

Overview of stochastic GW background searches and sources

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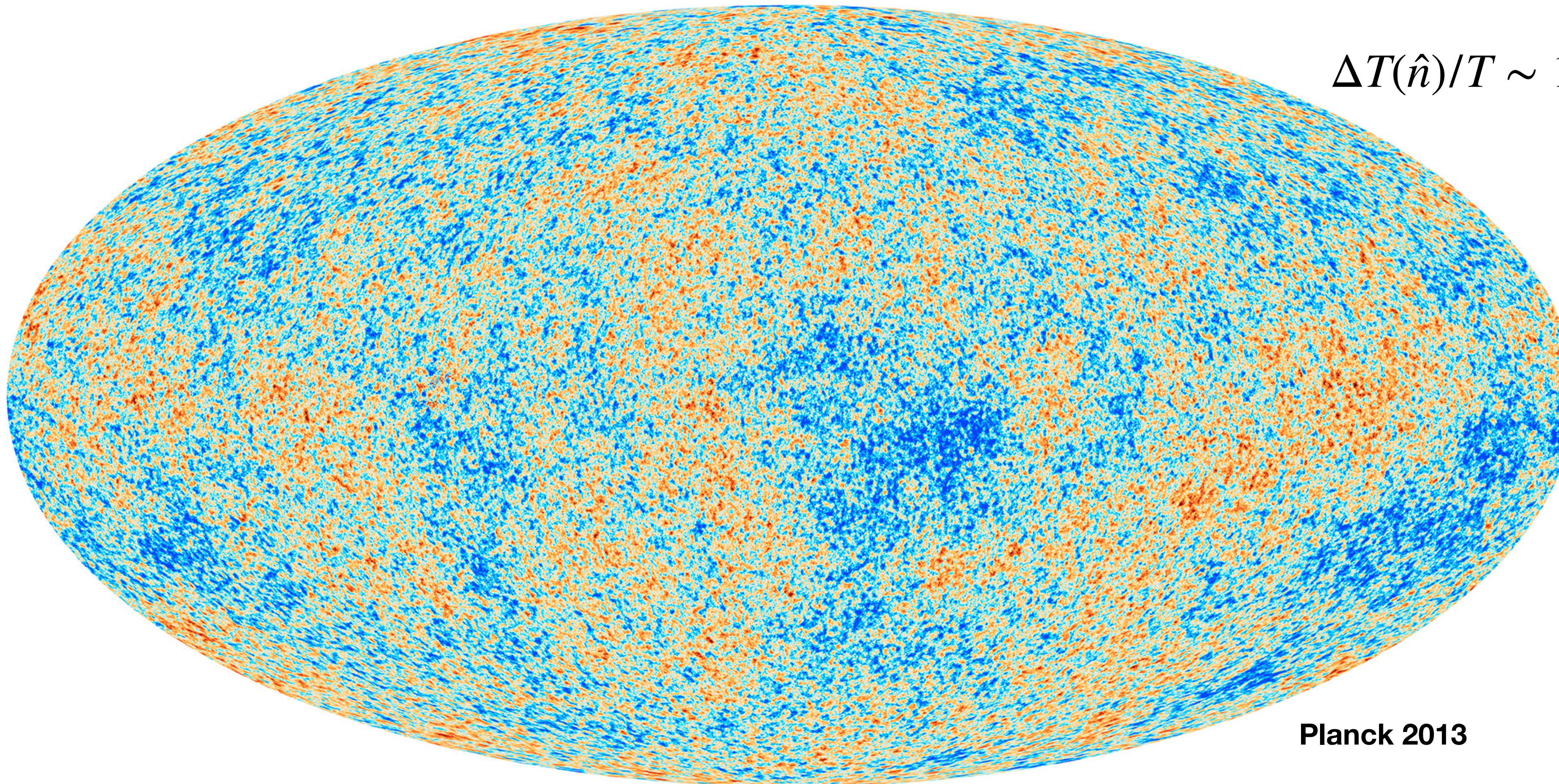
(HUST GW Summer School 2022, Lecture 2)

References

- Allen, Les Houches article, 1995
- Romano and Cornish, Living Reviews in Relativity article, 2017

I. Motivation

Ultimate goal - produce GW analogue of CMB sky maps



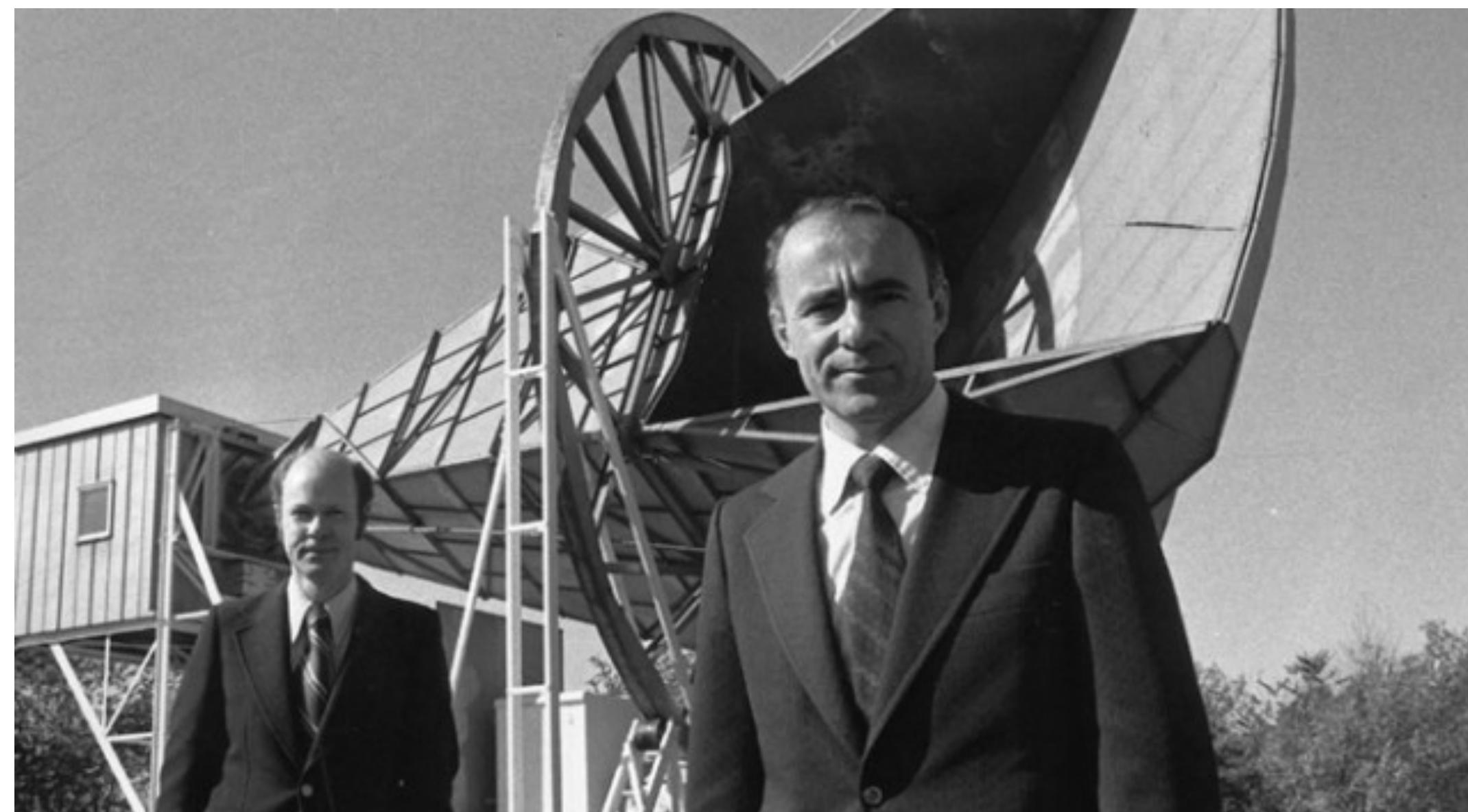
but there's a long road ahead....

1965: Penzias and Wilson (“excess noise” CMB discovery paper)

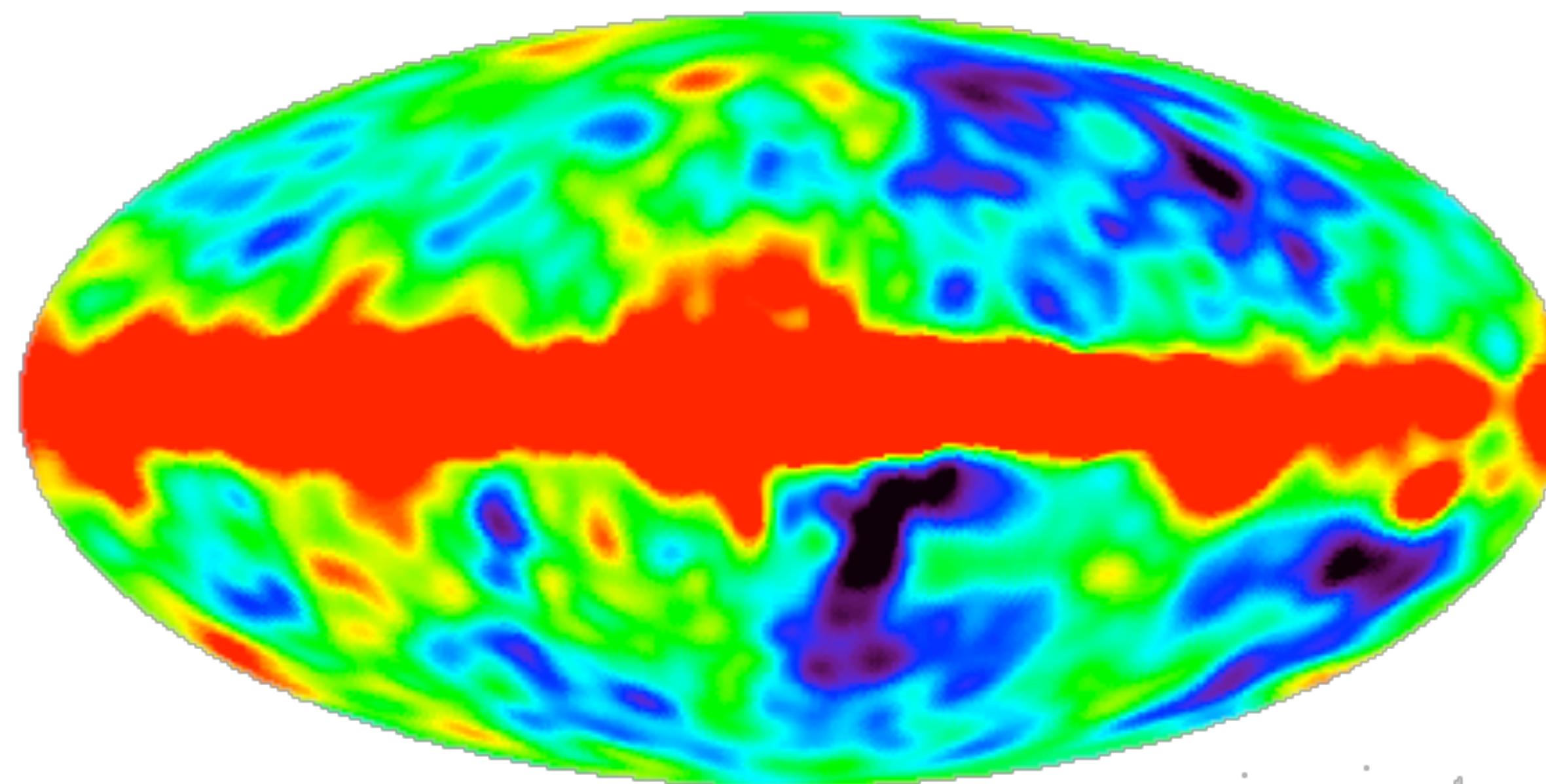
A MEASUREMENT OF EXCESS ANTENNA TEMPERATURE AT 4080 Mc/s

(4080 Mc/s \leftrightarrow 7.35 cm)

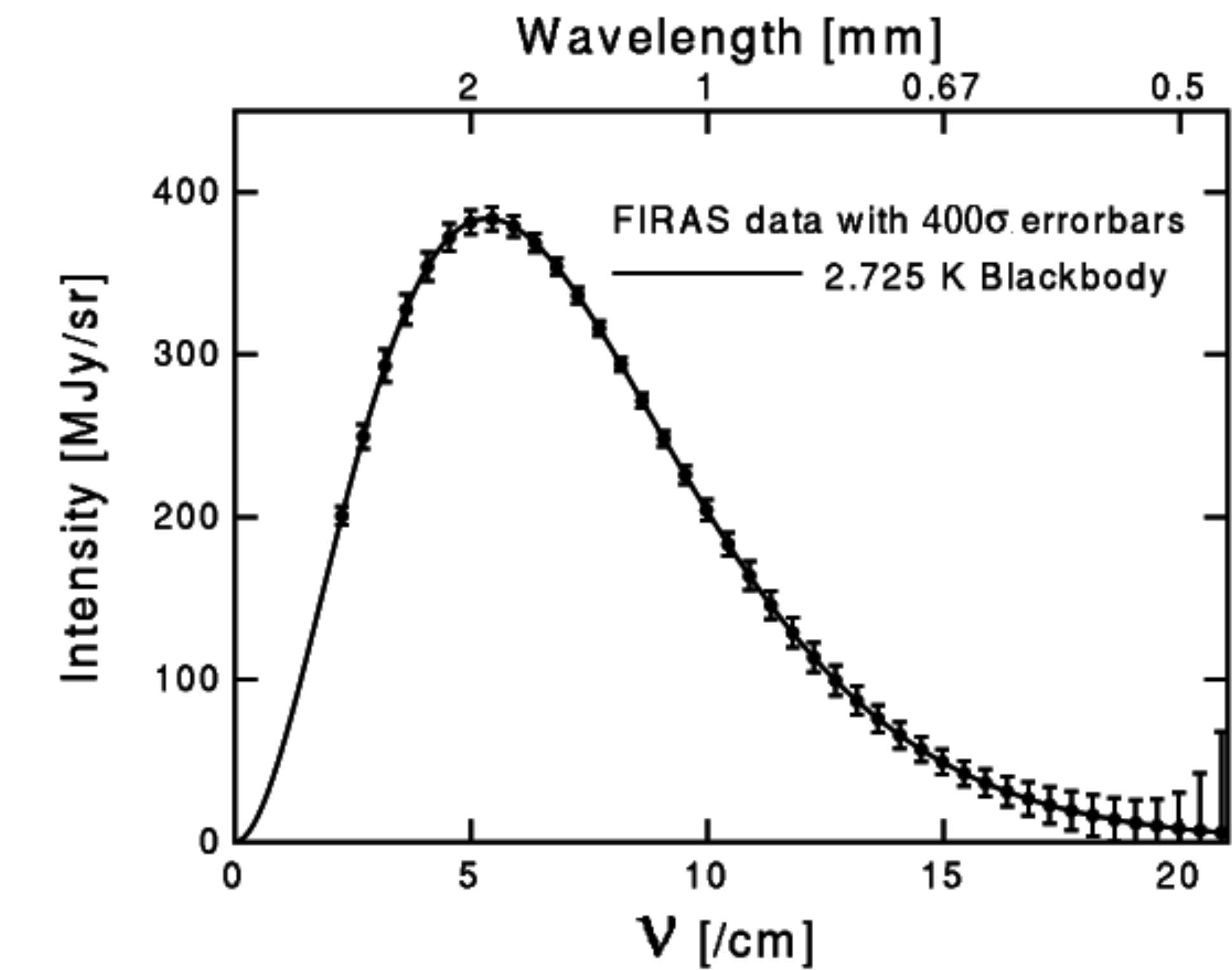
Measurements of the effective zenith noise temperature of the 20-foot horn-reflector antenna (Crawford, Hogg, and Hunt 1961) at the Crawford Hill Laboratory, Holmdel, New Jersey, at 4080 Mc/s have yielded a value about 3.5° K higher than expected. This excess temperature is, within the limits of our observations, isotropic, unpolarized, and free from seasonal variations (July, 1964–April, 1965). A possible explanation for the observed excess noise temperature is the one given by Dicke, Peebles, Roll, and Wilkinson (1965) in a companion letter in this issue.



