The Effects of Uncertainty and Reputation in Sub-contracting Networks

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Abstract

It is well known that modern production processes usually involve a large number of sub-contractors who each specialize in producing particular components that together form the final good. The efficiency afforded by the specialization, however, also incurs additional uncertainty in the production process, since each firm in the subcontract chain can fail to produce the component it is responsible for and jeopardize the entire production process. I construct a simulation model in which firms arranged on a network optimize their subcontracting decisions based on the local information available to them about their neighboring firms' reputations.

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Contents

Chapter 1

Introduction

Subcontracting is ubiquitous in modern global supply chains. For example, Airbus has 1500 contractors in 30 countries that provide them with the 4 million components that are required in the manufacture of an Airbus A380 (Airbus, 2012). Dell similarly uses more than 130 suppliers from 17 countries (Dell, 2012). Neither is subcontracting limited to the manufacturing sector. International outsourcing in the service industry has been growing since the 1990's. Industry surveys indicate that in 2011, 43% of US companies in the information technology services sector and 38% of those in the research and development sector outsourced some of their production processes internationally (SourcingLine, 2012).

Although the potential avenues for subcontracting have been greatly expanded by technological innovations such as those in information technology in the recent decades, it is not a recent phenomenon. The success of Japanese manufacturing in the Post-War years, especially of automakers such as Toyota, has been attributed to their innovative use of flexible networks of subcontractors (Womack et al., 2007; Shimokawa, 2010). Even as far back as the eighteenth century, networks of subcontractors have been documented in the manufacturing processes of French paper makers (Reynard, 1998).

At the core of modern economic theory is the idea of diminishing returns, and of the gains resulting from specialization and division of labor. Thus we expect countries, firms, and individuals to specialize in certain techniques or the production of certain goods, and exchange goods and services with others in a market setting in order to allow for a more efficient utilization of resources. Subcontracting is one of the ways in which this specialization and division of labor can occur, specifically, if firms specialize in certain stages of a multi-stage production process of a single good. The efficiency gains afforded by subcontracting, however, have to be weighed against the transaction costs that could potentially be incurred during the market exchanges. This insight was first explained by Ronald Coase in his essay, "The Nature of the Firm" (Coase, 1937).

This paper expands on a theoretical model by Kikuchi et al. (2012) which formalizes Coase's argument about transactions costs, asking the question, "what is the optimal amount of subcontracting that should take place given the trade-off between gains from specialization and losses from transaction costs?" I present a dynamic model consisting of a network of firms that collaboratively produce one unit of a good in each round by subcontracting to one another. In this model, transaction costs arise from the uncertainty associated with whether or not a subcontractor will deliver the finished goods.

In each round there is only a probability θ that the subcontractor will successfully deliver the goods, and firms have to factor in this uncertainty before deciding to subcontract to another firm. I solve the model computationally to reproduce the stylised facts observed in the model by Kikuchi et al. (2012), which are the following: the amount of subcontracting declines with increasing uncertainty; the downstream firms produce a larger proportion of the final good than the upstream firms; and the downstream firms have a higher value-added to the final good.

After that, I extend my model to an imperfect information setting, in which firms do not have objective knowledge about the uncertainty associated with subcontracting to other firms. Instead, each firm observes successes and failures from prior rounds of subcontracting to other firms, and use this information to update its beliefs about the uncertainty associated with these other firms. In other words, each firm assigns a reputation to all the other firms that it can subcontract to, and learns from experience about whether or not their subcontractors are reputable. The firms then factor in the other firms' reputation in deciding how much and to whom they should subcontract.

The results show that when reputation updating is involved, certain firms can dominate the production process, receiving the lion's share of the subcontracts. This happens even in a network of identical firms where each has the same uncertainty. Furthermore, the model shows that firms that are more interconnected with one another are more likely to dominate the subcontracting process. This suggests that the availability of potential subcontractors is one of the reasons for economies of agglomeration.

Networks and Agglomeration

Subcontracting requires that firms are interconnected with other firms. Hence, opportunities for subcontracting are most abundant in situations where either firms are in geographical proximity, or are closely knit together in social and professional networks. Transaction costs are a catch-all term that include transportation costs, search costs, and costs due to uncertainty. These costs are lowered when firms either locate near each other, or when they can communicate more effectively with one another. When a client can communicate with its subcontractor to make sure the goods are of adequate quality, and are delivered on time, this facilitates an increased usage of subcontracting. Ease of communication with subcontractors also enables flexibility. Last minute changes in the design of the product, or the quantity of goods ordered, can be more easily accommodated when the client can better communicate with it's subcontractor. An article in the *Atlantic Monthly* magazine from 2007 describes how the availability of a vast network of diverse subcontractors in Chinese manufacturing hubs such as the Pearl River Delta allows for this flexibility (Fallows, 2007):

> You have announced a major new product, which has gotten great buzz in the press. But close to release time, you discover a design problem that must be fixed—and no U.S. factory can adjust its production process in time.

> The Chinese factories can respond more quickly, and not simply because of 12-hour workdays. "Anyplace else, you'd have to import different raw materials and components," Casey told me. "Here, you've got nine different suppliers within a mile, and they can bring a sample over that afternoon. People think China is cheap, but really, it's fast."

This anecdotal evidence will be supported by a review of thorough empirical research in the next section. Nevertheless, the economic intuition is that, the more potential subcontractors are available, the more a client can spread the risk associated with subcontracting. Thus, the degree of interconnectedness in a network of firms increases the likelihood of subcontracting. In other words, an economy in which firms are more connected to other firms has the upper hand in subcontracting. The results from the model presented in this paper show this to be true.

Consider an economy with two regions such as in Fig. 1.1. It consists of Region A, in which all firms are connected to one another, and Region B, where each firm is only connected to two other firms. The two regions have the same number of identical firms, where each of those firms have the same uncertainty in delivering the finished goods (i.e. the same probability θ that they will successfully deliver the finished good after being subcontracted to). Thus, each region can divide a production process into six different steps, with each firm producing one step. When firms have objective knowledge about the uncertainty associated with each firm, the expected cost of the finished product would be the same in both regions. However, when firms gradually learn about the other firms in their region by updating their reputations from past experience, a different outcome is obtained. It turns out that firms in Region A can produce the good at a lower expected cost than those in Region B. The reason being that firms in Region A have more choices in whom they can subcontract to. If a string of bad outcomes ruins the reputation of a subcontractor in Region A, there are other subcontractors that are available, whereas in Region B, a tarnished reputation for one subcontractor may induce a firm to forgo subcontracting and thus reap lesser gains from the division of labor.

One way to interpret this result is that the region in which firms are more interconnected due to geographical proximity can out-compete regions in which firms are less interconnected. As explained in the precious example where the firms were identical in both regions, this can happen even without a necessarily more efficient production process. This offers an additional reason for the positive spillovers that result from agglomeration.

The next section reviews that theoretical and empirical literature on subcontracting, agglomeration, networks, and reputation. Chapter 2 describes the theoretical framework of the baseline model with perfect information and derives the analytical and computational results. Chapter 3 allows for imperfect information and adds the mechanism of reputation updates and concludes. It shows the computational results in models with complete networks with imperfect information and subsequently compares how complete and incomplete networks differ in situations with imperfect information.

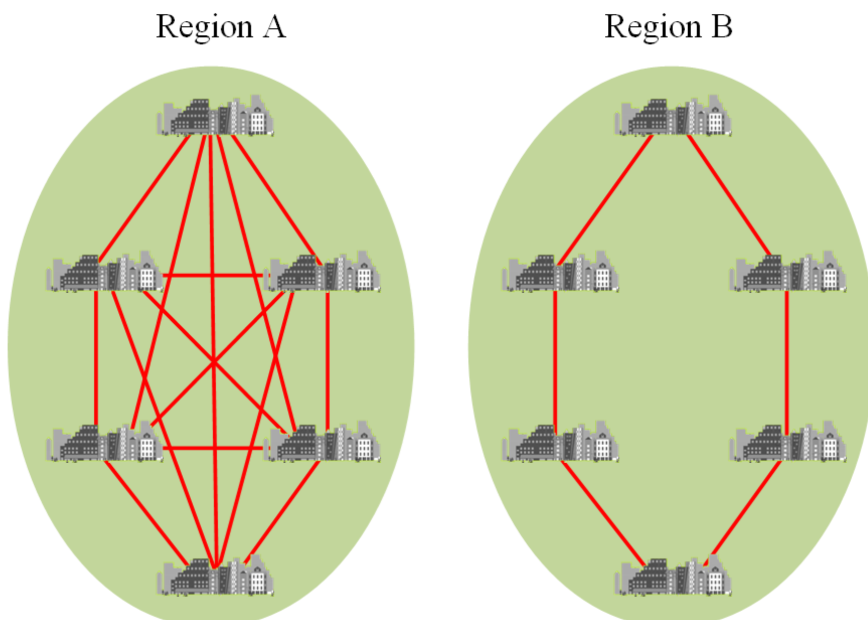


Figure 1.1: Two regions with interconnected firms. Region A shows a complete network where all firms are connected to one another. Region B shows an incomplete network where each firm is only connected to two other firms.

1.1 Literature Review

1.1.1 Subcontracting

Ever since Adam Smith wrote in the *Wealth of Nations* about the division of labor in a pin factory (Smith, 1776), the benefits of specialization has been part of economic theory. As mentioned in the introduction, this paper belongs to the strand of the literature that deals with the tradeoffs between gains from specialization and losses incurred through transaction costs. The literature on transaction costs begins with Coase's aforementioned essay (Coase, 1937). In it, he argues that the reason why market economies do not produce goods by simply subcontracting everything out to individuals and letting the price mechanism decide the optimal allocation of resources, is that every single transaction carried out via the market involves transaction costs. These transaction costs come in the form of search costs, bargaining costs, costs due to uncertainty, and the costs of enforcing contracts. Hence the need for islands of command economies, or firms, to exist within a market economy, since transactions within each firm do not have to incur these transaction costs. The flip side of the argument, then, is that if organizing production within a single firm can mitigate transaction costs, why does the economy not consist of a single giant firm? The answer, Coase argues, is that there are "decreasing returns to the entrepreneur function", or "diminishing returns to management". That is, large organizations become unwieldy to manage, and the decision making apparatus of a single organization cannot match the efficiency of the decentralized market in which resources are allocated via the price mechanism. Hence, Coase argues that there is an optimal size of firms where they are only big enough such that at the margin, the increase in costs due to diminishing returns of management is equal to the increase in costs incurred in the form of the transaction costs of a market exchange.

Following Coase, Oliver Williamson has been one of the main figures in the field of transaction costs economics. In a series of papers that are collected in the book *Economic Organization* (Williamson, 1986), he presents formal models that incorporate Coase's insights and also elaborates on the ways in which transaction costs occur in economies. One of them introduces a hierarchically organized structure of the firm where the workers at the lowest level, who supply the labor that goes into the production of goods, are supervised by managers one level up in the hierarchy, who in turn are supervised by managers who are an additional level up the hierarchy, and so on (Williamson, 1967). He shows that the optimal number of hierarchies, n^*, is dependent on several factors. For example, n^* increases as the span of control, i.e. the number of employees a supervisor can handle effectively, increases. Also, n^* increases as the degree of compliance to supervisor objectives increases. In another paper (Williamson, 1971), he argues that vertical integration of firms take place to mitigate the transaction costs ensuing from bargaining between upstream firms and downstream firms, contractual incompleteness, moral hazard, costs incurred when gathering and processing information, and institutional characteristics such as the level of trust. He conjectures that "vertical integration would be more complete in a low-trust than a high trust culture", which supports the results obtained in my model.

The paper on subcontracting by Kikuchi et al. (2012) is largely based on the formalization of the ideas by Coase and Williamson. In their paper, firms in a supply chain collaborate to produce one unit of a final good. This collaboration process starts off when an initial firm decides between 1) producing a certain portion of the final good in-house whilst facing diminishing marginal returns, in accordance with Coase's idea of diminishing returns to management; and 2) subcontracting the portion that was not produced in-house to a another firm. This subcontracting averts the costs due to diminishing returns to management but instead incurs transaction costs associated with market exchange. As firms recursively repeat this process of subcontracting, transaction costs are compounded as the number of firms in the supply chain grows. As such, the firms in the supply chain face an optimization problem in which they decide the best trade-off between diminishing marginal returns to in-house production and the increasing transaction costs as the more firms are added to the supply chain. Chapter 2 discusses Kikuchi et al.'s (2012) model in more detail, while Chapter 3 extends this model to a setting where firms do not know, ex-ante, the transaction costs that they will be facing when they choose to subcontract. In this setting, the firms rely on their past experience with various potential subcontractors to form expectations about the transaction costs involved in subcontracting.

This paper also differs from Kikuchi et al. (2012) in its methodology and the economic phenomenon that it seeks to explain. Firstly, in terms of methodology, Kikuchi et al. (2012) derives analytical proofs for their main results relying on Tarski's fixed point theorem and other methods from functional analysis. My methodology in this paper relies on computational simulations on an exogenously determined network of a finite fixed number of firms. Secondly, in terms of the economic phenomenon that is explained, Kikuchi et al.'s (2012) aim is mainly to formalize Coase's intuitive arguments, whereas my paper seeks to uncover a possible reason for agglomeration economies in networks of subcontractors. Sections 1.1.2 and 1.1.3 review the literature on agglomeration and the economics of networks respectively.

Hart and Moore (1990) also contributed to the literature on the theory of the firm with a model that formalizes Coase's insights. In the vein of Kikuchi et al. (2012), their paper also analyses the reasons why a firm would choose to carry out its production either in-house or through contracting to another firm. However, their approach to the problem is based on the allocation of property rights to the various parties. They argue that a firm in possession of an asset that is required in the production process will have more bargaining power over the labor that is needed for the production, whereas if the firm did not own the productive asset but instead contracted out the work to another firm that did, it will have less bargaining power over labor. In a dynamic setting where agents who can sell their labor make ex-ante investments in human capital, they would choose to invest differently depending on how the property rights are allocated. The authors give an example in which a yacht's skipper and a chef jointly provide a service to a rich client. If the chef could invest in human capital to increase his productivity, he would choose to make the said investment if the yacht was owned by the rich client, but he would not make the investment if the skipper owned the yacht. The reason for this is because in the first scenario, the chef needs both the client and the skipper to produce and sell his good and thus, in a symmetric bargaining outcome he has to share two thirds of his earnings with the skipper and the client. In the second scenario, however, since the yacht's skipper does not own the essential asset and thus does not have bargaining power, the chef will only have to share half his earnings with the client under symmetric bargaining. This means that he has a higher incentive to invest in human capital. From this brief outline, it can be seen that the motivation of their paper is different from that of Kikuchi et al. (2012) and this current paper, since our models do not rely on property rights or assume any explicit role for capital in the productive process.

Outside of economics, the field of operations research also has a large theoretical literature on subcontracting and supply chain management (Chopra and Meindl, 2007). These models often feature explicitly modelled networks of manufacturers, suppliers and retailers (Nagurney, 2006). They also feature computational simulation models such as those that use multi-agent systems, which are autonomous software agents which act as decision makers in the supply chain, often incorporating artificial intelligence techniques (Chaib-Draa and Müller, 2011). The approach differs from that of economists, however, in that the supply chain is already taken as exogenously determined, and the models focus only on deriving the optimal behaviour of firms within it, whereas economists seek to explain *why* production is organized in a multi-level supply chain in the first place.

In terms of the empirical literature on subcontracting, we will briefly examine Banerjee and Duflo's (2000) study of contracting in the software industry in India, and Arzaghi and Henderson's (2008) study of subcontracting in the advertising agency industry in Manhattan.

Banerjee and Duflo (2000) use data from interviews with 125 CEOs of Indian software firms and examined the extent to which reputation played a role in contracting in the software industry. They found that firms which have better proxies for reputation - such as having been established for a longer time, are ISO certified, or are subsidiaries of foreign companies - have to bear less of the overrun costs. These are costs which are *ex ante* unaccounted for when the contract is signed but are incurred by the contractor during the production process and are split between the client and contractor in *ex post* negotiation. For example, an overrun cost may be incurred when the contractor estimated that a project will only take 3 months but ended up taking 5 months instead, therefore the *ex ante* contract does not account the costs of the additional 2 months. The authors' interpretation is that, the more the client bears the overrun costs, the more reputable the contractor is. They also find that most clients rely on long established relations with contractors, this corroborates with the results of my model.

Arzaghi and Henderson (2008) use data from individual advertising agencies in lower Manhattan to measure the benefits of being located in an area with a cluster of other agencies. These clusters consist of firms specializing in different aspects of advertising and regularly subcontract to one another. They found the benefits to profitability of locating in a cluster was significant. These scale effects decrease rapidly with distance and are gone if a firm is located more than 750 metres away from a cluster. The authors describe an example of the process by which subcontracting occurs and the benefits accrued from locating in an area which has a cluster of similar firms:

> The executives said that their main goals in contacts are to supplement their limited in-house capacity, in terms of gathering both ideas in preparing proposals and sufficient materials and labour to fulfil a particular contract. As a simple example of the latter, agency A received work to redesign a set of presentation slides for a client. The people in agency A worked on the set of slides for a week and presented a sample to the client. The client was happy with the sample. Then the agency learnt that the work involved not only the 100 pages in the set of slides discussed in the initial meeting, but also that there were 10 other similar cases that needed to be done in about 10 days. This was beyond the capacity of the agency. To help keep the account, the head of the agency A utilized a contact in agency B he trusted could help with the job. That contact was currently two blocks away. They have been involved in a business relationship that started 10 years earlier.

