Overview of stochastic GW background searches and sources

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Wednesday, 20 July 2022

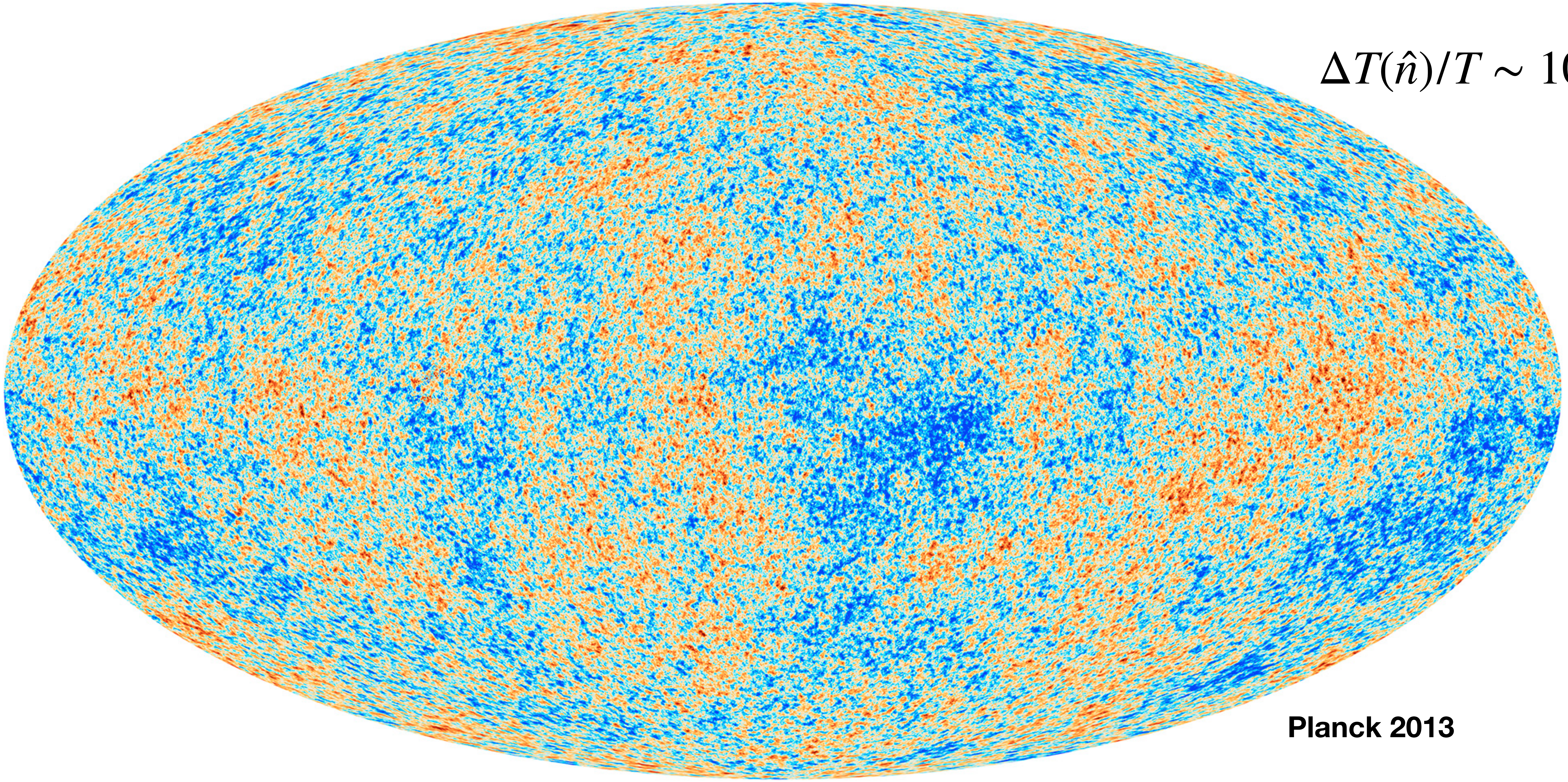
(HUST GW Summer School 2022, Lecture 2)

References

- Allen, Les Houches article, 1995
- Romano and Cornish, Living Reviews in Relativity article, 2017

I. Motivation

Ultimate goal - produce GW analogue of CMB sky maps



$\Delta T(\hat{n})/T \sim 10^{-5}$

Planck 2013

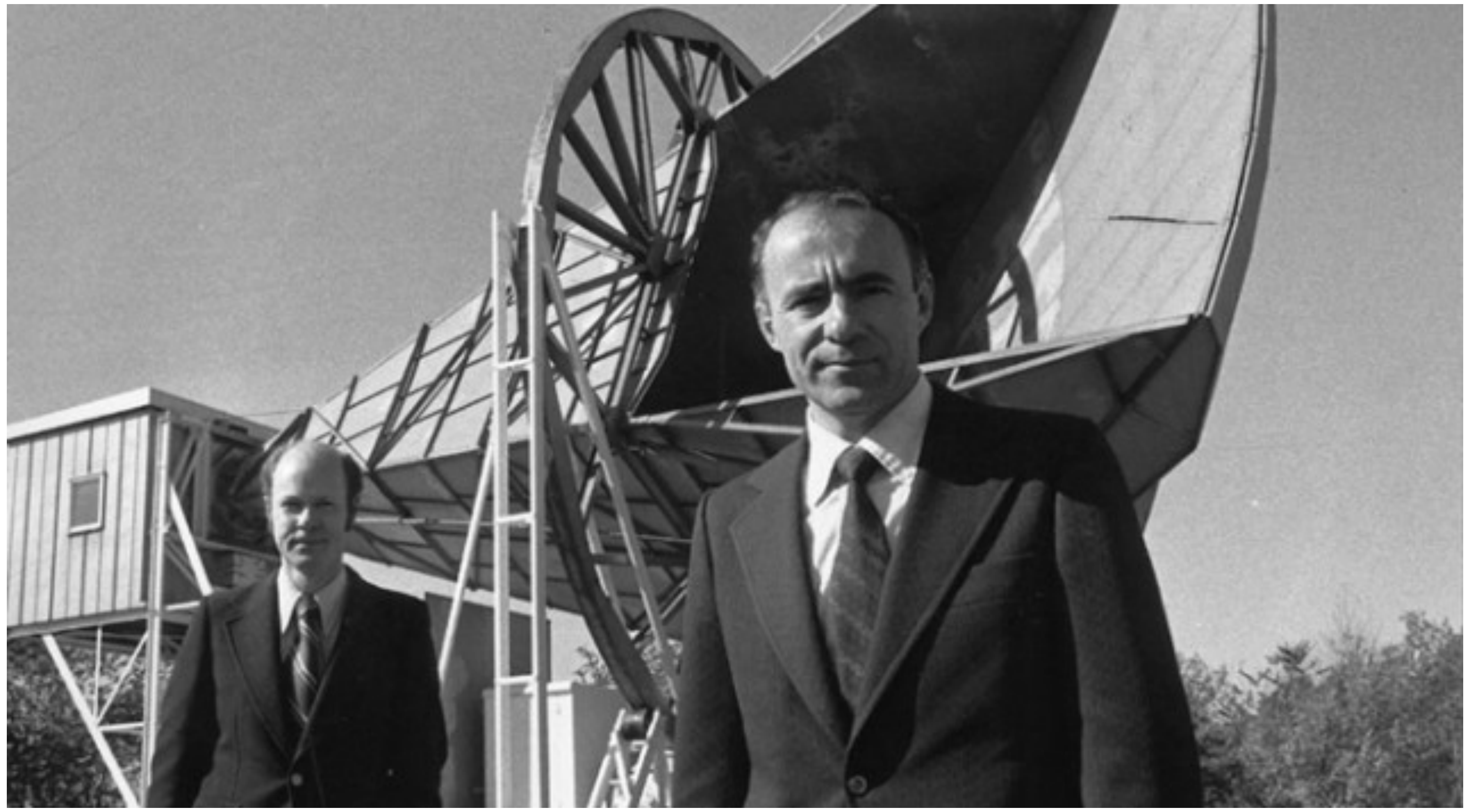
but there's a long road ahead….

1965: Penzias and Wilson ("excess noise" CMB discovery paper)

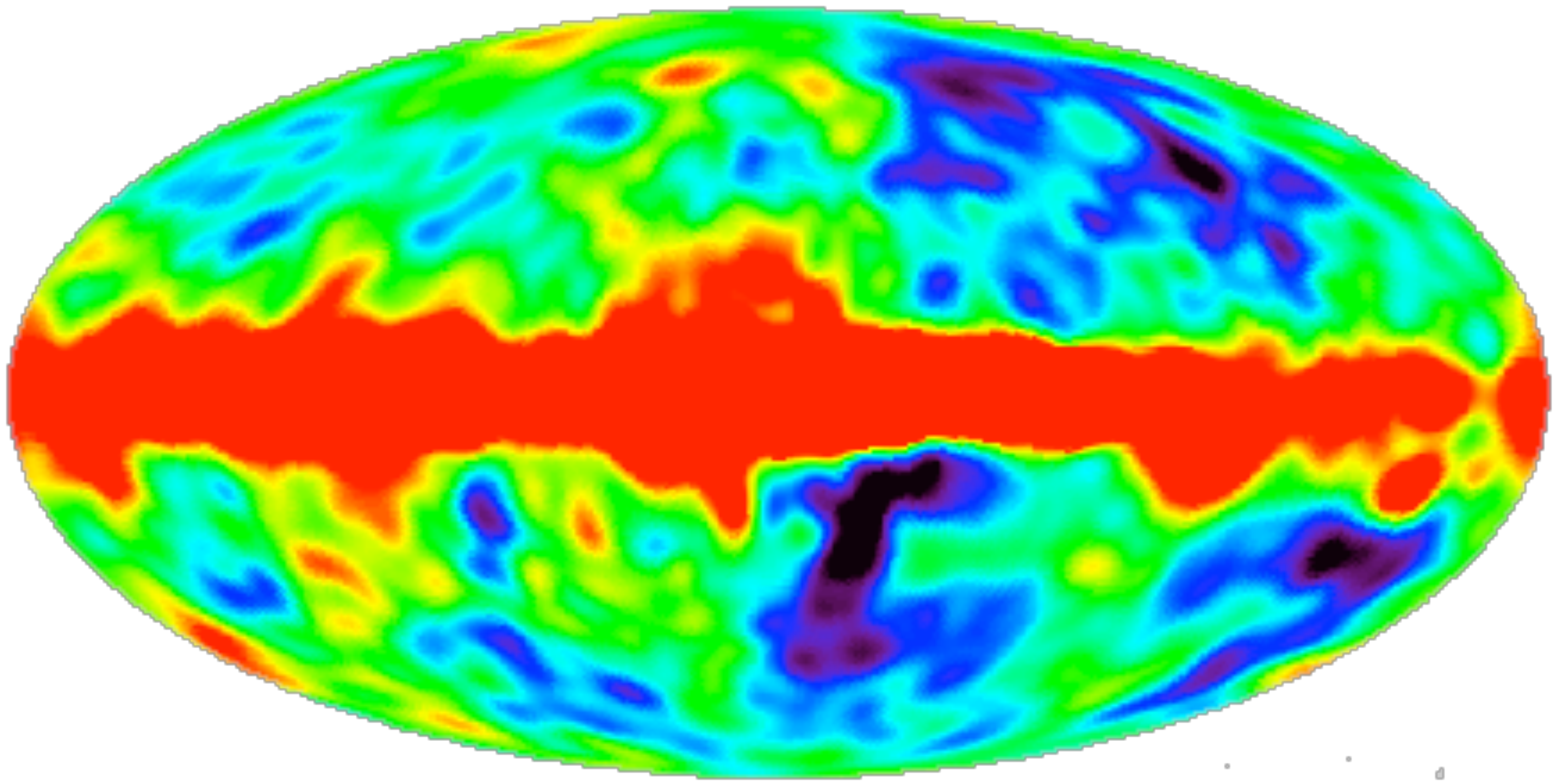
A MEASUREMENT OF EXCESS ANTENNA TEMPERATURE AT 4080 Mc/s

(4080 Mc/s <-> 7.35 cm)

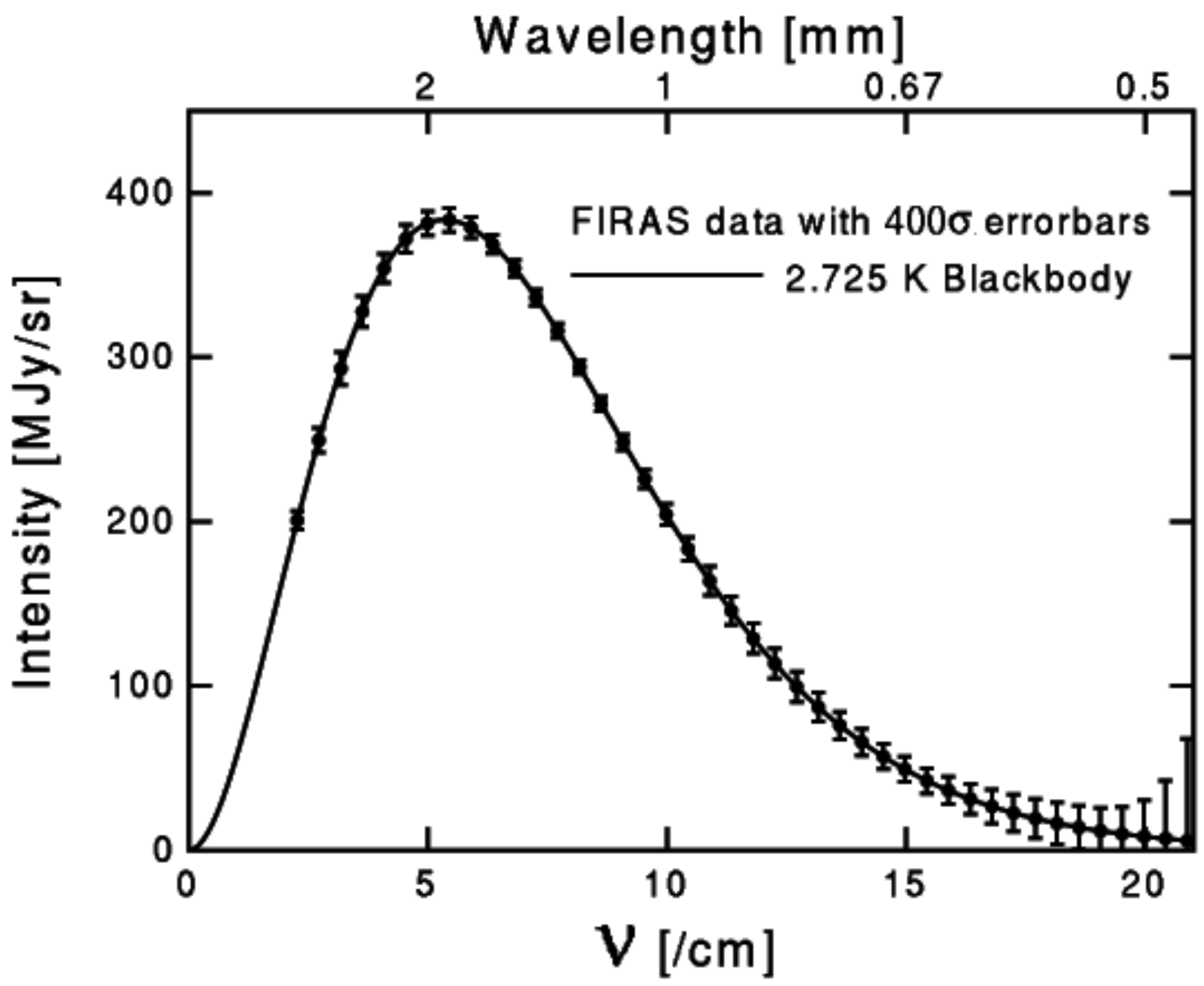
Measurements of the effective zenith noise temperature of the 20-foot horn-reflector antenna (Crawford, Hogg, and Hunt 1961) at the Crawford Hill Laboratory, Holmdel, New Jersey, at 4080 Mc/s have yielded a value about 3.5° K higher than expected. This excess temperature is, within the limits of our observations, isotropic, unpolarized, and free from seasonal variations (July, 1964–April, 1965). A possible explanation for the observed excess noise temperature is the one given by Dicke, Peebles, Roll, and Wilkinson (1965) in a companion letter in this issue.



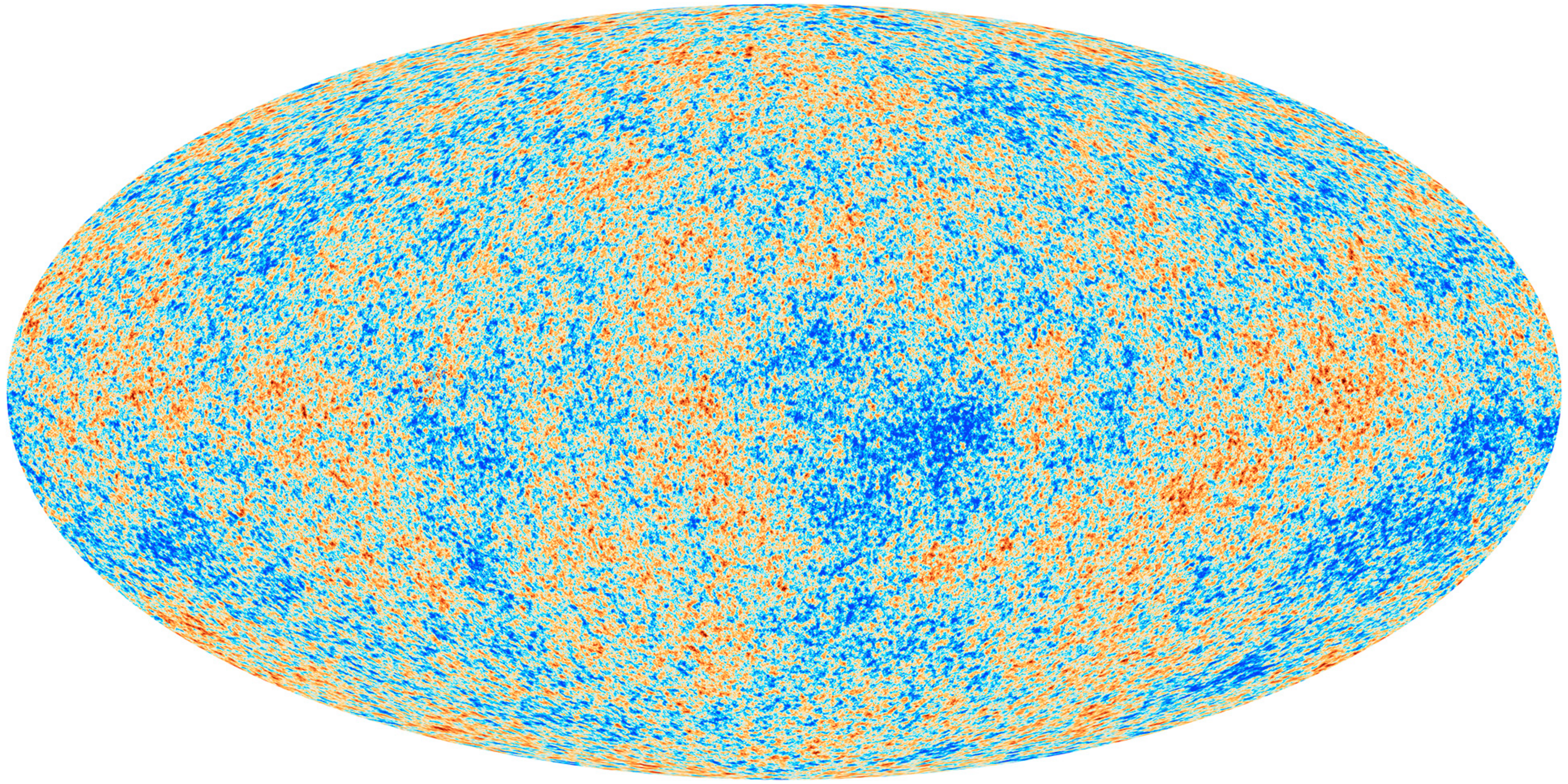
1992: COBE (1st sky maps)



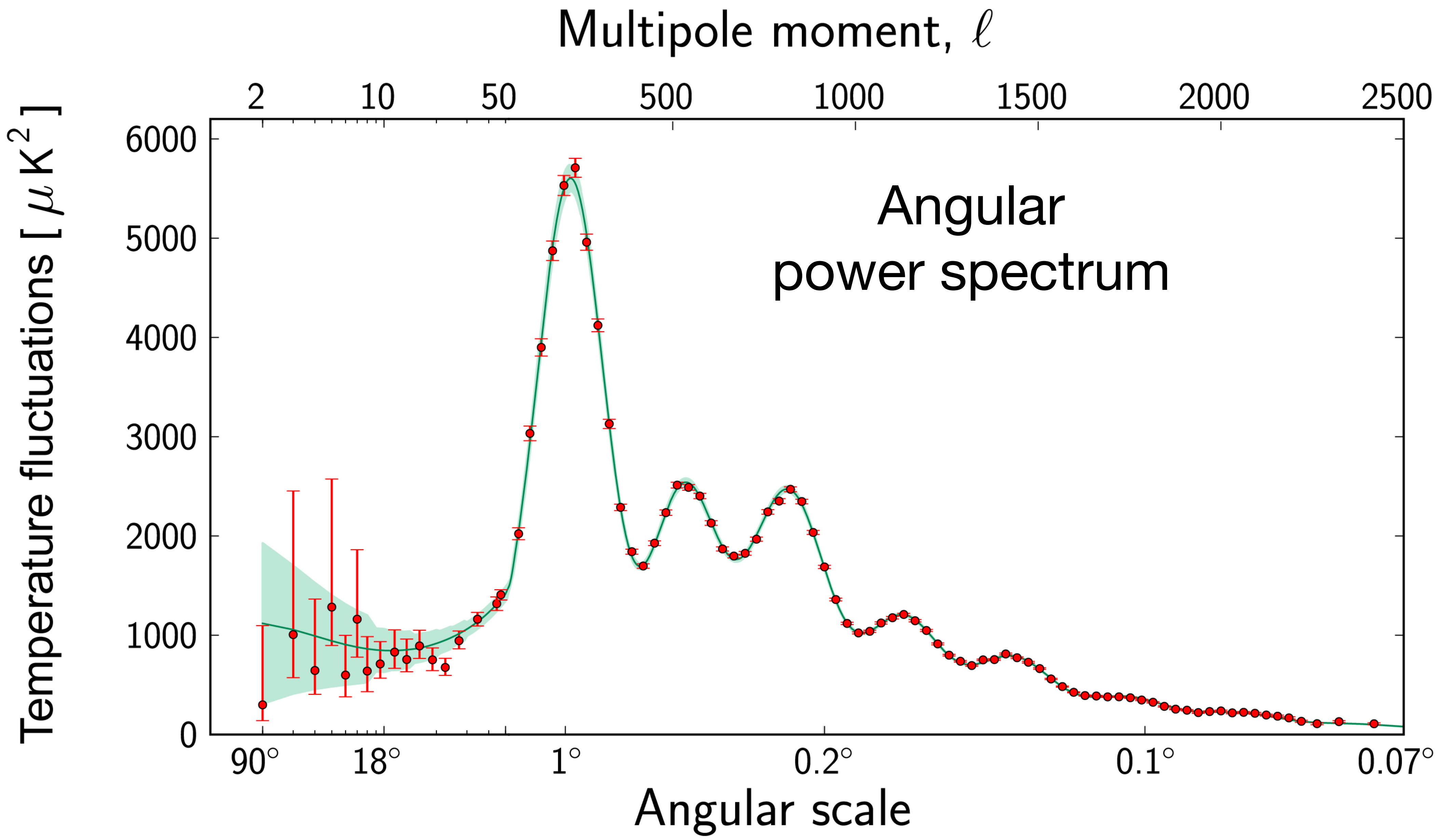
(ang resolution: ~10 degrees)



