ASC Appendix: Piezoelectric Admittance-based Damage Identification

In this Appendix, we present the formulation of damage identification utilizing piezoelectric impedance measurement. As the identification is model based, we first outline the finite element modeling of piezoelectric admittance prediction/simulation.

When a piezoelectric transducer is integrated to a host structure, the coupled structure-transducer interaction is characterized by the following equations (Shuai et al, 2017; Cao et al, 2023)

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} + \mathbf{K}_{12}Q = \mathbf{0} \tag{1a}$$

$$R\dot{Q} + k_c Q + \mathbf{K}_{12}^T \mathbf{q} = V_{\text{in}} \tag{1b}$$

where $\mathbf{q}$ is the displacement vector, k_c is the inverse of the capacitance of the piezoelectric transducer; $\mathbf{K}_{12}$ is the electromechanical coupling vector between the transducer and the structure, and $\mathbf{K}$, $\mathbf{C}$, and $\mathbf{M}$ are the stiffness, damping, and mass matrices, respectively. In this research, we use piezoelectric admittance which is the reciprocal of impedance as information carrier for damage identification. Specifically, we apply frequency-sweeping harmonic voltage excitation, denoted as V_{in}, to the transducer. Q is the electrical charge on the surface of the piezoelectric transducer, and R is the resistance in the measurement circuitry. It is worth noting that in the experimental implementation, we measure the voltage drop across the resistor, which is denoted as V_{out}, and subsequently obtain the current as $\dot{Q} = V_{\text{out}} / R$. As such, here we include an additional term $R\dot{Q}$ in Equation (1b), compared with those presented in previous research (Shuai et al, 2017; Cao et al, 2018). We let the excitation frequency be denoted as ω, and use overbar hereafter to indicate magnitude of the corresponding response variable. The piezoelectric admittance of the healthy structure, which is the ratio of the current magnitude and the voltage magnitude, can be derived as

$$y_h(\omega) = \frac{\dot{\bar{Q}}}{\bar{V}_{\text{in}}} = \frac{j\omega\bar{Q}}{\bar{V}_{\text{in}}} = \frac{j\omega}{j\omega R + k_c - \mathbf{K}_{12}^T\left(\mathbf{K} + j\omega\mathbf{C} - \omega^2\mathbf{M}\right)^{-1}\mathbf{K}_{12}} \tag{2}$$

where j is the imaginary unit.

In a finite element model for damage identification, the structure is generally divided into n segments to facilitate the identification of location and severity of damage. That is, each segment is susceptible of damage occurrence, and damage causes homogenized change of structural properties within a segment. In this research, without loss of generality, we assume damage causes stiffness reduction in one or multiple segments. As such, the stiffness matrix with structural damage occurrence, $\mathbf{K}_d$, can then be expressed as $\mathbf{K}_d = \sum_{i=1}^{n} \mathbf{K}_h^i (1 - \alpha_i)$, in which $\mathbf{K}_h^i$ is stiffness matrix of the i-th segment under the healthy status and the summation refers to the direct summation in finite element model assemblage. $\alpha_i \in [0,1]$ is the damage index indicating the possible stiffness loss of the *i*-th segment, which is to be identified. The piezoelectric admittance corresponding to the damaged structure can be written as

$$y_d(\omega) = \frac{j\omega}{j\omega R + k_c - \mathbf{K}_{12}^T \left(\mathbf{K}_d + j\omega\mathbf{C} - \omega^2\mathbf{M}\right)^{-1} \mathbf{K}_{12}} \tag{3}$$

Mathematically, the change of admittance can be derived based on Equations (2) and (3). To increase the computational efficiency, based on the assumption of damage being small in size, we apply the Taylor series expansion to the expression of the change of admittance and ignore the higher-order terms, i.e.,

$$\Delta y(\omega) = \sum_{i=1}^{n} [\, j\omega(j\omega R + k_c - \mathbf{K}_{12}^T \mathbf{Z}^{-1} \mathbf{K}_{12})^{-2} \mathbf{K}_{12}^T \mathbf{Z}^{-1} (\mathbf{L}_i^T \mathbf{K}_{ei} \mathbf{L}_i) \mathbf{Z}^{-1} \mathbf{K}_{12} \,]\, \alpha_i \tag{4}$$

where $\mathbf{Z} = \mathbf{K} - \mathbf{M}\omega^2 + j\omega\mathbf{C}$ represents the dynamic stiffness of the structure, and **L** is the Boolean matrix indicating how the segmental stiffness matrices are assembled into the global stiffness matrix. Once again, the term $j\omega R$ corresponds to the measurement resistance employed in experiment. This formulation is completely consistent with the actual experimental data acquisition procedure. In Equation (4), the change of admittance under excitation frequency ω is denoted as $\Delta y(\omega)$. In damage identification practice, we conduct frequency sweeping in the frequency range of interest, and acquire a series of admittance changes $\Delta y(\omega_1)$, $\cdots$, $\Delta y(\omega_m)$, under *m* different excitation frequencies ω_1, $\cdots$, ω_m. We introduce the following notations of excitation frequency vector, admittance change vector, and damage index vector,

$$\boldsymbol{\omega} = [\omega_1, \quad \cdots, \quad \omega_m]^T \tag{5a}$$

$$\Delta \boldsymbol{y}(\boldsymbol{\omega}) = \left[\Delta y(\omega_1), \quad \ldots, \quad \Delta y(\omega_m)\right]^T \tag{5b}$$

$$\boldsymbol{\alpha} = [\alpha_1, \quad \cdots, \quad \alpha_n]^T \tag{5c}$$

Based on Equation (4), we can then obtain the following relation,

$$\Delta \mathbf{y} = \begin{bmatrix} \Delta y(\omega_1) \\ \vdots \\ \Delta y(\omega_m) \end{bmatrix} = \mathbf{T}_{m\times n} \boldsymbol{\alpha}_{n\times 1} \tag{6}$$

where **T** is the finite element-based sensitivity matrix that links the admittance change vector with the damage index vector.

References

Shuai, Q., Zhou, K., Zhou, S. and Tang, J., 2017. Fault identification using piezoelectric impedance measurement and model-based intelligent inference with pre-screening. Smart Materials and Structures, 26(4), p.045007.

Cao, P., Zhang, S., Wang, Z. and Zhou, K., 2023, April. Damage identification using piezoelectric electromechanical impedance: a brief review from a numerical framework perspective. In *Structures* (Vol. 50, pp. 1906-1921). Elsevier.