Again, both the anecdotal and the econometric evidence show that: (i) reputation plays a large role in the assigning of subcontracts, (ii) firms subcontract to other firms with which they have long running relationships, and

(iii) locating your firm within a cluster of other firms enhances these relationships. All of this corroborates with the results of my model.

1.1.2 Agglomeration

There is a vast literature on the economies of agglomeration, that stretch back to Marshall's (1890) *Principles of Economics*, in which he argues that economies of agglomeration can arise from lower transport costs, lower labor costs due to labor pooling effects, and information spillovers. A recent paper by Ellison et al. (2010) finds empirical evidence for all three of these effects using data from US and UK manufacturing industries.

For the purposes of this current paper, I will only discuss the reasons for the third of Marshall's theories for agglomeration, that is, the information spillover effects. Specifically relating to my model is the agglomeration economies arising from the ease of subcontracting. Duranton and Puga (2004), and Gill and Goh (2010) provide surveys of the recent literature on agglomeration. The former argues that the theoretical literature (as of 2004) on agglomeration due to information spillovers is not solidly based on microfoundations, and usually ad-hoc assumptions are made regarding the nature of the information externality. The latter summarises the empirical literature on spatial agglomeration effects in different industries as follows: (i) spatial clustering is more pronounced in high-technology industries than light industries, (ii) services are more spatially concentrated than manufacturing as service industries are more codependent, e.g. banks need advertising, advertising firms need banks. Both of these pieces of evidence suggest that ease of subcontracting fosters clustering, since high-tech firms tend to be more specialized and rely more subcontracting than light industry, and firms in service industries need to subcontract due to the multi-faceted nature of their business, e.g. a bank cannot efficiently carry out an advertising campaign in-house.

Theoretical models which seek to provide an explanation for agglomeration in urban areas include Duranton and Puga (2001) and Harrigan and Venables (2006). The former uses a general equilibrium framework to explain the co-existence of different types of clusters. Some cities have clusters of diverse industries which fosters the development of new products and prototypes, while others have clusters of specialized industries to focus on mass production once a prototype is perfected. The latter uses a model similar to Kremer's (1993) O-Ring theory to explain that clustering may arise so that the costs arising due to the time taken to wait for deliveries of intermediate goods can be minimized.

1.1.3 Networks and Reputation

This paper uses a model that involves a network of firms and the reputations that these firms have of each other. The techniques used here are borrowed from the network model of labor markets by Calvo-Armengol and Jackson (2004), and the lecture notes on Bayesian reputation updating by Cabral (2005).

The economics of networks has been a thriving field in recent years. A survey can be found in Jackson (2010). Recent literature include the aforementioned Calvo-Armengol and Jackson (2004) on labor markets, Battiston et al. (2007) and Delli Gatti et al. (2010) on financial and credit networks, Hausmann and Hidalgo (2011) and Hausmann and Hidalgo (2011) on the network structure of international trade, Acemoglu and Ozdaglar (2011) on learning in social networks, and Acemoglu et al. (2011) on how input-output linkages in various sectors of an economy can propagate microeconomic shocks into aggregate fluctuations.

Chapter 2

Baseline Model

2.1 Model

The baseline model consists of a network of n firms which divide up a task to produce one unit of a good for an external client in each round. Figure 2.1 shows an example of a network with $n = 3$. The client has a choice of ordering the good from any of these 3 firms. When a firm receives an order from the client to produce one unit of the good, it chooses to produce a certain portion of the good in-house and is free to subcontract the remaining portion to firms in the rest of the network. Figure 2.2 shows how an order from the client might be processed by the firms. In this case, each firm produces $\frac{1}{3}$ of the product and passes it along to the next firm.

This structure can be represented linearly in order to better explain the notation and assumptions used in the model. The entire production process is seen as producing a unit measure of the good from $[0, 1]$. It is assumed

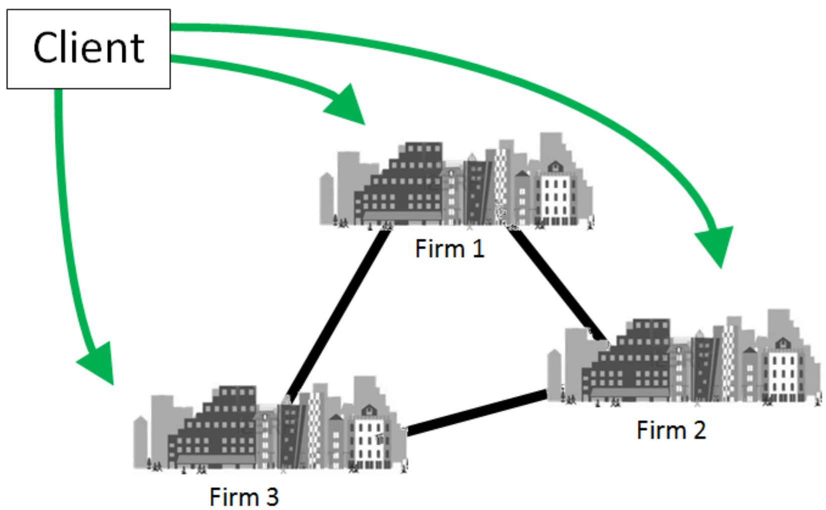


Figure 2.1: A complete network of 3 firms showing the client's contracting options.

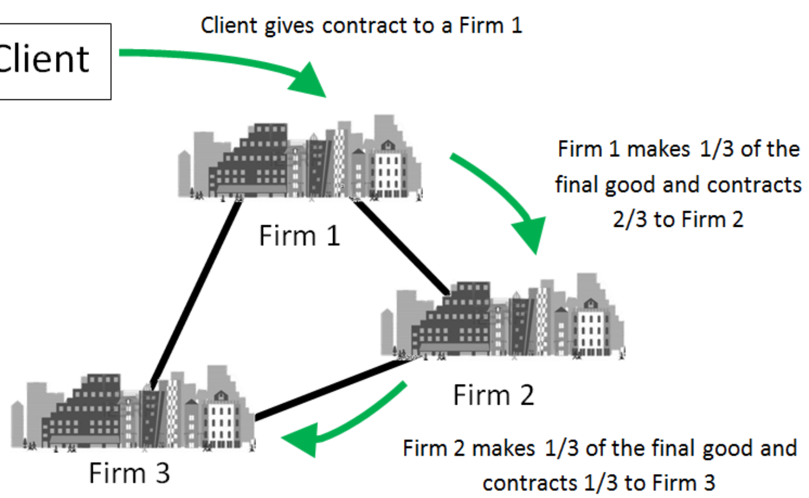


Figure 2.2: A possible chain of subcontracts in a 3 firm network.

that the production process can be divided into n parts of equal measure, $\{[0, \frac{1}{n}], (\frac{1}{n}, \frac{2}{n}], \ldots, (\frac{n-2}{n}, \frac{n-1}{n}], (\frac{n-1}{n}, 1]\}$. Upon getting the original order from the client, a firm can choose to produce $k_1 \in \{1, 2, \ldots, n\}$ parts in house, and subcontract $n - k_1$ parts to another firm. The parts that are produced by the first firm (most downstream) start from 1, i.e. if $k_1 = 1$, the first firm produces $(\frac{n-1}{n}, 1]$, if $k_1 = 2$, it produces $(\frac{n-2}{n}, 1]$, if $k_1 = n$, it produces $[0, 1]$, etc. The next firm in line then chooses to produce $k_2 \in \{1, 2, \ldots, n - k_1\}$, and subcontracts the remaining $n - k_1 - k_2$ parts to the next firm, and the process goes on until all n parts are produced. We denote the start of the interval that firm i produces as u_i and the end of the interval that it produces as s_i. Thus $(u_1, s_2] = (\frac{n-k_1}{n}, 1]$, $(u_2, s_2] = (\frac{n-k_1-k_2}{n}, \frac{n-k_1}{n}]$, etc.

A case for which $n = 3$ is shown in Figure 2.3, where each firm chooses to produce 1 out of 3 parts and thus is each responsible for an interval of measure $\frac{1}{3}$, i.e. $k_1 = k_2 = k_3 = 1$. This is one of the many possible paths to produce the good. The other possible paths are $\{k_1 = 3, k_2 = 0, k_3 = 0\}$, and $\{k_1 = 2, k_2 = 1, k_3 = 0\}$. Sections 2.1.4 and 2.3.1 discuss the possible permutations in detail, since the computational solution to the model evaluates the expected costs associated with all the possible paths and chooses the optimal one.

Before looking at how to determine the optimal path to produce the good in Section 2.1.3, the next section describes additional features of the model which provide the trade-off between the diminishing returns to in-house production, and the uncertainty associated with subcontracting.

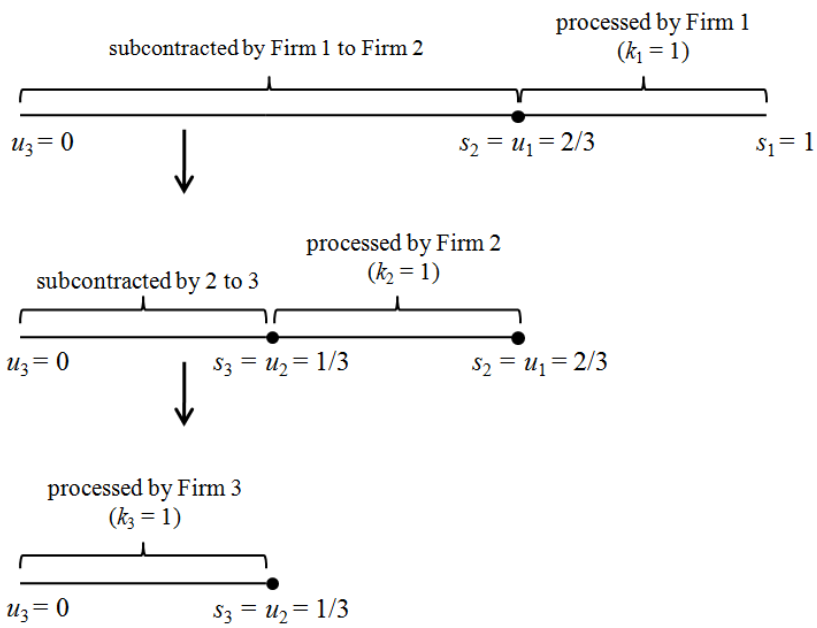


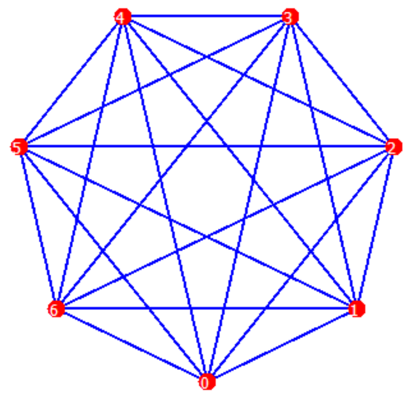
Figure 2.3: Notation showing the number of parts produced k, the starting point u, and the ending point s, in a possible chain of subcontracts in a 3 firm network.

2.1.1 Exogenous network and free entry

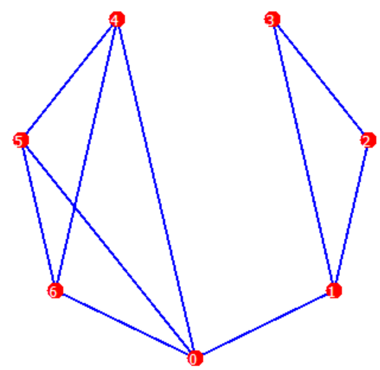
The network structure determines how each firm is connected to all the other firms and is given exogenously. A firm can only subcontract to the other firms that it is connected to. Also, it can only subcontract to firms that have not already been subcontracted to in that round. For instance, firm i, upon getting a subcontract from firm $i-1$, can in turn choose to subcontract to any firm in the set $\{i+1, i+2, \ldots, n\}$ that it is connected to in the network. All the connections are bilateral. So, a firm i can subcontract to a firm j and vice versa as long as they are connected in the network.

Figure 2.4 shows some examples of networks. In Figure 2.4a, every firm is potentially able to subcontract to every other firm, this is called a complete network. In Figure 2.4b, some firms such as the ones labeled 3 and 4 at the top, cannot subcontract to each other. In Figure 2.4c, the firms are seperated into two sub-networks, where the firms from one sub-network cannot subcontract to the firms in the other. These latter two are called incomplete networks.

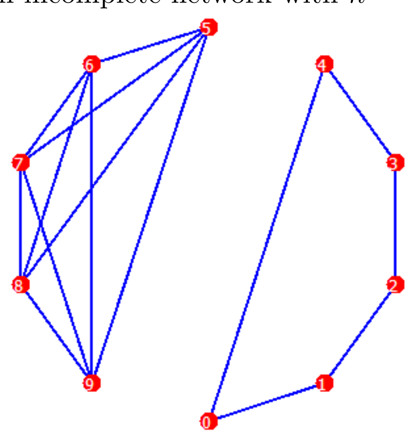
We now look at the economic interpretation behind the network. Each firm in the network can be thought of as a location, in which a competitive market of identical potential entrants exist. We assume that there are no barriers to entry. Thus, in each round, a firm will occupy each location and take part in the production, making an expected profit of zero.



(a) A complete network with $n = 7$ firms



(b) An incomplete network with $n = 7$ firms



(c) An incomplete network with $n = 10$ firms. Divided into a 2 sub-networks

Figure 2.4: Examples of networks of firms.

A possible interpretation is that each firm on the network occupies a location in a city, which is represented by the network. Some locations are connected to others, whereas some are not. The "connections" can be interpreted liberally, either as physical transportation links, or as interpersonal contacts between entrepreneurs living in different neighbourhoods.

The scale of the network can also be interpreted in different ways. Instead of the firms being located in different neighborhoods and the network representing a city, the firms might represent individual cities in a network of cities. Another scale at which the model can also be seen is one where firms that represent individual countries are connected together in an international trade network.

Each firm has the resources to produce the entire unit measure of the good, or it can produce any number of parts as explained above. However, since the resources available to the firm in each location is limited, each firm faces diminishing returns in the form of a twice differentiable, convex production function $c(x)$, with $c'(x) > 0$ and $c''(x) > 0$. In the computational solution to the model, we assume $c(x) = x^2$. Hence, there is an incentive for the firm to subcontract to other firms, i.e. only produce a portion of the good in-house and buy the rest from another firm, in order to reduce costs.

2.1.2 Uncertainty and the production process

Whenever a firm (contractee) subcontracts to another firm, there is a probability θ that the subcontractor will successfully deliver the finished goods, and therefore a $1-\theta$ chance of failure. We assume that all the firms have the same probability of success. Therefore θ is a constant across all firms and in every round. It is also assumed that all firms know the true θ of every other firm on the network. This latter assumption of perfect information will be relaxed in the extended model in Chapter 3.

Note also, as explained earlier, that each firm is one amongst many of the potential entrants which may occupy a particular location in the network. Hence, the θ can be thought of as location specific, and not firm specific. This can be interpreted as different firms within the same neighborhood all having the same uncertainty.

The uncertainty due to θ can be interpreted as arising from multiple possible sources. It could be the firm's fault that the goods manufactured are not up to standard, it could factors such as corruption or bad transportation which prevents the finished goods from being delivered successfully.

This uncertainty creates a limit to the extent that firms should optimally subcontract, since every additional level of subcontracting compounds the probability of failure. The optimal amount of subcontracting trades off the diminishing returns of in-house production to the diminishing returns of additional subcontracting costs accrued due to uncertainty.

Additional details of the process need to be examined before presenting the firms' profit maximization problem. It it important to consider what happens when all the stages of production are successful and what happens when they are not. Figure 2.5 shows how a successful production process might take place, when $n = 3$ and $k_1 = k_2 = k_3 = 1$. The production takes

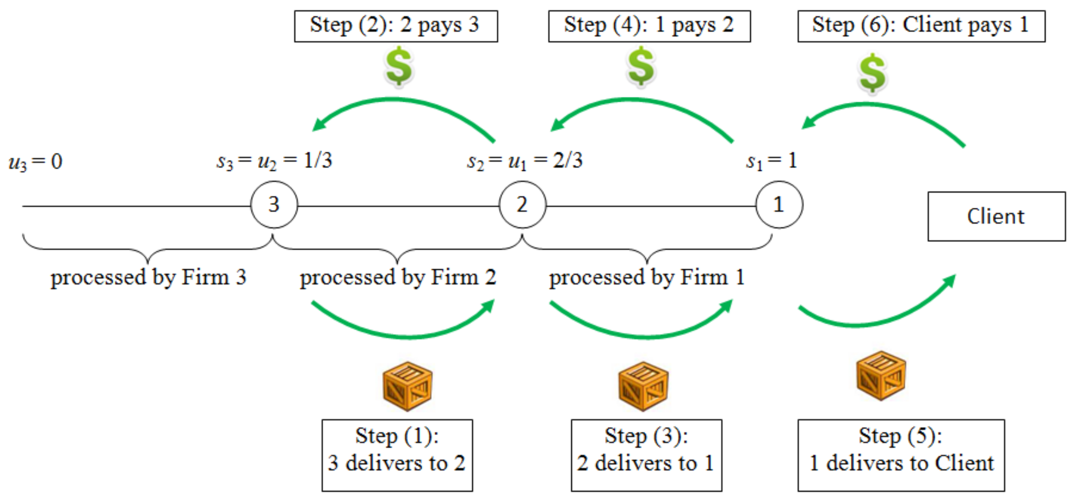


Figure 2.5: The steps of production and payment in a 3 stage process when every stage is successfully carried out.

place starting from the last firm in the subcontracting chain, in this case, firm 3. Firm 2 waits for a successful deliverly from firm 3 before making the payment. Once firm 2 receives the goods from firm 3, it will in turn produce its portion, and if successfully delivered to firm 1, will receive its payment. In turn, firm 1 only starts production when it receives the intermediate goods from firm 2, and produces its in house component, which it sends to the client, and is subsequently paid.

