

Unemployment Fluctuations

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Objective :

- countercyclical unemployment
- procyclical tightness
- "large" unemployment fluctuating

→ elasticity of unemployment

rate with shock is the same as
in data

Two wage functions:

- Rigid wage
- Bargained wage

Matching model with rigid wage

$$W = \omega \cdot \alpha^\gamma \quad \gamma \in [0, 1]$$

$$L^s(\theta) = \frac{f(\theta)}{\alpha + f(\theta)} \quad H$$

$$L^d(\theta) = \left[\frac{\alpha \cdot \omega}{\omega \cdot \alpha^r \cdot [1 + \tau(\theta)]^2} \right]^{1/(1-\alpha)}$$

$$L^d(\theta) = \left[\frac{\alpha^{1-r} \cdot \omega}{\omega \cdot [1 + \tau(\theta)]^2} \right]^{1/(1-\alpha)} \quad L > 0$$

equilibrium condition : $L^s(\theta) = L^d(\theta)$

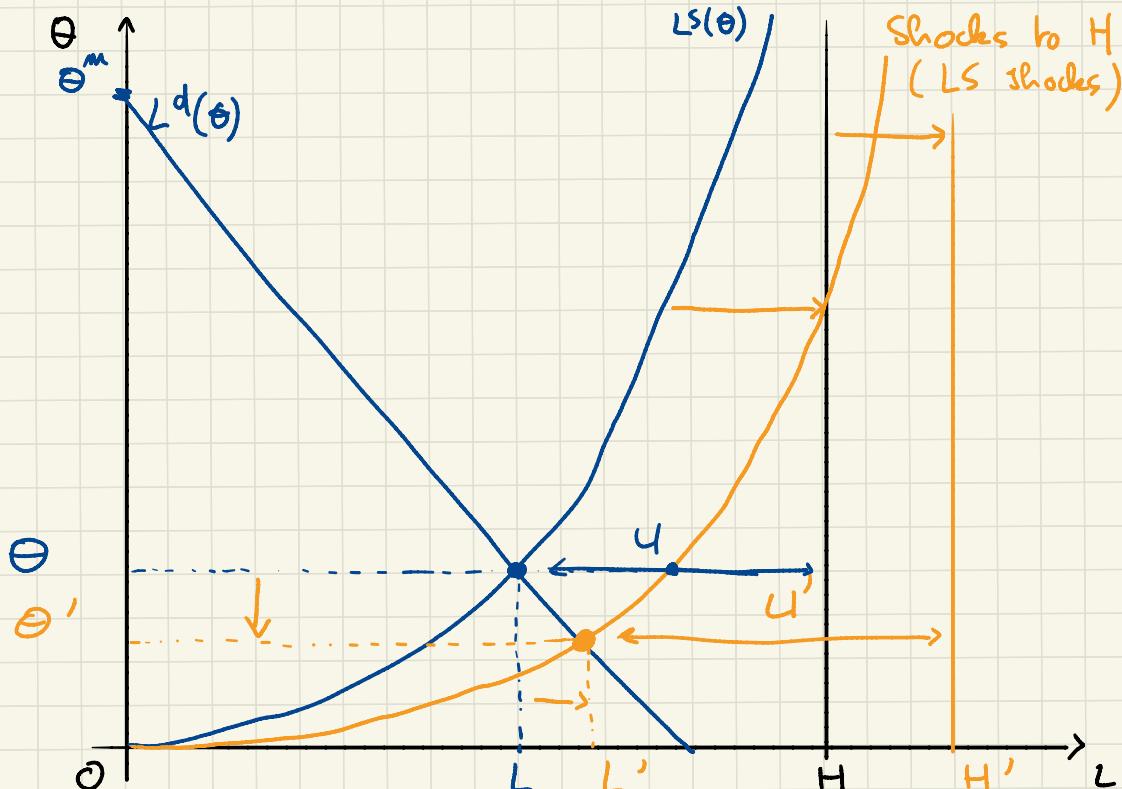
→ gives equilibrium θ .

Which shock can explain fluctuations .

Compare labo-demand shock α with
labo-supply shock H .

