EE140 Introduction to Communication Systems

Lecture 7

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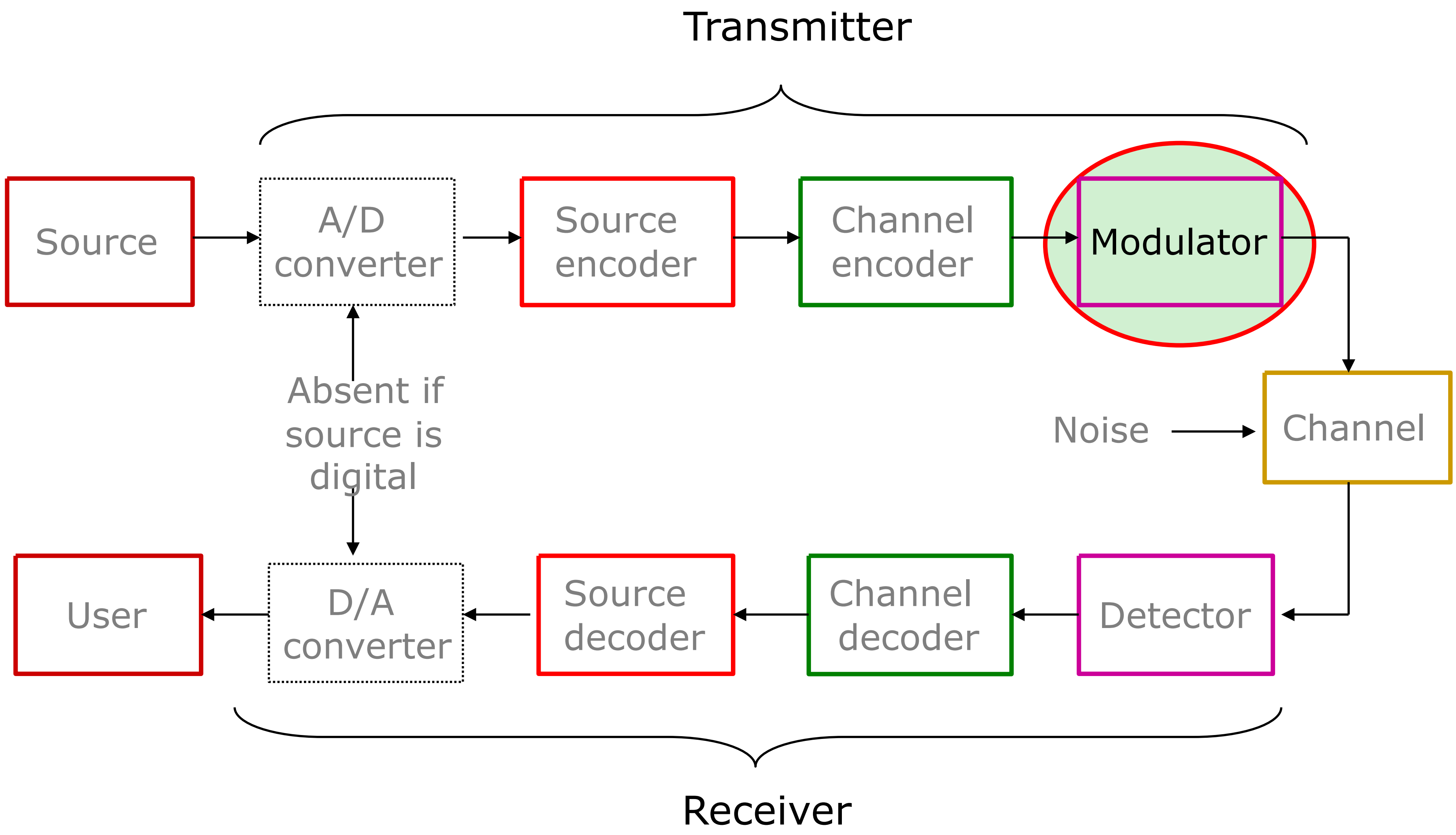
ShanghaiTech University, Fall 2025

Architecture of a (Digital) Communication System



Contents

- Analog Modulation
  - Amplitude modulation
  - Pulse modulation
  - Angle modulation (phase/frequency)

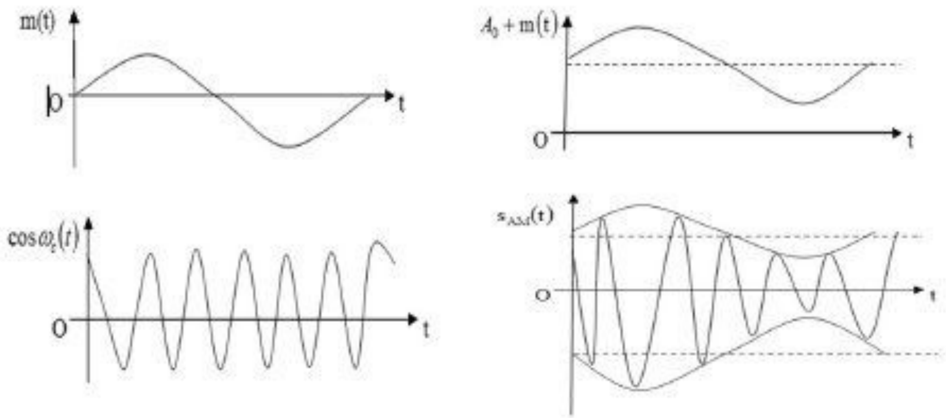
Analog Modulation

- Characteristics that can be modified in the carrier

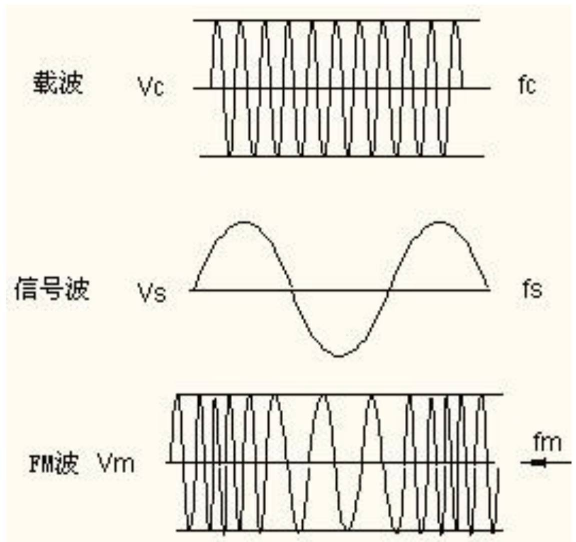
$$C(t) = A(t)\cos(2\pi f(t)t + \theta(t))$$

  - Amplitude $\Rightarrow$ Amplitude modulation
  - Frequency
  - Phase

  Frequency, Phase $\Rightarrow$ Angle modulation



AM



FM

Angle Modulation

- Either phase or frequency of the carrier is varied according to the message signal
- General form

$c(t) = A_c \cos 2\pi f_c t$ FM/PM

$$x_c(t) = A_c \cos[\,2\pi f_c t + \phi(t)]$$

  - Instantaneous phase

$$\theta_i(t) = 2\pi f_c t + \phi(t)$$

phase deviation (PD)

  - Instantaneous frequency

$$\omega(t) = \frac{d\theta_i(t)}{dt} = \omega_c + \frac{d\phi(t)}{dt}$$

frequency deviation (FD)

$$f(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = f_c + \frac{1}{2\pi}\frac{d\phi(t)}{dt}$$

Hz

PM and FM

FD: $\frac{1}{2\pi}\frac{d\phi(t)}{dt} = \frac{k_p}{2\pi}\frac{dm(t)}{dt}$

- Phase modulation

  $\phi(t) = k_p m(t)$, where k_p is phase deviation constant（调相灵敏度)

  – Overall output $x_c(t) = A_c \cos[\,2\pi f_c t + k_p m(t)]$

- Frequency modulation

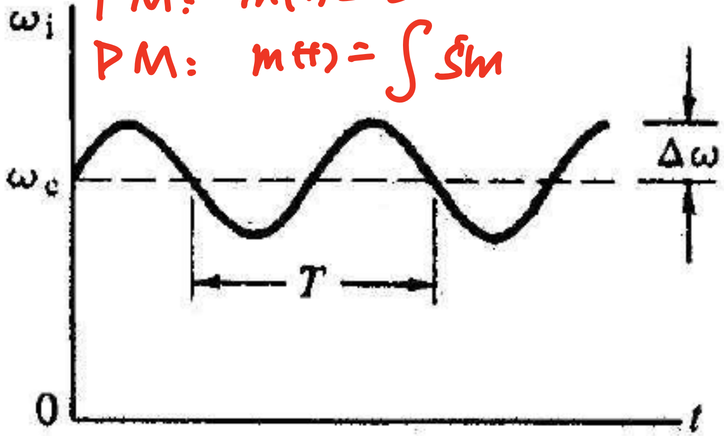
  rad/s $\frac{d\phi(t)}{dt} = k_f m(t) = 2\pi f_d m(t)$, where f_d is frequency deviation constant (调频灵敏度)