Figures 2.6 and 2.7 illustrate what happens when some firm fails in the subcontracting process. When firm 2 fails, as shown in Figure 2.6, firm 2 still has to pay firm 3 for its portion, but firm 1 incurs no costs either in production or in having to pay its subcontractor, firm 2. Similarly the client does not have to pay firm 1 either. As another example, consider firm 1

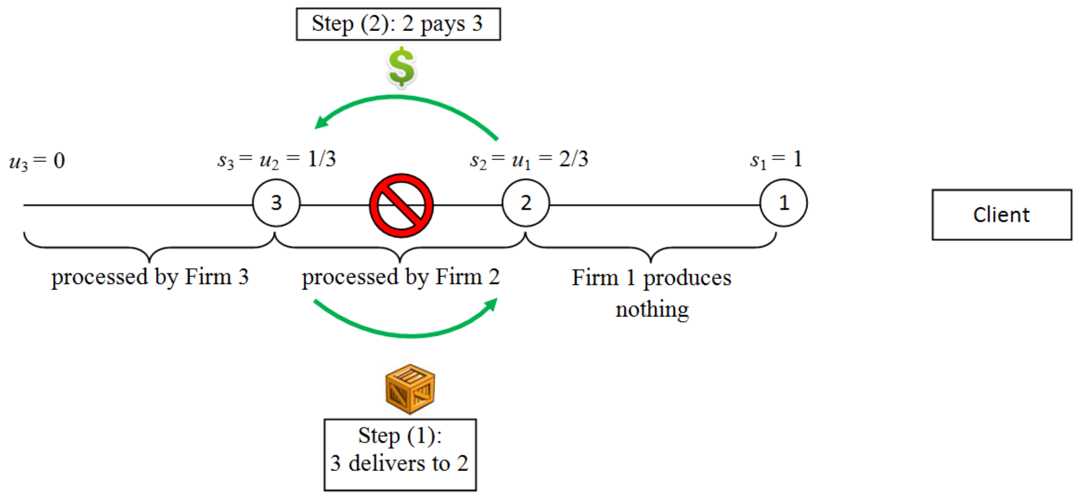


Figure 2.6: The steps of production and payment in a 3 stage process when firm 2 fails.

failing while firms 2 and 3 are successful. In this case, firm 1 still pays firm 2, and firm 2 still pays firm 3, but the client does not pay firm 1.

2.1.3 Profit maximization

Since each firm is maximizing expected profits, it will have to mark up the price it charges its contractee so as to make up for the possibility of failure of its in-house production, but each firm does not have to take into account the failure of its subcontractor's chance of failure as this will already be reflected in the price the subcontractor charges.

Figure 2.8 shows the set-up of the problem faced by firm i in a production process involving n firms. Firm i's problem of maximizing expected profit

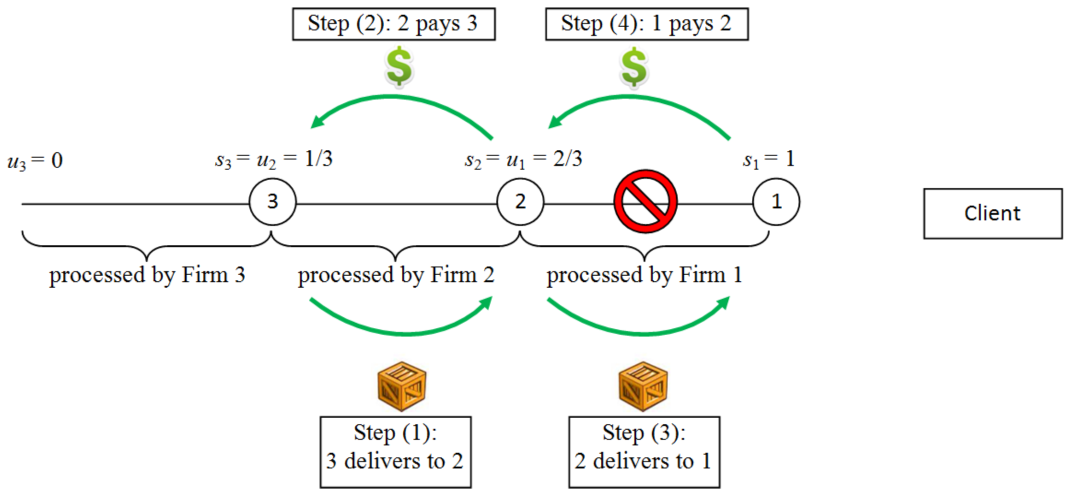


Figure 2.7: The steps of production and payment in a 3 stage process when firm 1 fails.

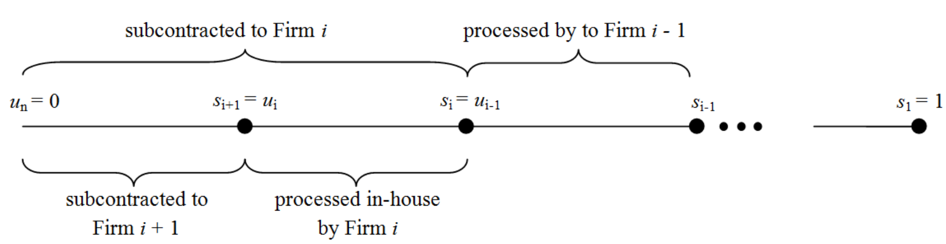


Figure 2.8: Notation for the recursive production process.

can be stated as

$$E\left(\pi_i\right)=\theta p_i\left(s_i\right)-c\left(s_i-u_i\right)-p_j\left(u_i\right), \tag{2.1}$$

where $p_i(x)$, with $x \in [0,1]$, is the intermediate price for the $[0,x]$ interval sub-portion of the final good that is charged by firm i, and $c(x)$ is the cost of producing a sub-portion of measure x of the good in-house. The expected revenue firm i receives is $\theta p_i(s_i)$, factoring in its uncertainty. Its costs consists of the in-house cost $c\left(s_i-u_i\right)$ and the subcontracting cost $p_j\left(u_i\right)$. Since it only has to incur costs when its subcontractor is successful, θ does not factor into the expected costs.

Each firm i has to make the choice of the number of steps to produce, k_i, and which firm j to subcontract to. The starting point u_i is determined by k_i, as $u_i(k_i)=\frac{s_i-k_i}{n}$, and s_i is determined by the firm's contractee. Since the maximum expected profits is zero after optimization,

$$\max_{k_i,j}\left\{\theta p_i\left(s_i\right)-c\left(s_i-u_i(k_i)\right)-p_j\left(u_i(k_i)\right)\right\}=0. \tag{2.2}$$

Rearranging, we have

$$p_i\left(s_i\right)=\frac{1}{\theta}\min_{k_i,j}\left\{c\left(s_i-u_i(k_i)\right)+p_j\left(u_i(k_i)\right)\right\}, \tag{2.3}$$

which is the equilibrium price function for every intermediate portion $[0,s_i)$ of the good. Thus, (2.3) gives a recursive definition of the price function at every possible value of $s_i \in \left\{0,\frac{1}{n},\frac{2}{n},\ldots,1\right\}$.

Since the network structure determines which firms j, the solution to the model requires that we work out all the possible production paths available for any given network. An analytical method of calculating the possible production paths for a complete network is given in Section 2.1.4 and a more general algorithmic method for any network structure is given in Section 2.3.1.

2.1.4 Possible production paths for a complete network

Consider a complete network of n firms. Dividing n into its integer partitions and taking all the permutations of the parts of each partition will give the number of ways the n steps can be divided among n firms. For example, when $n = 3$, the process can be divided into 3, $1+2$, $2+1$ or $1+1+1$ steps. The partitions can be translated into production quotas for each of the three firms as follows:

- 3 means the most downstream firm produces everything, i.e. $\{k_1 = 3, k_2 = 0, k_3 = 0\}$;
- $1 + 2$ means $\{k_1 = 1, k_2 = 2, k_3 = 0\}$;
- $2 + 1$ means $\{k_1 = 2, k_2 = 1, k_3 = 0\}$;
- and $1 + 1 + 1$ means $\{k_1 = 1, k_2 = 1, k_3 = 1\}$.

For each permutation of the integer partition of n involving k partitions, we have ${}_nP_m$ ways of allocating m out of the n firms to the production

n	Number of possible paths
1	1
2	4
3	21
4	136
5	1,045
6	9,276
7	93,289
8	1,047,376
9	12,975,561
10	175,721,140
11	2,581,284,541
12	40,864,292,184
13	693,347,907,421
14	12,548,540,320,876
15	241,253,367,679,185
16	4,909,234,733,857,696
17	105,394,372,192,969,489
18	2,380,337,795,595,885,156
19	56,410,454,014,314,490,981
20	1,399,496,554,158,060,983,080

Table 2.1: Number of possible production paths for complete networks with n firms.

process. Using the $n = 3$ example, in the case of the partition $1 + 2$, which uses $m = 2$ out of the $n = 3$ firms, we have ${}_3P_2 = 6$ allocations. We then have

$$\text{number of possible production paths} = \sum_{m=1}^{n} \mathcal{P}(n, m)\left({}_nP_m\right) \tag{2.4}$$

where $\mathcal{P}(n, m)$ is the number of integer partitions of n with m parts. Table 2.1 shows how the number of possible paths increases factorially with n.

In the analytical solution to the model presented in Section 2.2, we assume that the firms are identical to one another. Hence, for any given integer partition of the quotas for each firm in the production process, it does not matter which particular firms are involved in the production. Thus, the number of possible paths reduces to

$$\text{number of possible production paths} = \sum_{m=1}^{n} \mathcal{P}(n, m). \tag{2.5}$$

2.2 Analytical solution

2.2.1 An example solution for $p(\bullet)$

This section gives an example of how to solve for the pricing function $p(\frac{m}{n})$ for $m = 0, 1, 2, 3$ in a complete network. Since all firms are identical, the client as well as each firm along the production chain is indifferent about which firm j it should subcontract to, and the optimization problem in (2.3) just involves choosing k_i. We can thus drop the subscripts for the $p(\bullet)$ function.

We have the following values for equilibrium prices:

$$p\,(0) = 0, \tag{2.6}$$

$$\begin{aligned}
p\left(\frac{1}{n}\right) &= \frac{1}{\theta}\min_{k_i\in\{1\}}\left\{c\left(\frac{1}{n}-u_i(k_i)\right)+p\,(u_i(k_i))\right\} \\
&= \frac{1}{\theta}\min_{k\in\{1\}}\left\{c\left(\frac{k}{n}\right)+p\left(\frac{1-k}{n}\right)\right\} \\
&= \frac{1}{\theta}\left\{c\left(\frac{1}{n}\right)+p\,(0)\right\} \\
&= \frac{1}{\theta}c\left(\frac{1}{n}\right),
\end{aligned} \tag{2.7}$$

$$\begin{aligned}
p\left(\frac{2}{n}\right) &= \frac{1}{\theta}\min_{k\in\{1,2\}}\left\{c\left(\frac{k}{n}\right)+p\left(\frac{2-k}{n}\right)\right\} \\
&= \frac{1}{\theta}\min\left\{\begin{array}{c} c\left(\frac{2}{n}\right), \\ \frac{1}{\theta}c\left(\frac{1}{n}\right)+c\left(\frac{1}{n}\right) \end{array}\right\},
\end{aligned} \tag{2.8}$$

$$\begin{aligned}
p\left(\frac{3}{n}\right) &= \frac{1}{\theta}\min_{k\in\{1,2,3\}}\left\{c\left(\frac{k}{n}\right)+p\left(\frac{3-k}{n}\right)\right\} \\
&= \frac{1}{\theta}\min\left\{\begin{array}{c} c\left(\frac{3}{n}\right), \\ \frac{1}{\theta}c\left(\frac{1}{n}\right)+c\left(\frac{2}{n}\right), \\ \min\left\{\frac{1}{\theta}c\left(\frac{2}{n}\right),\frac{1}{\theta^2}c\left(\frac{1}{n}\right)+\frac{1}{\theta}c\left(\frac{1}{n}\right)\right\}+c\left(\frac{1}{n}\right) \end{array}\right\} \\
&= \frac{1}{\theta}\min\left\{\begin{array}{c} c\left(\frac{3}{n}\right), \\ \frac{1}{\theta}c\left(\frac{1}{n}\right)+c\left(\frac{2}{n}\right), \\ \frac{1}{\theta}c\left(\frac{2}{n}\right)+c\left(\frac{1}{n}\right), \\ \frac{1}{\theta^2}c\left(\frac{1}{n}\right)+\frac{1}{\theta}c\left(\frac{1}{n}\right)+c\left(\frac{1}{n}\right) \end{array}\right\}.
\end{aligned} \tag{2.9}$$

We can observe from (2.6) to (2.9) that:

- A downstream firm will produce a larger portion of the good than an upstream firm, since any production carried out upstream has higher costs due to compounding uncertainty. For example, compare $\frac{1}{\theta}c\left(\frac{1}{n}\right)+c\left(\frac{2}{n}\right)$ and $\frac{1}{\theta}c\left(\frac{2}{n}\right)+c\left(\frac{1}{n}\right)$, in line (2.9). Since $\frac{1}{\theta}>0$, and $c\left(\frac{2}{n}\right)>c\left(\frac{1}{n}\right)>0$, we have

$$\begin{aligned}\frac{1}{\theta}\left[c\left(\frac{2}{n}\right)-c\left(\frac{1}{n}\right)\right] &> c\left(\frac{2}{n}\right)-c\left(\frac{1}{n}\right)\\ \frac{1}{\theta}c\left(\frac{2}{n}\right)+c\left(\frac{1}{n}\right) &> \frac{1}{\theta}c\left(\frac{1}{n}\right)+c\left(\frac{2}{n}\right).\end{aligned}$$

- The value added, $p(s_i)-p(u_i)$, is higher for a downstream firm than for an upstream firm. This follows from the second point, and also from the compounding effect of the uncertainty. To illustrate, consider the case where $k_1=k_2=k_3=1$. Compare the last options in (2.7), (2.8), and (2.9). The values added are $\frac{1}{\theta^3}c\left(\frac{1}{n}\right)$, $\frac{1}{\theta^2}c\left(\frac{1}{n}\right)$, and $\frac{1}{\theta}c\left(\frac{1}{n}\right)$, for firms 1, 2 and 3 respectively, where 3 is the furthest upstream and 1 is the furthest downstream. This shows that the compounding effect of the uncertainty creates a higher value added for the downstream firms than the upstream firms.

- A higher uncertainty (smaller θ) leads the firms to subcontract less, since the coefficients $\frac{1}{\theta}$, $\frac{1}{\theta^2}$, $\frac{1}{\theta^3}$, etc. increase when θ decreases.

These three results are proven in the following section for a complete network with any $n\in\mathbb{N}^+$. A more generalized version of these three results is mentioned in Kikuchi et al. (2012). The difference between their model and the one presented here, is that this model only allows the firms to pick from a discrete set of production quotas k_i, whereas their model allows firms to pick any real value for the production quota.

2.2.2 Properties of the solution

Generalizing from (2.6) to (2.9), we get an intermediate price function $p\left(\frac{m}{n}\right)$ defined by (2.10).

Lemma 1. *For every $k \in \{1, 2, \ldots m\}$, $\exists$ a set of possible sequences $\{x_i\}_{i=1}^{k}$ where each is a sequence of positive integers such that $x_1 + x_2 + \ldots + x_k = m$, which gives the price function*

$$p\left(\frac{m}{n}\right) = \min_{k, \{x_i\}_{i=1}^{k}} \left\{ \sum_{i=1}^{k} \frac{1}{\theta^i} c\left(\frac{x_i}{n}\right) \right\}_{k=1}^{m}, \tag{2.10}$$

where $1 \leq m \leq n$, and $m, n \in \mathbb{N}^+$.

Proof. For $m = 1$, $p\left(\frac{1}{n}\right) = \frac{1}{\theta} c\left(\frac{1}{n}\right)$, where $k \in \{1\}$, and $\{x_i\} = \{1\}$.
Assume $p\left(\frac{m}{n}\right) = \min_{k,\{x_i\}_{i=1}^{k}} \left\{\sum_{i=1}^{k} \frac{1}{\theta^i} c\left(\frac{x_i}{n}\right)\right\}_{k=1}^{m}$ for all $m \in \{1, 2, \ldots, k'\}$.

