Foundations of Artificial Intelligence: Homework 2
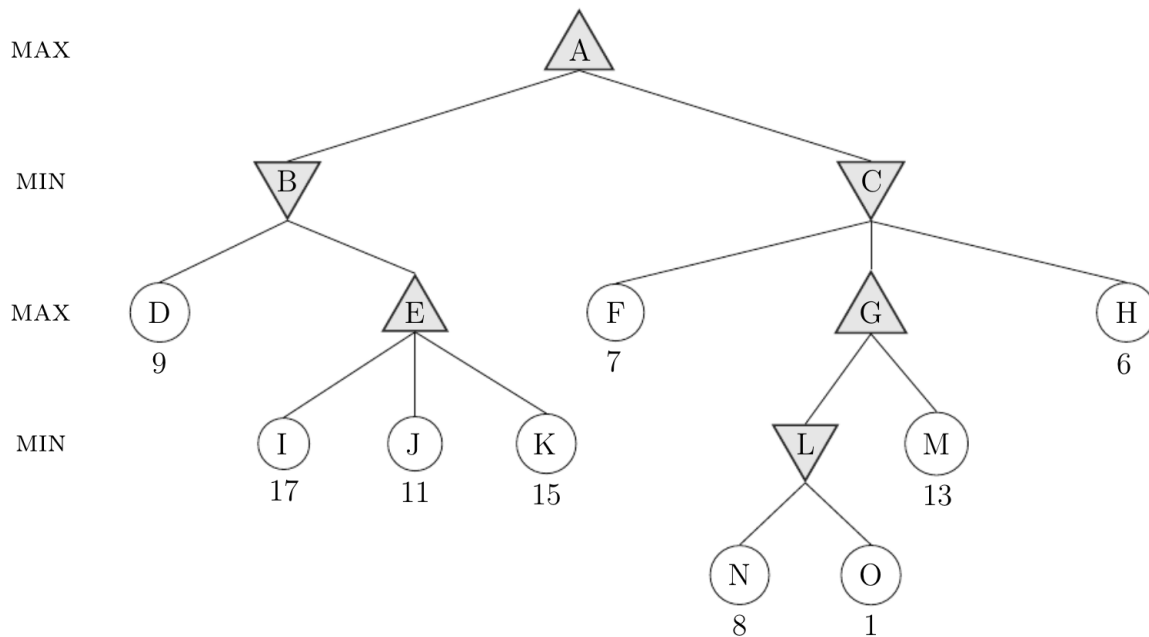
Instructor: Shang-Tse Chen & Yun-Nung Chen

Problem 1   (10 points)

Consider the `MAX-MIN` game tree shown below where the numbers underneath the leaves of the tree are utility values from the first player's point of view (`MAX`).

a) Draw a copy of the tree on paper and perform the **minimax** algorithm algorithm on it by hand. Write the resulting minimax values next to every node

b) Do the same, but with **left-to-right alpha-beta** pruning. Write the final values for α and β next to every node, and indicate which nodes are not examined due to pruning.

c) Do the same, but with **right-to-left alpha-beta** pruning. Write the final values for α and β next to every node, and indicate which nodes are not examined due to pruning.

Problem 2   (12 points)

(a) Rewrite the following propositional formulae into Conjunctive Normal Form.

1. $((((p \rightarrow q) \rightarrow \neg p) \rightarrow \neg q) \rightarrow \neg r) \rightarrow r$

2. $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow \neg r) \rightarrow (p \rightarrow \neg q))$

(b) Construct a formula A such that the formula $(A \wedge q) \rightarrow \neg p) \rightarrow ((p \rightarrow \neg q) \rightarrow A)$ is always true.
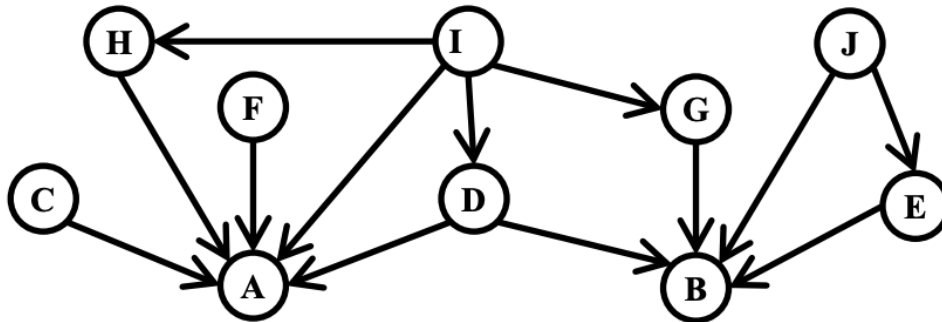
(c) Construct a formula with variables p, q, r that is true if and only if at least two of the variables are true.

