EE140 Introduction to Communication Systems

Lecture 8

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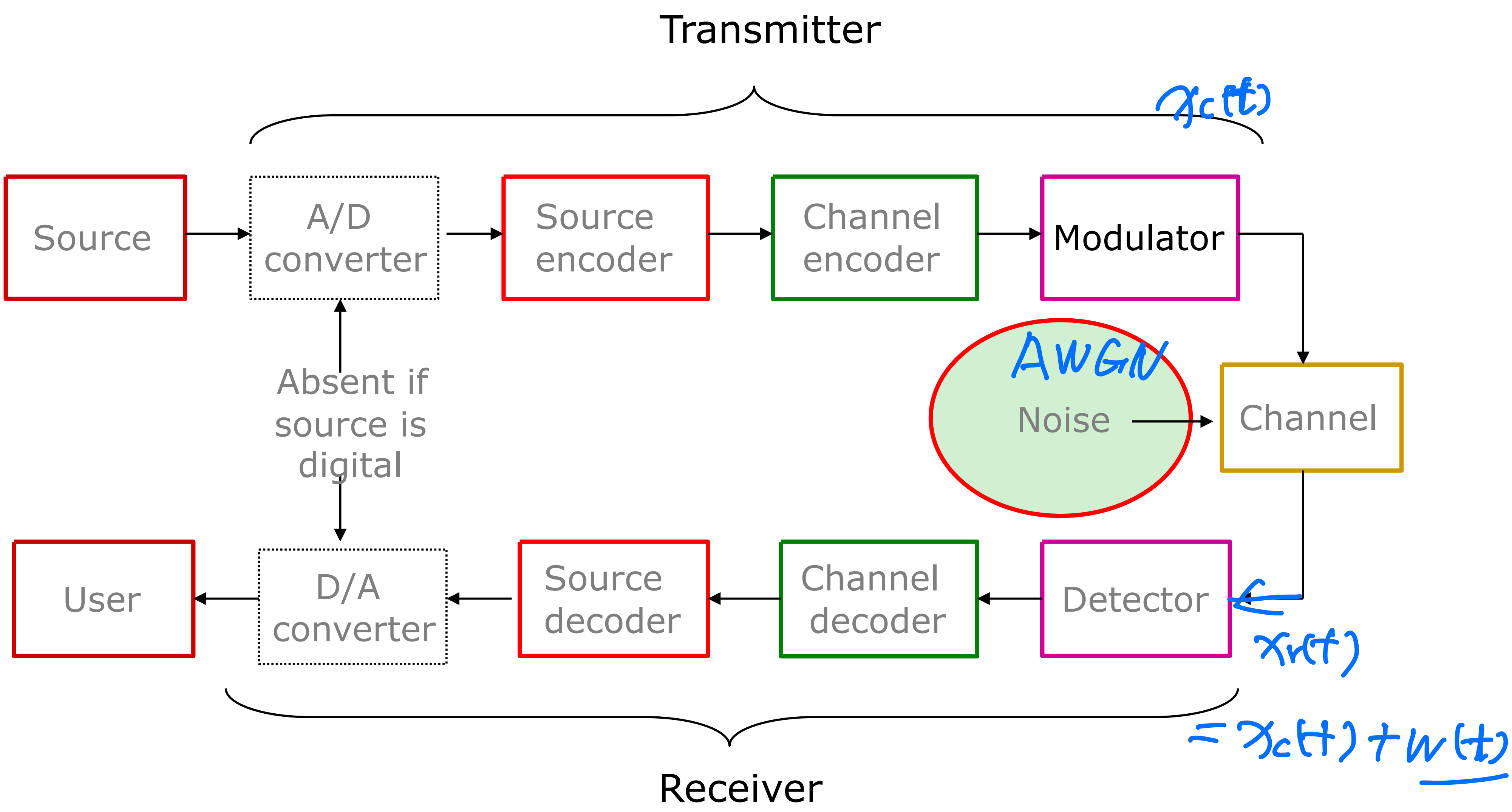
ShanghaiTech University, Fall 2025

Architecture of a (Digital) Communication System



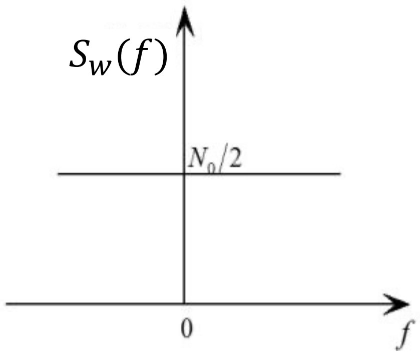
Contents

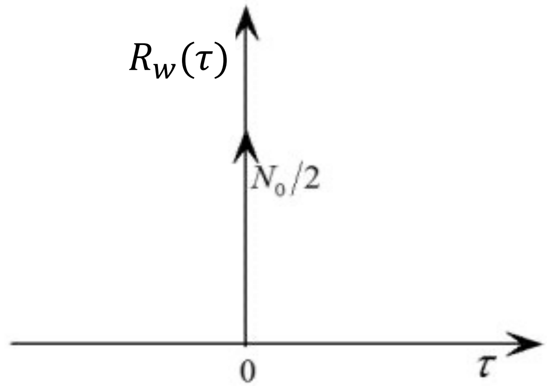
- Noise in Modulation Systems
  - Review
  - Noise in DSB-SC Receiver
  - Noise in SSB Receiver
  - Noise in AM Receiver
  - Noise in Angle Modulation

Noise

- White Gaussian Noise

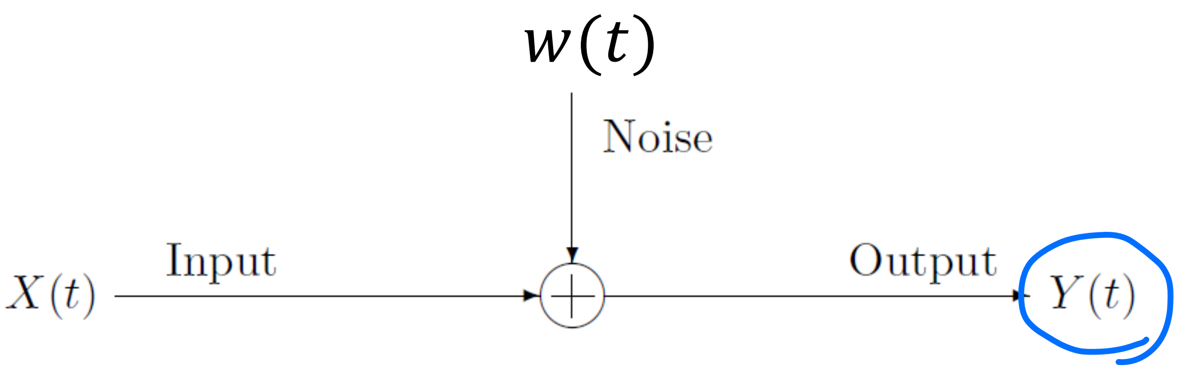
$$S_w(f) = \frac{N_0}{2} \qquad \Longrightarrow \qquad R_w(\tau) = \frac{N_0}{2}\delta(\tau)$$





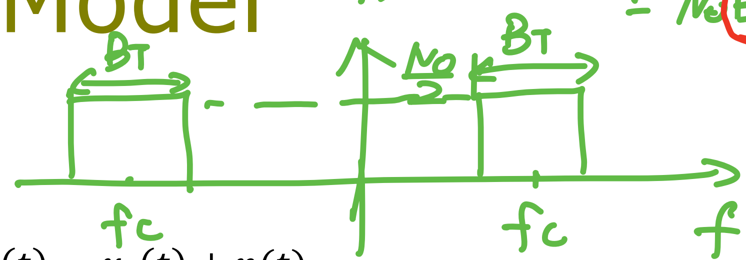
White Gaussian noise is completely independent!

- Additive white Gaussian noise (AWGN) model



$= X(t) + w(t)$

Noisy Receiver Model



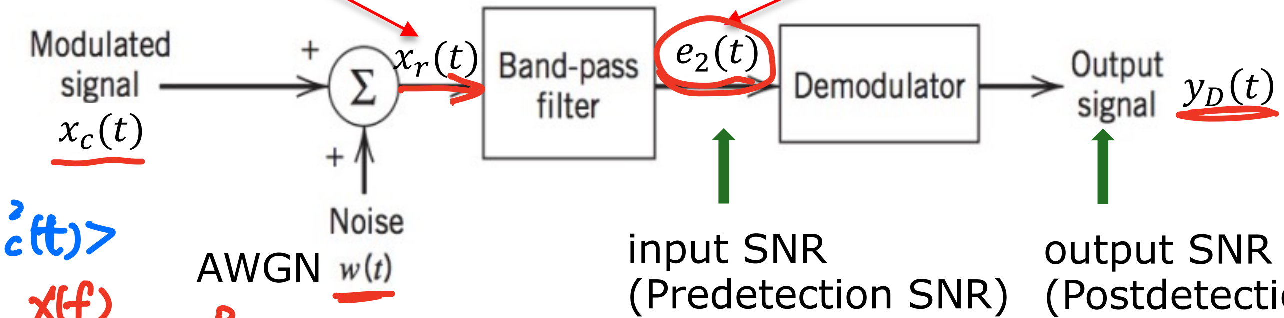
$P = \frac{N_0}{2} 2B_T$
$= N_0 B_T$

- Noisy receiver model

$x_r(t) = x_c(t) + w(t)$

Ideal, bandwidth= B_T, mid-band frequency f_c

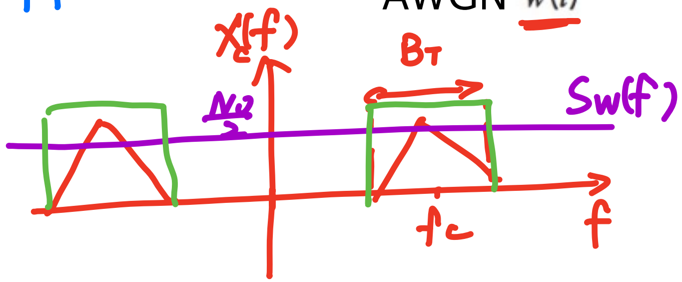
$e_2(t) = x_c(t) + n(t)$



input SNR
(Predetection SNR)

output SNR
(Postdetection SNR)

$P_T = \langle x_c^2(t) \rangle$



$x_r(t)$: $SNR = \frac{P_T}{\infty} = 0$

$e_2(t)$: $SNR = \frac{P_T}{N_0 B_T}$

$y_D(t)$: $SNR =$ ______

Filter: BPF, LPF

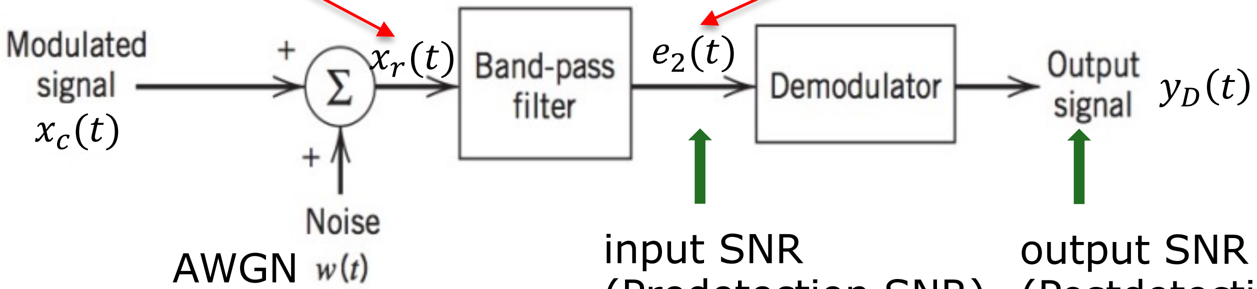
Noise: $w(t)$, $n(t)$, noise in $y_D(t)$

SNR: P_T, $S_n(f)$, B_T

Noisy Receiver Model

- Noisy receiver model

$x_r(t) = x_c(t) + w(t)$ Ideal, bandwidth= B_T, mid-band frequency f_c $e_2(t) = x_c(t) + n(t)$



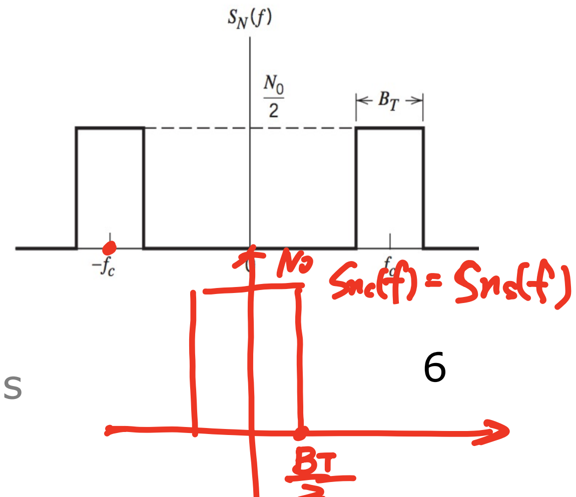
input SNR (Predetection SNR) output SNR (Postdetection SNR)

- Narrowband Noise:
  - $f_c \gg B_T$, filtered noise n(t): stationary narrowband Gaussian noise

$$\mathrm{n}(t) = n_c(t)\cos(2\pi f_c t + \theta) - n_s(t)\sin(2\pi f_c t + \theta)$$