  PD: $\phi(t) = 2\pi f_d \int^t m(\tau)d\tau$

  – Overall output $x_c(t) = A_c \cos[\,2\pi f_c t + 2\pi f_d \int_0^t m(\tau)d\tau + \phi_0]$

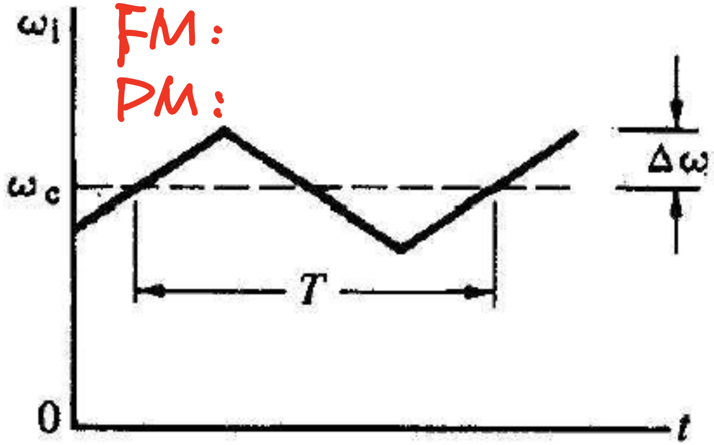
FM: $m(t) = \sin$

PM: $m(t) = \int \sin$



FM:

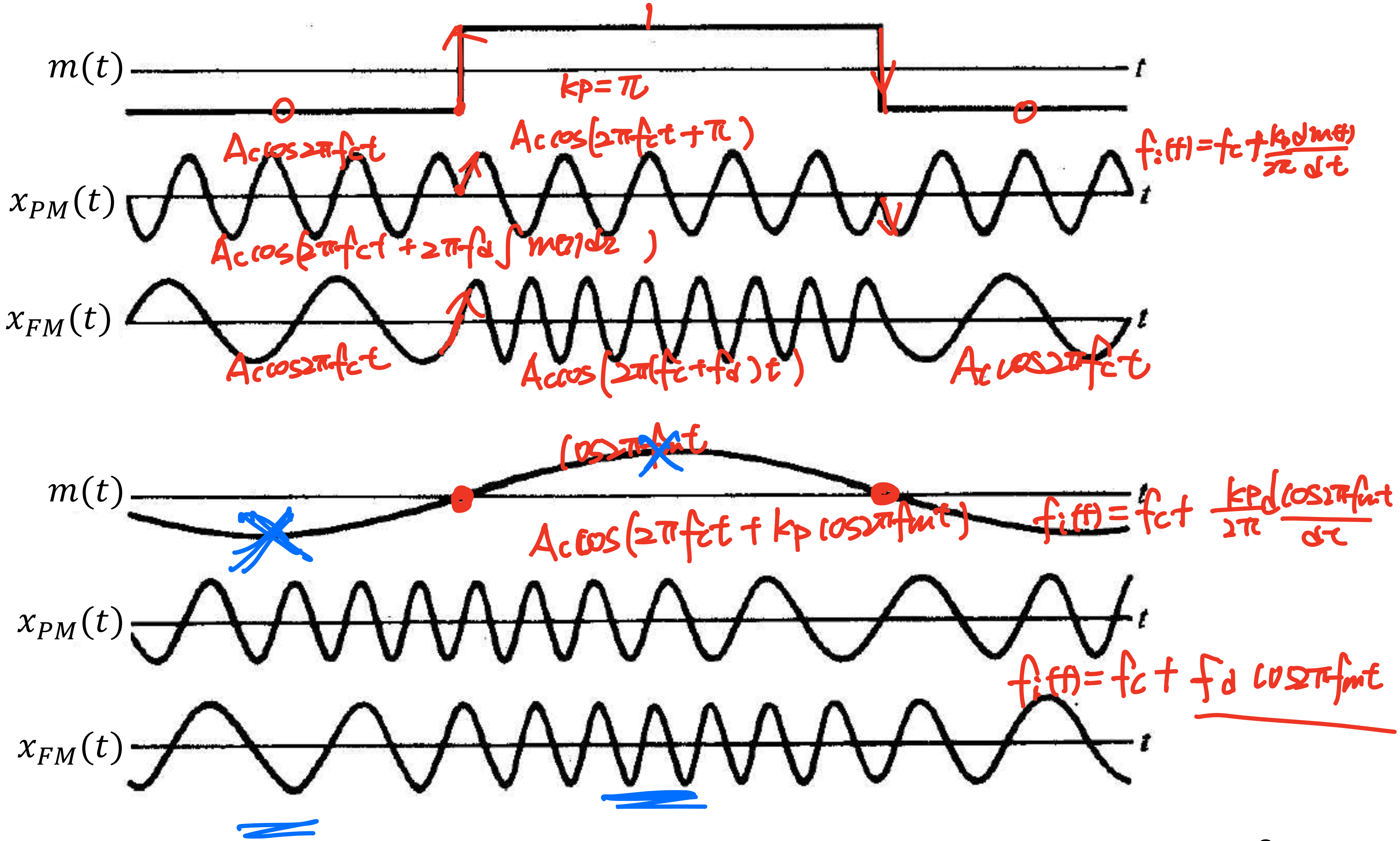
PM:



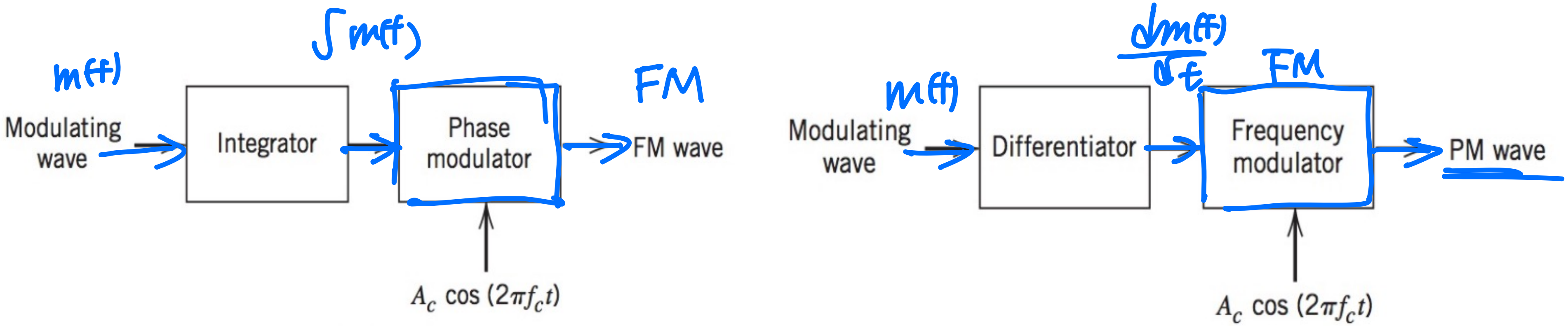
$$x_c(t) = A_c \cos\left(2\pi f_c t + \phi(t)\right)$$

		PD	FD (Hz)
PM:	$A_c \cos\left(2\pi f_c t + k_p m(t)\right)$	$\phi(t) = \underline{k_p}\, m(t)$	$\frac{1}{2\pi}\frac{d\phi(t)}{dt} = \frac{k_p}{2\pi}\frac{dm(t)}{dt}$
FM:	$A_c \cos\left(2\pi f_c t + 2\pi f_d \int m(\tau)d\tau\right)$	$\phi(t) = 2\pi f_d \int m(\tau)d\tau$	$\frac{1}{2\pi}\frac{d\phi(t)}{dt} = \underline{f_d}\, m(t)$

PM and FM: Graphic Interpretation

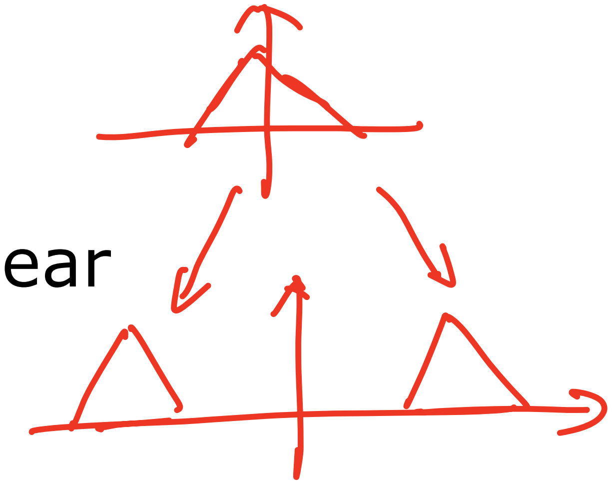


FM and PM



- Transform between FM and PM
  - FM: PM with the modulation wave $\int_0^t m(\tau)d\tau$.
  - PM: FM with the modulation wave $\frac{dm(t)}{dt}$.
  - Deduce the property of PM from FM.
  - We concentrate on FM signal.

