Intelligent Agents

Paper Exercise: Coalitions and Group Decisions

Solutions

Question 1: Consider the following characteristic function for coalitions of four agents A, B, C and D:

A	1	AB	3	BD	4	ACD	6
B	1	AC	4	CD	4	BCD	6
C	2	AD	4	ABC	6	ABCD	8
D	1	BC	3	ABD	7		

Figure 1: Game in characteristic function form.

Is this game superadditive? If not, what set of coalitions would be a counterexample?

Answer: No - counterexample: ABD and C has larger payoff than ABCD.

Is the grand coalition (ABCD) the best coalition structure? If not, what is the best coalition structure?

Answer: no, (ABD),© is the best structure.

Question 2: In the game of Figure 1, suppose that the payoff of the grand coalition is changed from 8 to 9. Is the game superadditive?

Answer: yes

Is the game convex? If not, what set of coalitions would be a counterexample?

Answer: no, counterexample:
ABC, ABD: p(ABC)+p(ABD)-p(AB) = 6+7-3 = 10 > 9 = p(ABCD).

What is the Shapley value of each agent in the coalition (ABD)?
Is this payoff distribution in the core?

Answer: (13/6, 13/6, 16/6), in the core.

For the following questions, consider the following voter preferences:

No. of voters	1st choice	2nd choice	3rd choice
20	A	C	B
18	B	C	A
5	C	B	A

Question 3:
Is there a Condorcet winner? (yes – C)
Who wins in plurality voting? (A)
Who wins if alternative B is removed? (C wins)
Who wins if alternative C is removed? (B wins)
Who wins using plurality voting with elimination? (B)
Who wins using Borda counts? (C)

Question 4: How many voters would have to change to that C would win in plurality voting?

Answer: 10 would have to make C their most preferred choice, 6 from group 1 and 4 from group 2

Question 5: What manipulation will make candidate B win in plurality voting?

Answer: three members of third group vote for B instead of C

Question 6: Compute the Slater ranking for this example! How many voters would need to change to manipulate it?

Answer: (
C,B,A), 4 voters have to rank A first to get (C,A,B)…however this is unlikely.