Compare labo-demand & labo-supply shocks
from labor-market diagram -

Labo-supply shocks :



increase in H :

- $L \uparrow$: boom / expansion

- $\theta \downarrow$

- $u = s / s + f(\theta)$ so $u \uparrow$

- $y = a \cdot N^d$

$$N = L / [1 + \tau(\theta)]$$

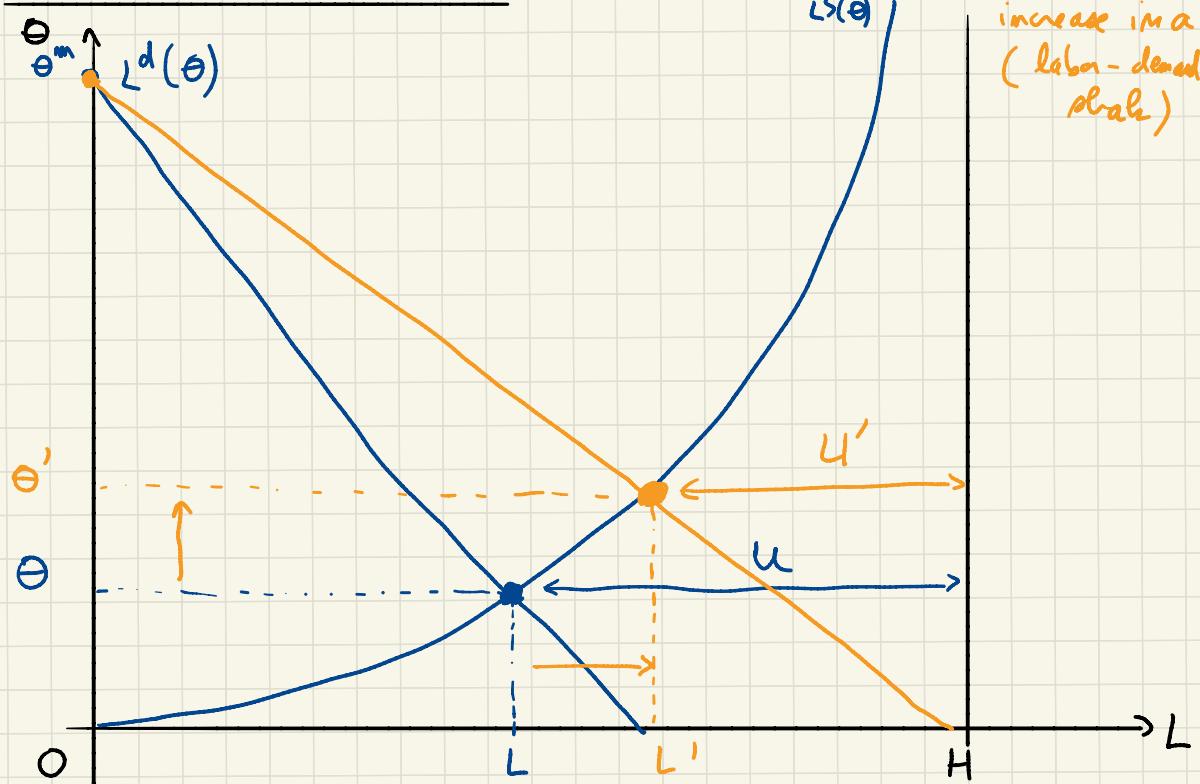
$$\theta \downarrow \Rightarrow \begin{cases} \tau(\theta) \downarrow \\ L \uparrow \end{cases} \Rightarrow N \uparrow \Rightarrow y \uparrow$$

- u is pro cyclical
 - Θ is counter cyclical
- if H causes business cycles.