FM and PM



- Amplitude modulation (AM) is linear
  - $x_c(t) = (A_c + m(t)) \cos 2\pi f_c t$

$$\Downarrow$$

$$\frac{dx_c(t)}{dm(t)} \text{ is independent of } m(t)$$

- Angle modulation (PM and FM) is nonlinear

$$x_c(t) = A_c \cos[2\pi f_c t + \phi(t)] = \mathrm{Re}\{A_c e^{j2\pi f_c t} e^{j\phi(t)}\}$$

$$= \mathrm{Re}\left\{A_c e^{j2\pi f_c t}\left[1 + j\phi(t) - \frac{1}{2!}\phi^2(t) - j\frac{1}{3!}\phi^3(t) + \cdots\right]\right\}$$

$$= A_c\left[\cos(2\pi f_c t) - \phi(t)\sin(2\pi f_c t) - \frac{\phi^2(t)}{2!}\cos(2\pi f_c t) + \frac{\phi^3(t)}{3!}\sin(2\pi f_c t) + \cdots\right]$$

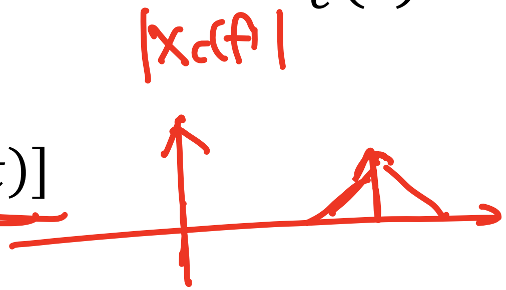
$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$\frac{d x_c(t)}{d\phi(t)} =$$

"Linear" Angle Modulation

- Nonlinear angle modulation: the sidebands arising in angle modulation do not obey the principle of superposition.

- However, if $|\phi(t)| << 1$, the high-order terms in $x_c(t)$ can be ignored



$|\phi(t)| >> 1$

↓

Nonlinear Angle Mod.

↓

Wide band Angle Mod.

(WBFM, WBPM)

$$x_c(t) \approx A_c\,[\cos(2\pi f_c t) - \phi(t)\sin(2\pi f_c t)]$$

⇩

Approximately linear!

⇩

Narrowband Angle Modulation

$B = 2W$

(NBFM, NBPM)

Narrowband FM (NBFM)

FD: $\frac{1}{2\pi}\frac{d\phi(t)}{dt} = f_d\, m(t)$

$= f_d A_m \cos 2\pi f_m t$

- FM– sinusoidal modulating signal

$$x_c(t) = A_c \cos[\,2\pi f_c t + \overset{2\pi f_d}{k_f} \int_0^t m(\tau)d\tau + \phi_0]$$

$\Delta f = \max\left|\frac{1}{2\pi}\frac{d\phi(t)}{dt}\right|$

$= f_d A_m$

- Given $m(t) = A_m \cos 2\,\pi f_m t$ and $\phi_0 = 0$

$$\phi(t) = \overset{2\pi f_d}{k_f} \int_0^t m(\tau)d\tau = \frac{A_m k_f}{2\pi f_m} \sin 2\,\pi f_m t$$

$$= \frac{A_m f_d}{f_m} \sin 2\,\pi f_m t = \frac{\Delta f}{f_m} \sin 2\,\pi f_m t = \beta \sin 2\,\pi f_m t$$

$\beta = \max|\phi(t)|$

$= \frac{A_m f_d}{f_m} = \frac{\Delta f}{f_m}$

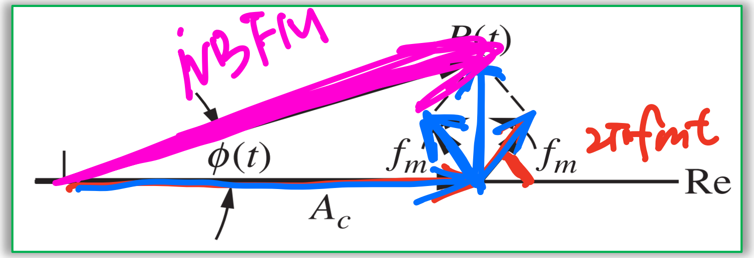
  - Where $\Delta f = A_m f_d$ is the peak frequency deviation, and $\beta = \frac{A_m f_d}{f_m}$ is the modulation index.

- FM signal $x_c(t) = A_c \cos[\,2\pi f_c t + \beta \sin 2\,\pi f_m t]$
  - Narrowband FM (NBFM): $0 < \beta \ll 1$ (small β)
  - Wideband FM (WBFM): $\beta \gg 1$ (large β)

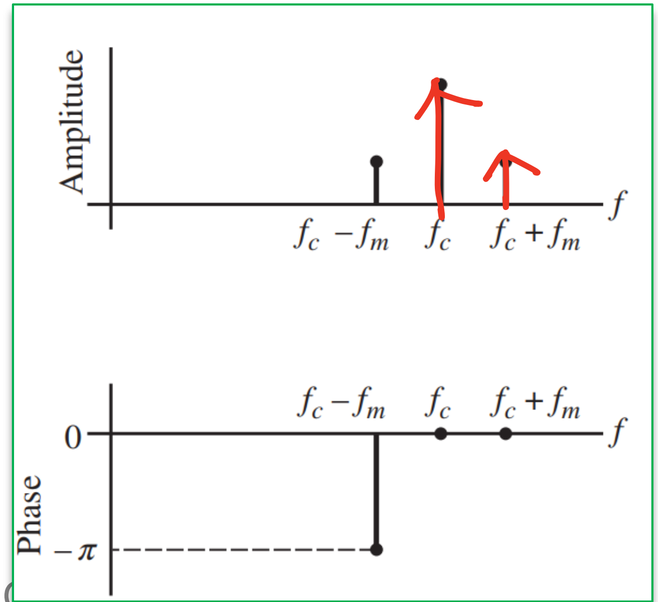
NBFM (Cont'd)

- If $0 < \beta << 1$ (i.e. $|\phi(t)| << 1$), the narrowband FM signal is given by

$$
\begin{aligned}
x_c(t) &= A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t] \\
&\approx A_c \cos 2\pi f_c t - A_c\, \beta \sin 2\pi f_m t\, \sin 2\pi f_c t \\
&= A_c \cos 2\pi f_c t + \frac{1}{2} A_c \beta \{\cos[2\pi(f_c + f_m)t] - \cos[2\pi(f_c - f_m)t]\} \\
&= A_c \mathrm{Re}\left\{ e^{j2\pi f_c t} \left(1 + \frac{\beta}{2} e^{j2\pi f_m t} - \frac{\beta}{2} e^{-j2\pi f_m t}\right)\right\}
\end{aligned}
$$



Phasor diagram

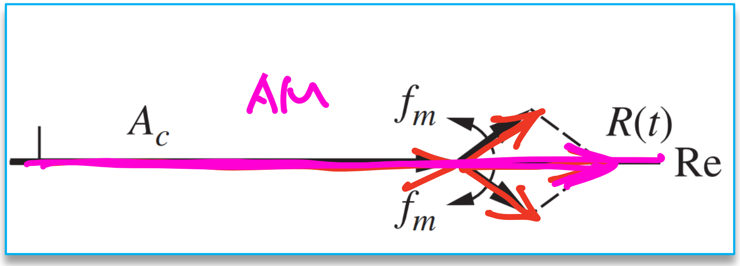


Spectrum

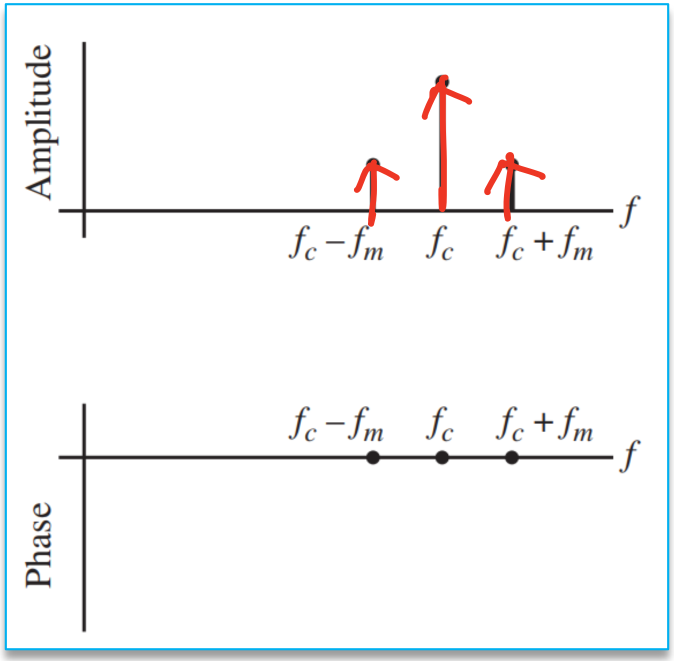
NBFM (Cont'd)

