] {
        variable = $p1 + $p2;
    }
}
```

If then else


```
GAL iteExample {
    int variable = 0 ;

    transition invert [variable == 0 || variable == 1] {
        if (variable == 0) {
            variable = 1;
        } else {
            variable = 0;
        }
    }
}
```

Equivalent to xor : variable = variable ^ 1

For loop

- Limited iteration

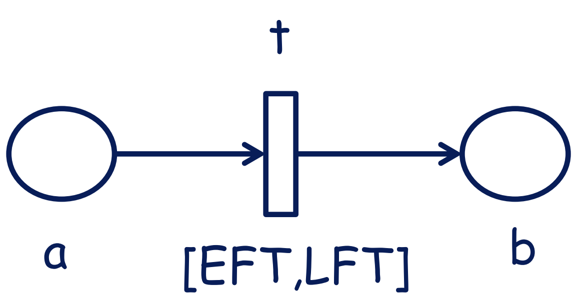
```
GAL forLoop  {
    typedef Dom = 0..2;

    array [3] tab  = (0,0,0);

    transition forExample [true] {
        for ($i : Dom) {
            tab[$i] = $i;
        }
    }
}
```

```
GAL forLoop_inst {
    array [3] tab = (0, 0, 0) ;

    transition forExample [true] {
        tab [0] = 0 ;
        tab [1] = 1 ;
        tab [2] = 2 ;
    }
}
```

```
GAL callExample {
    int variable = 0 ;

    transition NDassignX [variable == 0 || variable == 1] {
        self."setX" ;
    }

    transition callee1 [true] label "setX" {
        variable = 1 ;
    }

    transition callee2 [true] label "setX" {
        variable = 0 ;
    }
}
```

Abort

```
GAL abortExample ($EFT = 1, $LFT = 3) {
    int a = 1 ;
    int b = 0 ;
    int t.clock = 0 ;

    transition t [a >= 1 && t.clock >= $EFT] {
        a = a - 1 ;
        b = b + 1 ;
        t.clock = 0 ;
    }
```



Abort (2)


```
transition elapse [true] label "elapse" {
    // is t enabled ?
    if (a >= 1) {
        // is t's clock strictly less than
        // its latest firing time ?
        if (t.clock < $LFT) {
            // if yes increment t clock
            t.clock = t.clock + 1 ;
        } else {
            // otherwise, time cannot elapse,
            //  kill exploration
            abort ;
        }
```

Fixpoint : sort example

```
GAL sortEx {
  typedef index = 0..3;                       // 0 to  n-2
  array [5] tab = (3,1,2,4,5);
  int tmp = 0;

  transition swap (index $i) [ tab[$i] > tab[$i+1] ]  label "sort"
  { tmp = tab[$i];  tab[$i] = tab[$i+1]; tab[$i+1] = tmp; tmp = 0;   }

  transition sorted  label "sort" [true] {
        for ($i : index) {
                if (tab[$i] > tab[$i+1]) { abort; } } }

