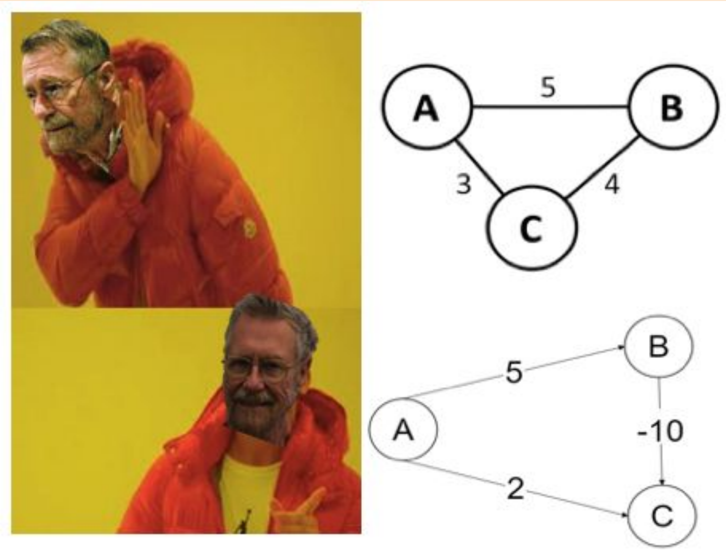
Bellman–Ford Algorithm



Presented by Group 4:
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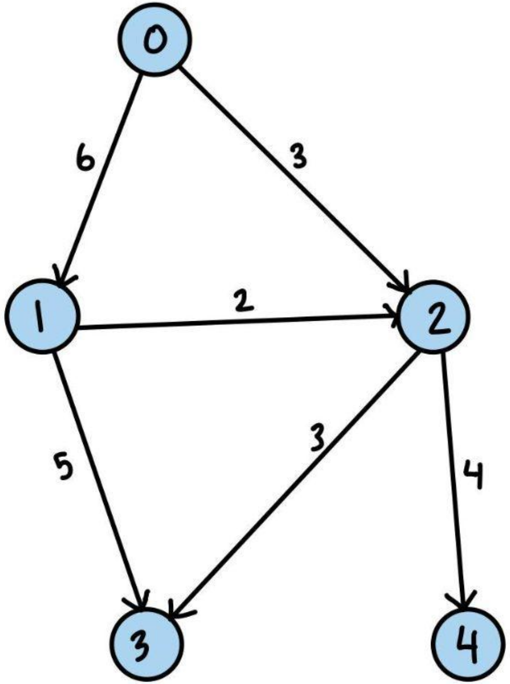
Algorithm Introduction

- Shortest path algorithm
- Performs shortest path traversal on a source node from a weighted digraph
- Edges can have negative weights and the algorithm can detect negative weight cycles
- Real life application: Negative weights can exist in an transportation graph or a discount or rebate is offered for a specific flight or route

Review



Node	MinDist	lastStep
0	0	[0,0]
1	6	[0, 1]
2	3	[0, 2]
3	6	[2, 3]
4	7	[2,4)

Dijkstra's Algorithm

BMF:

1) Initalize all weights to ∞

2) Source = 0

3) Relax all edges - Inner loop runs E times

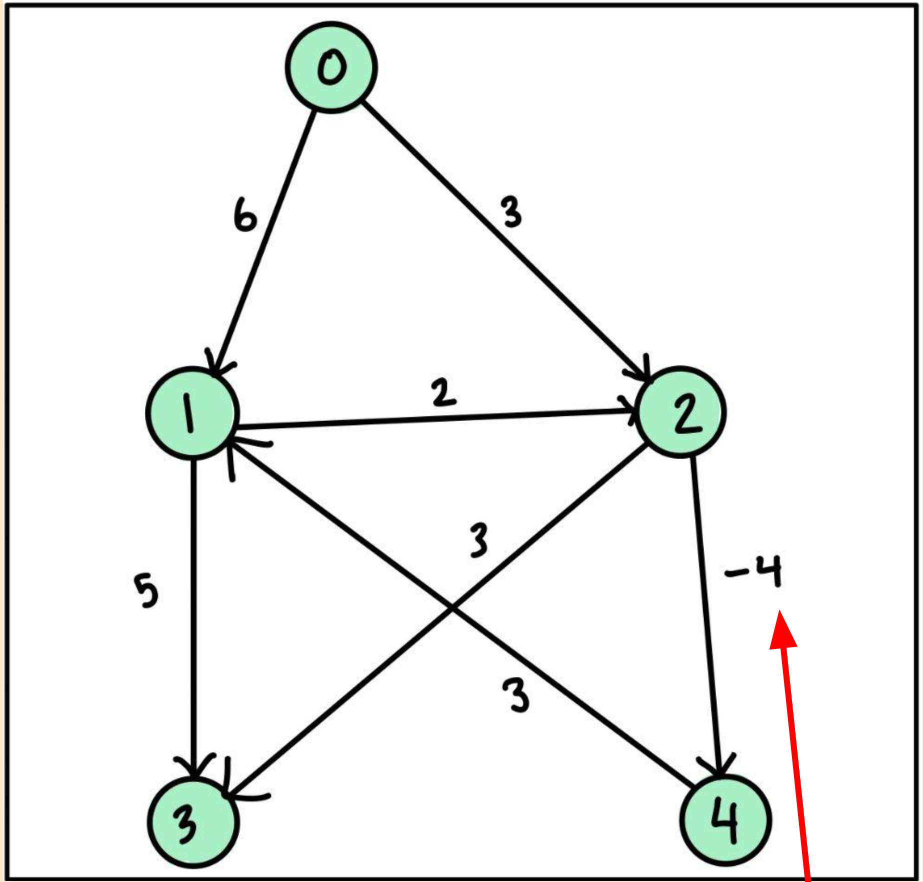
4) Outer loop runs V-1 times

5) Negative weight check

Algorithm should detect shortest path in V-1 loops

Otherwise:

