Intelligent Agents

Paper Exercise: Uncertainty and Negotiation

Solutions

Question 1: Consider the game in Figure 1. Let α and β be the uncertain types of agents 1 and 2; they characterize the payoff for playing action A to that agent.

Agent 1	Agent 2: A	Agent 2: B
A	α, β	α, 1
B	1, β	1, 1

Figure 1. *An uncertain game*. Each of two players chooses either action A or B. Payoffs α and β are uncertain.

Consider first that for both agents, the type is distributed among the 2 values [0.5, 2.0] with equal probability, and that this distribution is common knowledge. Derive the ex-ante Bayes-Nash equilibria of the game. Does the game have an ex-post Bayes-Nash equilibrium?

Next, consider that agent 1 knows its own type α=0.5. What are the ex-interim Bayes-Nash equilibria?

Consider another variant where the type is distributed among [2,3] with equal probability. Now does the game have an ex-post Bayes-Nash equilibrium?

Answer:

Here is the game with expected utilities:

Agent 1	Agent 2: A	Agent 2: B
A	1.25, 1.25	1.25, 1
B	1, 1.25	1, 1

There is a unique dominant strategy (and thus Nash equilibrium) to play (A,A). There is no ex-post equilibrum as the two different types would either lead to A or B being the preferred action.

When agent 1 knows its own type, it will not average for its own utilities, but take the actual value, thus obtain this game:

Agent 1 \ Agent 2	A	B
A	0.5, 1.25	0.5, 1
B	1, 1.25	1, 1

The unique *ex-interim* equilibrium is for agent 1 to play B and agent 2 to play A. Note that here agent 2's strategy is not affected and so it does not have to know agent 1's type. Otherwise, however, agent 2 could make the wrong prediction about agent 1's preferences and action. Thus, the ex-interim notion can be problematic.

When the types are distributed between 2 and 3, for both agents the utility of playing A is higher than that of B, no matter what their type actually is. Thus, the strategies (A,A) is a dominant strategy equilibrium in all cases: it holds *ex-post*.

Question 2: Consider the game in Figure 2. What are the equilibria of this game? What is the Nash Bargaining solution for the case of non-transferable utility?

Player A \ Player B	0	1
0	7,8	3,15
1	10,3	5,5

Figure 2. A game that can use cooperation.

Answer:
(1,1) is a dominant strategy equilibrium, but has the lowest combined payoff. It defines the conflict deal for the bargaining process. Now the agents will pick the strategies that maximize the product of utility gains, which means they play (0,0) (since in both other strategies this product is negative).

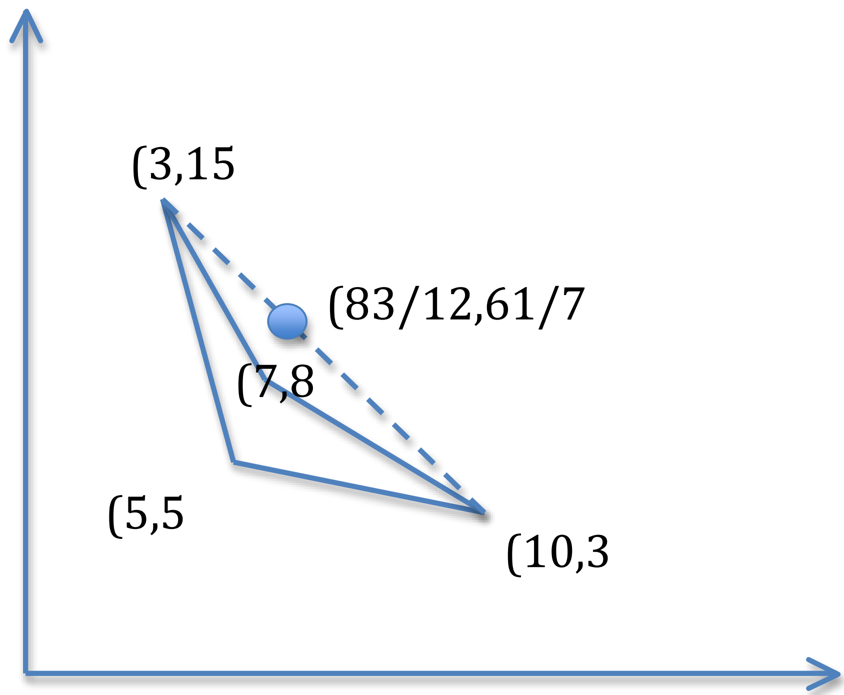
Question 3: For the game in Figure 2, consider a version where utility can be transferred from one agent to another. How does this change the space of possible bargaining solutions? What is the new Nash Bargaining solution?

Answer:
With transferable utility, the agents can pick the strategies that give the highest combined utility and redistribute among them. In this case, they would play (0,1) for a joint payoff of 18 and B would pay 6 to A so that the effective utility to both of them is (9,9).

Question 4:
For the game in Figure 2, what is space of utilities achieved by co-related strategies? Suppose, the set of feasible utilities in the bargaining game is the space of utilities obtained by co-related strategies, what is Nash Bargaining solution?

Answer:

The space of utilities realized by correlated strategies in the following figure.



Nash Bargaining solution will be (83/12,61/7).
This can be achieved by playing a correlated strategy as follows>
The players toss a coin whose probability of H is 37/84. If H, they play (0,1) and T, they play (1,0). However this strategy is not a equilibrium as Player 1's best response this strategy is to play 1.

Question 5: How would you modify the Nash bargaining scheme to so that each agent has a different importance? Hint: consider that agents act for a group of agents and that importance is proportional to the size of the group.

Answer:
Assume there are two agents A and B and that they should carry a weight of a and b, respectively. Then we consider that they represent groups of a and b agents, and to find the Nash bargaining solution we maximize the product of a copies of A's utility function and b copies of B's utility function. Thus, the function to maximize is u_a^a * u_b^b.