2013: WMAP, Planck (high precesion cosmology)

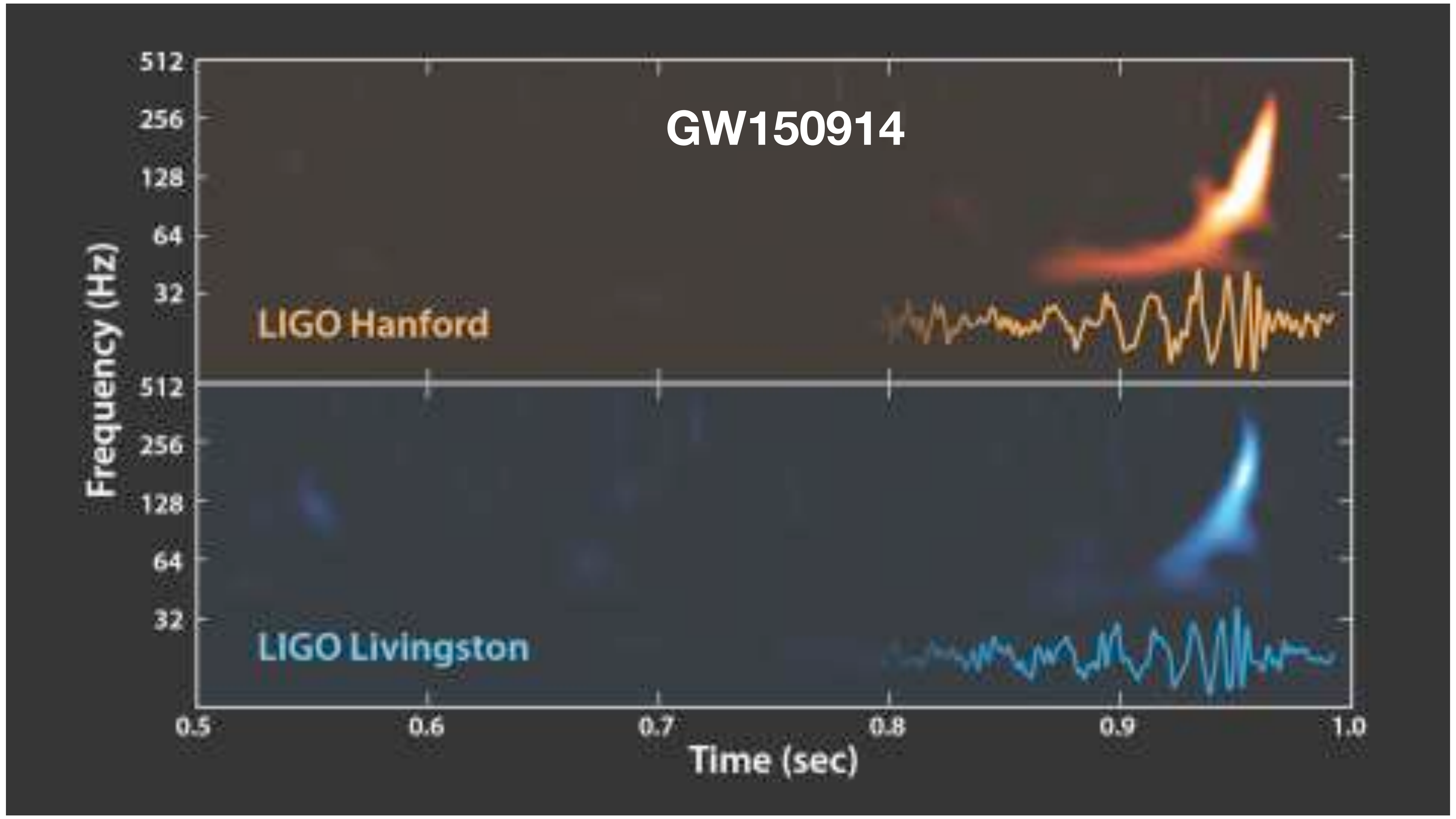


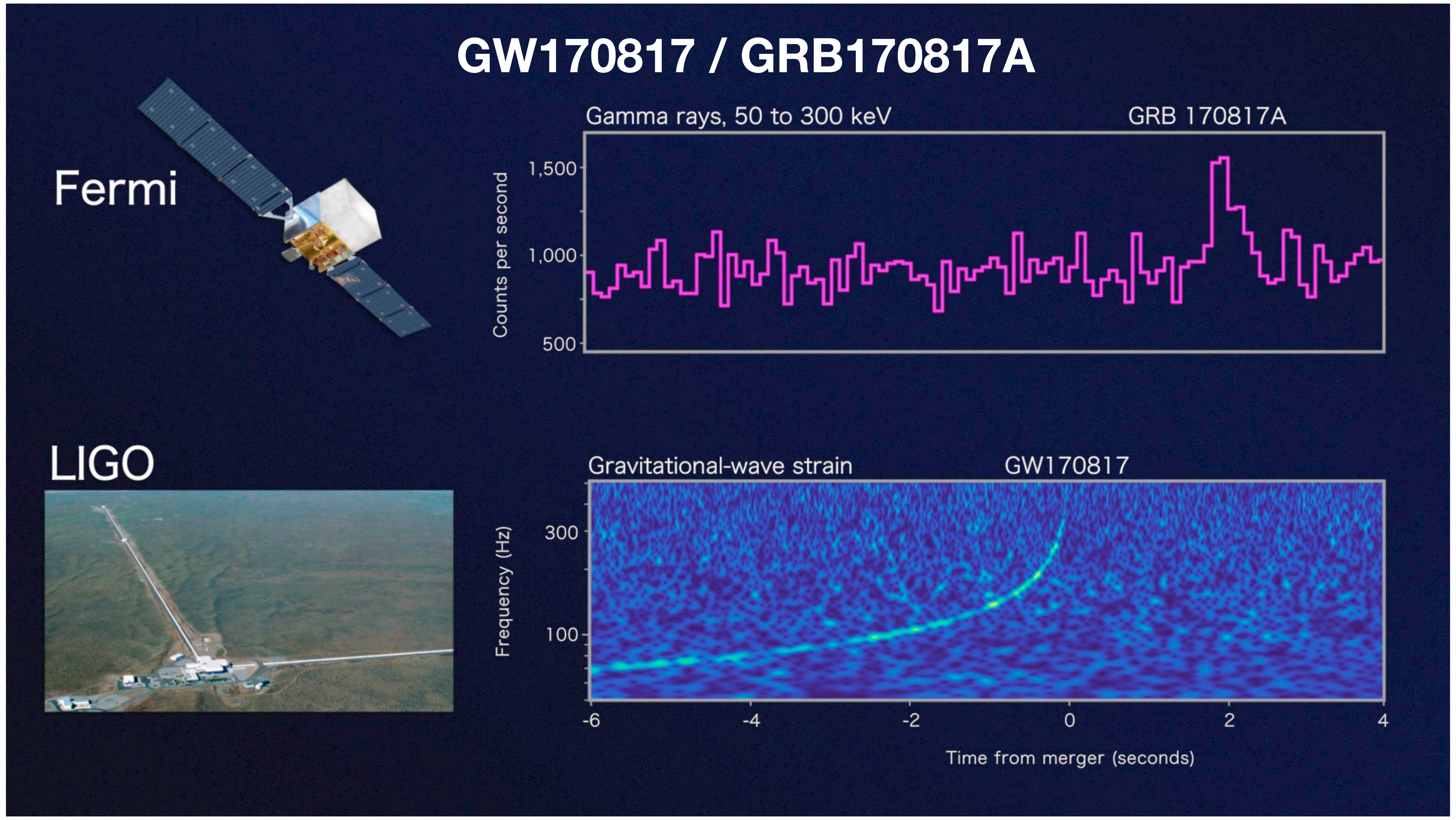
(ang resolution: ~10 arcmin)



It took ~50 years for the CMB community to go from first detection to precision sky maps, but we have not yet detected the isotropic component of the GW background!!

… at least we have detected other (loud) signals





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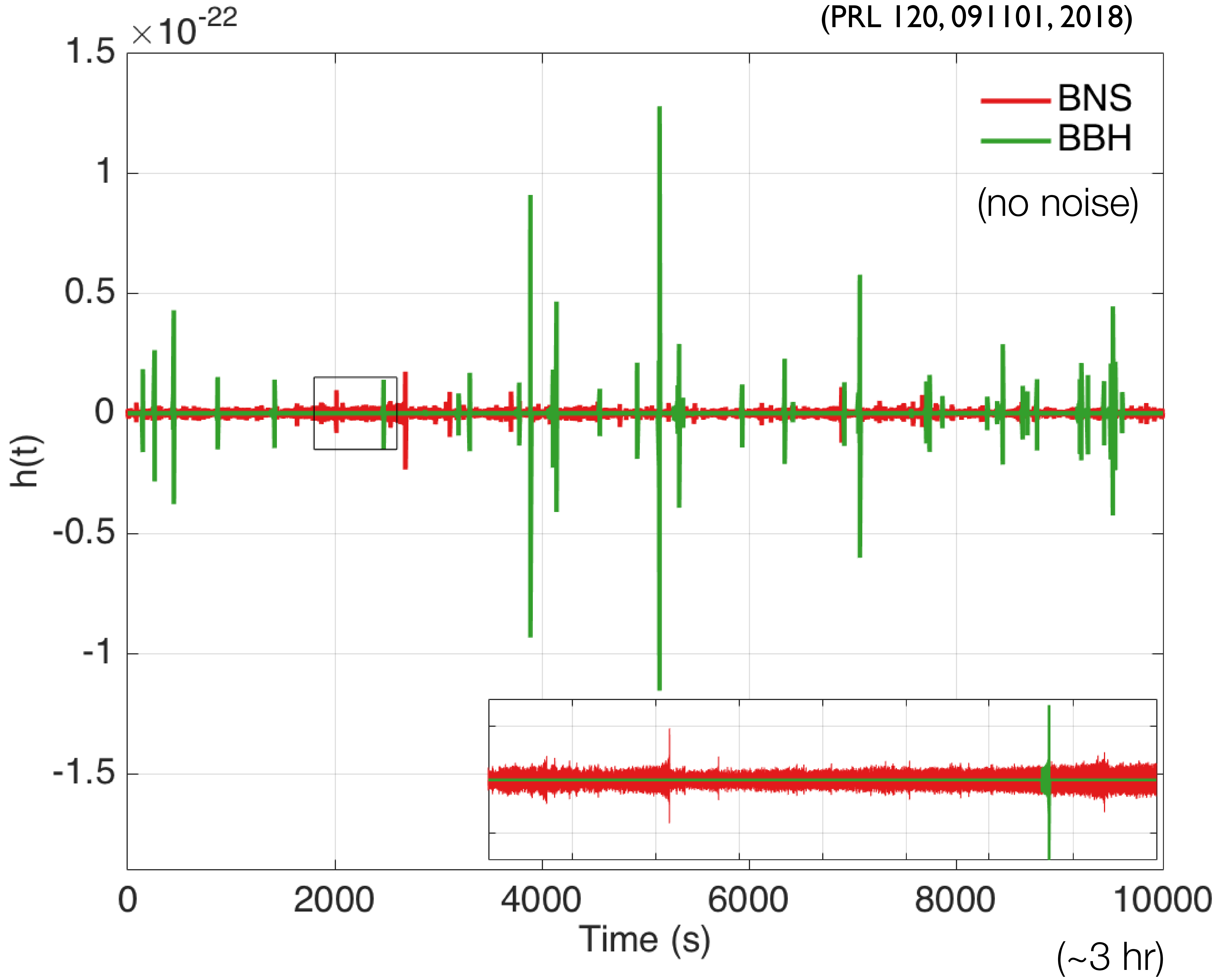
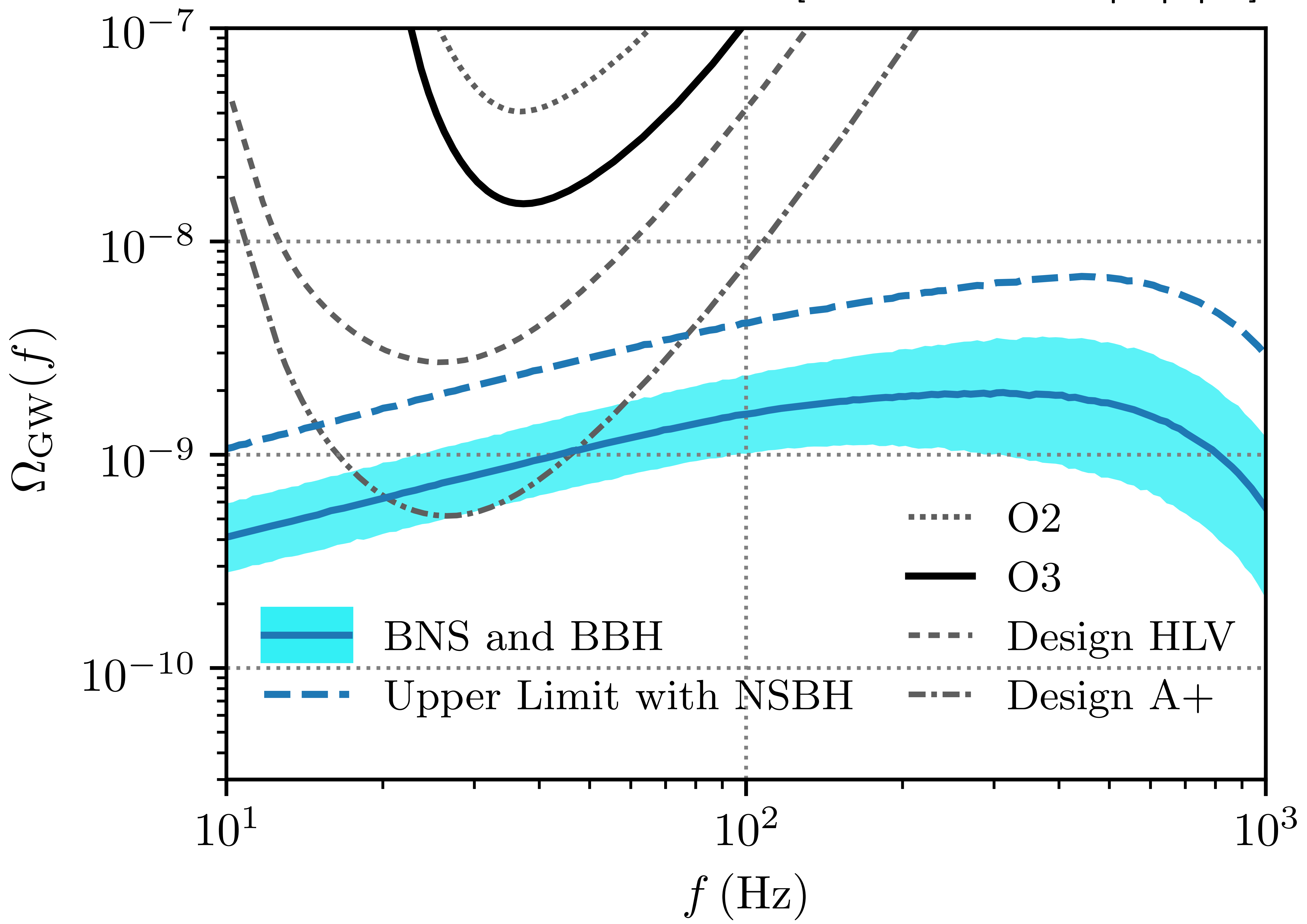


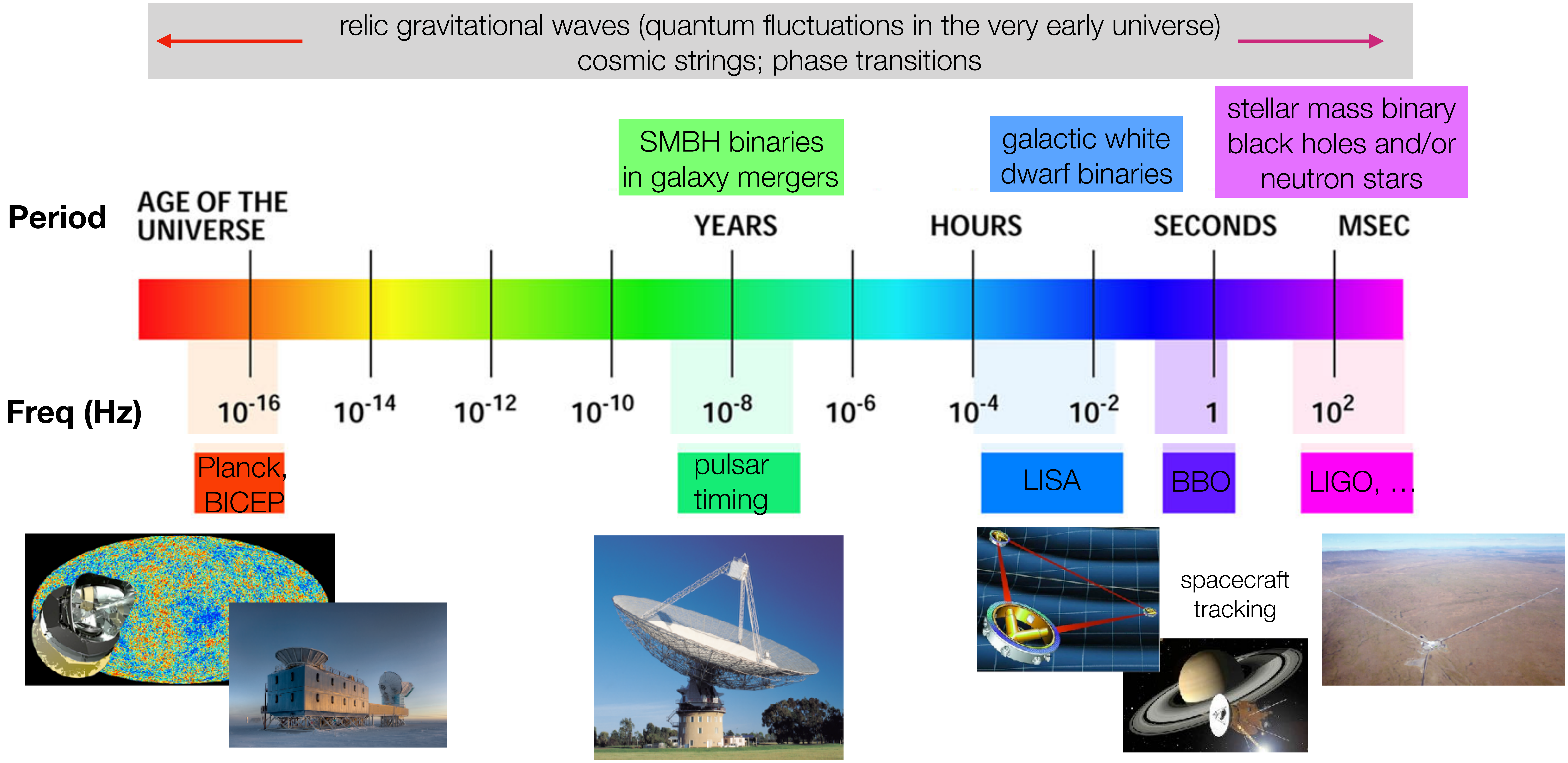
Potentially detectable with Advanced LIGO/Virgo or A+

[credit: LVK O3 Isotropic paper]



Based on standard search, but there exists a better method!
(Smith & Thrane, PRX 8, 021019, 2018)

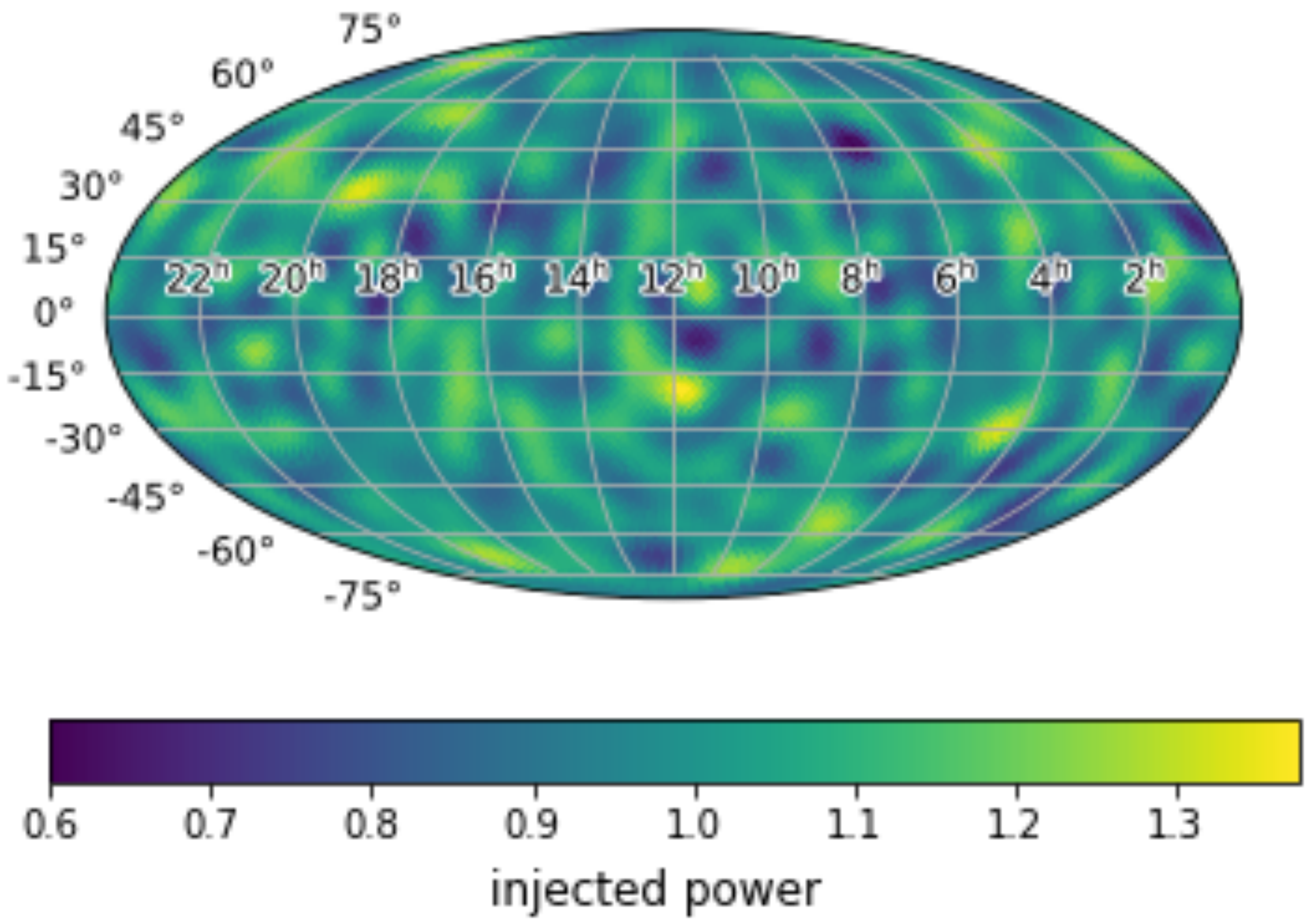
The bigger picture - GWB sources and detectors