\Rightarrow predictions are countercyclical

\Rightarrow H / labor-supply shocks cannot cause business cycles (in matching model)

Labor-demand shocks



increase in a
(labor-demand
shock)

increase in a (positive labor-demand shock)

- $L \uparrow$: boom / expansion / peak

- $\Theta \uparrow$

- $u = s / s + f(\Theta)$ so $u \downarrow$

- $U = u \cdot H$ so $U \downarrow$

- balanced flows : inflows = outflows

$$s \times L = m(U, V)$$

↑ ↓ ↑

$V \uparrow$ and $v = V/H \uparrow$

under shocks to labor productivity:

- Θ is procyclical

- v is procyclical

- u is countercyclical

Beveridge curve

In matching model: shocks to productivity generate realistic business cycles -
[under rigid wages]

↑
labour-demand
shocks

Michaillat (2012, p. 1741) : elasticity of θ

wrt to productivity is $\Sigma_a^\theta \approx \theta$

→ when productivity goes up by 1%, θ increases
goes up by $\theta\%$.

- elasticity of u wrt a is $\Sigma_a^u \approx 1$
- elasticity of v wrt a is $\Sigma_a^v \approx 1$

Can we get Σ_a^θ of θ in matching model
with rigid wages?

equilibrium condition: $L^s(\theta) = L^d(\theta)$

$$\Leftrightarrow \frac{f(\theta)}{\alpha + f(\theta)} \cdot H = \left[\frac{\alpha \cdot a^{1-\eta}}{\omega [1 + \tau(\theta)]^\alpha} \right]^{1/\alpha-\eta}$$

→ θ is an implicit function of a : $\theta(a)$

→ (implicit function theorem)

$$\Sigma_a^\theta \equiv \frac{d \ln \theta}{d \ln a} .$$

$$m(U, V) = N \cdot U \cdot V^{\eta - 1 - \eta} \quad \eta \in (0, 1)$$

$$f(\theta) = N \cdot \theta^{1-\eta} \rightarrow \frac{d \ln f}{d \ln \theta} = 1 - \eta$$

$$g(\theta) = N \cdot \theta^{-\eta} \rightarrow \frac{d \ln g}{d \ln \theta} = -\eta$$

$$\frac{f(s)}{s+f(\theta)} = 1-u = \ell(\theta) : \underline{\text{employment rate.}}$$

$$\frac{d \ln \ell}{d \ln \theta} = \frac{d \ln f}{d \ln \theta} - \frac{d \ln (s+f)}{d \ln \theta}$$

$$= 1-\eta - \frac{s}{s+f} \times \frac{d \ln f}{d \ln \theta}$$

$$= (1-\eta) \left(1 - \frac{s}{s+f} \right)$$

$$= (1-\eta) \times \frac{s}{s+f}$$

$$\boxed{\frac{d \ln \ell}{d \ln \theta} = (1-\eta) \times u}$$

Results on elasticities

Leibniz's notation: $\frac{dx}{df}$

$f(x)$: any differentiable function

$$\boxed{df = f'(x) \cdot dx \rightarrow \frac{df}{dx} = f'(x)}$$

elasticity of f with respect to x :

$$\boxed{\sum_x \frac{f}{x} = \frac{d \ln f}{d \ln x}}$$

Interpretation

x increases by 1% : what happens to f ?

$$dx = 1\% \times x$$

$$df = f'(x) dx = f'(x) \times x \times 1\%$$

percentage change in f :

$$\rightarrow \frac{df}{f} \times 100 = f'(x) \times \frac{x}{f} \times 1\% \times 1.05$$

$$\text{percentage change in } f = \frac{x}{f} \quad f'(x) = \frac{x}{f} \cdot \frac{df}{dx}$$

$$d\ln f = \frac{1}{f} \cdot df \quad ; \quad d\ln x = \frac{1}{x} \cdot dx$$

derivative of \ln

$$\boxed{\frac{d\ln f}{d\ln x} = \frac{x}{f} \cdot \frac{df}{dx}}$$

$$\boxed{\frac{d\ln f}{d\ln x} = \frac{x}{f} \cdot f'(x)}$$

percentage change in f given by $\frac{d\ln f}{d\ln x} = \varepsilon_f^x$

Results about elasticities:

$$\bullet f(x) = x^\alpha \longrightarrow$$

$$\boxed{\varepsilon_x^f = \alpha}$$

$$\underline{\text{proof:}} \quad \ln(f) = \alpha \cdot \ln(x) \rightarrow \frac{d\ln f}{d\ln x} = \alpha$$