  transition sort [true] {
        fixpoint { self."sort"; }
  }
}
```

Fonction de Canonisation…

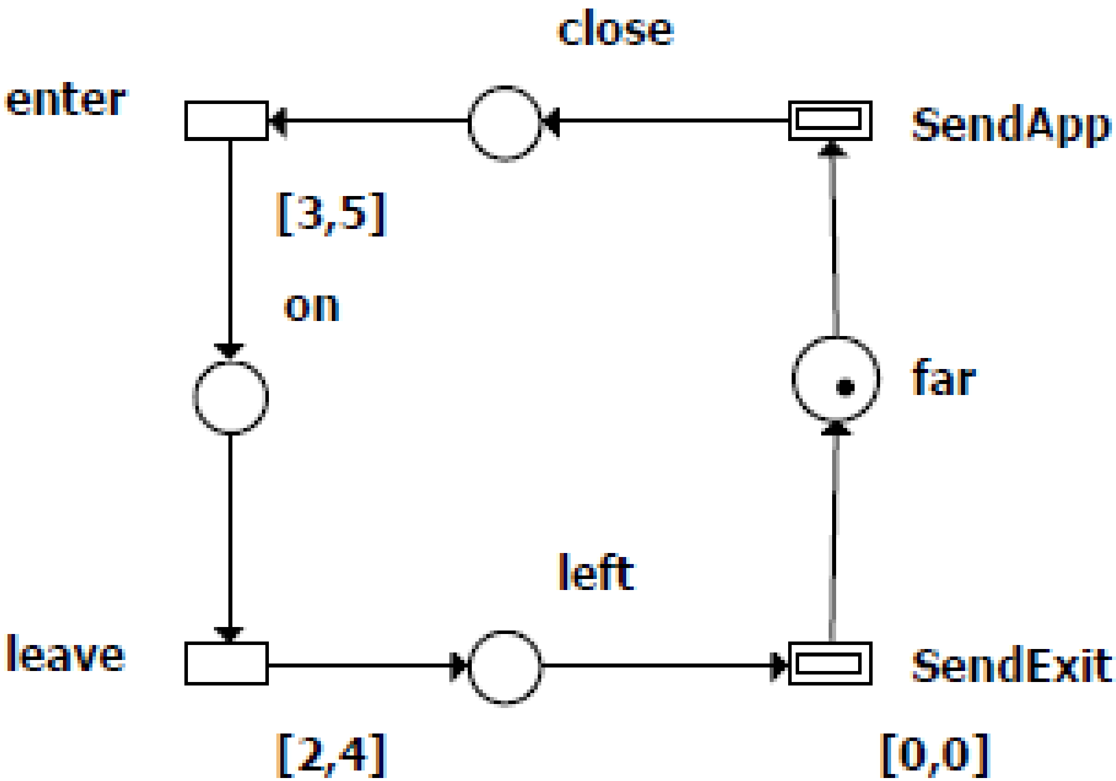
Some applications of GAL

- Petri nets
- Discrete Time Petri nets
- Colored Petri Nets
- Divine (Promela-like) models
- CCSL clock logic
- …

Encoding Petri nets

- Each place => a variable
- Each transition => a transition
  - Guard tests enabling conditions
  - Actions update state variables
- Easy to support many extensions of PN
  - Test arcs
  - Reset arcs
  - Inhibitor
  - Capacity places
  - …

Labeled Discrete TPN model of a train



Discrete Time Petri Net Semantics

- Place -> integer variable,
  - initial value=initial marking
- Transitions -> define variable t.clock
  - unless [0,0] or [0,inf[
- Time elapse -> additional transition labeled « elapse »
  - Sequence, for each transition t

```
If (enabled(t)) {
    If ( t.clock < lft(t) ) {
        t.clock=t.clock+1;
    } else {
        abort;
    }
}
```
General case

```
If (enabled(t)) {
    abort;
}
```
[0,0] urgent case

```
If (enabled(t)) {
    If ( t.clock < eft(t) ) {
        t.clock=t.clock+1;
    }
}
```
[a,inf[infinite lft case

DTPN : Transitions

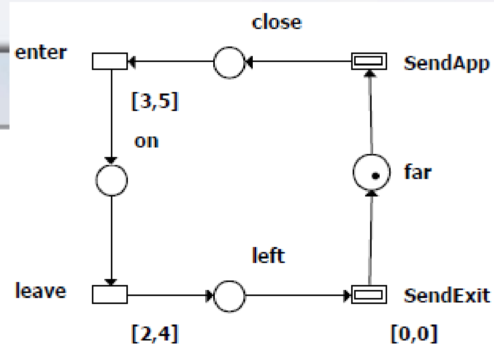
- Transition t -> transition <l,g,b>
  - l: Label is copied from input transition,
  - g: enabled(t) and t.clock >= eft(t)
  - b:
    - *update place markings according to arc types and inscriptions,*
    - *reset current clock,*
    - *reset any disabled transition clocks (call(reset))*
- Reset disabled transitions: <reset,true,b> for each transition t :