- Compared with DSB-LC (AM)

$$x_c(t) = A_c(1 + a\cos 2\pi f_m t)\cos 2\pi f_c t$$
$$= A_c \cos 2\pi f_c t + A_c a\cos 2\pi f_m t \cos 2\pi f_c t$$
$$= A_c \cos 2\pi f_c t + \frac{1}{2} A_c a\{\cos[2\pi(f_c + f_m)t] + \cos[2\pi(f_c - f_m)t]\}$$
$$= \mathrm{Re}\left\{A_c e^{j2\pi f_c t}\left(1 + \frac{a}{2}e^{j2\pi f_m t} + \frac{a}{2}e^{-j2\pi f_m t}\right)\right\}$$



Phasor diagram



Spectrum

Comparison between NBFM & AM

1. Same transmission bandwidth (B=2fm)
2. NBFM: diff phase with carrier, approximately same amplitude
3. AM: same phase, different amplitude

Narrowband PM (NBPM)

- PM– sinusoidal modulating signal

$$x_c(t) = A_c \cos[\,2\pi f_c t + k_p m(t)]$$

FD : $\frac{1}{2\pi}\frac{d\phi(t)}{dt}$

$= -f_m k_p A_m . \sin(2\pi f_m t)$

$\Delta f = k_p f_m A_m$

- Given $m(t) = A_m \cos 2\pi f_m t$ and $\phi_0 = 0$

$$\phi(t) = k_p A_m \cos 2\pi f_m t = \beta \cos 2\pi f_m t$$

$\beta = k_p A_m$

$\beta = \frac{\Delta f}{f_m}$

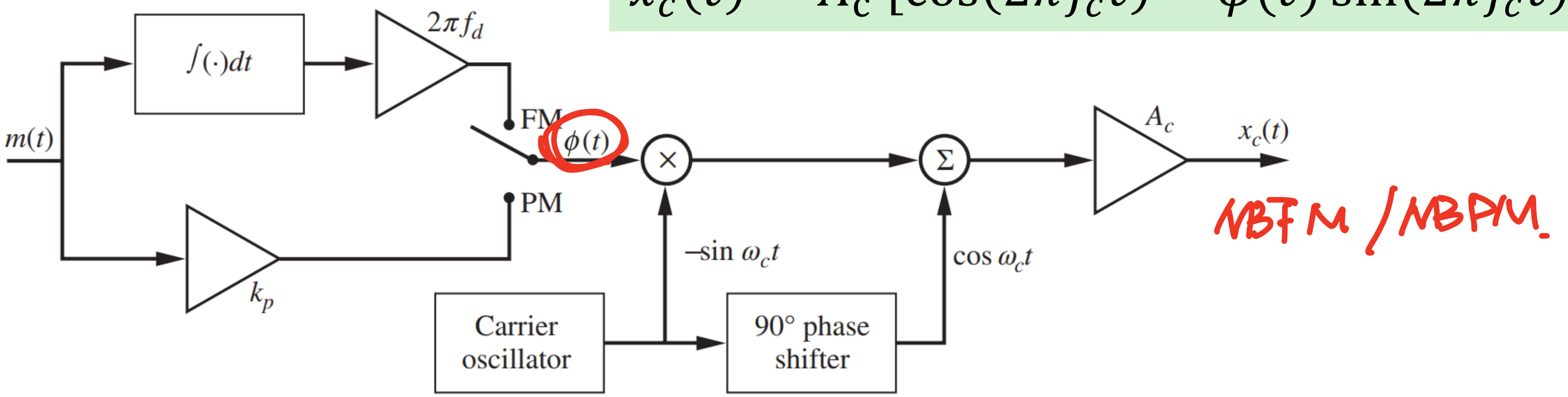
  - $\beta = k_p A_m$ is the modulation index.

- PM signal $x_c(t) = A_c \cos[\,2\pi f_c t + \beta \cos 2\pi f_m t]$
  - Narrowband PM (NBPM): $0 < \beta \ll 1$ (small β)
  - Wideband PM (WBPM): $\beta \gg 1$ (large β)

Narrowband Angle Modulation

- If $0 < \beta << 1$, $x_c(t)$ is approximately linear.
- DSB-LC(AM), NBPM and NBFM are examples of linear modulation.
- If the modulating signal bandwidth is f_m, the narrowband angle-modulated signal will have a bandwidth of $2f_m$.
- Generation of Narrowband angle modulation

$$x_c(t) = A_c\,[\cos(2\pi f_c t) - \phi(t)\sin(2\pi f_c t)]$$



Wideband FM

- If modulation index is NOT small, the spectral density of a general angle-modulated signal cannot be obtained by Fourier transform.

$$x_c(t) = A_c \cos[\,2\pi f_c t + 2\pi f_d \int_0^t m(\tau)d\tau + \phi_0]$$

- Wideband FM: given $m(t) = A_m \cos 2\,\pi f_m t$ and $\phi_0 = 0$

$$x_c(t) = A_c \cos\left[2\pi f_c t + \frac{A_m f_d}{f_m} \sin 2\,\pi f_m t\right] = A_c \cos[2\pi f_c t + \beta \sin 2\,\pi f_m t]$$
$$= \mathrm{Re}\{A_c e^{j2\pi f_c t} e^{j\beta \sin 2\pi f_m t}\}$$

  - Modulation index

$$\boxed{\beta = \frac{\Delta f}{f_m} = \frac{f_d A_m}{f_m}}$$

Wideband FM

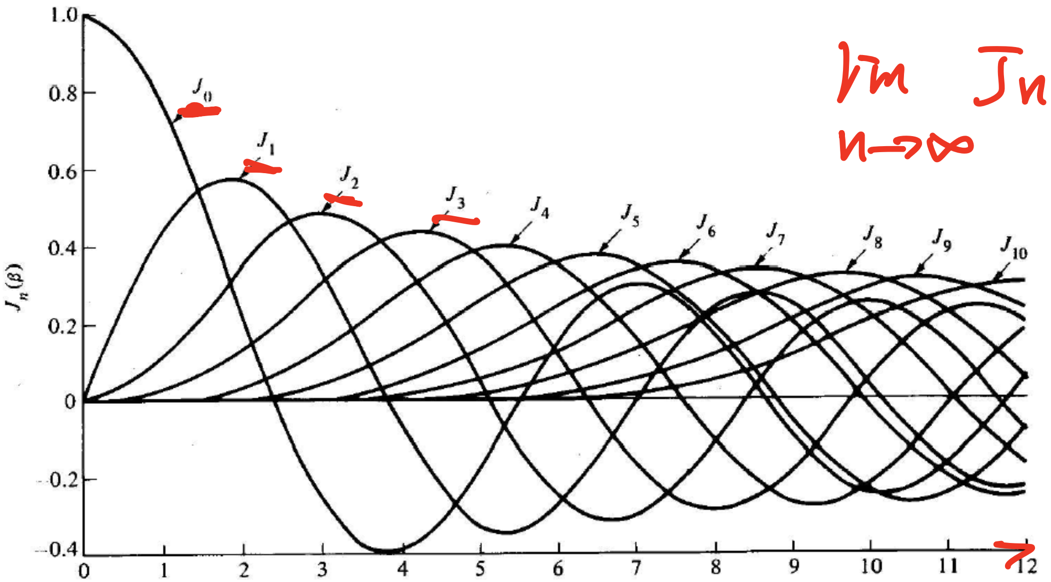
- $e^{j\beta \sin 2\pi f_m t}$ is a periodic function of time with a fundamental frequency of f_m. Its Fourier series representation is

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} F_n \, e^{j2\pi n f_m t}, \quad \text{where } F_n = \frac{1}{T}\int_{-T/2}^{T/2} e^{j\beta \sin 2\pi f_m t} e^{-j2\pi n f_m t} dt$$