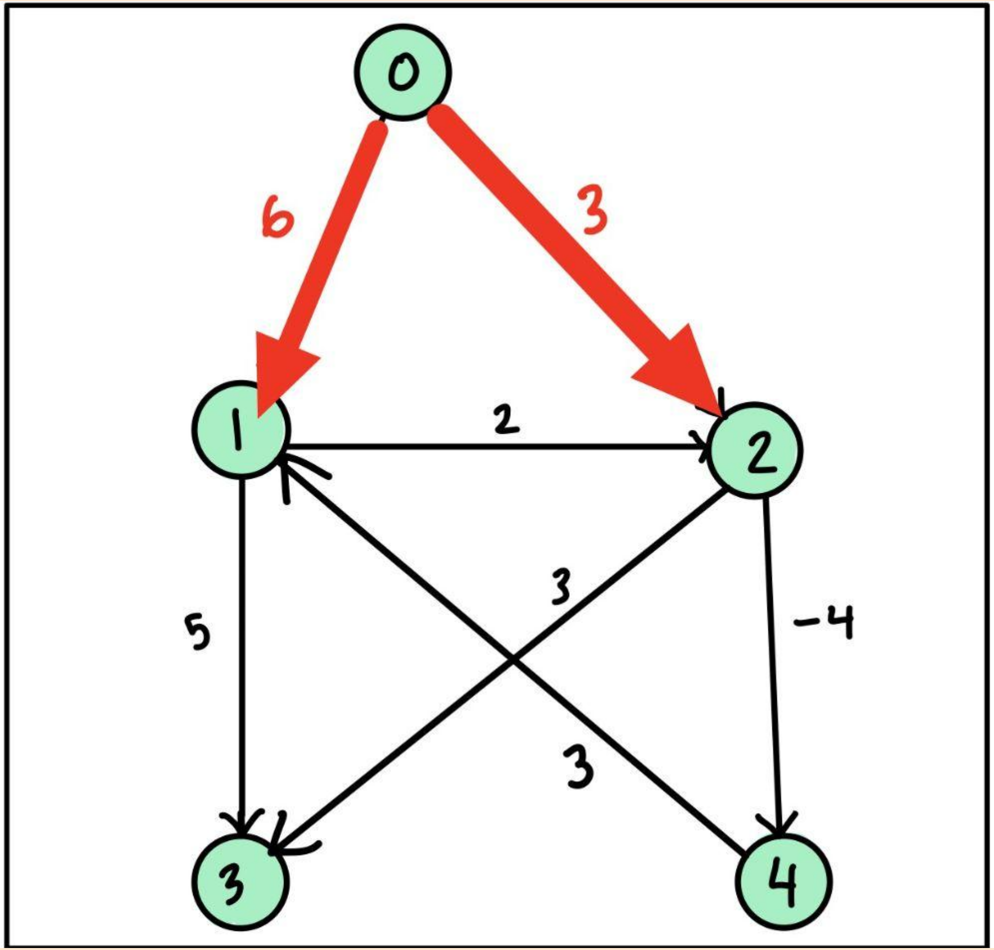
A negative cycle exists



New: Handles negative edge weights

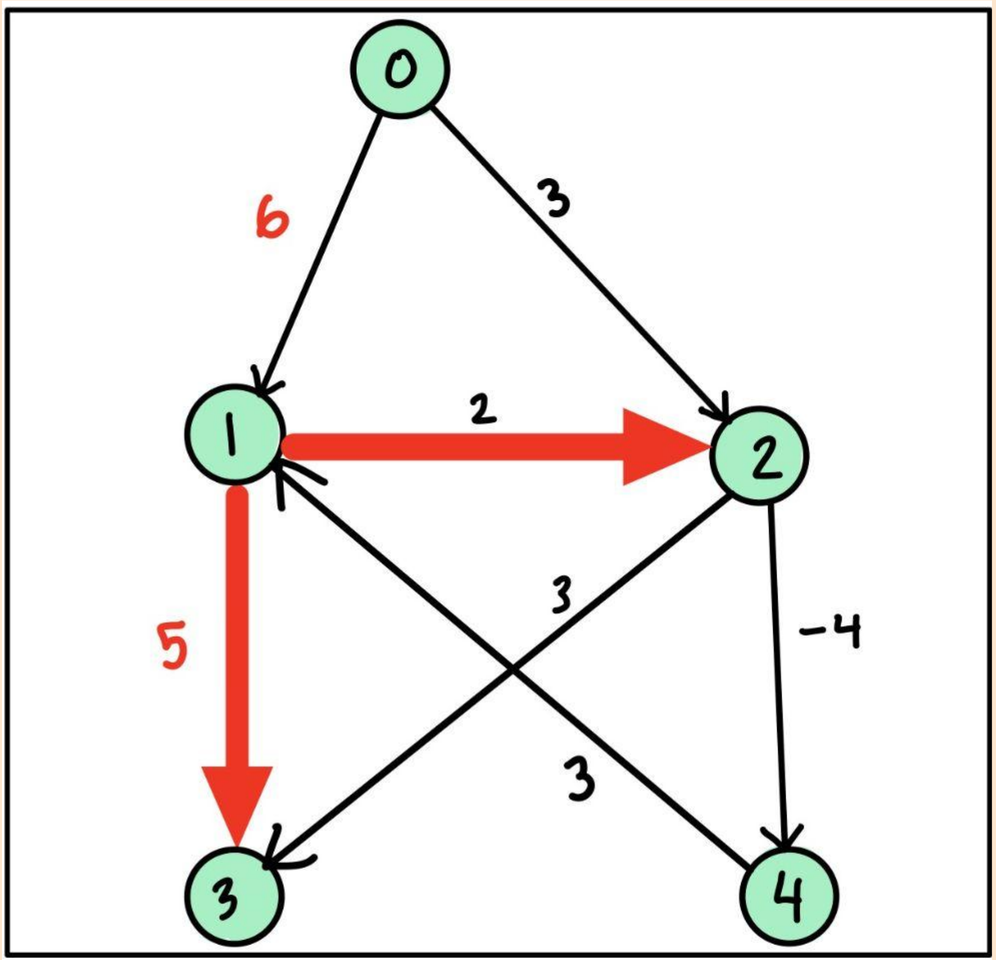
Initialization

Edges	Node	Min Dist
	0	0
[0,1] [4,1]	1	∞
[0,2] [1,2]	2	∞
[1,3] [2,3]	3	∞
[2,4]	4	∞