II. Different types of stochastic signals

(i) differ in terms of spatial distribution

<u>(statistically) isotropic</u>



(like *cosmic microwave background*)

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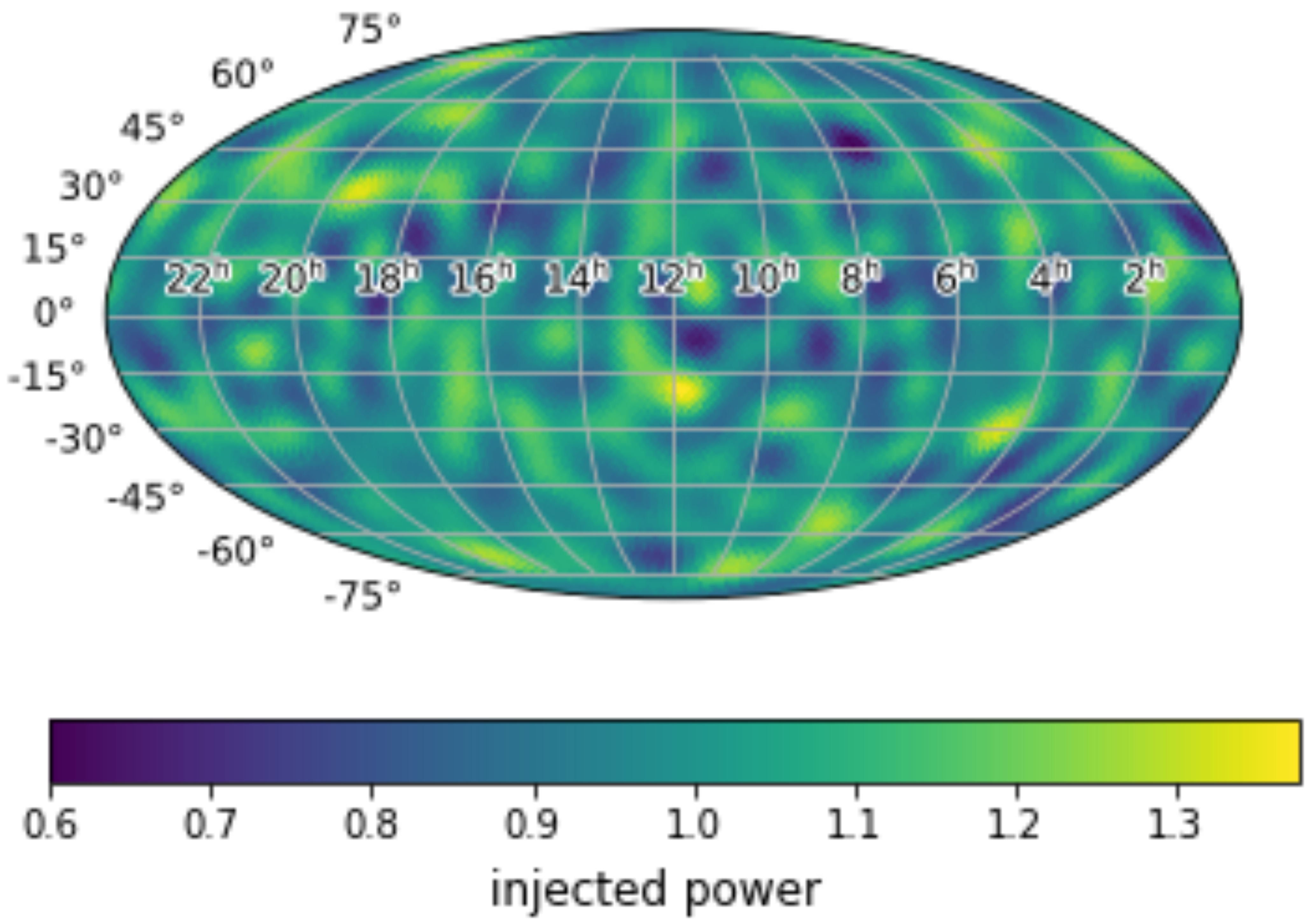
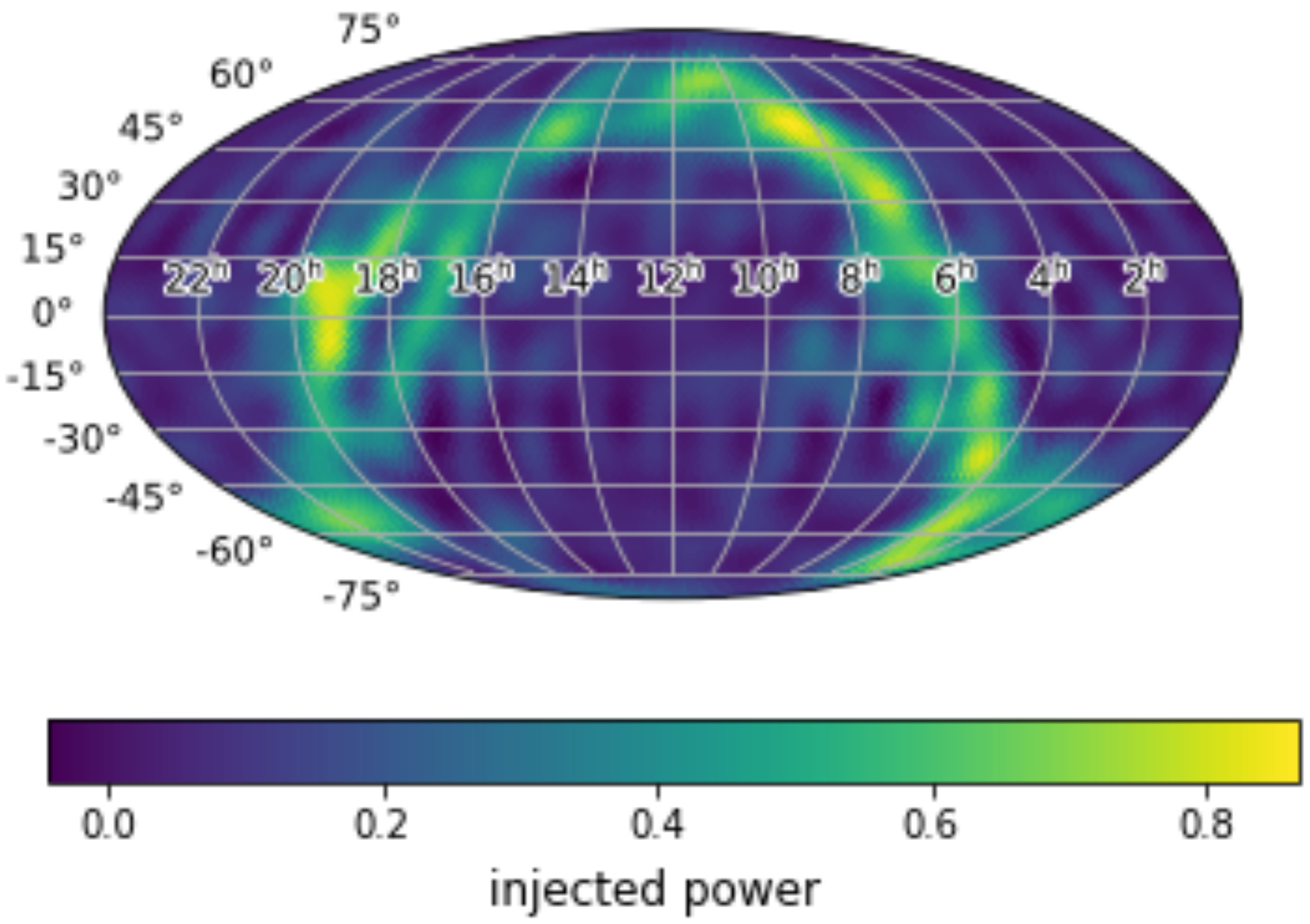


(like *cosmic microwave background*)

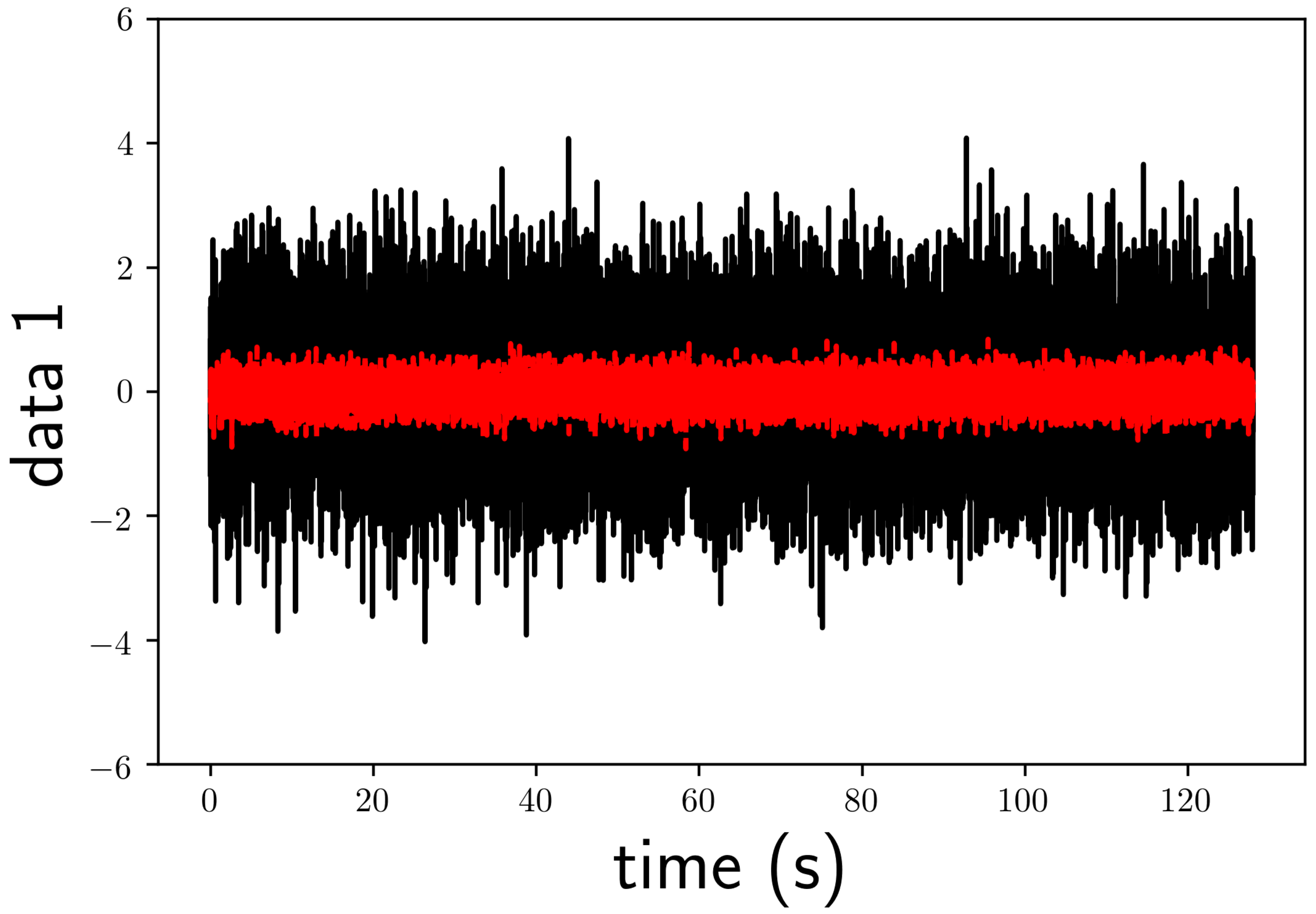
anisotropic

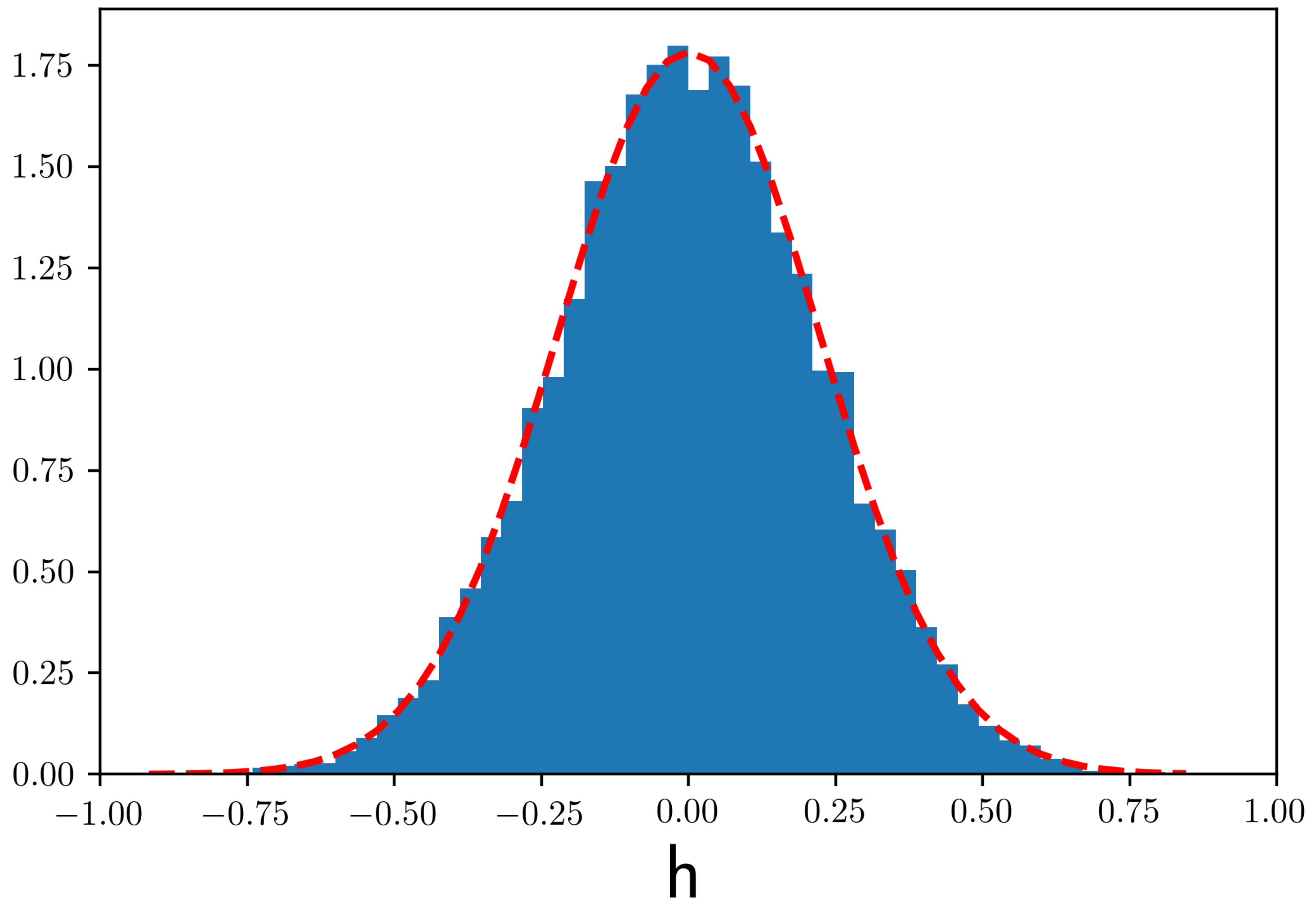


(galactic plane in equatorial coords)

(ii) differ in terms of temporal distribution

<u>Continuous (Gaussian)</u>





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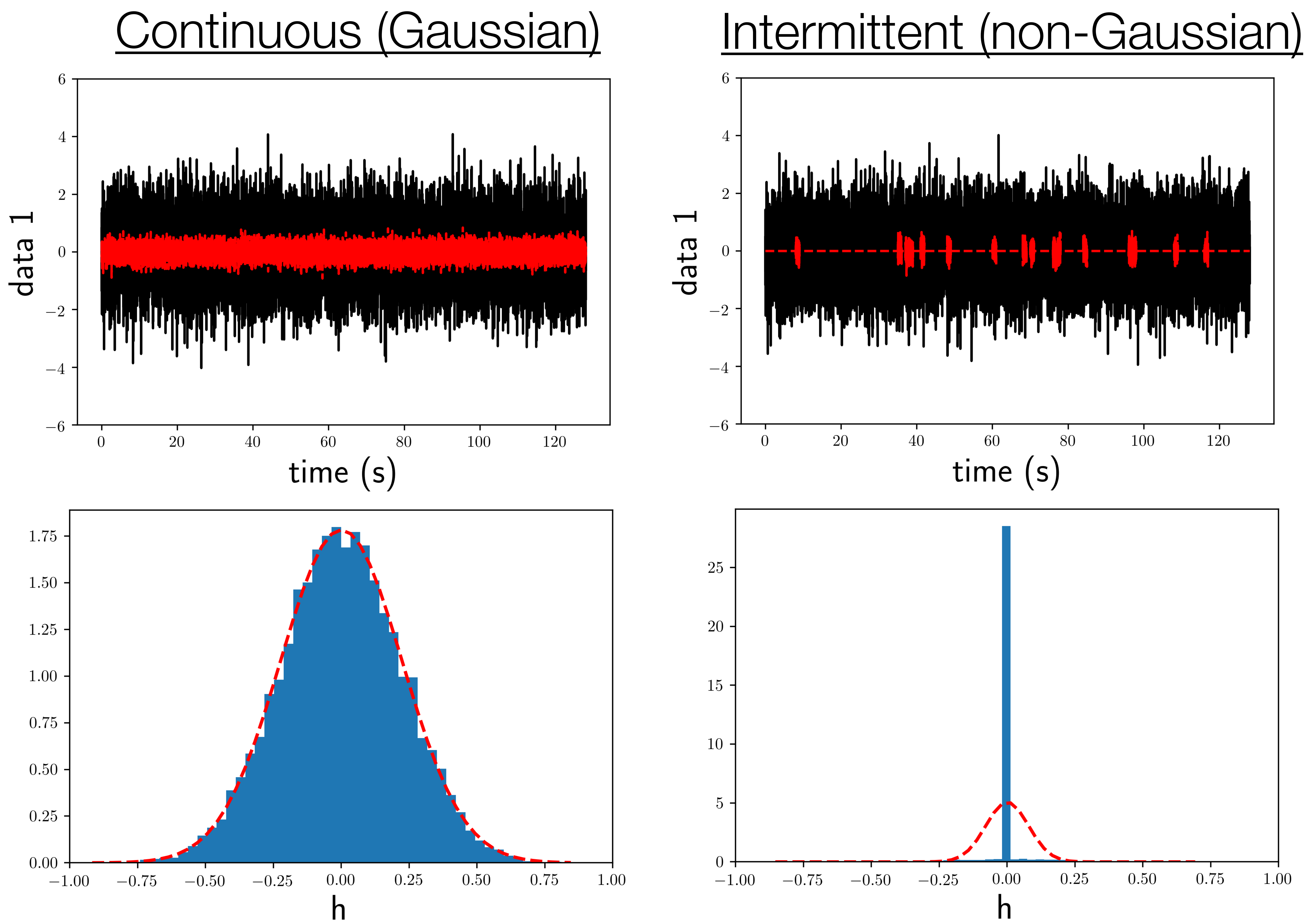