```
If (! enabled(t)) {
        t.clock=0;
}
```

General case

If no clock do nothing

Train Example, GAL



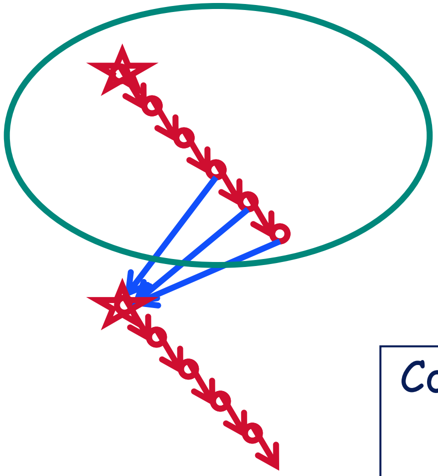

```
GAL train {
	int far = 1 ;
	int close = 0 ;
	int on = 0 ;
	int left = 0 ;
	int enter.clock = 0 ;
	int leave.clock = 0 ;

	transition SendApp [ far >= 1 ] label "SendApp" {
		far = far - 1 ;
		clo
		sel
	}

	transitio
		clo
		on
		ent
		sel
	}

	transitio
		on
		lef
		lea
		sel
	}

	transition SendExit [ left >= 1 ] label "SendExit" {
		left = left - 1 ;
		far = far + 1 ;
		self.reset ;
	}
```

```
transition elapse [ True ] label "elapse" {
	if (close >= 1) {
		if (enter.clock < 5) {
			enter.clock = enter.clock + 1 ;
		} else {
			abort ;
		}
	}
	{
	eave.clock + 1 ;

	eset" {

	if (! on >= 1) {
		leave.clock = 0 ;
	}
}
}
```

```
transition enter [ close >= 1 && enter.clock >= 3 ] {
	close = close - 1 ;
	self.reset ;
	on = on + 1 ;
	enter.clock = 0 ;
}
```

Using a fixpoint

- Essential states construction [Popova]
  - Letting time elapse cannot disable transitions
  - Let time progress, only consider states that immediately follow a discrete transition



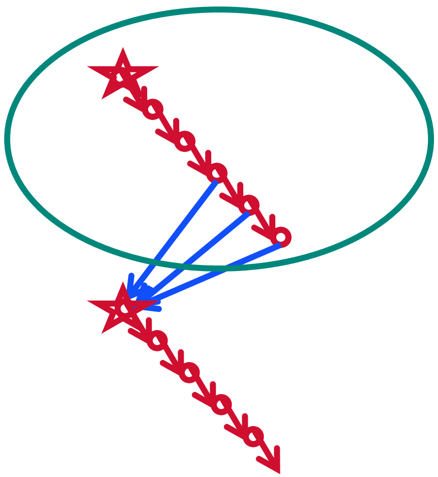
Compute green set :
    let time elapse if it can
    cumulate states
Fire transitions (succ) from resulting set

Fixpoint for « Essential States » of TPN


```
GAL tpnModel ($EFT = 3, $LFT = 5) {
    int a = 1 ;
    int b = 0 ;
    int t.clock = 0 ;

    transition t [a >= 1 && t.clock >= $EFT] label "succ"
    {
        a = a - 1 ;
        b = b + 1 ;
        t.clock = 0 ;
    }
```

Essential States (2)


