

Yiqiao Chen 2024/3/30

1. Donoho's Algorithm

- ① $g(t)$: noise-free signal
- ② $z(t)$: white noise
- ③ $f(t)$: noise-bearing signal

$$f(t) = g(t) + \underbrace{\sigma_n}_{\text{"additive"}} \underbrace{z(t)}_{\text{normal distribution } \mathcal{N}(0,1)}$$

noise variance

④

step 1: Discretize $f(t)$

$$\begin{array}{ccc} f(t) & \xrightarrow{\text{sampling}} & f(i) \\ \text{C-T} & & \text{D-T} \end{array}$$

step 2: Transform $f(i)$ to orthogonal domain

$$\begin{array}{ccc} f(i) & \xrightarrow{\text{DWT}} & \varphi, \psi \\ \text{time-domain} & & \text{orthogonal domain} \end{array}$$

step 3: Apply soft thresholding or hard thresholding to wavelet coefficients

a. threshold: $\lambda = \sqrt{2\sigma_n^2 \log n}$ \leftarrow All decided by $f(i)$
length($f(i)$)

b. soft thresholding function:

$$S_\lambda \stackrel{\text{def}}{=} \text{sign}(x) \times \max(|x| - \lambda, 0) \leftarrow \text{shrinks}$$

c. hard thresholding function:

$$h_\lambda \stackrel{\text{def}}{=} x \times 1\{|x| > \lambda\} \leftarrow \begin{cases} \text{input} > \lambda : \text{keep} \\ \text{input} < \lambda : \text{set } 0 \end{cases}$$

step 4: Reconstruction

$$\varphi, \psi \xrightarrow{\text{inverse DWT}} \text{denoised signal } f'(i)$$

⑤ a. wavelet transform has the compaction property of having only a small number of large coefficient

b. the denoising is done only on the detail coefficients of wavelet transform.