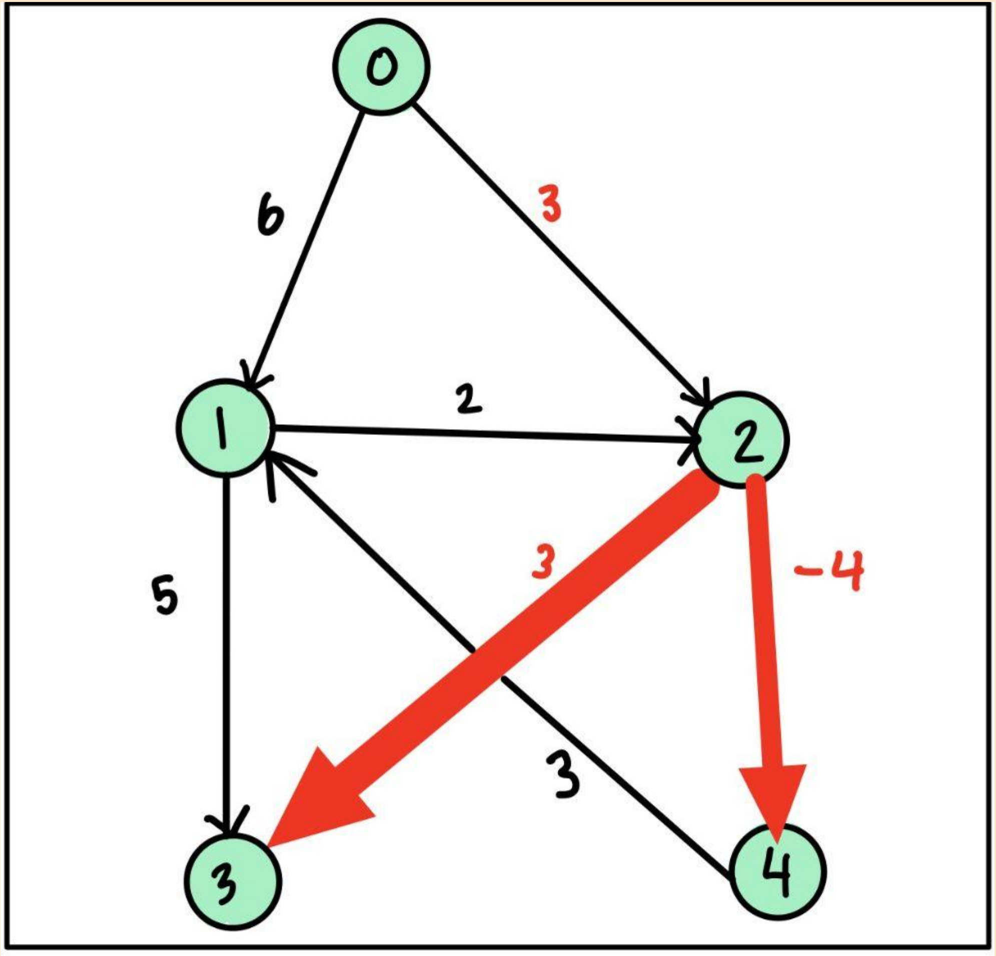
1st Iteration

Edges	Node	Min Dist
	0	0
[0,1] [4,1]	1	6
[0,2] [1,2]	2	3
[1,3] [2,3]	3	∞
[2,4]	4	∞



1st Iteration

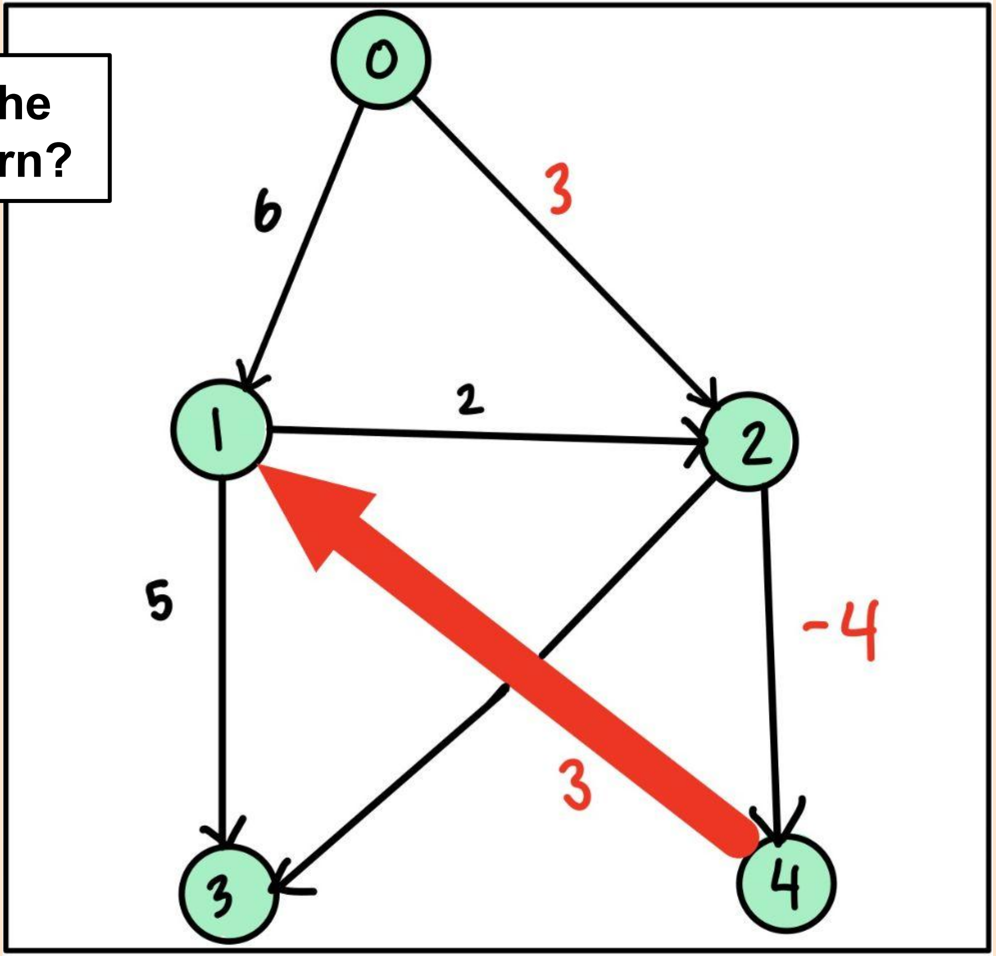
Edges	Node	Min Dist
	0	0
[0,1] [4,1]	1	6
[0,2] [1,2]	2	3
[1,3] [2,3]	3	11
[2,4]	4	∞