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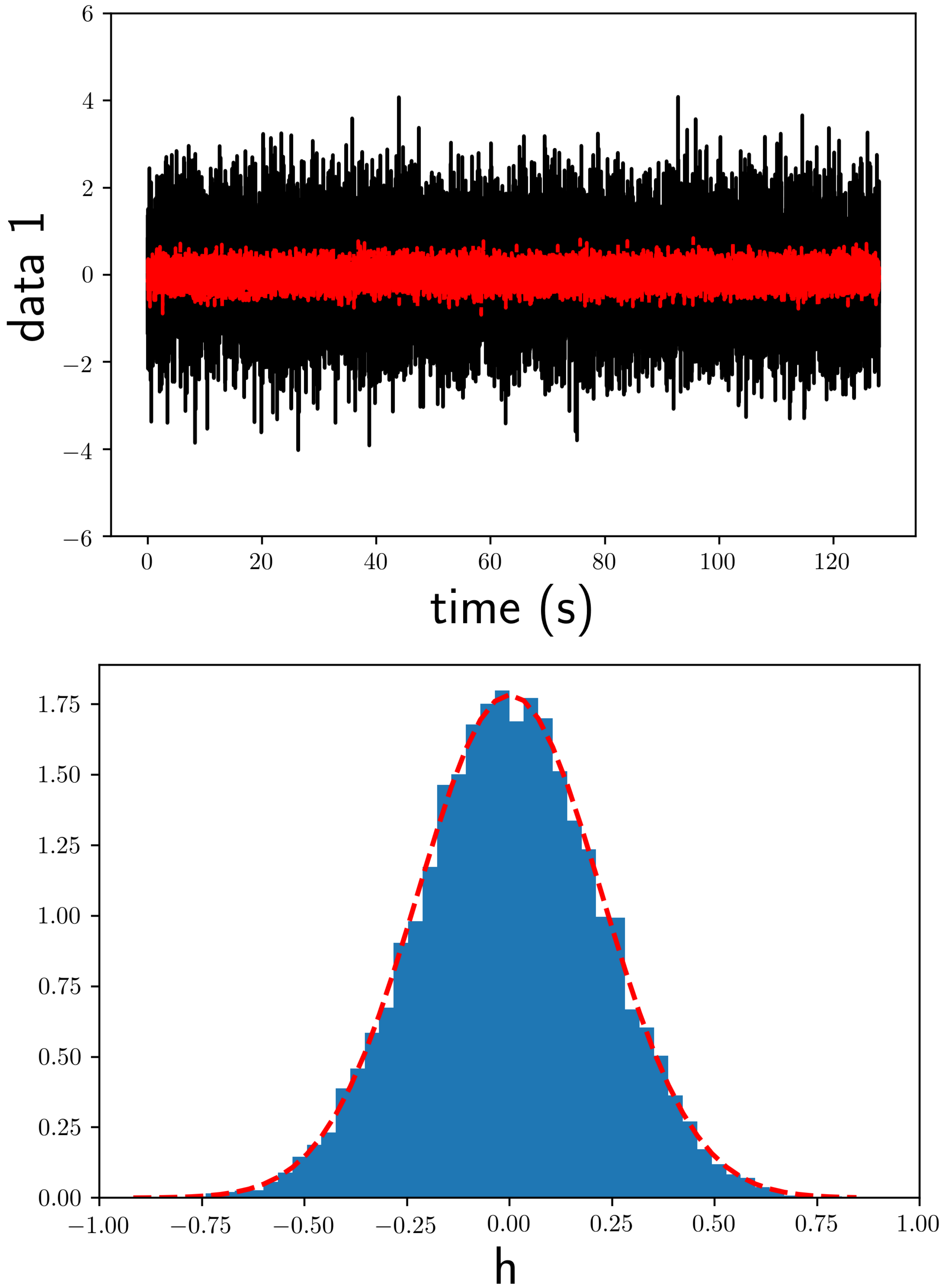
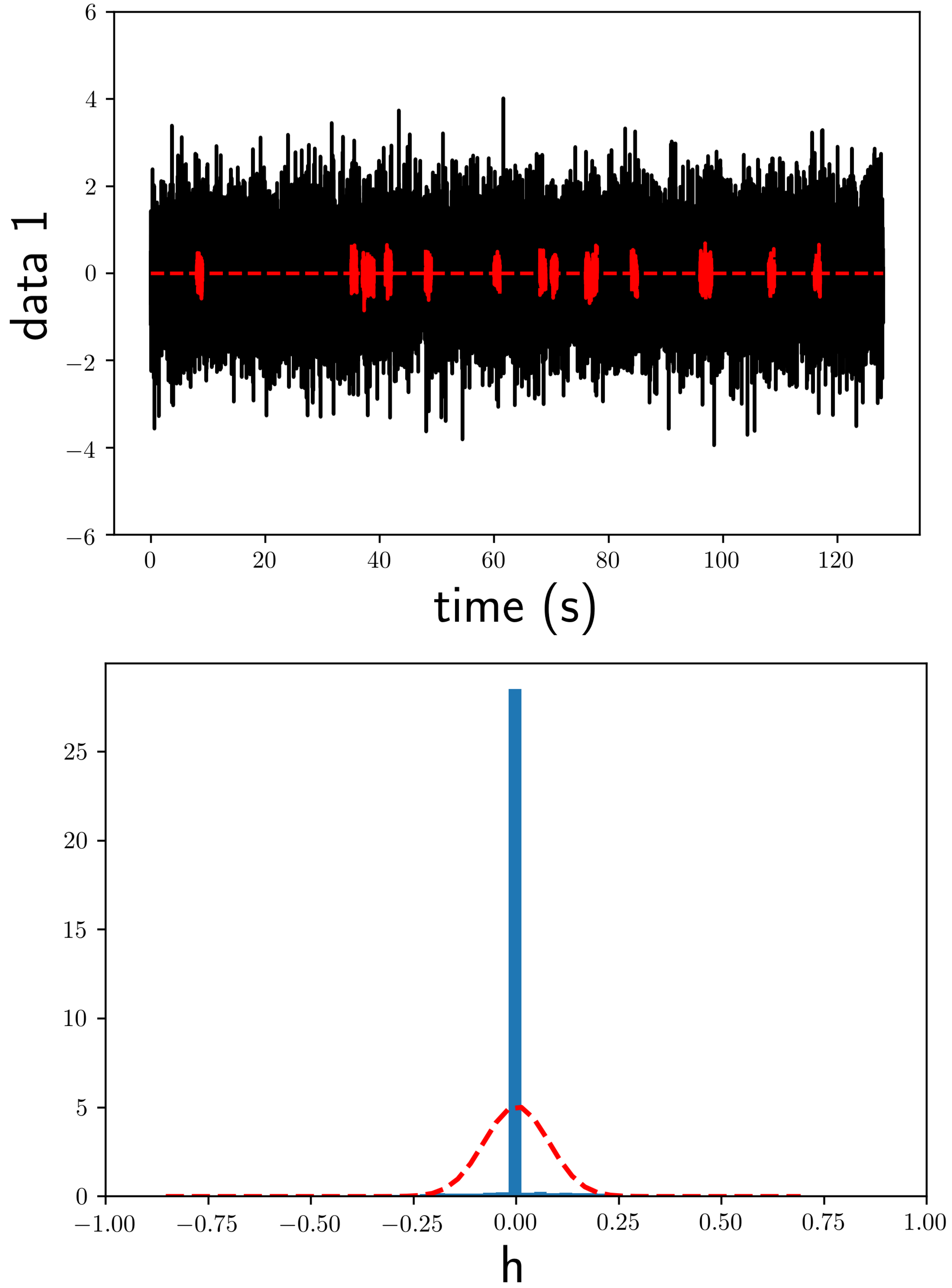
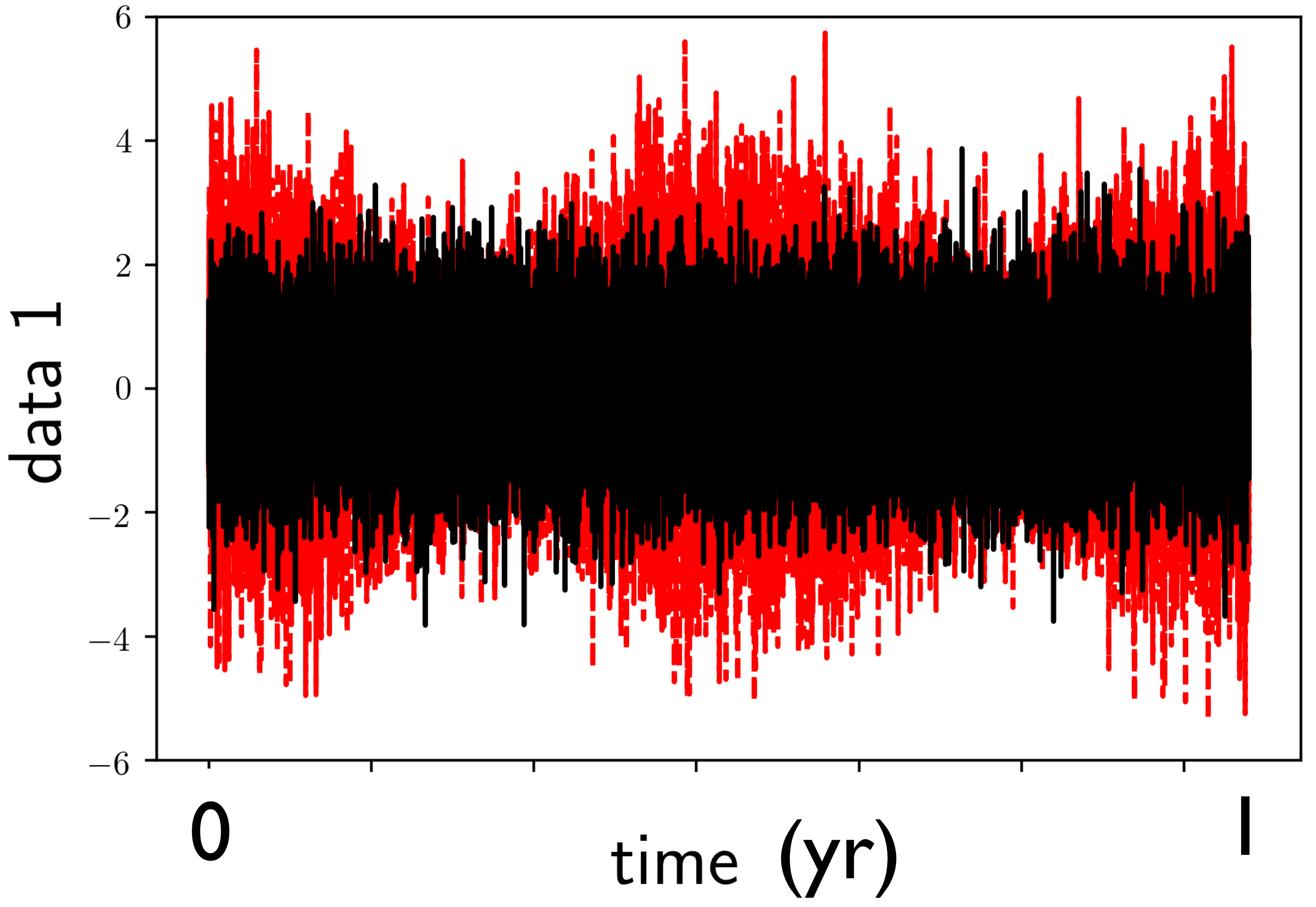


Intermittent (non-Gaussian)



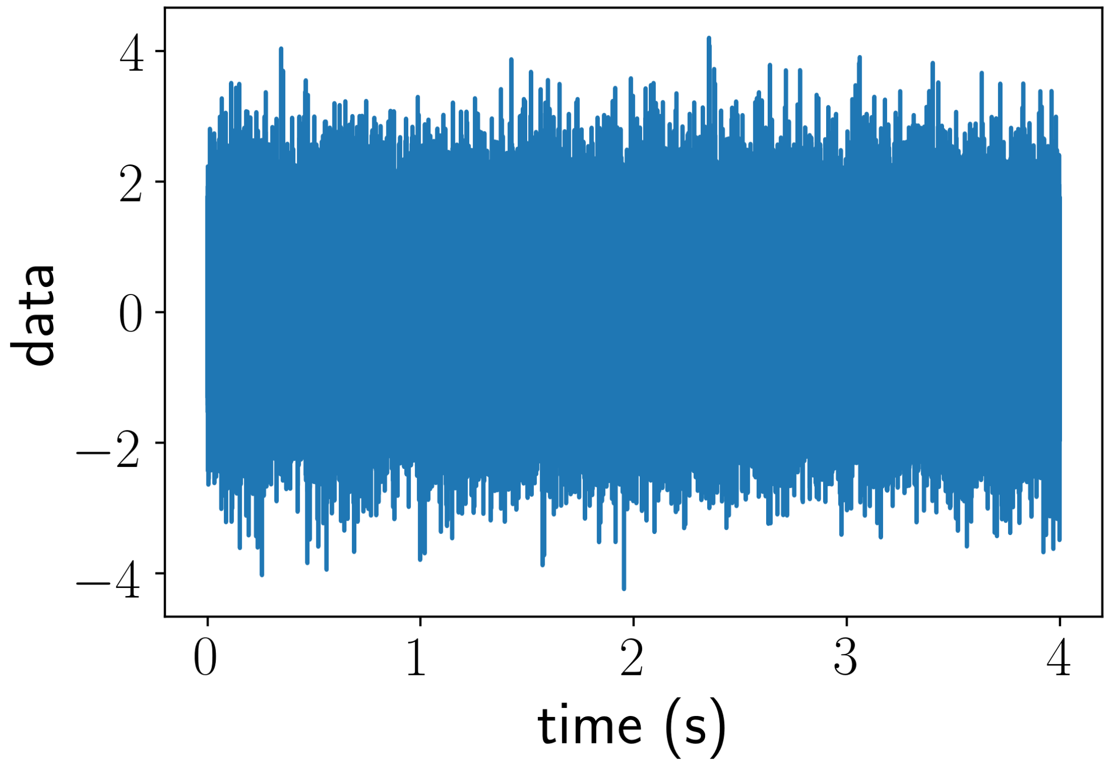
Foreground



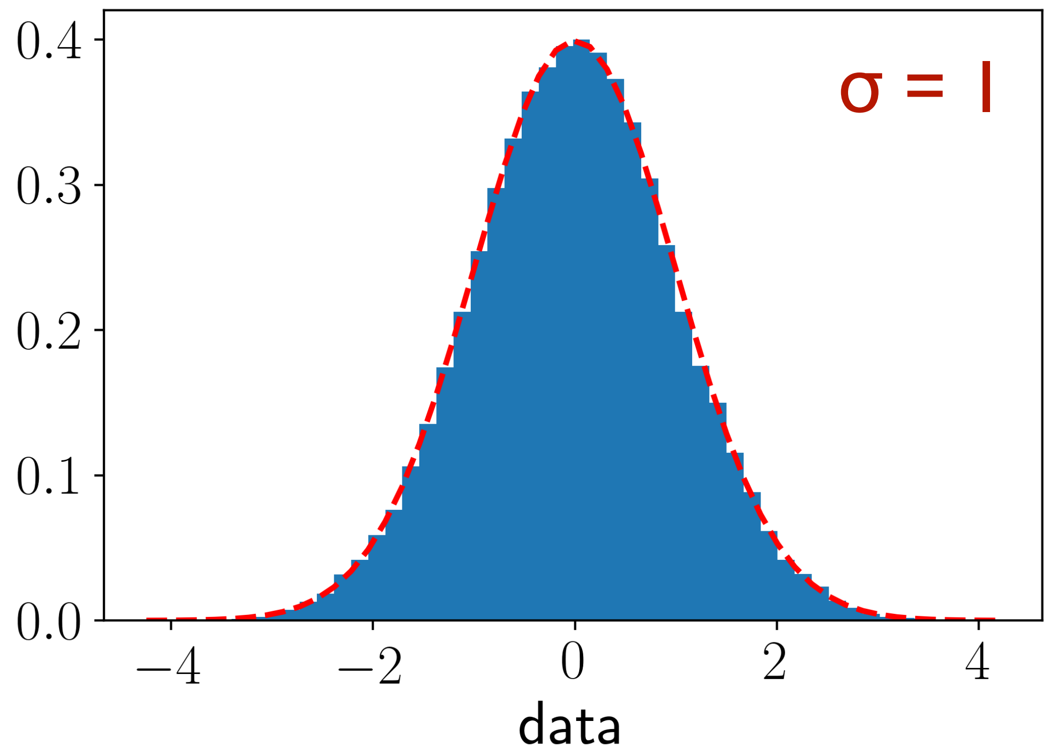
(e.g., from galactic white dwarf binaries; modulated by LISA's orbital motion)

(iii) differ in terms of spectral distribution (power spectra)

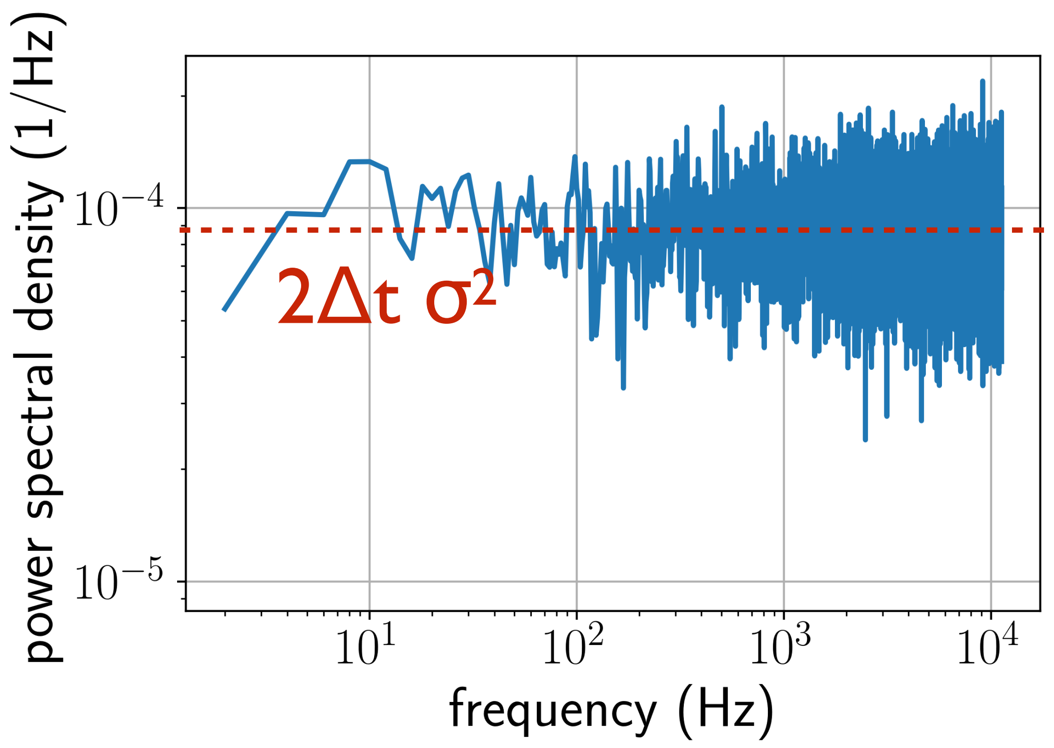
white noise



histograms



power spectra



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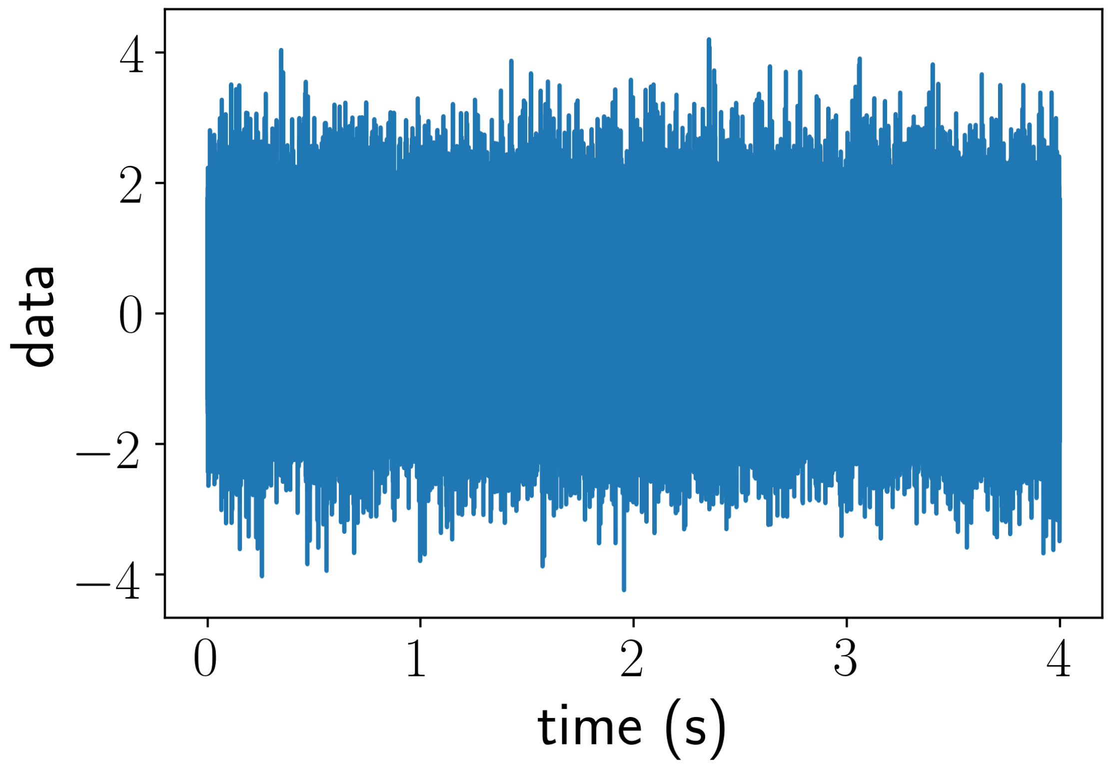
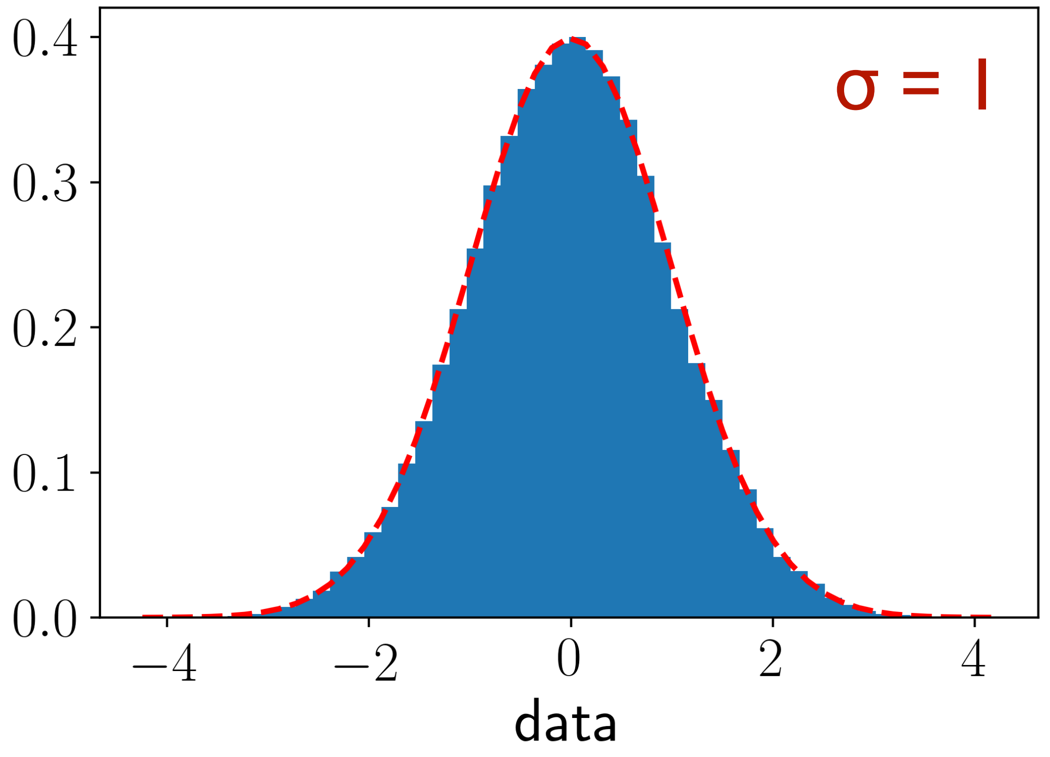
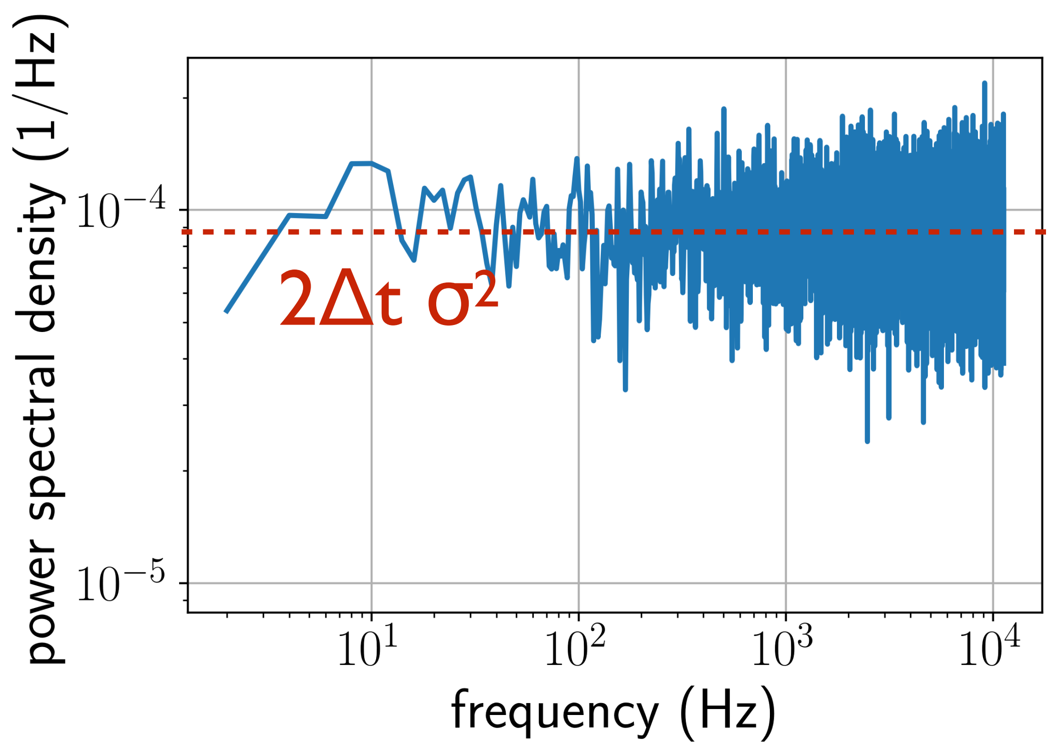
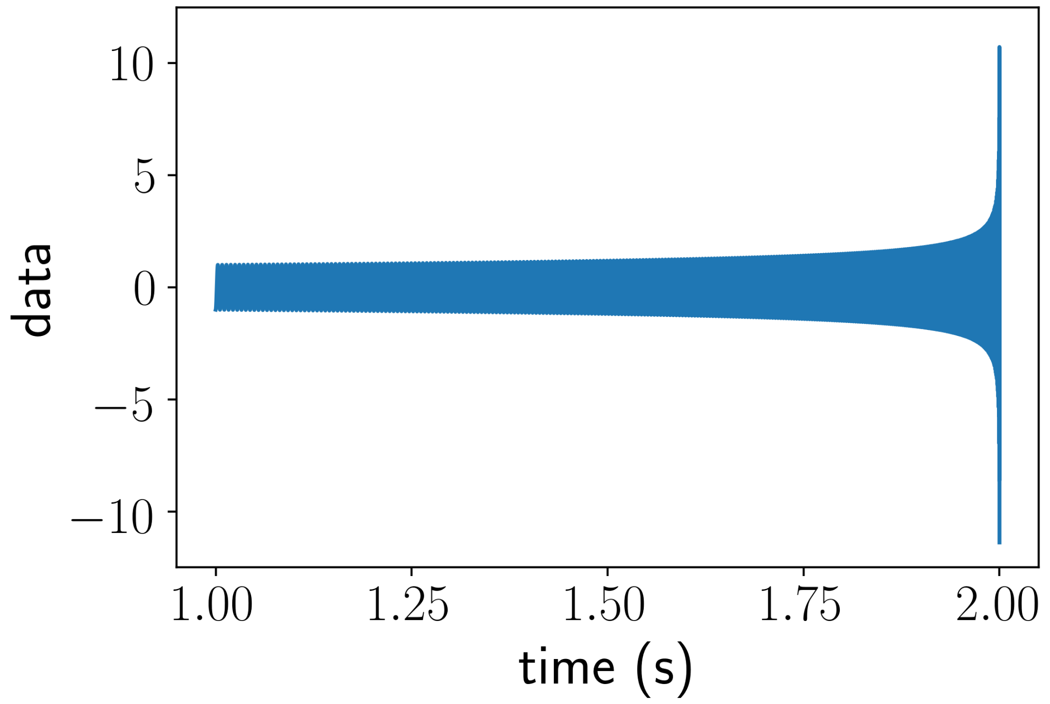
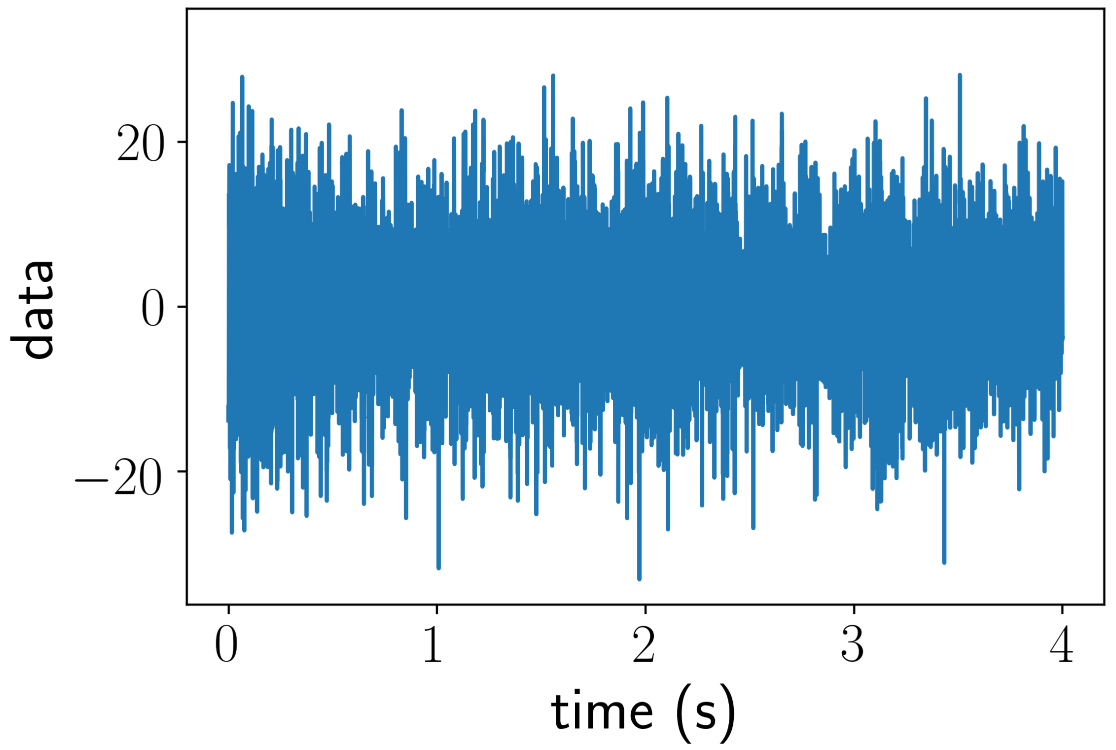