From (2.3),

$$
\begin{aligned}
p\left(\frac{k'+1}{n}\right) &= \frac{1}{\theta} \min_{k\in\{1,2,\ldots,k'+1\}} \left\{ c\left(\frac{k}{n}\right) + p\left(\frac{k'+1-k}{n}\right) \right\} \\
&= \frac{1}{\theta} \min \left\{ \begin{array}{c} c\left(\frac{1}{n}\right) + p\left(\frac{k'}{n}\right), \\ c\left(\frac{2}{n}\right) + p\left(\frac{k'-1}{n}\right), \\ \ldots \\ c\left(\frac{k'}{n}\right) + p\left(\frac{1}{n}\right), \\ c\left(\frac{k'+1}{n}\right) \end{array} \right\} \\
&= \min_{k,\{x_i\}_{i=1}^{k}} \left\{ \sum_{i=1}^{k} \frac{1}{\theta^i} c\left(\frac{x_i}{n}\right) \right\}_{k=1}^{k'+1} \qquad (2.11)
\end{aligned}
$$

Hence, we have shown by induction that the Lemma 1 is true for all $m \in \{1, 2, \ldots, n\}$. $\square$

Next, we show that any integer partition of m can be attributed to a certain sequence of possible production quotas for a production chain involving m firms.

Lemma 2. *For a complete network, any sequence $\{x_i\}_{i=1}^{k}$ such that $\sum_{i=1}^{k} x_i = m$, with $x_i \in \{1, 2, \ldots, m\}$ is a possible division of production quotas for a production chain involving m firms.*

Proof. Let each firm i produce a quota $\frac{x_i}{n}$ of the final good. For k firms, $\frac{\sum_{i=1}^{k} x_i}{n} = \frac{m}{n}$. Hence, $\{x_i\}_{i=1}^{k}$ represents a feasible sequence of production quotas for producing a fraction $\frac{m}{n}$ of the final good. $\square$

2.2.3 Downstream firms produce more

The first theorem shows that in an optimal production chain, firms that are more downstream produce a larger fraction of the final good.

Theorem 1. *The minimizer* $\{x_i^\star\}_{i=1}^k$ *for (2.10) for each* $k \in \{1, 2, \ldots, n\}$ *is such that* $x_i^\star \geq x_j^\star$ *for all* $i < j$*, where* $i, j \in \{1, 2, \ldots, k\}$*.*

Proof. By way of contradiction, assume $\exists$ $\{x_i^\star\}_{i=1}^k$ that is a minimizer for (2.10) such that $\exists$ k' such that $x_{k'}^\star < x_{k'+1}^\star$, where $1 \leq k' < k'+1 \leq k$.

$$\sum_{i=1}^{k} \frac{1}{\theta^i} c\left(\frac{x_i^\star}{n}\right) = \sum_{i=1}^{k'-1} \frac{1}{\theta^i} c\left(\frac{x_i^\star}{n}\right) + \frac{1}{\theta^{k'}} c\left(\frac{x_{k'}^\star}{n}\right) + \frac{1}{\theta^{k'+1}} c\left(\frac{x_{k'+1}^\star}{n}\right) + \sum_{i=k'+2}^{k} \frac{1}{\theta^i} c\left(\frac{x_i^\star}{n}\right)$$

Since $c\left(\frac{x_{k'+1}^\star}{n}\right) - c\left(\frac{x_{k'}^\star}{n}\right) > 0$, and $\frac{1}{\theta} > 1$,

$$\begin{aligned}
\frac{1}{\theta}\left[c\left(\frac{x_{k'+1}^\star}{n}\right) - c\left(\frac{x_{k'}^\star}{n}\right)\right] &> c\left(\frac{x_{k'+1}^\star}{n}\right) - c\left(\frac{x_{k'}^\star}{n}\right) \\
\frac{1}{\theta} c\left(\frac{x_{k'+1}^\star}{n}\right) + c\left(\frac{x_{k'}^\star}{n}\right) &> \frac{1}{\theta} c\left(\frac{x_{k'}^\star}{n}\right) + c\left(\frac{x_{k'+1}^\star}{n}\right) \\
\frac{1}{\theta^{k'}} c\left(\frac{x_{k'}^\star}{n}\right) + \frac{1}{\theta^{k'+1}} c\left(\frac{x_{k'+1}^\star}{n}\right) &> \frac{1}{\theta^{k'}} c\left(\frac{x_{k'+1}^\star}{n}\right) + \frac{1}{\theta^{k'+1}} c\left(\frac{x_{k'}^\star}{n}\right).
\end{aligned}$$

Thus, $\{x_i^\star\}_{i=1}^k$ cannot be a minimizer, since there exists another sequence $\{x_i'\}_{i=1}^k$ for which $x_k' = x_{k+1}^\star$, $x_{k+1}' = x_k^\star$, $\sum_{i=1}^k \frac{1}{\theta^i} c\left(\frac{x_i^\star}{n}\right) > \sum_{i=1}^k \frac{1}{\theta^i} c\left(\frac{x_i'}{n}\right)$, and by Lemma 2, $\{x_i'\}_{i=1}^k$ is a feasible sequence of production quotas. $\square$

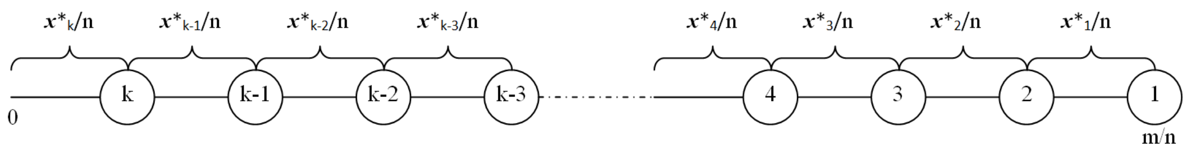


Figure 2.9: An optimal production chain showing the four most upstream and four most downstream firms in the production of the intermediate good from $\left[0, \frac{m}{n}\right]$.

2.2.4 Downstream firms have higher value-added

The value added by each firm in an optimal production chain increases as we move from upstream to downstream firms.

Theorem 2. *In a network with $n \geq 3$ firms, where $\frac{x_i^\star}{n}$ is the optimal quota chosen by each firm $i \in \{1, 2, 3, ..., k\}$, where $i = 1$ is the most downstream firm, and firm $i = 1$'s intermediate price is $p\left(\frac{m}{n}\right)$, $m \leq n$, we have*

$$p\left(\frac{m}{n}\right) - p\left(\frac{m - x_1^\star}{n}\right) \geq p\left(\frac{m - x_1^\star}{n}\right) - p\left(\frac{m - x_1^\star - x_2^\star}{n}\right). \tag{2.12}$$

Proof. With reference to Figure 2.9,

$$
\begin{aligned}
p\left(0\right) &= 0 && (2.13)\\
p\left(\frac{m-\sum_{i=1}^{k-1} x_i^\star}{n}\right) &= \frac{1}{\theta}\left[c\left(\frac{x_k^\star}{n}\right)+p\left(0\right)\right]\\
&= \frac{1}{\theta}c\left(\frac{x_k^\star}{n}\right) && (2.14)\\
p\left(\frac{m-\sum_{i=1}^{k-2} x_i^\star}{n}\right) &= \frac{1}{\theta}\left[c\left(\frac{x_{k-1}^\star}{n}\right)+p\left(\frac{m-\sum_{i=1}^{k-1} x_i^\star}{n}\right)\right]\\
&= \frac{1}{\theta}c\left(\frac{x_{k-1}^\star}{n}\right)+\frac{1}{\theta^2}c\left(\frac{x_k^\star}{n}\right) && (2.15)\\
p\left(\frac{m-\sum_{i=1}^{k-3} x_i^\star}{n}\right) &= \frac{1}{\theta}\left[c\left(\frac{x_{k-2}^\star}{n}\right)+p\left(\frac{m-\sum_{i=1}^{k-2} x_i^\star}{n}\right)\right]\\
&= \frac{1}{\theta}c\left(\frac{x_{k-2}^\star}{n}\right)+\frac{1}{\theta^2}c\left(\frac{x_{k-1}^\star}{n}\right)+\frac{1}{\theta^3}c\left(\frac{x_k^\star}{n}\right). && (2.16)
\end{aligned}
$$

Hence we can show that value added is increasing as we go downstream starting from the most upstream firm $i = k$,

$$
\begin{aligned}
& p\left(\frac{m-\sum_{i=1}^{k-3} x_i^\star}{n}\right)-p\left(\frac{m-\sum_{i=1}^{k-2} x_i^\star}{n}\right)\\
&= \frac{1}{\theta}c\left(\frac{x_{k-2}^\star}{n}\right)+\frac{1-\theta}{\theta}\left[\frac{1}{\theta}c\left(\frac{x_{k-1}^\star}{n}\right)+\frac{1}{\theta^2}c\left(\frac{x_k^\star}{n}\right)\right]\\
&\geq \frac{1}{\theta}c\left(\frac{x_{k-1}^\star}{n}\right)+\frac{1-\theta}{\theta}\left[\frac{1}{\theta}c\left(\frac{x_k^\star}{n}\right)\right]\\
&= p\left(\frac{m-\sum_{i=1}^{k-2} x_i^\star}{n}\right)-p\left(\frac{m-\sum_{i=1}^{k-1} x_i^\star}{n}\right), && (2.17)
\end{aligned}
$$

where $c\left(\frac{x_{k-2}^\star}{n}\right) \geq c\left(\frac{x_{k-1}^\star}{n}\right)$ due to Theorem 1.

Assuming $p\left(\frac{m-x_1^\star}{n}\right)-p\left(\frac{m-\sum_{i=1}^{2}x_i^\star}{n}\right) \geq p\left(\frac{m-\sum_{i=1}^{2}x_i^\star}{n}\right)-p\left(\frac{m-\sum_{i=1}^{3}x_i^\star}{n}\right)$, we now show (2.12) to be true.

$$\begin{aligned}
&p\left(\frac{m}{n}\right)-p\left(\frac{m-x_1^\star}{n}\right)\\
&=\frac{1}{\theta}\left[c\left(\frac{x_1^\star}{n}\right)-c\left(\frac{x_2^\star}{n}\right)\right]+\frac{1}{\theta}\left[p\left(\frac{m-x_1^\star}{n}\right)-p\left(\frac{m-x_1^\star-x_2^\star}{n}\right)\right]\\
&\geq\frac{1}{\theta}\left[c\left(\frac{x_2^\star}{n}\right)-c\left(\frac{x_3^\star}{n}\right)\right]+\frac{1}{\theta}\left[p\left(\frac{m-x_1^\star}{n}\right)-p\left(\frac{m-x_1^\star-x_2^\star}{n}\right)\right]\\
&\geq\frac{1}{\theta}\left[c\left(\frac{x_2^\star}{n}\right)-c\left(\frac{x_3^\star}{n}\right)\right]+\frac{1}{\theta}\left[p\left(\frac{m-\sum_{i=1}^{2}x_i^\star}{n}\right)-p\left(\frac{m-\sum_{i=1}^{3}x_i^\star}{n}\right)\right]\\
&=\frac{1}{\theta}\left[c\left(\frac{x_2^\star}{n}\right)+p\left(\frac{m-\sum_{i=1}^{2}x_i^\star}{n}\right)\right]-\frac{1}{\theta}\left[c\left(\frac{x_3^\star}{n}\right)+p\left(\frac{m-\sum_{i=1}^{3}x_i^\star}{n}\right)\right]\\
&=p\left(\frac{m-x_1^\star}{n}\right)-p\left(\frac{m-x_1^\star-x_2^\star}{n}\right), \qquad (2.18)
\end{aligned}$$

where the first inequality is due to Theorem 1 and the second inequality is due to the assumption above.

Since m is arbitrary, we have shown by induction that (2.12) is true. □

2.2.5 More subcontracting takes place as θ increases

As the success probability θ increases, firms take advantage of the gains from specialization and choose to subcontract more, resulting in production chains that involve more firms. In order to show this, first we prove that the price of the final good is decreasing with respect to θ. Next, we look at how two production chains that are initially producing the final good at the same price will change their prices with respect to θ. We show in Theorem 3 that the production chain utilizes more firms will decrease its final price by more than the the chain which utilizes less firms, given a marginal increase in θ. Theorem 3 thus shows that it benefits firms to increase the amount of subcontracting as θ increases.

We first show that the price function of the final good $p(1)$ is decreasing with respect to θ.

Lemma 3. *$p(1)$ is decreasing with respect to θ.*

Proof. For a given $\{x_i\}_{i=1}^{k}$, with $\sum_{i=1}^{k} \frac{x_i}{n} = 1$,

$$p(1) = \sum_{i=1}^{k} \frac{1}{\theta^i} c\left(\frac{x_i}{n}\right)$$

$$\frac{dp(1)}{d\theta} = -\sum_{i=1}^{k} \frac{i}{\theta^{i+1}} c\left(\frac{x_i}{n}\right) < 0$$

$\square$

Next we consider the situation when two production chains $\{x_i\}$ and $\{x'_i\}$ are available and have identical prices for the final good, i.e. $p_{\{x_i\}}(1) = p_{\{x'_i\}}(1)$. We show that for a given increase in θ, the price for the chain utilizing a greater number of firms decreases more than the one utilizing less firms.

Theorem 3. *For a given $\theta \in (0, 1]$ such that $p_{\{x_i\}}(1) = p_{\{x'_i\}}(1)$, that is $\sum_{i=1}^{k} \frac{1}{\theta^i} c\left(\frac{x_i}{n}\right) = \sum_{i=1}^{k'} \frac{1}{\theta^i} c\left(\frac{x'_i}{n}\right)$, where $\{x_i\}_{i=1}^{k}$ and $\{x'_i\}_{i=1}^{k'}$ represent two possible production paths as explained in Lemma 2, with $k' > k$, and $x_i \geq x'_i$ for* *every* $i \in \{1, 2, \ldots, k\}$,

$$\frac{d}{d\theta}\left[\sum_{i=1}^{k'} \frac{1}{\theta^i} c\left(\frac{x_i'}{n}\right)\right] \leq \frac{d}{d\theta}\left[\sum_{i=1}^{k} \frac{1}{\theta^i} c\left(\frac{x_i}{n}\right)\right] < 0 \tag{2.19}$$

Proof. The second inequality in (2.19) is shown in Lemma 3. The proof of the first inequality is below. Since

$$\begin{aligned}
\sum_{i=1}^{k} \frac{1}{\theta^i} c\left(\frac{x_i}{n}\right) &= \sum_{i=1}^{k'} \frac{1}{\theta^i} c\left(\frac{x_i'}{n}\right) \\
\sum_{i=1}^{k} \frac{1}{\theta^i} c\left(\frac{x_i}{n}\right) &= \sum_{i=1}^{k} \frac{1}{\theta^i} c\left(\frac{x_i'}{n}\right) + \sum_{i=k+1}^{k'} \frac{1}{\theta^i} c\left(\frac{x_i'}{n}\right) \\
\sum_{i=k+1}^{k'} \frac{1}{\theta^i} c\left(\frac{x_i'}{n}\right) &= \sum_{i=1}^{k} \frac{1}{\theta^i}\left[c\left(\frac{x_i}{n}\right) - c\left(\frac{x_i'}{n}\right)\right],
\end{aligned}$$

$$\text{we have} \sum_{i=k+1}^{k'} \frac{k+1}{\theta^{i+1}} c\left(\frac{x_i'}{n}\right) = \sum_{i=1}^{k} \frac{k+1}{\theta^{i+1}}\left[c\left(\frac{x_i}{n}\right) - c\left(\frac{x_i'}{n}\right)\right] \tag{2.20}$$

In addition, we have

$$\begin{aligned}
\sum_{i=k+1}^{k'} \frac{i}{\theta^{i+1}} c\left(\frac{x_i'}{n}\right) &\geq \sum_{i=k+1}^{k'} \frac{k+1}{\theta^{i+1}} c\left(\frac{x_i'}{n}\right) \\
&= \sum_{i=1}^{k} \frac{k+1}{\theta^{i+1}}\left[c\left(\frac{x_i}{n}\right) - c\left(\frac{x_i'}{n}\right)\right] \\
&\geq \sum_{i=1}^{k} \frac{i}{\theta^{i+1}}\left[c\left(\frac{x_i}{n}\right) - c\left(\frac{x_i'}{n}\right)\right]
\end{aligned} \tag{2.21}$$

where the second inequality is from the assumption that $x_i \geq x_i'$ for every

$i \in \{1, 2, \ldots, k\}$. Thus,

$$\sum_{i=k+1}^{k'} \frac{i}{\theta^{i+1}} c\left(\frac{x_i'}{n}\right) \geq \sum_{i=1}^{k} \frac{i}{\theta^{i+1}} \left[c\left(\frac{x_i}{n}\right) - c\left(\frac{x_i'}{n}\right)\right]$$

$$-\sum_{i=1}^{k'} \frac{i}{\theta^{i+1}} c\left(\frac{x_i'}{n}\right) \leq -\sum_{i=1}^{k} \frac{i}{\theta^{i+1}} c\left(\frac{x_i}{n}\right)$$

$$\frac{d}{d\theta}\left[\sum_{i=1}^{k'} \frac{1}{\theta^i} c\left(\frac{x_i'}{n}\right)\right] = -\sum_{i=1}^{k'} \frac{i}{\theta^{i+1}} c\left(\frac{x_i'}{n}\right) \leq -\sum_{i=1}^{k} \frac{i}{\theta^{i+1}} c\left(\frac{x_i}{n}\right) = \frac{d}{d\theta}\left[\sum_{i=1}^{k} \frac{1}{\theta^i} c\left(\frac{x_i}{n}\right)\right] \tag{2.22}$$

□

2.3 Computational solution

2.3.1 Possible production paths for any network

This section and the next describes the computational solution to the model which allows for a wide range of heterogeneity. The model can accommodate heterogeneous θ, incomplete networks, or heterogeneous cost functions. It can also be easily extended to an imperfect information setting.