$$\bullet f(x) = A(x) \times B(x) \rightarrow$$

$$\boxed{\varepsilon_x^f = \varepsilon_x^A + \varepsilon_x^B}$$

$$\underline{\text{proof:}} \quad \ln f = \ln A + \ln B$$

$$\frac{d\ln f}{d\ln x} = \frac{d\ln A}{d\ln x} + \frac{d\ln B}{d\ln x}$$

$$\cdot f(x) = A(x) / B(x) \rightarrow \varepsilon_f^x = \varepsilon_x^A - \varepsilon_x^B$$

proof: $\ln f = \ln A - \ln B$

$$\frac{d \ln f}{d \ln x} = \frac{d \ln A}{d \ln x} - \frac{d \ln B}{d \ln x}$$

$$\cdot f(x) = \alpha \cdot A(x) \rightarrow \varepsilon_f^x = \varepsilon_x^A$$

proof: $\ln f = \ln \alpha + \ln A$

$$\frac{d \ln f}{d \ln x} = \frac{d \ln \alpha}{d \ln x} + \frac{d \ln A}{d \ln x} \leftarrow \varepsilon_x^A$$

$\varepsilon_x^f \nearrow$ $\uparrow 0$

$$\cdot f(x) = A(x) + B(x) \rightarrow \varepsilon_x^f = \frac{A}{A+B} \cdot \varepsilon_x^A + \frac{B}{A+B} \cdot \varepsilon_x^B$$

"elasticity of sum = weighted sum of elasticities"

proof: $\ln f = \ln (A+B)$

$$\begin{aligned} \frac{d \ln f}{d \ln x} &= \frac{1}{A+B} \times \left(\frac{dA}{d \ln x} + \frac{dB}{d \ln x} \right) \\ &= \frac{A}{A+B} \cdot \frac{1}{A} \frac{dA}{d \ln x} + \frac{B}{A+B} \cdot \frac{1}{B} \frac{dB}{d \ln x} \end{aligned}$$

$$d \ln A = \frac{dA}{A} ; \quad d \ln B = \frac{dB}{B}$$

$$\frac{d \ln f}{d \ln x} = \frac{A}{A+B} \cdot \frac{d \ln A}{d \ln x} + \frac{B}{A+B} \cdot \frac{d \ln B}{d \ln x}$$

$\uparrow \varepsilon_A$ $\uparrow \varepsilon_B$

- $\underbrace{1 + \tau(\theta)}_{g(\theta)} : \varepsilon_\theta^q = \frac{\tau}{1+\tau} \cdot \varepsilon_\theta^\tau$

- $\tau(\theta) = \frac{rs}{q(\theta) - rs}$ $\varepsilon_\theta^\tau = - \left(\frac{q}{1-rs} \times \varepsilon_\theta^q \right)$

- $\varepsilon_\theta^q = -\eta$

- $\varepsilon_\theta^\tau = \eta \cdot \frac{q}{1-rs} = \eta (1+\tau)$

- $\varepsilon_\theta^q = \frac{\tau}{1+\tau} \cdot \eta \cdot (1+\tau)$

$$\Rightarrow \boxed{\varepsilon_\theta^{1+\tau} = \tau \cdot \eta}$$

equilibrium: $\frac{f(\theta)}{s_1 f(\theta)} \cdot H = \left[\frac{\alpha \cdot \alpha^{1-\alpha}}{\omega [1+\tau(\theta)]^\alpha} \right]^{\frac{1}{1-\alpha}}$

$\Theta(a)$. Consider small change in productivity $\Delta \theta$

$\rightarrow d\ln \theta$: induced, small change in θ

$$df = f'(x) dx$$

$$\rightarrow \frac{df}{f} = \frac{1}{f} f'(x) dx$$

$$\rightarrow \frac{df}{f} = \frac{x}{f} \cdot f'(x) \frac{dx}{x}$$

$$\rightarrow d\ln f = \varepsilon_x^f \cdot d\ln x$$

• $d\ln L^S = \varepsilon_\theta^{L^S} \cdot d\ln \theta$

• extends to multivariate functions.