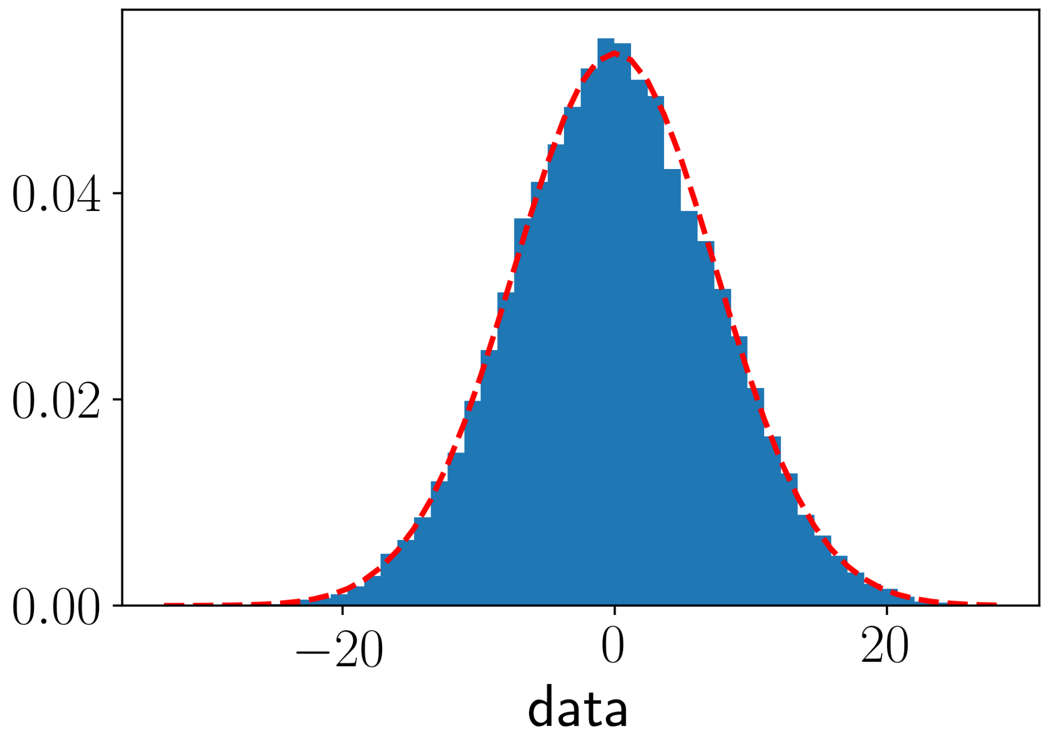


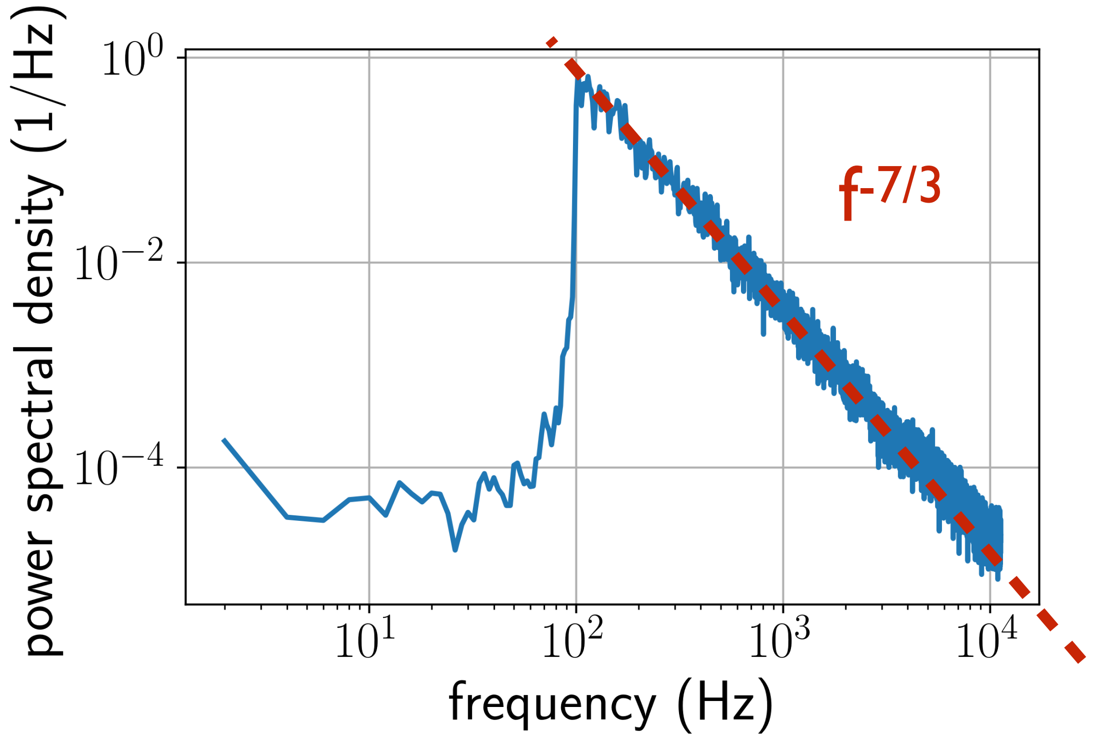


BNS chirp









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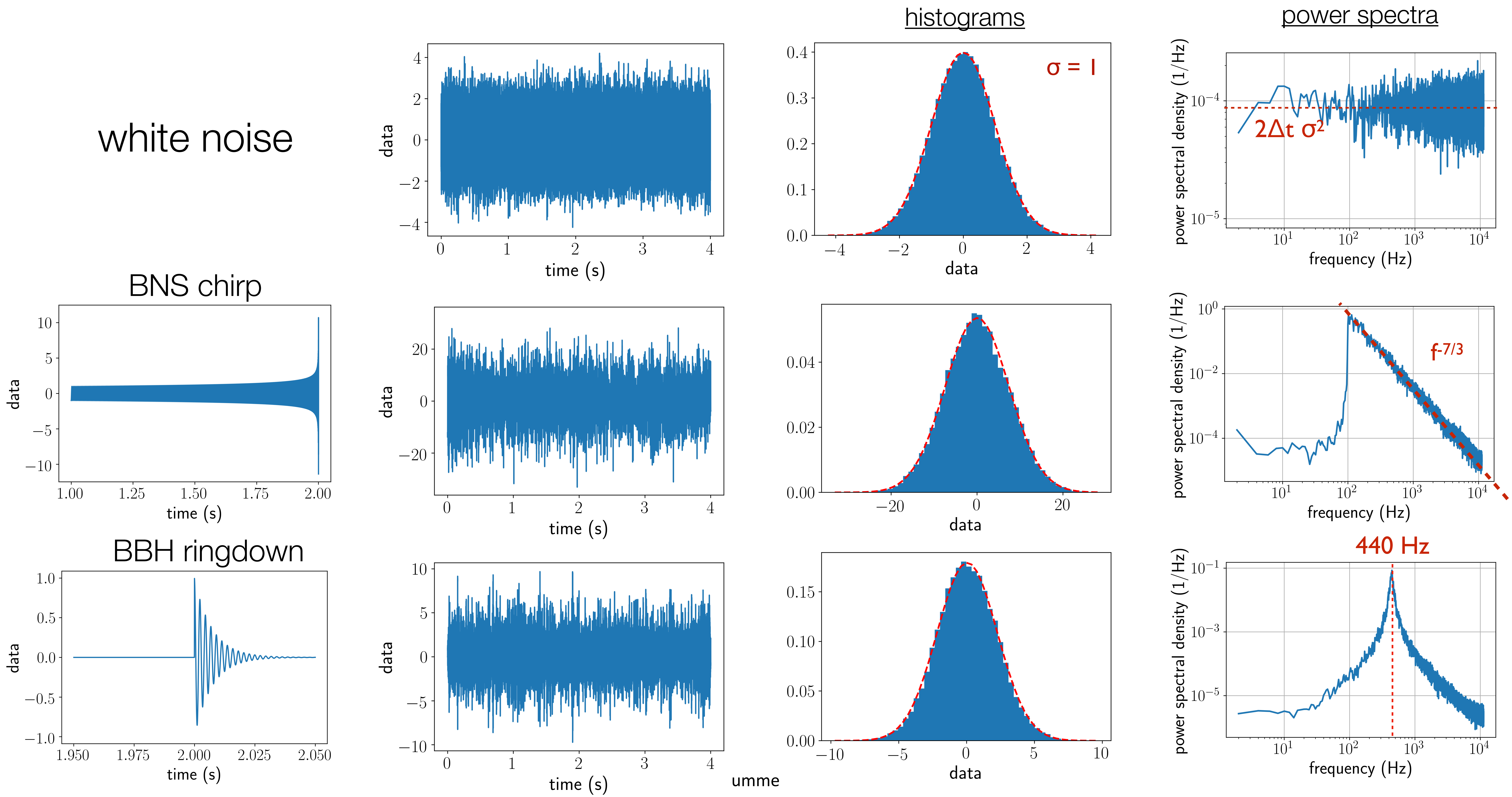

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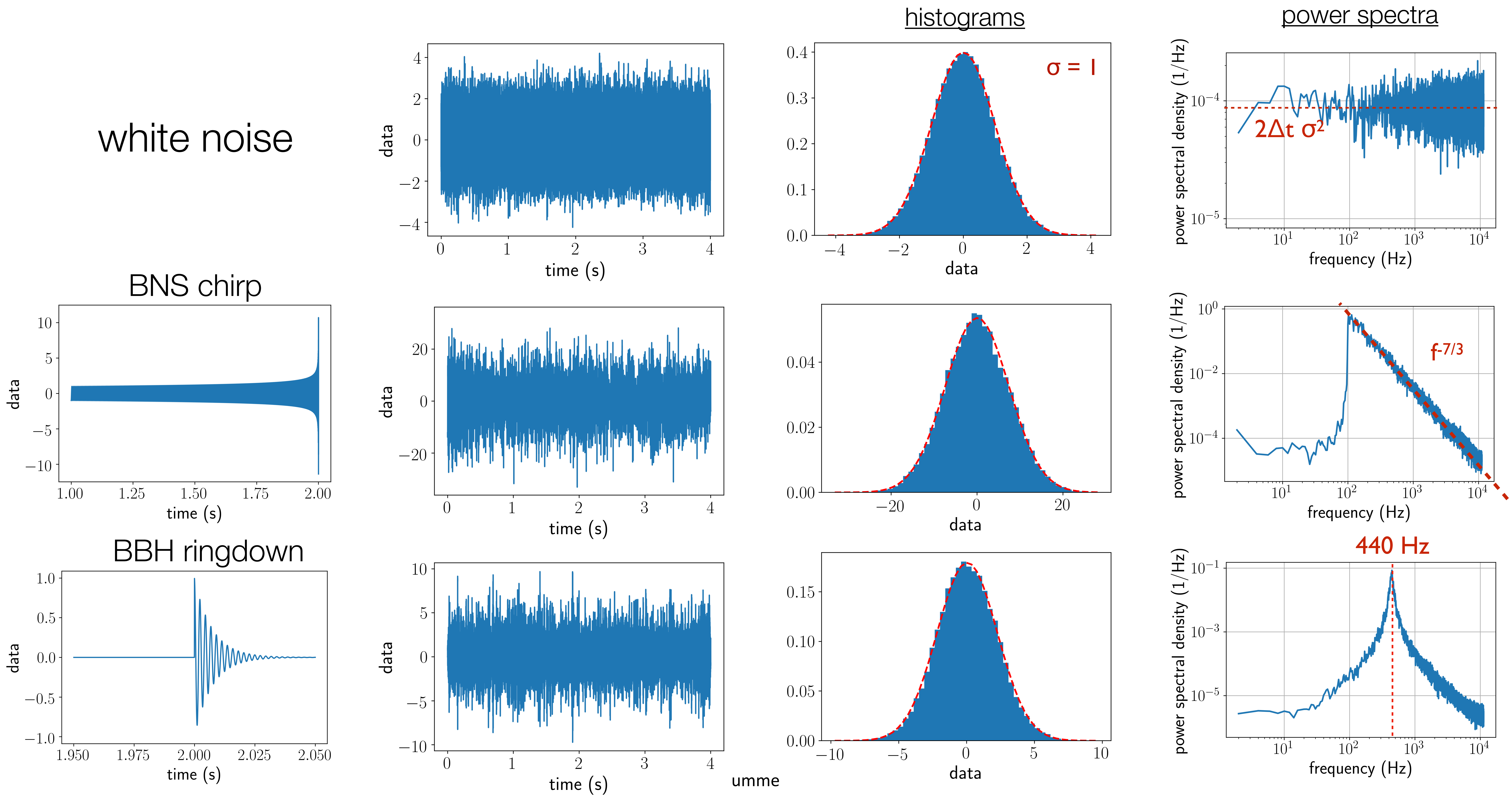

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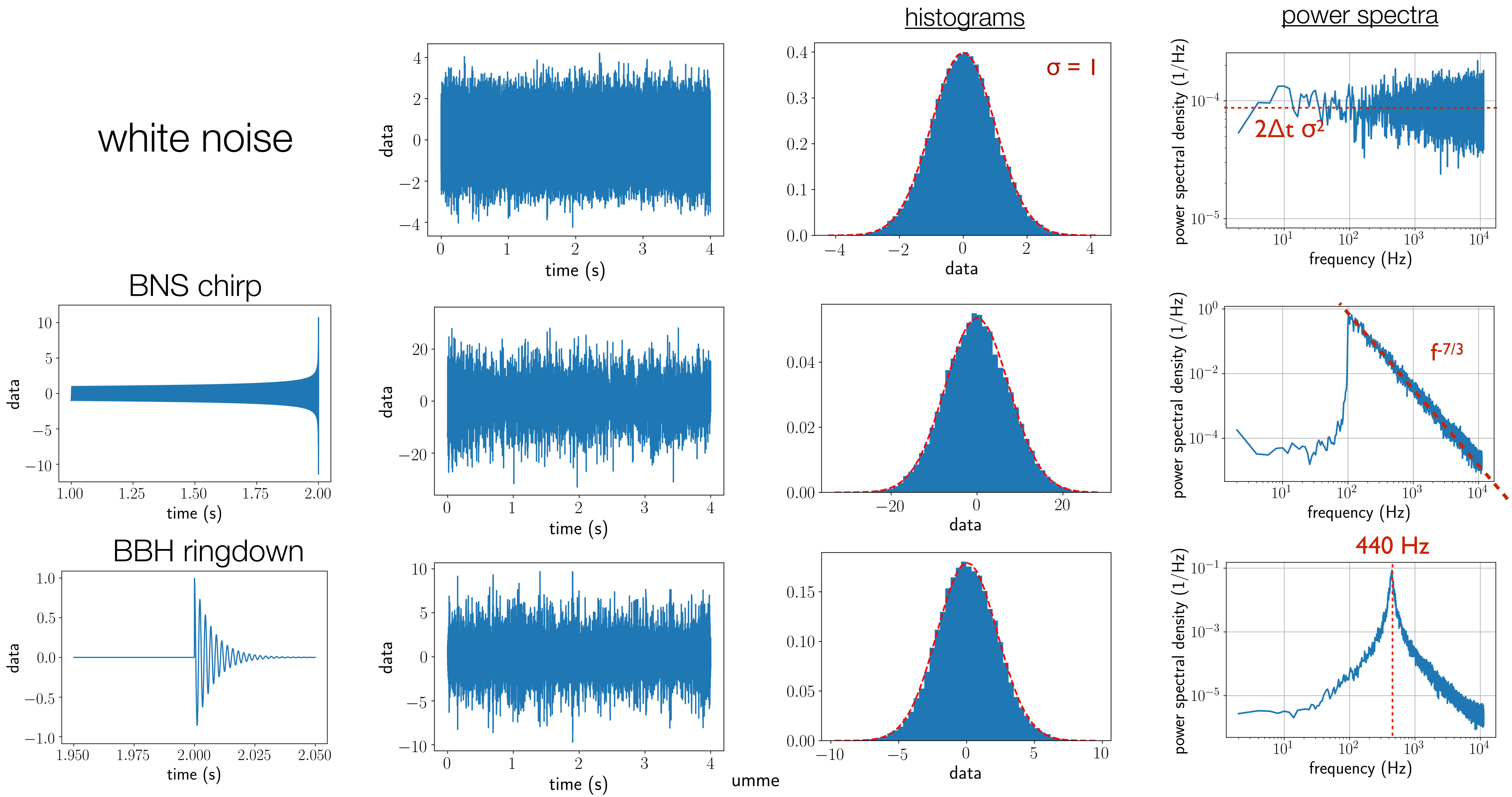