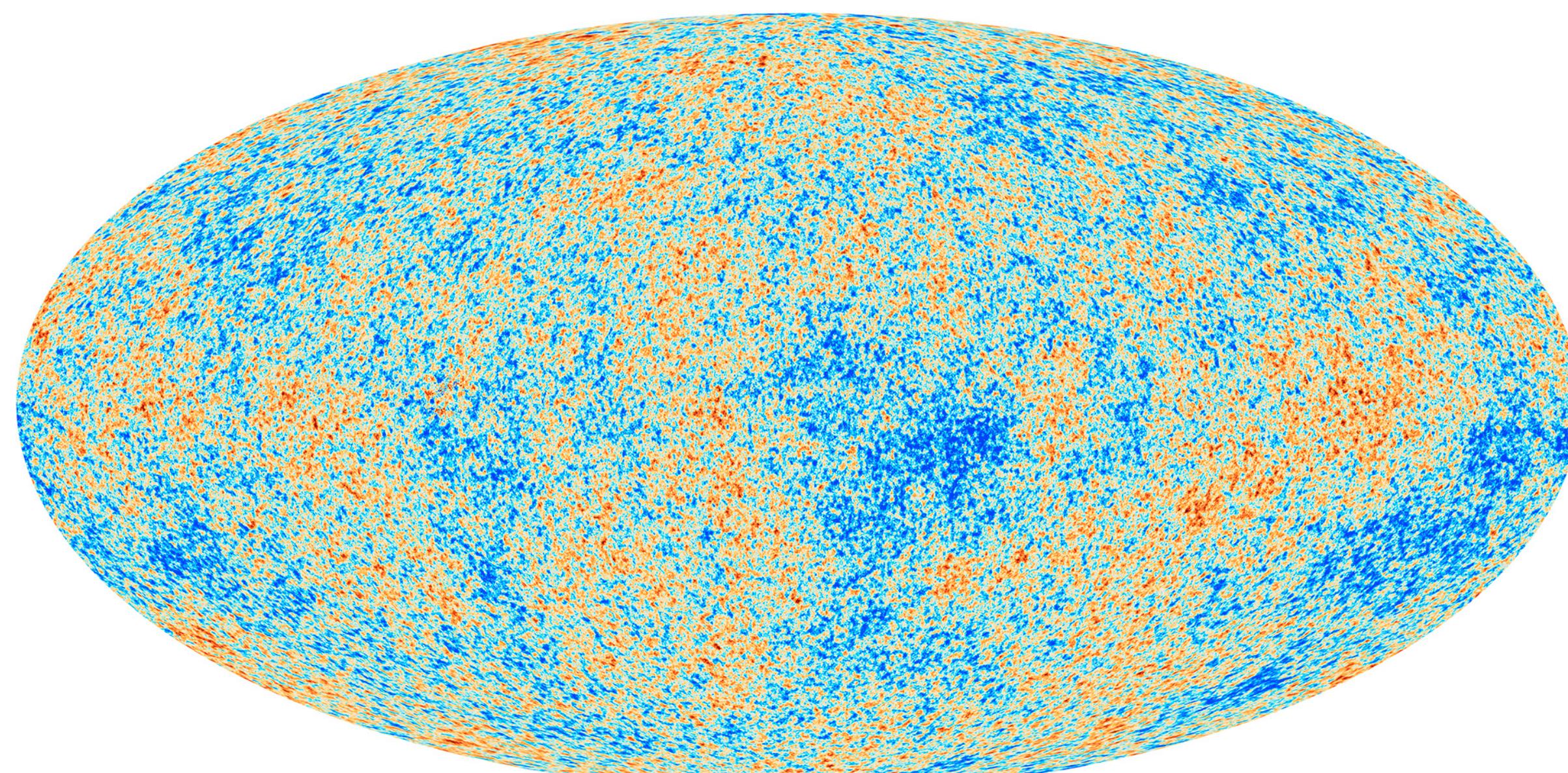
1992: COBE (1st sky maps)



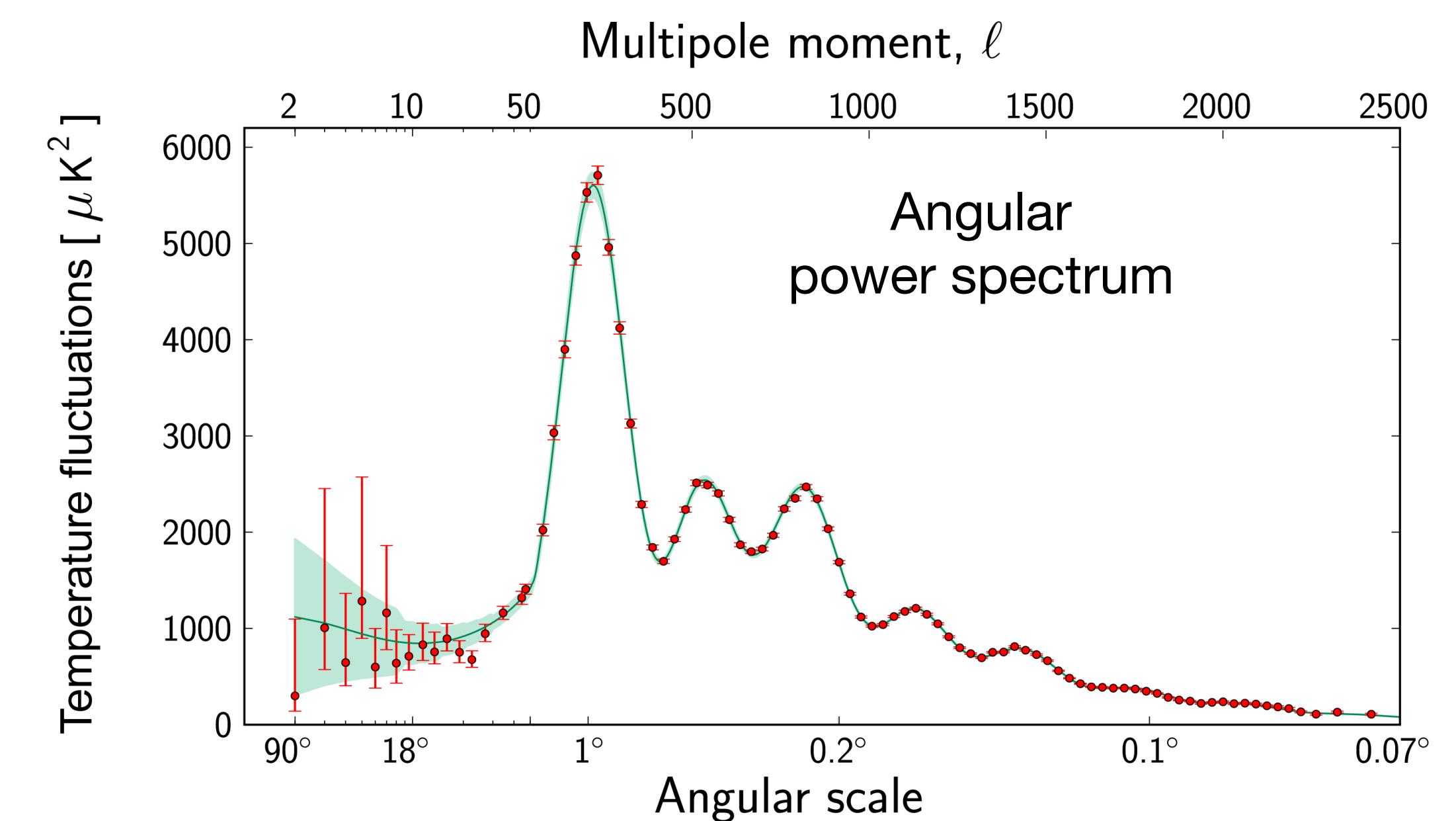
(ang resolution: ~10 degrees)



2013: WMAP, Planck (high precision cosmology)

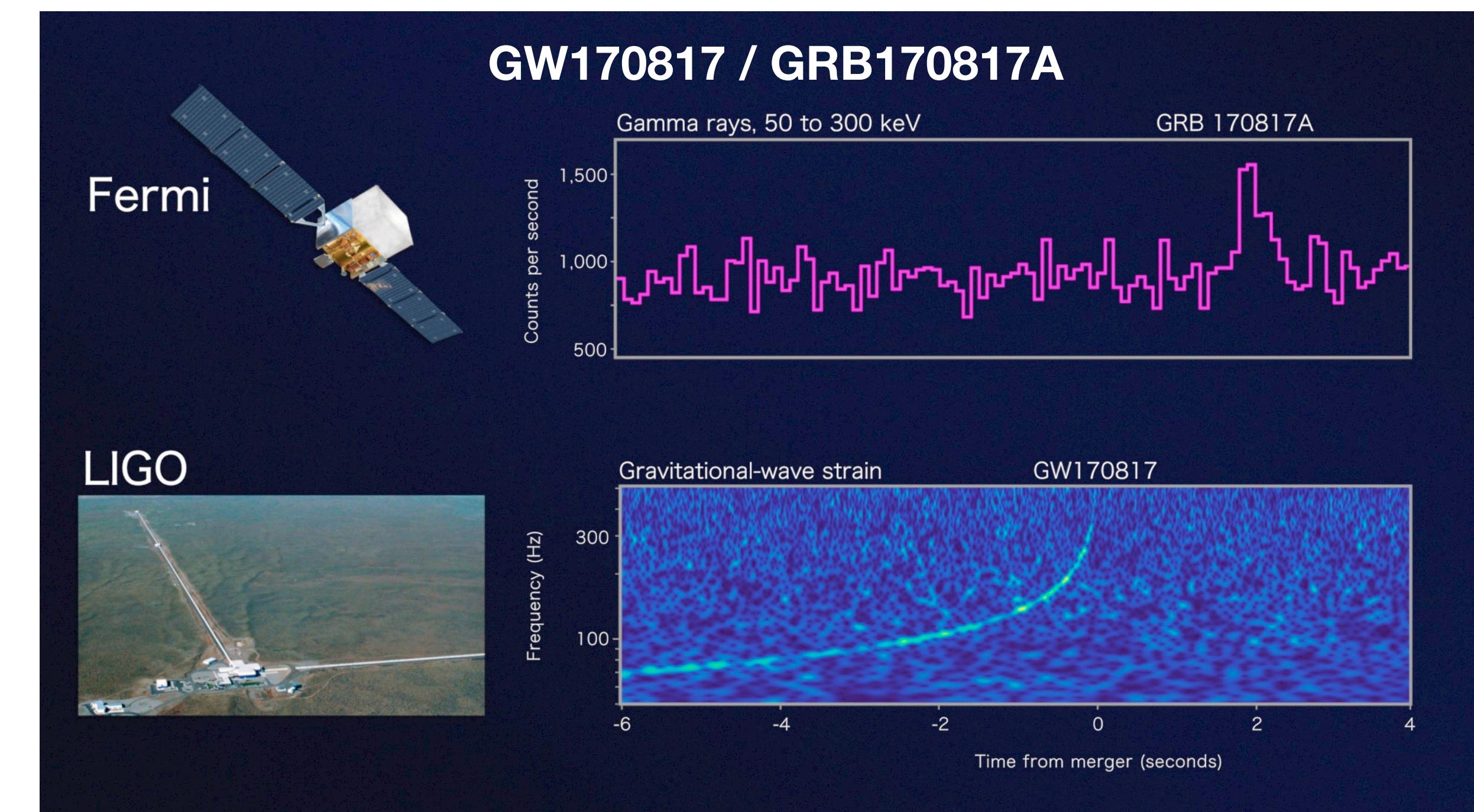
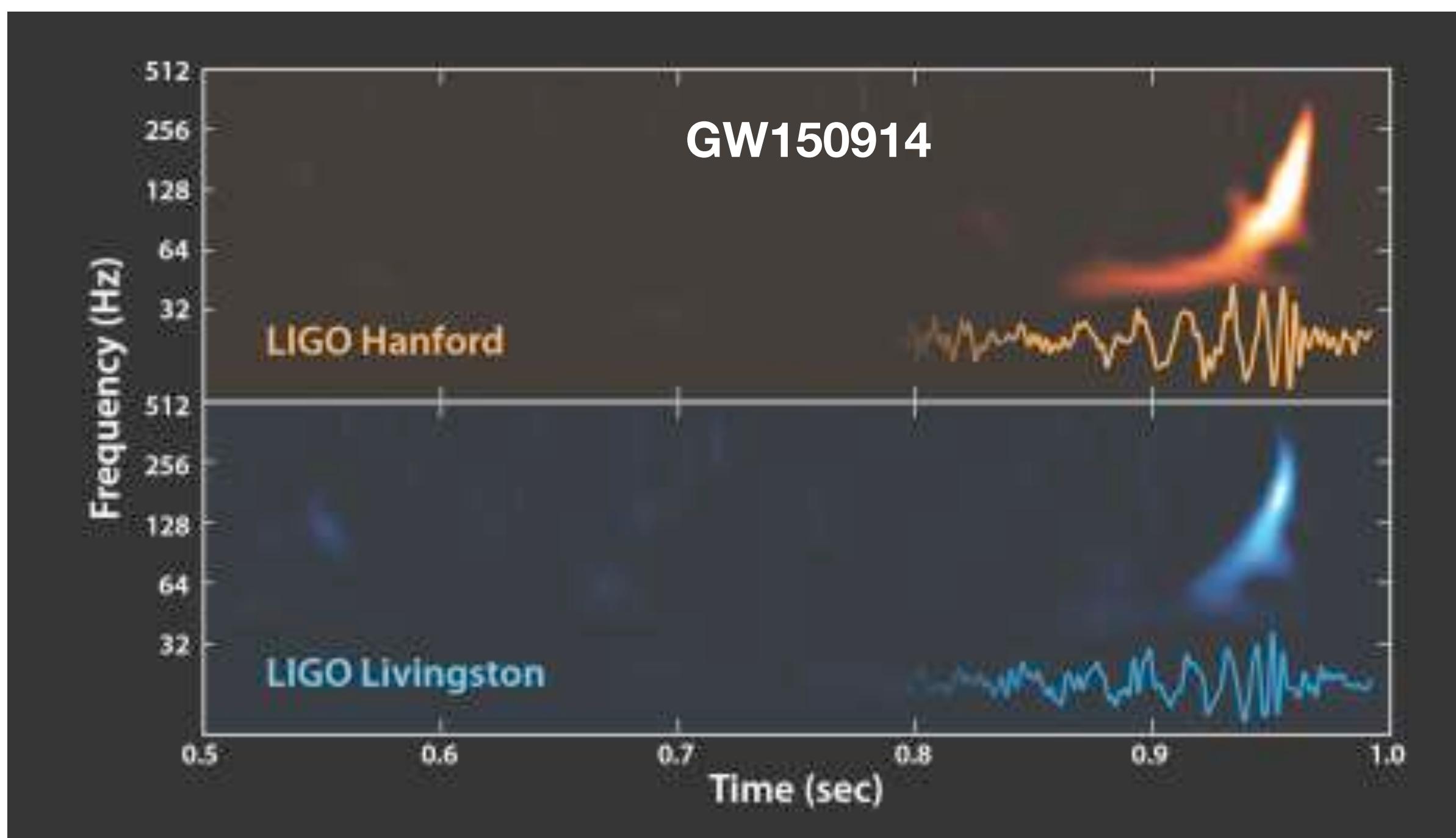


(ang resolution: ~ 10 arcmin)



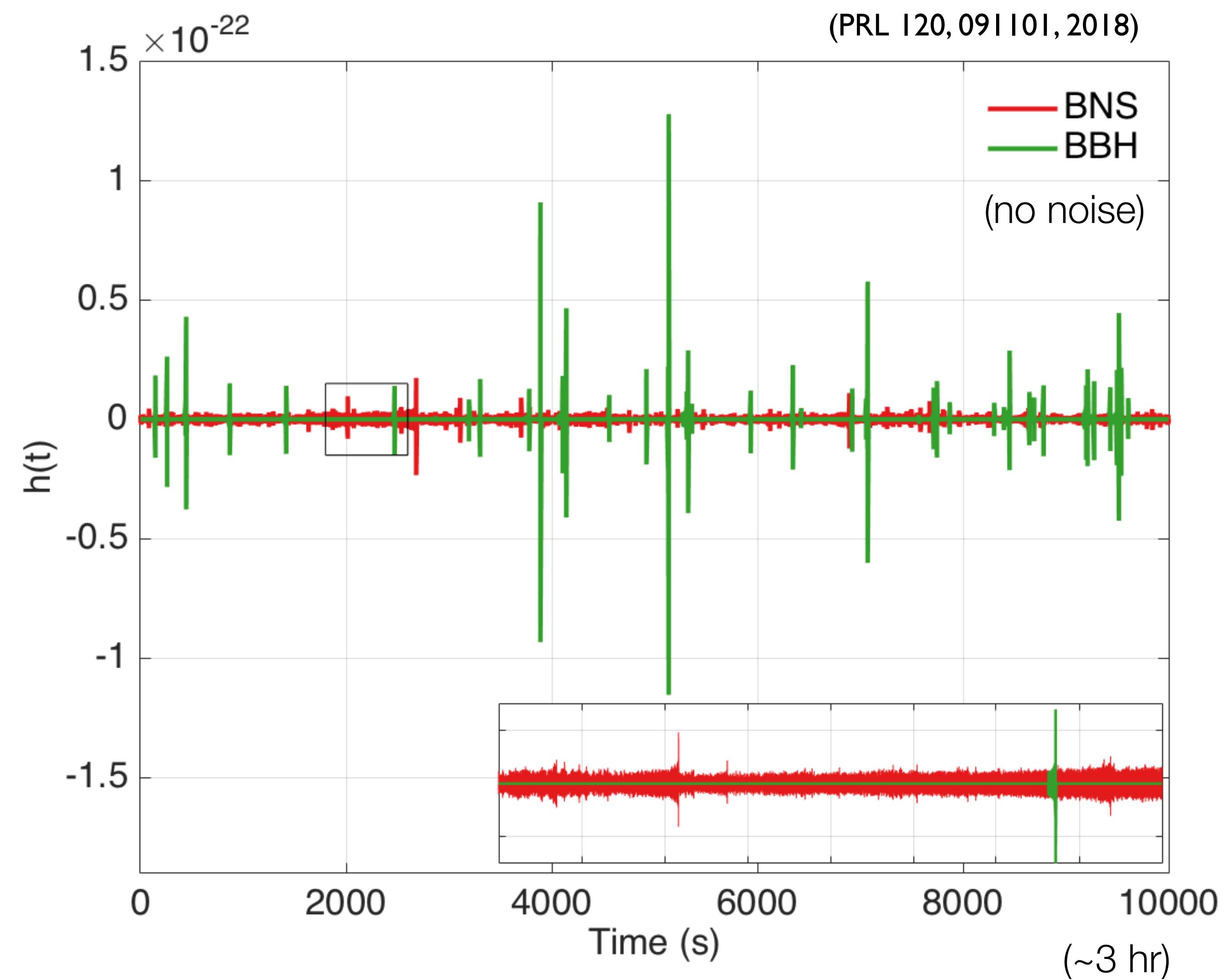
**It took ~50 years for the CMB community to go
from first detection to precision sky maps, but we
have not yet detected the isotropic component of
the GW background!!**