```
   transition elapseIfpossible
      [ a >= 1 && t.clock < $LFT] label "elapseIP" {
       t.clock = t.clock + 1 ;
   }
    transition id [true] label "elapseIP" {
   }
 transition nextState [true] {
    fixpoint {
      self."elapseIP" ;
    }
   self."succ" ; // Any discrete transition
  }
}
```

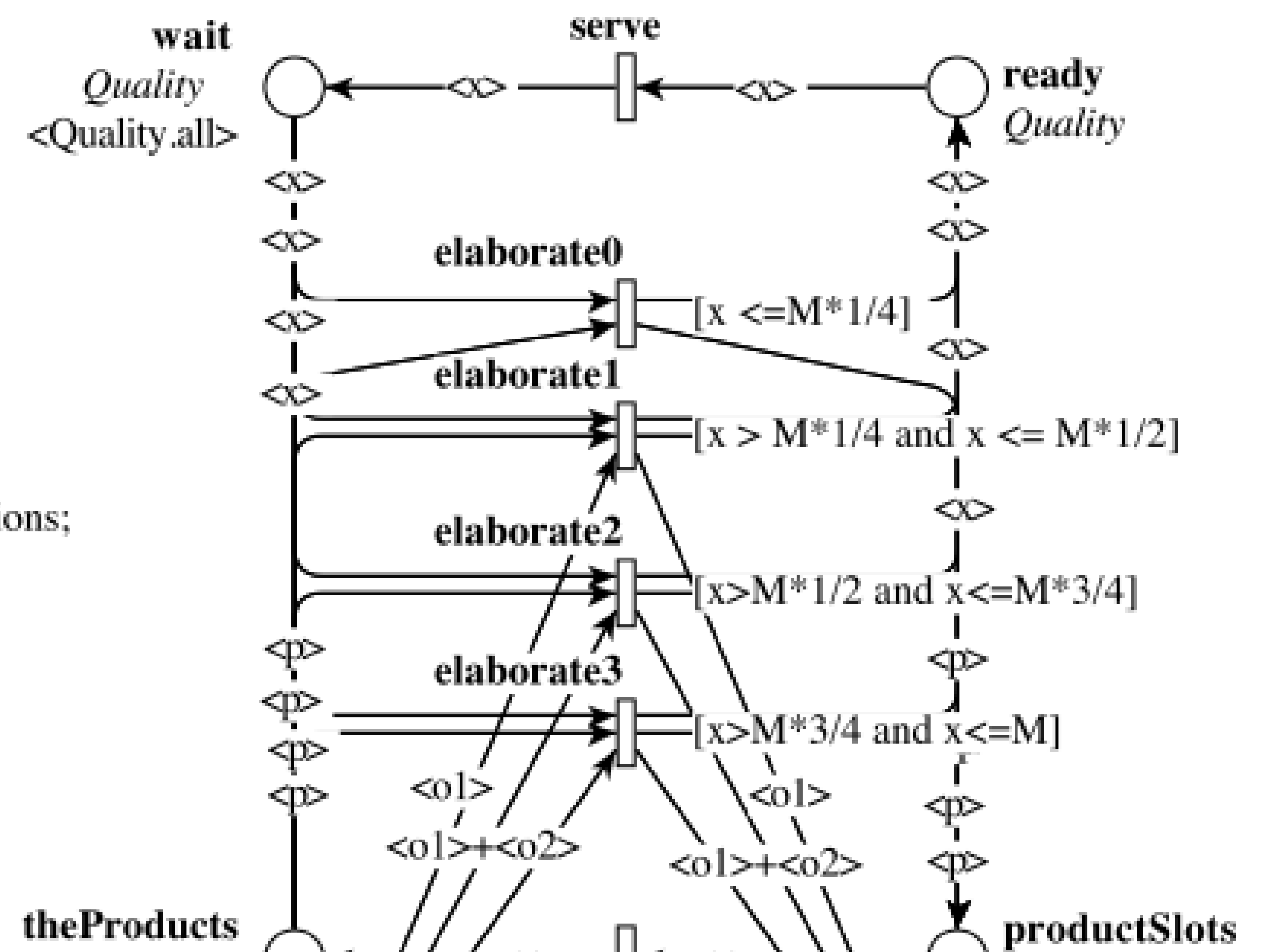


Colored Petri nets

- Each place => an array of dimension proportional to domain
  - Uncolored places => size 1
- Formal parameters => typedef of a range
- Each transition => transition with parameters

An example

Class
Quality is 1..M;
Products is 1..N;
Options is 1..N;
Var
p in Products;
o1, o2, o3 in Options;
x in Quality;



CPN example