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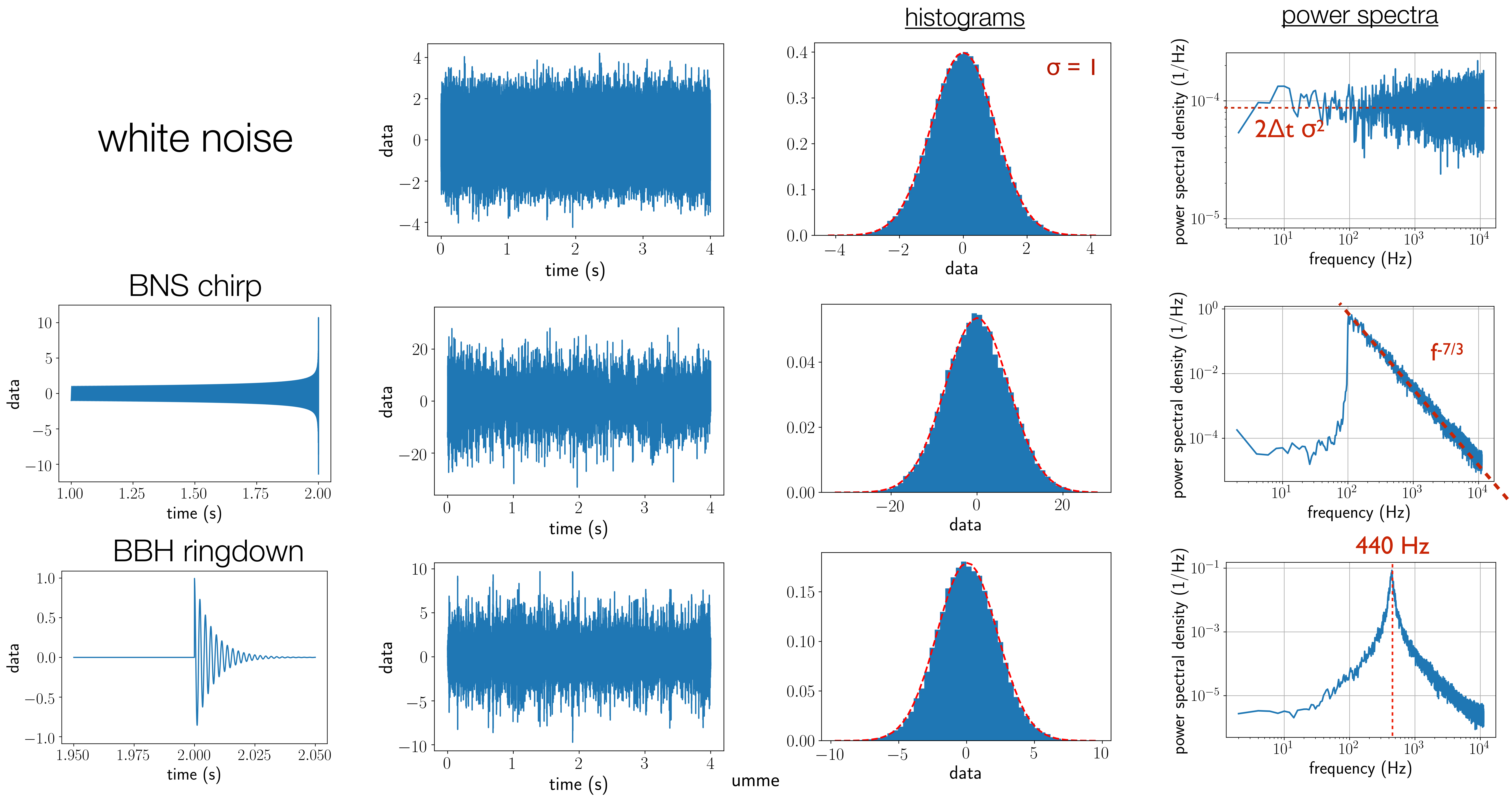
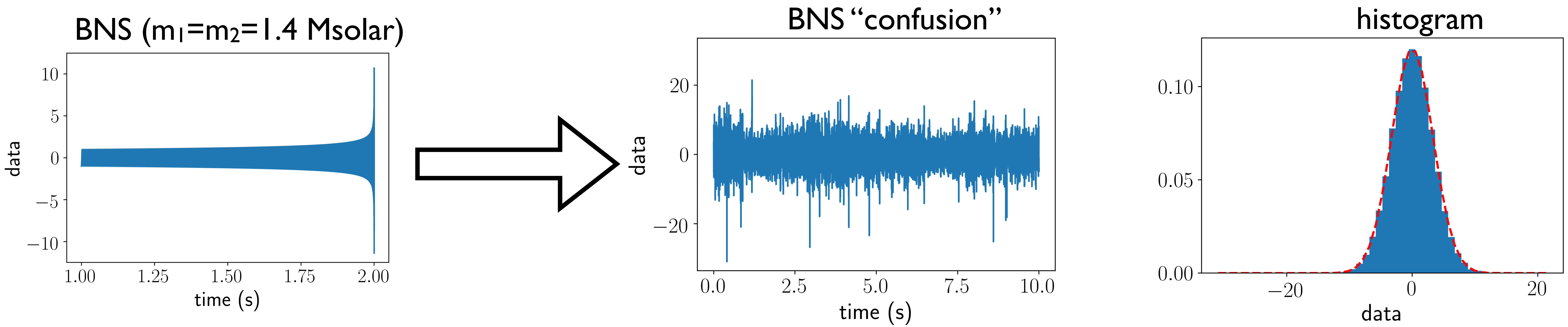


Rate estimate predict BNS “confusion” and BBH “popcorn”



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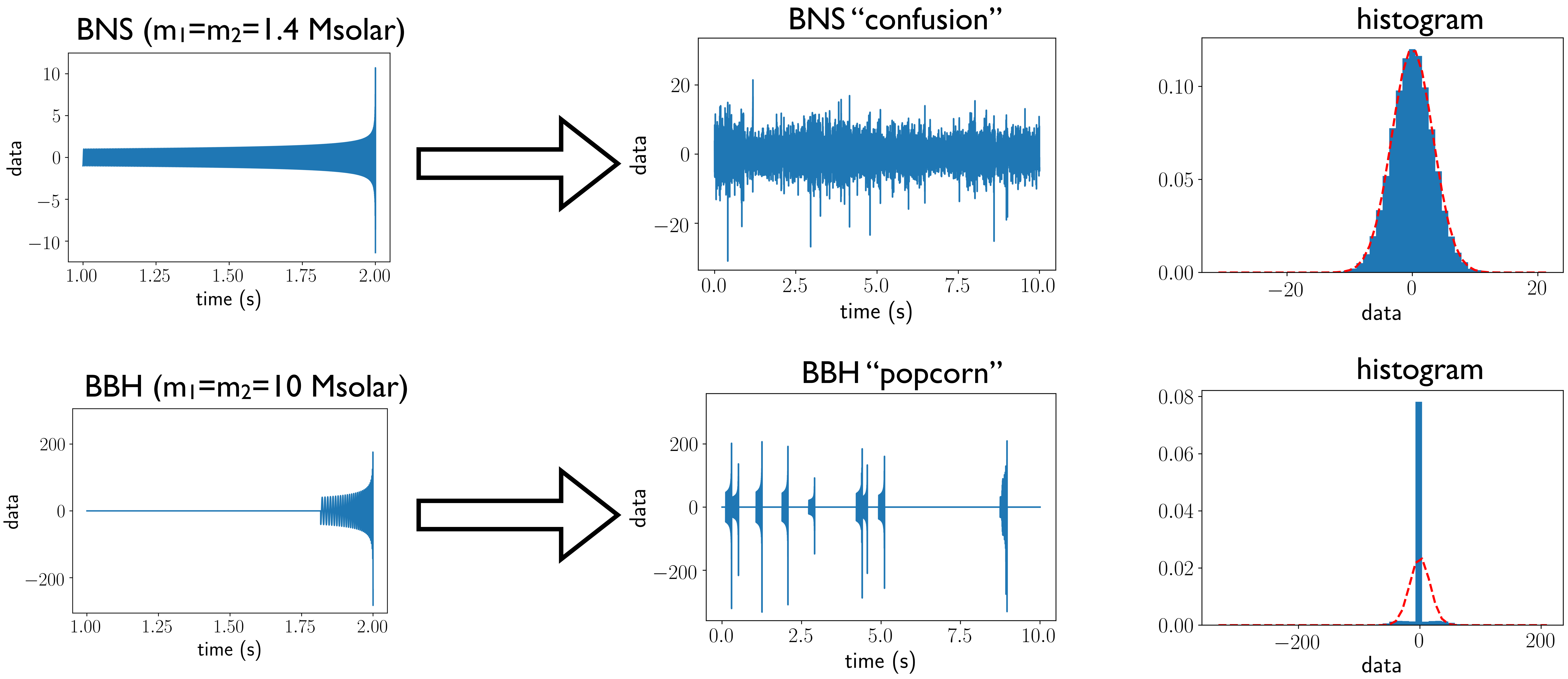

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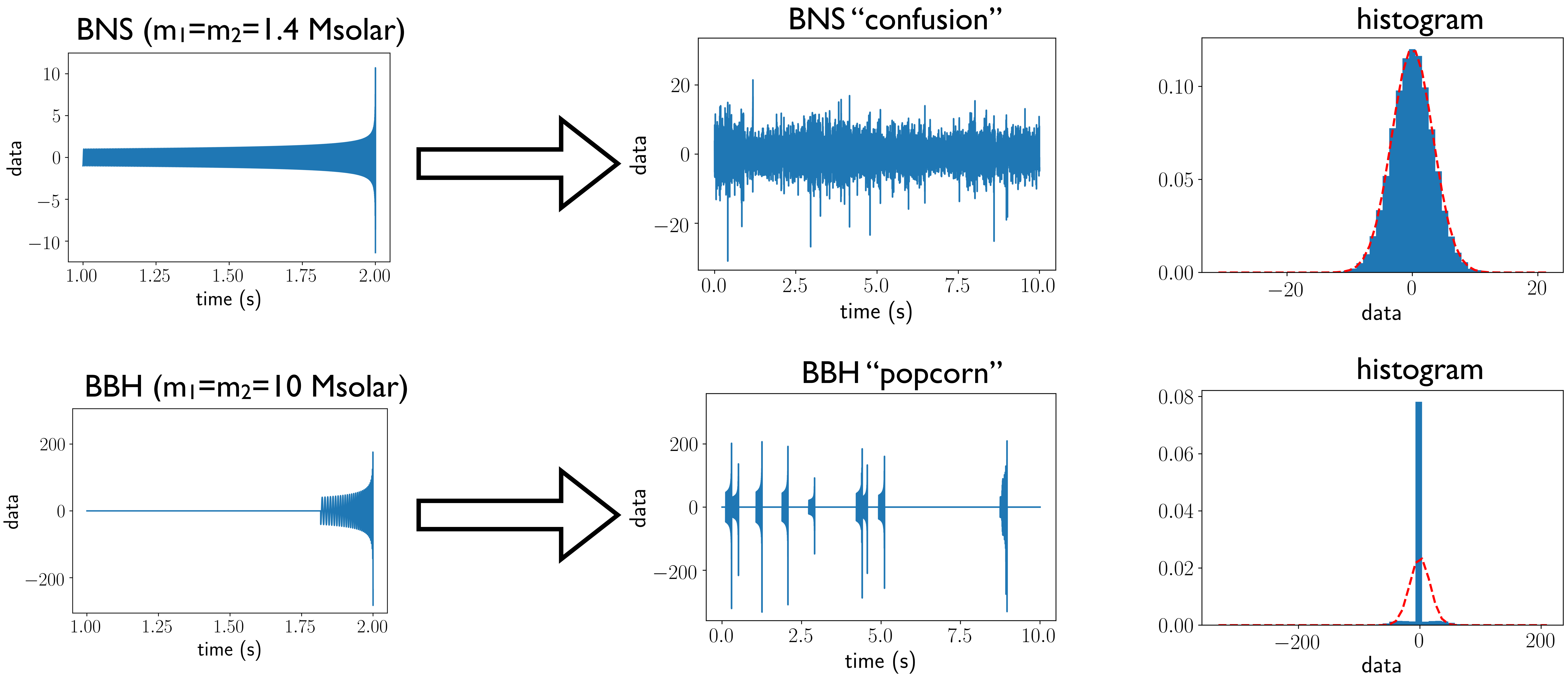

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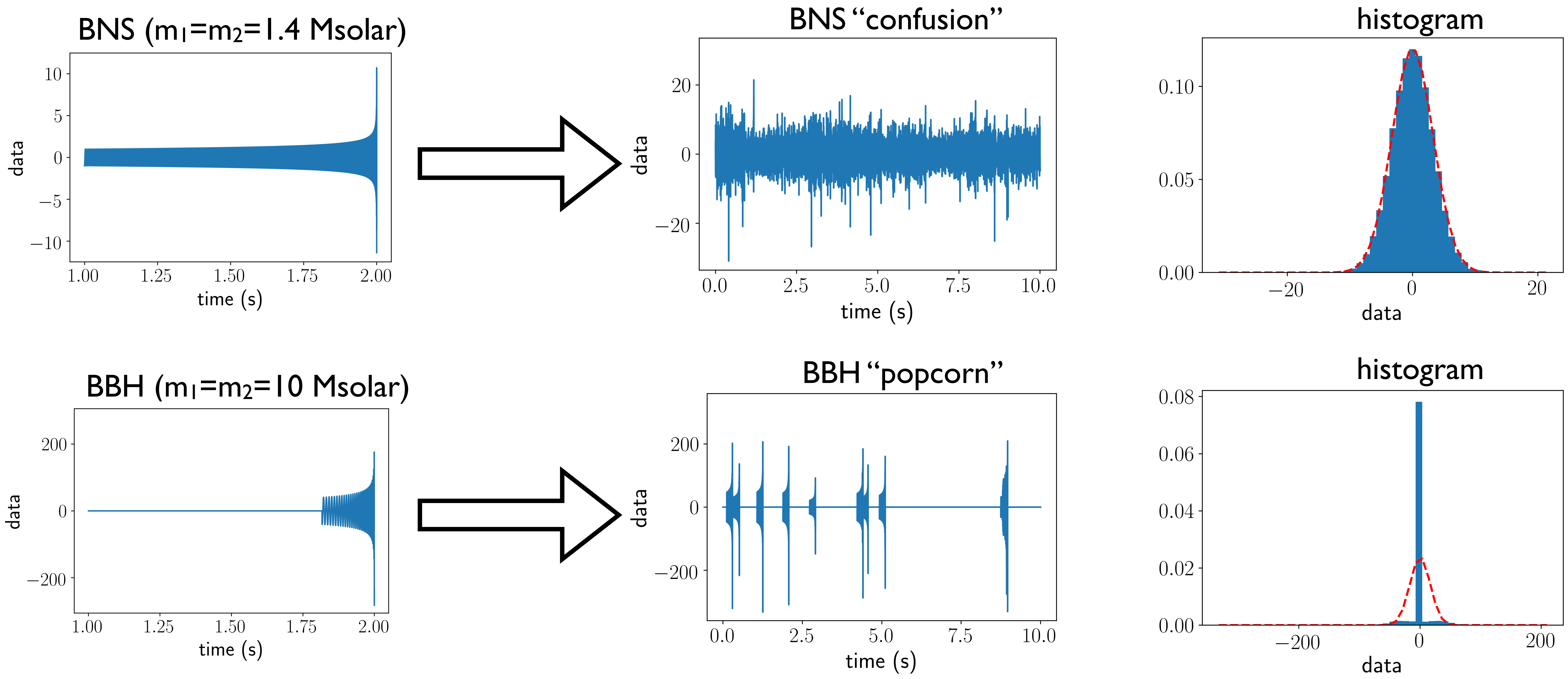

III. How do we characterize a stochastic background??

Definition

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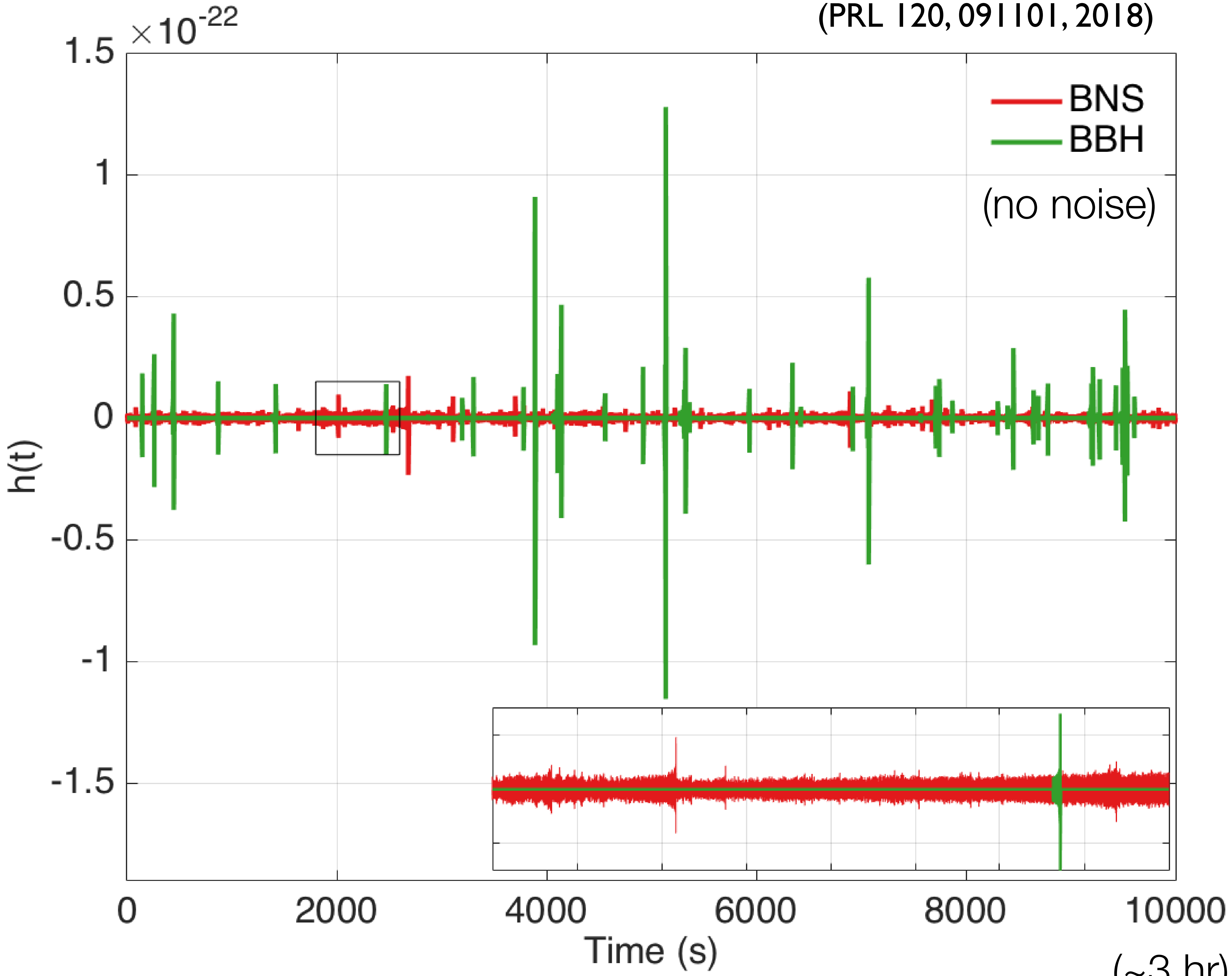
(PRL 120, 091101, 2018)

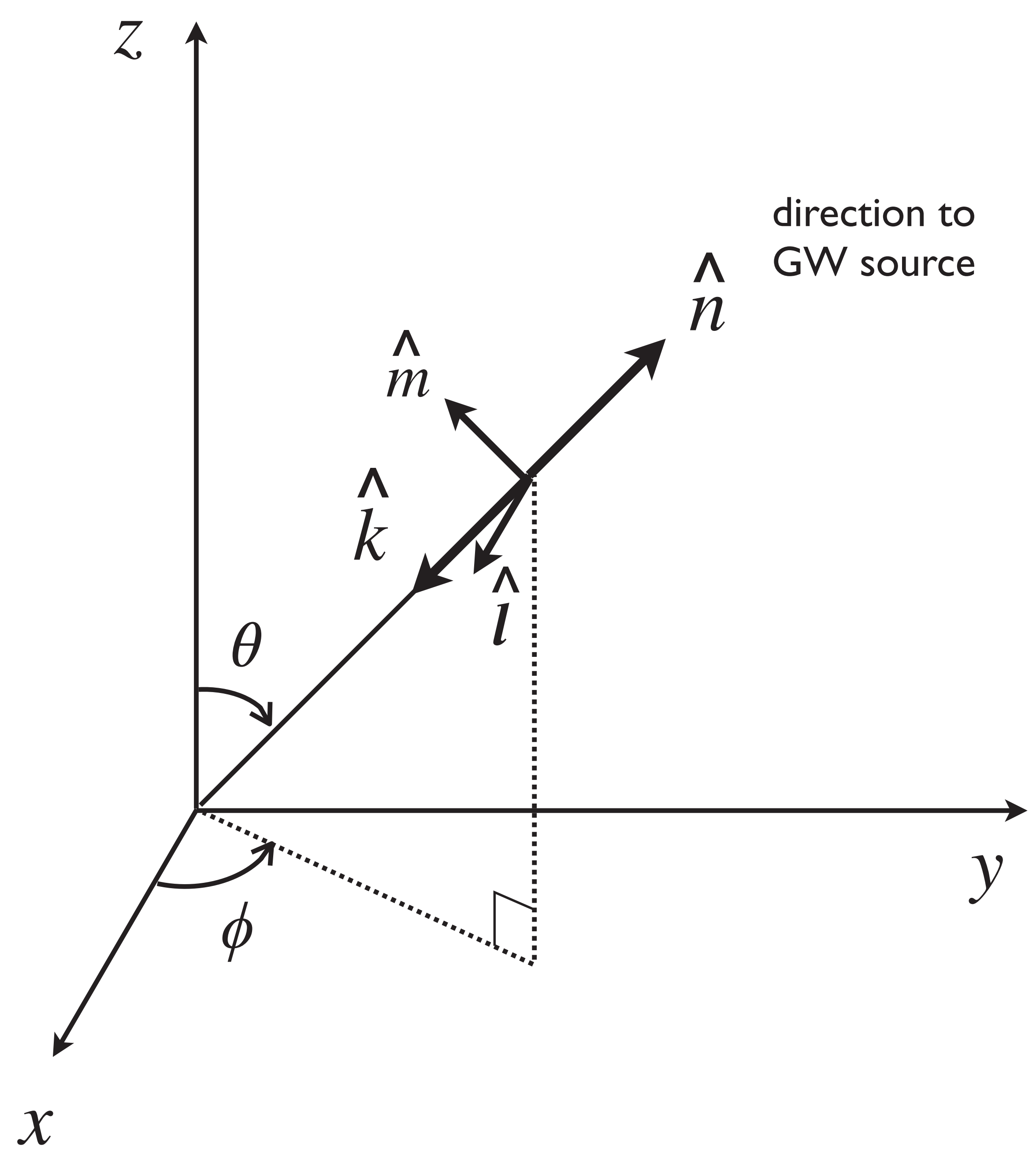


(~3 hr)

Plane wave expansion, ensemble average

- Plane wave expansion:

$$h_{ab}(t, \vec{x}) = \int_{-\infty}^{\infty} \mathrm{d}f \int \mathrm{d}^2\Omega_{\hat{k}} \sum_{A=+,\times} h_A(f, \hat{k}) e_{ab}^A(\hat{k})\, \mathrm{e}^{i2\pi f(t - \hat{k}\cdot\vec{x}/c)}$$