$$L^S = \frac{\rho(\theta)}{\alpha + f(\theta)} \cdot H \rightarrow \varepsilon_\theta^{L^S} = (1-\eta) \cdot u$$

$$d\ln L^S = (1-\eta) \cdot u \cdot d\ln \theta$$

• $d\ln L^d(\theta, a) = \varepsilon_\theta^{L^d} \cdot d\ln \theta + \varepsilon_a^{L^d} \cdot d\ln a -$
 $(df(x, y) = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy)$

$$L^d(\theta, a) = \left[\frac{\alpha \cdot a^{1-\gamma}}{w \cdot (1 + T(\theta))^\alpha} \right]^{1/(1-\gamma)}$$

$$\rightarrow \varepsilon_a^{L^d} = \frac{1}{1-\gamma} \cdot 1-\gamma = \frac{1-\gamma}{1-\alpha}$$

$$\rightarrow \varepsilon_\theta^{L^d} = \frac{-1}{1-\alpha} \cdot \alpha \cdot \varepsilon_\theta^{1+\gamma}$$

$$\xi_0^L = \frac{-\alpha}{1-\alpha} \cdot \eta \cdot \tau$$

$$d \ln L^d = \frac{1-\gamma}{1-\alpha} d \ln a - \frac{\alpha}{1-\alpha} \cdot \eta \cdot \tau \cdot d \ln \theta$$

before shock: $L^s(\theta) = L^d(\theta, a)$

after shock: $L^s(\theta') = L^d(\theta', a')$

$$d \ln L^s = d \ln L^d$$

$$(1-\eta) u d \ln \theta = \frac{1-\gamma}{1-\alpha} d \ln a - \frac{\alpha}{1-\alpha} \eta \tau d \ln \theta$$

$$(1-\eta) u \frac{d \ln \theta}{d \ln a} = \frac{1-\gamma}{1-\alpha} - \frac{\alpha}{1-\alpha} \eta \tau \frac{d \ln \theta}{d \ln a}$$

$$\left[(1-\eta) u + \frac{\alpha}{1-\alpha} \eta \tau \right] \frac{d \ln \theta}{d \ln a} = \frac{1-\gamma}{1-\alpha}$$

$$\frac{d \ln \theta}{d \ln a} = \frac{1-\gamma}{(1-\alpha)(1-\eta)u + \alpha \eta \tau}$$

- $\gamma = 1$ (wage $\propto a$: flexible wage)

$$\frac{d \ln \theta}{d \ln a} = 0 : \text{no fluctuations}$$

→ need somewhat rigid wage to obtain business-cycle fluctuations → $\gamma < 1$.

- $\gamma < 1$ (wages are rigid): $\frac{d \ln \theta}{d \ln a} > 0$

- necessity of wage rigidity to obtain business cycle fluctuations in matching model
- in data: wages are rigid ($\delta \approx 0.5$)
so model can generate business-cycle fluctuations.

Calibration of model

$$\begin{aligned}\gamma &: 0.5 \\ \eta &: 0.5 \\ \alpha &: 2/3 \\ u &: 6\% \\ \tau &: 3\%\end{aligned}$$

$$\begin{aligned}\varepsilon_a^\theta &= \frac{1-\gamma}{(1-\lambda)(1-\eta)u + \alpha\eta\tau} \\ &= \frac{0.5}{1/3 \cdot 0.5 \cdot 6\% + 2/3 \cdot 0.5 \cdot 3\%} \\ &= \frac{1}{2\% + 2\%} \\ \varepsilon_a^\theta &= \frac{1}{u\%} \\ \boxed{\varepsilon_a^\theta = 25}\end{aligned}$$

if we want $\varepsilon_a^\theta = \delta$: what is γ ?

$$\delta = \frac{1-\gamma}{1/3 \cdot 1/2 \cdot 6\% + 2/3 \cdot 1/2 \cdot 3\%}$$

$$\delta \cdot [1\% + 1\%] = 1 - \gamma$$

$$16\% = 1 - \gamma$$

$$\boxed{\gamma = 0.84}$$

Matching model with bargained wage

Wage bargaining solution: surplus sharing

$$W = (1-\beta) z + \beta \cdot MPL \cdot (1 + \tau(\theta))$$

$$MPL = \alpha \cdot a \cdot N^{d-1}$$

Linear production function: $y = a \cdot N$

$$\underline{d=1}$$

$$\rightarrow MPL = a$$

$$\rightarrow W = (1-\beta) z + \beta a (1 + \tau(\theta))$$

$$\text{Labor demand: } (L^d)^{1-d} = \left[\frac{\alpha}{w [1 + \tau(\theta)]} \right]^{\frac{1-d}{1-\alpha}}$$