1st Iteration

Edges	Node	Min Dist
	0	0
[0,1] [4,1]	1	6
[0,2] [1,2]	2	3
[1,3] [2,3]	3	6
[2,4]	4	-1

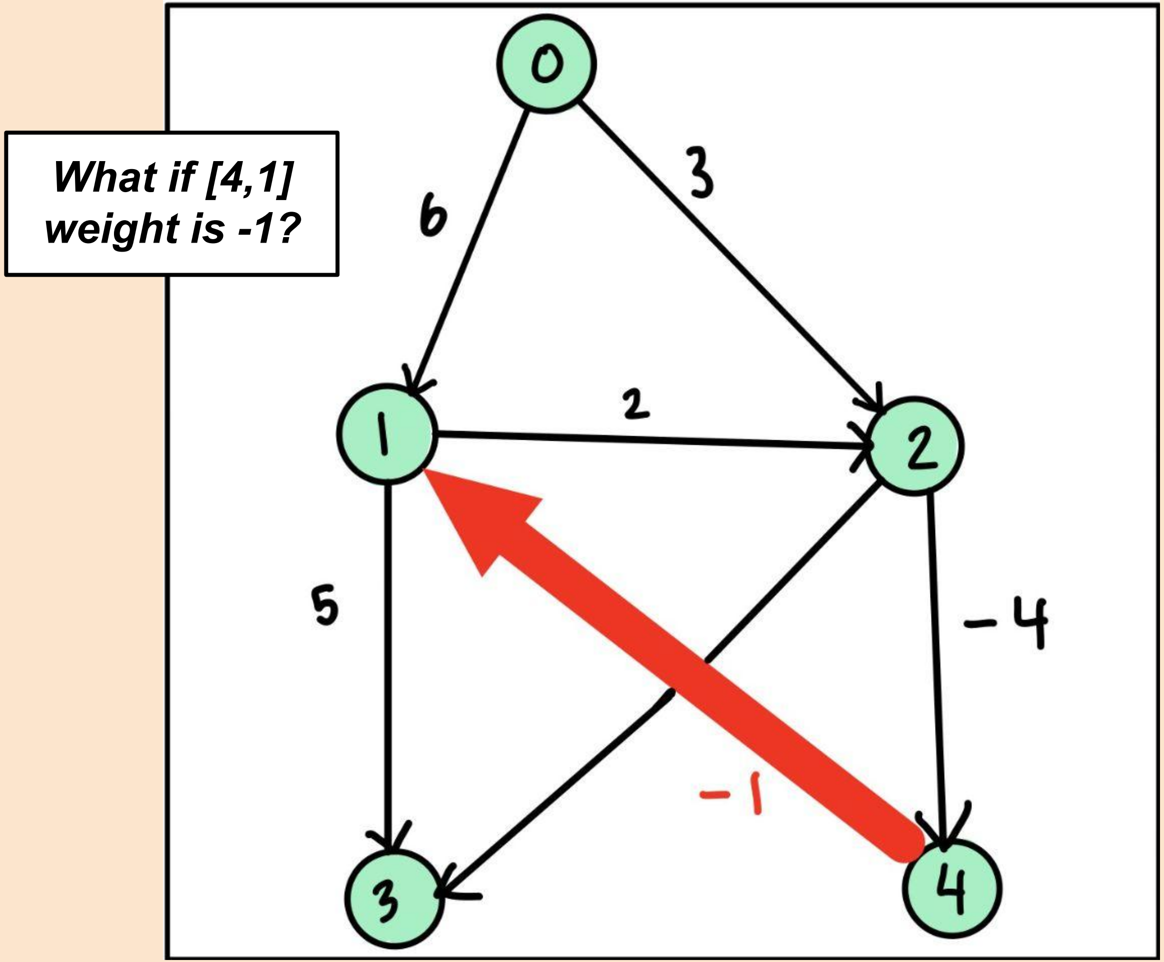
See the pattern?