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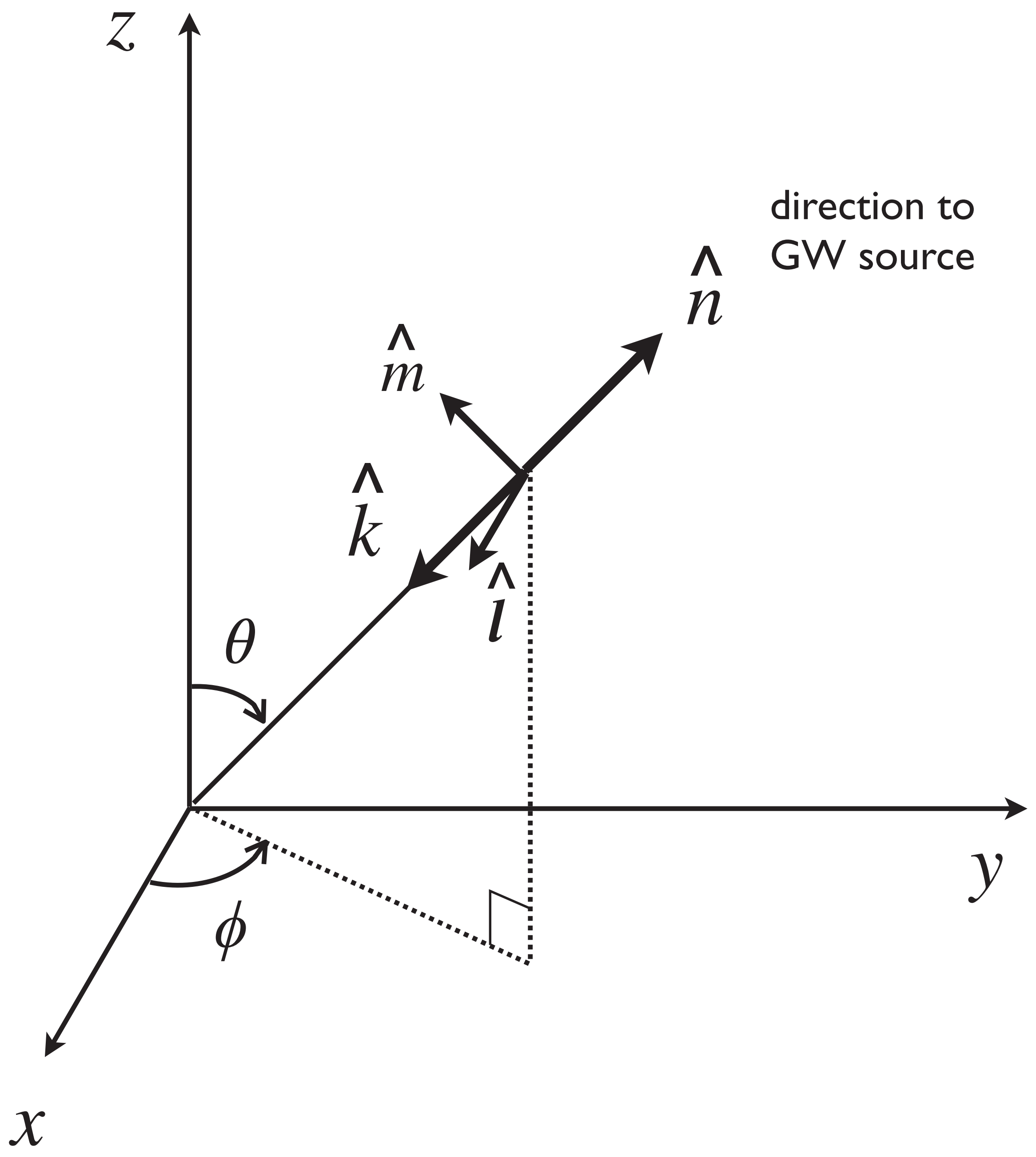
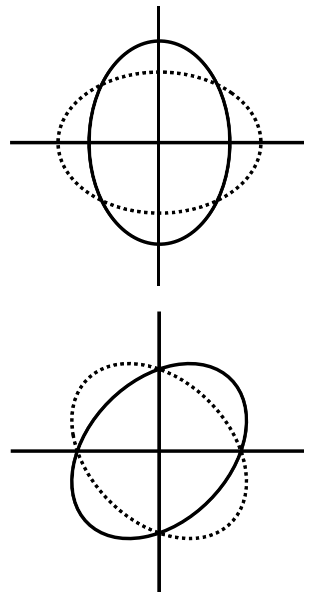


- Polarization tensors:

$$e^+_{ab}(\hat{k}) = \hat{l}_a\hat{l}_b - \hat{m}_a\hat{m}_b\,, \qquad e^\times_{ab}(\hat{k}) = \hat{l}_a\hat{m}_b + \hat{m}_a\hat{l}_b$$

$$\hat{k} = -\,\hat{r}\,, \quad \hat{l} = -\,\hat{\phi}\,, \quad \hat{m} = -\,\hat{\theta}$$



A=+

A=x

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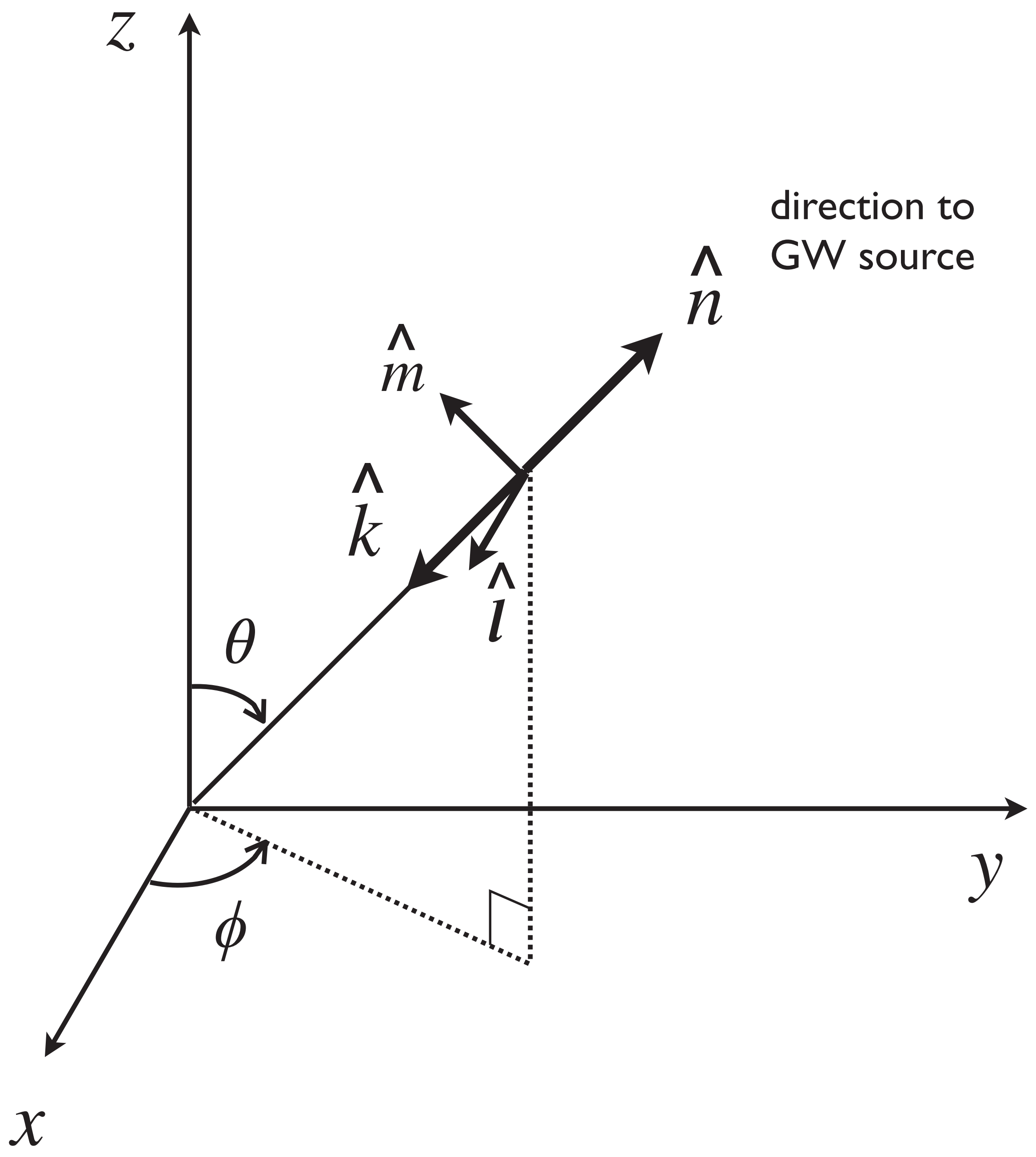

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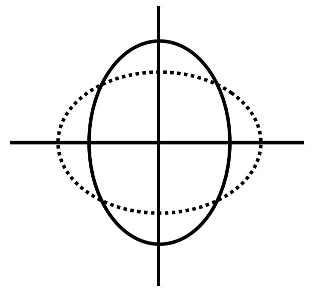
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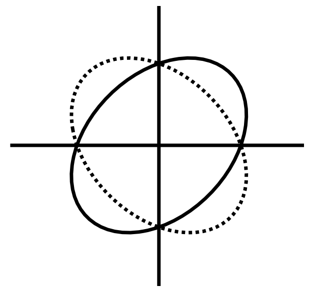
A=x

- Statistical properties encoded in:

$$\langle h_A(f,\hat{k})\rangle\,, \quad \langle h_A(f,\hat{k})h_{A'}(f',\hat{k}')\rangle\,, \quad \langle h_A(f,\hat{k})h_{A'}(f',\hat{k}')h_{A''}(f'',\hat{k}'')\rangle\,, \quad \cdots$$

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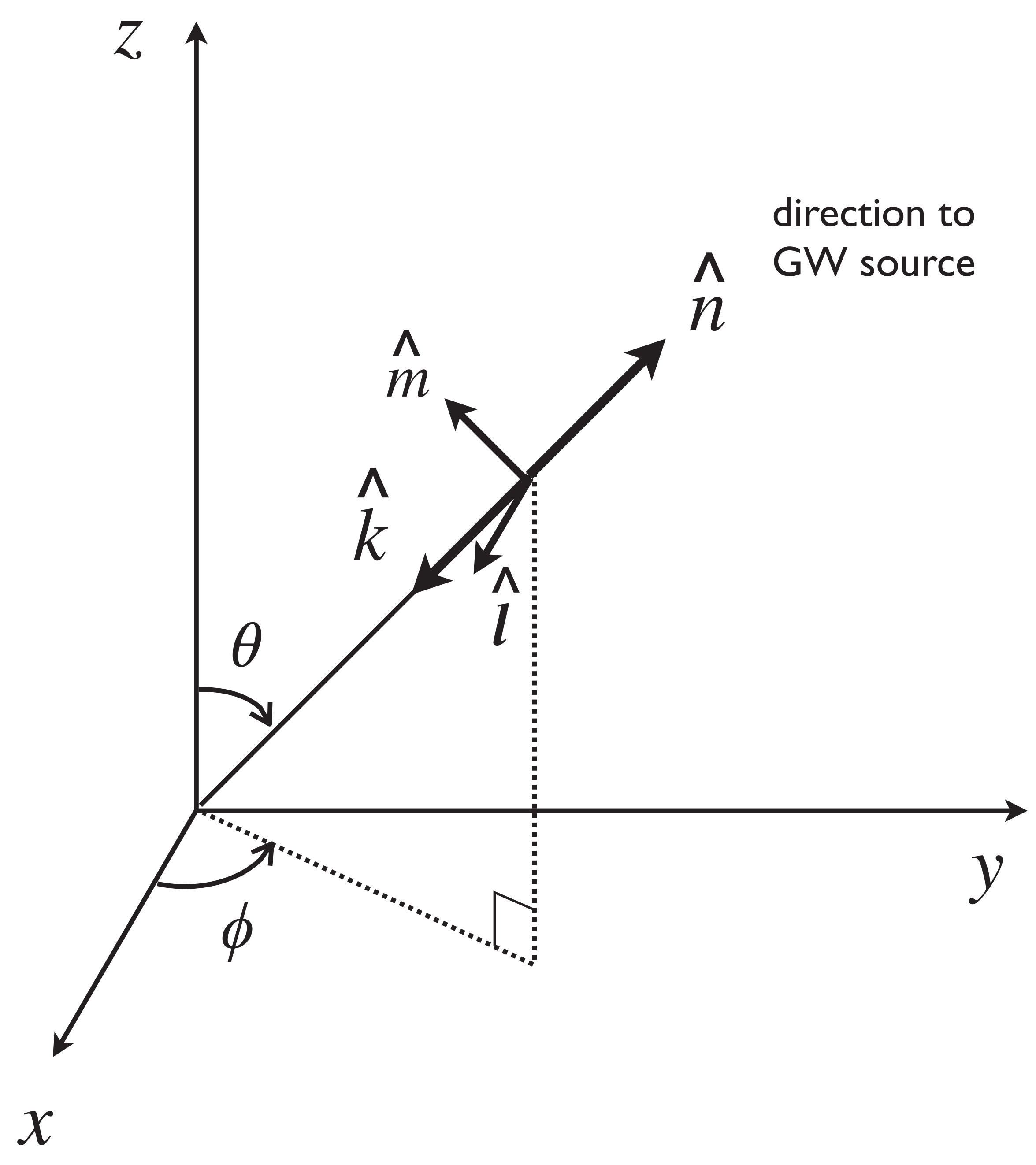

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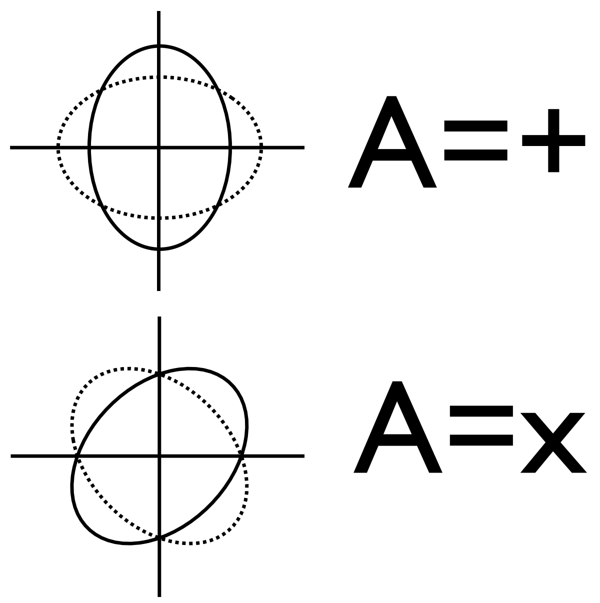

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0

(no loss of generality)

in terms of quadratic expectation values

(if Gaussian)

Quadratic expectation values specify different types of Gaussian stochastic GW backgrounds

- stationary, unpolarized and isotropic:

$$\langle h_A(f,\hat{k}) h^*_{A'}(f',\hat{k}') \rangle = \frac{1}{16\pi} S_h(f) \delta(f-f') \delta_{AA'} \delta^2(\hat{k},\hat{k}')$$

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power spectral density (Hz^{-1})

$$S_h(f) = \frac{3H_0^2}{2\pi^2}\frac{\Omega_{\rm gw}(f)}{f^3}$$

energy density spectrum (dimensionless)

$$\Omega_{\rm gw}(f) \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm gw}}{d\ln f} = \frac{f}{\rho_c}\frac{d\rho_{\rm gw}}{df}$$

$$\rho_c \equiv \frac{3H_0^2c^2}{8\pi G} \qquad \rho_{gw} = \frac{c^2}{32\pi G}\langle \dot{h}_{ab}(t,\vec{x})\dot{h}^{ab}(t,\vec{x})\rangle$$

characteristic strain (dimensionless)

$$h_c(f) \equiv \sqrt{fS_h(f)} = A_\alpha\left(\frac{f}{f_{\rm ref}}\right)^\alpha$$

For a collection of astrophysical sources

- "Phinney formula" (1991):

$$\Omega_{\rm gw}(f) = \frac{1}{\rho_c}\int_0^\infty \mathrm{d}z\, n(z)\, \frac{1}{1+z}\left(f_{\rm s}\frac{\mathrm{d}E_{\rm gw}}{\mathrm{d}f_{\rm s}}\right)\Bigg|_{f_{\rm s}=f(1+z)}$$

$$f_s = f(1+z)$$

For a collection of astrophysical sources

- “Phinney formula” (1991):

$$\Omega_{\rm gw}(f) = \frac{1}{\rho_c}\int_0^\infty {\rm d}z\, n(z)\,\frac{1}{1+z}\left(f_{\rm s}\frac{{\rm d}E_{\rm gw}}{{\rm d}f_{\rm s}}\right)\Bigg|_{f_{\rm s}=f(1+z)}$$

$$f_s = f(1+z)$$

- In terms of event rate:

$$n(z)\,dz = R(z)\,|\,dt\,|_{\rm s} \qquad \left|\frac{{\rm d}t}{{\rm d}z}\right|_s = \frac{1}{(1+z)H_0E(z)} \qquad E(z) = \sqrt{\Omega_{\rm m}(1+z)^3+\Omega_\Lambda} \longleftarrow$$

cosmology

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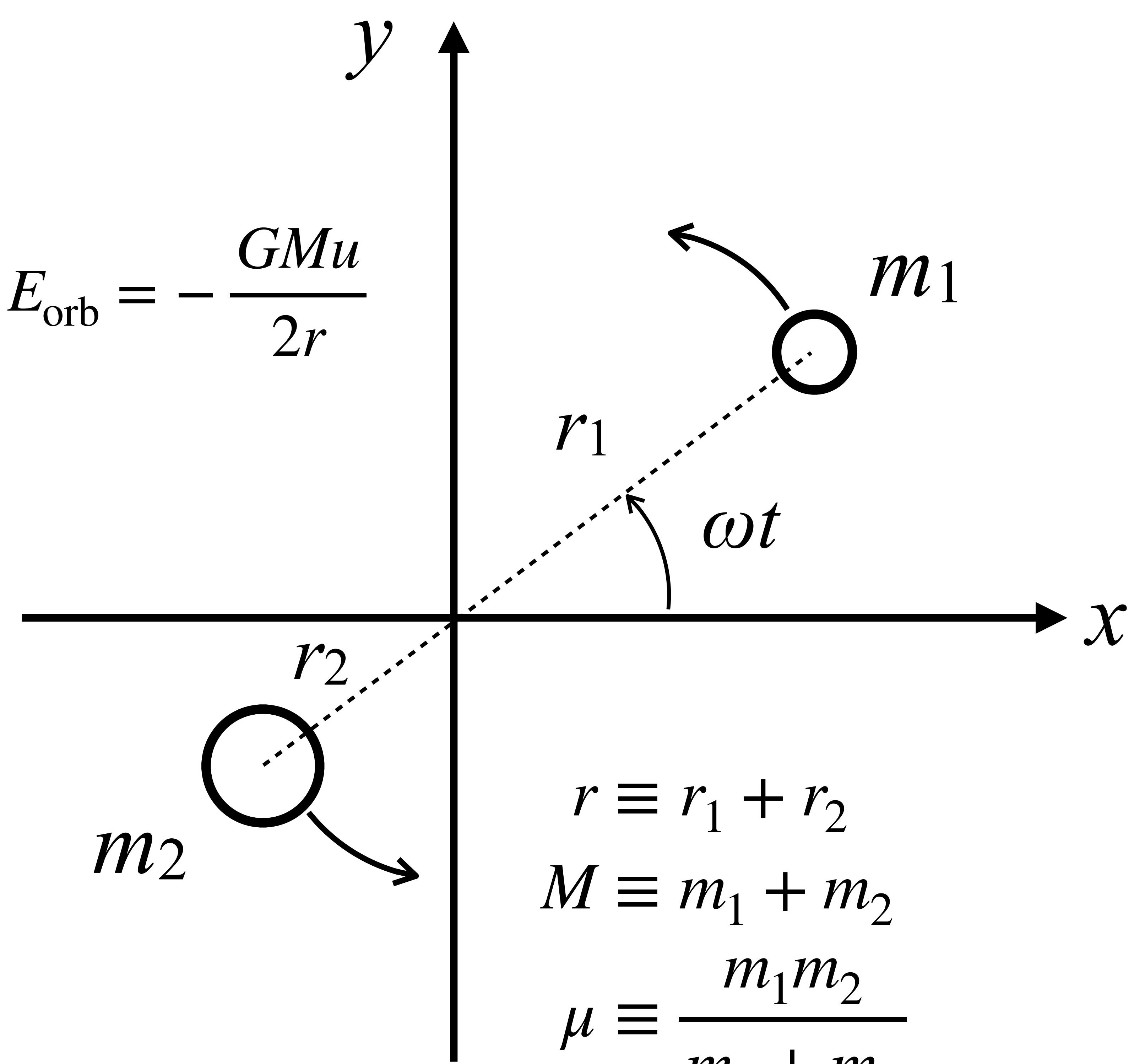
Example: circular binaries



$$E_{\rm orb} = -\frac{GM\mu}{2r}$$

$$r \equiv r_1 + r_2$$

$$M \equiv m_1 + m_2$$

$$\mu \equiv \frac{m_1 m_2}{m_1 + m_2}$$

$$\mathscr{M}_c \equiv \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \mu^{3/5} M^{2/5}$$

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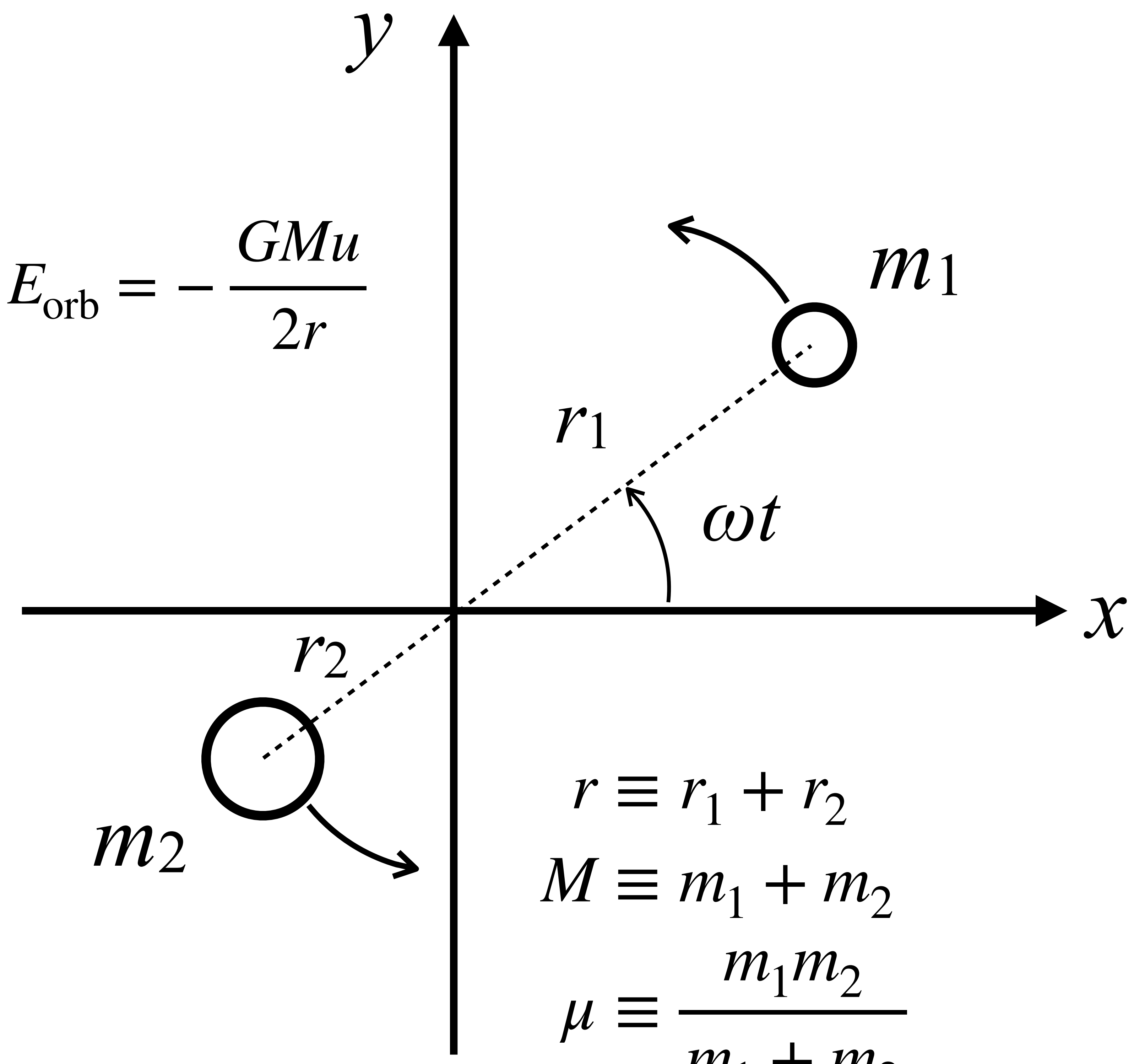

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Units: $G = c = 1$

Kepler's law: $\omega^2 r^3 = GM \implies r \sim M^{1/3}\omega^{-2/3}, \quad \dot{r} \sim -r\dot{\omega}/\omega$