In-phase component Quadrature component

Low-pass noise process

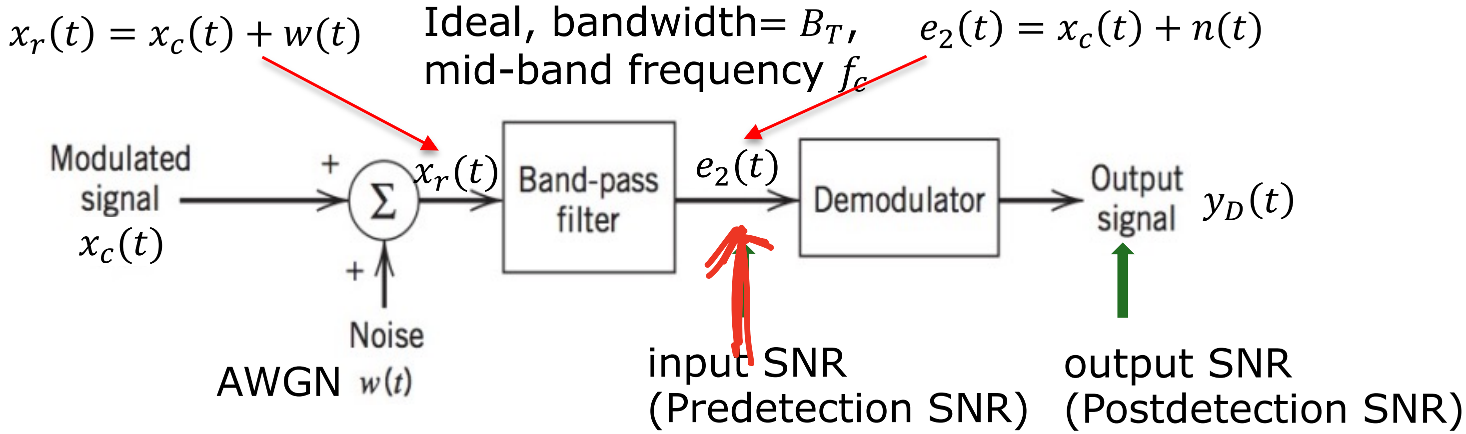


Noisy Receiver Model

- Narrowband Noise:
  - Let n(t) be a zero-mean, stationary and Gaussian noise, then $n_c(t)$ and $n_s(t)$ satisfy the following properties
    - $n_c(t)$ and $n_s(t)$ are zero-mean, jointly stationary and jointly Gaussian process
    - Means: $E[n(t)] = E[n_c(t)] = E[n_s(t)] = 0$
    - Variances(power): $E[n^2(t)] = E[n_c^2(t)] = E[n_s^2(t)] = N_0 B_T$ $\mathcal{I}(f)$
    - PSD: $S_{n_c}(f) = S_{n_s}(f) = \mathrm{Lp}[S_n(f - f_c) + S_n(f + f_c)]$
    - Correlation function:
      - $R_{n_c}(\tau) = R_{n_s}(\tau)$, $R_n(0) = R_{n_c}(0) = R_{n_s}(0)$
      - $R_{n_c n_s}(\tau) = -R_{n_c n_s}(-\tau)$ (odd), $R_{sc}(0) = R_{cs}(0) = 0.$
    - Cross-PSD: $S_{n_c n_s}(f) = j\mathrm{Lp}[S_n(f - f_c) - S_n(f + f_c)]$
      - $R_{n_c n_s}(\tau) \equiv 0, \forall\tau,$ if $\mathrm{Lp}[S_n(f - f_c) - S_n(f + f_c)] = 0.$

Noisy Receiver Model

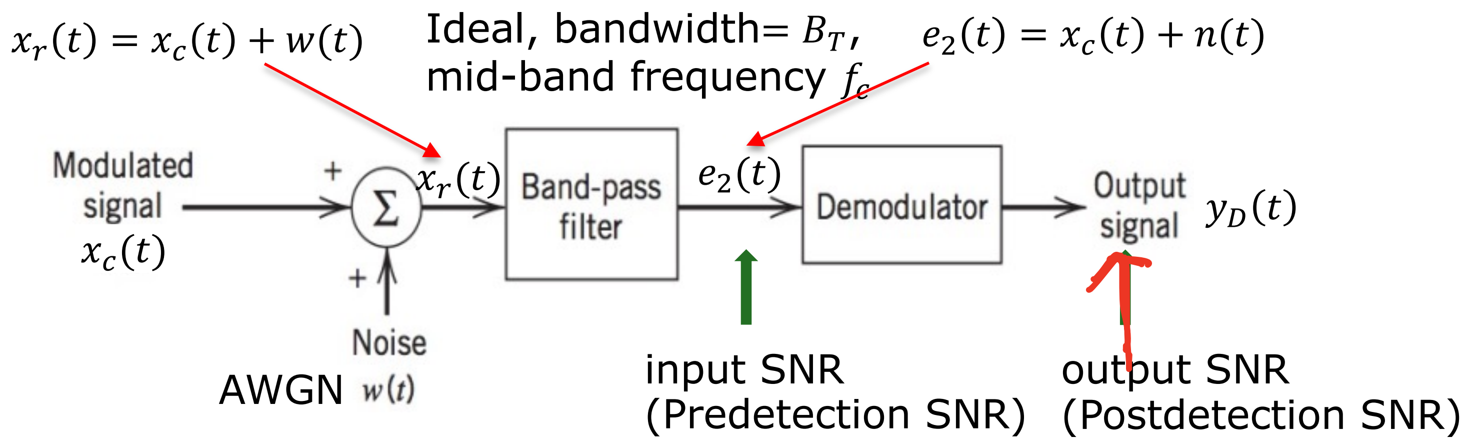
- Noisy receiver model



- Input SNR (Predetection SNR):
  - The ratio of the average power of the modulated signal $x_c(t)$ to the average power of the filtered noise $n(t)$, both measured at the receiver input.
  - $SNR_i = \frac{P_T}{N_0 B_T}$ or $SNR_T = \frac{P_T}{N_0 B_T}$

Noisy Receiver Model

- Noisy receiver model

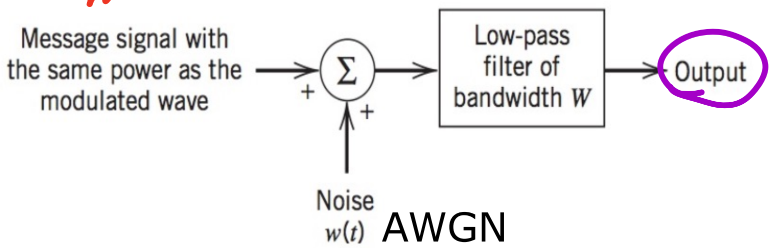


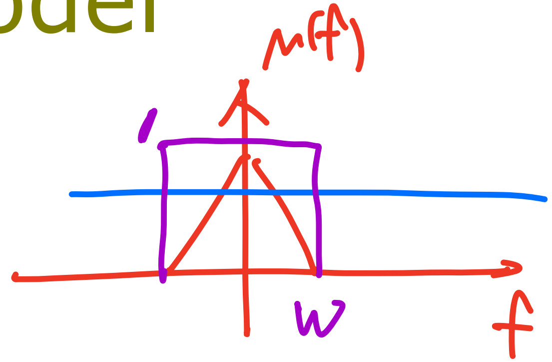
- Output SNR (Postdetection SNR):
  - The ratio of the average power of the demodulated signal to the average power of the noise, both measured at the receiver output.
  - Type of modulation, type of demodulation
  - SNR_o or SNR_D

Noisy Receiver Model

- Baseband model

$m(t)$ P_T





$P_n = \frac{N_0}{2} \cdot 2W$

$= N_0 W$

$m(t)$

- Baseband SNR (Channel SNR):
  - The ratio of the average power of the modulated signal to the average power of the noise in the message bandwidth P_T
    - The total noise power in the message bandwidth: $\frac{N_0}{2} 2W = N_0 W$
    - The total signal power P_T
  - Baseband SNR: $\mathrm{SNR}_c = \frac{P_T}{N_0 W}$

Performance Comparison

- Detection Gain (SNR Gain):

$$\frac{SNR_o}{SNR_i} \quad \text{or} \quad \frac{SNR_D}{SNR_T} \qquad SNR_T = \frac{P_T}{N_0 B_T}$$ Bandwidth of Modulated signal

- Figure of Merit for the receiver

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$$\frac{SNR_o}{SNR_c} \quad \text{or} \quad \frac{SNR_D}{SNR_c} \qquad SNR_c = \frac{P_T}{N_0 W}$$ Message bandwidth

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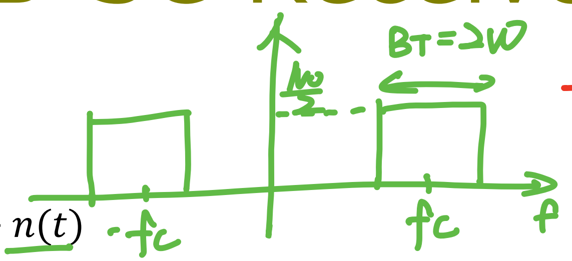
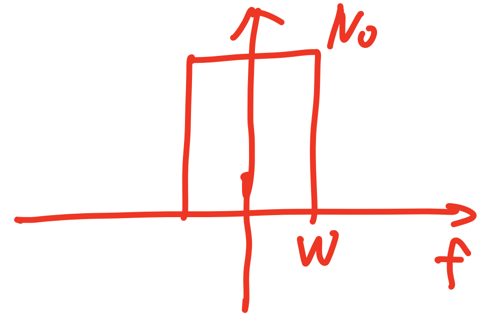
Noise in DSB-SC Receiver

$P = N_0 B_T = 2WN_0$

$S_n(f)$



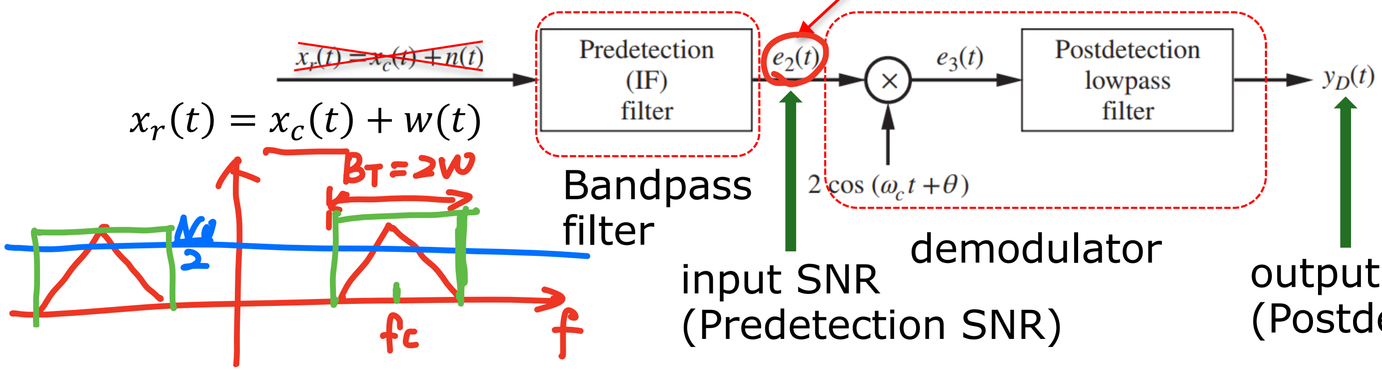
$S_{nc}(f) = S_{ns}(f)$



$P = 2WN_0$

- Coherent DSB detector

$e_2(t) = x_c(t) + n(t)$



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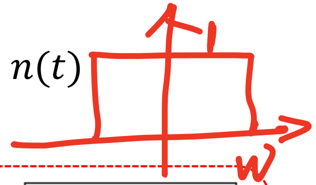
  - DSB Signal: $x_c(t) = A_c m(t)\cos(2\pi f_c t + \theta)$
  - White Gaussian Noise: $w(t)$
  - Predetection filter: $B_T = 2W$
  - $e_2(t) = A_c m(t)\cos(2\pi f_c t + \theta) + n_c(t)\cos(2\pi f_c t + \theta) - n_s(t)\sin(2\pi f_c t + \theta)$
  - Predetection SNR:

$$SNR_T = \frac{\frac{1}{2}A_c^2 P}{2N_0 W}, \quad P = \overline{m^2}$$

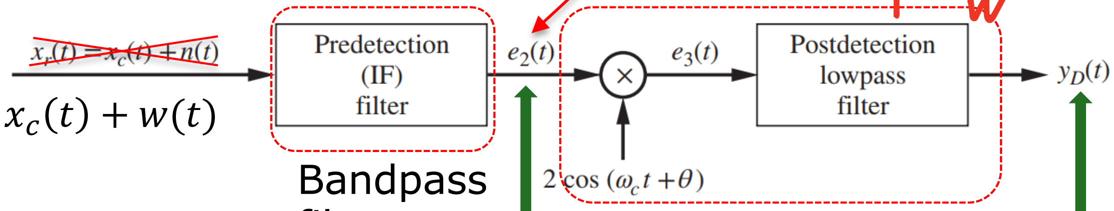
Noise in DSB-SC Receiver

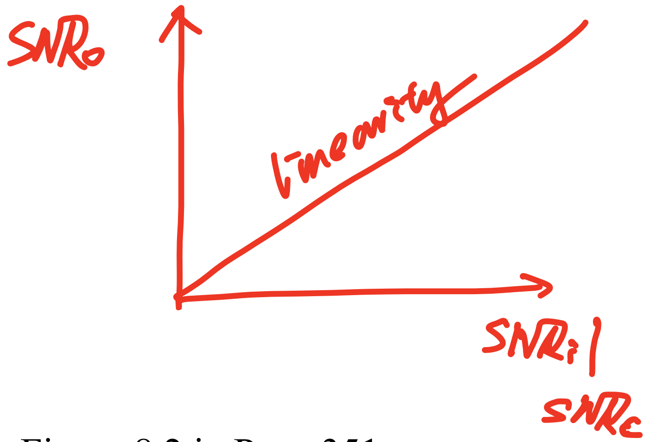
- Coherent DSB detector

$e_2(t) = x_c(t) + n(t)$



$x_r(t) = x_c(t) + w(t)$





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- $e_3(t) = [A_c m(t)\cos(2\pi f_c t + \theta) + n_c(t)\cos(2\pi f_c t + \theta) - n_s(t)\sin(2\pi f_c t + \theta)]2\cos(2\pi f_c t + \theta) = A_c m(t) + A_c m(t)\cos(4\pi f_c t + 2\theta) + n_c(t) + n_c(t)\cos(4\pi f_c t + 2\theta) - n_s(t)\sin(4\pi f_c t + 2\theta)$

$m(t) + n_c(t)$

- $y_D(t) = A_c m(t) + n_c(t)$ (linearity)

$x_c(t) + n(t) \rightarrow \boxed{\text{Linear Demodulator}} \rightarrow$

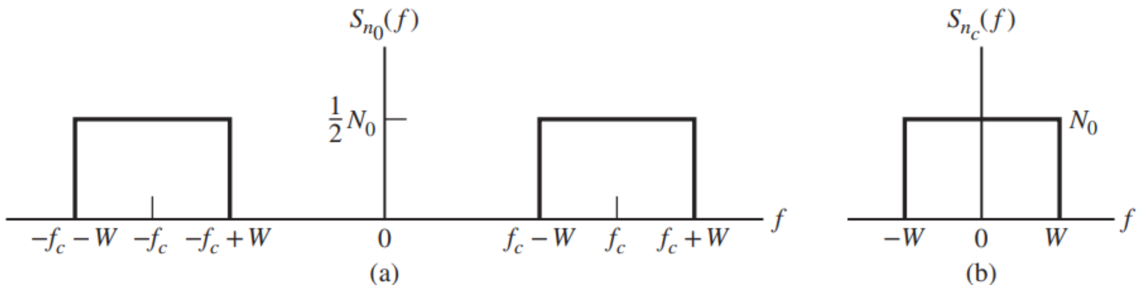
Coherent Dem.

