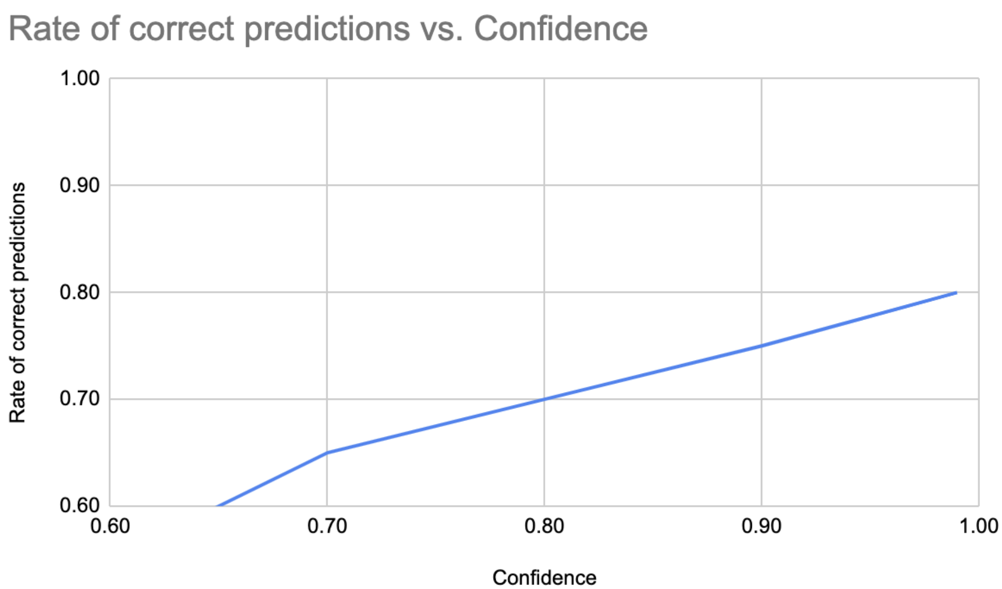
CMSC 320 Homework 2

1. You pay $10 to play the following game of chance. There is a bag containing 12 balls, five are red, three are green and the rest are yellow. You are to draw one ball from the bag. You will win $14 if you draw a red ball and you will win $12 if you draw a yellow ball. How much do you expect to win or lose if you play this game 100 times?
   Red: 5/12. Win $14
   Green: 3/12. Win $0
   Yellow: 4/12. Win $12
   E[X] = 5/12 * 14 + 3/12 * 0 + 4/12 * 12 = $9.83
   9.83 * 100 = $983
   10 * 100 = $1000
   1000-983 = $17
   You are expected to win $983/lose $17 if you play this game 100 times.

2. A detective figures that he has a one in nine chance of recovering stolen property. His out-of-pocket expenses for each investigation are $9,000. If he is paid his fee only if he recovers the stolen property, what should he charge clients in order to break even?
   Detective spends $9000 each investigation and is only paid when he recovers stolen property (1/9 chance)
   E[X] = 1/9 * 9000 = $1000
   Detective needs to make 9 investigations to break even, since 9 * 1000 = 9000
   Therefore, the detective should charge clients 9 * 9000 = $81000

3. A person makes a number of predictions at different confidence levels, shown on the x-axis. The rate of how often those predictions at each confidence level turn out to be true is shown on the y-axis. (So, for example, of the predictions they make with 80% confidence, 70% of them come true.) Is this person overconfident or underconfident?



This person is overconfident.

4. You are collecting data about the US, and decide to graph what the distribution of **bankruptcies declared in a given week** is. Keep in mind, each bankruptcy is going to be its own individual event, governed by some random process, and we are aggregating these based on time. Which distribution would this graph (x-axis number of bankruptcies, y-axis number of occurrences) most likely follow and why?
A poisson distribution. A poisson distribution gives the probability of an event (bankruptcy) happening within a given interval of time (a week), which makes it the most likely distribution for the graph.

5. A doctor is called to see a sick child. The doctor has prior information that 90% of sick children in that neighborhood have the flu, while the other 10% are sick with measles. Let F stand for an event of a child being sick with flu and M stand for an event of a child being sick with measles. Assume for simplicity that $F \cup M = \Omega$, i.e., that there are no other maladies in that neighborhood. A well-known symptom of measles is a rash (the event of having which we denote R). Assume that the probability of having a rash if one has measles is $P(R \mid M) = 0.8$. However, occasionally children with flu also develop rash, and the probability of having a rash if one has flu is $P(R \mid F) = 0.08$. Upon examining the child, the doctor finds a rash. What is the probability that the child has measles?
   P(F) = 90% = 0.9
   P(M) = 10% = 0.1
   P(R|M) = 0.8
   P(R|F) = 0.08
   Probability of having measles given rash found: P(M|R) = (P(R|M) * P(M)) / P(R)
   P(M|R) = (0.8 * 0.1) / (0.08 + 0.8) = 0.08/0.88 = 0.1 = 10%