The computational methodology is to calculate the price functions of all the possible production paths in a network and pick the optimal one. Hence, we first need to find all the combinatorial possibilities with which the subcontracting process may take place. I have presented an analytical way to calculate the number of possible paths in a complete network in Section 2.1.4, and in this section I will describe an algorithm which will calculate the paths for any kind of network.

To handle an incomplete network, or any generalized form of network, Algorithm 1 is used to build a tree that maps out all the possible pathways. The tree has the “Client node” at the root and initially branches off to each of the firms in the network, each firm forms a “firm node”, which decides how many parts of the good it should produce in-house. To represent this decision, the tree branches at each firm node to all the possible choices of the number of parts it can produce in-house. Each of these choices are represented by a “task node”. Having chosen the number of parts to produce in house, the production process can either come to an end, in which case that current task node is a leaf of the tree, or it may be that the firm chose to not producing everything in-house and instead subcontract some of the production to another firm. In this case, the tree then branches at the task node to the available subcontractors for that firm. These consist of the firms which are: (i) connected to the current firm in the network, and (ii) not already occupied in the production process further downstream. The tree then branches to all these available firms, and the procedure recursively begins again just like at the first level of firm nodes.

An example of the resulting production tree for a complete network of $n = 3$ is shown in Figure 2.10. The circles show the firms, and the rectangles show the portions of the good that each is producing. The 3 firms in the network are numbered 0, 1, and 2.

Algorithm 1: Building a production tree.

```
Data: Boolean matrix showing the n firms' interconnections
Result: a production tree with n firms
set integer s = 0;
declare firmsUsed as an empty array;
if Algorithm is called at Client node then
    for i ← 0 to n − 1 do
        build a child firm node and call it firm i;
        call this algorithm recursively to firm i;
    end
else if called at a firm node i then
    insert self into firmsUsed array;
    for t ← 1 to s − 1 do
        build a child firm node and call it task t;
        call this algorithm recursively to task t;
    end
else if called at a task node t then
    s ← s + t ;
    if s != n then
        foreach firm i that is connected to parent firm node AND not
        yet in the firmsUsed array do
            build a child firm node and call it firm i;
            call this algorithm recursively for firm i;
        end
    end
end
```

2.3.2 Algorithm for computational solution

The computational solution simply traces the tree produced in Section 2.3.1 and illustrated in Figure 2.10, from the client located at the root of the tree to each of the paths that lead to the leaves of the tree. Along the this path, intermediate prices are calculated and firms in the intermediate stages of the production process will eliminate the branches of the tree with higher

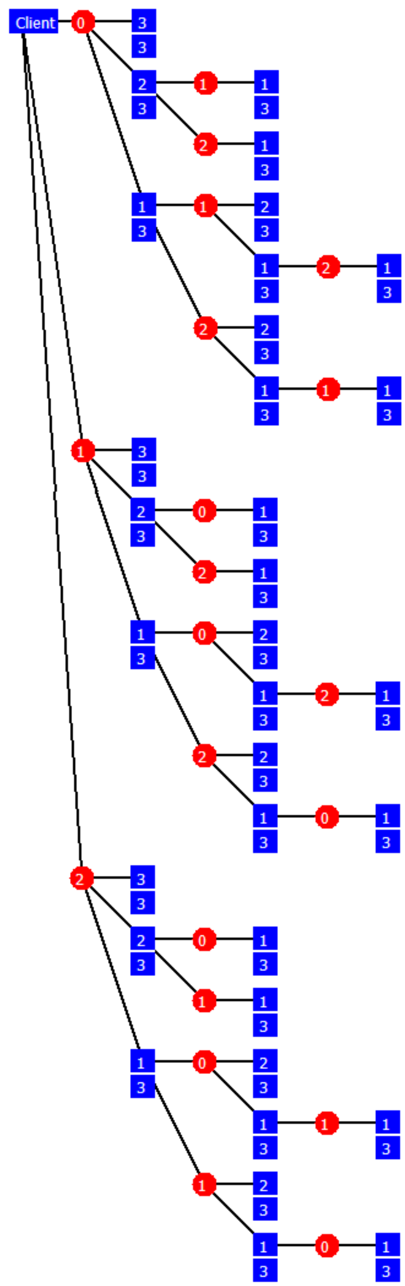


Figure 2.10: Production tree for a complete network of $n = 3$.

expected costs. As the result of this recursive elimination of branches, the final path that is chosen by the client is the one with the lowest expected expected cost at each stage in the production process. It is described in Algorithms 2 and 3.

Algorithm 2: Solving for the optimal production path (Part 1).

Data:
The tree produced by Algorithm 1
The success probability θ
The cost function $c(\bullet)$
Result:
The expected final price of the good p
The *productionChain* specifiying: (i) the firms involved in the production process, (ii) the success or failure at each stage, (iii) the number of parts produced by each firm, and (iv) the intermediate prices

if *Algorithm is called at Client node* **then**
    for $j \leftarrow 0$ **to** $n-1$ **do**
        call this algorithm recursively for firm j;
        the algorithm will return: (i) p_j, i.e. the expected cost for firm j, and (ii) the *productionChain* that was used;
    end
    pick a firm j_{pick} that offers the lowest p_j, if several firms are tied, select one at random;
    return lowest p_j, and *productionChain*;
continued in Algorithm 3...

Algorithm 3: Solving for the optimal production path (Part 2).

Data: See Algorithm 2.

Result: See Algorithm 2.

else if *called at a firm node i* **then**
    foreach *task node t of firm node i* **do**
        calculate $p_{inhouse}$, i.e. in house cost for producing fraction of good defined by task node t;
        pick a random boolean draw with probability θ to decide if the subcontract was a success or failure;
        if *t is a leaf node* **then**
            declare a suitable data structure for the *productionChain*;
            add self (firm i), outcome (success or failure), p, and number of parts produced to the *productionChain*;
            return $p_{inhouse}$ and *productionChain*;
        else if *t has children, which are firm nodes* **then**
            foreach *children firm node j of task node t* **do**
                call this algorithm recursively for firm j;
                the algorithm will return: (i) p_j, i.e. the expected cost for firm j, and (ii) the *productionChain* that was used;
            end
            pick a firm j_{pick} that offers the lowest p_j, if several firms are tied, select one at random;
            $p \leftarrow p_{inhouse} + p_{j_{pick}}$;
            add self (firm i), outcome (success or failure), p, and number of parts produced to the *productionChain* of j_{pick};
            return p, and *productionChain*;
        end
    end
end

2.3.3 Computational Results

This section presents the results obtained from solving the model computationally using the algorithms shown in Sections 2.3.1 and 2.3.2. The solutions were computed for complete networks consisting of 2 to 7 firms. For each network, θ was varied from 0.1 to 1.0, in increments of 0.01.

Tables A.1, A.2, and A.3 in Appendix A show respectively for networks of $n = 3, 5$ and 7 firms how the production process is divided amongst the firms. As θ increases, it can be seen that more firms are used in the subcontracting process. Also, for every θ, the downstream firms are seen to produce larger portions of the final good than the upstream firms. This result can also be seen in Figures A.1 to A.6 in Appendix A, which show the number of firms used in the production as theta varies from $\theta = 0.01$ to 1.0 in increments of 0.01. These results corroborate with those presented in Section 2.2 and Kikuchi et al. (2012).

Tables A.4, A.5, and A.6 in Appendix A show respectively for networks of $n = 3, 5$ and 7 firms how the value added varies amongst the firms. For every θ, it can be seen that the value added by each firm increases as we go from upstream to downstream firms. This result can also be seen in Figures A.7 to A.12 in Appendix A, which show value added by each firm as a percentage of the final price of the good, as theta varies from $\theta = 0.01$ to 1.0 in increments of 0.01. These results also corroborate with those presented in Section 2.2 and Kikuchi et al. (2012).

Chapter 3

Extended Model

This chapter presents the extended version of the model in which neither the firms nor the client have objective knowledge of the probability of success θ associated with one another. Instead, they learn about θ by observing the successes and failures from the outcomes of previous subcontracts. This adds a dynamic element to the model, where firms learn about the true value of θ after multiple rounds of interaction.

When firms and the client update their beliefs about θ, they attribute a different θ for each firm that they interact with and update each one separately. I use the notation $\theta_{i \to l}$ to denote l's belief about i's probability of success. Henceforth, I will refer to $\theta_{i \to l}$ as i's reputation to l.

As explained in Section 2.1.2, should a subcontracting process fail at a certain point in the supply chain, losses are only incurred by a firm which has failed in its in-house production of goods. Once a subcontractor A's in-house production fails, it will not obtain payment from it's contractee, but it still has to pay its subcontractor which successfully delivered the intermediate goods. Thus, when we model the reputation updates as occurring for a firm's immediate subcontractor. For example, if the subcontracts go from "client → Firm A → Firm B → Firm C", the client will only update A's reputation to it, A will only update B's reputation to A, and B will only update C's reputation to B. Also, a contractee will only update a subcontractor's reputation if the subcontrator was the one responsible for the success or failure of the production. In other words, firms are not punished for the failures of subcontractors who are further upstream. In the context of the example above, where we have the path "client → Firm A → Firm B → Firm C", a failure by the most upstream firm, C, will not result in A punishing B by lowering B's reputation to A. The only update which takes place will be for C's reputation to B.

There are various assumptions we can make to justify this method of updating reputations. One is to assume that a mechanism exists for verifying who was at fault when a failure occurs.

Following the example above with the structure "client → Firm A → Firm B → Firm C", following a failure by C to deliver goods to B, a subcontractor B can point out to A that the failure was not caused during B's in-house production. We can assume that A can then verify that this is true. The same goes for the A's reputation to the client. Thus the A's reputation to the client remains intact and only B's reputation to A also remains intact, whereas C's reputation to B is lowered.

In order to implement the reputation updating process, we modify the

Algorithm 2 and 3 presented in 2.3.2 as shown in Algorithm 4 and 5. The only changes being that the production takes place repeatedly over multiple rounds as defined by the *numRounds* variable, and after the optimal path has been chosen at the Client node, a round of reputation updates take place depending on the success or failure of the subcontractors.

Algorithm 4: Solving for the optimal production path with reputation updates (Part 1).

Data:
The tree produced by Algorithm 1
The success probability θ
The cost function $c(\bullet)$
Result:
The expected final price of the good p
The *productionChain* specifiying: (i) the firms involved in the production process, (ii) the success or failure at each stage, (iii) the number of parts produced by each firm, and (iv) the intermediate prices

set int *numRounds*;
for $r \leftarrow 0$ **to** *numRounds* **do**
if *Algorithm is called at Client node* **then**
    for $j \leftarrow 0$ **to** $n-1$ **do**
        call this algorithm recursively for firm j;
        the algorithm will return: (i) p_j, i.e. the expected cost for firm j, and (ii) the *productionChain* that was used;
    end
    pick a firm j_{pick} that offers the lowest p_j, if several firms are tied, select one at random;
    foreach *firm or Client j and its subcontractor firm k in productionChain starting from most upstream k* **do**
        update $\theta_{k \to j}$, i.e. k's reputation to j;
        if *k's production failed* **then**
            exit “foreach” loop;
        end
    end
    return lowest p_j, and *productionChain*;
continued in Algorithm 5...

Algorithm 5: Solving for the optimal production path with reputation updates (Part 2).

```
Data: See Algorithm 4.
Result: See Algorithm 4.
else if called at a firm node i then
    foreach task node t of firm node i do
        calculate p_inhouse, i.e. in house cost for producing fraction of
        good defined by task node t;
        pick a random boolean draw with probability θ to decide if the
        subcontract was a success or failure;
        if t is a leaf node then
            declare a suitable data structure for the productionChain;
            add self (firm i), outcome (success or failure), p, and
            number of parts produced to the productionChain;
            return p_inhouse and productionChain;
        else if t has children, which are firm nodes then
            foreach children firm node j of task node t do
                call this algorithm recursively for firm j;
                the algorithm will return: (i) p_j, i.e. the expected cost
                for firm j, and (ii) the productionChain that was used;
            end
            pick a firm j_pick that offers the lowest p_j, if several firms are
            tied, select one at random;
            p ← p_inhouse + p_{j_pick} ;
            add self (firm i), outcome (success or failure), p, and
            number of parts produced to the productionChain of j_pick;
            update firm i's reputation for firm j_pick;
            return p, and productionChain;
        end
    end
end
end main "for" loop
```

3.1 Bayesian updating

The reputations are updated through a Bayesian updating process, in which the client, as well as each firm, l, assigns a probability distribution of $\theta_{i\to l}$ to each potential subcontractor, i. This initial distribution is known as the prior distribution $p_{\theta_{i\to l}}(\theta_k) = P(\theta_{i\to l} = \theta_k)$ over some set of values of θ_k. For the purposes of this model, we use a discrete distribution of which allows 11 distinct values of $\theta_k = \{0.0, 0.1, 0.2, \ldots, 1.0\}$. Let $\mu_{\theta_k} = P(\theta_{i\to l} = \theta_k)$ be the prior probability assigned by l that i's reputation to l is θ_k. After gathering evidence about firm i's number of successes while working as a subcontractor for l, $S_{i\to l}$, and failures, $F_{i\to l}$, l proceeds to form an updated posterior probability $\mu'_{\theta_k} = P(\theta_{i\to l} = \theta_k | S_{i\to l}, F_{i\to l})$, where μ'_{θ_k} is updated using Bayes' Rule as

$$\mu'_{\theta_k} = \frac{\mu_{\theta_k}\theta_k^{S_{i\to l}}(1-\theta_k)^{F_{i\to l}}}{\sum_{\theta_k}\left\{\mu_{\theta_k}\theta_k^{S_{i\to l}}(1-\theta_k)^{F_{i\to l}}\right\}}, \tag{3.1}$$

which gives a new posterior distribution for $p_{\theta_{i\to l}}(\theta_k)$.

For the purposes of this model, the initial prior distribution $p_{\theta_{i\to i}}(\theta_k)$ is assumed to be uniform over $\theta_k = \{0.0, 0.1, 0.2, \ldots, 1.0\}$.

Using this method for updating the $\theta_{i\to l}$ values means that the firm i's expected profit maximization problem, which was formulated in Equation 2.1 becomes

$$E(\pi_i) = E[\theta_{i\to l}]\, p_i(s_i) - c(s_i - u_i) - p_j(u_i). \tag{3.2}$$

Solving for the equilibrium price, we reformulate Equation 2.3 to obtain

$$p_i\left(s_i\right) = \frac{1}{E\left[\theta_{i\to l}\right]} \min_{k_i,j} \left\{c\left(s_i - u_i(k_i)\right) + p_j\left(u_i(k_i)\right)\right\}, \tag{3.3}$$

where $E\left[\theta_{i\to l}\right] = \sum_{\theta_k} \theta_k P\left(\theta_{i\to l} = \theta_k \middle| S_{i\to l}, F_{i\to l}\right)$ is the contractee l's expectation of $\theta_{i\to l}$.

The next section discusses the circumstances in which $E\left[\theta_{i\to l}\right]$ converges to the true probability of success θ, that is actually homogeneous across every firm in the network.

3.2 Results for extended model

In order to show that the Bayesian updating will result in the convergence of $E\left[\theta_{i\to l}\right]$ to θ after repeated interactions between the contractee l and the subcontractor i, I present the results on a complete network of $n = 5$ firms with a homogeneous $\theta = 0.8$ across all firms. At this θ value, we know from section 2.3.3, and Tables A.2 and A.5 that the optimal production process involves utilizing all 5 firms, each producing $\frac{1}{5}$ parts of the final good.

From the computational results, it is observed that all five firms are used in the production, and that the value added by each firm converges to the same result as in the baseline model. This is shown in Figure 3.1. It can be seen that the values added converge to exactly those calculated in Table A.5.