- Postdetection SNR:

$$SNR_D = \frac{A_c^2 P}{2N_0 W}, \quad P = \overline{m^2}, \quad \overline{n^2(t)} = \overline{n_c^2(t)} = N_0 B_T = 2N_0 W$$

Noise in DSB-SC Receiver

- Coherent DSB detector



Predection noise

postdection noise

$$P_T = \frac{1}{2}A_c^2 P$$

$$SNR_T = \frac{P_T}{2N_0 W}$$

$$SNR_D = \frac{P_T}{N_0 W}$$

$$SNR_c = \frac{P_T}{N_0 W}$$

Detection gain: $\frac{SNR_D}{SNR_T} = 2$

Figure of Merit: $\frac{SNR_D}{SNR_c} = 1$

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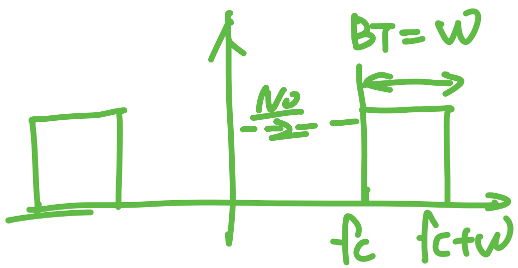
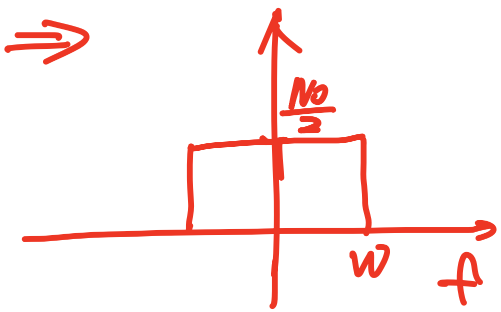
Noise in SSB Receiver

$S_n(f)$ $P = N_0 B_T = N_0 W$ $S_{nc}(f) = S_{ns}(f)$

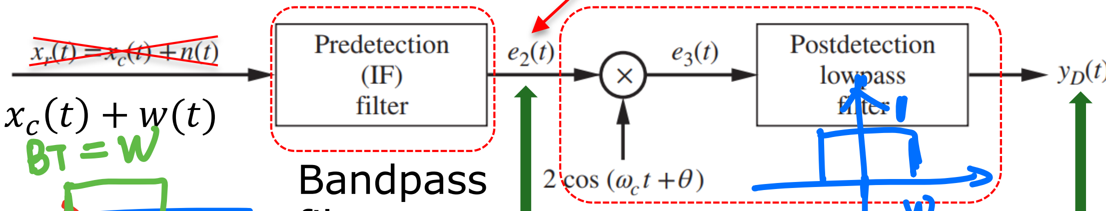
- Coherent detector

$e_2(t) = x_c(t) + n(t)$

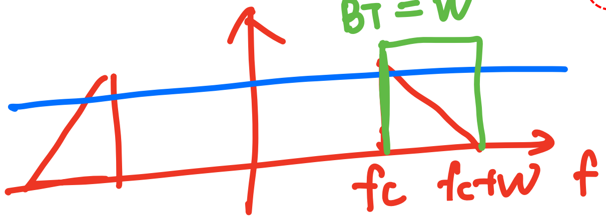




$P = N_0 W$



$x_r(t) = x_c(t) + w(t)$



Bandpass filter

input SNR (Predetection SNR)

demodulator

output SNR (Postdetection SNR)

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- SSB Signal: $x_c(t) = A_c[m(t)\cos(2\pi f_c t + \theta) \pm \hat{m}(t)\sin(2\pi f_c t + \theta)]$
- Predetection filter: $B_T = W$
- $e_2(t) = A_c[m(t)\cos(2\pi f_c t + \theta) \pm \hat{m}(t)\sin(2\pi f_c t + \theta)] + n_c(t)\cos(2\pi f_c t + \theta) - n_s(t)\sin(2\pi f_c t + \theta)$

$\times\, 2\cos(2\pi f_c t + \theta)$

$y_D(t) = A_c m(t) + n_c(t)$

- Predetection SNR: $SNR_T = \frac{A_c^2 P}{N_0 W}$

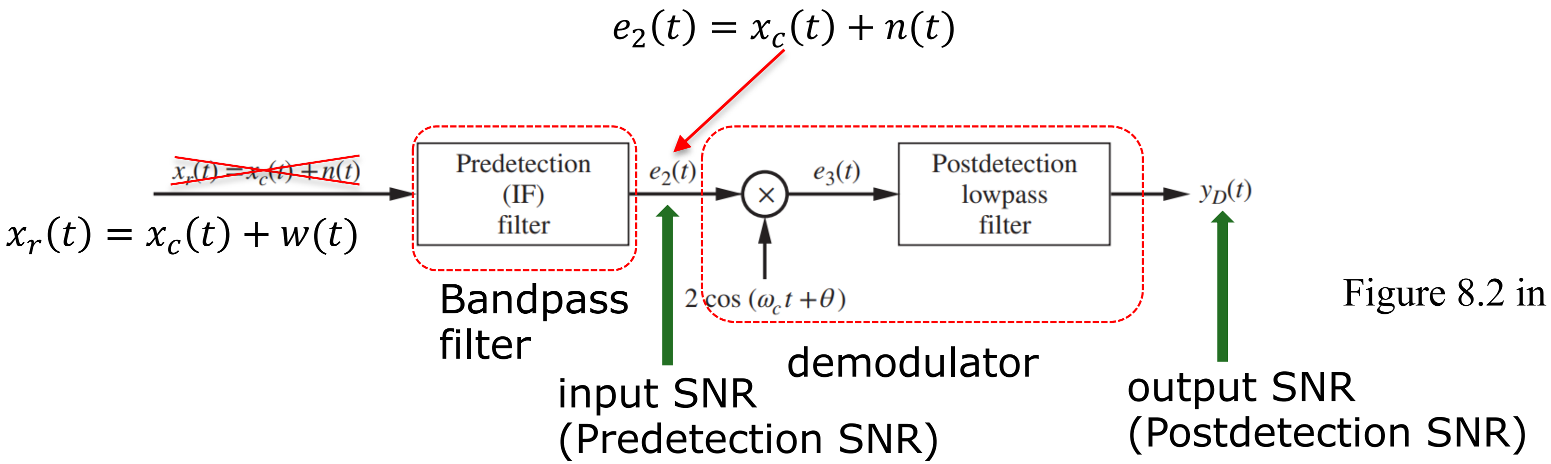
$$S_T = A_c^2 \overline{[m(t)\cos(2\pi f_c t + \theta) \pm \hat{m}(t)\sin(2\pi f_c t + \theta)]^2}$$
$$= A_c^2 \left[\frac{1}{2}\overline{m(t)^2} + \frac{1}{2}\overline{\hat{m}(t)^2}\right]$$
$$= A_c^2\, \overline{m(t)^2}$$

$A_c^2 P$

$N_T = N_0 B_T = N_0 W$

Noise in SSB Receiver

- Coherent detector



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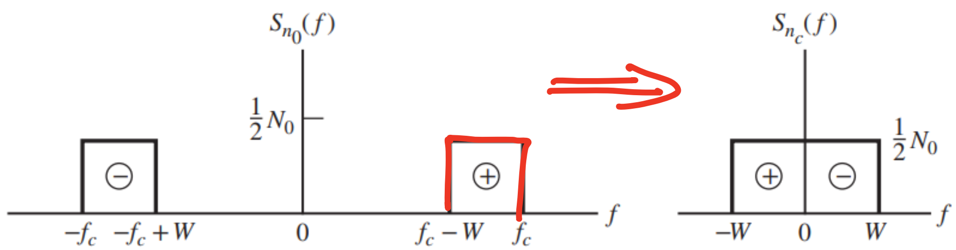
  - $y_D(t) = A_c m(t) + n_c(t)$
  - postdetection SNR: $SNR_D = \frac{A_c^2 P}{N_0 W}$



Predection noise   postdection noise

Noise in SSB Receiver

- Coherent detector

$$P_T = A_c^2 P$$

$$SNR_T = \frac{P_T}{N_0 W}$$

$$SNR_D = \frac{P_T}{N_0 W}$$

$$SNR_c = \frac{P_T}{N_0 W}$$

Detection gain: $\frac{SNR_D}{SNR_T} = 1$

Figure of Merit: $\frac{SNR_D}{SNR_c} = 1$

Coherent demodulation of both DSB and SSB results in performance equivalent to baseband.

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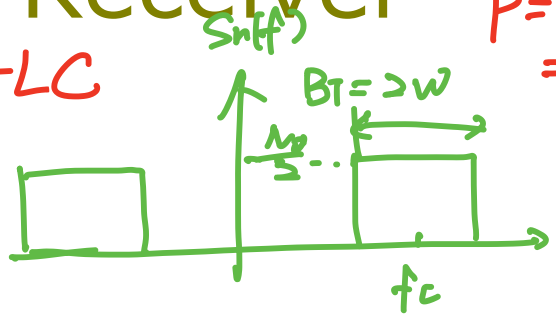
Noise in AM Receiver

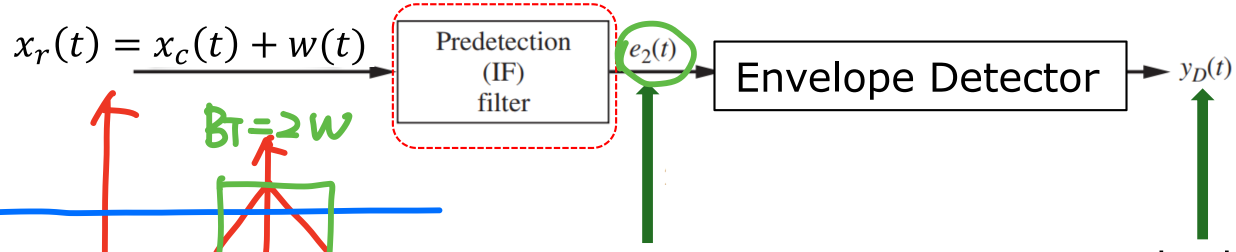
DSB-LC

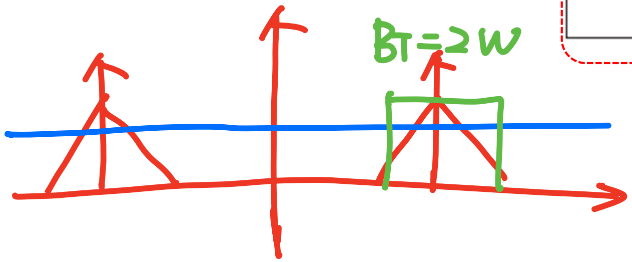
$P = N_0 B_T$
$= 2N_0 W$



- Envelope detector

$e_2(t) = x_c(t) + n(t)$





$A_c[1 + a m_n(t)] + n_c(t)$

$\begin{cases} r(t)\cos\phi(t) = \\ r(t)\sin\phi(t) = n_s(t) \end{cases}$

– AM Signal: $x_c(t) = A_c[1 + a m_n(t)]\cos(2\pi f_c t + \theta)$ $0 < a \leq 1$

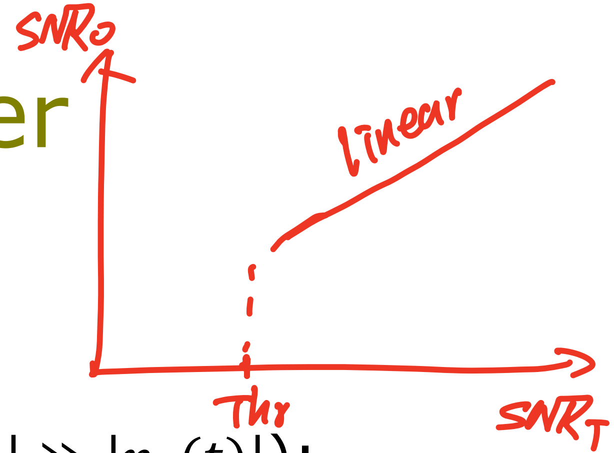
– Predetection filter: $B_T = 2W$

– $e_2(t) = A_c[1 + a m_n(t)]\cos(2\pi f_c t + \theta) + n_c(t)\cos(2\pi f_c t + \theta) - n_s(t)\sin(2\pi f_c t + \theta) = r(t)\cos[2\pi f_c t + \theta + \phi(t)]$,

$\begin{cases} r(t) = \sqrt{(A_c[1 + a m_n(t)] + n_c(t))^2 + n_s^2(t)} \\ \phi(t) = \tan^{-1}(\quad) \end{cases}$

- Predetection SNR: $SNR_T = \dfrac{\frac{A_c^2}{2}\left[1 + a^2\overline{m_n^2}\right]}{2N_0 W}$
- Envelope: $r(t) = \sqrt{\{A_c[1 + a m_n(t)] + n_c(t)\}^2 + n_s(t)^2}$

Noise in AM Receiver



- Envelope detector   non linear
  - $y_D(t) = \sqrt{\{A_c[1 + am_n(t)] + n_c(t)\}^2 + n_s(t)^2}$
  - When SNR$_T$ is large $(|A_c[1 + am_n(t)] + n_c(t)| \gg |n_s(t)|)$:
    $y_D(t) \cong A_c a m_n(t) + n_c(t)$ (after removal of DC component)
    - Postdetection SNR: $SNR_D = \frac{A_c^2 a^2 \overline{m_n^2}}{2N_0 W}$
    - When SNR$_T$ is large

$\mu = \frac{a^2 \langle m_n^2 \rangle}{1 + a^2 \langle m_n^2(t) \rangle}, \ \mu < 1$

$\langle m_n^2(t) \rangle \uparrow, a \uparrow, \ \mu \uparrow$

$m_n(t) = \cos$___ $\quad \mu = \frac{1}{3}$

$$SNR_T = \frac{\frac{A_c^2}{2}[1 + a^2\overline{m_n^2}]}{2N_0 W}$$

Detection gain: $\frac{SNR_D}{SNR_T} = \frac{2a^2\overline{m_n^2}}{1 + a^2\overline{m_n^2}} = 2\mu$