  - Fourier coefficients: Bessel functions of the first kind

$$F_n = J_n(\beta)$$

P163
Table 4.1



$\lim_{n\to\infty} J_n(\beta) \to 0$

β

Wideband FM

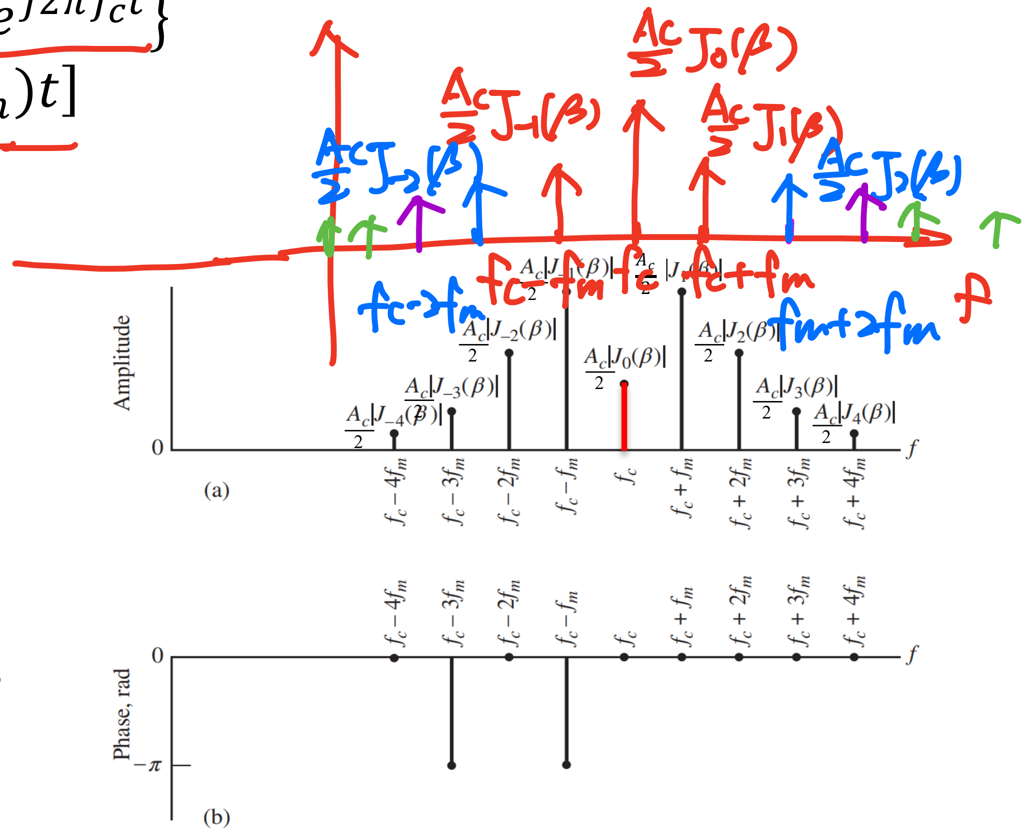
- $e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} F_n \, e^{j2\pi n f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}$
- Modulated signal

$$x_c(t) = A_c \cos[2\pi f_c t + \beta \sin 2\,\pi f_m t] = \mathrm{Re}\{A_c e^{j2\pi f_c t} e^{j\beta \sin 2\pi f_m t}\}$$
$$= \mathrm{Re}\{(A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}) e^{j2\pi f_c t}\}$$
$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t]$$

- Spectrum

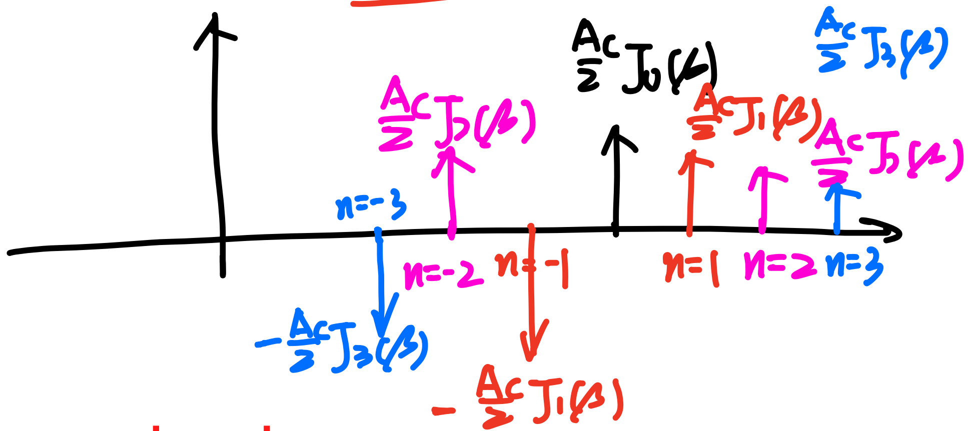
$$X_c(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \, [\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m)]$$

  - n=0: carrier component with amplitude $\frac{A_c}{2} J_0(\beta)$
  - n=1,2,…: side frequencies with amplitude $\frac{A_c}{2} J_n(\beta)$



Wideband FM

- The spectrum of angle modulated signal
  - Properties of Bessel function $J_n(\beta)$
    - Even n: $J_n(\beta) = J_{-n}(\beta)$; odd n: $J_n(\beta) = -J_{-n}(\beta)$.
    - If $\beta \ll 1$: $J_0(\beta) \simeq 1; J_1(\beta) \simeq \frac{\beta}{2}; J_n(\beta) \simeq 0, n \geq 2$. NBFM: $B = 2f_m$
    - $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$.



  - For $\beta \ll 1$: narrowband FM
  - Total Average Power

$$P = \frac{A_c^2}{2} \sum_{n=-\infty}^{\infty} J_n^2(\beta) = \frac{A_c^2}{2} = P_c$$ constant

$\Delta f = \beta \cdot f_m$

$\beta = \frac{A_m f_d}{f_m} = \frac{\Delta f}{f_m}$

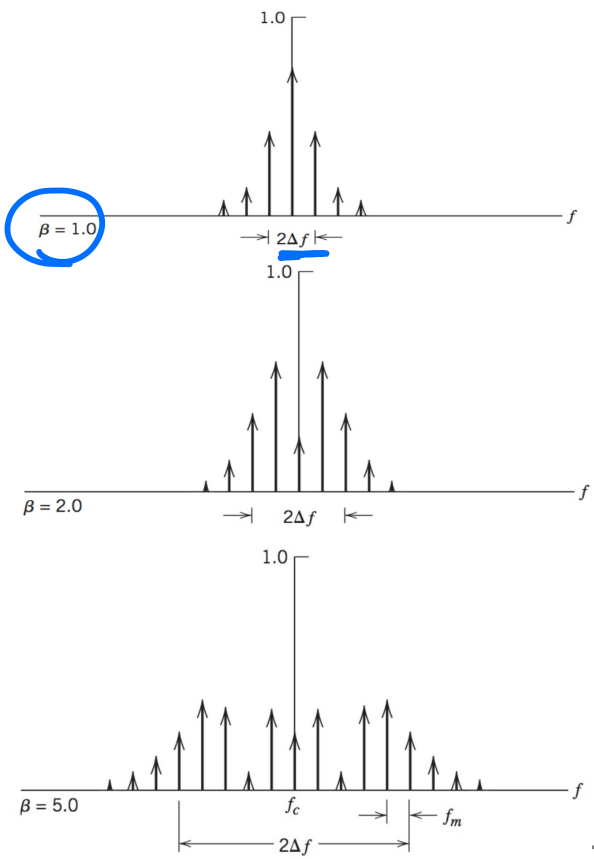
WBFM: Spectra of FM Signal

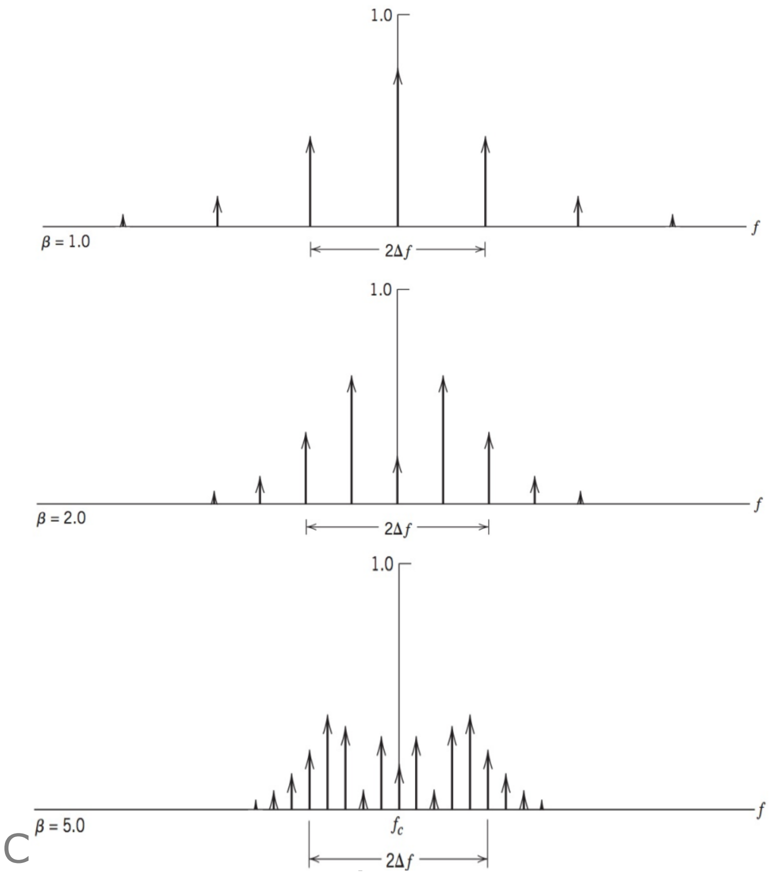
$\Delta f = \beta \cdot f_m$

- Total average power in an FM signal is a constant.