1st Iteration

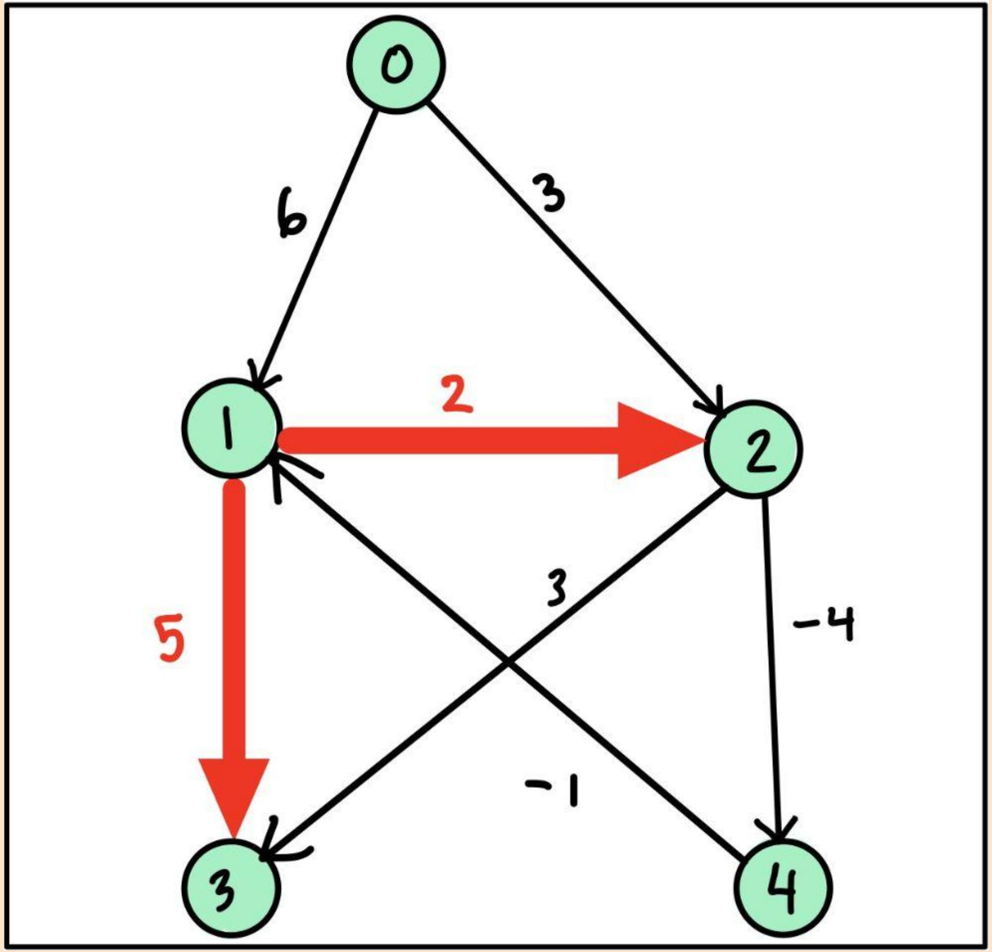
Edges	Node	Min Dist
	0	0
[0,1] [4,1]	1	2
[0,2] [1,2]	2	3
[1,3] [2,3]	3	6
[2,4]	4	-1