... at least we have detected other (loud) signals

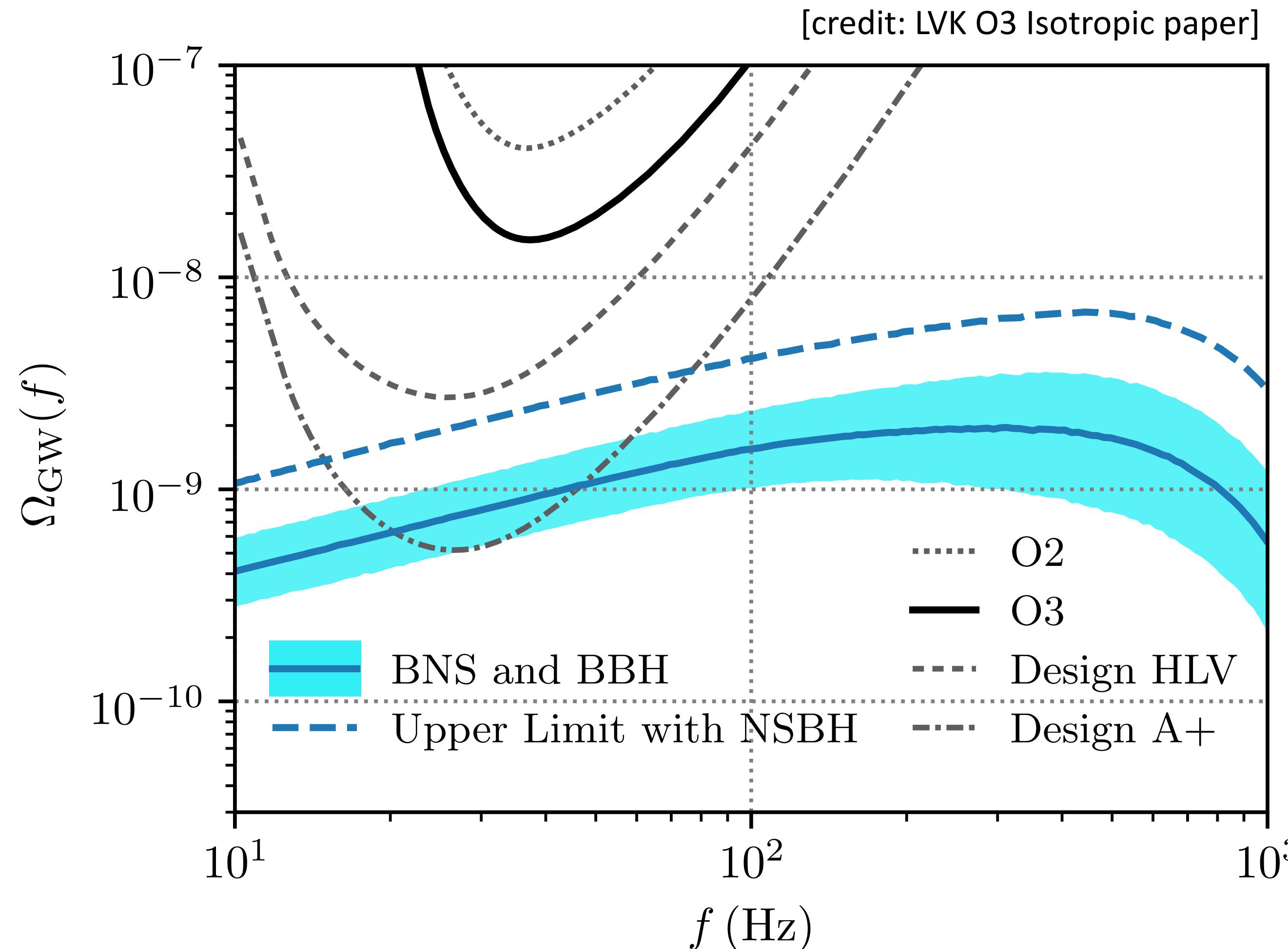


... we know that there also exist weaker signals

- individually undetectable (subthreshold)
- detectable as an aggregate via their **common influence on multiple detectors**
- combined signal described **statistically** — stochastic gravitational-wave background

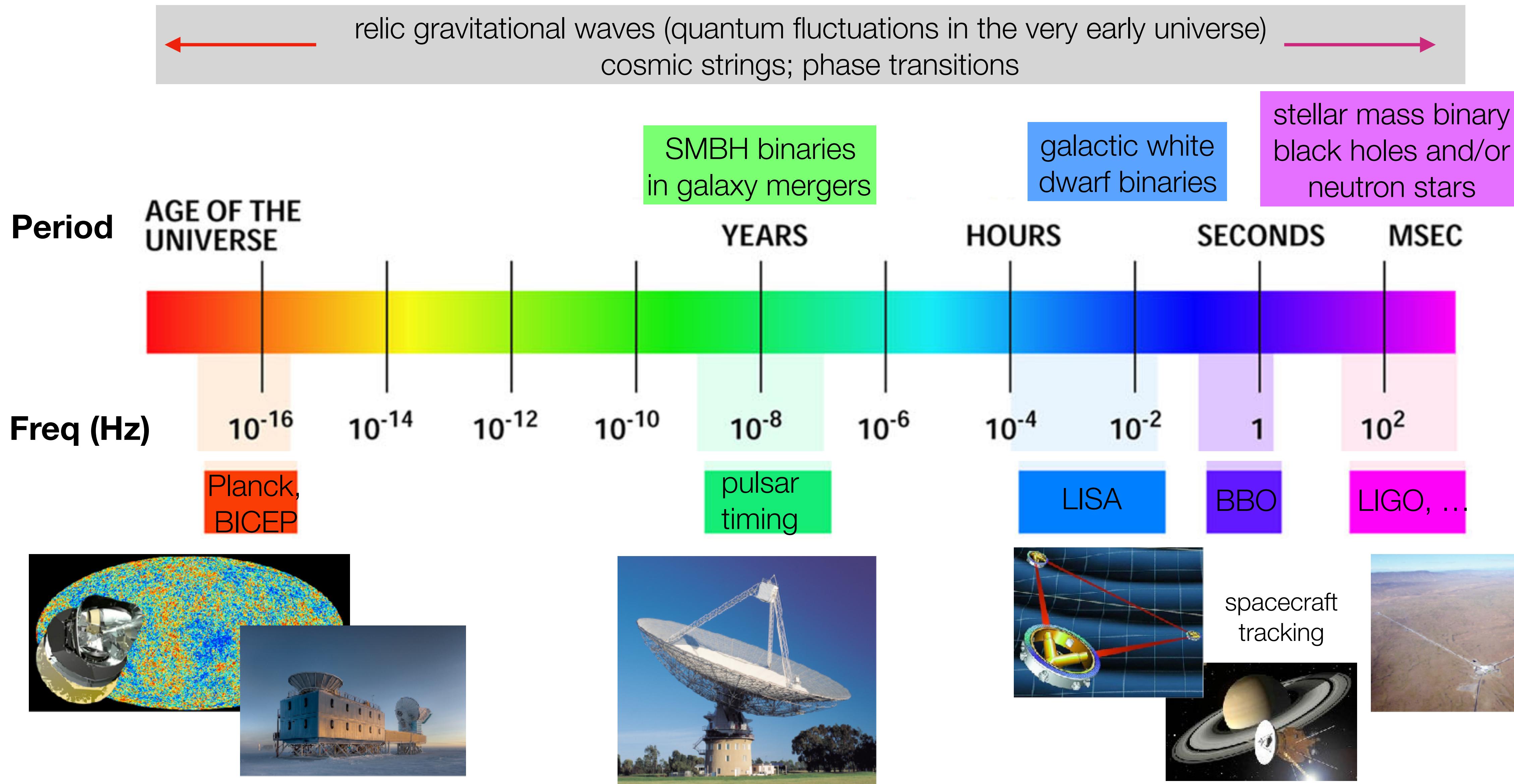


Potentially detectable with Advanced LIGO/Virgo or A+



Based on standard search, but there exists a better method!
(Smith & Thrane, PRX 8, 021019, 2018)

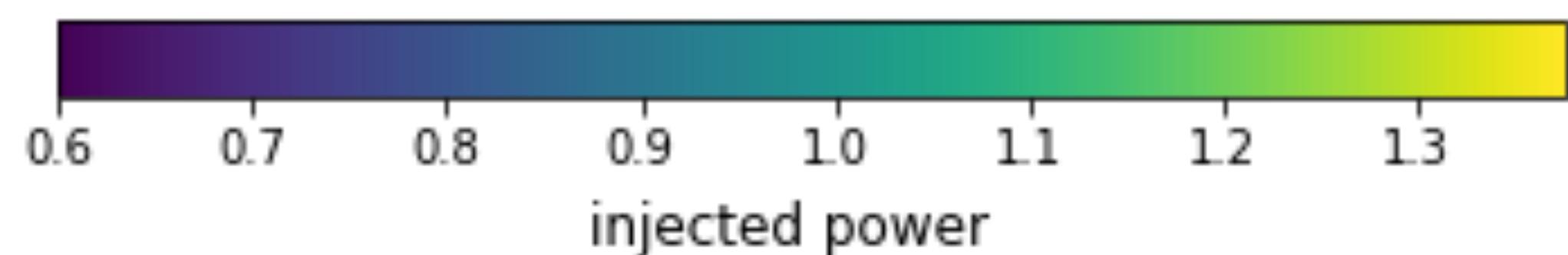
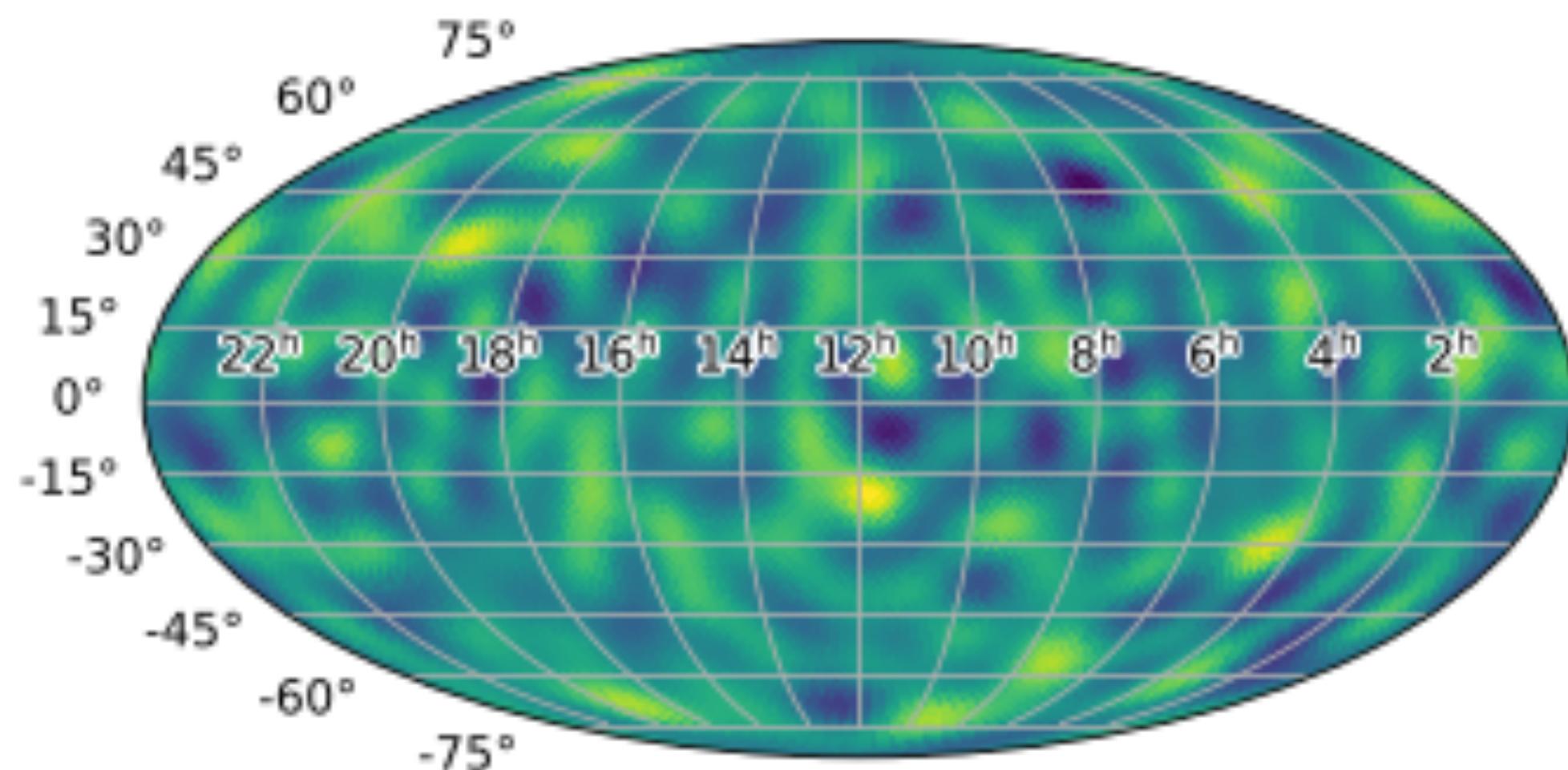
The bigger picture - GWB sources and detectors



II. Different types of stochastic signals

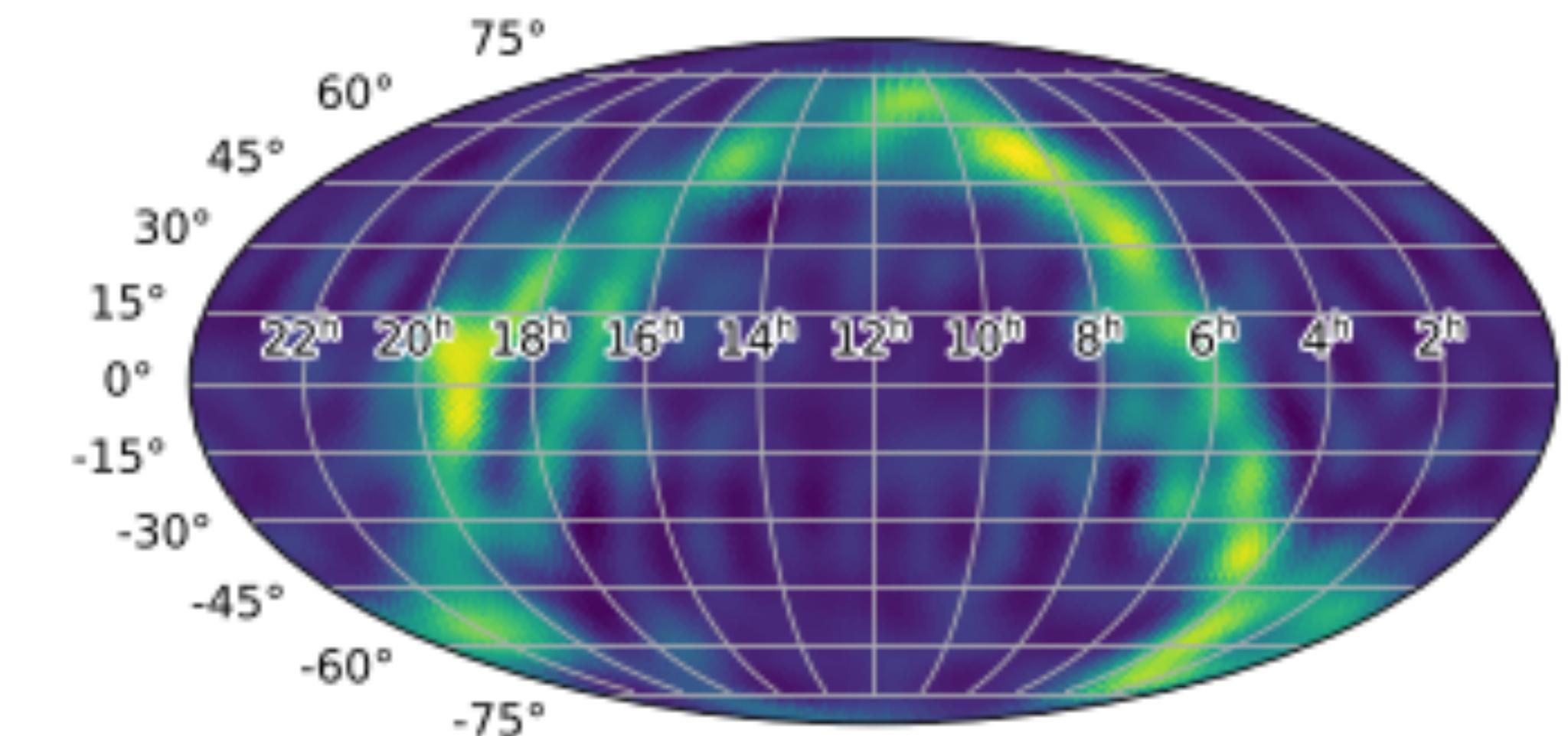
(i) differ in terms of spatial distribution

(statistically) isotropic



(like *cosmic microwave background*)