$$\beta = \frac{A_m f_d}{f_m} = \frac{\Delta f}{f_m} \qquad m(t) = A_m \cos 2\pi f_m t$$

Case 1: Fix f_m, increase $A_m (\Delta f)$	Case 2: Fix $A_m (\Delta f)$, decrease f_m





WBFM: Bandwidth of FM Signals

- Bandwidth of FM signals, theoretically unlimited.
- Significant sideband $B = 2n\, f_m$
  - For large β: $J_n(\beta)$ diminish rapidly for $n > \beta$. Assume there are $n = \beta$ significant sidebands, $B = 2n\, f_m \approx 2\beta\, f_m = 2\,\Delta f$. (wideband FM)
  - For small β: only $J_0(\beta)$ and $J_1(\beta)$ have significant magnitude. Assume $n = 1$, $B \approx 2 f_m$. (Narrowband FM)
- Carson's rule:

$$B \approx 2\,(\Delta f + f_m) = 2(1+\beta)\, f_m = 2(1 + 1/\beta)\,\Delta f$$

  - An approximation of the bandwidth.
- Arbitrary m(t)
  - Deviation Ratio: $D = \frac{f_d \max|m(t)|}{W}$, $B \approx 2\,(1 + D)W$

$\beta = \frac{\Delta f}{f_m}$

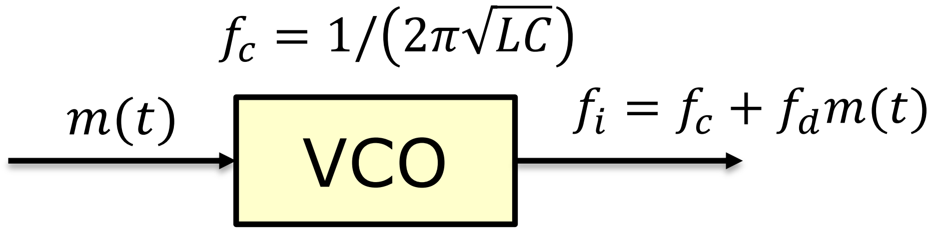
Example

$\Delta f = 50k$

- A 10 MHz carrier is frequency-modulated by a sinusoidal signal such that the peak frequency deviation is 50 kHz. Determine the approximate bandwidth of the FM signal when modulating frequency is (a) 500 kHz; (b) 500 Hz; (c) 10 kHz. f_m
- Solution:
  - (a) $\beta = \dfrac{\Delta f}{f_m} = \dfrac{50}{500} = 0.10 << 1$ ➔ $B \approx 2f_m = 1\ MHz$
    - Carson's rule gives: $B \approx 2f_m(1+\beta) = 1.1\ MHz$ $<<1$
  - (b) $\beta = \dfrac{\Delta f}{f_m} = \dfrac{50000}{500} = 100 >> 1$ ➔ $B \approx 2\,\Delta f = 100\ kHz$
    - Carson's rule gives: $B \approx 2f_m(1+\beta) = 101\ kHz$ $>>1$
  - (c) $\beta = 50/10 = 5$. Check Bessel function table(P163), we have
    1% basis: $n = 8, J_8(5) = 0.018$, $B = 2nf_m = 160\ kHz$ $J_n(\beta) > 0.01$
    - Carson's rule gives: $B \approx 2f_m(1+\beta) = 120\ kHz$ n_{max}:

Generation of Wideband FM Signals

- Direct method: vary the carrier frequency directly with the modulating signal *m(t)* by using the voltage-controlled oscillator (VCO).

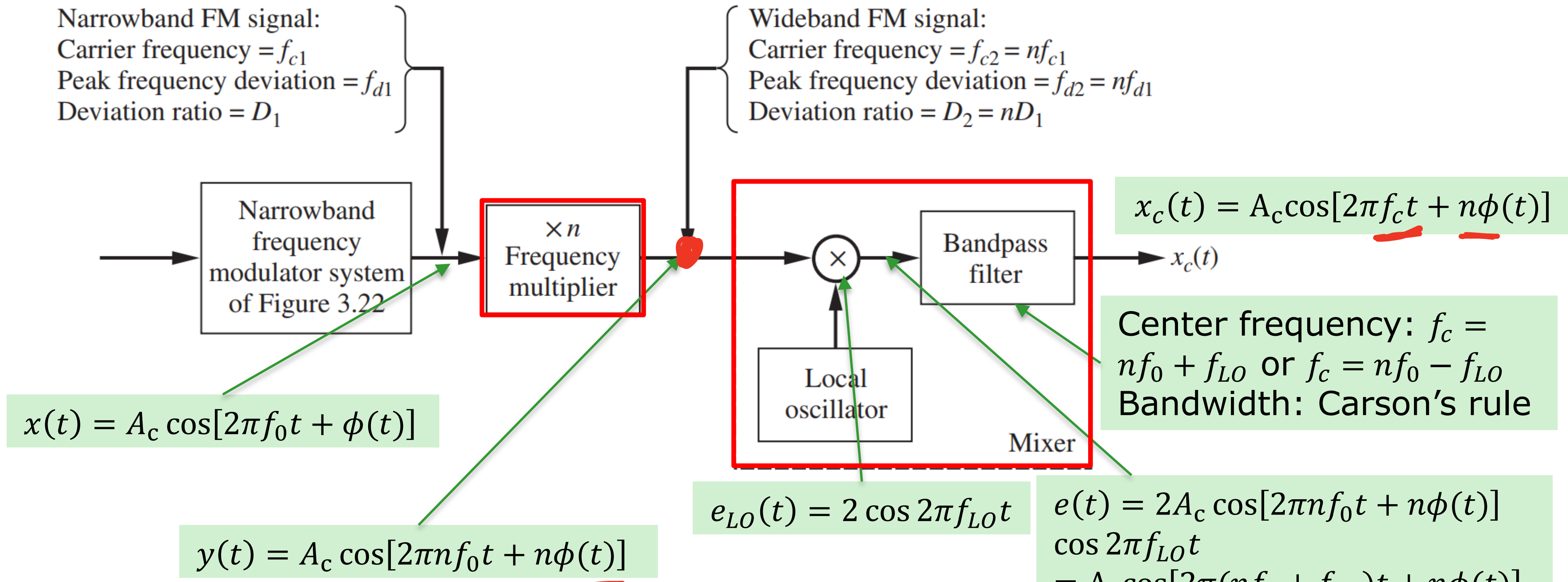


- Requirement of direct method:
  - The long-term frequency stability is not as good as the crystal-stabilized oscillators so that frequency stabilization is needed.
  - The percentage frequency deviation that can be attained in this method is quite small. (say $\beta < 0.2$ in theory)

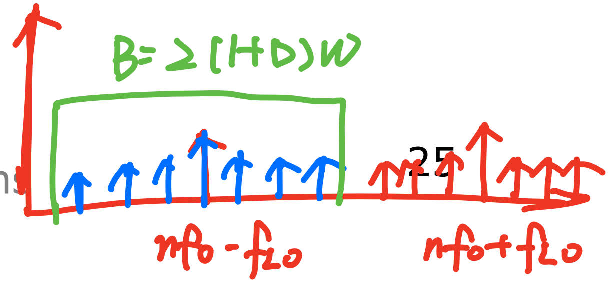
Generation of Wideband FM Signals (Cont'd)

- Indirect method: Armstrong indirect FM transmitter
  - produce a narrowband FM signal.