Now let's look at a <u>bad</u> implementation…



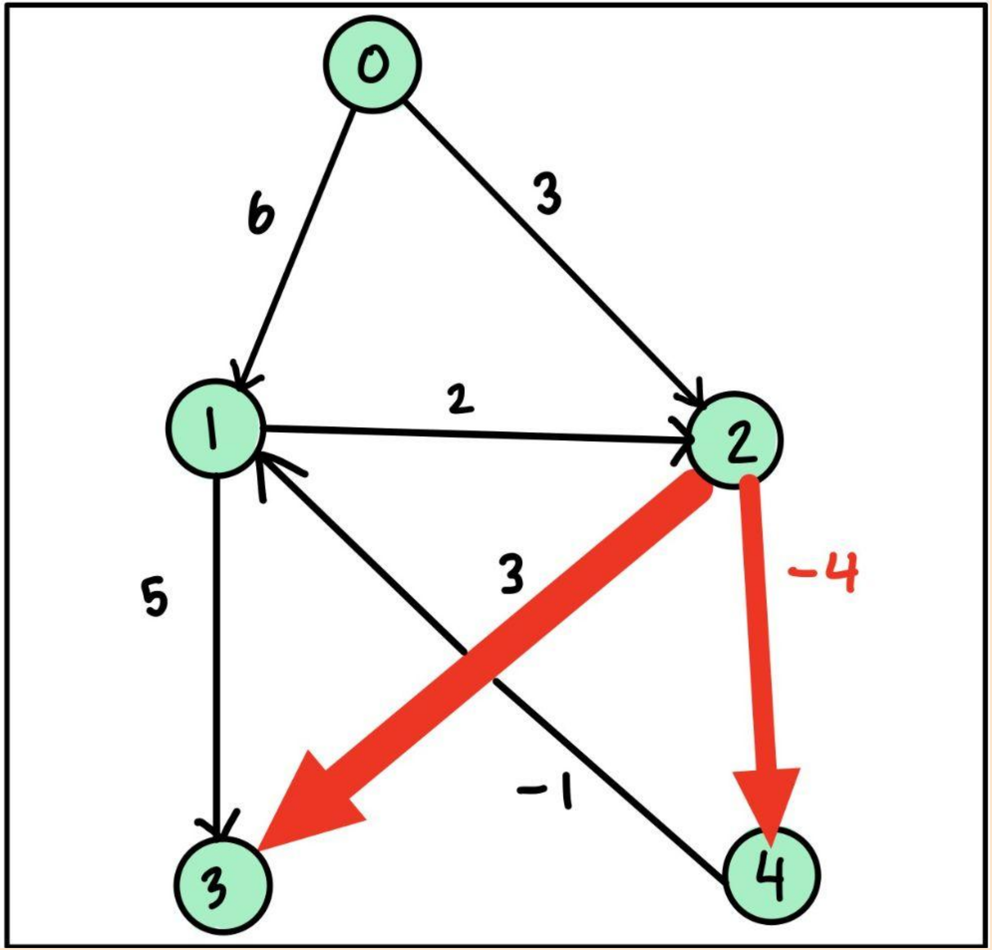
1st Iteration

Edges	Node	Min Dist
	0	0
[0,1] [4,1]	1	-2
[0,2] [1,2]	2	3
[1,3] [2,3]	3	6
[2,4]	4	-1



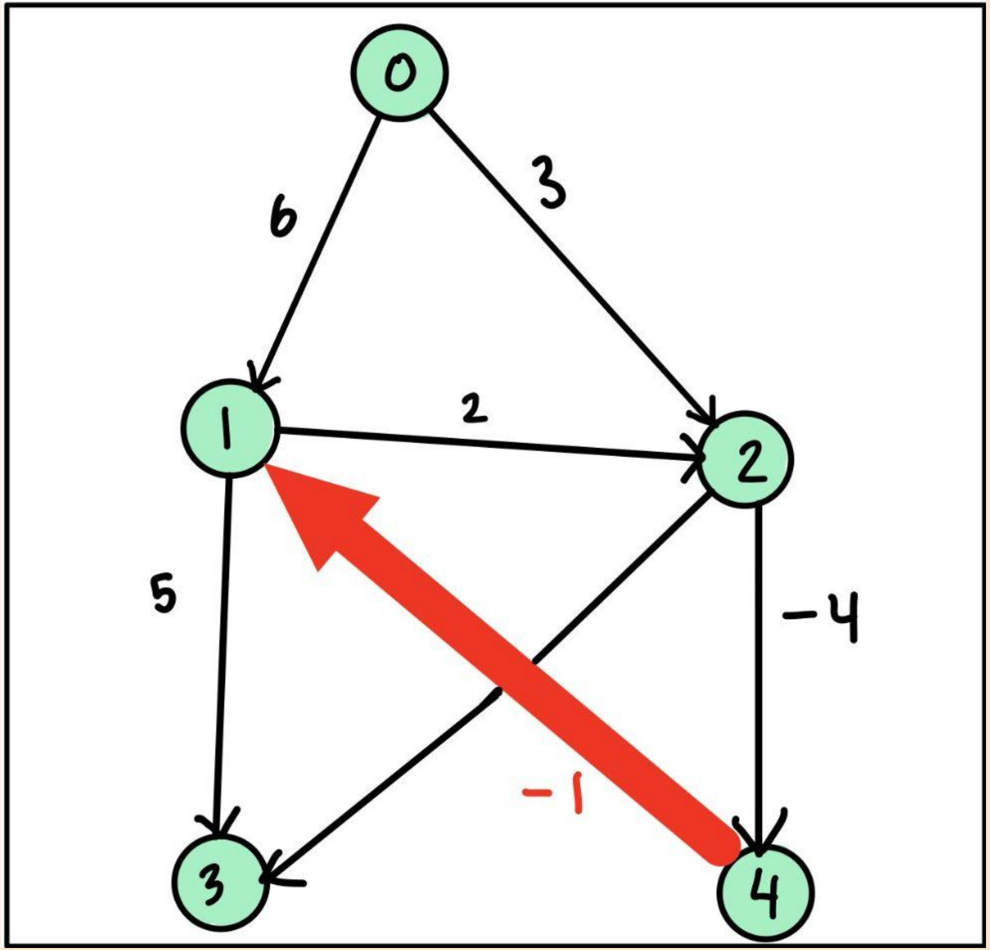
2nd Iteration

Edges	Node	Min Dist
	0	0
[0,1] [4,1]	1	-2
[0,2] [1,2]	2	0
[1,3] [2,3]	3	3
[2,4]	4	-1