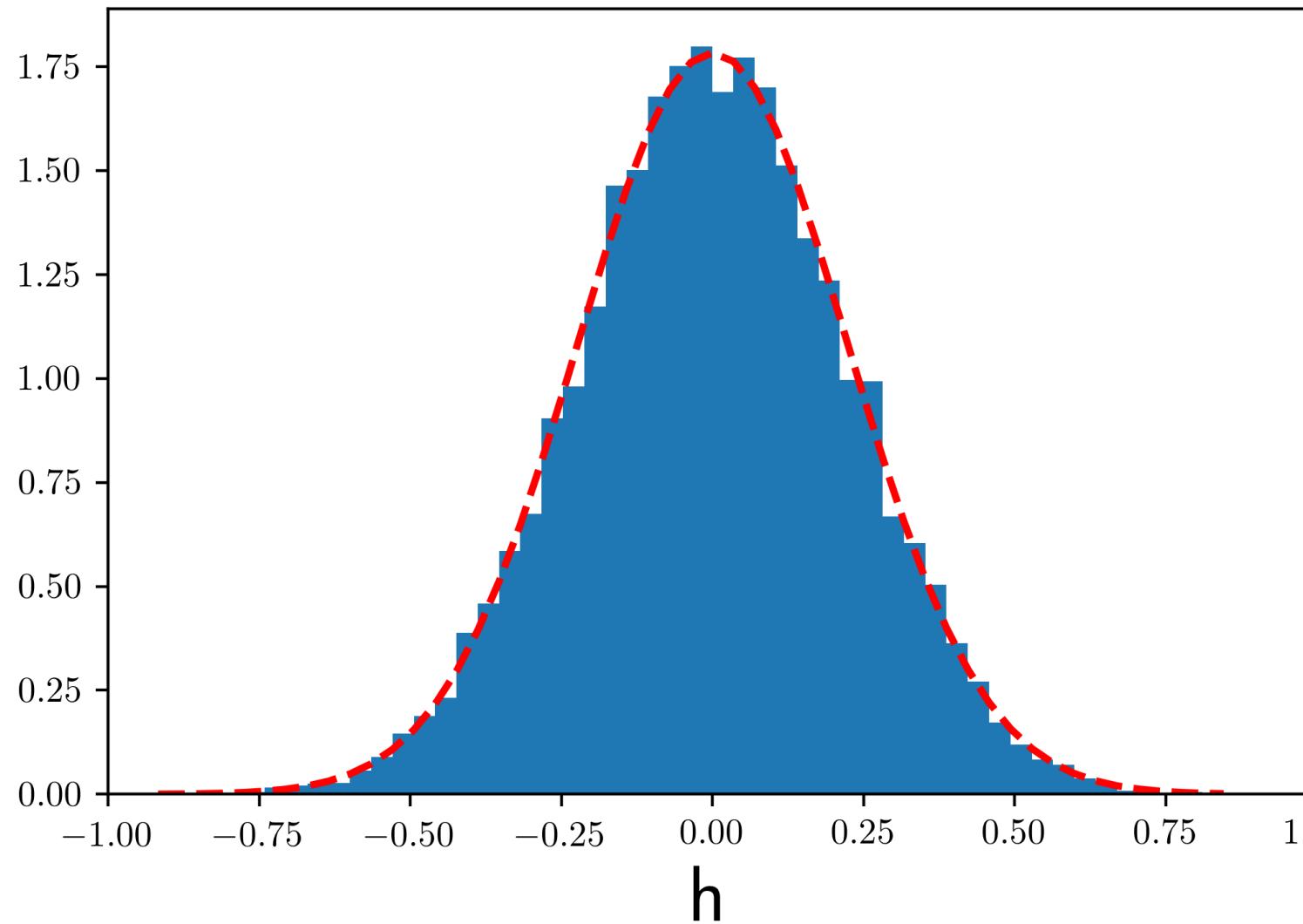
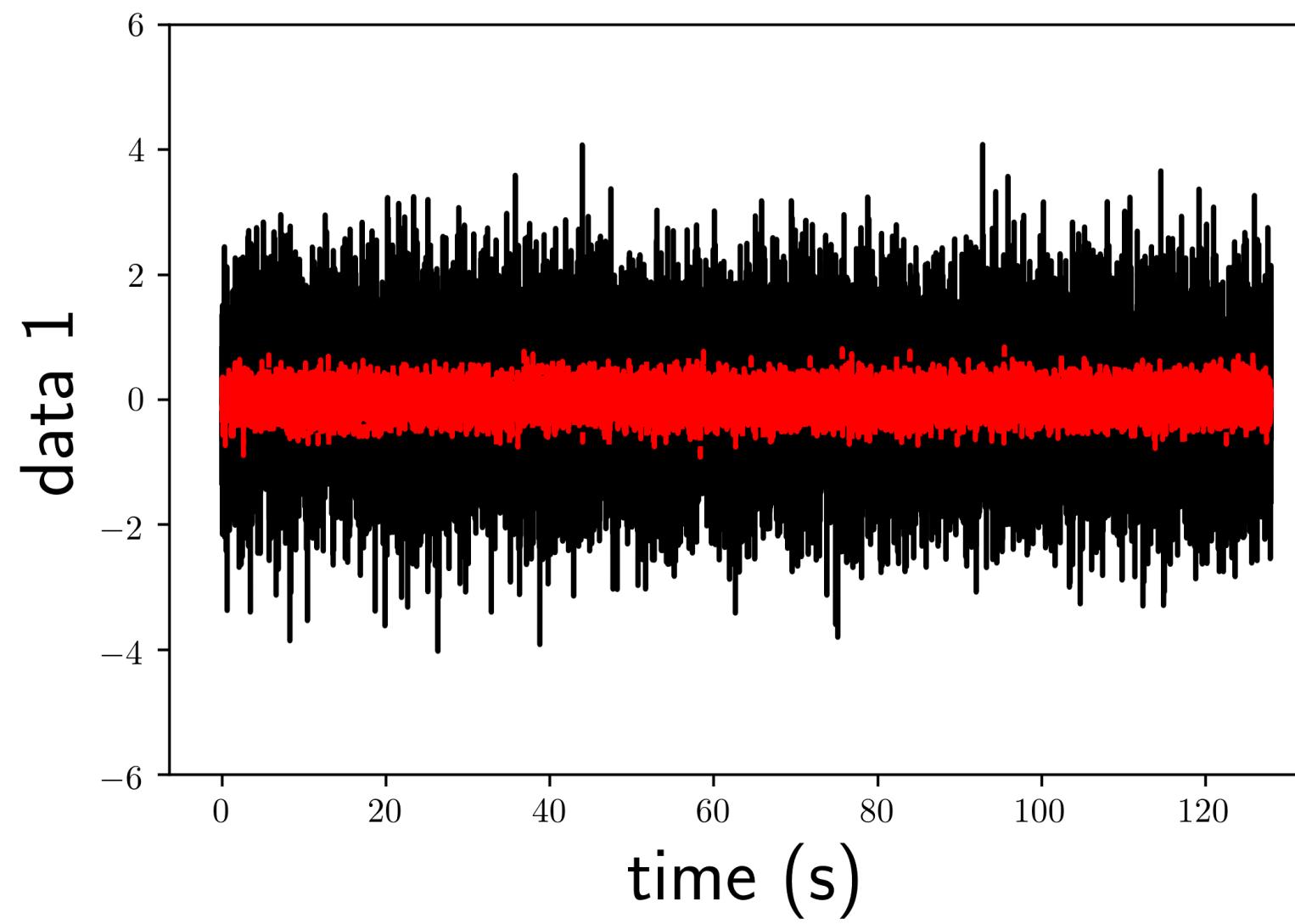
anisotropic



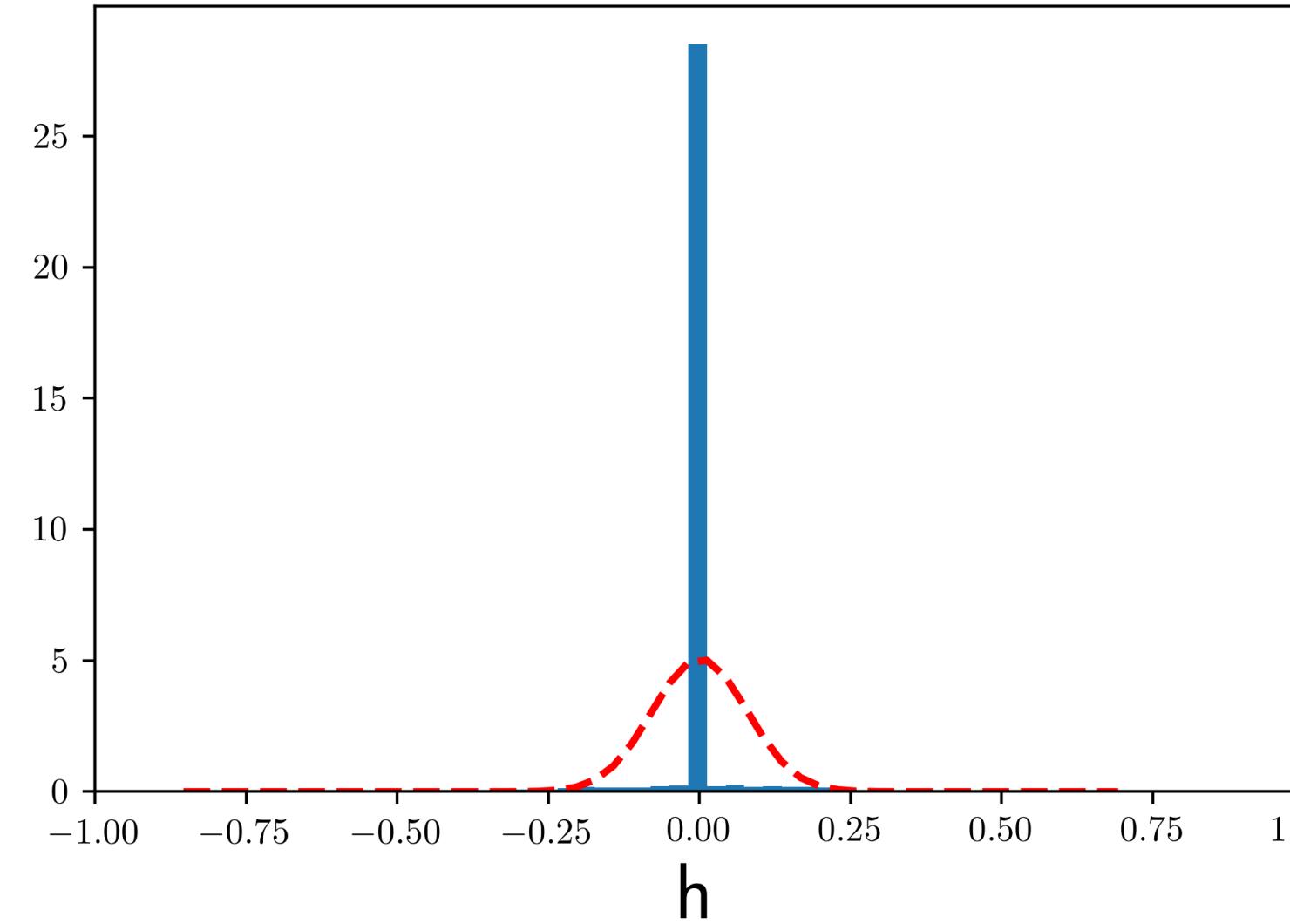
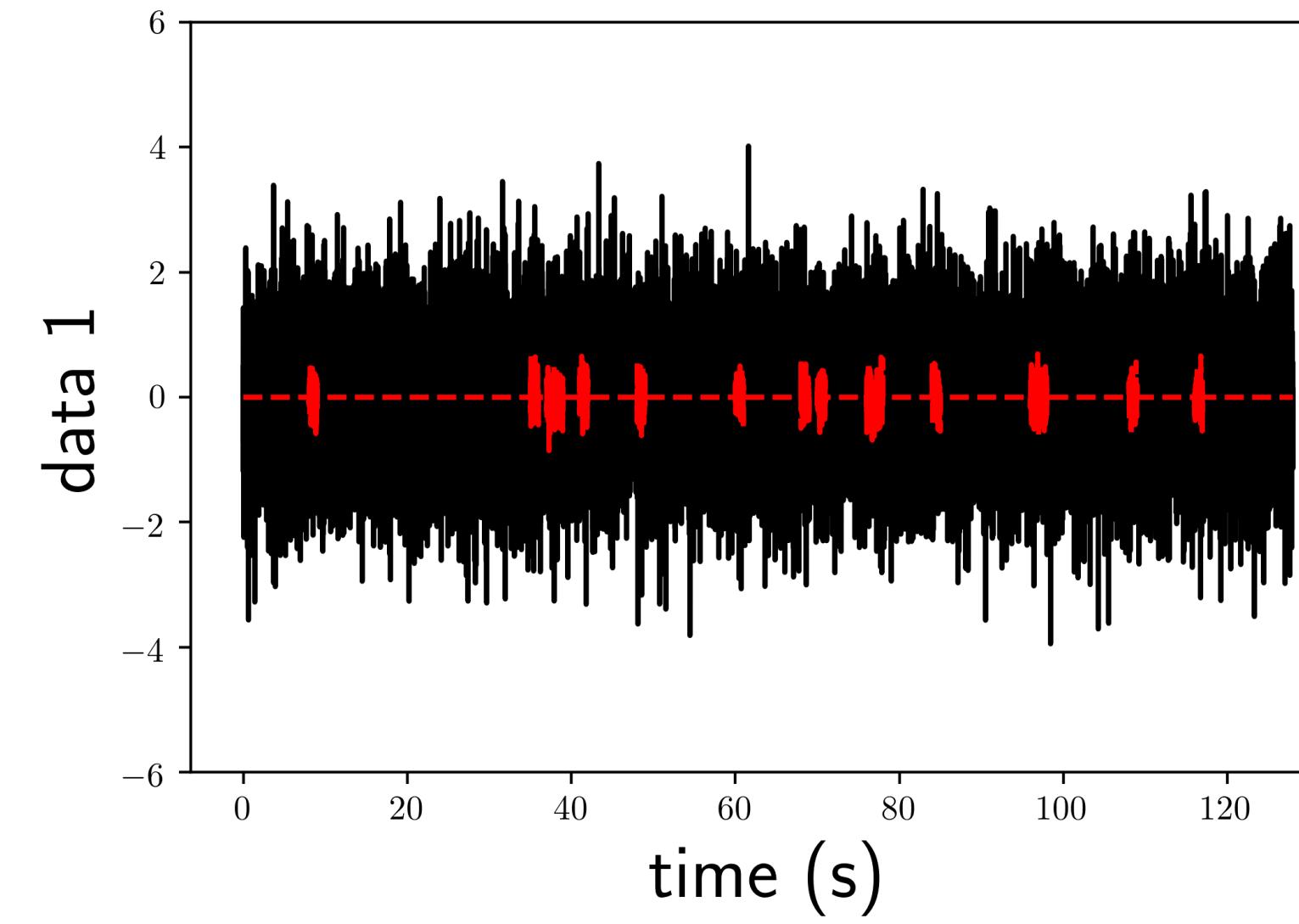
(galactic plane in equatorial coords)

(ii) differ in terms of temporal distribution

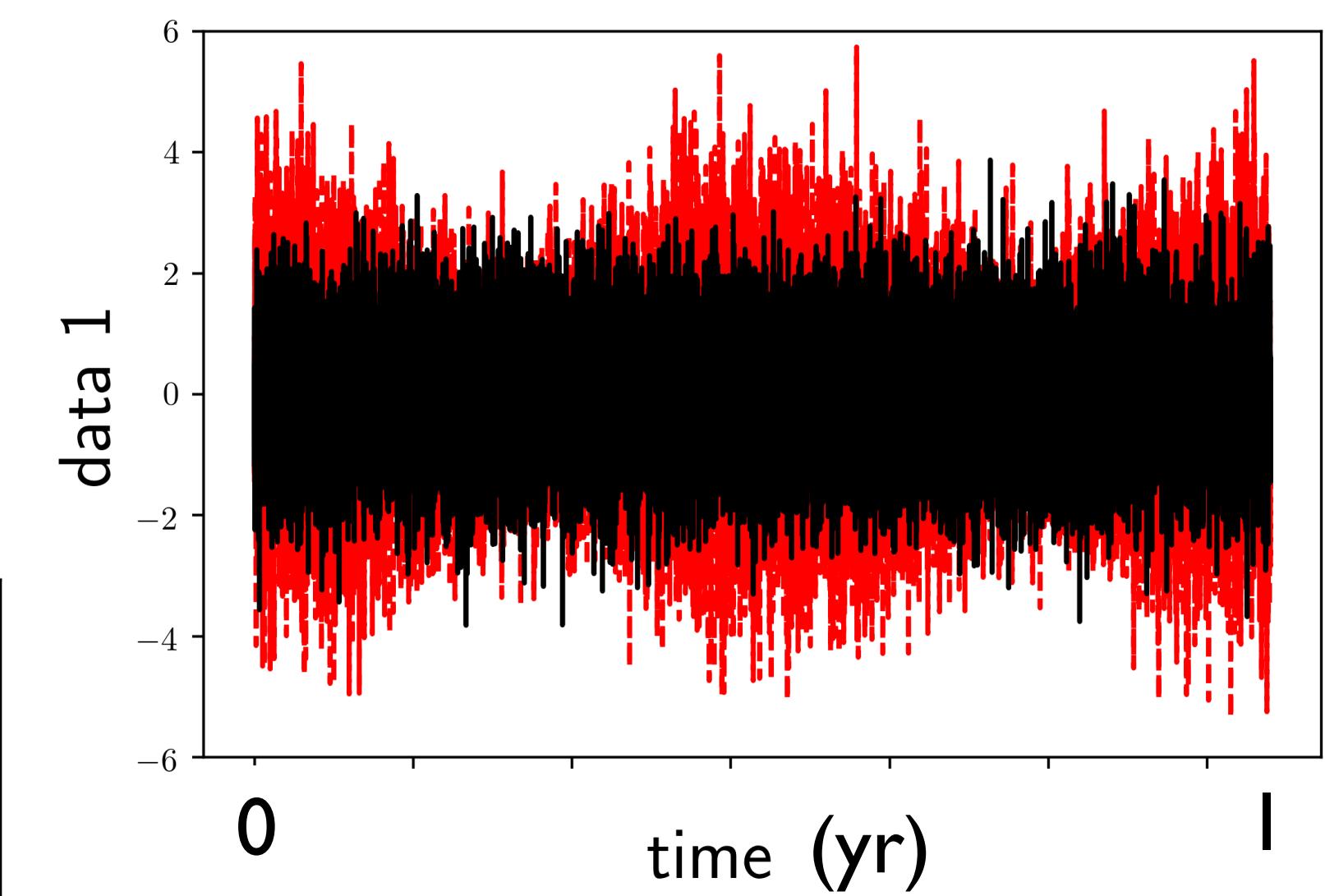
Continuous (Gaussian)



Intermittent (non-Gaussian)



