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1. Donoho's Algorithm

- ① $g(t)$: noise-free signal
- ② $z(t)$: white noise
- ③ $f(t)$: noise-bearing signal

$$f(t) = g(t) + \underbrace{\sigma_n}_{\text{"additive"}} \underbrace{z(t)}_{\substack{\text{noise variance} \\ \text{normal distribution } \mathcal{N}(0,1)}}$$

④

step 1: Discretize $f(t)$

$$\begin{array}{ccc} f(t) & \xrightarrow{\text{sampling}} & f(i) \\ C-T & & D-T \end{array}$$

step 2: Transform $f(i)$ to orthogonal domain

$$\begin{array}{ccc} f(i) & \xrightarrow{\text{DWT}} & \varphi, \psi \\ \text{time-domain} & & \text{orthogonal domain} \end{array}$$

step 3: Apply soft thresholding or hard thresholding to wavelet coefficients

a. threshold: $\lambda = \sqrt{2\sigma_n^2 \log n}$ \leftarrow All decided by $f(i)$
 $\text{length}(f(i))$

b. soft thresholding function:

$$S_\lambda \stackrel{\text{def}}{=} \text{sign}(x) \times \max(|x| - \lambda, 0) \leftarrow \text{shrinks}$$

c. hard thresholding function:

$$h_\lambda \stackrel{\text{def}}{=} x \times \mathbb{1}_{\{|x| > \lambda\}} \leftarrow \begin{cases} \text{input} > \lambda : \text{keep} \\ \text{input} < \lambda : \text{set } 0 \end{cases}$$

step 4: Reconstruction

$$\varphi, \psi \xrightarrow{\text{inverse DWT}} \text{denoised signal } f'(i)$$

⑤ a. wavelet transform has the compaction property of having only a small number of large coefficient

b. the denosing is done only on the detail coefficients of wavelet transform.