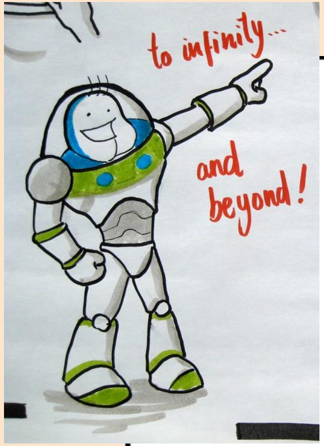
2nd Iteration

Edges	Node	Min Dist
	0	0
[0,1] [4,1]	1	-2
[0,2] [1,2]	2	0
[1,3] [2,3]	3	3
[2,4]	4	-4

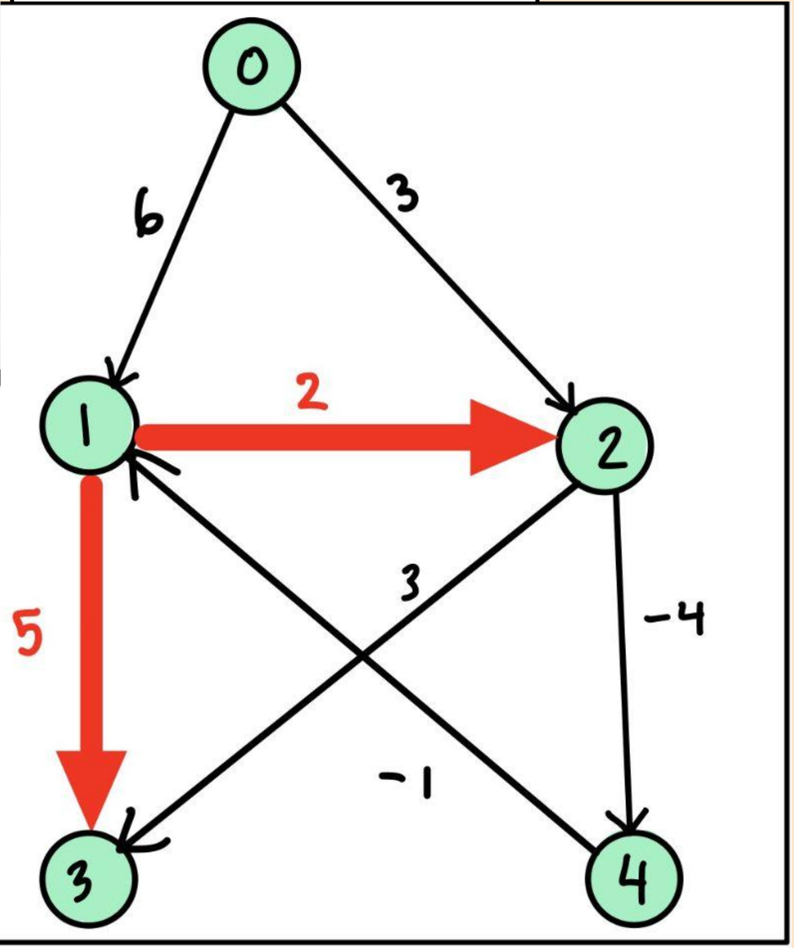


2nd Iteration

Edges	Node	Min Dist
	0	0
[0,1] [4,1]	1	-5
[0,2] [1,2]	2	0
[1,3] [2,3]	3	3
[2,4]	4	-4



negative weight cycle



3rd Iteration

Edges	Node	Min Dist
	0	0
[0,1] [4,1]	1	-5
[0,2] [1,2]	2	-3
[1,3] [2,3]	3	0
[2,4]	4	-4

Bellman-Ford Pseudocode:

```
function BellmanFord(vertices, edges, source):
    // Step 1: Initialize distances from source to all other vertices as infinity
    dist = {}
    for each vertex in vertices:
        dist[vertex] = infinity
    dist[source] = 0

    // Step 2: Relax edges repeatedly
    for i from 1 to |vertices|-1:
        for each edge (u, v, w) in edges:
            if dist[u] + w < dist[v]:
                dist[v] = dist[u] + w

    // Step 3: Check for negative-weight cycles
    for each edge (u, v, w) in edges:
        if dist[u] + w < dist[v]:
            throw "Graph contains a negative-weight cycle"

    return dist
```

Time and Space Complexity

- Worst case time complexity: **O (V^3)**

- Average case time complexity: **O(|V| * |E|)**

- Best case time complexity: **O(E)**

- Space Complexity: **O(V + E) for all cases**

Thanks for learning with us!

References

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