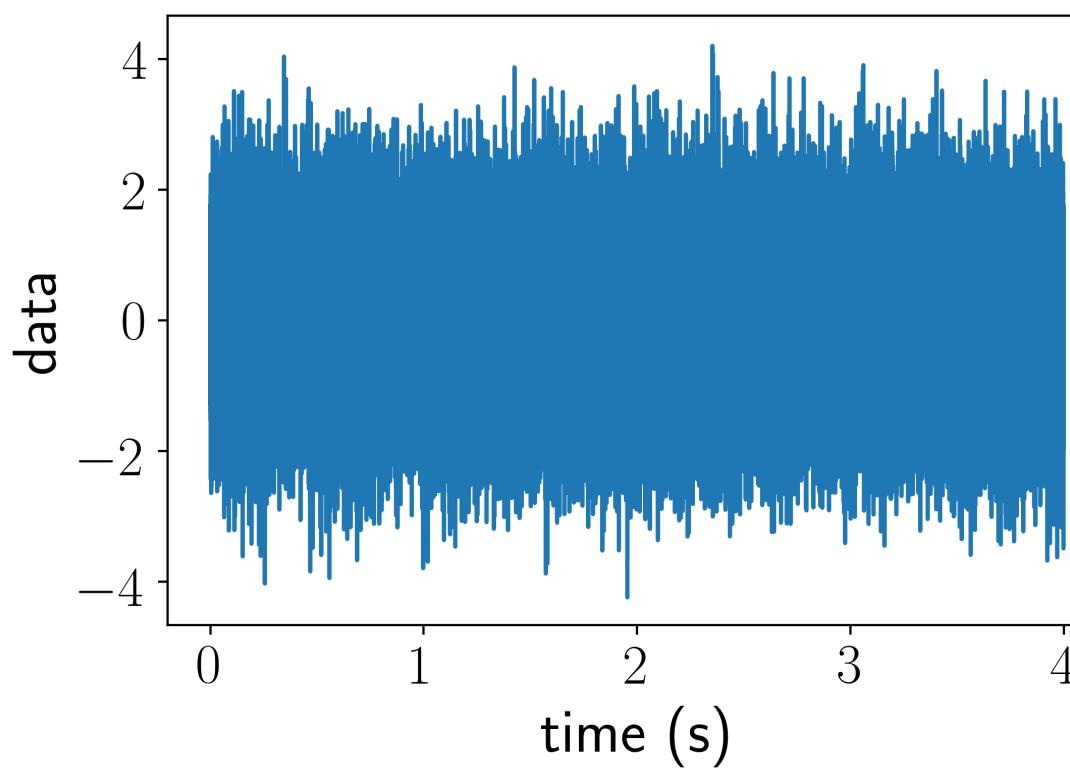
Foreground



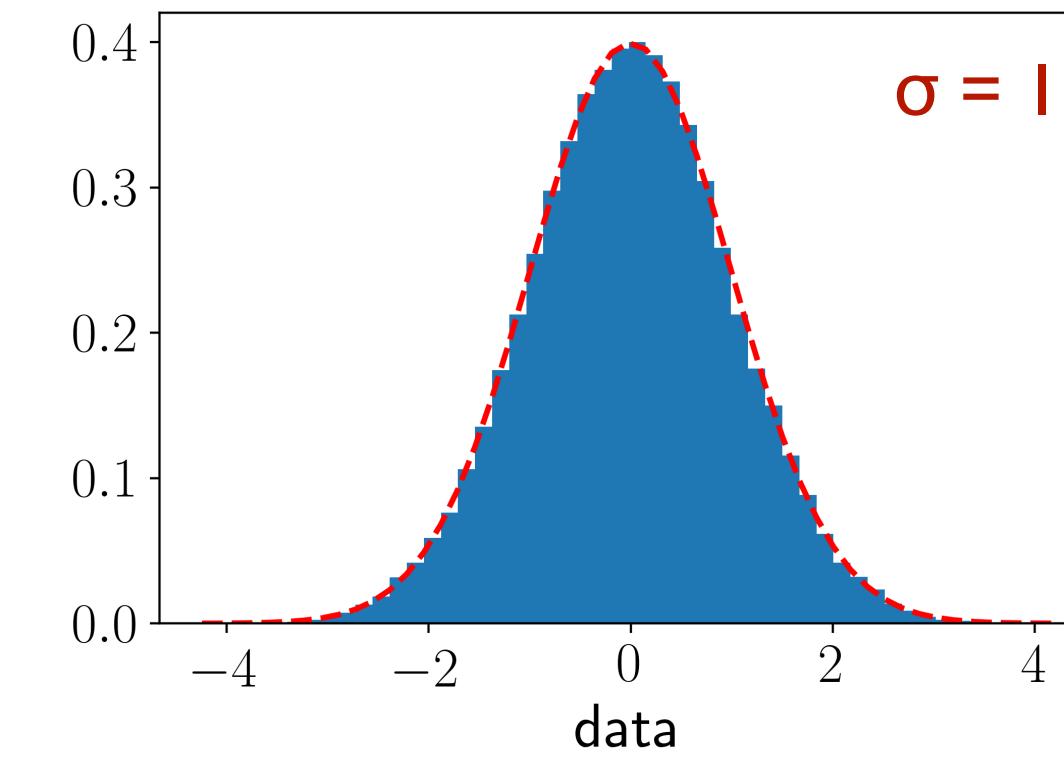
(e.g., from galactic white dwarf binaries;
modulated by LISA's orbital motion)

(iii) differ in terms of spectral distribution (power spectra)

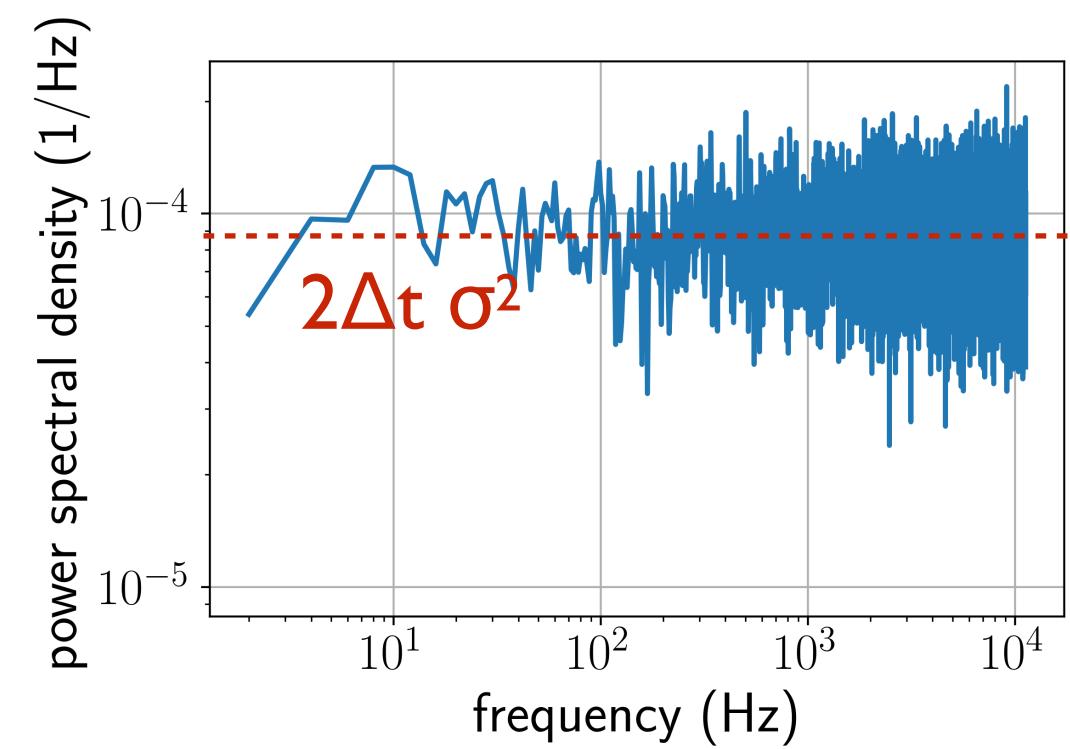
white noise



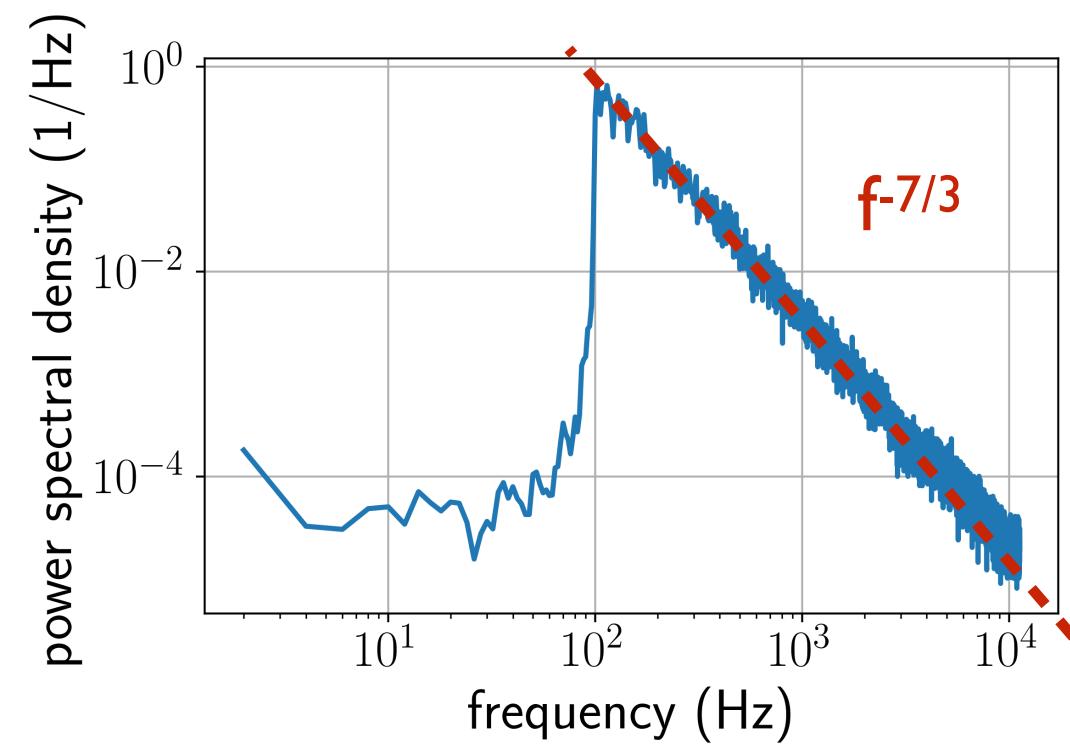
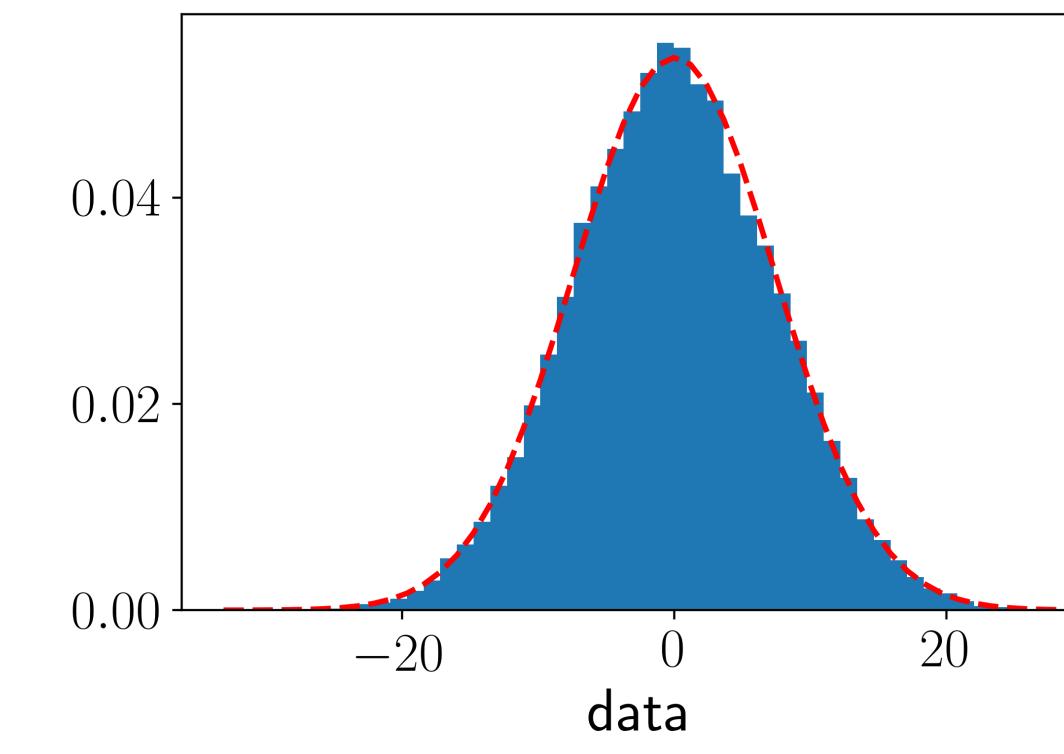
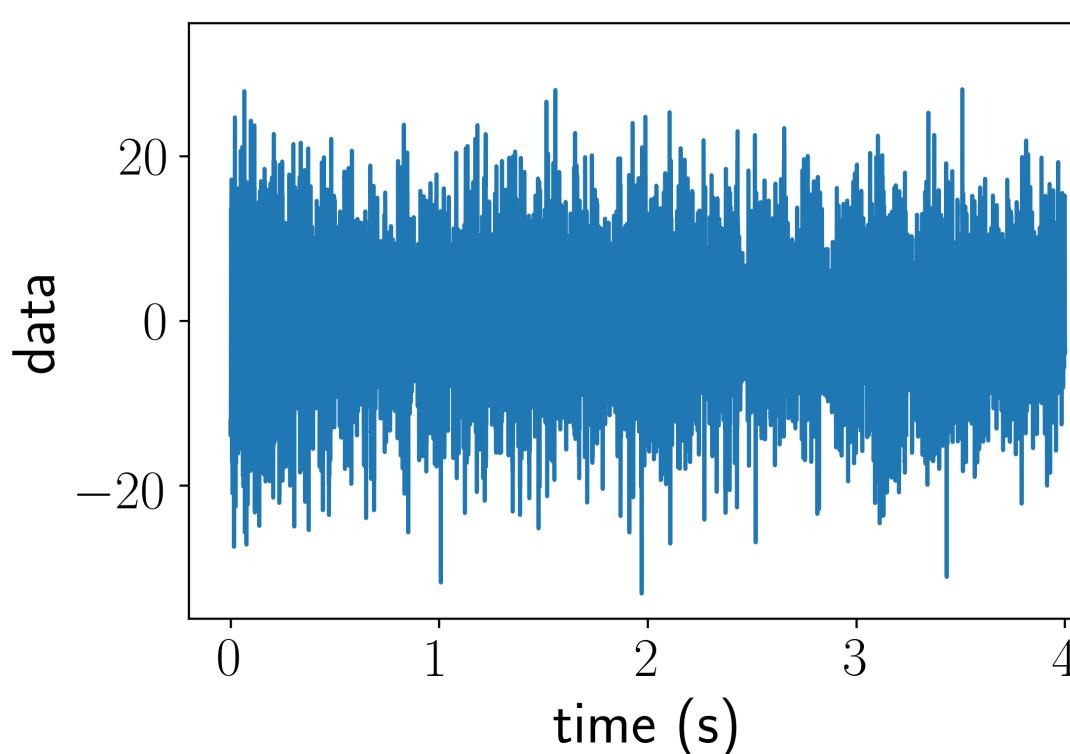
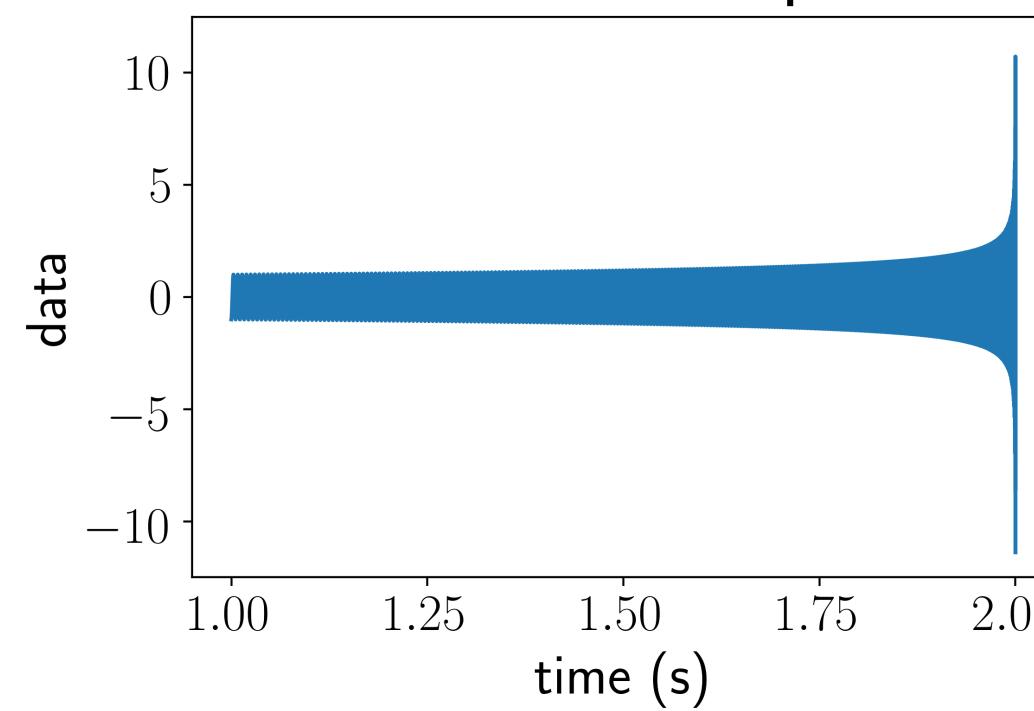
histograms