```
Class
  Quality is 1..M;
  Products is 1..N;
  Options is 1..N;
Var
  p in Products;
  o1, o2, o3 in Options;
  x in Quality;
```

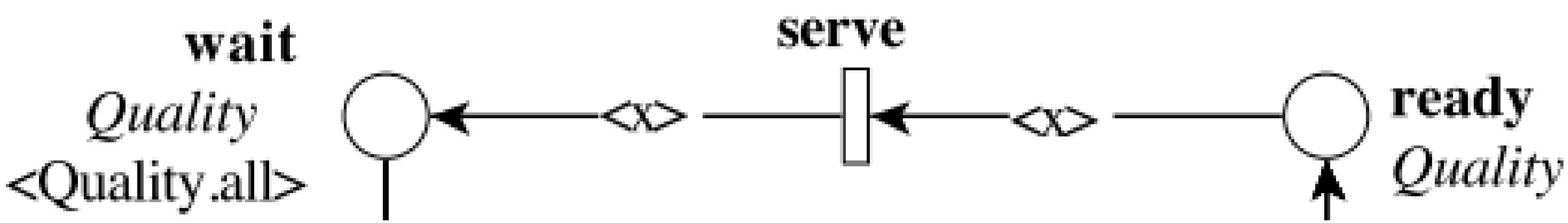
```
GAL DrinkVending2 {
typedef Options = 0 .. 1 ;
typedef Products = 0 .. 1 ;
typedef Quality = 0 .. 7 ;
array [8] ready = (0, 0, 0, 0, 0, 0, 0, 0) ;
array [8] wait = (1, 1, 1, 1, 1, 1, 1, 1) ;
array [2] theProducts = (1, 1) ;
array [2] productSlots = (0, 0) ;
array [2] theOptions = (1, 1) ;
array [2] optionSlots = (0, 0) ;
```

Color Domains
M=8, N=2

Places

CPN example (2)

```
transition serve (Quality $x) [ready [$x] >= 1] {
        ready [$x] = ready [$x] - 1 ;
        wait [$x] = wait [$x] + 1 ;
}
```



Divine : a language for concurrent process