6. Scientists have created a test to verify if people are infected with a new disease that turns people into adorable kittens. 1% of the population is currently infected with this horrible plague. The test has the following behavior:

	Infected Person	**Non-infected Person**
Reports Positive	70%	10%
Reports Negative	30%	90%

   A person is pulled off the street and given a test, and they test positive. What are the chances they have the disease?
   Chance person is infected given they tested positive = P(I|P)
   P(I|P) = (P(P|I) * P(I)) / P(P) by Bayes' Theorem
   P(I) = Chance a person is infected = 1% = 0.01
   P(~I) = Chance a person is non-infected = 99% = 0.99
   P(P) = Chance any person reports positive = P(P|I) * P(I) + P(P|~I) * P(~I) = 0.7 * 0.01 + 0.3 * 0.99 = 0.106
   P(I|P) = (0.7 * 0.01)/0.106 = 0.007/0.106 = 0.066 = 6.6%

7. You are an insurance adjuster working for an auto insurance company. From historical data, you know that 70% of accidents involve only one vehicle (Single-Vehicle Accidents), and 30% involve two or more vehicles (Multi-Vehicle Accidents). For Single-Vehicle Accidents, 80% are caused by the driver's error (Driver's Fault), and 20% are due to external factors (External Factors). For Multi-Vehicle Accidents, 60% are caused by the driver's error, and 40% are due to external factors. Additionally, the expected cost of processing a Single-Vehicle Accident claim is $5,000, while the expected cost for a Multi-Vehicle Accident claim is $8,000.

   (a) In the current scenario, where you have just received an accident claim, what is the total expected cost of processing the claim, taking into account both types of accidents?

   (b) BONUS In a new scenario where the Single-Vehicle Accident occurs during adverse weather conditions, resulting in an extra cost of $1,000, what is the total expected cost of processing the claim for both types of accidents?

   (c) BONUS In another new scenario where the Multi-Vehicle Accident occurs during adverse weather conditions, leading to an extra cost of $2,000, what is the total expected cost of processing the claim for both types of accidents?

   (d) BONUS In a scenario where both Single-Vehicle and Multi-Vehicle accidents occur during adverse weather conditions, with additional costs of $1,000 for Single-Vehicle and $2,000 for Multi-Vehicle, what is the total expected cost of processing the claim for both types of accidents?

	Single-Vehicle Accident	**Multi-Vehicle Accident**
Driver's Fault	80%	60%
External Factors	20%	40%

(a) E[X] = 0.7 * 5000 + 0.3 * 8000 = $5900
(b) E[X] = 0.7 * (5000 + 1000) + 0.3 * 8000 = $6600
(c) E[X] = 0.7 * 5000 + 0.3 * (8000 + 2000) = $6500
(d) E[X] = 0.7 * (5000 + 1000) + 0.3 * (8000 + 2000) = $7200

8. A modified version of the Monty Hall problem is as follows: You are on a game show and shown four doors. Behind one door is a prize; behind the other three is nothing. You pick a door. Before the door can be opened, the game show host selects two doors with nothing behind them and opens them, revealing nothing. You are given the option to switch doors. What are the chances of getting the prize if you switch? Show your work.
   Original chance of getting prize on chosen door = ¼ = 25%
   Original chance of prize being behind unchosen three doors = ¾ = 75%

   New chance of getting prize on chosen door = ¼ = 25%
   New chance of prize being behind single unchosen door = ¾ = 75%

   The chance of getting the prize if you switch is ¾ = 75%

9. You are in the same situation as problem 7, but instead of the game show host opening the doors, the set of the game show malfunctions, opening two doors at random and revealing nothing behind them. What is the probability of getting the prize if you choose to switch?
   Original chance of getting prize on chosen door = ¼ = 25%
   Original chance of prize being behind unchosen three doors = ¾ = 75%

   Randomly open two doors, could be the chosen door if it has nothing.
   Scenario A: Two doors randomly opened, one is your chosen door.
   In this scenario, since there's only two doors left, 50% chance of getting the prize if switch.
   Scenario B: Two doors randomly opened, your door is not chosen.
   In this scenario, same probability as problem 8. 75% chance of getting the prize if switch.

   50% chance of originally chosen door being randomly opened. 50% chance of not being opened.
   50% * 50% + 50% * 75% = 62.5%