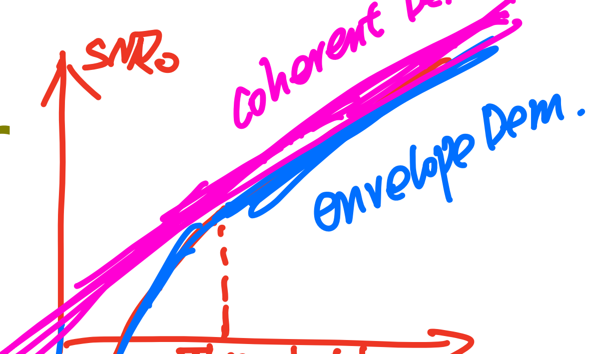
$$SNR_D = \frac{A_c^2 a^2 \overline{m_n^2}}{2N_0 W}$$

$$P_T = \frac{A_c^2}{2}[1 + a^2\overline{m_n^2}] \qquad SNR_c = \frac{\frac{A_c^2}{2}[1 + a^2\overline{m_n^2}]}{N_0 W}$$

Figure of Merit: $\frac{SNR_D}{SNR_c} = \frac{a^2\overline{m_n^2}}{1 + a^2\overline{m_n^2}} = \mu \le 1$

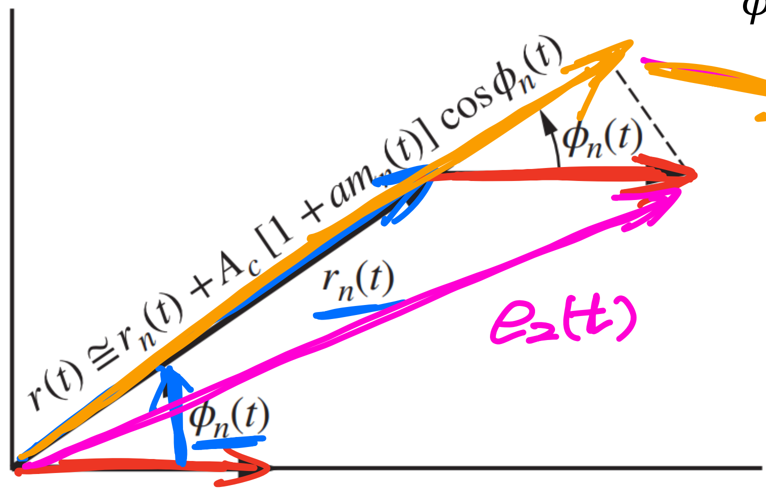
The noise performance of an AM receiver is always inferior to that of a DSB-SC, SSB receiver.

Noise in AM Receiver



- Envelope detector
  - $e_2(t) = A_c[1 + am_n(t)]\cos(2\pi f_c t + \theta) + n_c(t)\cos(2\pi f_c t + \theta) - n_s(t)\sin(2\pi f_c t + \theta) = A_c[1 + am_n(t)]\cos(2\pi f_c t + \theta) + r_n(t)\cos[2\pi f_c t + \theta + \phi_n(t)]$
  - When SNRT is small $(|A_c[1 + am_n(t)]| \ll |r_n(t)|)$:
    - $y_D(t) \cong r_n(t) + A_c[1 + am_n(t)]\cos[\phi_n(t)]$

$\phi_n(t) \sim U[0,2\pi]$



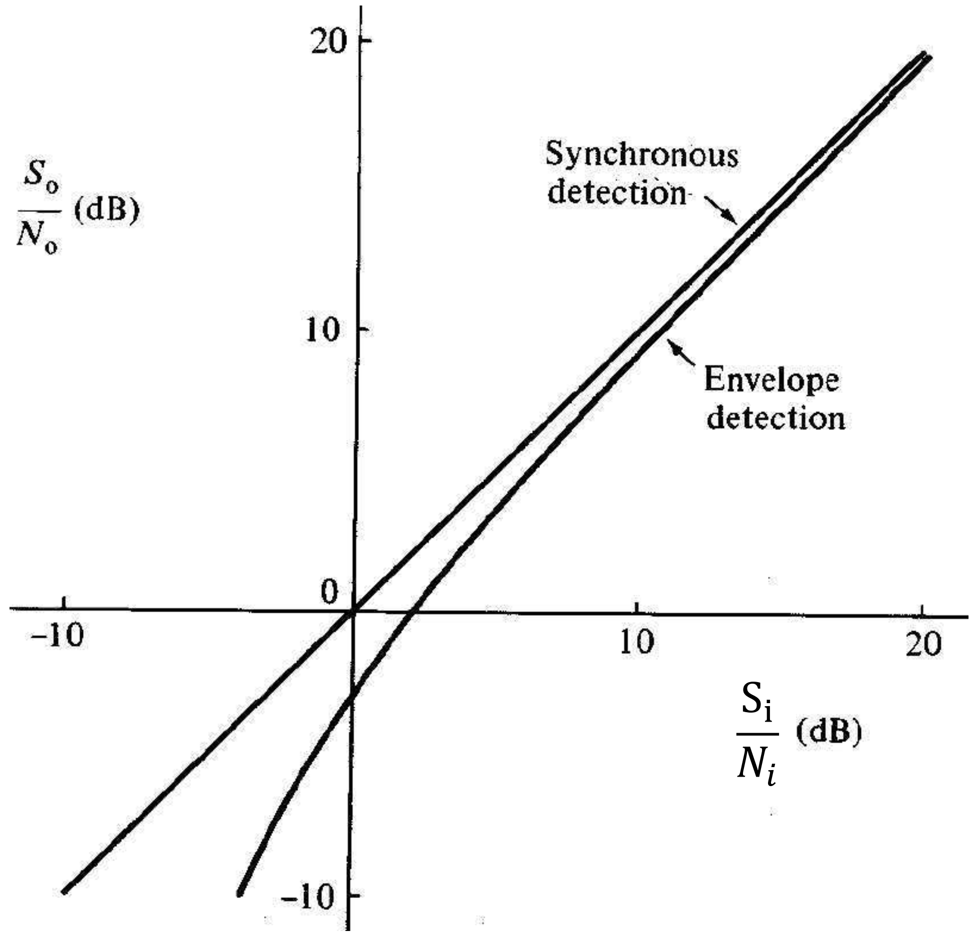
$y_n(t) + A_c(1+a\,m_n(t))\cos\phi_n(t)$

$\phi_n(t) \sim$ Uniform $(0, 2\pi]$

Threshold Effect (loss of message at low SNR): Every nonlinear detector exhibits a threshold effect.

Performance of AM Demod

- Synchronous detection vs envelope detection
  - In synchronous detection, the output signal and noise always remain additive and the curve-slope is a constant, independent of input SNR.
  - The nonlinear behavior of envelope detection declines the SNR performance when input noise increases.



Contents

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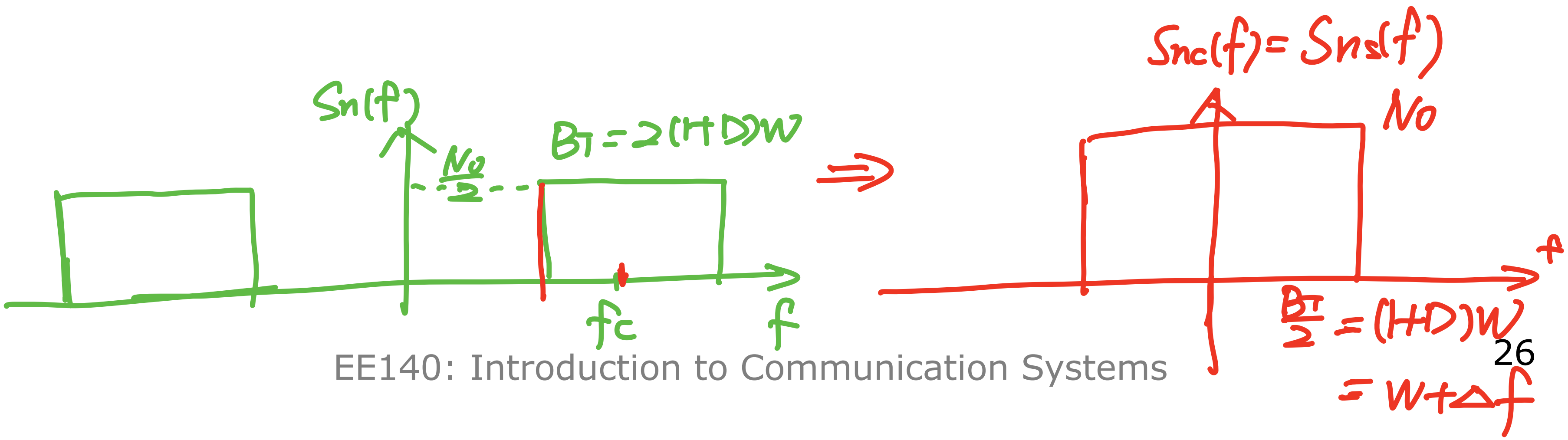
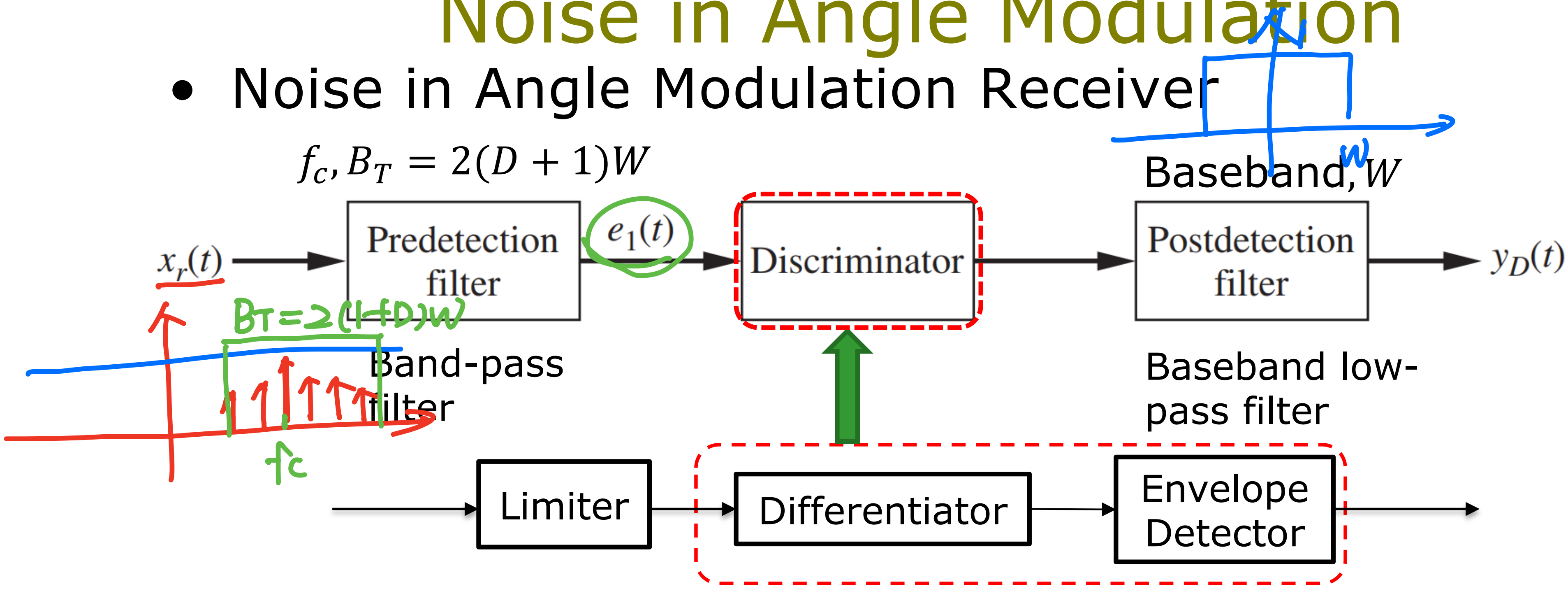
  - Noise in Angle Modulation



Noise in Angle Modulation

- Noise in Angle Modulation Receiver



  - Received signal: $x_r(t) = A_c \cos[2\pi f_c t + \theta + \phi(t)] + w(t)$
  - Bandpass filter: $f_c, B_T = 2(D+1)W$
  - $e_1(t) = A_c \cos[2\pi f_c t + \theta + \phi(t)] + n_c(t)\cos(2\pi f_c t + \theta) - n_s(t)\sin(2\pi f_c t + \theta) = A_c \cos[2\pi f_c t + \theta + \phi(t)] + r_n(t)\cos[2\pi f_c t + \theta + \phi_n(t)]$
    - $r_n(t)$: Rayleigh-distributed noise envelope; $\phi_n(t)$: uniformly distributed noise phase
    - Predetection SNR (input SNR): $SNR_T = \frac{\frac{A_c^2}{2}}{N_0 B_T}$ $B_T = 2(1+D)W$

Noise in Angle Modulation

- Noise in Angle Modulation Receiver
  - $e_1(t) = A_c \cos[2\pi f_c t + \theta + \phi(t)] + r_n(t)\cos[2\pi f_c t + \theta + \phi_n(t)]$
    $= R(t)\cos[2\pi f_c t + \theta + \phi(t) + \phi_e(t)]$
  - $\phi_e(t) = \tan^{-1}\left\{\frac{r_n(t)\sin[\phi_n(t)-\phi(t)]}{A_c + r_n(t)\cos[\phi_n(t)-\phi(t)]}\right\}$
  - Phase Deviation of the receiver input:
    - $\psi(t) = \phi(t) + \phi_e(t)$ (phase error due to noise)

$e_1(t) = A_c \cos(2\pi f_c t + \theta + \phi(t)) + r_n(t)\cos(2\pi f_c t + \theta + \phi(t) + \phi_n(t) - \phi(t))$

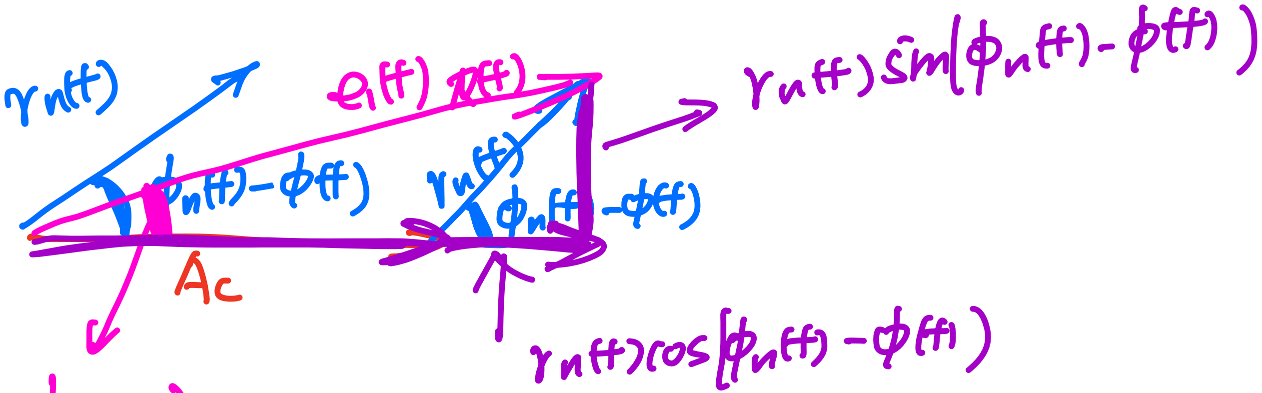
$r_n(t)\cos(2\pi f_c t + \theta + \phi(t))\cos(\phi_n(t) - \phi(t))$
$- r_n(t)\sin(2\pi f_c t + \theta + \phi(t))\sin(\phi_n(t) - \phi(t))$