The expected values for $\theta_{i\to l}$, however, only converge for some of the "contractee-contractor" relationships. It is found that the sequence of sub-

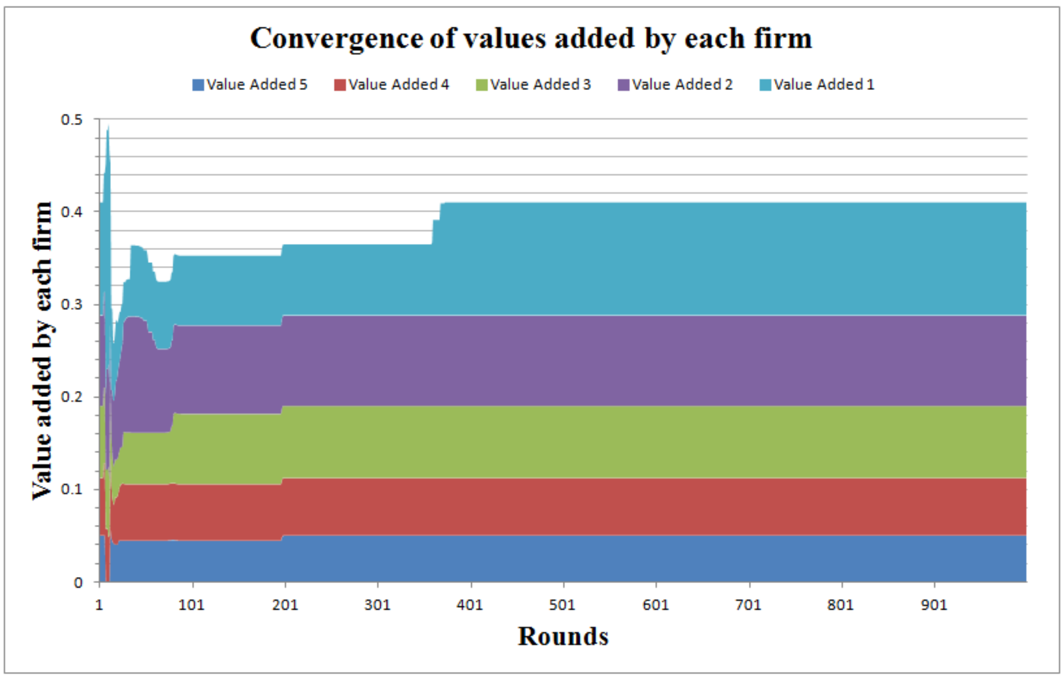


Figure 3.1: Convergence of value added by firms over 1000 rounds with reputation updates in a complete network of $n = 5$ with $\theta = 0.8$.

contractors from the most upstream to the most downstream converge to the same sequence of firms. In this particular example, the most upstream is Firm #4, followed by #3, #2, #0, and finally #1 who occupies the most downstream spot. As a consequence of the repeated interactions between those particular pairs of firms, the expected values for $\theta_{4\to3}$, $\theta_{3\to2}$, $\theta_{2\to0}$, $\theta_{0\to1}$, and $\theta_{1\to Client}$ converge to the true value $\theta = 0.8$. This is shown in Figure 3.2.

To look at the phenomenon in more detail, consider Figure 3.3, which traces the prior distribution of $\theta_{0\to1}$, i.e. $p_{\theta_{0\to1}}(\theta_k)$ for each value of $\theta_k = \{0.0, 0.1, 0.2, \ldots, 1.0\}$. The thickness of each band of color represents the prior probability for each θ_k. It can be seen that after about 65 out of the 1000 rounds, $\theta_{0\to1}$, i.e. $p_{\theta_{0\to1}}(\theta_k = 0.8) = 1.0$ whereas all the others are 0.

Contrast this with other values of $\theta_{i\to l}$ for inter-firm relationships that are not in the subcontracting path #4 $\to$ #3 $\to$ #2 $\to$ #0 $\to$ #1 $\to$ Client. For example, Figure 3.4 shows how the distribution for $\theta_{2\to1}$, which is not one of the pairs which are continually updated, does not converge to the true value of $\theta = 0.8$. The figure shows only the first 100 rounds but the distribution failed to converge even after 1000 rounds.

This selective convergence of θ values account for the main result of this paper, namely that more interconnected networks are favoured over less interconnected ones. The reason for this selective convergence is that in the initial rounds of subcontracting, some firm may get a string of failures while processing a subcontract and thus will result in contractee assigning a low reputation for them. This in turn means that the expected costs associated

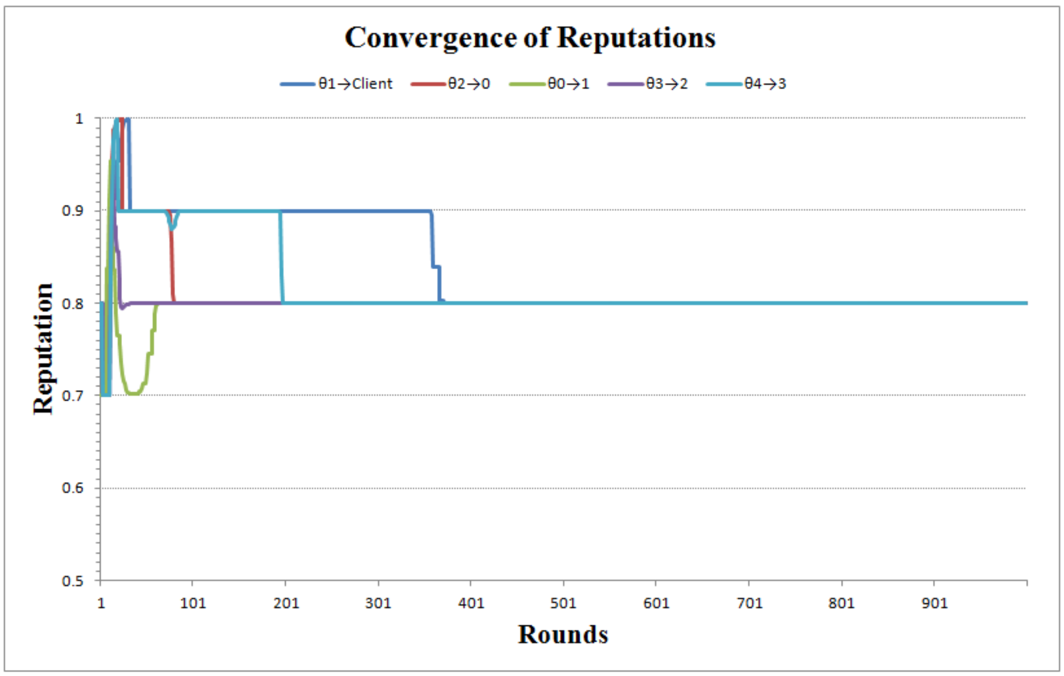


Figure 3.2: Convergence of reputations for firms with repeated interactions over 1000 rounds with reputation updates in a complete network of $n = 5$ with $\theta = 0.8$.

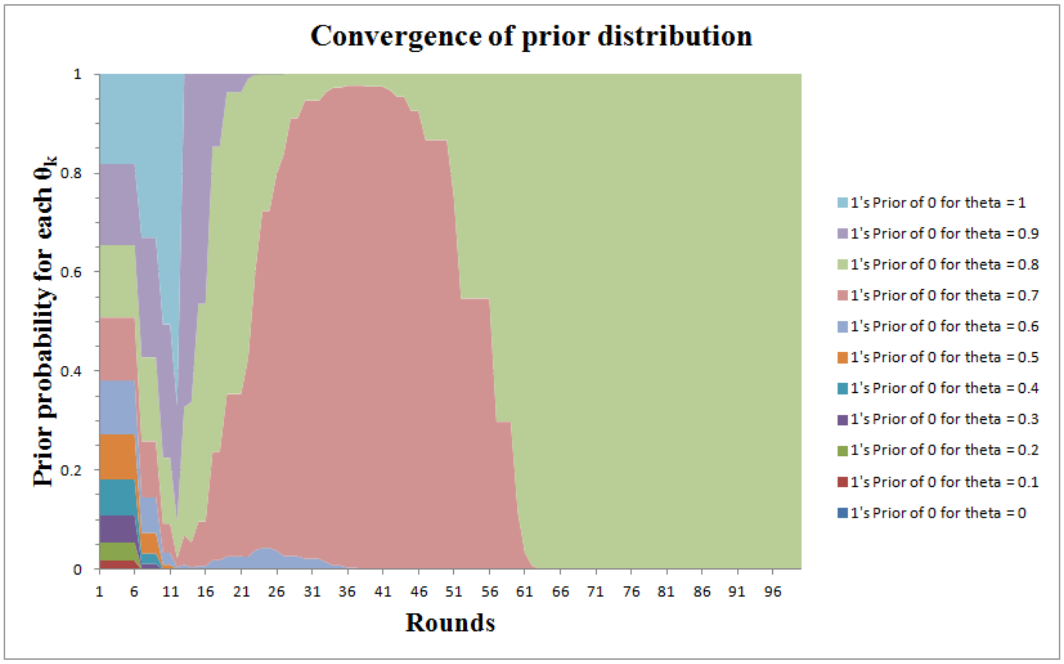


Figure 3.3: An example of the convergence of the prior distribution for firms with repeated interactions. Recorded over 100 rounds with reputation updates in a complete network of $n = 5$ with $\theta = 0.8$. In this case, it is the reputation of Firm #0 to Firm #1, $\theta_{0\to 1}$.

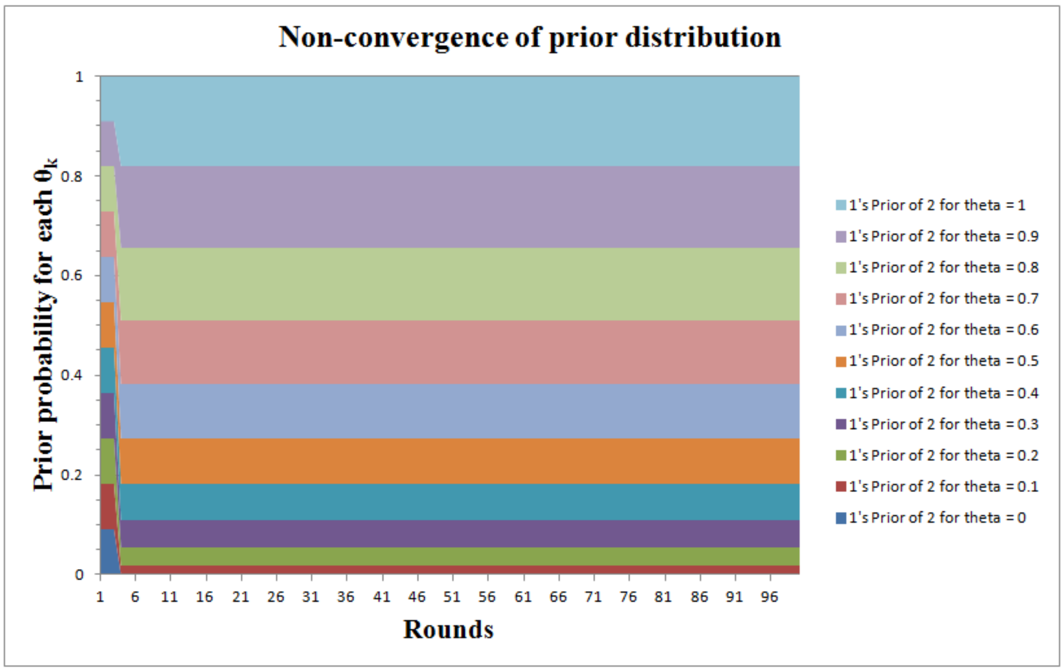


Figure 3.4: An example of the non-convergence of the prior distribution for firms without repeated interactions. Recorded over 100 rounds with reputation updates in a complete network of $n = 5$ with $\theta = 0.8$. In this case, it is the reputation of Firm #2 to Firm #1, $\theta_{2\to1}$.

with these low reputation firms are now higher than the expected costs associated with some other firms which might have gotten a string of initial successes. This creates a path dependence in the updating process for $\theta_{i \to l}$.

When a network has more interconnections between the firms, an initial string of failures by some of the firms does not necessarily raise expected costs for all possible production paths. The more numerous interconnections allow for other production paths to be used, which have not yet earned bad reputations. To put in another way, each firm in a more interconnected network has more potential firms to subcontract to, hence, if some of these potential subcontractors have earned a bad reputation due to a string of failures, it can easily subcontract to the other potential subcontractors, whereas in a less interconnected network, firms might be stuck with subcontractors who have earned a bad reputation, hence incurring increased expected costs. This ability of firms in a more interconnected networks to spread risk across multiple potential subcontractors allows for a more interconnected network to produce the good with a lower expected cost.

The result mentioned above is obtained via simulations on a network shown in Figure 3.5. The actual success probability θ is set at 0.8 homogenously across all the 10 firms. 100 simulations were run, with each simulation lasting 1000 rounds. Towards the end of each simulation, the selective convergence explained above causes a particular sequence of firms to be the "dominant" contractors for every round. It is observed that for 72 out of the 100 simulations, the sequence of firms that became the dominant contractors were in the more interconnected network, while for 28 out of the 100

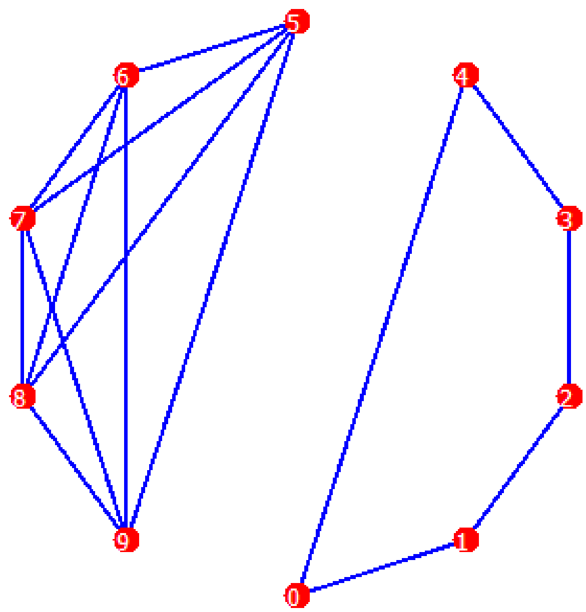


Figure 3.5: Two networks with $n = 5$ firms each. The one on the left is a complete network where each firm is connected to every other firm. The one on the right is an incomplete network where each firm only has two connections.

simulations, a sequence from the incomplete network became dominant. As explained in Chapter 1, this phenomenon can be interpreted as a reason for agglomeration economies.

3.3 Conclusion

I have presented a theoretical model of subcontracting among firms in a network. In Chapter 2, it was shown that in a complete network setting with complete information, certain analytical results can be obtained, as described in Section 2.2. A computational solution to the model was introduced in Section 2.3, which could handle any network structure.

In Chapter 3, I extended the computational model to take into account a dynamic Bayesian updating process for θ, which allows each of the firms to learn the other firms' θ from their past experiences. Simulation results show that there is a path dependent effect in the updating of θ where firms that initially obtain a string of successes become the dominant contractors. This effect gives an advantage to networks in which firms are more interconnected, suggesting a reason for agglomeration economies to take place.

For further research, analytical solutions to the extended model showing the selective convergence would present a stronger case for the phenomenon. Another change to the model might be to allow each firm at each stage of the production process to not only subcontract to one other firm, but to multiple firms at once. Empirical research linking reputation effects as a factor that causes agglomeration would also give more validation to the model.

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Appendix A

Solutions to baseline model

θ	Number of firms used	Number of parts produced by each firm				
0.1	1	3				
0.2	2	2	1			
0.3	2	2	1			
0.4	2	2	1			
0.5	2	2	1			
0.6	3	1	1	1		
0.7	3	1	1	1		
0.8	3	1	1	1		
0.9	3	1	1	1		
1.0	3	1	1	1		

Table A.1: Number of parts produced by each firm as θ varies for a complete network with 3 firms.

θ	Number of firms used	Number of parts produced by each firm					
0.1	1	5					
0.2	2	4	1				
0.3	2	4	1				
0.4	3	3	1	1			
0.5	3	3	1	1			
0.6	4	2	1	1	1		
0.7	4	2	1	1	1		
0.8	5	1	1	1	1	1	
0.9	5	1	1	1	1	1	
1.0	5	1	1	1	1	1	

Table A.2: Number of parts produced by each firm as θ varies for a complete network with 5 firms.

θ	Number of firms used	Number of parts produced by each firm							
0.1	2	6	1						
0.2	2	6	1						
0.3	2	5	2						
0.4	3	4	2	1					
0.5	3	4	2	1					
0.6	4	3	2	1	1				
0.7	5	2	2	1	1	1			
0.8	6	2	1	1	1	1	1		
0.9	7	1	1	1	1	1	1	1	
1.0	7	1	1	1	1	1	1	1	

Table A.3: Number of parts produced by each firm as θ varies for a complete network with 7 firms.

θ	Number of firms used	Final price	Value added by each firm			
0.1	1	10	10			
0.2	2	5	4.444	0.556		
0.3	2	2.716	2.346	0.370		
0.4	2	1.806	1.528	0.278		
0.5	2	1.333	1.111	.0222		
0.6	3	1.008	0.514	0.309	0.185	
0.7	3	0.709	0.324	0.226	0.159	
0.8	3	0.530	0.218	0.173	0.139	
0.9	3	0.413	0.152	0.138	0.123	
1.0	3	0.333	0.111	0.111	0.111	

Table A.4: Number of parts produced by each firm as θ varies for a complete network with 3 firms.

θ	Number of firms used	Final price	Value added by each firm				
0.1	1	10	10				
0.2	2	4.200	4.000	0.200			
0.3	2	2.578	2.248	0.330			
0.4	3	1.775	1.425	0.250	0.100		
0.5	3	1.200	0.960	0.160	0.080		
0.6	4	0.872	0.509	0.185	0.111	0.067	
0.7	4	0.593	0.338	0.116	0.082	0.057	
0.8	5	0.410	0.122	0.097	0.079	0.062	0.050
0.9	5	0.277	0.067	0.061	0.055	0.050	0.044
1.0	5	0.200	0.040	0.040	0.040	0.040	0.040

Table A.5: Number of parts produced by each firm as θ varies for a complete network with 5 firms.

