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1. Donoho's Algorithm

① $g(t)$: noise-free signal

② $z(t)$: white noise

③ $f(t)$: noise-bearing signal

noise variance

$$f(t) = g(t) + \sigma_n z(t)$$

"additive" normal distribution $N(0,1)$

④

step1: Discretize $f(t)$

$$\underset{\text{C-T}}{f(t)} \xrightarrow{\text{sampling}} \underset{\text{D-T}}{f(i)}$$

step2: Transform $f(i)$ to orthogonal domain

$$\underset{\text{time-domain}}{f(i)} \xrightarrow{\text{DWT}} \underset{\text{orthogonal domain}}{\varphi, \psi}$$

step3: Apply soft thresholding or hard thresholding to wavelet coefficients

a. threshold: $\lambda = \sqrt{2\sigma_n^2 \log n}$ ⇐ All desided by $f(i)$

n: Length($f(i)$)

b. soft thresholding function:

$$S_\lambda \overset{\text{def}}{=} \operatorname{sign}(x) \times \max(|x|-\lambda, 0) \Leftarrow \text{shrinks}$$

c. hard thresholding function:

$$h_\lambda \overset{\text{def}}{=} x \times 1\{|x| > \lambda\} \Leftarrow \begin{cases} \text{input} > \lambda : \text{keep} \\ \text{input} < \lambda : \text{set } 0 \end{cases}$$

step4: Reconstruction

$$\varphi, \psi \xrightarrow{\text{inverse DWT}} \text{denoised signal } f'(i)$$

⑤ a. wavelet transform has the compaction property of having only a small number of large coefficient

b. the denoising is done only on the detail coefficients of wavelet transform.