$B = 2(1+D)W$



Frequency multiplier: increase modulation index
Mixer: control the value of the carrier frequency

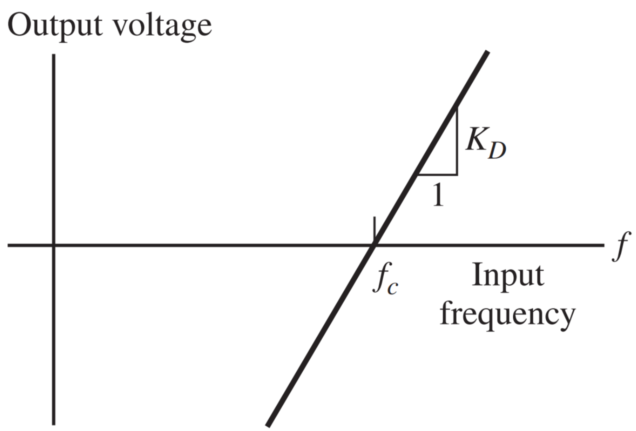


Example

- A NB to WB converter, the output of the narrowband frequency modulator is given by $x(t) = A_c \cos[2\pi f_0 t + \phi(t)]$ with $f_0 = 100$ kHz. The peak frequency deviation is 50 Hz and the bandwidth of $\phi(t)$ is 500Hz. The wideband output $x_c(t)$ is to have a carrier frequency of 85 MHz and a deviation ratio of 5. In this example we determine the frequency multiplier factor n, two possible local oscillator frequencies and the center frequency and the bandwidth of the BP filter.
- Sol:
  - $D_1 = \frac{f_{d_1}}{W} = \frac{50}{500} = 0.1$, → $n = \frac{D_2}{D_1} = 50.$
  - $f_{c_2} = nf_{c_1} = 5$ MHz, $f_{LO} = f_c - f_{c_2} = 85 - 5 = 80$ MHz, or $f_{LO} = f_c + f_{c_2} = 85 + 5 = 90$ MHz.
  - The center frequency of the BP filter is 85 MHz, the bandwidth of the BP filter is $B = 2W(1 + D) = 2 * 500 * (1 + 5) = 6$ kHz.

Demodulation of Wideband FM Signals

- Demodulation: to provide an output signal whose amplitude is linearly proportional to the frequency deviation of the input FM signal.
- Direct method: use frequency discriminator 鉴频器
  - Frequency discriminator is the system that has a linear frequency-to-voltage transfer characteristic.



input $x_r(t) = A_c \cos[\,2\pi f_c t + 2\pi f_d \int_0^t m(\tau)\ d\tau + \phi_0]$

Frequency discriminator

output $y_D(t) = K_D f_d m(t)$

K_D: discriminator constant

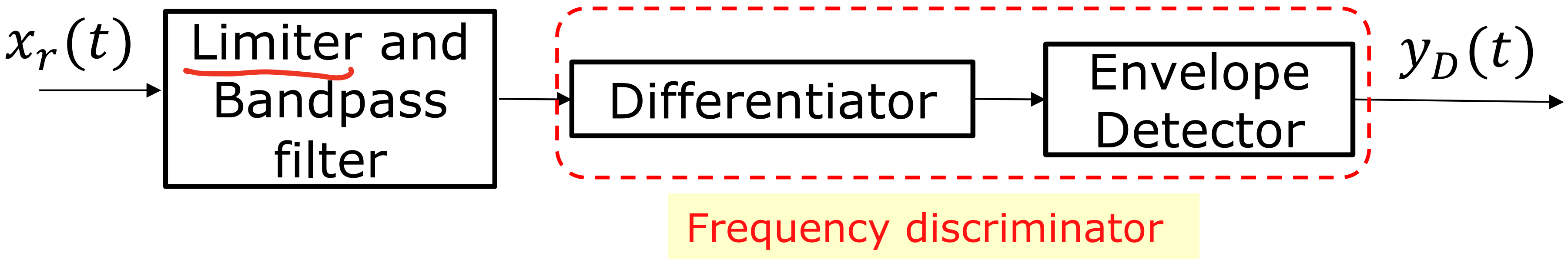
Demodulation of Wideband FM Signals

- Direct method: use frequency discriminator
  - Ideal differentiator has a linear amplitude versus frequency characteristic and therefore is a frequency discriminator.

    Input: $x_r(t) = A_c \cos[2\pi f_c t + 2\pi f_d \int_0^t m(\tau)\, d\tau + \phi_0]$

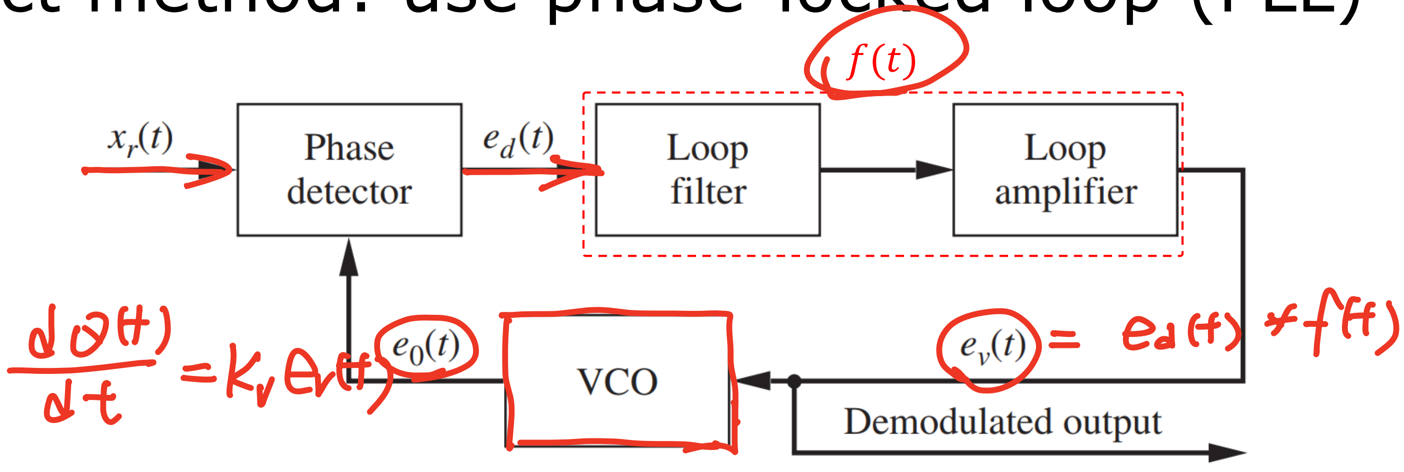
    Output: $\frac{d}{dt} x_r(t) = -A_c[2\pi f_c + 2\pi f_d m(t)] \sin\left[2\pi f_c t + 2\pi f_d \int_0^t m(\tau)\, d\tau + \phi_0\right]$

    Envelope: $A_c[2\pi f_c + 2\pi f_d m(t)]$
  - If $f_c > -f_d m(t), \forall t$, the modulating signal can then be detected by an envelope detector.
  - The output of envelope detector: $y_D(t) = 2\pi A_c f_d m(t)$



Demodulation of Wideband FM Signals (Cont'd)

- Indirect method: use phase-locked loop (PLL)

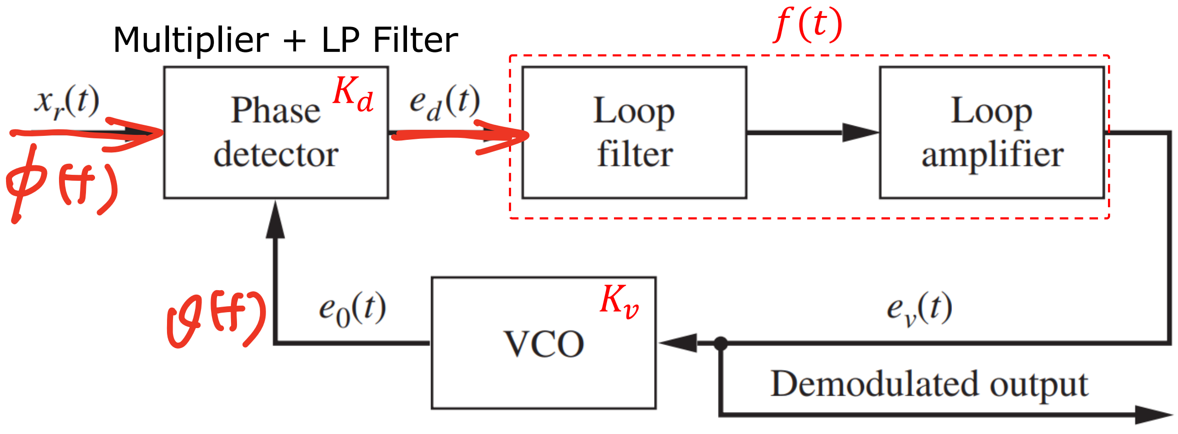


  - Phase detector detects the timing difference between the two periodic signals (with the same fundamental frequency) and produces an output voltage that is proportional to this difference.
  - Loop filter controls the dynamic response of the PLL. We have

    $e_v(t) = e_d(t) * f(t)$
  - Voltage-controlled oscillator (VCO) generates a constant-amplitude periodic waveform whose frequency deviation is proportional to the input voltage, i.e., $\frac{d\theta(t)}{dt} = K_v e_v(t)$.

