LTL in MCC
Definition of Semantics

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One-Step Reachability Relation

Assume for the whole document that N is a fixed (PT or CP) net.

Let m, m' be markings of N.

Define m →m' if

- (PT net:) there is a transition t such that firing t in m leads to m'
- (CP net:) there is a transition t and a binding d of t such that firing (t,d) in m leads to m'

m is called *deadlock* if there is no m' with m → m'

Trace

A trace is an infinite sequence of markings connected by the one-step firing relation. The sequence may contain a deadlock marking which is then repeated forever.

Formally:

Infinite sequence of markings (m_1 m_2 m_3 ....) is a *trace* if, for all $i \geq 1$:

- $m_i \rightarrow m_{i+1}$, or
- m_i is a deadlock and $m_{i+1}=m_i$

Suffix

A suffix of a trace is obtained from a given trace by removing the first n markings of the sequence, for some natural number n.

Formally:

Let π = (m_1 m_2 m_3 ....) be a trace and n≥0 a natural number.

Trace π+n is defined as (m_{n+1} m_{n+2} m_{n+3} ...)

For all n, π+n is called *suffix* of π.

Satisfaction of LTL formulas in traces

Let π = (m_1 m_2 m_3 ...) be a trace.

- For an atomic proposition a, $\pi \vDash$ a iff $m_1 \vDash$ a ("m $\vDash$ a" is defined elsewhere in the MCC manual)
- $\pi \vDash \varphi \land \psi$ iff $\pi \vDash \varphi$ and $\pi \vDash \psi$
- $\pi \vDash \varphi \lor \psi$ iff $\pi \vDash \varphi$ or $\pi \vDash \psi$
- $\pi \vDash \neg \varphi$ iff not $\pi \vDash \varphi$
- $\pi \vDash$ X φ iff π+1 $\vDash \varphi$ (X "nextstep" operator)
- $\pi \vDash$ F φ iff there exists an i, i ≥ 0 with π+i $\vDash \varphi$ (F "finally" / "future" operator)
- $\pi \vDash$ G φ iff for all i, i ≥ 0: π+i $\vDash \varphi$ (G "globally" / "invariantly" operator)
- $\pi \vDash \varphi$ U ψ iff there exists an i, i ≥ 0 with π+i $\vDash \psi$ and, for all j (0 ≤ j < i): π+j $\vDash \varphi$ (U "until" operator)

Satisfaction of LTL Formula in Petri Net

Let m_0 be the initial marking of N.

Trace π = (m_1 m_2 m_3 ...) is called *initial trace* if $m_0 = m_1$.

For an LTL formula φ, N $\vDash \varphi$ (equivalently: N $\vDash$Aφ, where A is the universal path quantifier) if, for all initial traces π, $\pi \vDash \varphi$.

LTL category in MCC

Problem description:

Given

- A (PT or CP) net N and
- an LTL formula φ,

decide whether N $\vDash \varphi$!

Remarks

Our definitions implicitly transform the reachability graph of N into a Kripke structure. A Kripke structure is a transition system where every state has at least one successor state. The transformation consists of adding, for each deadlock marking m, m as its own successor (self-loop). We have implemented this transformation in our definition of "trace". For Kripke structures, the definition of LTL semantics is undisputed, to our best knowledge.