As we set $d=1$:

$$1 = \frac{a}{w \cdot [1 + \tau(\theta)]}$$

Labor demand relation:

$$\boxed{a = [1 + \tau(\theta)] \cdot w}$$

MPL

MC

Combined bargained wage & labor demand:

$$a = [1 + \tau(\theta)] \times [(1-\beta) z + \beta a (1 + \tau(\theta))]$$

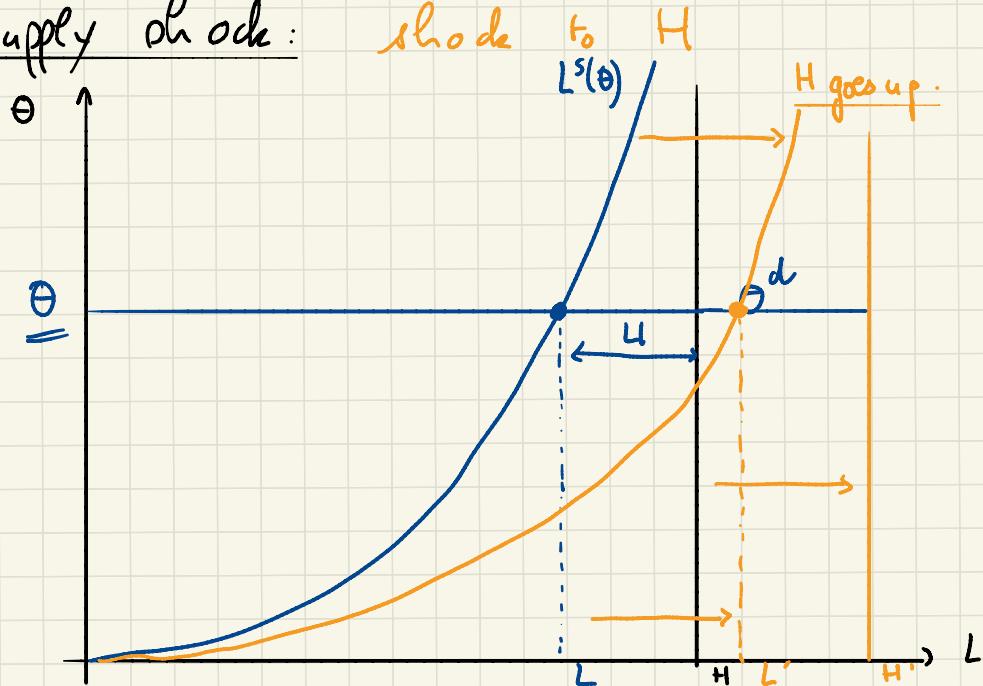
$$1 = [1 + \tau(\theta)] \left[(1 - \beta) \frac{z}{a} + \beta (1 + r\theta) \right]$$

↳ new labor demand: linear prod. fun. dr. in + bargained wage.

↳ . perfectly elastic labor demand
• horizontal labor demand.