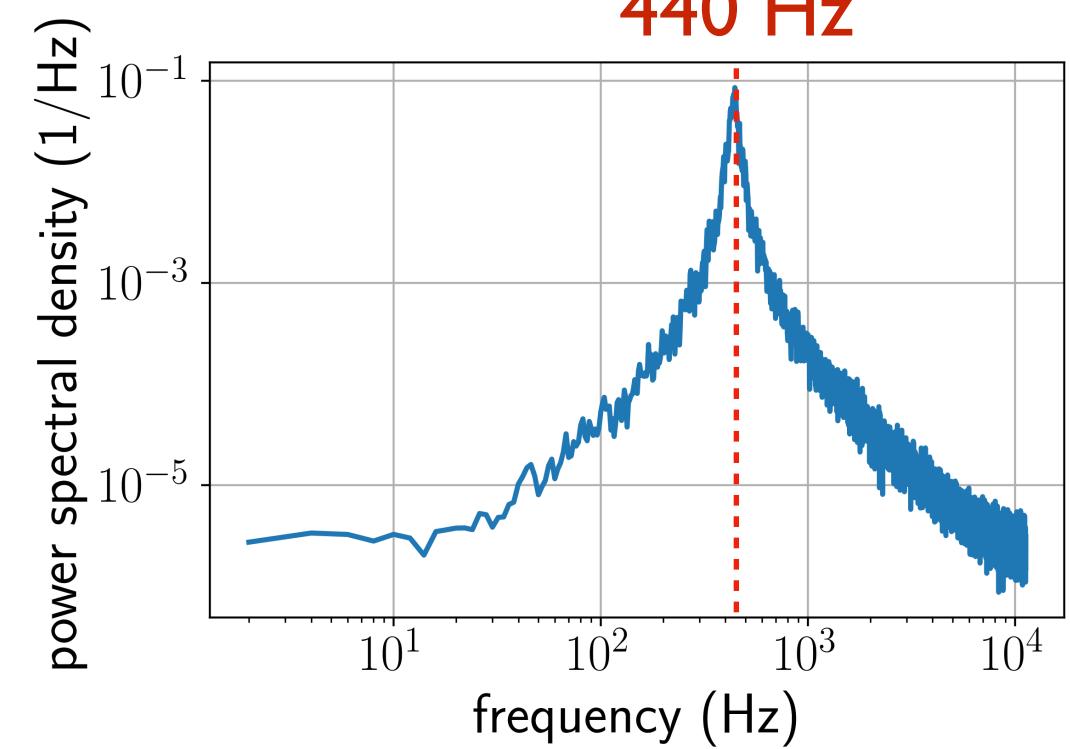
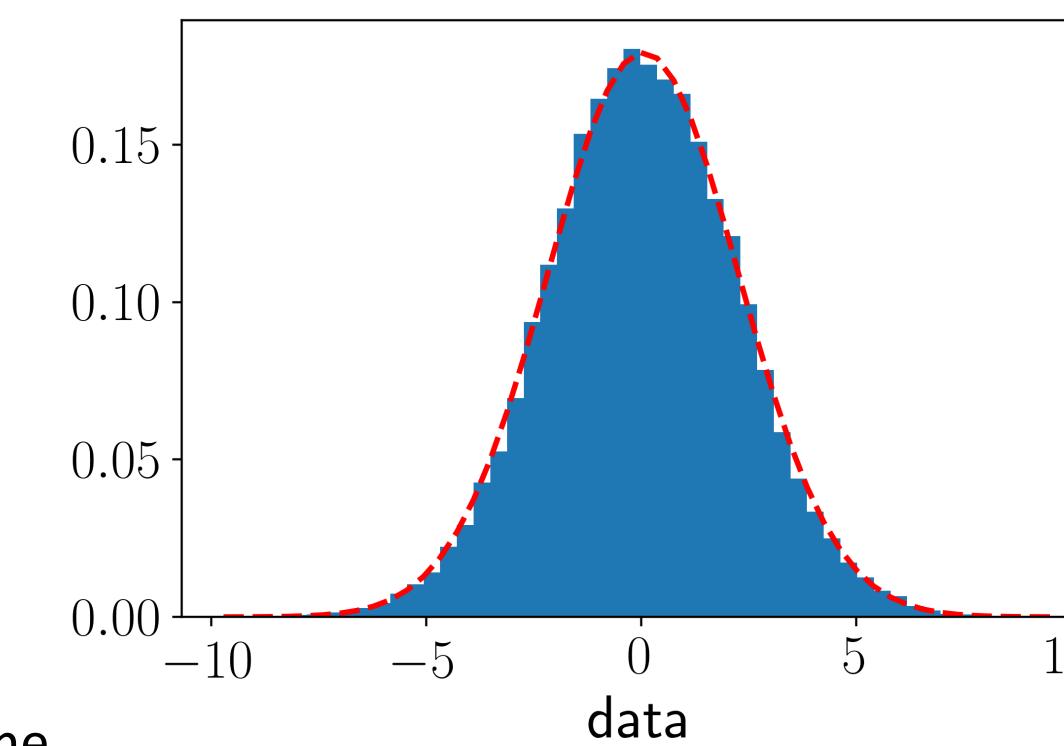
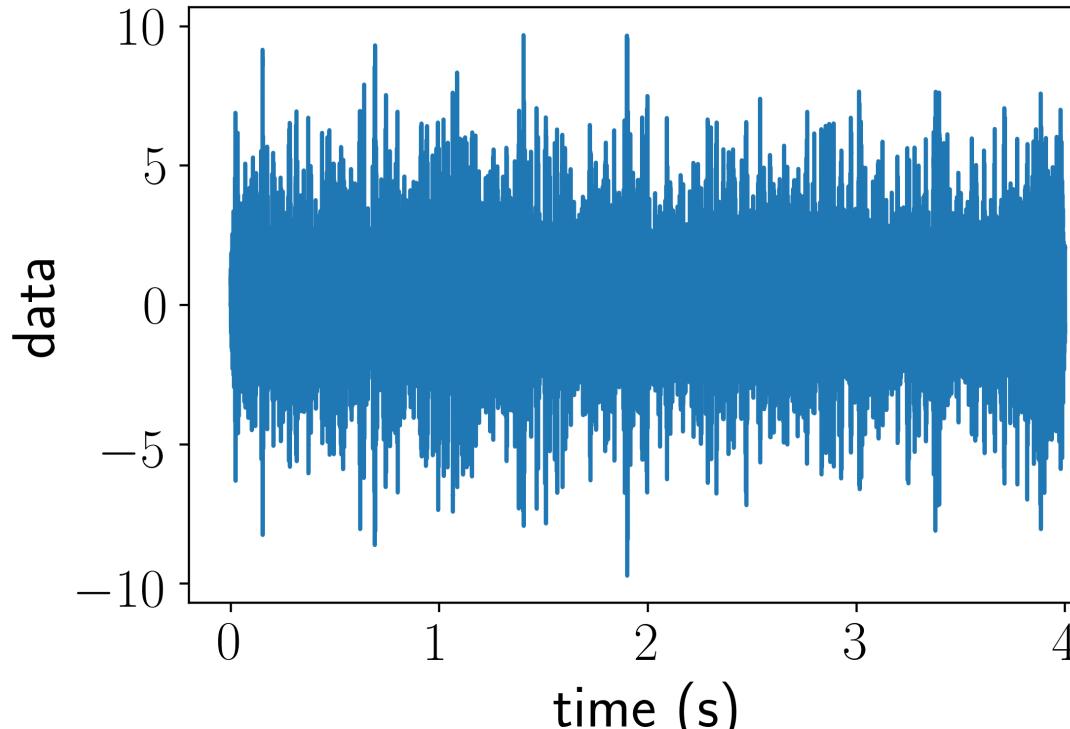
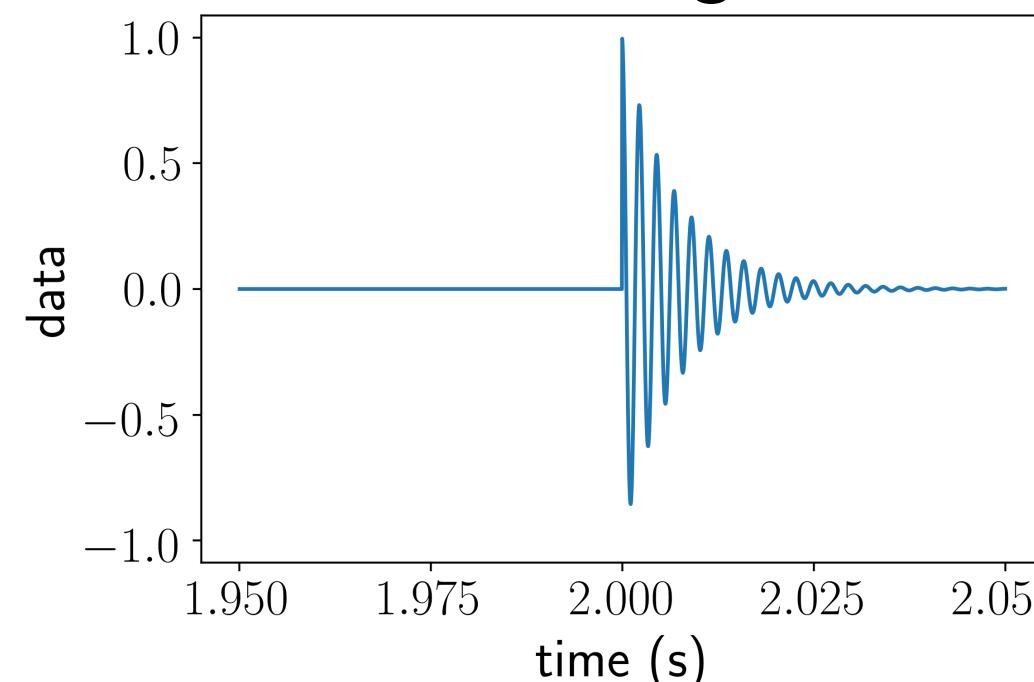
power spectra



