Statistics 360: Advanced R for Data Science

MARS, part II

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Recursive partitioning

- In the week 4 exercises we will implement the forward part of an algorithm called recursive partitioning.
- Friedman (1991) recasts recursive partitioning in terms of a forward stepwise regression procedure that selects products of mirror-image step functions $I((x_v - t) \geq 0) = I(x_v \geq t)$ and $I(-(x_v - t) \geq 0) = I(x_v \leq t)$.
- Exercise: draw these two step functions to see why they are mirror image.
- Note: I think the first mirror-image step function should be $I(x_v > t)$, but will not make any changes to the discussion in the paper.

Region splitting as a product of step functions

- Take the case of two covariates and a region $R = [a_1, b_1] \times [a_2, b_2]$.
- Claim: R is the set of points (x_1, x_2) such that the basis function

$$B_R(x) = I(x_1 \geq a_1) \times I(x_1 \leq b_1) \times I(x_2 \geq a_2) \times I(x_2 \leq b_2) > 0$$

- Splitting R on variable x_v at point $t \in [a_v, b_v]$ means removing a "parent" basis function $B_R(x)$ from the model and replacing it with two "children" basis functions $B_R(x)I(x_v \leq t)$ and $B_R(x)I(x_v \geq t)$.

Recursive partitioning forward algorithm

- See page 11 of the paper.
- Outer loop 1 over the number of model terms, from 1 to some maximum number
  - Outer loop 2 over parent basis functions to replace by splitting
    - Inner loops to choose variables v, splits t in the region where $B_R(x) > 0$, and coefficients for $B_R(x)I(x_v \leq t)$ and $B_R(x)I(x_v \geq t)$ to minimize a LOF criterion. (This is like our recursive partitioning.)

Week 5 lab

- Re-implement recursive partitioning with the forward stepwise regression algorithm outlined above.
- A more detailed breakdown of the tasks will follow.

MARS generalization

- Replace the step functions with hinge functions $h(t - x_v)$ and $h(x_v - t)$, where $h(x) = \max(0, x)$.
- Do not remove a parent basis function, just add pairs of children.
- Restrict the product that defines a basis function to distinct variables; i.e., no variable appears twice in the product.

MARS forward algorithm

- See page 17 of the paper.
- Outer loop 1 over the number of model terms, M, from 1 to some maximum number
    - Outer loop 2 over parent basis functions B_m to generate children
        - Inner loops over variables v not part of B_m, splits t such that B_m is positive for $x_v = t$ and coefficients for the child basis functions to minimize a LOF criterion.