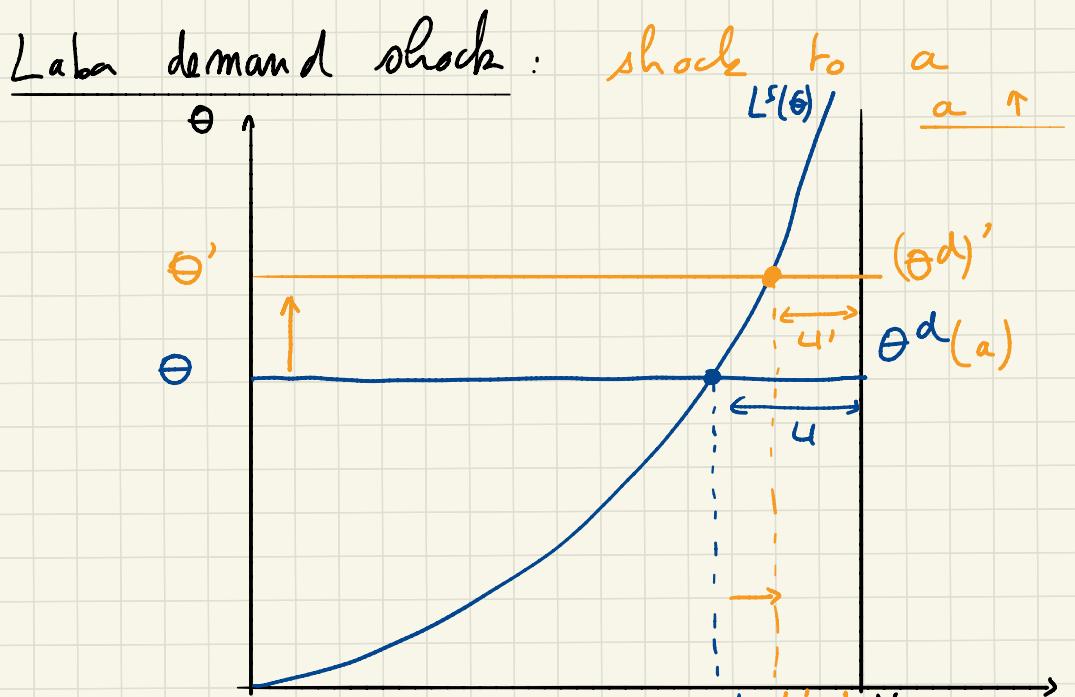
↳ labor demand $\theta^d(a)$

Labor supply shock:



H goes up:

- $L \uparrow$: boom / expansion ($\gamma \uparrow$)
- $\Theta \rightarrow$
- $u = s/[s + f(\theta)]$ $\begin{matrix} \text{rate} \\ u \text{ remains same} \end{matrix}$
- $U = u \times H$ $U \uparrow$
- $v = \Theta \times u$ $v \text{ remains same}$
- $V = v \times H$ $V \uparrow$
- Θ , u , v are acyclical
- not a realistic business cycle



Labour demand θ^d : $I = [1 + \tau(\theta)] \left[(1 - \beta) \frac{\bar{z}}{\alpha} + \beta (1 + r\theta) \right]$

if $a \uparrow$: $\theta^d(a) \uparrow$

After an increase in a :

- $L \uparrow$: boom / expansion
- $\Theta \uparrow$: Θ procylical
- $\begin{cases} u \downarrow \\ U \downarrow \end{cases}$: u countercyclical
- $\delta \times L = m(U, V)$
(inflows) ↓ (outflows)
- $\begin{cases} V \uparrow \\ v \uparrow \end{cases}$: v procylical

Under labor-demand shocks, fluctuations have realistic features (qualitatively)

Not the case with labor-supply shocks.

Objective: compute Σ_a^Θ .

Lab demand relation:

$$LHS(\Theta, a) = [1 + \tau(\Theta)] \left[(1 - \beta) \frac{z}{a} + \beta (1 + r\Theta) \right]$$

Study impact of an infinitesimal change in productivity : $d \ln a$

→ Generates infinitesimal change in tightness:
 $d\ln \Theta$

$$d\ln \text{RHS} = \sum_a^{RHS} \cdot d\ln a + \sum_{\Theta}^{RHS} \cdot d\ln \Theta = 0$$

so $\boxed{\frac{d\ln \Theta}{d\ln a}} = - \frac{\sum_a^{RHS}}{\sum_{\Theta}^{RHS}} -$

• First, compute $\sum_a^{RHS} = \underbrace{\frac{\partial \ln \text{RHS}}{\partial \ln a}}$.

$$\text{RHS} = \underbrace{[1 + \tau(\Theta)]}_{(1-\beta)} \cdot \left[\underbrace{(1-\beta) \frac{z}{a}}_{+ \beta (1+r\Theta)} \right]$$

$$\frac{\partial \ln \text{RHS}}{\partial \ln a} = \frac{(1-\beta)(z/a)}{(1-\beta)(z/a) + \beta(1+r\Theta)} \cdot (-1)$$

$$\frac{\partial \ln \text{RHS}}{\partial \ln a} = \frac{-(1-\beta) z}{(1-\beta) z + \beta a (1+r\Theta)} = \sum_a^{RHS} .$$