$= R(t)\cos(2\pi f_c t + \theta + \phi(t) + \phi_e(t))$

$\begin{cases} R(t)\cos\phi_e(t) = A_c + r_n(t)\cos(\phi_n(t) - \phi(t)) \\ R(t)\sin\phi_e(t) = r_n(t)\sin(\phi_n(t) - \phi(t)) \end{cases}$

$R(t) =$ ______

$\Rightarrow \tan(\phi_e(t)) = \frac{r_n(t)\sin(\phi_n(t) - \phi(t))}{A_c + r_n(t)\cos(\phi_n(t) - \phi(t))}$



$$\tan(\phi_e(t)) = \frac{r_n(t)\sin(\phi_n(t)-\phi(t))}{A_c + r_n(t)\cos(\phi_n(t)-\phi(t))}$$

$$e_1(t) = R(t)\cos(\underline{2\pi f_c t + \theta + \phi(t) + \phi_e(t)}) \rightarrow \boxed{} \rightarrow y_D(t)$$

PM: $y_D(t) = k_D[\phi(t) + \phi_e(t)]$ $\quad \phi(t) = k_p m(t)$

$$= k_D k_p m(t) + k_D \phi_e(t)$$

$$\frac{1}{2\pi}\frac{d\phi(t)}{dt} = f_d m(t)$$

FM: $y_D(t) = k_D \dfrac{1}{2\pi}\dfrac{d(\phi(t)+\phi_e(t))}{dt} = k_D f_d m(t) + \dfrac{k_D}{2\pi}\dfrac{d\phi_e(t)}{dt}$

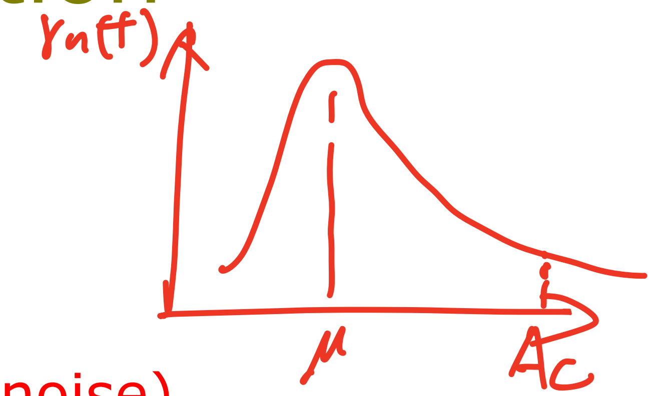
$SNR_T \uparrow$

$SNR_T \downarrow$ $\Leftarrow$ Threshold effect $\Leftarrow$ Nonlinearity

SNR↑

$P(r_n(t) \ll A_c) \to 1$

Noise in Angle Modulation



- Noise in Angle Modulation Receiver
  - Phase Deviation of the receiver input:
    - $\phi_e(t) = \tan^{-1}\left\{\frac{r_n(t)\sin[\phi_n(t)-\phi(t)]}{A_c + r_n(t)\cos[\phi_n(t)-\phi(t)]}\right\}$
    - $\psi(t) = \phi(t) + \phi_e(t)$ (phase error due to noise)
  - When SNR$_T$ is large ($A_c \gg r_n(t)$ most of the time):
    - $\phi_e(t) \approx \tan^{-1}\left\{\frac{r_n(t)\sin[\phi_n(t)-\phi(t)]}{A_c}\right\}$

when $x \ll 1, \tan^{-1}(x) \approx x$

$$\approx \frac{r_n(t)\sin[\phi_n(t)-\phi(t)]}{A_c}$$

$$= \frac{n_s(t)}{A_c}$$

SNR$_T$↑ Linearity

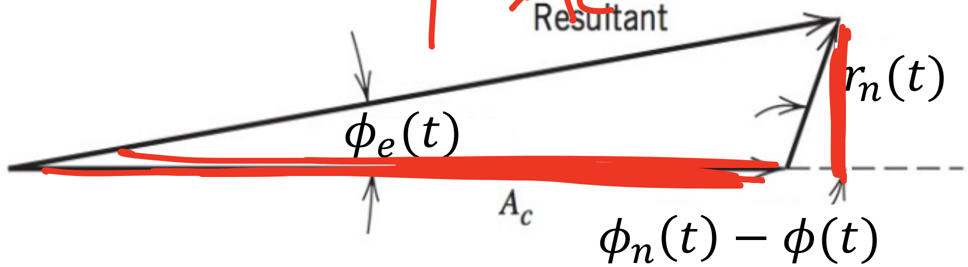
$n_s(t)$

$\frac{n_s(t)}{A_c}$

$\phi_n(t) \sim U[0, 2\pi]$

$\phi'(t) = \phi_n(t) - \phi(t) \sim U[0, 2\pi]$

$r_n(t)\sin\phi'(t) = n_s(t)$



$$\begin{cases} n_c(t) = r_n(t)\cos\phi_n(t) \\ \underline{n_s(t)} = \underline{r_n(t)}\,\underline{\sin}\,\underline{\phi_n(t)} \end{cases}$$

$$e_i(t) = A_c\cos[2\pi f_c t + \theta + \phi(t)] + r_n(t)\cos[2\pi f_c t + \theta + \phi_n(t)]$$

$$= R(t)\cos[2\pi f_c t + \theta + \phi(t) + \underline{\phi_e(t)}]$$

$\underline{\phi_e(t)} = f(m(t))$

nonlinear

$\longrightarrow$ PM, DeM $\longrightarrow$ $y_D(t)$

$$\phi_e(t) \approx \frac{n_s(t)}{A_c}$$

$SNR_T \uparrow$ $\xrightarrow{PM}$ $y_D(t) = k_D(\phi(t) + \phi_e(t)) = k_D k_p m(t) + k_D \phi_e(t)$

$$= k_D k_p m(t) + k_D \frac{n_s(t)}{A_c}$$

$\boxed{\text{linearity}}$

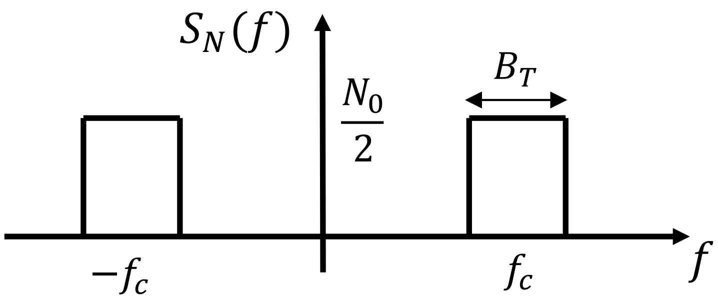
$\xrightarrow{FM}$ $y_D(t) = \frac{k_D}{2\pi}\frac{d[\phi(t) + \phi_e(t)]}{dt} = k_D f_d m(t) + \frac{k_D}{2\pi}\frac{1}{A_c}\frac{dn_s(t)}{dt}$

Nonlinearity DeM.

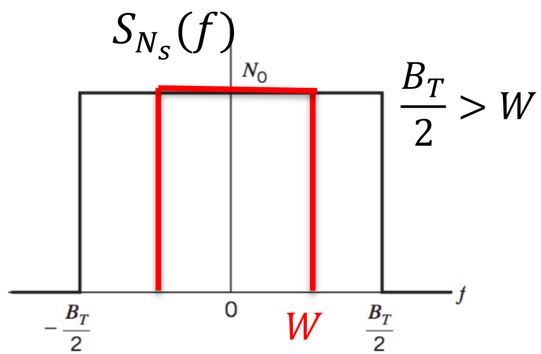
$SNR_T \downarrow$ $\longrightarrow$ Threshold effect

Noise in Phase Modulation

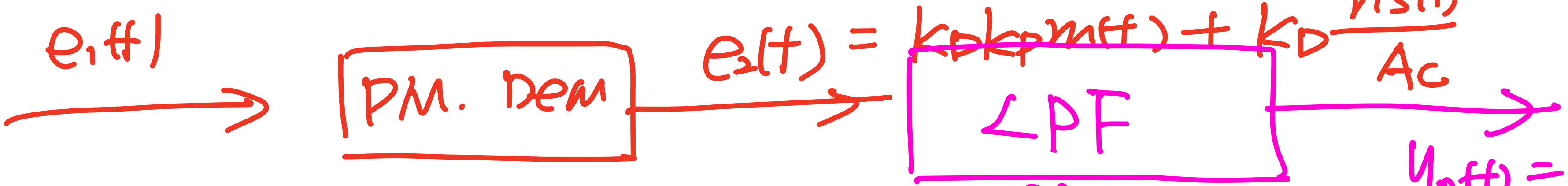
- PM Demodulator $SNR_T \uparrow$
  - $e_1(t) = R(t)\cos[2\pi f_c t + \theta + \psi(t)]$
    - Phase deviation: $\psi(t) = \phi(t) + \phi_e(t)$
    - Signal: $\phi(t) = k_p m(t)$
    - Phase error: $\phi_e(t) = \dfrac{n_s(t)}{A_c}$
  - Demodulated output of PM:
    - $y_D(t) = K_D\psi(t) = K_D k_p m(t) + K_D \dfrac{n_s(t)}{A_c}$
    - Output signal power: $S_{DP} = K_D^2 k_p^2 \overline{m^2}$
    - Bandwidth of the $n_s(t)$: $\dfrac{B_T}{2} > W \rightarrow$ postdetection lowpass filter with bandwidth W



Predetection noise



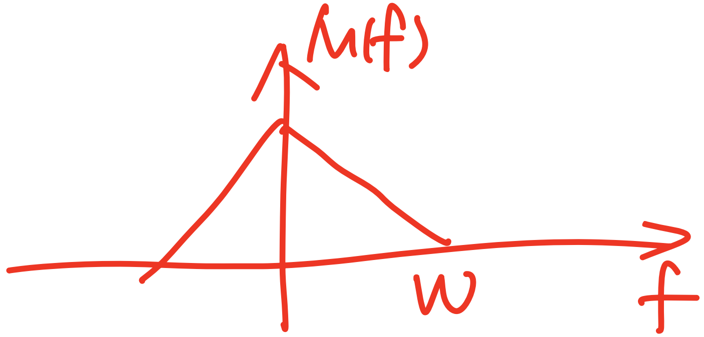
Quadrature component

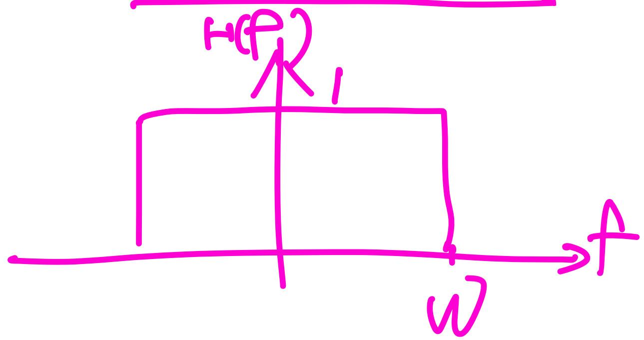


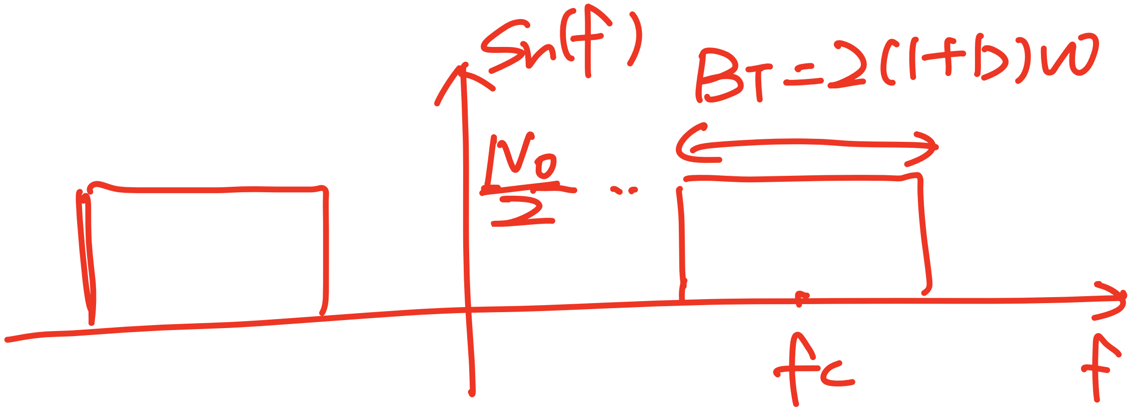
$$e_2(t) = k_D k_p m(t) + k_D \frac{n_s(t)}{A_c}$$

