Adaptive Vision-Based Control of Redundant Robots with Null-Space Interaction for Human-Robot Collaboration

Xiangjie Yan, Chen Chen, and Xiang Li

In the presence of the uncalibrated camera, the unknown depth and image Jacobian matrix are represented as $\hat{z}(\boldsymbol{q})$ and $\hat{\boldsymbol{J}}_s(\boldsymbol{r})$, respectively. It is obtained that

$$\hat{z}(\boldsymbol{q})\dot{\boldsymbol{x}} = \boldsymbol{Y}_z(\dot{\boldsymbol{x}}, \boldsymbol{q})\hat{\boldsymbol{\theta}}_z, \tag{1}$$

$$\hat{\boldsymbol{J}}_s(\boldsymbol{r})\dot{\boldsymbol{r}} = \boldsymbol{Y}_k(\dot{\boldsymbol{r}}, \boldsymbol{r})\hat{\boldsymbol{\theta}}_k, \tag{2}$$

where $\hat{\boldsymbol{\theta}}_z$ and $\hat{\boldsymbol{\theta}}_k$ denote the estimates of $\boldsymbol{\theta}_z$ and $\boldsymbol{\theta}_k$, respectively.

The position of the feature point in camera frame and end-effector frame is ${}^c r$ and ${}^e r$, the overline $\bar{\cdot}$ denotes homogeneous coordinate.

$${}^c\overline{r} = {}^c_e T {}^b_e T {}^e\overline{r} \tag{3}$$

the depth $z(\boldsymbol{q})$, which is the 3rd element of ${}^c\overline{r}$, can be formulated as

$$z(\boldsymbol{q}) = \begin{bmatrix} t_1 & t_2 & t_3 & t_4 \end{bmatrix} {}^b_e T(\boldsymbol{q}) {}^e\overline{r} \tag{4}$$

where $\begin{bmatrix} t_1 & t_2 & t_3 & t_4 \end{bmatrix}$ is the third row of ${}^c_e T(\boldsymbol{q})$. We denote:

$${}^b_e T(\boldsymbol{q}) = \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{5}$$

Because the feature point is placed exactly on the end-effector (i.e.${}^e\overline{r} = [0, 0, 0, 1]^T$), only $\begin{bmatrix} t_1 & t_2 & t_3 & t_4 \end{bmatrix}$ is unknown.

$$\hat{z}(\boldsymbol{q}) = \begin{bmatrix} f_{14} & f_{24} & f_{34} & 1 \end{bmatrix} \begin{bmatrix} t_1 & t_2 & t_3 & t_4 \end{bmatrix}^T \tag{6}$$

$$\hat{\boldsymbol{\theta}}_z = \begin{bmatrix} t_1 & t_2 & t_3 & t_4 \end{bmatrix}^T \tag{7}$$

$$\boldsymbol{Y}_z(\dot{\boldsymbol{x}}, \boldsymbol{q}) = \begin{bmatrix} \dot{u}f_{14} & \dot{u}f_{24} & \dot{u}f_{34} & \dot{u} \\ \dot{v}f_{14} & \dot{v}f_{24} & \dot{v}f_{34} & \dot{v} \end{bmatrix} \tag{8}$$

where $\boldsymbol{x} = [u, v]^T$ and $\dot{\boldsymbol{x}} = [\dot{u}, \dot{v}]^T$.

We use (6) to recover $\hat{z}$ from $\hat{\boldsymbol{\theta}}_z$.

==

Utilizing ${}^e\overline{r} = [0, 0, 0, 1]^T$), we have

$$\hat{\boldsymbol{J}}_s(\boldsymbol{r})\dot{\boldsymbol{r}} = \begin{bmatrix} f & 0 & -(u-u_0) \\ 0 & f & -(v-v_0) \end{bmatrix} {}^c\dot{r} \tag{9}$$

$$= \begin{bmatrix} f & 0 & -(u-u_0) \\ 0 & f & -(v-v_0) \end{bmatrix} {}^c_b R {}^b\dot{r} \tag{10}$$

$$= \begin{bmatrix} f & 0 & -(u-u_0) \\ 0 & f & -(v-v_0) \end{bmatrix} {}^c_b R \frac{d}{dt}\{{}^b_e T {}^e\overline{r}\}_{1:3} \tag{11}$$

$$= \begin{bmatrix} f & 0 & -(u-u_0) \\ 0 & f & -(v-v_0) \end{bmatrix} {}^c_b R {}^b\dot{r}, \tag{12}$$

where the 1st and 2nd matrix are unknown. We denote

$${}^c_b R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}, \tag{13}$$

and

$${}^b\dot{r} = \begin{bmatrix} v_x & v_y & v_z \end{bmatrix}^T, \tag{14}$$

we obtain

$$\hat{\boldsymbol{\theta}}_k = [\ fr_{11} \quad fr_{12} \quad fr_{13} \quad fr_{21} \quad fr_{22} \quad fr_{23} \quad r_{31} \quad r_{32} \quad r_{33} \quad u_0r_{31} \quad u_0r_{32} \quad u_0r_{33} \quad v_0r_{31} \quad v_0r_{32} \quad v_0r_{33}]^T \tag{15}$$

$$\boldsymbol{Y}_k(\dot{\boldsymbol{r}}, \boldsymbol{r}) = \left[\begin{matrix} v_x & v_y & v_z & 0 & 0 & 0 & -uv_x & -uv_y & -uv_z & v_x & v_y & v_z & 0 & 0 & 0 \\ 0 & 0 & 0 & v_x & v_y & v_z & -vv_x & -vv_y & -vv_z & 0 & 0 & 0 & v_x & v_y & v_z \end{matrix} \right] \tag{16}$$

The way we recover $\hat{J}_s$ from $\hat{\boldsymbol{\theta}}_k$ is as follows:

$$\hat{J}_s = \left[\begin{matrix} fr_{11} - (u-u_0)r_{31} & fr_{12} - (u-u_0)r_{32} & fr_{13} - (u-u_0)r_{33} \\ fr_{21} - (v-v_0)r_{31} & fr_{22} - (v-v_0)r_{32} & fr_{23} - (v-v_0)r_{33} \end{matrix} \right]$$

$$= \left[\begin{matrix} \hat{\boldsymbol{\theta}}_k(1) - u\hat{\boldsymbol{\theta}}_k(7) + \hat{\boldsymbol{\theta}}_k(10) & \hat{\boldsymbol{\theta}}_k(2) - u\hat{\boldsymbol{\theta}}_k(8) + \hat{\boldsymbol{\theta}}_k(11) & \hat{\boldsymbol{\theta}}_k(3) - u\hat{\boldsymbol{\theta}}_k(9) + \hat{\boldsymbol{\theta}}_k(12) \\ \hat{\boldsymbol{\theta}}_k(4) - v\hat{\boldsymbol{\theta}}_k(7) + \hat{\boldsymbol{\theta}}_k(13) & \hat{\boldsymbol{\theta}}_k(5) - v\hat{\boldsymbol{\theta}}_k(8) + \hat{\boldsymbol{\theta}}_k(14) & \hat{\boldsymbol{\theta}}_k(6) - v\hat{\boldsymbol{\theta}}_k(9) + \hat{\boldsymbol{\theta}}_k(15) \end{matrix} \right] \tag{17}$$