Demodulation of Wideband FM Signals (Cont'd)

- Indirect method: output of PLL



  - Assume $x_r(t) = A_c \cos[2\pi f_c t + \phi(t)]$ and $e_0(t) = A_v \sin[2\pi f_c t + \theta(t)]$ the phase detector output is then

$$e_d(t) \propto \{A_c \cos[2\pi f_c t + \phi(t)]\, A_v \sin[2\pi f_c t + \theta(t)]\}_{LP}$$
$$\propto \frac{1}{2} A_c A_v K_d \sin[\phi(t) - \theta(t)]$$

  - If $\phi(t) - \theta(t)$ is small and we have $e_d(t) \approx \frac{1}{2} A_c A_v K_d [\phi(t) - \theta(t)]$.

Demodulation of Wideband FM Signals (Cont'd)

- Indirect method: output of PLL

$$\Psi(f) = \frac{\cancel{j2\pi} f}{\cancel{j2\pi} f + \boxed{\dfrac{2\pi K_t F(f)}{j2\pi f}}} \Phi(f)$$

$\triangleq L(f)$

$$= \frac{1}{1+L(f)} \Phi(f)$$

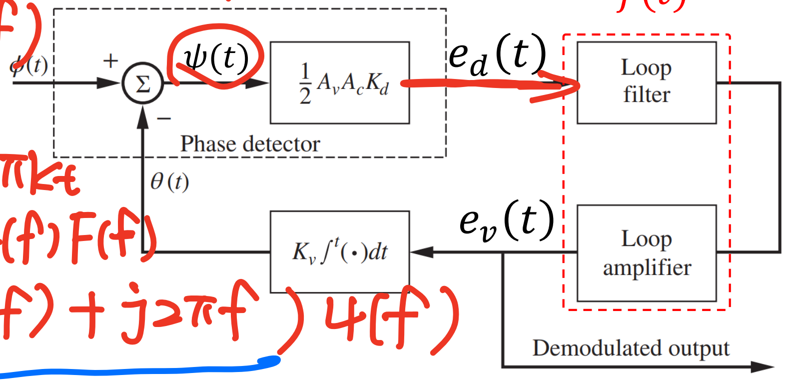
$$L(f) = \frac{K_t F(f)}{jf}$$

$= \phi(t) - \theta(t)$

$j2\pi f\, \Theta(f) = 2\pi K_t \Psi(f) F(f)$

$j2\pi f\,(\Phi(f) - \Psi(f)) = 2\pi K_t \Psi(f) F(f)$

$j2\pi f\, \Phi(f) = (2\pi K_t F(f) + j2\pi f)\, \Psi(f)$



$\psi(t)$

$$\frac{d\theta(t)}{dt} = 2\pi K_v e_v(t) = 2\pi K_v e_d(t) * f(t) = 2\pi K_t[\phi(t) - \theta(t)] * f(t)$$

Fourier Transform

Phase error: $\phi(t) - \theta(t) = \psi(t)$

Total loop gain: $K_t = \frac{1}{2} A_c A_v K_d K_v$

$$\Psi(f) = \frac{1}{L(f)+1} \Phi(f),\ L(f) = \frac{K_t F(f)}{jf}$$

$|L(f)| \gg 1$

$\theta(t) \rightarrow \phi(t)$

$\Phi(f) - \Theta(f) = \Psi(f) \rightarrow 0$

Phase lock is established

Demodulation of Wideband FM Signals (Cont'd)

- Indirect method: output of PLL (with loop)

$$\phi(t) \approx \theta(t) = 2\pi K_v \int_0^t e_v(\tau)\, d\tau$$

$$\boxed{\frac{d\phi(t)}{dt} = 2\pi f_d\, m(t)}$$

$$e_v(t) \approx \frac{1}{2\pi K_v} \frac{d}{dt} \phi(t)$$

$$\approx \underline{\quad} m(t)$$

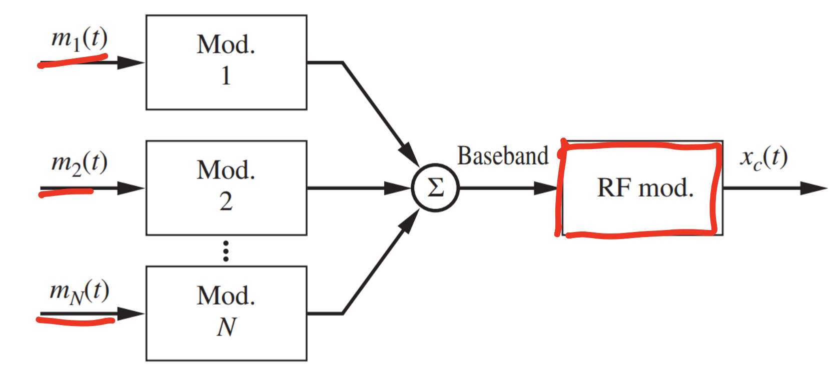
  - Output voltage is proportional to the frequency deviation (referred to the carrier) of the input wideband FM signal.
  - The PLL demodulates the input wideband FM signal!

$$m(t) \rightarrow \boxed{\boxed{D} \xrightarrow{\frac{dm(t)}{dt}} \boxed{VCO}} \rightarrow x_{PM}(t) \qquad \text{(PM)}$$

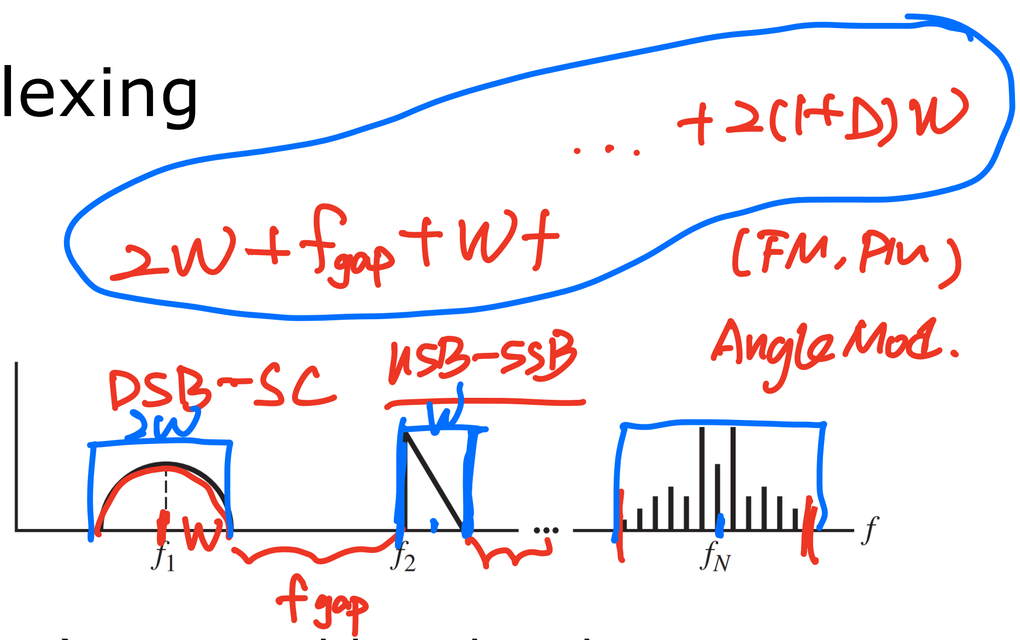
$$x_{PM}(t) \rightarrow \boxed{\boxed{PLL} \xrightarrow{\frac{dm(t)}{dt}} \boxed{I}} \rightarrow m(t) \qquad \text{(PM Demodulation)}$$

Multiplexing

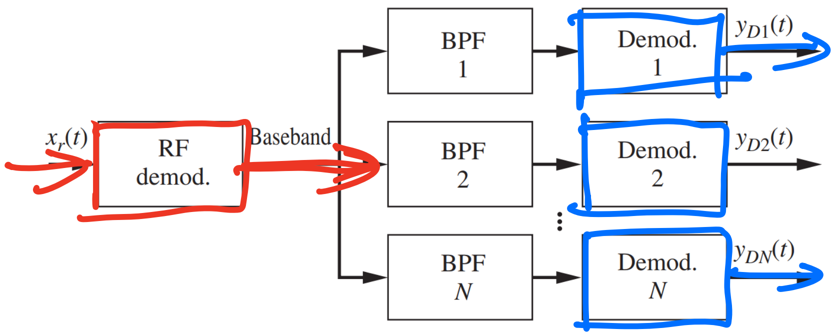
- Frequency-Division Multiplexing



a) FDM modulator



c) Assumed baseband spectrum



b) FDM demodulator

Thanks for your kind attention!

Questions?