$$y_o(t) = k_D k_p m(t) + k_D \frac{n_s(t)}{A_c}$$

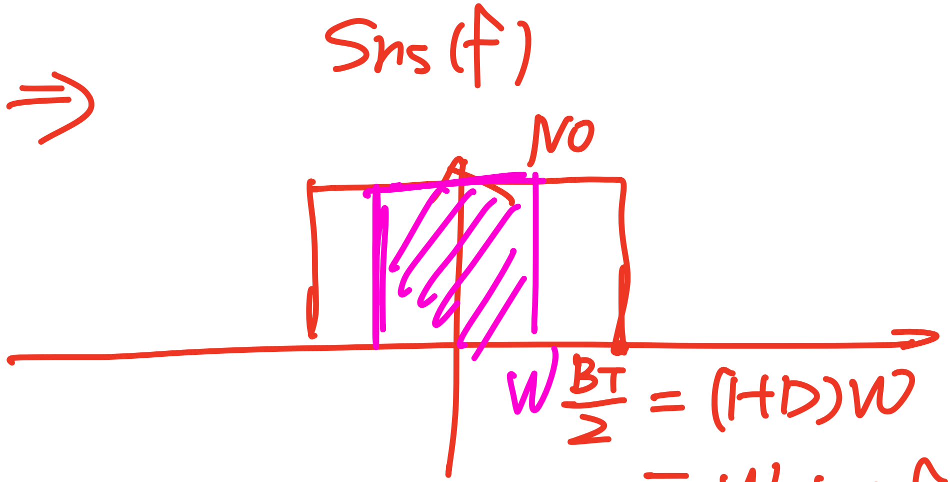
$|f| \le W$







$B_T = 2(1+D)W \Rightarrow$

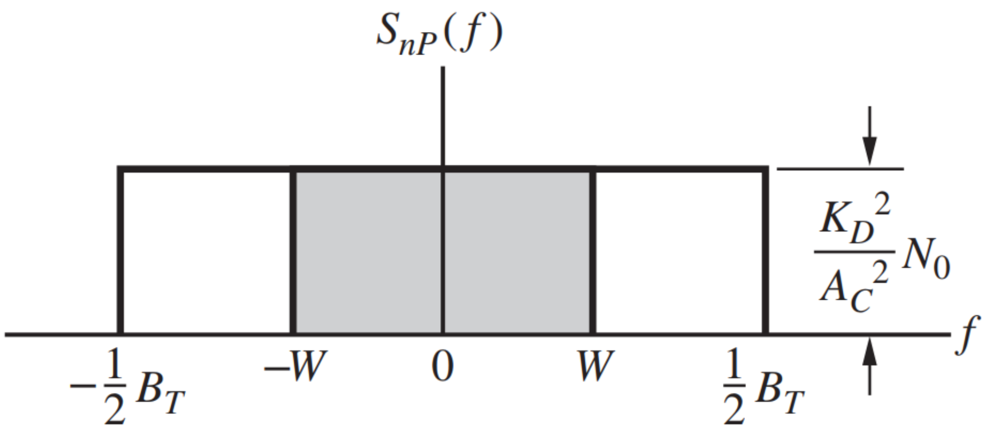


$$\frac{B_T}{2} = (1+D)W = W + \Delta f$$

$$SNR_o = \frac{\cancel{k_D^2} k_p^2 \langle m^2(t) \rangle}{\frac{\cancel{k_D^2}}{A_c^2} 2 N_o W} = \frac{A_c^2 k_p^2 P}{2 N_o W}$$

Noise in Phase Modulation

- PM Demodulator
  - Demodulated output of PM:
    - $y_{DP}(t) = K_D \psi(t) = K_D k_p m(t) + K_D \frac{n_s(t)}{A_c}$
    - Output signal power: $S_{DP} = K_D^2 k_p^2 \overline{m^2}$
    - Output noise power: $N_{DP} = 2\frac{K_D^2}{A_c^2} N_0 W$



- Postdetection SNR(output SNR): $SNR_D = \frac{K_D^2 k_p^2 \overline{m^2}}{2\frac{K_D^2}{A_c^2} N_0 W} = \frac{A_c^2 k_p^2 \overline{m^2}}{2 N_0 W}$

$D = \frac{\Delta f}{W}$

$B = 2(1+D)W = 2W + 2\Delta f$

Noise in Phase Modulation

- PM Demodulator
  - When SNRT is large

$\Delta\phi$

$k_p m(t) = \boxed{k_p|m(t)|_{max}} m_n(t)$

$\Delta f = \left|\frac{1}{2\pi}\frac{d\phi(t)}{dt}\right|_{max}$

$= \frac{1}{2\pi}\boxed{k_p|m(t)|_{max}}\left|\frac{dm_n(t)}{dt}\right|_{max}$

$\Delta\phi$

$$P_T = \frac{A_c^2}{2} \qquad SNR_T = \frac{P_T}{N_0 B_T}$$

Detection gain: $\frac{SNR_D}{SNR_T} = \frac{k_p^2 \overline{m^2} B_T}{W}$

$k_p \uparrow \{ \Delta\phi \uparrow \rightarrow \Delta f \uparrow \rightarrow B \uparrow$

Detection gain / F.O.M $\uparrow$

$$SNR_D = \frac{P_T k_p^2 \overline{m^2}}{N_0 W}$$

Figure of Merit: $\frac{SNR_D}{SNR_c} = k_p^2 \overline{m^2}$

$= \left(k_p|m(t)|_{max}\right)^2 \overline{m_n^2}$

$$SNR_c = \frac{P_T}{N_0 W}$$

For $k_p >> 1$, $B_T \approx 2\Delta f \propto k_p|m(t)|_{max}$

Tradeoff between bandwidth and noise performance in PM system

AM.

① Bandwidth $= 2W / W$

F.O.M

② Noise Perf. $= 1/\mu$

PM

① $k_p \uparrow \longrightarrow \Delta f \longrightarrow B \uparrow$

② $k_p \uparrow$ F.O.M. $= \underline{k_p^2}\ \overline{m^2} \uparrow$

① Noise Performance : WBPM > AM

② PM: Tradeoff.

Noise in Frequency Modulation

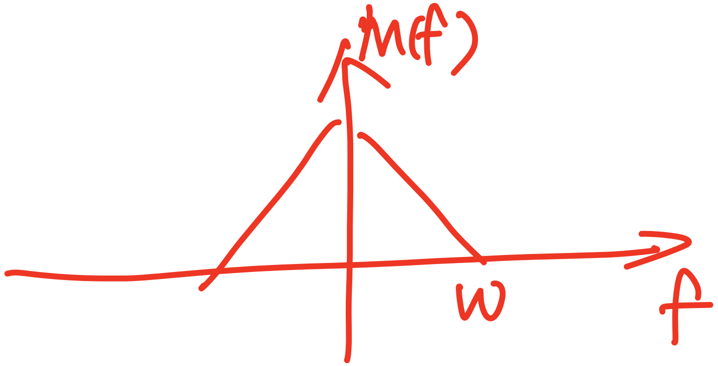
- FM Demodulator $SNR_T \uparrow$
  - $e_1(t) = R(t)\cos[2\pi f_c t + \theta + \psi(t)]$
    - Phase deviation: $\psi(t) = \phi(t) + \phi_e(t)$
    - Signal: $\phi(t) = 2\pi f_d \int^t m(\tau)d\tau$
    - Phase error: $\phi_e(t) = \frac{n_s(t)}{A_c}$
  - Demodulator output:
    - $y_D(t) = \frac{1}{2\pi} K_D \frac{d\psi(t)}{dt} = K_D f_d m(t) + \boxed{\frac{K_D}{2\pi A_c}\frac{dn_s(t)}{dt}}$
    - Output signal power: $S_{DF} = K_D^2 f_d^2 \overline{m^2}$
    - Noise at the output of discriminator:
      - $S_{nF}(f) = \left(\frac{K_D}{2\pi A_c}\right)^2 |j2\pi f|^2 S_{n_s}(f) = \frac{K_D^2}{A_c^2} N_0 f^2,\ |f| < \frac{1}{2} B_T$

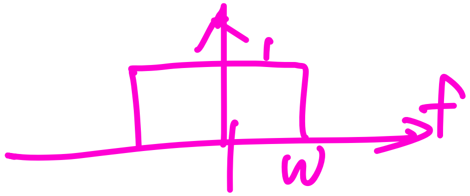
$x(t) \longrightarrow \boxed{\text{H(t)}} \longrightarrow y(t) = \frac{K_D}{2\pi A_c}\frac{dx(t)}{dt}$

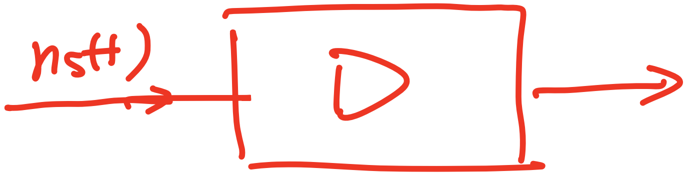
$$S_y(f) = \left|\frac{K_D}{2\pi A_c} \cdot j2\pi f\right|^2 S_x(f)$$

$$e_2(t) = k_D f_d m(t) + \frac{k_D}{2\pi A_c}\frac{dn_s(t)}{dt}$$

$$y_D(t) = k_D f_d m(t) + \bigcirc$$

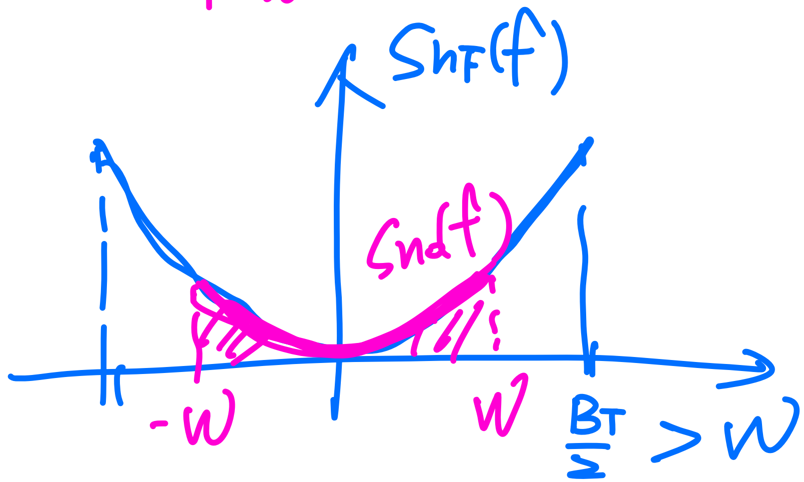


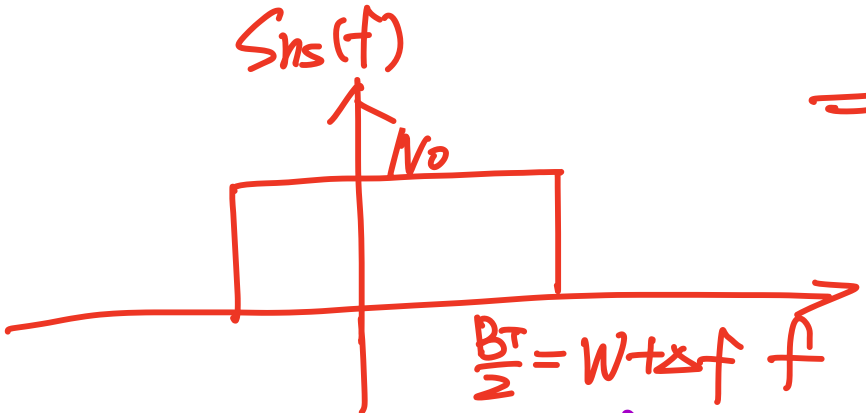




$$\frac{k_D}{2\pi A_c}\frac{dn_s(t)}{dt}$$

$$H(f) = \frac{k_D}{2\pi A_c} j2\pi f$$





$$\Longrightarrow \quad S_{nF}(f) = S_{ns}(f)\frac{k_D^2}{(2\pi)^2 A_c^2}|j2\pi f|^2$$

$$= \frac{N_0 k_D^2 f^2}{A_c^2} \qquad |f| \le \frac{B_T}{2}$$

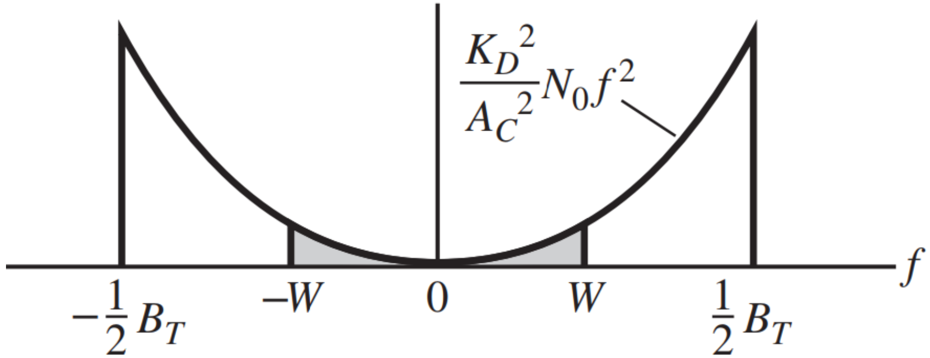
$$S_{nd}(f) = \frac{N_0 k_D^2 f^2}{A_c^2} \qquad |f| \le W$$

$$SNR_o = \frac{k_D^2 f_d^2 P}{\int_{-W}^{W} N_0 k_D^2 \ldots} = \frac{f_d^2 P}{2 N_0 \ldots} = \frac{3 A_c^2 f_d^2 P}{2 N_0 W^3}$$

$\int_{-W}^{W} \frac{N_0 K_D^2}{A_c^2} f^2 df$ $= \frac{2}{3}\frac{N_0}{A_c^2} W^3$ $2N_0 W$

Noise in Frequency Modulation

- FM Demodulator
  - Output noise:
    - – Noise at the output of discriminator:
      - » Power spectral density: $S_{nF}(f) = \frac{K_D^2}{A_c^2} N_0 f^2$, $|f| < \frac{1}{2} B_T$
    - – $\frac{B_T}{2} > W \rightarrow$ postdetection lowpass filter with bandwidth W to remove the out-of-band noise
    - – Output noise power: $N_{DF} = \frac{K_D^2}{A_c^2} N_0 \int_{-W}^{W} f^2 df = \frac{2K_D^2 N_0 W^3}{3A_c^2}$
  - Postdetection SNR (output SNR):
    - – $SNR_D = \frac{K_D^2 f_d^2 \overline{m^2}}{\frac{2K_D^2 N_0 W^3}{3A_c^2}} = \frac{3A_c^2 f_d^2 \overline{m^2}}{2N_0 W^3}$