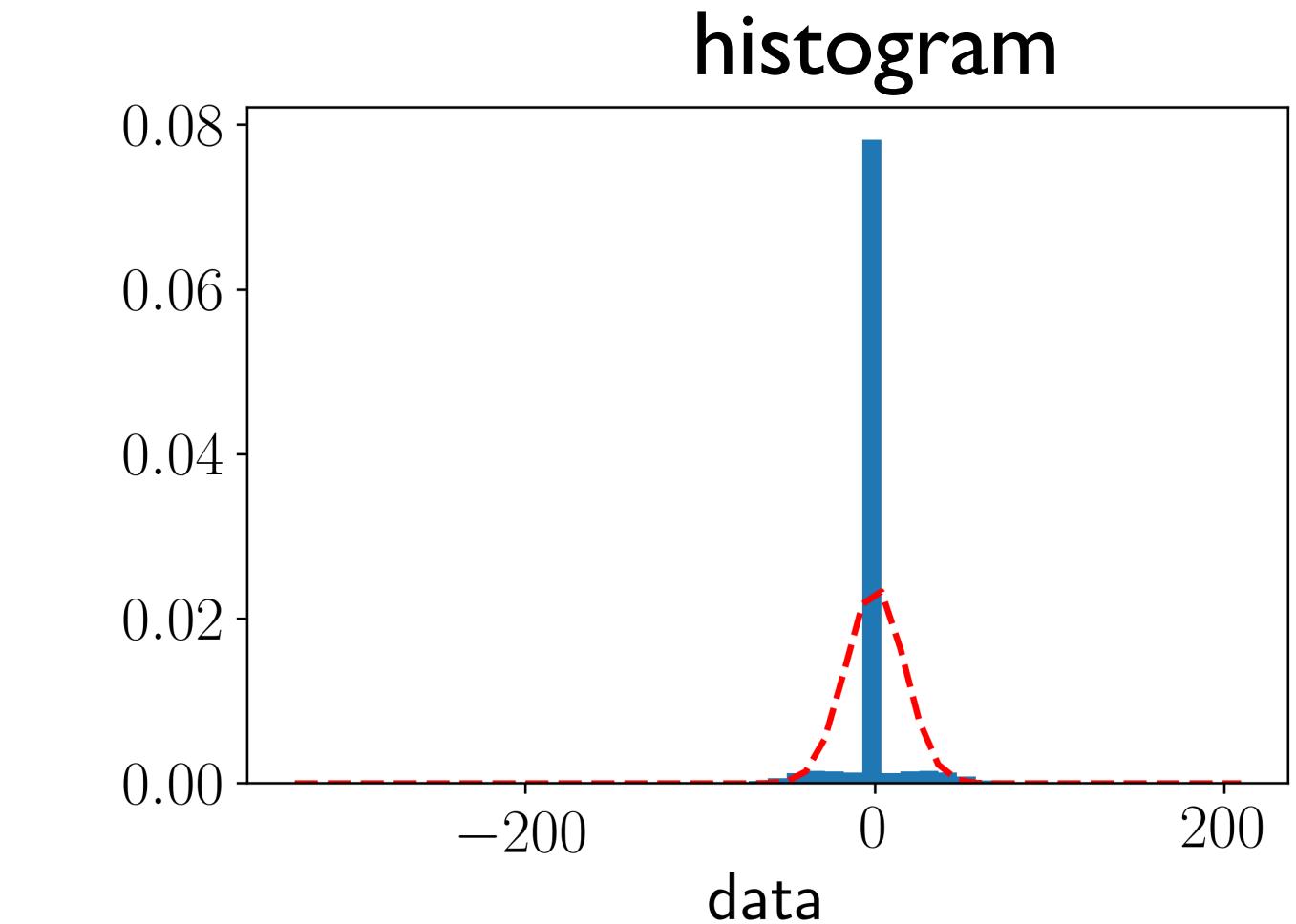
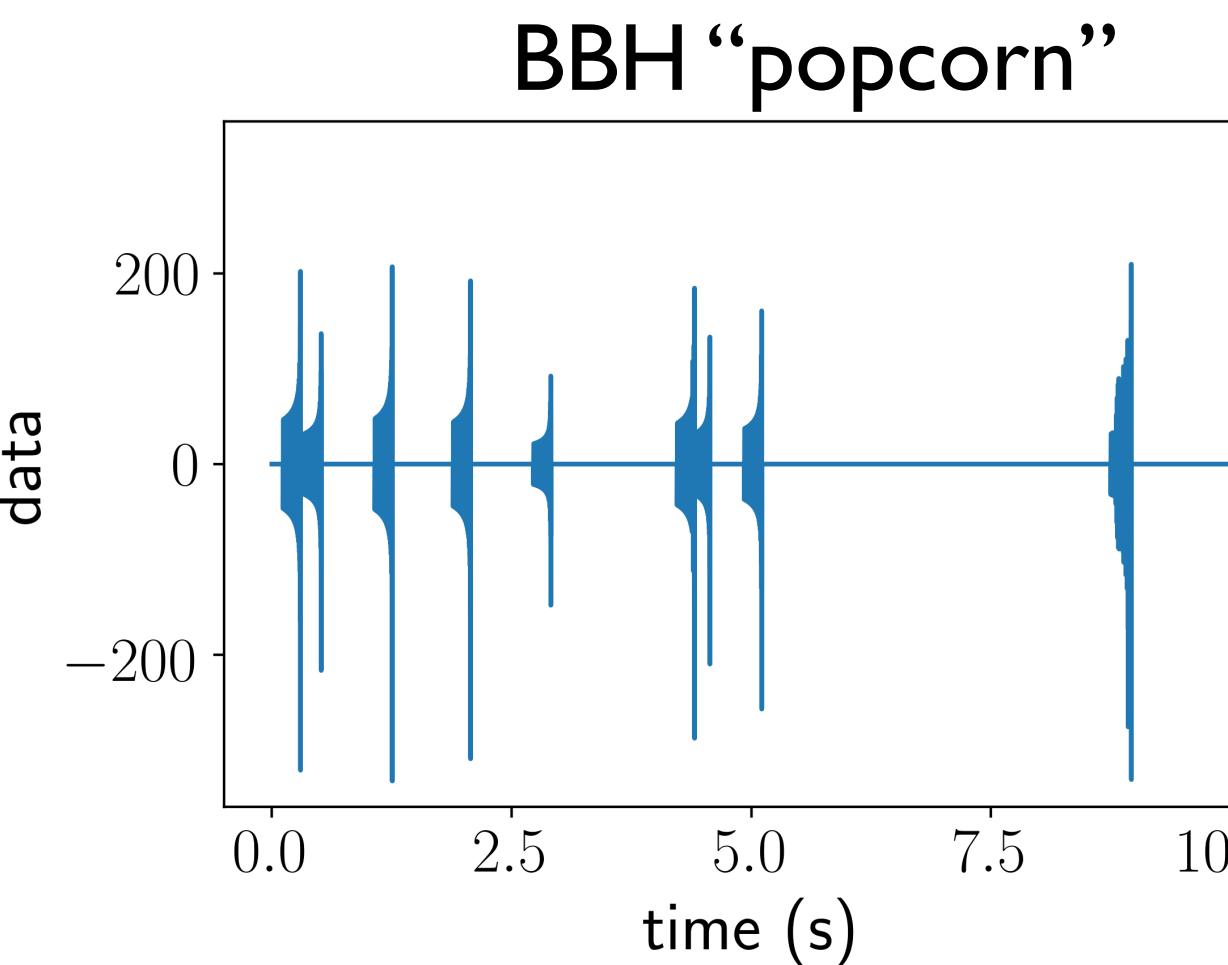
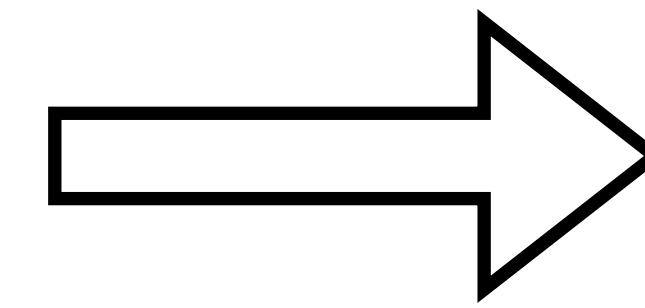
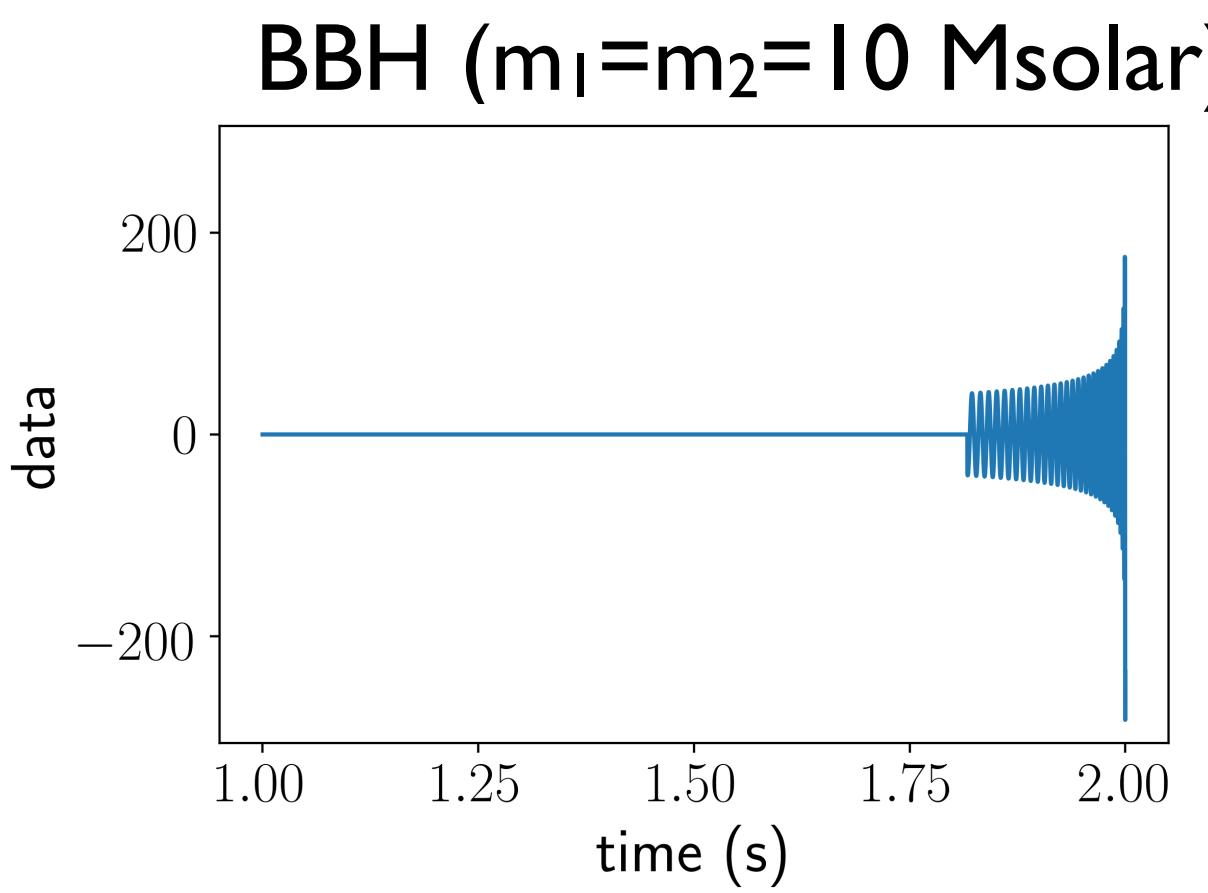
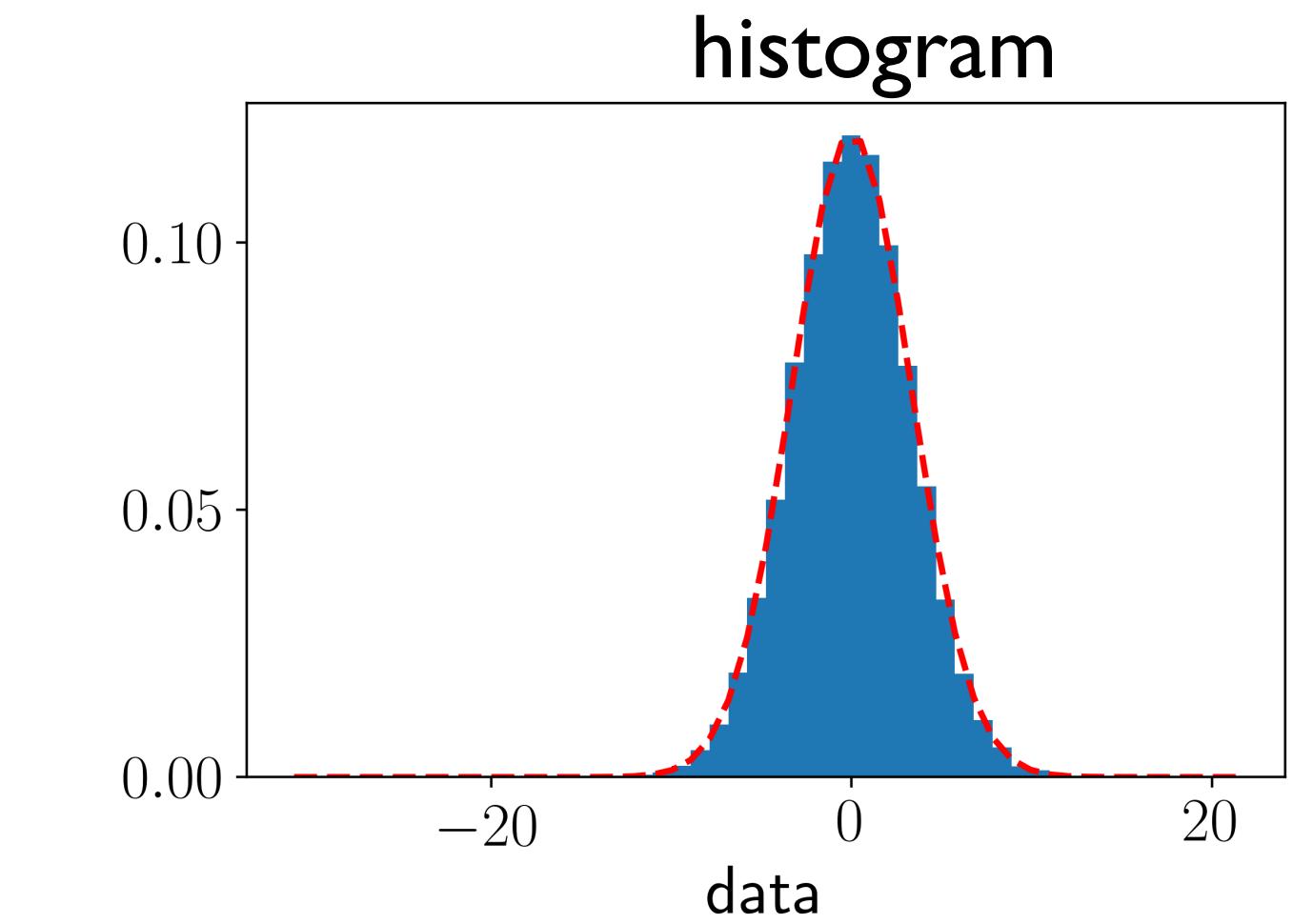
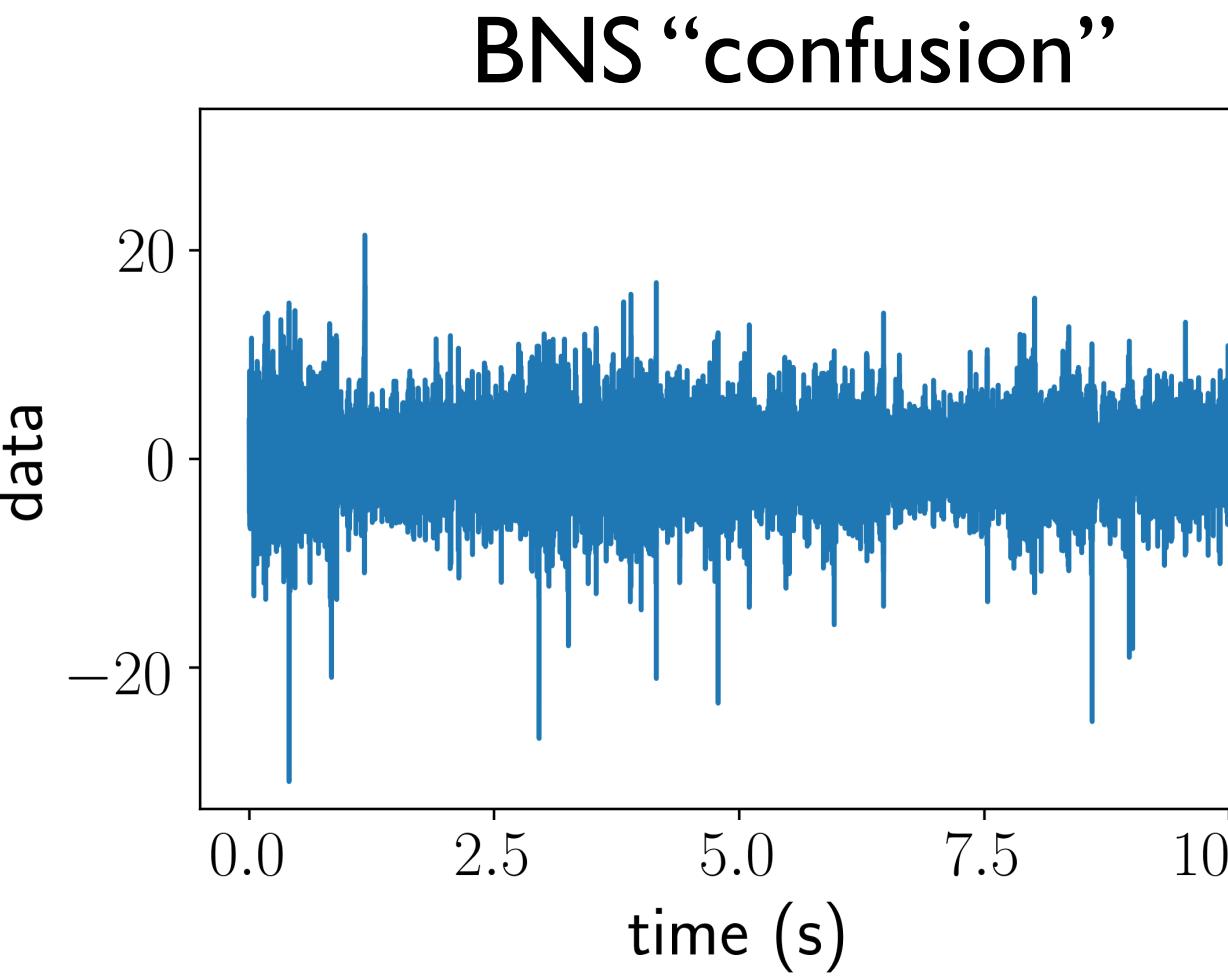
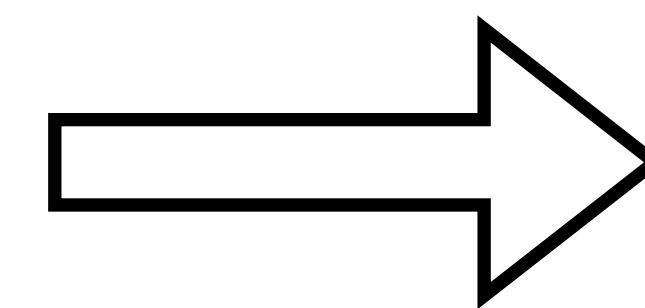
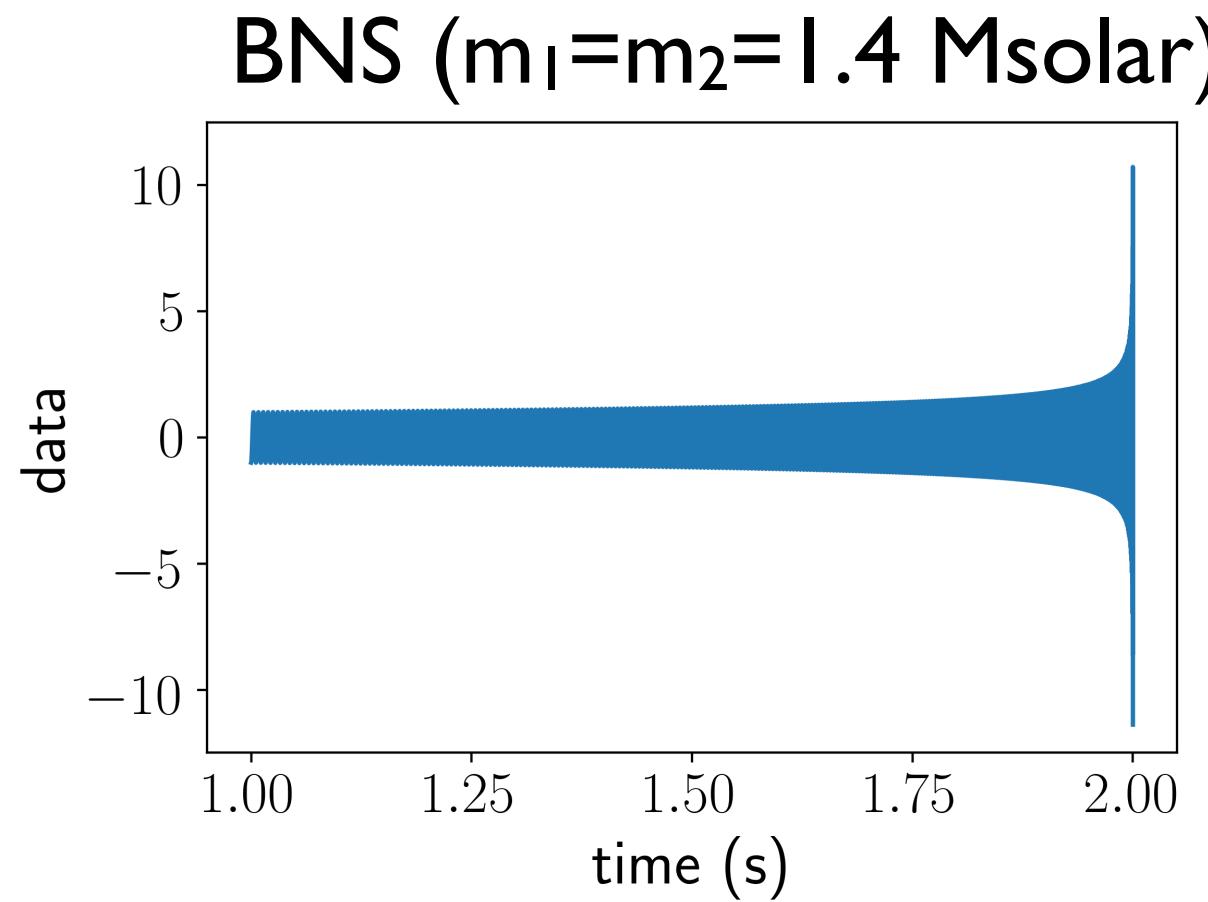
BNS chirp



BBH ringdown



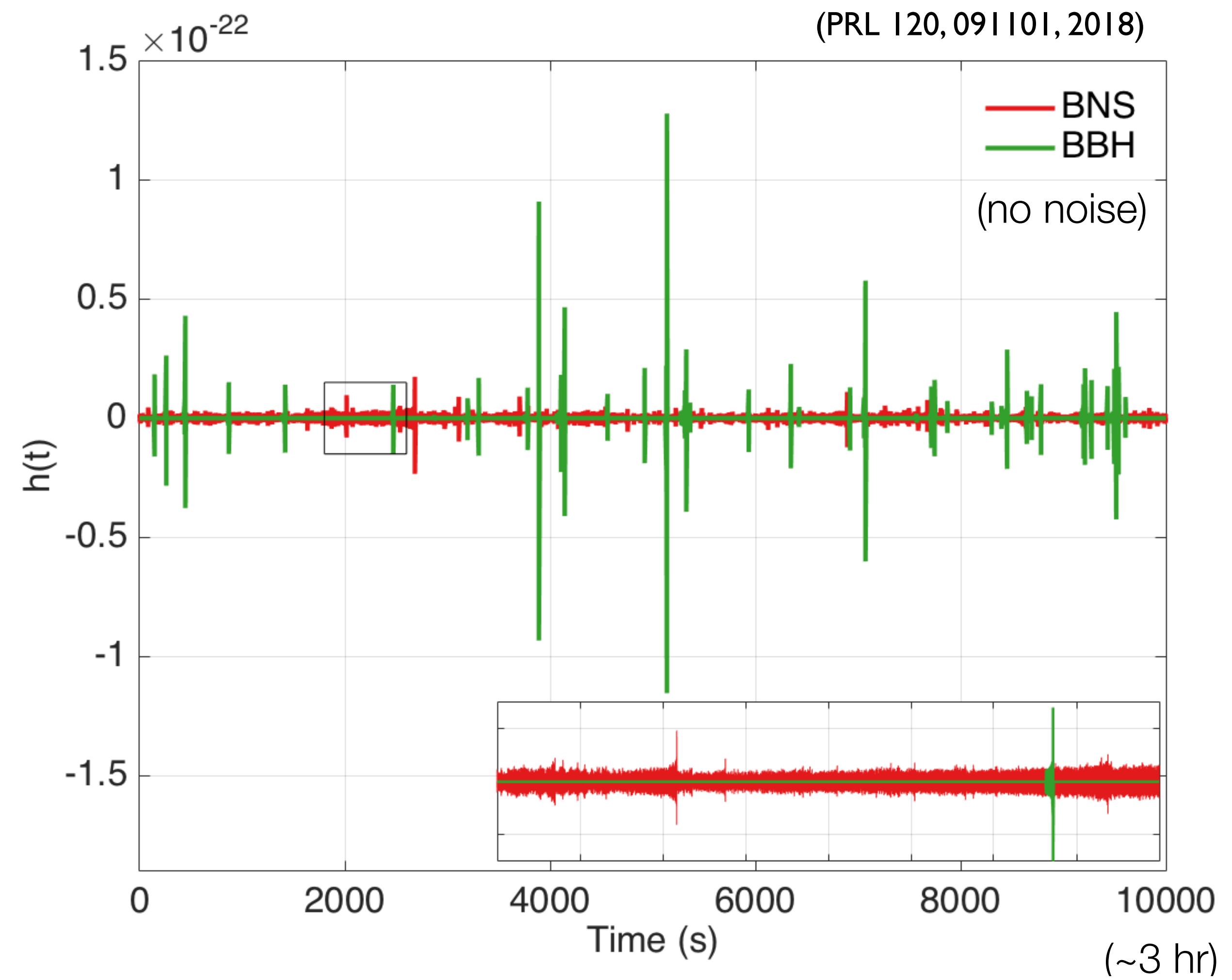
Rate estimate predict BNS “confusion” and BBH “popcorn”



III. How do we characterize a stochastic background??

Definition

- Superposition of GW signals that are either **too weak or too numerous** to individually detect
- Confusion-limited GW signal **looks like noise** in an individual detector
- **Characterized statistically** in terms of moments (ensemble averages) of the metric perturbations