θ	Number of firms used	Final price	Value added by each firm						
0.1	2	9.388	9.368	0.020					
0.2	2	4.184	4.082	0.102					
0.3	2	2.608	2.336	0.272					
0.4	3	1.645	1.313	0.281	0.051				
0.5	3	1.143	0.898	0.204	0.041				
0.6	4	0.785	0.498	0.196	0.057	0.034			
0.7	5	0.549	0.246	0.173	0.059	0.042	0.029		
0.8	6	0.364	0.155	0.062	0.050	0.040	0.031	0.026	
0.9	7	0.233	0.043	0.038	0.035	0.031	0.028	0.025	0.023
1.0	7	0.143	0.021	0.020	0.020	0.021	0.020	0.021	0.020

Table A.6: Number of parts produced by each firm as θ varies for a complete network with 7 firms.

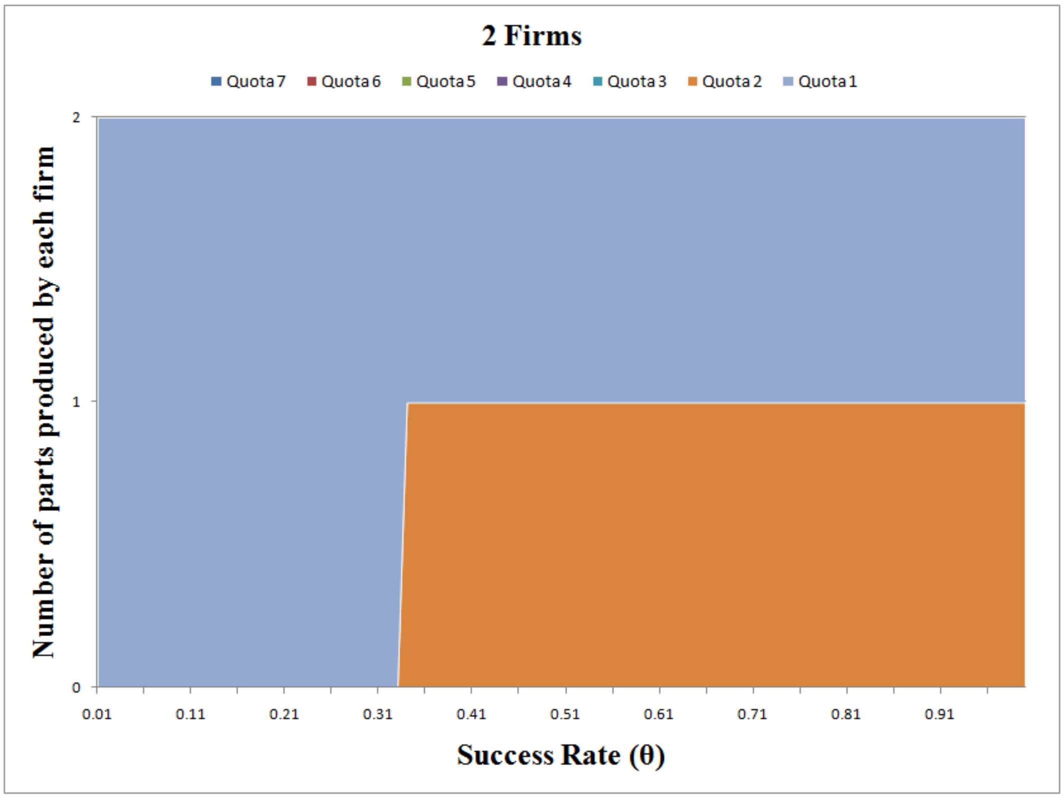

Figure A.1: Graph showing number of parts produced by each firm varies as a function of θ in a complete network with 2 firms.

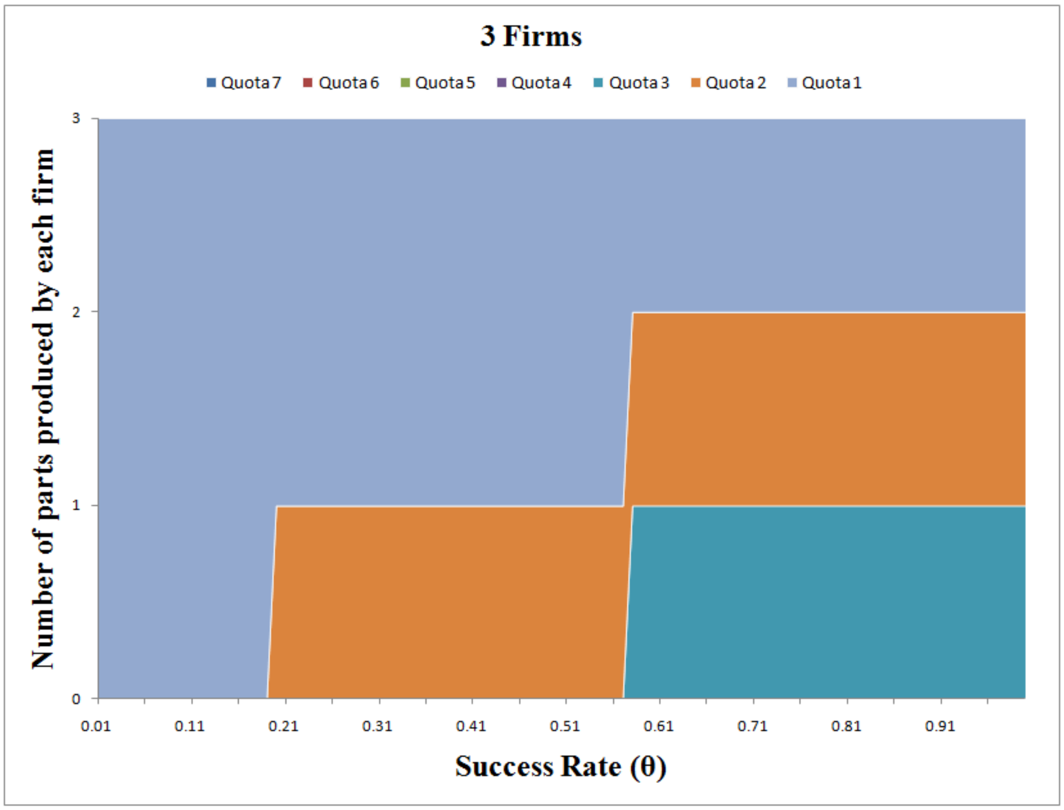

Figure A.2: Graph showing number of parts produced by each firm varies as a function of θ in a complete network with 3 firms.

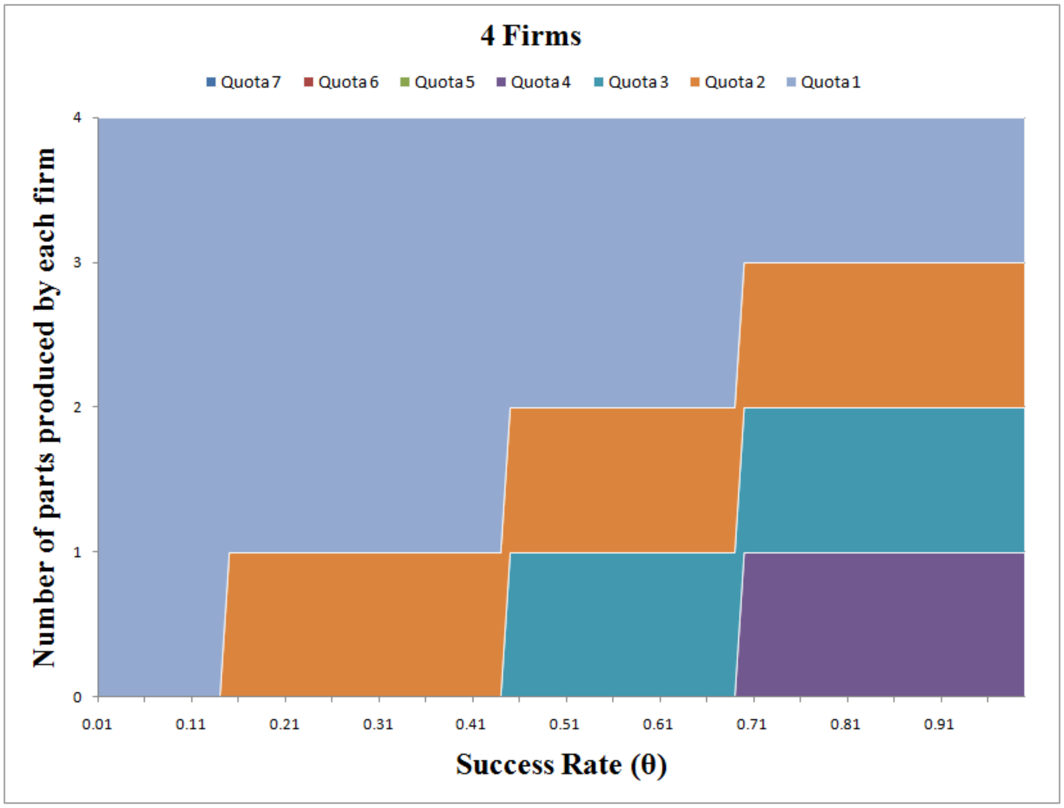

Figure A.3: Graph showing number of parts produced by each firm varies as a function of θ in a complete network with 4 firms.

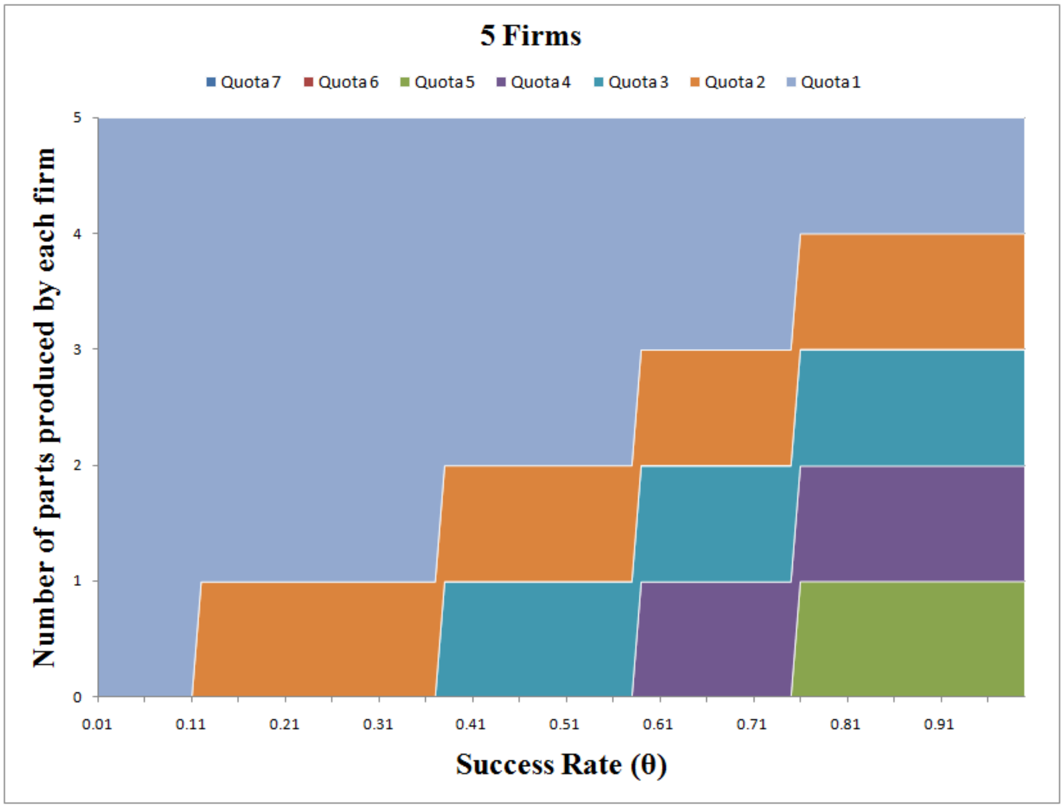

Figure A.4: Graph showing number of parts produced by each firm varies as a function of θ in a complete network with 5 firms.

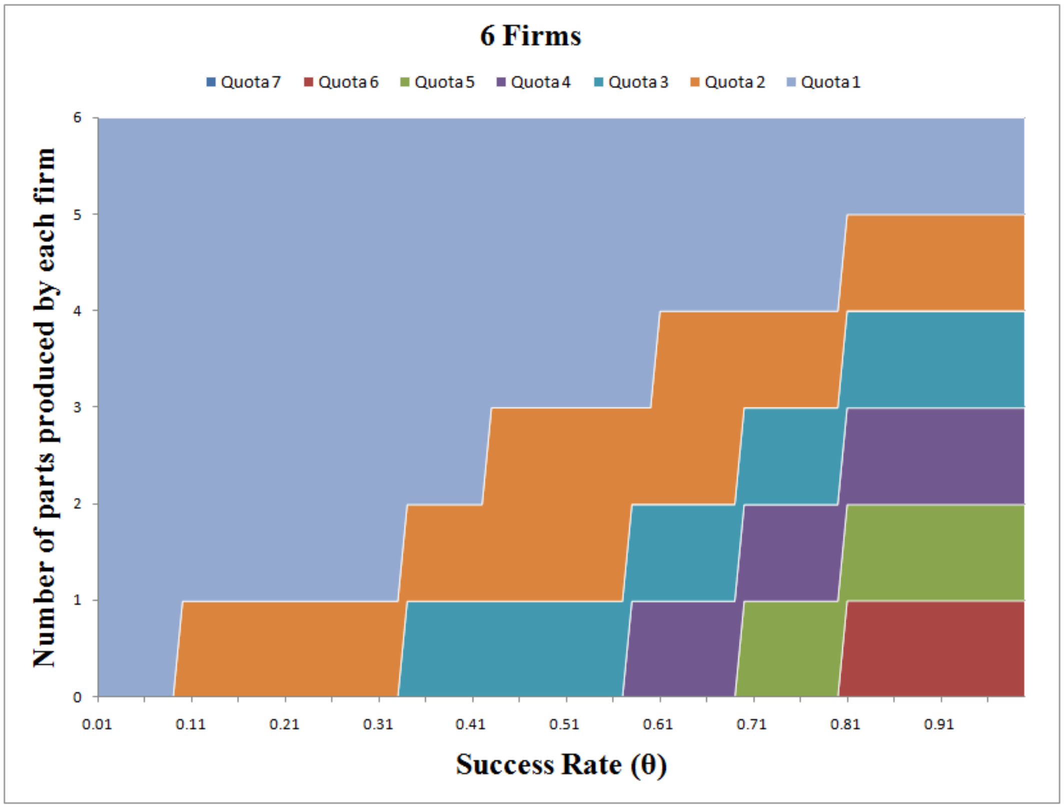

Figure A.5: Graph showing number of parts produced by each firm varies as a function of θ in a complete network with 6 firms.

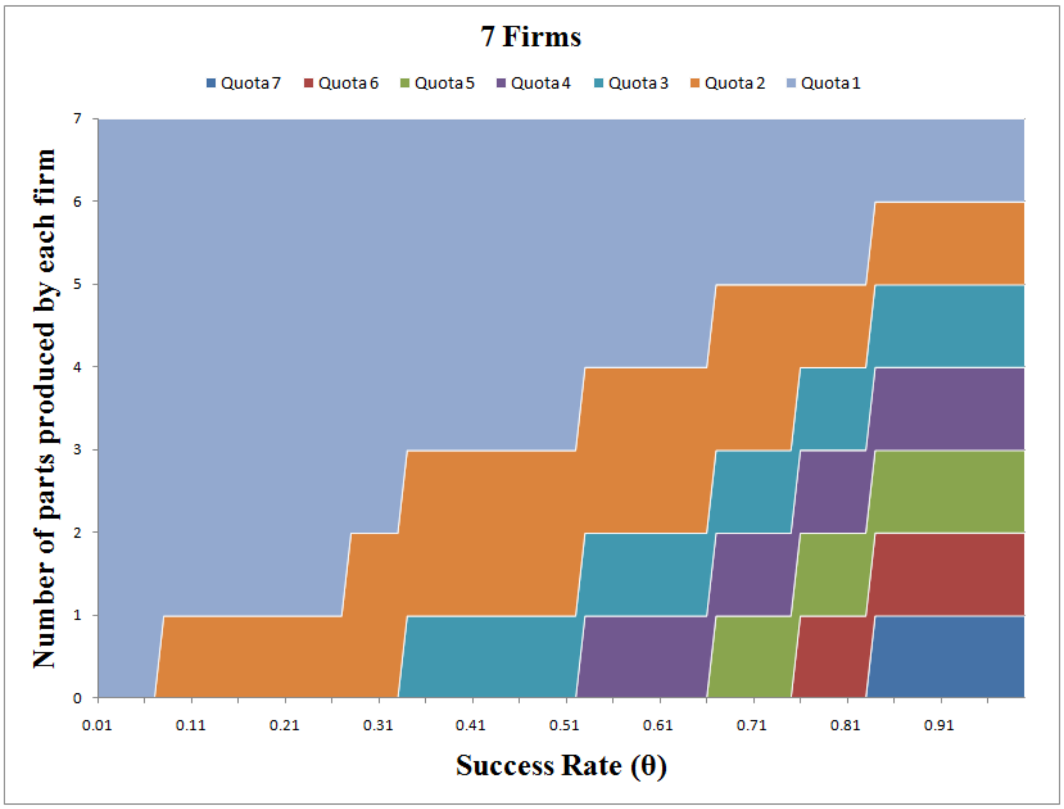

Figure A.6: Graph showing number of parts produced by each firm varies as a function of θ in a complete network with 7 firms.