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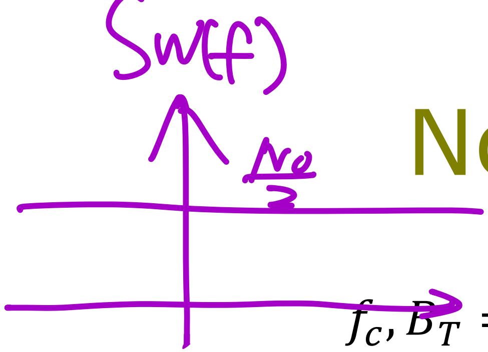
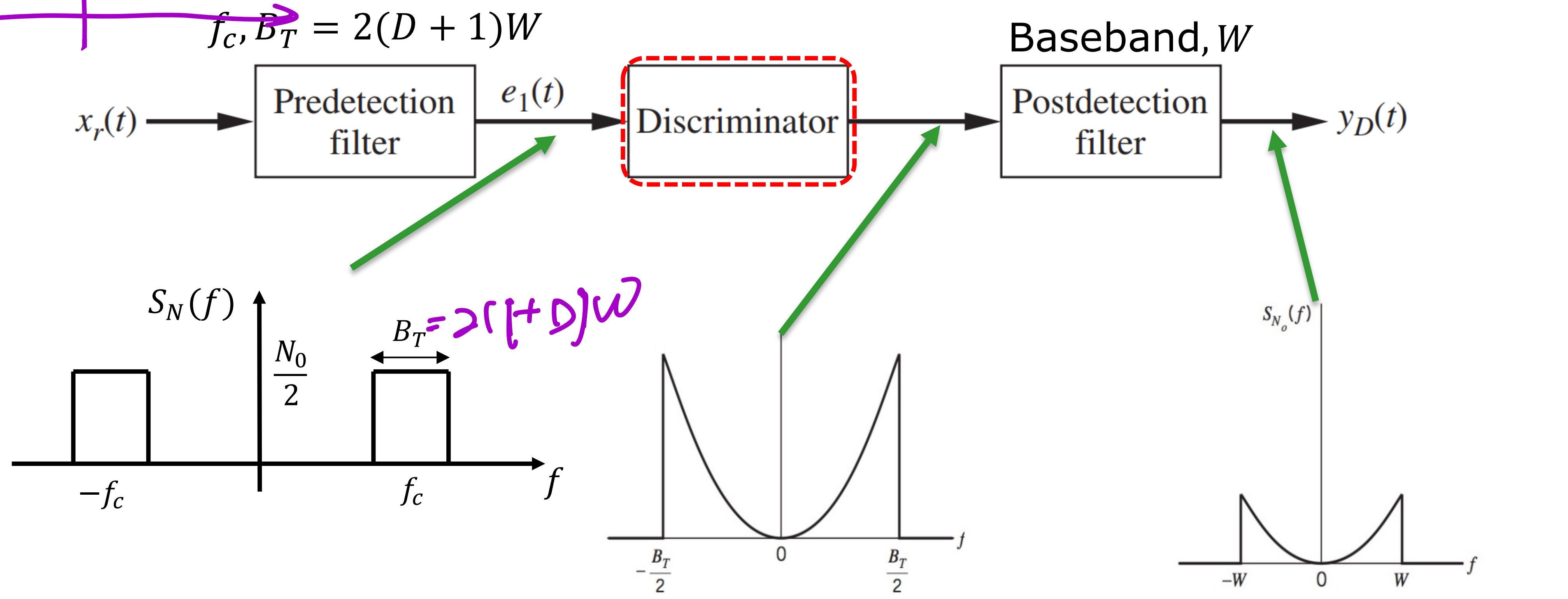


Noise in Frequency Modulation



$$S_{nF}(f) = \frac{K_D^2}{A_c^2} N_0 f^2, \; |f| < \frac{1}{2} B_T \qquad S_{N_o}(f) = \frac{K_D^2}{A_c^2} N_0 f^2, \; |f| < W$$

$D = \frac{\Delta f}{W}$

Noise in Frequency Modulation

- FM Demodulator
  - When SNR_T is large

$$f_d m(t) = \boxed{f_d |m(t)|_{max}}^{\Delta f} m_n(t)$$

$$\frac{f_d^2 \overline{m^2}}{W^2} = \left(\frac{f_d |m(t)|_{max}}{W}\right)^2 \overline{m_n^2} = D^2 \overline{m_n^2}$$

$$\left(\frac{\Delta f}{W}\right)^2$$

$$P_T = \frac{A_c^2}{2}$$

$$SNR_T = \frac{P_T}{N_0 B_T}$$

$$SNR_D = \frac{3 P_T f_d^2 \overline{m^2}}{N_0 W^3}$$

$$SNR_c = \frac{P_T}{N_0 W}$$

Detection gain: $\frac{SNR_D}{SNR_T} = \frac{3 f_d^2 \overline{m^2} B_T}{W^3} = \frac{3 D^2 2(1+D) \overline{m_n^2}}{\cancel{W^2}}$

$= 6D^2(D+1)\overline{m_n^2}$

Figure of Merit: $\frac{SNR_D}{SNR_c} = \frac{3 f_d^2 \overline{m^2}}{W^2}$

$= 3D^2 \overline{m_n^2}$

$D\uparrow \begin{cases} B\uparrow \\ D.G. \\ F.O.M.\uparrow \end{cases}$

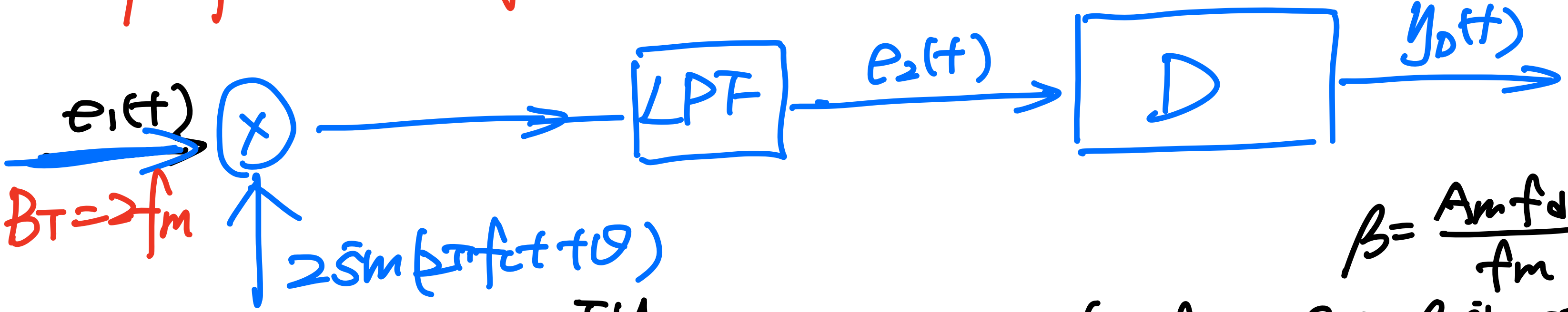
For D>>1, $B_T \approx 2DW$ $\Longrightarrow$ $\frac{SNR_D}{SNR_c} = \frac{3}{4}\left(\frac{B_T}{W}\right)^2 \overline{m_n^2}$

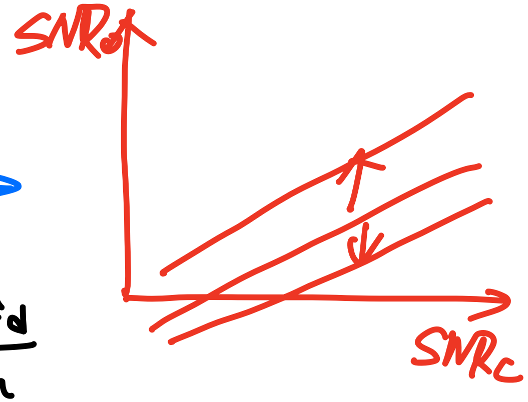
Tradeoff between bandwidth and noise performance in FM system

Noise in Frequency Modulation

- Comparison of FM and AM
  - When SNR$_T$ is large, and $m(t) = A_m \cos(2\pi f_m t)$
  - FM: Postdetection SNR (Output SNR):
    - $SNR_D = \frac{3A_c^2 f_d^2 \overline{m^2}}{2N_0W^3} = \frac{3A_c^2 f_d^2 A_m^2}{4N_0W^3} = \frac{3A_c^2 \Delta f^2}{4N_0W^3} = \frac{3A_c^2 \beta^2}{4N_0W}$ $SNR_C = \frac{\frac{1}{2}A_c^2}{N_0 W}$
    - Figure of merit: $\frac{SNR_D}{SNR_C} = \frac{3}{2}\beta^2$ $\boxed{\Delta f = f_d A_m, \beta = \frac{\Delta f}{W}}$
    - Bandwidth: $B_{FM} \approx 2\beta f_m$ $\beta \gg 1$ WBFM
  - AM: 100 percent modulation (a=1)
    - Figure of merit: $\frac{SNR_D}{SNR_C} = \frac{a^2\overline{m_n^2}}{1+a^2\overline{m_n^2}} = \frac{1}{3}$ (μ) $\frac{3}{2}\beta^2 > \frac{1}{3}$ $\Rightarrow \beta > 0.47$
    - Bandwidth: $B_{AM} = 2f_m$
  - Noise performance: FM>AM at the cost of excessive bandwidth.
  - FM: Exchange of bandwidth for improved noise performance.

Noise performance of NBFM ($\beta \ll 1$)



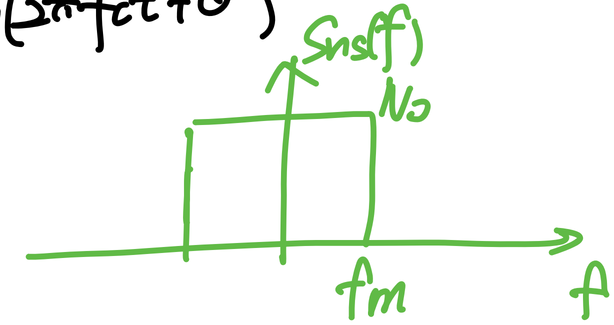


$$\beta = \frac{A_m f_d}{f_m}$$

$$m(t) = A_m \cos 2\pi f_m t \xrightarrow{FM} x_c(t) = A_c \cos\left(2\pi f_c t + \theta + \underbrace{\beta \sin 2\pi f_m t}_{\phi(t)}\right)$$

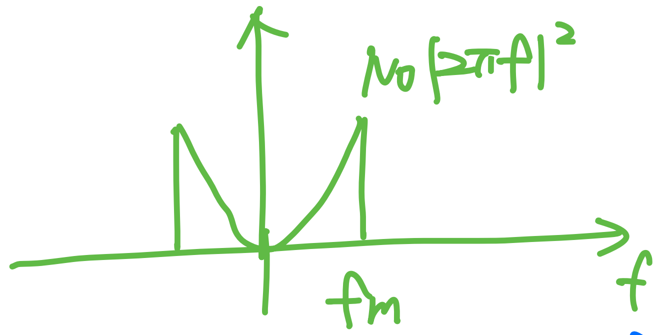
$$\beta \ll 1 \Longrightarrow x_c(t) = A_c \cos(2\pi f_c t + \theta) - A_c \sin(2\pi f_c t + \theta) \underbrace{\boxed{\beta \sin 2\pi f_m t}}_{\phi(t)}$$

$$e_1(t) = x_c(t) + n_c(t) \cos(2\pi f_c t + \theta) - n_s(t) \sin(2\pi f_c t + \theta)$$



$$e_2(t) = -A_c \underbrace{\boxed{\beta \sin 2\pi f_m t}}_{\phi(t)} - n_s(t)$$

$$y_D(t) = -A_c \underbrace{\boxed{\beta\, 2\pi f_m \cos 2\pi f_m t}}_{\frac{d\phi(t)}{dt}} - \frac{d n_s(t)}{dt}$$



$$SNR_o = \frac{\frac{1}{2} A_c^2 \beta^2 \cancel{4\pi^2} f_m^2}{\int_{-f_m}^{f_m} 4 N_0 \pi^2 f^2 \, df = \cancel{4} N_0 \cancel{\pi^2} \frac{2}{3} f_m^3} = \frac{\frac{1}{2} A_c^2 \beta^2}{\frac{2}{3} N_0 f_m}$$

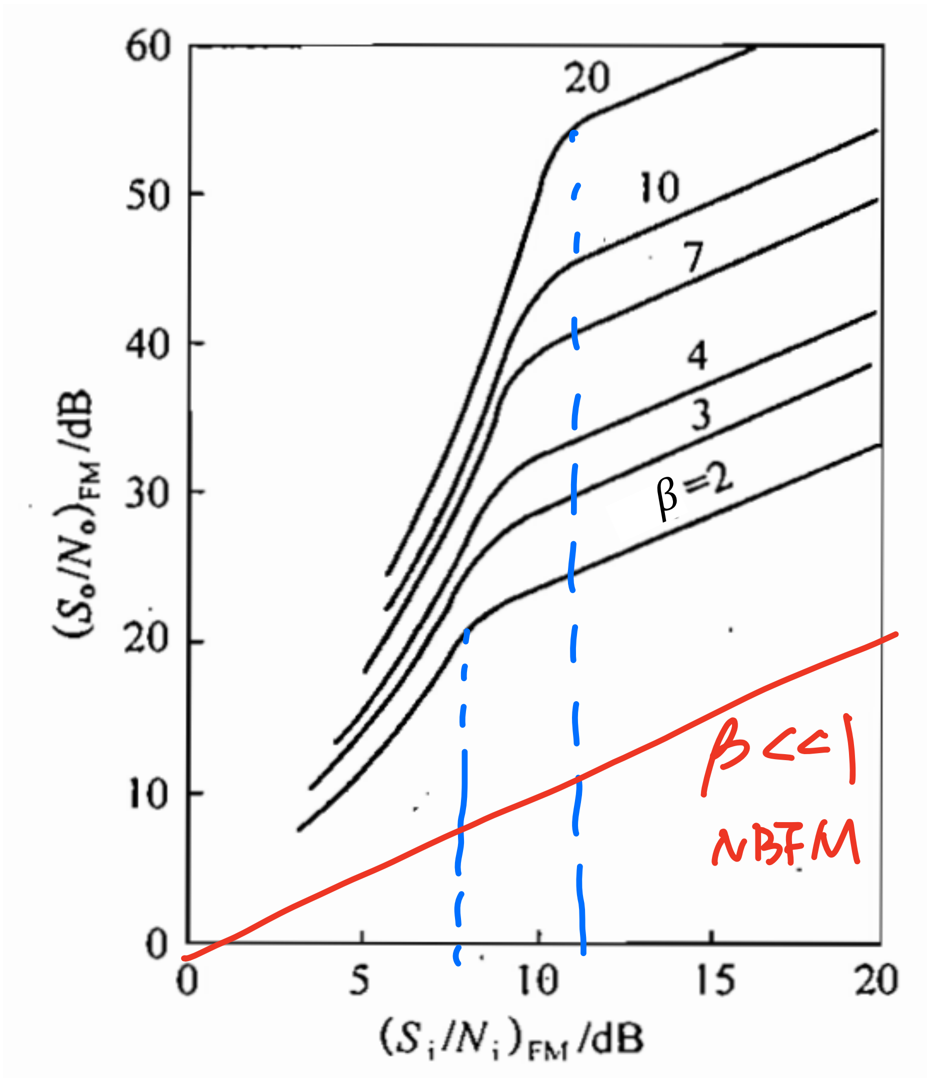
$$\frac{3}{2}\beta^2 > \frac{1}{3}$$

$$\beta > 0.471$$

$SNR_C = \frac{\frac{1}{2}A_c^2}{N_0 f_m}$ $F.O.M._{NBFM} = \frac{3}{2}\beta^2$

Noise in Frequency Modulation

- When SNRT is small (Threshold effect)

