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① Two types of changes:

I. Changes in the availability of the resources: rhs of the constraints

II. Changes in the unit profit or unit cost: coefficients of the objective function

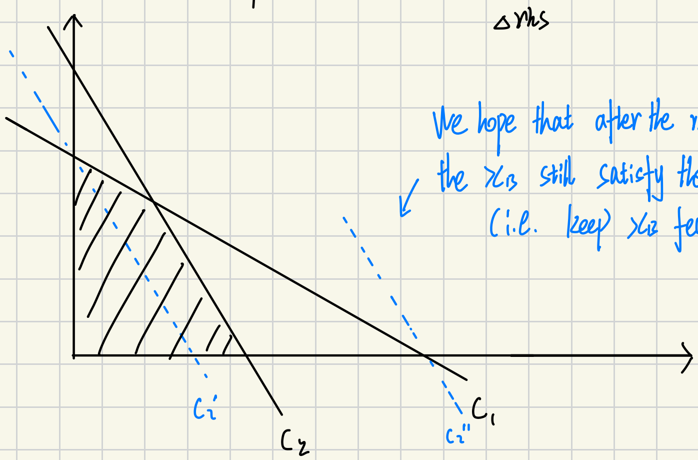
② Changes on the rhs

This will affect the values of the basic variables, and hence also the optimal objective value.

I. Shadow price (dual price)

Increase the rhs by one unit, the optimal objective value changes.

$$\text{Shadow price} = \frac{Z(rhs+\Delta rhs) - Z(rhs)}{\Delta rhs}$$



We hope that after the rhs changes, the x_B still satisfy the non-negative constraint. (i.e. keep x_B feasible)

II. rhs Economic Decisions:

1) the constraints with higher shadow price will receive higher priority.

2) profit = shadow price - rhs increase cost,
if profit > 0, advisable to increase rhs (capacities)

3) Steps of making decision:

i. test if Δrhs makes x_B still feasible

(falling the feasibility range does not mean that the problem has no solution, it only means that the available information is not sufficient to make a complete decision)

ii. Calculate the new x_B value and new objective function value.

III. Algebraic analysis in rhs changes

Prior knowledge : optimal solution is obtained

basis	x_N	x_B	rhs
z	$C_B^T B^{-1}N - C_N^T$	0^T	$C_B^T B^{-1} b$
x_B	$B^{-1}N$	I	$B^{-1}b$

let $b' := b + \Delta b$ be the new rhs in constraints.

Noticed that:

1) changes in b do not effect the coefficients of x_N.
Which means the choses of basis won't change.

2) Considering we need to keep x_B feasible.
we have value of x_B' as:

$$x_B' = B^{-1} b'$$
$$= B^{-1}(b + \Delta b) \geq 0$$

i.e. test if $B^{-1}b + B^{-1}\Delta b \geq 0$

3) Solving workflow

Step1: find B^{-1}

Identity the variables that originally corresponded to identity matrix coefficients I. And in the optimal tableau, find new coefficients matrix for these variable, this matrix is B^{-1}.

Step2: test if $B^{-1}b + B^{-1}\Delta b$ still nonnegative.

Step3: Calculate the 1. new x_B 2. new z value.

a. new x_B

$$x_B' = B^{-1}b + B^{-1}\Delta b$$

b. new z value

$$z' = C_B^T B^{-1} b + \boxed{C_B^T B^{-1}} \Delta b$$

this additional coefficients for Δb is the shadow price.

e.g. $z' = C_B^T B^{-1} b + a\Delta b_1 + b\Delta b_2 \quad \cdots$

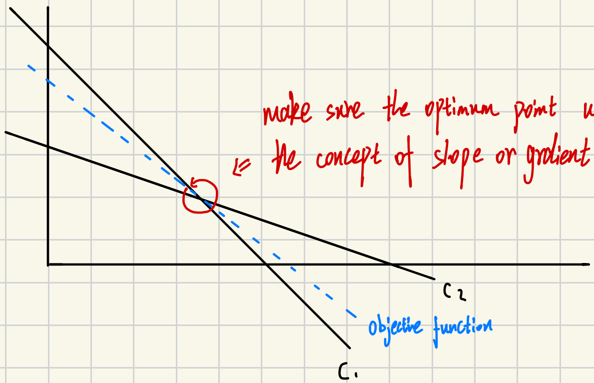
shadow price

and shadow price with zero value means such resoures is surplus.

③ Changes on the coefficients

coefficient is associated with a basic or a nonbasic variable in optimal bfs

(i.e. Considering the new $\hat{C_N}$)



make sure the optimum point unchanged.

⇐ the concept of slope or gradient

I. Algebraic analysis

Set $c' := c + \Delta c$ as the changed coefficients in objective function

And for the Lp problem.

$$z - C^T x = 0$$
$$Ax = b$$

we can expand and divided the vector and matrix

$$x = [x_B^T \mid x_N^T]^T$$

$$c' = [c_B^T + \Delta c_B^T \mid c_N^T + \Delta c_N^T]^T$$

$$A = [\, B \mid N \,]$$

thus we have,

$$z - (c_B^T + \Delta c_B^T)x_B - (c_N^T + \Delta c_N^T)x_N = 0 \qquad \text{--- (1)}$$

$$Bx_B + Nx_N = b \qquad \text{--- (2)}$$

build the mapping relationship between x_B and x_N

$$x_B = B^{-1}b - B^{-1}Nx_N$$

substitute $x_B = B^{-1}b - B^{-1}Nx_N$ into equ(1), we have

$$z - (c_B^T + \Delta c_B^T)(B^{-1}b - B^{-1}Nx_N) + (c_N^T + \Delta c_N^T)x_N = 0$$

$$z + (c_B^T B^{-1}b - c_N^T)x_N + \underbrace{(\Delta c_B^T B^{-1}N - \Delta c_N^T)}_{\text{new additional coefficients for } x_N}^{\text{Changing 1:}} x_N = c_B^T B^{-1} b + \underbrace{\Delta c_B^T B^{-1} b}_{\text{new additional coefficients for rhs}}^{\text{Changing 2:}}$$

And for each $c_j \in c_N$, we will have

$$c_j' = \hat{c}_N^{[j]} + [\Delta c_B^T [B^{-1}N]^{[j]} - \Delta c_N^{[j]}]$$

Graph1:

$$\overset{\Delta c_B^T}{[\Delta c_B^{[1]}\ \Delta c_B^{[2]} \cdots \Delta c_B^{[m]}]} \ \overset{B^{-1}N}{\underset{m\times n}{\Big[\, [B^{-1}N]^{[1]} \ [B^{-1}N]^{[2]} \cdots [B^{-1}N]^{[n]} \,\Big]}} - \Delta c_N^{[j]}$$

Graph2: original optimal tableau

	basis	x_B	x_N	rhs
			$[\Delta c_B\ c_N\ \square]$	
	z	0	$c_B^T B^{-1}N - c_N^T$	$c_B^T B^{-1} b$
$[\Delta c_B]$	x_B	1	$[B^{-1}N]$	$B^{-1}b$

for new rhs,

$z = c_B^T B^{-1} b + \Delta c_B^T [B^{-1}b]$

each column respect to one x

for all $\hat{c}_N'$. if we want basis unchanged.

$\hat{c}_N'$ stay nonnegative (if LP is maximum)

nonpositive (if LP is minimum)

And all $\hat{c}_B'$ will continue to be 0.