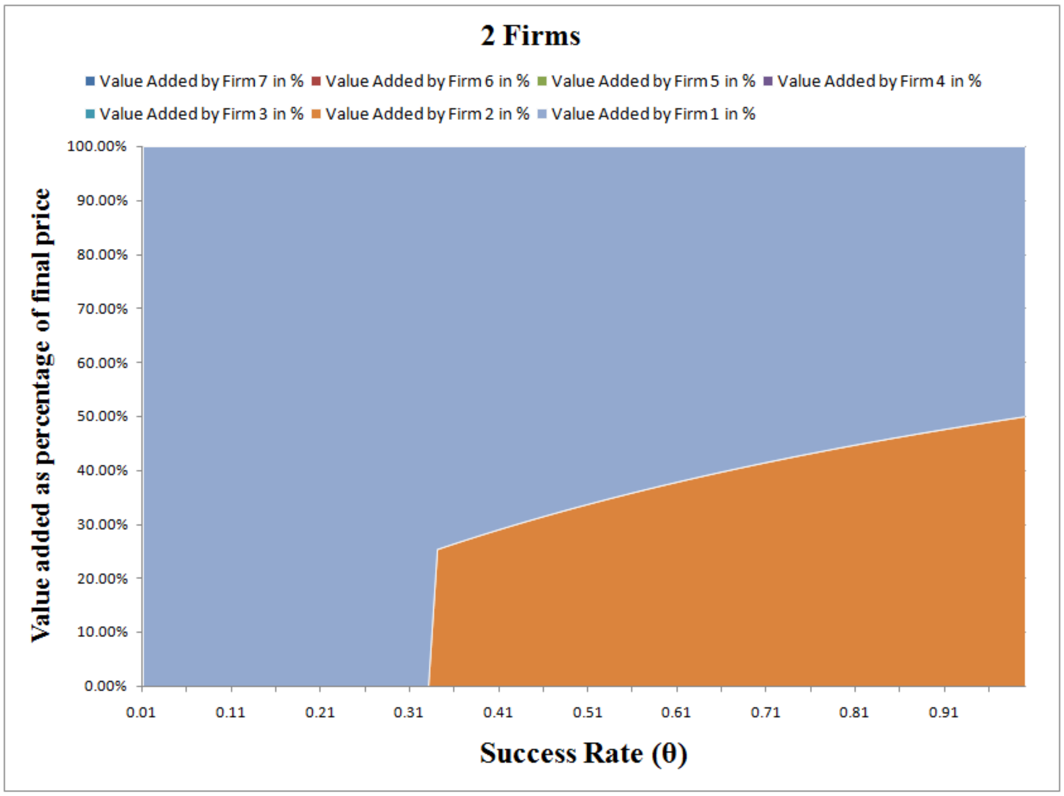

Figure A.7: Graph showing value added by each firm varies as a function of θ in a complete network with 2 firms.

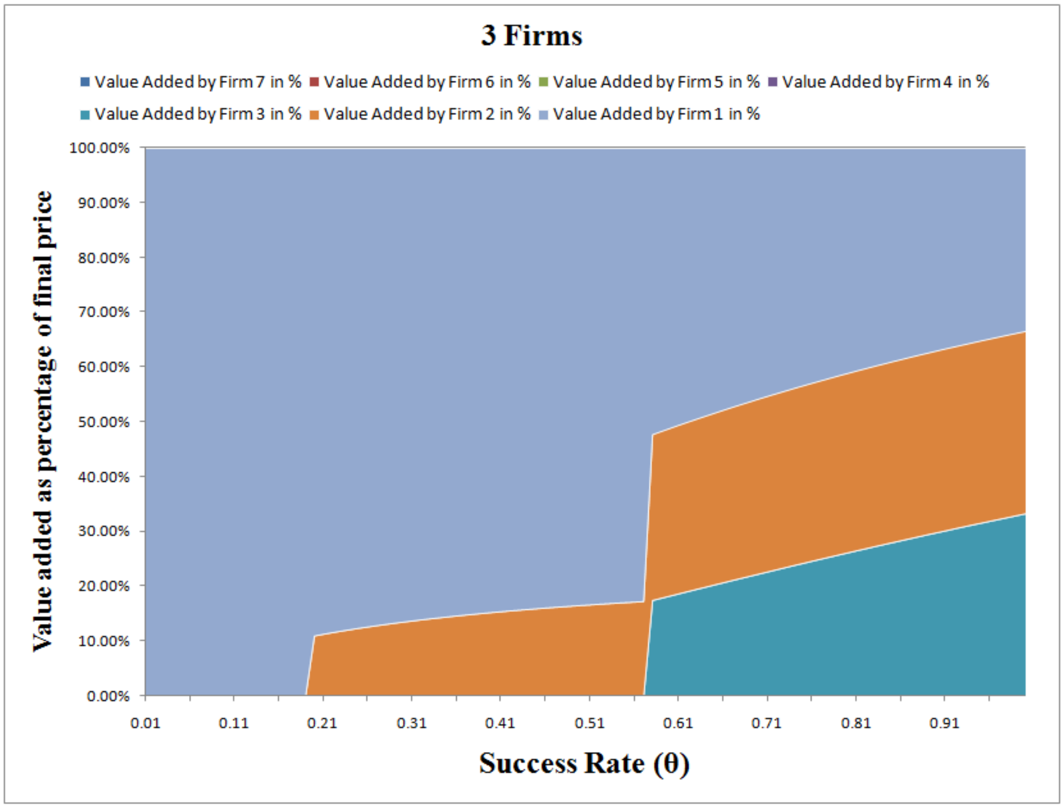

Figure A.8: Graph showing value added by each firm varies as a function of θ in a complete network with 3 firms.

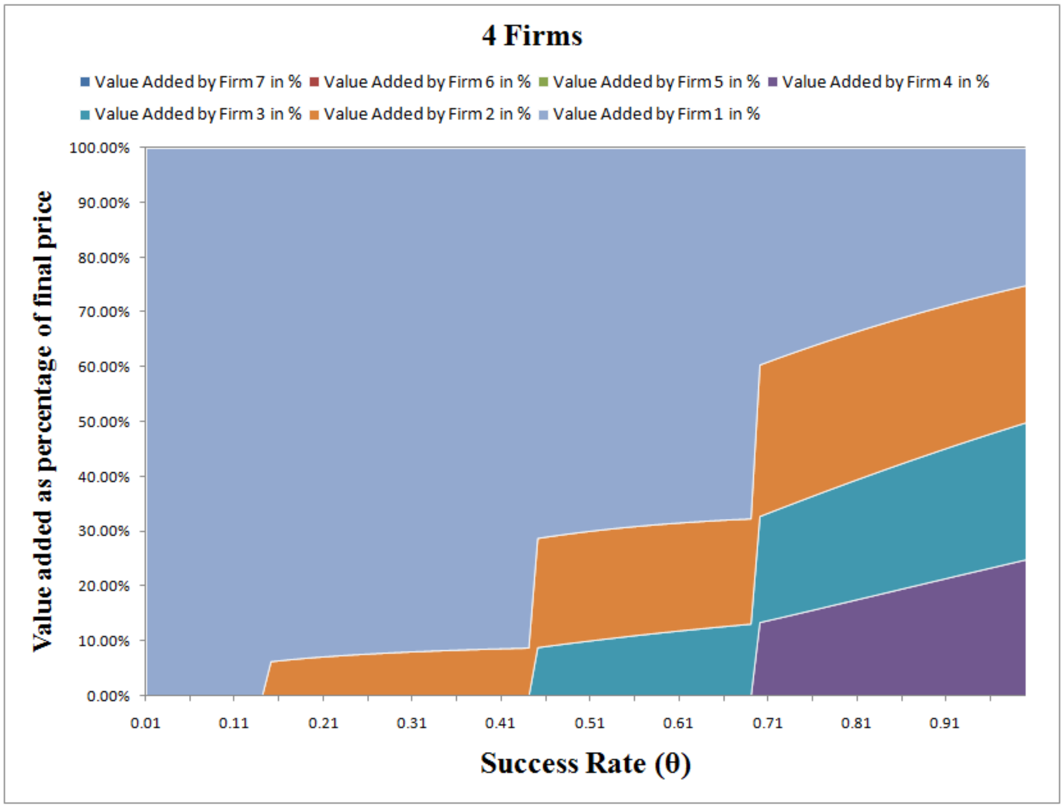

Figure A.9: Graph showing produced by each firm varies as a function of θ in a complete network with 4 firms.

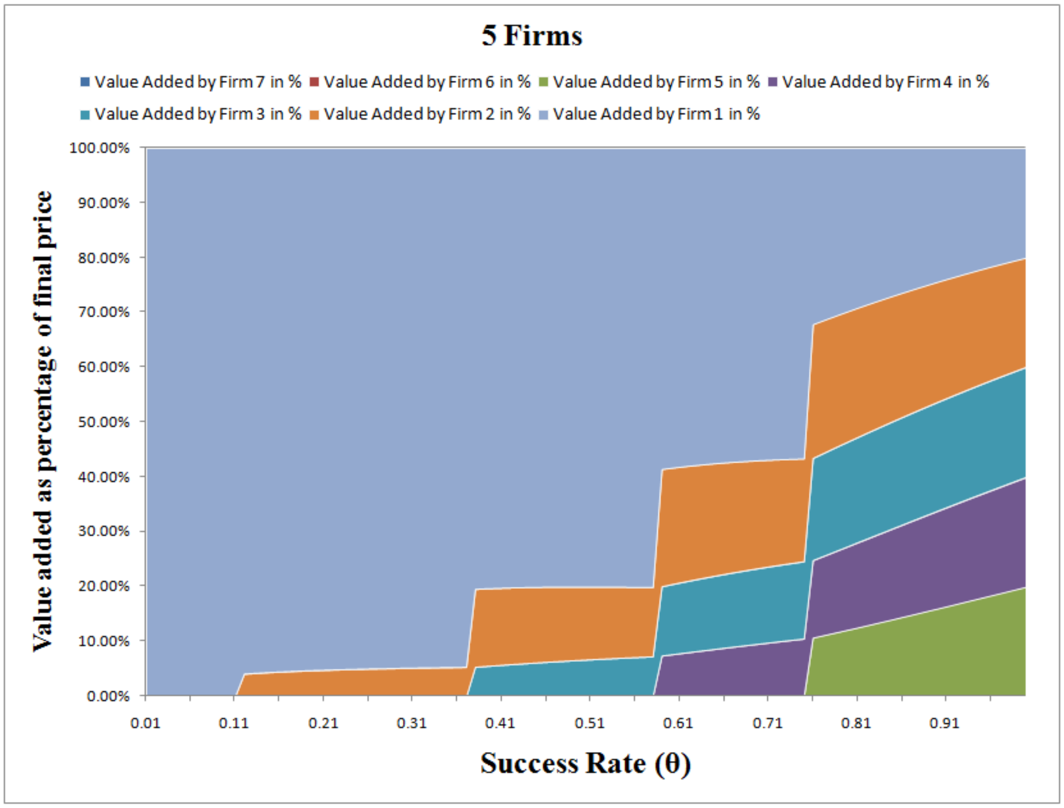

Figure A.10: Graph showing produced by each firm varies as a function of θ in a complete network with 5 firms.

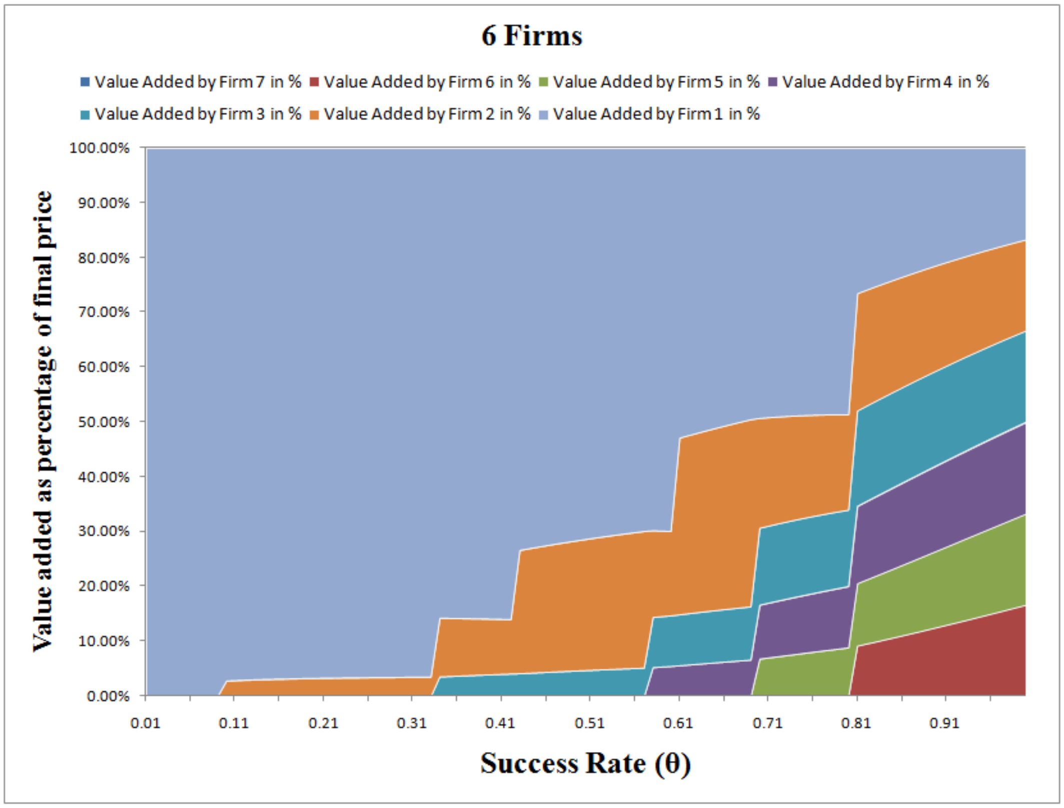

Figure A.11: Graph showing produced by each firm varies as a function of θ in a complete network with 6 firms.

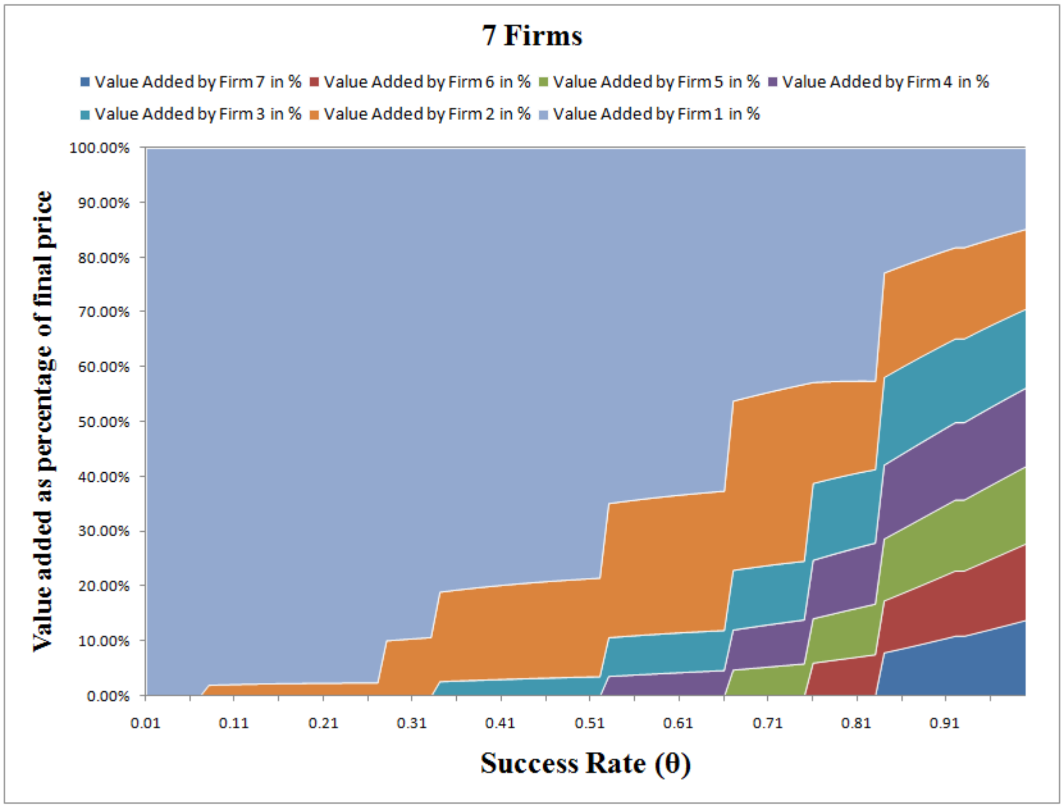

Figure A.12: Graph showing produced by each firm varies as a function of θ in a complete network with 7 firms.