```
byte id;
byte t[3] = { 255 ,255 ,255  };
process P_0 {
        state NCS, try, wait, CS;
        init NCS;
trans
        NCS -> try
{ guard id == 0; effect t[0] = 2;},
        try -> wait
{ effect t[0] = 3, id =0 +1; },
        wait -> wait
{ guard t[0] == 0; effect t[0] = 255;}, …
```

Global Variables

Process + Channels

Analyzing Divine models

- Translate Divine concepts to GAL
  - Process state => variable
  - Divine variables and arrays => GAL equivalent
  - Guards, Instructions => GAL equivalent
  - Synchronizations => use GAL call semantics
  - Channels => GAL arrays + variable for size

Symbolic analysis of expressions

- Symbolic data structures: BDD, MDD
  - k boolean variables => 2^k
- Encode transitions with sets : $2^k \times 2^k$
- Use the support (only k' vars) of transitions
  - Build clusters of transitions
- Reorder evaluations in fixpoint computation (chaining, saturation)
- But :
  - Exponential worst case complexity when k' grows
  - In general, necessary to invoke an explicit solver for each new state in $2^{k'}$

LTSMin PINS interface

- A system to relate explicit and symbolic engines
- Interface :
  - System is a fixed set of integer variables
  - A transition is declared as an opaque function through its *support*, *i.e.* set of variables impacted
- Algorithm for each transition :
  - Store as a DD projection of encountered states on support
  - Execute (explicit) transitions on new states only, store resulting states and transition DD

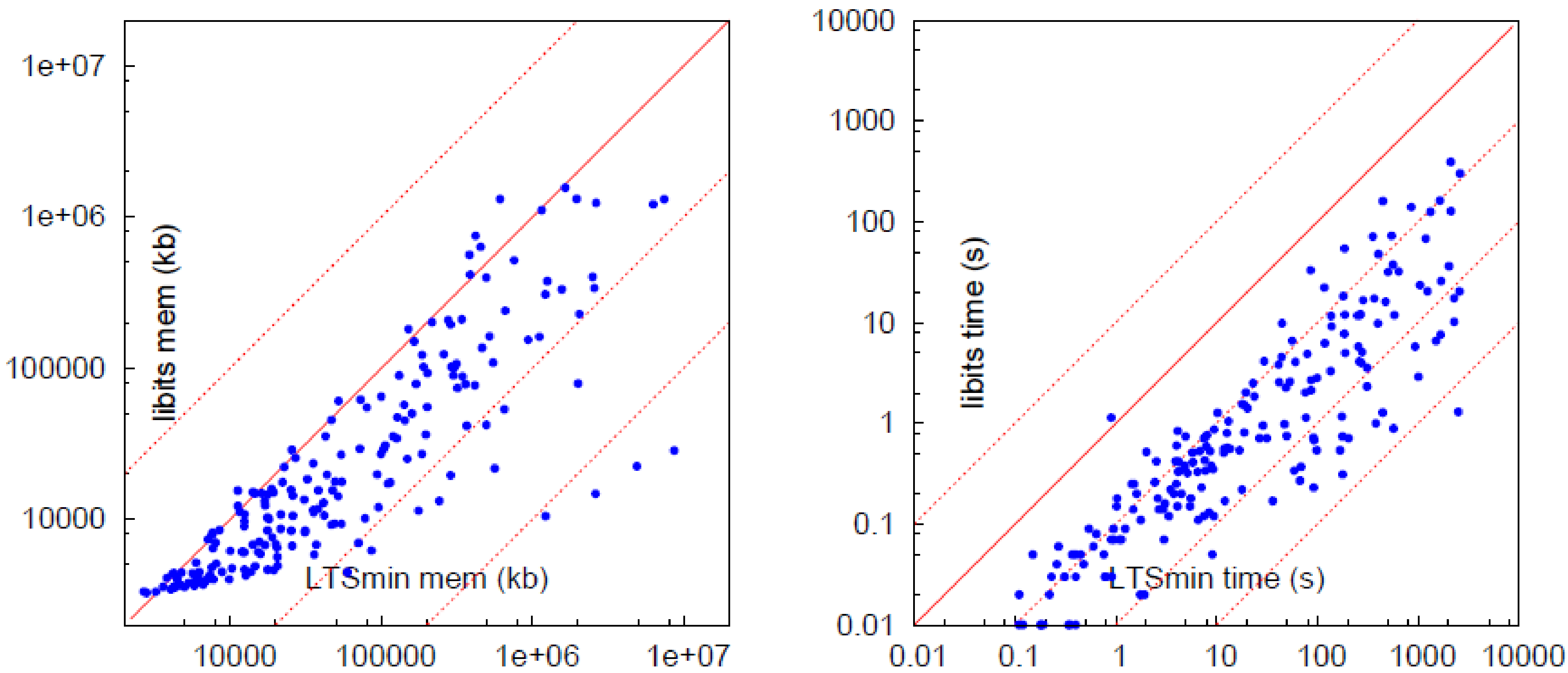
DDD support high level transition relations (NEW CAV'2013)

- Computing the support
  - A[x + y] => pessimistic assumptions
  - x=x+1; y=y+1 => Atomic sequence of updates produce artificially large support
- What if we could compute this on the fly ?
  - Carry the expression in a dedicated operation
  - Traverse a state -> path
  - Resolve variables as they are encountered
  - Drop pessimistic assumptions ASAP
- But we must still reason with sets !

An equivalence relation ?

- Partial expression evaluation
  - f = a + b
  - States : s1:(a=1, b=0) s2:(a=0,b=1)
  - If both a and b are known : f(s1)=f(s2),
    s1 and s2 are equivalent
  - If only a is known, f(s1)=1+b f(s2)=0+b
    s1 and s2 are NOT equivalent
- Algorithm discovers variable values and builds equivalence classes on the fly
  - Split a node into a partition w.r.t the value of the expression