Plane wave expansion, ensemble average

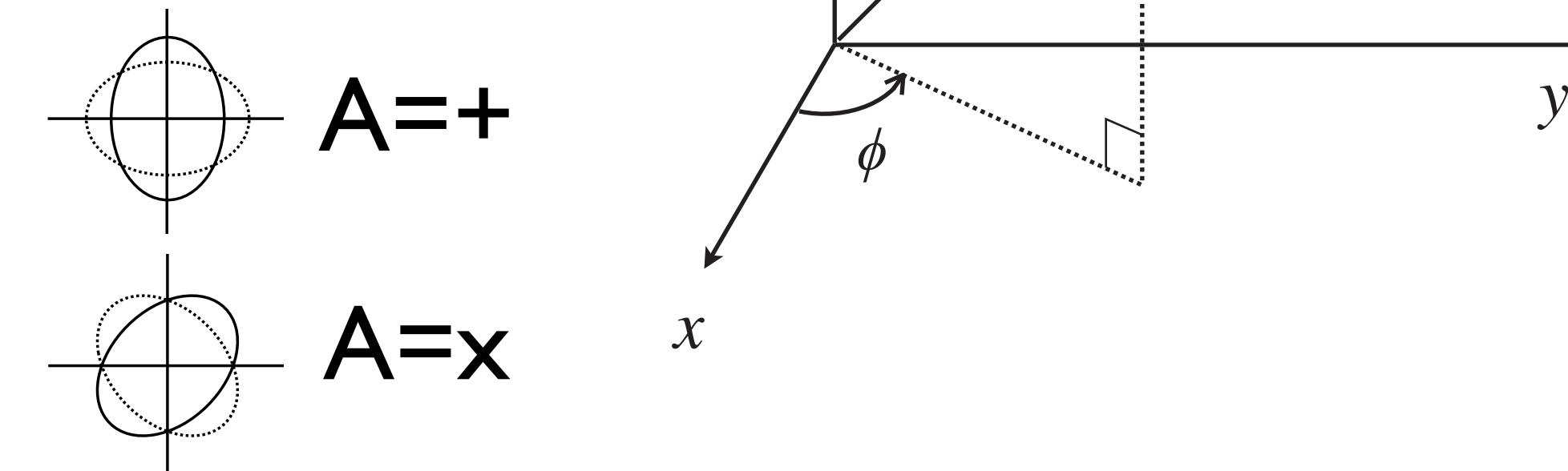
- Plane wave expansion:

$$h_{ab}(t, \vec{x}) = \int_{-\infty}^{\infty} df \int d^2\Omega_{\hat{k}} \sum_{A=+, \times} h_A(f, \hat{k}) e_{ab}^A(\hat{k}) e^{i2\pi f(t - \hat{k} \cdot \vec{x}/c)}$$

- Polarization tensors:

$$e_{ab}^+(\hat{k}) = \hat{l}_a \hat{l}_b - \hat{m}_a \hat{m}_b, \quad e_{ab}^\times(\hat{k}) = \hat{l}_a \hat{m}_b + \hat{m}_a \hat{l}_b$$

$$\hat{k} = -\hat{r}, \quad \hat{l} = -\hat{\phi}, \quad \hat{m} = -\hat{\theta}$$



- Statistical properties encoded in:

$$\cancel{\langle h_A(f, \hat{k}) \rangle}, \quad \langle h_A(f, \hat{k}) h_{A'}(f', \hat{k}') \rangle, \quad \langle h_A(f, \hat{k}) h_{A'}(f', \hat{k}') h_{A''}(f'', \hat{k}'') \rangle, \quad \ddots$$

(no loss of generality)

in terms of quadratic expectation values
(if Gaussian)

Quadratic expectation values specify different types of Gaussian stochastic GW backgrounds

- stationary, unpolarized and isotropic:

$$\langle h_A(f, \hat{k}) h_{A'}^*(f', \hat{k}') \rangle = \frac{1}{16\pi} S_h(f) \delta(f - f') \delta_{AA'} \delta^2(\hat{k}, \hat{k}')$$

- stationary, unpolarized and anisotropic:

$$\langle h_A(f, \hat{k}) h_{A'}^*(f', \hat{k}') \rangle = \frac{1}{4} \mathcal{P}(f, \hat{k}) \delta(f - f') \delta_{AA'} \delta^2(\hat{k}, \hat{k}') \quad \text{where} \quad S_h(f) = \int d^2 \Omega_{\hat{k}} \mathcal{P}(f, \hat{k})$$

power spectral density (Hz⁻¹)

$$S_h(f) = \frac{3H_0^2}{2\pi^2} \frac{\Omega_{\text{gw}}(f)}{f^3}$$

energy density spectrum
(dimensionless)

$$\Omega_{\text{gw}}(f) \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{gw}}}{d \ln f} = \frac{f}{\rho_c} \frac{d\rho_{\text{gw}}}{df}$$

characteristic strain
(dimensionless)

$$h_c(f) \equiv \sqrt{f S_h(f)} = A_\alpha \left(\frac{f}{f_{\text{ref}}} \right)^\alpha$$

$$\rho_c \equiv \frac{3H_0^2 c^2}{8\pi G} \quad \rho_{\text{gw}} = \frac{c^2}{32\pi G} \langle \dot{h}_{ab}(t, \vec{x}) \dot{h}^{ab}(t, \vec{x}) \rangle$$

For a collection of astrophysical sources

- “Phinney formula” (1991):

$$\Omega_{\text{gw}}(f) = \frac{1}{\rho_c} \int_0^\infty dz n(z) \frac{1}{1+z} \left(f_s \frac{dE_{\text{gw}}}{df_s} \right) \Big|_{f_s=f(1+z)}$$

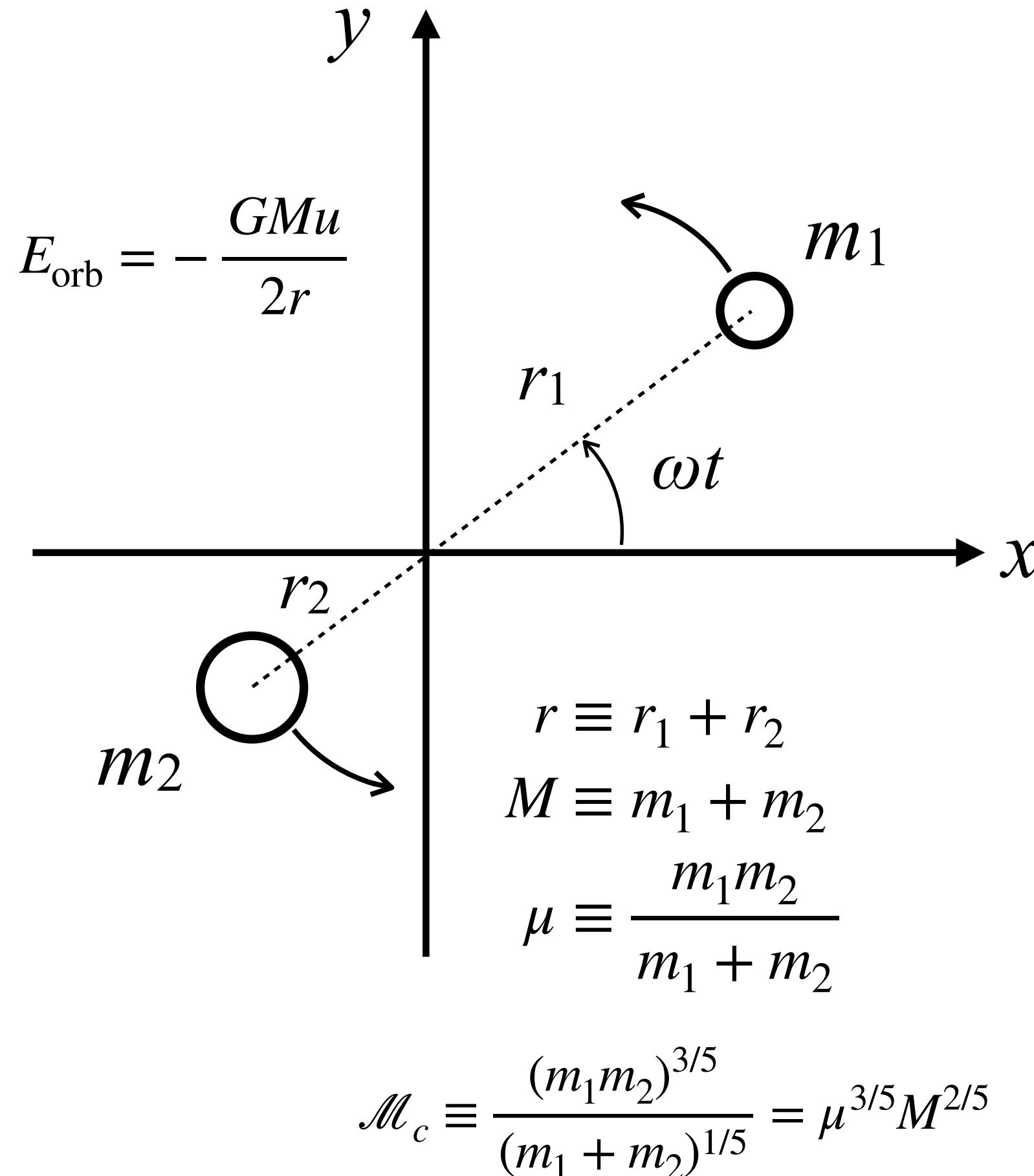
$f_s = f(1+z)$

- In terms of event rate:

$$n(z) dz = R(z) |dt|_s \quad \left| \frac{dt}{dz} \right|_s = \frac{1}{(1+z)H_0 E(z)} \quad E(z) = \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda} \leftarrow \text{cosmology}$$

$$\Omega_{\text{gw}}(f) = \frac{f}{\rho_c H_0} \int_0^\infty dz R(z) \frac{1}{(1+z)E(z)} \left(\frac{dE_{\text{gw}}}{df_s} \right) \Big|_{f_s=f(1+z)}$$

Example: circular binaries



Units: $G = c = 1$

Kepler's law: $\omega^2 r^3 = GM \implies r \sim M^{1/3} \omega^{-2/3}, \dot{r} \sim -r\dot{\omega}/\omega$

Energy balance: $\frac{dE_{\text{gw}}}{dt} = -\frac{dE_{\text{orb}}}{dt}$

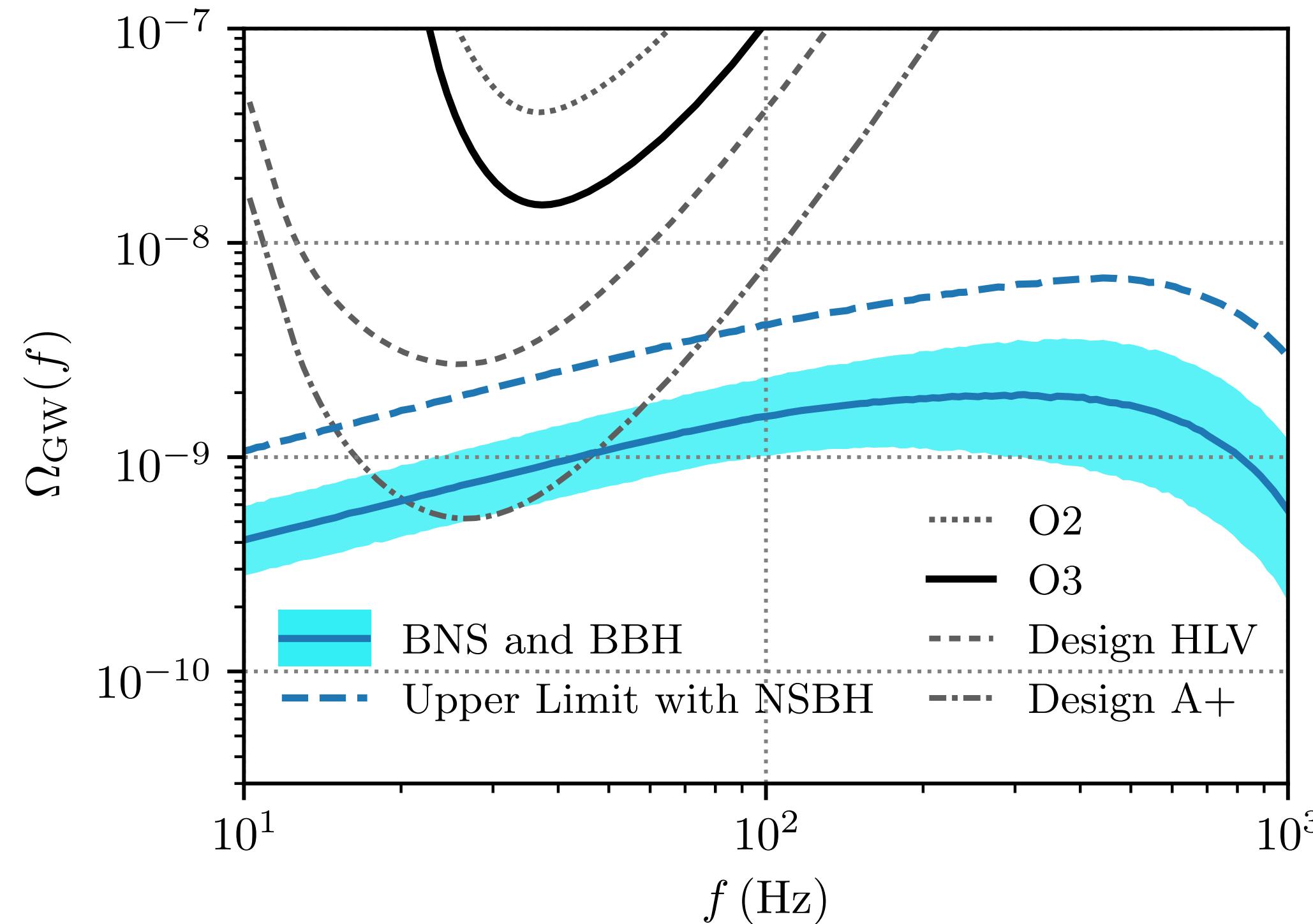
$$\implies \frac{dE_{\text{gw}}}{dt} \sim -M\mu\dot{r}/r^2 \sim \mathcal{M}_c^{5/3} \omega^{-1/3} \dot{\omega}$$

$$\implies \frac{dE_{\text{gw}}}{df} = \frac{dt}{df} \frac{dE_{\text{gw}}}{dt} \sim \frac{1}{\dot{\omega}} \frac{dE_{\text{gw}}}{dt} \sim \mathcal{M}_c^{5/3} f^{-1/3}$$

$$\implies \boxed{\Omega_{\text{gw}}(f) \propto f^{2/3}, h_c(f) \propto f^{-2/3}}$$

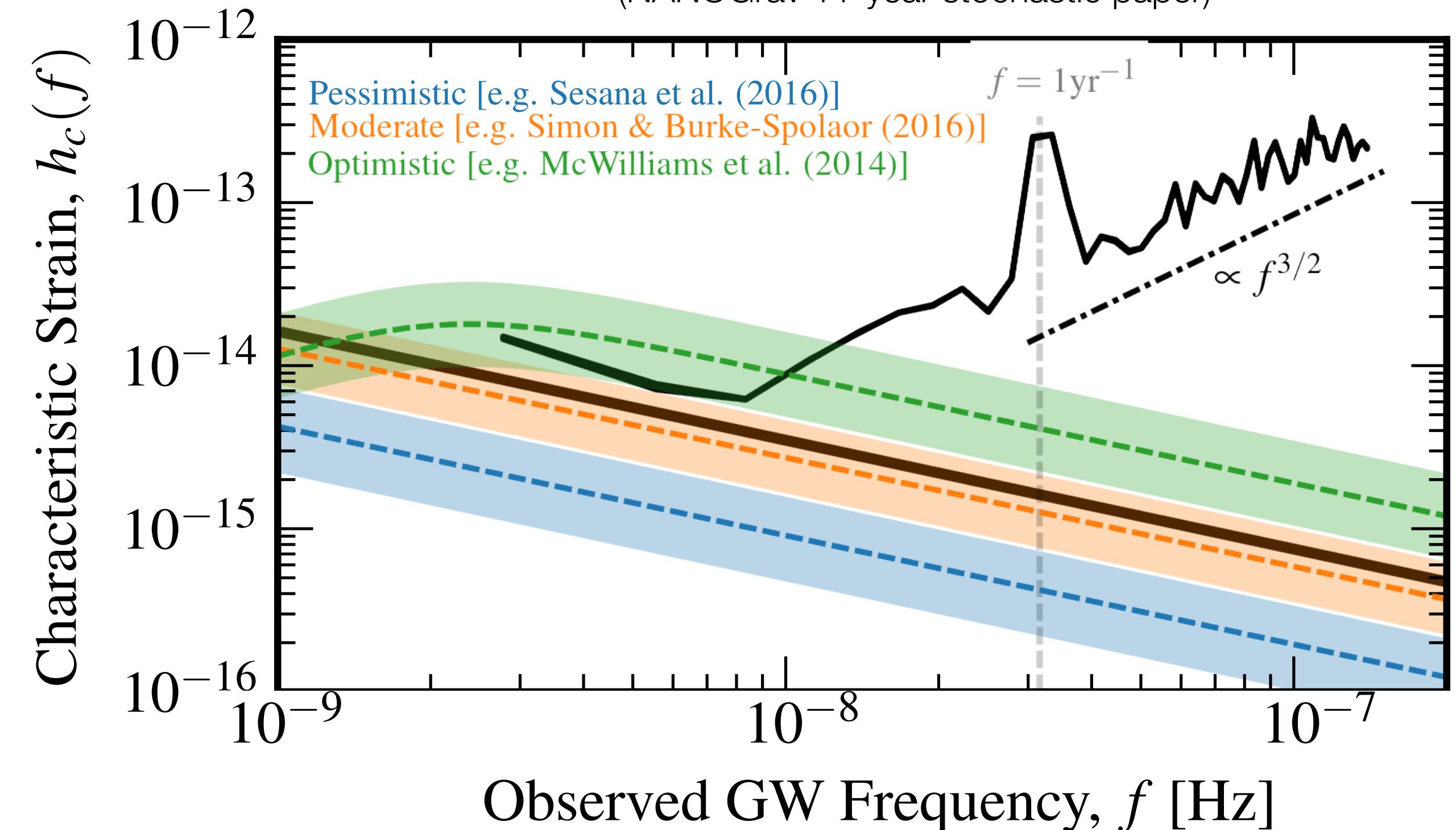
Ground-based interferometers

(LVK O3 Isotropic paper)



Pulsar Timing

(NANOGrav 11-year stochastic paper)



$$\Omega_{\text{gw}}(f) \propto f^{2/3}, \quad h_c(f) \propto f^{-2/3}$$

Exercises

1. Verify the expected rate of stellar mass BBH mergers is between ~ 1 per minute and a few per hour.
2. Derive the relationship between the strain spectral density $S_h(f)$ and the fractional energy density spectrum $\Omega_{\text{gw}}(f)$ using the plane-wave expansion for the metric perturbations and the quadratic expectation values for the Fourier components.
3. Verify the expression for $|dt/dz|$ as well as the “Phinney formula” in terms of the rate density $R(z)$.