

ZDT test instances

ZDT1

$$\begin{aligned}g(x) &= 1 + 9(\sum_{i=2}^n x_i) / (n-1) \\F_1(x) &= x_1 \\F_2(x) &= g(x)[1 - \sqrt{x_1/g(x)}] \quad x \in [0,1]\end{aligned}$$

ZDT2

$$\begin{aligned}g(x) &= 1 + 9(\sum_{i=2}^n x_i) / (n-1) \\F_1(x) &= x_1 \\F_2(x) &= g(x)[1 - (x_1/g(x))^2] \quad x \in [0,1]\end{aligned}$$

ZDT3

$$\begin{aligned}g(x) &= 1 + 9(\sum_{i=2}^n x_i) / (n-1) \\F_1(x) &= x_1 \\F_2(x) &= g(x)[1 - \sqrt{x_1/g(x)} - x_1/g(x)\sin(10\pi x_1)] \quad x \in [0,1]\end{aligned}$$

ZDT4

$$\begin{aligned}g(x) &= 91 + \sum_{i=2}^n [x_i^2 - 10\cos(4\pi x_i)] \\F_1(x) &= x_1 \\F_2(x) &= g(x)[1 - \sqrt{x_1/g(x)}] \quad x_1 \in [0,1], \quad x_i \in [-5,5]\end{aligned}$$

ZDT6

$$\begin{aligned}g(x) &= 1 + 9[(\sum_{i=2}^n x_i) / (n-1)]^{0.25} \\F_1(x) &= 1 - \exp(-4x_1)\sin^6(6\pi x_1) \\F_2(x) &= g(x)[1 - (F_1(x)/g(x))^2] \quad x \in [0,1]\end{aligned}$$

UF test instances

UF1: The search space is $x_1 \in [0,1], x_j \in [-1,1]$

$$f_1 = x_1 + \frac{2}{|J_1|} \sum_{j \in J_1} [x_j - \sin(6\pi x_1 + \frac{j\pi}{n})]^2$$

$$f_2 = 1 - \sqrt{x_1} + \frac{2}{|J_2|} \sum_{j \in J_2} [x_j - \sin(6\pi x_1 + \frac{j\pi}{n})]^2$$

$$J_1 = \{j \mid j \text{ is odd and } 2 \leq j \leq n\}$$

$$J_2 = \{j \mid j \text{ is even and } 2 \leq j \leq n\}$$

UF2: The search space is $x_1 \in [0,1], x_j \in [-1,1]^{n-1}$

$$f_1 = x_1 + \frac{2}{|J_1|} \sum_{j \in J_1} y_j^2$$

$$f_2 = 1 - \sqrt{x_1} + \frac{2}{|J_2|} \sum_{j \in J_2} y_j^2$$

$$J_1 = \{j \mid j \text{ is odd and } 2 \leq j \leq n\}$$

$$J_2 = \{j \mid j \text{ is even and } 2 \leq j \leq n\}$$

$$y_j = \begin{cases} x_j - [0.3x_1^2 \cos(24\pi x_1 + \frac{4j\pi}{n}) + 0.6x_1] \cos(6\pi x_1 + \frac{j\pi}{n}) & j \in J_1 \\ x_j - [0.3x_1^2 \cos(24\pi x_1 + \frac{4j\pi}{n}) + 0.6x_1] \sin(6\pi x_1 + \frac{j\pi}{n}) & j \in J_2 \end{cases}$$

UF3: The search space is $[0,1]$

$$f_1 = x_1 + \frac{2}{|J_1|} (4 \sum_{j \in J_1} y_j^2 - 2 \prod_{j \in J_1} \cos(\frac{20y_j\pi}{\sqrt{j}}) + 2)$$

$$f_2 = 1 - \sqrt{x_1} + \frac{2}{|J_2|} (4 \sum_{j \in J_2} y_j^2 - 2 \prod_{j \in J_2} \cos(\frac{20y_j\pi}{\sqrt{j}}) + 2)$$

$$J_1 = \{j \mid j \text{ is odd and } 2 \leq j \leq n\}$$

$$J_2 = \{j \mid j \text{ is even and } 2 \leq j \leq n\}$$

UF4: The search space is $x_1 \in [0,1], x_j \in [-2,2]^{n-1}$

$$\begin{aligned}
f_1 &= x_1 + \frac{2}{|J_1|} \sum_{j \in J_1} h(y_j) \\
f_2 &= 1 - x_1^2 + \frac{2}{|J_2|} \sum_{j \in J_2} h(y_j) \\
J_1 &= \{j \mid j \text{ is odd and } 2 \leq j \leq n\} \\
J_2 &= \{j \mid j \text{ is even and } 2 \leq j \leq n\} \\
h(t) &= \frac{|t|}{1 + e^{2|t|}} \\
y_i &= x_j - \sin(6\pi x_1 + \frac{j\pi}{n}), j = 2, \dots, n
\end{aligned}$$

UF5: The search space is $x_1 \in [0,1], x_j \in [-1,1]^{n-1}$. N is an integer and $\varepsilon > 0$

$$\begin{aligned}
f_1 &= x_1 + (\frac{1}{2N} + \varepsilon) |\sin(2N\pi x_1)| + \frac{2}{|J_1|} \sum_{j \in J_1} h(y_j) \\
f_2 &= 1 - x_1 + (\frac{1}{2N} + \varepsilon) |\sin(2N\pi x_1)| + \frac{2}{|J_2|} \sum_{j \in J_2} h(y_j) \\
J_1 &= \{j \mid j \text{ is odd and } 2 \leq j \leq n\} \\
J_2 &= \{j \mid j \text{ is even and } 2 \leq j \leq n\} \\
h(t) &= 2t^2 - \cos(4\pi t) + 1 \\
y_i &= x_j - \sin(6\pi x_1 + \frac{j\pi}{n}), j = 2, \dots, n
\end{aligned}$$

IMOP test instances

IMOP1: The search space is $[0,1]$

$$\begin{aligned}
f_1 &= g + \cos^8(\frac{\pi}{2} y_1) \\
f_1 &= g + \sin^8(\frac{\pi}{2} y_1) \\
y_1 &= (\frac{1}{L} \sum_{l=1}^L x_l)^{a_1} \\
g &= \sum_{l=L+1}^D (x_l - 0.5)^2
\end{aligned}$$

IMOP2: The search space is $[0,1]$

$$f_1 = g + \cos^{0.5}(\frac{\pi}{2} y_1)$$

$$f_1 = g + \sin^{0.5}(\frac{\pi}{2} y_1)$$

$$y_1 = (\frac{1}{L} \sum_{l=1}^L x_l)^{a_1}$$

$$g = \sum_{l=L+1}^D (x_l - 0.5)^2$$

IMOP3: The search space is [0,1]

$$f_1 = g + 1 + 0.2 \cos(10\pi y_1) - y_1$$

$$f_1 = g + y_1$$

$$y_1 = (\frac{1}{L} \sum_{l=1}^L x_l)^{a_1}$$

$$g = \sum_{l=L+1}^D (x_l - 0.5)^2$$