  - Cache the partition to reuse result in different computations

Performance evaluation (BEEM)



(a) comparison on peak memory (b) comparison on time

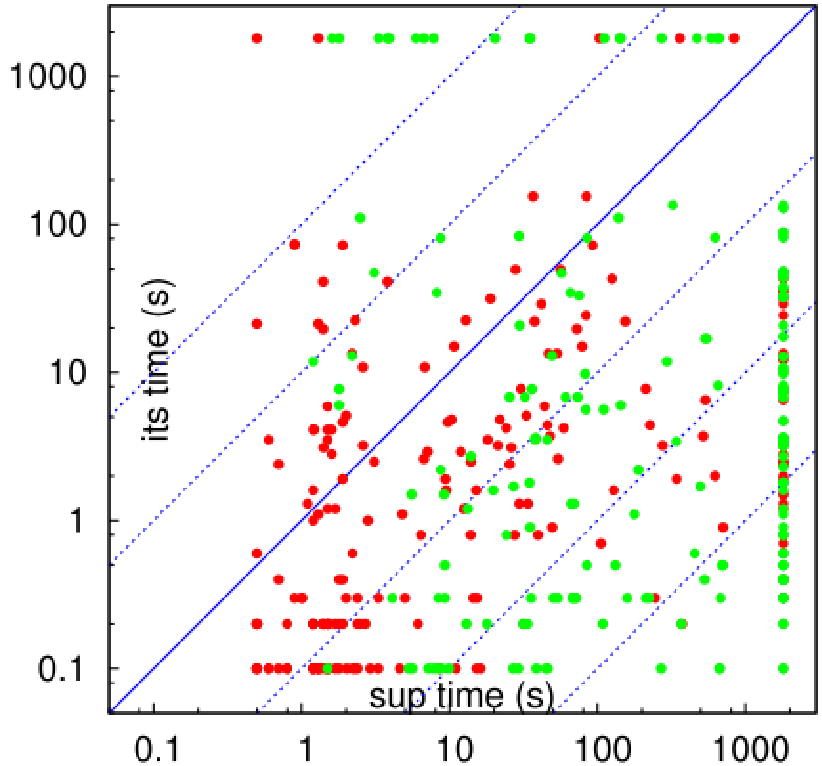
Comparison with super_prove

- reachability properties: 4 cores, 900s wall-clock, 1Gb (HWMCC)
- there are difficult instances for both tools

UNSAT

SAT

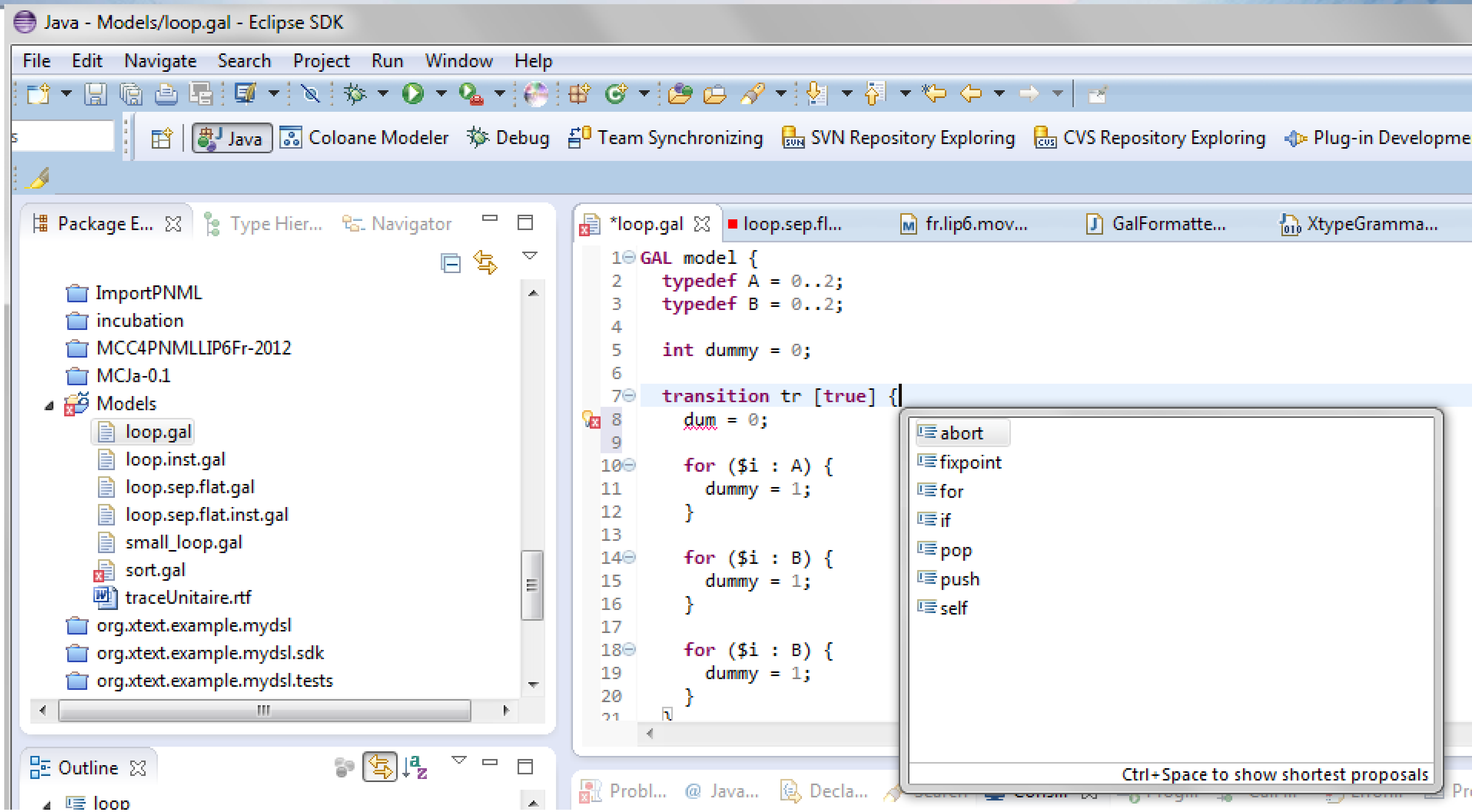
instances	456
libits	376
super_prove	282
both	258
none	56



Conclusion

- GAL modeling
  - A natural model for many discrete semantics
  - Efficient symbolic solution :
    - *More transparency = more optimizations*
- ITS Composite for compositional modeling
  - Modular and hierarchical specifications
  - Efficient support for symmetric models
- Model checking engine
  - Reachability (shortest traces)
  - CTL (Forward algorithms, traces)
  - LTL with Spot (Fully symbolic or hybrid)

GAL Eclipse plugin (Thanks Xtext !)



Thank you for your attention !

SDD and ITS-tools are distributed as an open-source LGPL/GPL C++ source and pre-compiled tools :

http://ddd.lip6.fr

Eclipse plugin for GAL/ITS manipulation and CTL model-checking

http://coloane.lip6.fr/night-updates