• Second, compute $\sum_{\Theta}^{RHS} = \frac{\partial \ln \text{RHS}}{\partial \ln \Theta}$.

$$\sum_{\Theta}^{RHS} = \eta \cdot \tau(\Theta) + \frac{\beta (1+r\Theta)}{(1-\beta)(z/a) + \beta(1+r\Theta)}$$

$$\sum_{\Theta}^{1+\tau} * \frac{r\Theta}{1+r\Theta} = 1$$

$$\sum_{\theta}^{RHS} = \underbrace{\gamma \cdot \tau(\theta)} + \underbrace{\frac{\beta r \theta \alpha}{(1-\beta) z + \beta \alpha (1+r\theta)}}.$$

• Third, combine results to compute \sum_{θ} :

$$\frac{d \ln \theta}{d \ln a} = \frac{(1-\beta) z}{(1-\beta) z + \beta \alpha (1+r\theta)} \cdot \frac{(1-\beta) z + \beta \alpha (1+r\theta)}{\beta r \theta \alpha + \gamma \cdot \frac{z \cdot \alpha}{1+z}}$$

$$a = (1+\tau)((1-\beta)z + \beta \alpha (1+r\theta)) \quad (\text{labor demand})$$

$$\frac{a}{1+z} = (1-\beta)z + \beta \alpha (1+r\theta)$$

$$\boxed{\frac{d \ln \theta}{d \ln a} = \frac{(1-\beta) \cdot z}{a \cdot [\beta r \theta + \gamma \cdot \frac{z}{1+z}]}}$$

• \exists : value from unemployment

$$\underline{z=0} : \frac{d \ln \theta}{d \ln a} = 0 : \text{no business-cycle fluctuations}$$

θ, L, u, v : do not respond to productivity a .

Intuition: if $z=0$: W is proportional to a

→ W is flexible: W absorbs fluctuations in a so θ^d is independent of a

Algebraically: $z = 0 \Rightarrow w = \beta a (1 + r\theta)$

[Lab demand relation]: $a = [1 + \tau(\theta)] w$

$$\Rightarrow 1/a = [1 + \tau(\theta)] \cancel{w} / \cancel{a} (1 + r\theta)$$

$$\Rightarrow 1 = [1 + \tau(\theta)] \cancel{(1 + r\theta)}$$

\hookrightarrow independent of a

Shimer (2005): $z = 0.4$
 $a = 1$ (normalization)

$$\eta = 0.5$$

$$\beta = 0.5 \text{ (tradition)}$$

$$\tau = 3\%$$

$$r\theta = 0.6$$

Lab demand relation:

$$d = (1 + \tau) ((1 - \beta) z + \beta a (1 + r\theta))$$

$$1 = (1.03) \left(\frac{1}{2} \cdot 0.4 + \frac{1}{2} \cdot 1 \cdot (1 + r\theta) \right)$$

$$1 = (1.03) (0.2 + 0.5 (1 + r\theta))$$

$$r\theta = \left[\frac{1}{1.03} - 0.2 \right] \times 2 - 1 \approx 0.6$$

Calibrated value of $\frac{d \ln \theta}{d \ln a}$?

$$\frac{d \ln \theta}{d \ln a} = \frac{0.4 \times 0.5}{0.5 \times 0.6 + 0.5 \cdot 0.03 / 1.03} = \frac{0.2}{0.3 + 0.015}$$

$$\frac{d \ln \theta}{d \ln u} \approx \frac{2}{3} > 0 \quad \text{once } z > 0$$

BUT

$$\frac{d \ln \theta}{d \ln u} \ll \delta$$

Fluctuations in θ , u , v are much smaller than in US data.

- Model with surplus sharing as wage function is inappropriate to describe business cycle on labor market.
- Violates criterion # 1 for a good model
see T. Bentley (1999). (Kuhn)