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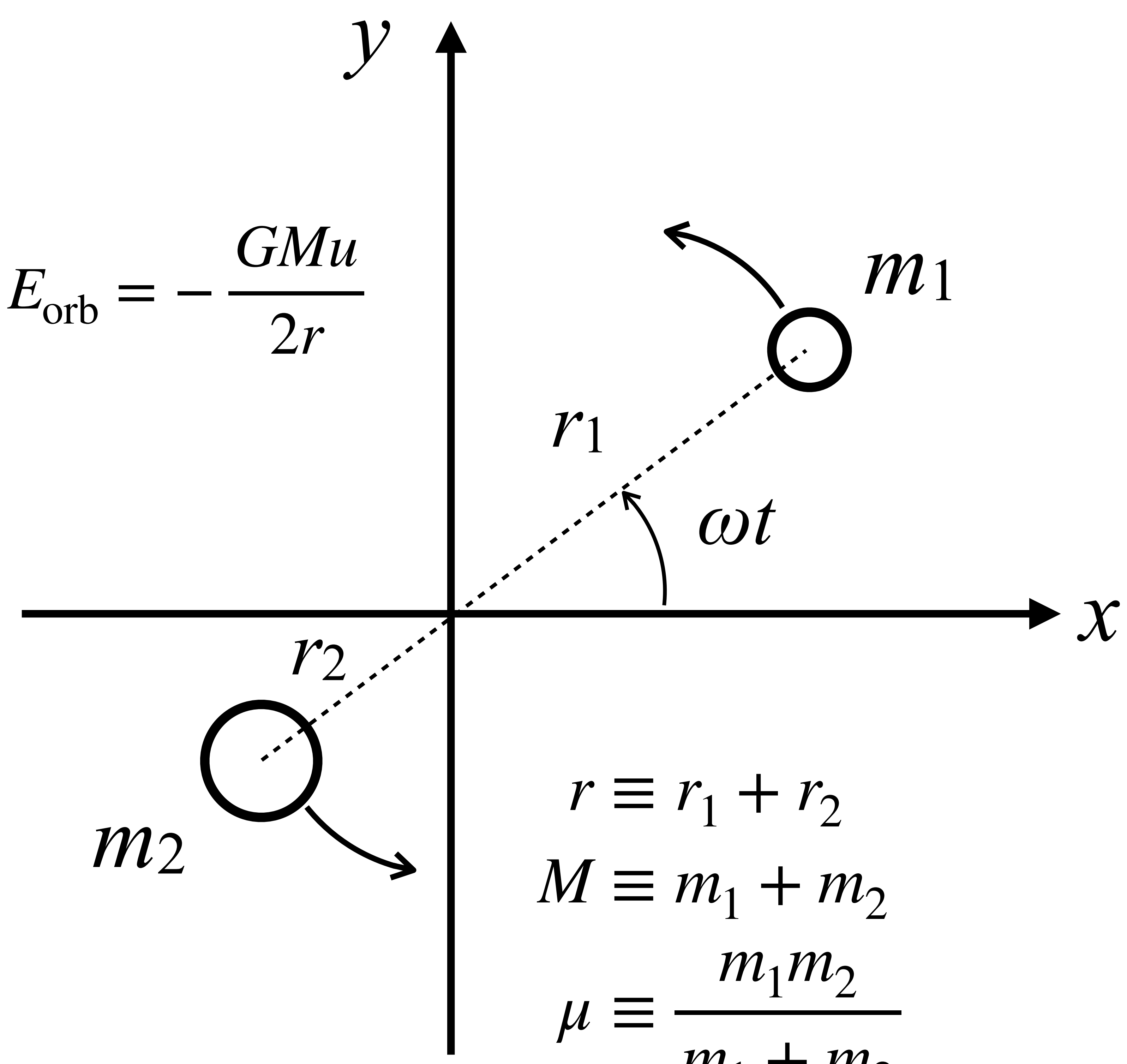

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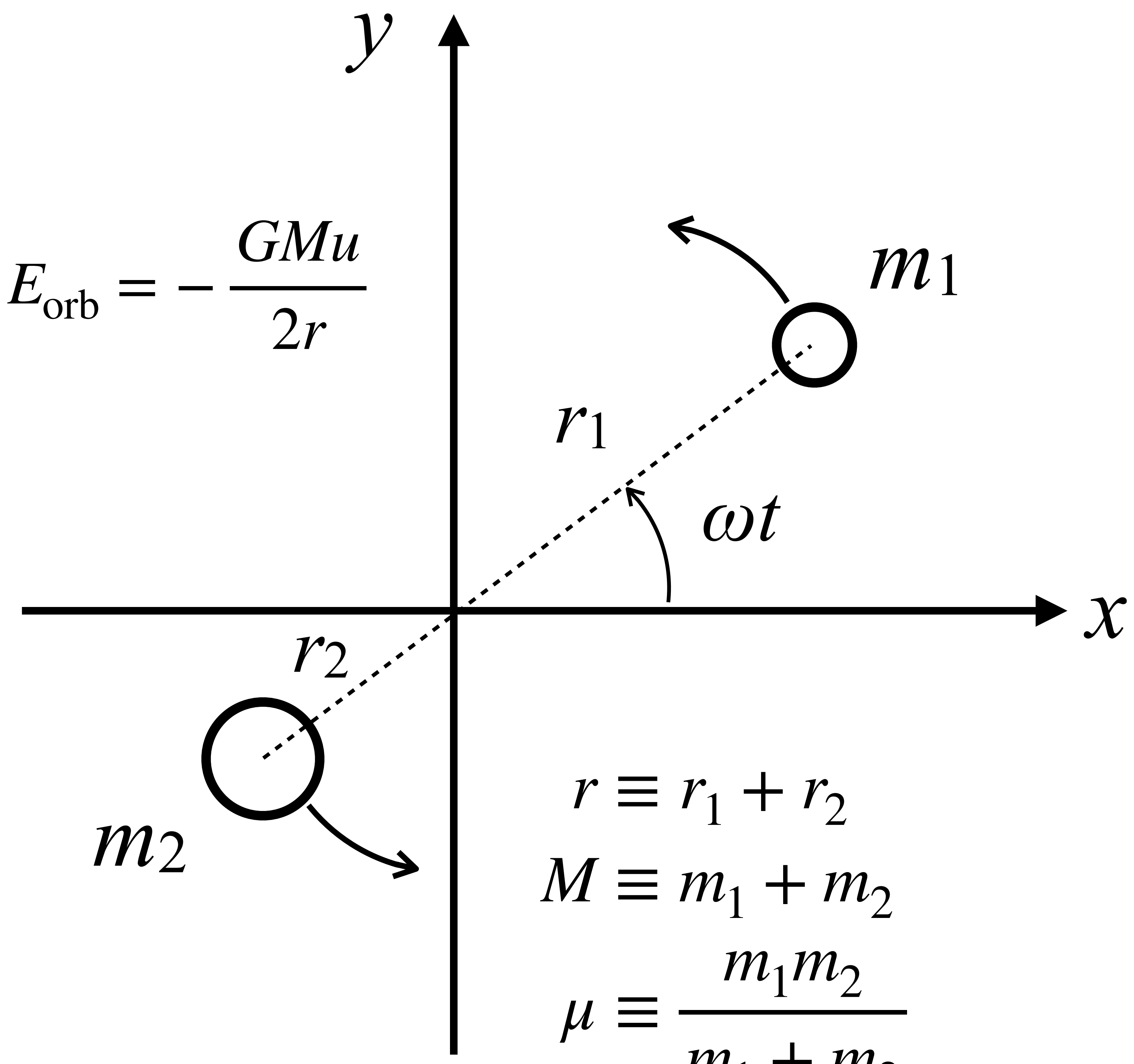

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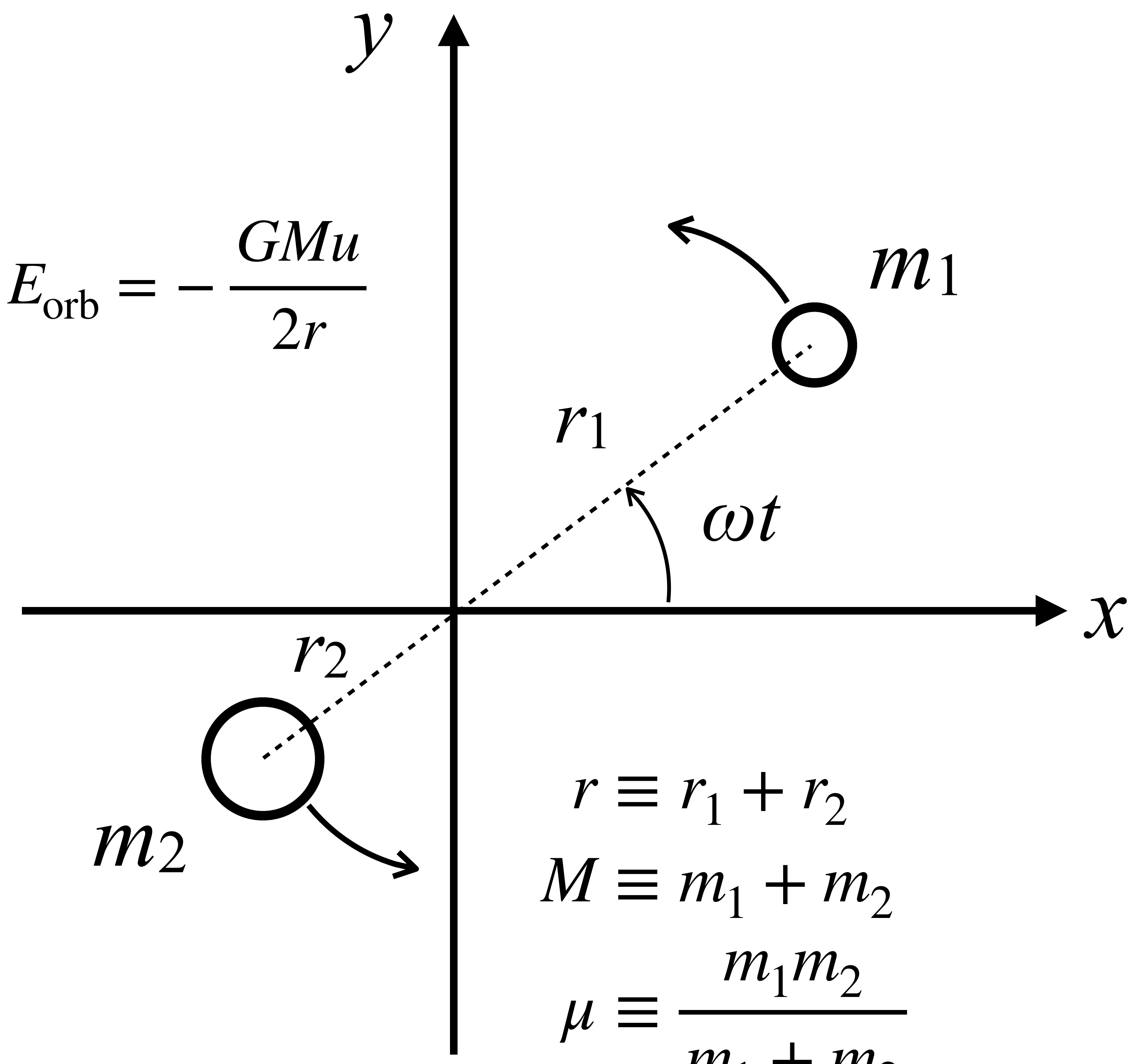

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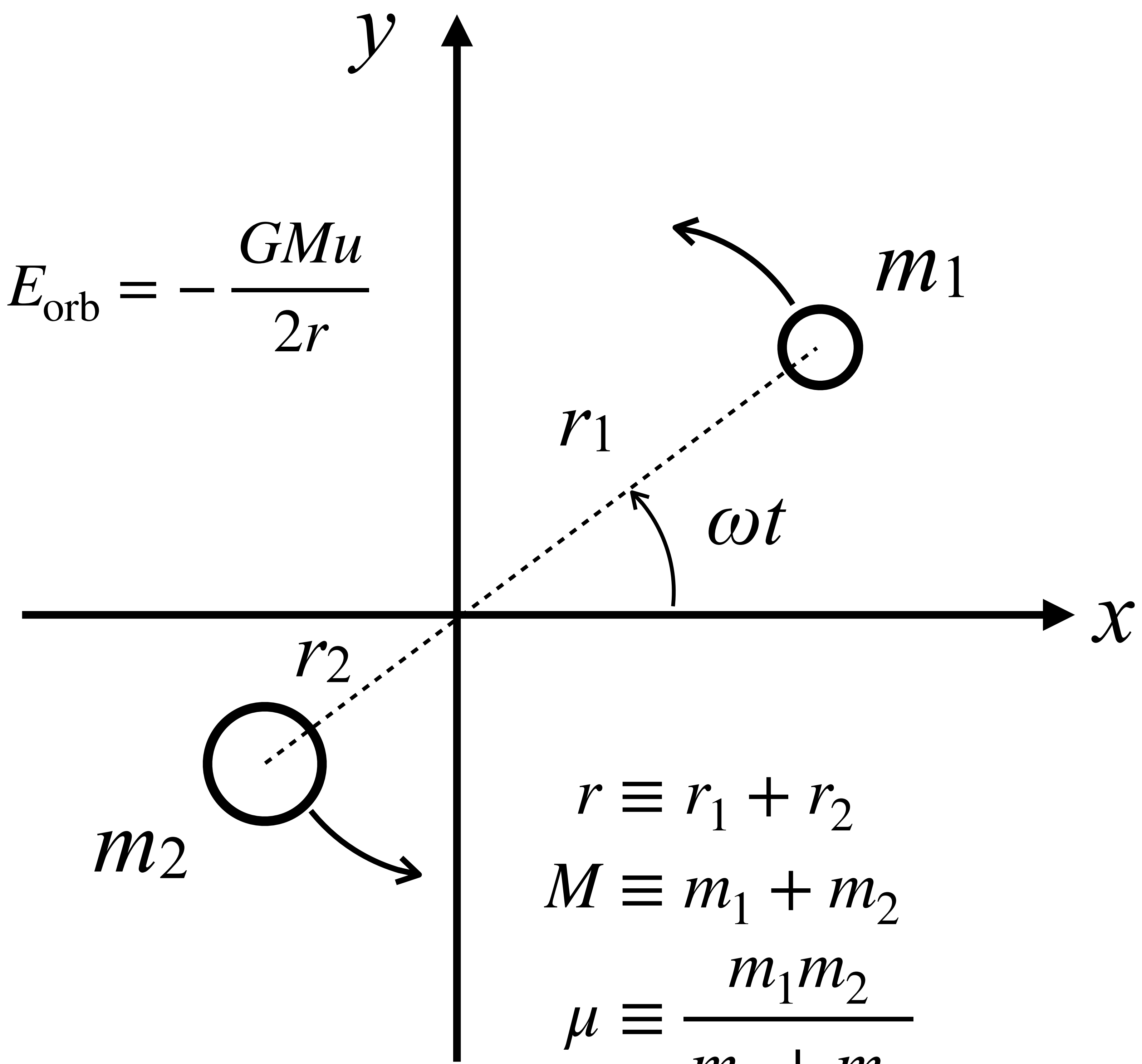

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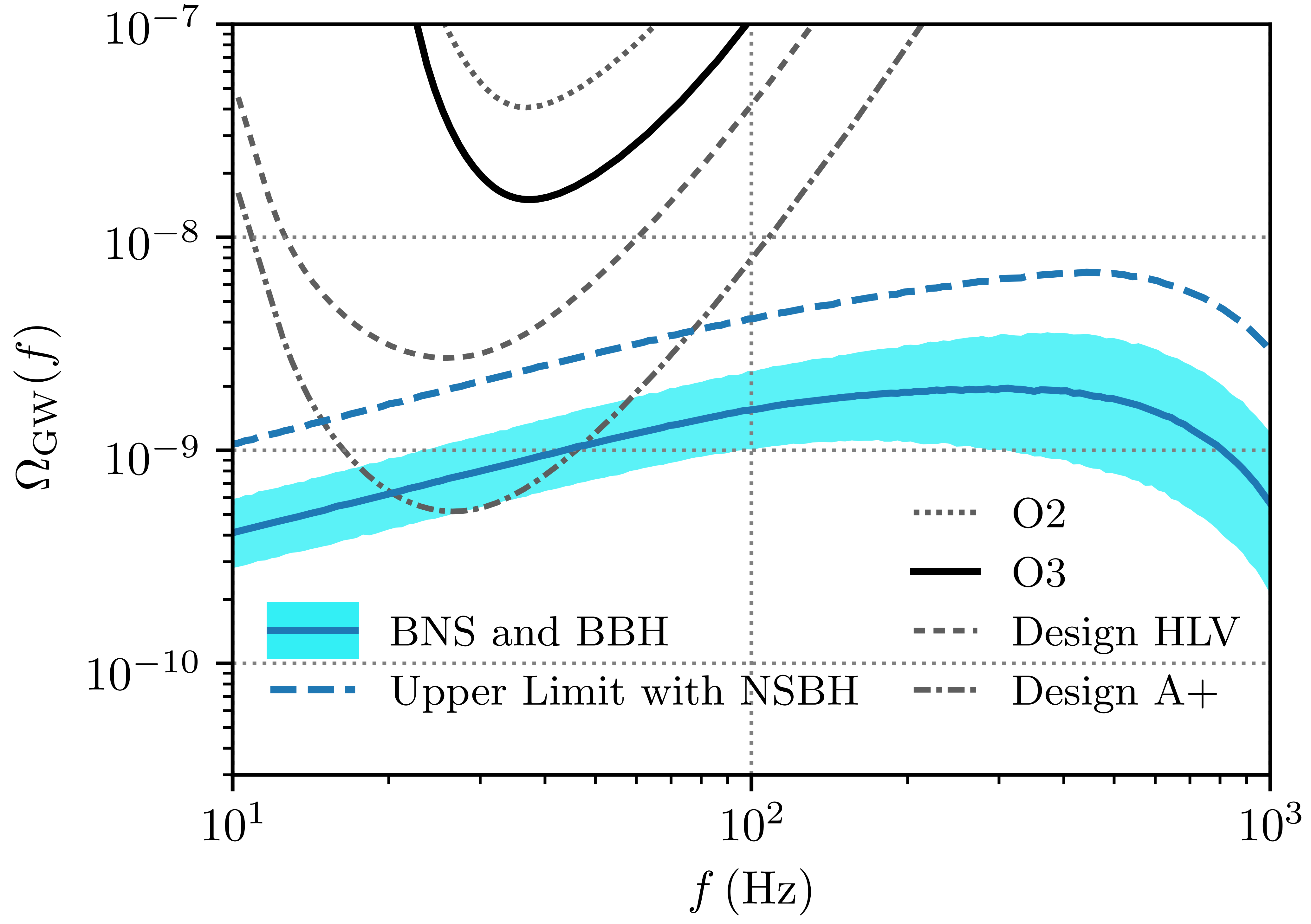
$$\implies \boxed{\Omega_{\rm gw}(f) \propto f^{2/3}, \quad h_c(f) \propto f^{-2/3}}$$

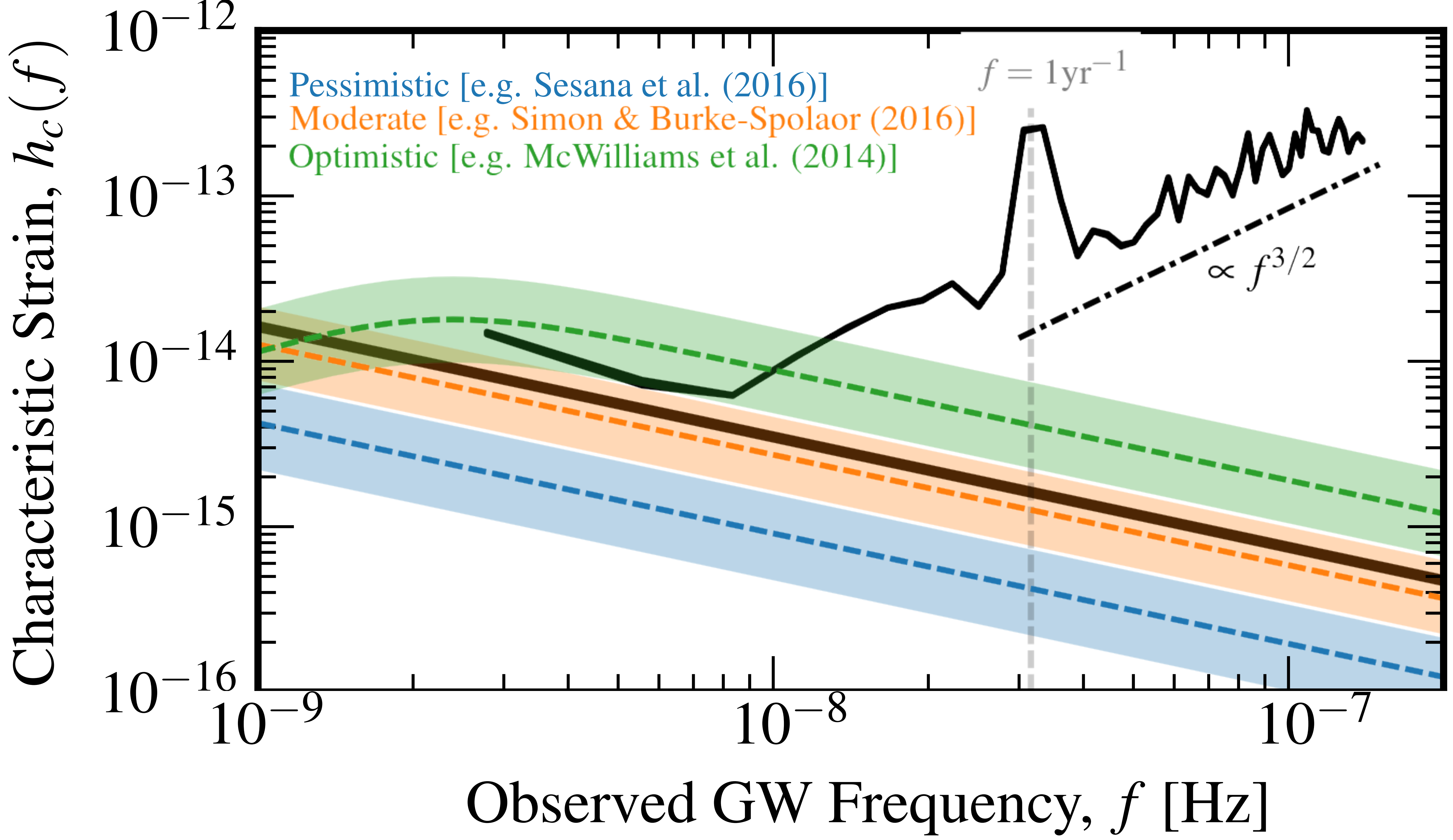
Ground-based interferometers

(LVK O3 Isotropic paper)



Pulsar Timing

(NANOGrav 11-year stochastic paper)



$$\boxed{\Omega_{\rm gw}(f) \propto f^{2/3}, \quad h_c(f) \propto f^{-2/3}}$$

Exercises

1. Verify the expected rate of stellar mass BBH mergers is between ~1 per minute and a few per hour.
2. Derive the relationship between the strain spectral density $S_h(f)$ and the fractional energy density spectrum $\Omega_{\rm gw}(f)$ using the plane-wave expansion for the metric perturbations and the quadratic expectation values for the Fourier components.
3. Verify the expression for $|\mathrm{d}t/\mathrm{d}z|$ as well as the "Phinney formula" in terms of the rate density $R(z)$.