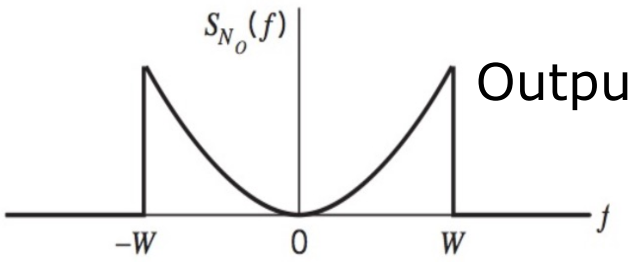


1.Threshold is related to β, which is 8dB~11dB, and it increases as β increases.
2. Above threshold, SNR_o increases linearly with SNR_i. Larger $\beta \rightarrow$ Larger SNR_o.
3. Below threshold, SNR_o will be significantly deteriorated when SNR_i decreases.
4. FM threshold reduction: Phase-locked loop demodulator (on the order of 2 to 3 dB)

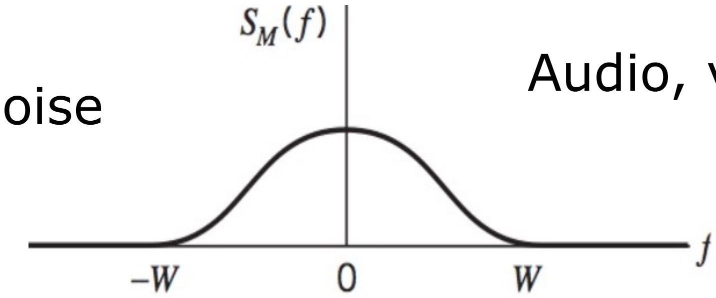
Noise in Frequency Modulation

预加重 去加重

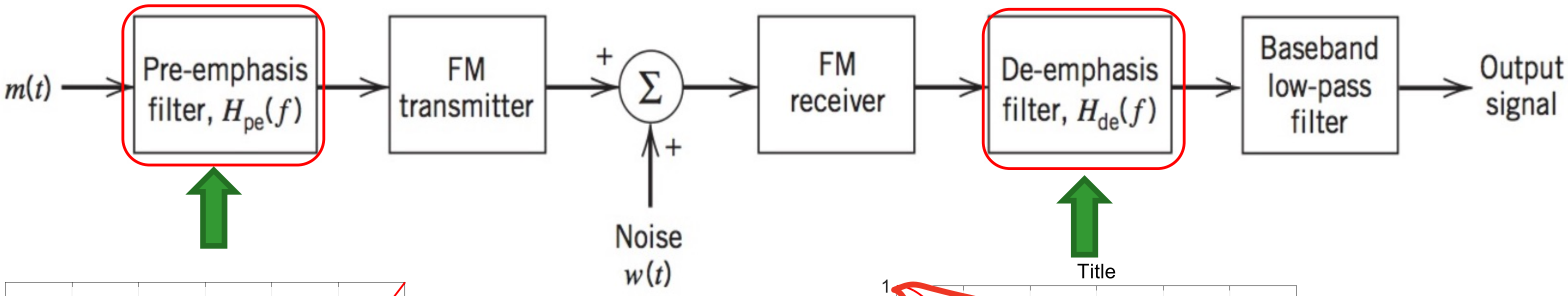
- Pre-emphasis and De-emphasis

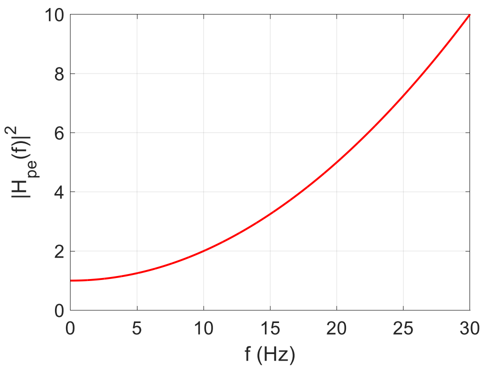


Output noise

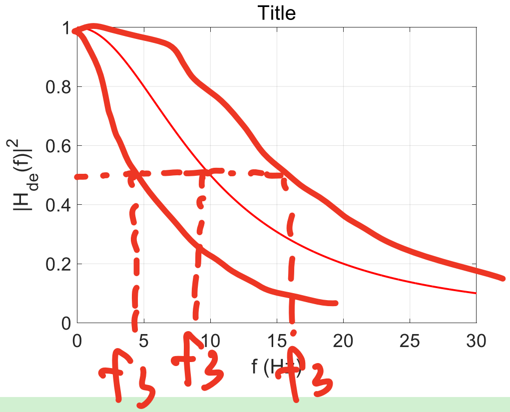


Audio, video signals





$$H_{pe}(f) = \frac{1}{H_{de}(f)}$$



$|H_{de}(f)| = \frac{1}{\sqrt{1+\left(\frac{f}{f_3}\right)^2}}$, f_3 is the 3dB frequency

Pre-emphasis and de-emphasis: message signal unchanged, effectively increase the output SNR of the FM system

Noise in Frequency Modulation

- Pre-emphasis and de-emphasis
  - Total noise power output: $\int_{-W}^{W} \frac{K_D^2}{A_c^2} N_0 f^2 \frac{1}{1+\left(\frac{f}{f_3}\right)^2} df$
  - $N_{DF} = \int_{-W}^{W} |H_{de}(f)|^2 S_{nF}(f) df$

    $= \frac{K_D^2}{A_c^2} N_0 f_3^2 \int_{-W}^{W} \frac{f^2}{f_3^2+f^2} df = 2\frac{K_D^2}{A_c^2} N_0 f_3^3 \left(\frac{W}{f_3} - \tan^{-1}\frac{W}{f_3}\right) = 2\frac{K_D^2}{A_c^2} N_0 W f_3^2 \left(1 - \frac{f_3}{W}\tan^{-1}\frac{W}{f_3}\right)$
  - If $f_3 \ll W$, $N_{DF} \approx 2\frac{K_D^2}{A_c^2} N_0 W f_3^2$

    $P_S = K_D^2 f_d^2 \overline{m^2(t)}$

    $SNR_C = \frac{\frac{1}{2}A_c^2}{N_0 W}$
  - Output SNR: $\mathrm{SNR_D} = \frac{A_c^2 f_d^2 \overline{m^2}}{2N_0 W f_3^2}$
  - Figure of merit: $\frac{SNR_D}{SNR_C} = \frac{f_d^2 \overline{m^2}}{f_3^2}$

    Without emphasis: $\frac{SNR_D}{SNR_C} = \frac{3 f_d^2 \overline{m^2}}{W^2}$

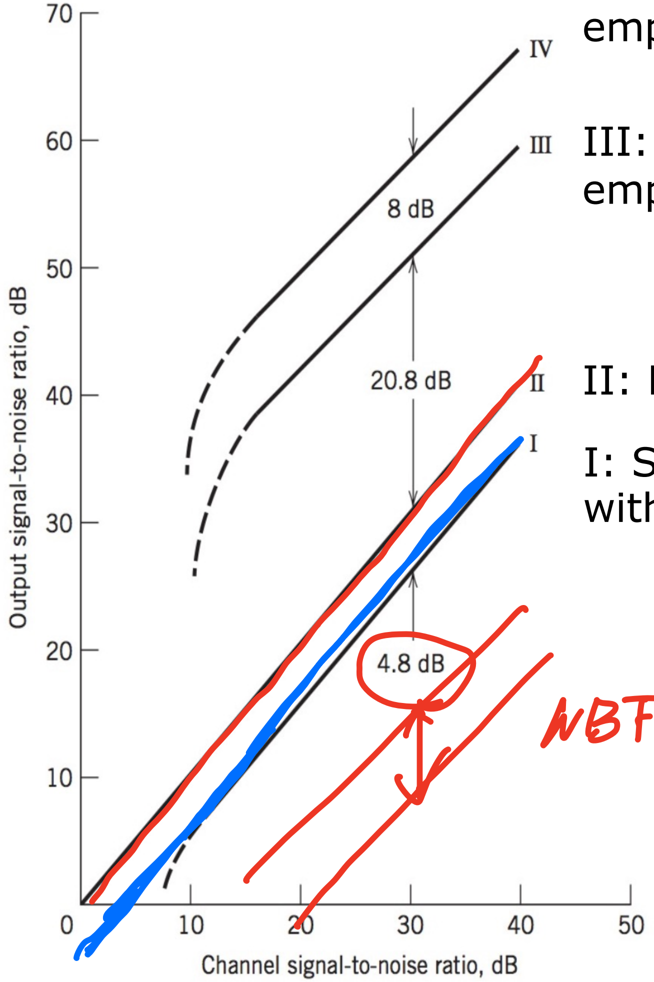
    $\frac{F.O.M(w)}{F.O.M.(w/o)} \propto \frac{W^2}{f_3^2}$
  - When $f_3 \ll W$, the improvement gained through the use of pre-emphasis and de-emphasis is about $\frac{W^2}{f_3^2}$, which can be very significant in noisy environment.
  - Emphasis is widely used in the commercial FM radio transmission and reception.

Comparison of CW Modulation Systems

Modulation system	Output SNR	Transmission Bandwidth	Equipment Complexity	Typical Applications
Baseband	$\frac{P_T}{N_0 W}$	W		
DSB with coherent demodulation	$\frac{P_T}{N_0 W}$	$2W$	Medium	Analog instrumentation, multiplexing
SSB with coherent demodulation	$\frac{P_T}{N_0 W}$	W	Complicated	Point-to-point voice, multiplexing
AM with envelope detection (above threshold) or AM with coherent demodulation	$\mu \frac{P_T}{N_0 W}$ ($\frac{1}{3}\frac{P_T}{N_0 W}$ if a=1)	$2W$	Simple	Broadcast radio, point-to-point voice
PM above threshold	$k_p^2 \overline{m^2} \frac{P_T}{N_0 W}$ $(\frac{1}{2}\beta^2 \frac{P_T}{N_0 W})$	$2(D+1)W$ $(2(\beta+1)f_m)$	Medium	Telemetry, digital data
FM above threshold (without preemphasis)	$3D^2 \overline{m_n^2} \frac{P_T}{N_0 W}$ $(\frac{3}{2}\beta^2 \frac{P_T}{N_0 W})$	$2(D+1)W$ $(2(\beta+1)f_m)$	Medium	Broadcast radio, mobile radio
FM above threshold (with preemphasis)	$D^2 \overline{m_n^2} \frac{W^2}{f_3^2} \frac{P_T}{N_0 W}$ $(\frac{1}{2}\beta^2 \frac{W^2}{f_3^2} \frac{P_T}{N_0 W})$	$2(D+1)W$ $(2(\beta+1)f_m)$	Medium	Broadcast radio, mobile radio

Comparison of CW Modulation Systems

- Sinusoidal modulating wave, same baseband SNR



IV: FM with $\beta = 5$ and 13 dB emphasis improvement

III: FM with $\beta = 2$ and 13 dB emphasis improvement

$\frac{3}{2}\beta^2$ 7.8dB

(SSB,VSB)

II: DSB-SC F.O.M.=1

I: Standard AM with a=1.

$\frac{1}{3}$

WBFM(β<<1)

Noise performance:
WBFM>DSB(SSB,VSB)>AM

Frequency Efficiency:
SSB(VSB)>DSB(AM)>WBFM

Equipment Complexity
AM>DSB (WBFM,VSB)>SSB

Thanks for your kind attention!

Questions?

RP

- $X(t, \xi)$
- $f_{X(t)}(x)$
  - E
  - Var
  - (t)
- $f_{X(t)X(t+\tau)}(x_1, x_2) \Longrightarrow R_{X(t)X(t+\tau)}(x_1, x_2)$
- stationary (WSS)
- ergodic $\boxed{\text{Power}}$
- $R_{XY}(\tau)$
- PSD
- frequency. $S_X(f) \longleftrightarrow R_X(\tau)$
- $X(t) \longrightarrow \boxed{LTI} \longrightarrow Y(t)$
  - E, Var
  - $R_Y(\tau)$
  - $S_Y(f) = S_X(f)|H(f)|^2$

Noise

- WGN
  - PSD $\frac{N_0}{2}$
  - $R_n(\tau)$
- LP Noise
- Narrow band Noise
  - $n_c(t)$ $n_s(t)$: PSD, Power, $R_{n_c n_s}(\tau)$
    - Cross PSD, E, Var
  - $R_n(t), \phi_n(t)$:
    - Rayleigh
    - Uniform

Modulation

$m(t) \rightarrow \boxed{\text{Mod}} \xrightarrow{x_c(t)} \boxed{\text{DeM}} \rightarrow y_D(t)$

- Time
- freq.

Linear Mod (AM)

- DSB-SC → $m(t)c(t)$
  - $LP(2x_c(t)\cos 2\pi f_c t)$
- AM. → $A_c(1 + a\,m_n(t))\cos 2\pi f_c t$
  - $0 < a \leq 1$, $\mu = \dfrac{a^2 \overline{m_n^2}}{1 + a^2 \overline{m_n^2}}$
  - E.D.
- SSB → Sideband Filter / P.S.
- VSB → $W \leq B \leq 2W$ $\boxed{\text{VSB Filter}}$

Nonlinear Mod (FM, PM)

$$x_c(t) = A_c \cos(2\pi f_c t + \phi(t))$$

$$\theta_i(t) = \underline{\qquad\qquad} \text{ (rad)}$$

$$f_i(t) = f_c + \frac{1}{2\pi}\frac{d\phi(t)}{dt} \text{ (Hz)}$$

- FM: $\dfrac{1}{2\pi}\dfrac{d\phi(t)}{dt} = f_d m(t)$
  - $\beta = \dfrac{\Delta f}{f_m} = \dfrac{f_d A_m}{f_m}$
- PM $\phi(t) = k_p m(t)$
  - $\beta = k_p A_m$

$\beta \ll 1$ NBFM (AM)

$\beta \gg 1$ WBFM $J_n(\beta)$

$$B = 2(1+\beta) f_m$$

$$m(t) = A_m \cos(2\pi f_m t)$$

$m(t)$, $D = \frac{\Delta f}{W} = \frac{f_d \cdot |m(t)|_{max}}{W}$

$B = 2(1+D)W$