(d) Construct a formula A such that $(r \to A) \equiv (r \to (p \wedge q))$ and $(A \to r) \equiv (\neg(p \vee q) \to r)$.

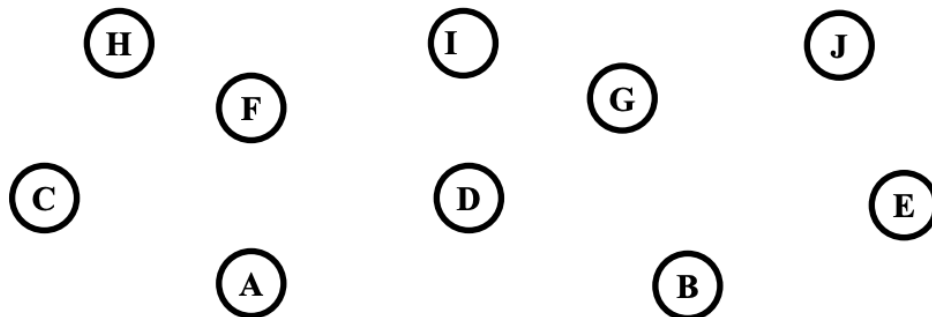**Problem 3**   (10 points)

(a) Write down the factored joint probability distribution according to the following Bayesian Network.
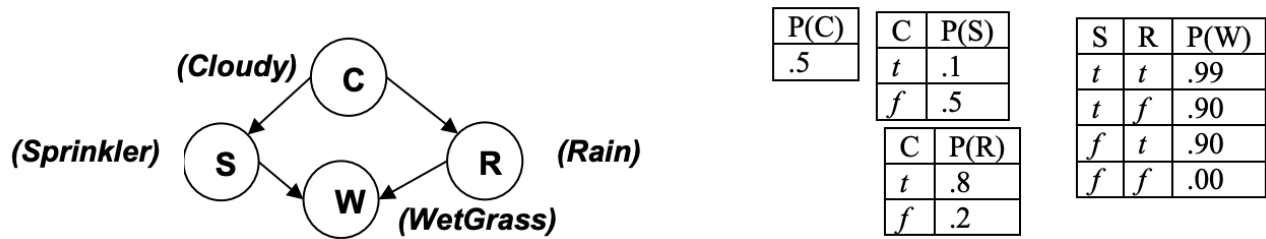

(b) Draw the Bayesian Network that corresponds to this conditional probability:

$$P(A|C, D, F, H)P(B|D, E, J)P(C|H)P(D|G, J)P(E)P(F|G, I)P(G|I, J)P(H)P(I)P(J)$$

(c) Below is the Bayesian network for the WetGrass problem.

P(C)
.5

C	P(S)
t	.1
f	.5

C	P(R)
t	.8
f	.2

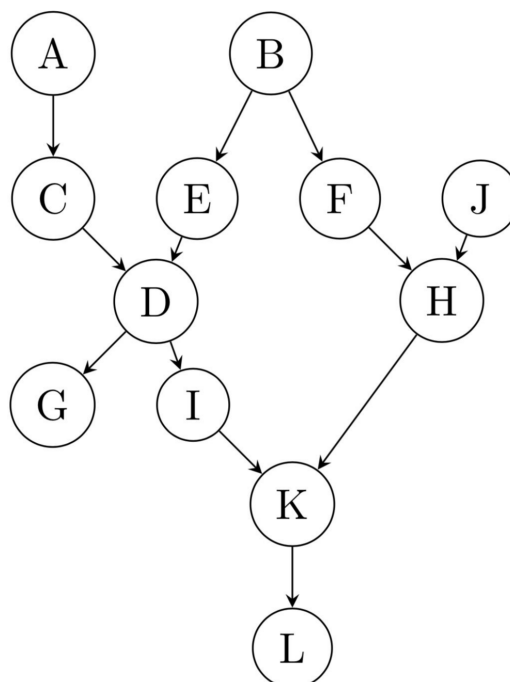| S	R	P(W)
t	*t*	.99
t	*f*	.90
f	*t*	.90
f	*f*	.00

Write down an expression that will evaluate to

$$P(C = f \land R = f \land S = t \land W = t).$$

You do not need to carry out the multiplication to produce a single number (probability).

Problem 4 (8 points)

According to the following Bayesian Network,

(a) List all the variables that are d-separated from F given E.

(b) List all the variables that are d-separated from F given E and K.