Universal Exact Compression of Differentially Private Mechanisms

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Background

Differential Privacy (DP) [1].

Local randomizer $\mathcal{A}: \mathcal{X} \to \mathcal{Z}$ with induced distribution $P_{Z|X}$ satisfies (ε, δ)-local DP if for any $x, x' \in \mathcal{X}$ and measurable set $\mathcal{S} \subseteq \mathcal{Z}$,

$$\Pr(Z \in \mathcal{S} | X = x) \le e^{\varepsilon} \cdot \Pr(Z \in \mathcal{S} | X = x') + \delta.$$

Compression of DP Mechanisms.

Objective: Compress **arbitrary** DP mechanisms **exactly** (i.e., $Z \sim P_{Z|X}$) to near-optimal sizes, while ensuring privacy guarantees.

Prior works:

· [2-5]: Compress ε-local DP mechanism **approximately**.

· [6,7]: Dithered quantization tools ensure a correct simulated distribution, but only for **additive noise** mechanisms.

Channel Simulation via Poisson Functional Representation [8]

- Let $(T_i)_i$ be a Poisson process with rate 1, independent of $Z_i \overset{\text{i.i.d.}}{\sim} Q$.
- Then $(Z_i, T_i)_i$ is a Poisson process with intensity measure $Q \times \lambda_{[0,\infty)}$.
- Fix distribution P absolutely continuous w.r.t Q. Let

$$\tilde{T}_i \triangleq T_i \cdot \left(\frac{\mathrm{d}P}{\mathrm{d}Q}(Z_i) \right)^{-1}.$$

Theorem: $K \triangleq \arg\min_i \tilde{T}_i$ and $Z = Z_K$, then $Z \sim P$.

Our Contributions

Poisson Private Representation is the first DP compressor that achieves:

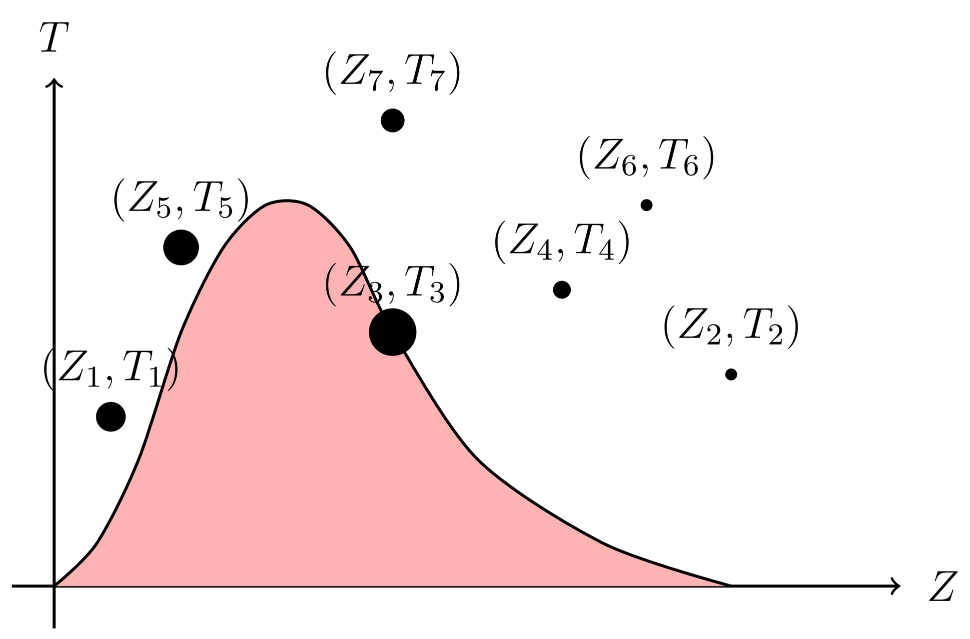
(a) **Exactness**: it simulates $P_{Z|X}$ exactly;

(b) **Universality**: it simulates **arbitrary** DP mechanism;

(c) **Communication-efficiency**: it compresses $P_{Z|X}$ to a near-optimal size:

$$I(X;Z) + \log\big(I(X;Z) + 1\big) + O(1) \text{ bits.}$$

(d) **Privacy**: it ensures both local and central DP.



Poisson Private Representation (PPR)

Algorithm:

Input: private $x \in \mathcal{X}$, (ε, δ)-local DP mechanism $P_{Z|X}$, reference distribution Q, parameter $\alpha > 1$.

(a) Generate shared randomness between user and server

$$(Z_i)_{i=1,2,\ldots} \overset{\text{i.i.d.}}{\sim} Q.$$

(b) The user knows $(Z_i)_i$, x, $P_{Z|X}$ and performs:

(1) Generate the Poisson process $(T_i)_i$ with rate 1.

(2) Compute $\tilde{T}_i \triangleq T_i \cdot \left(\frac{dP_{Z|X}(\cdot|x)}{dQ}(Z_i) \right)^{-1}$.

(3) Generate $K \in \mathbb{Z}_+$ with

$$\Pr(K = k) = \tilde{T}_k^{-\alpha} \Big/ \left(\sum_{i=1}^{\infty} \tilde{T}_i^{-\alpha} \right).$$

(4) Compress and send K.

(c) The server, which knows $(Z_i)_i$ and K, outputs $Z = Z_K$.

Remarks

- In short: given a DP-mechanism $P_{Z|X}$, PPR *simulates* it by $P_{(Z_i)_i, K|X}$.
- The exactness of PPR ($Z \sim P_{Z|X}$) follows from the PFR [8].
- While the algorithm requires infinite samples, it can be reparametrized to terminate in finite steps.
- When $\alpha = \infty$, PPR reduces to PFR.

Privacy

- **Thm 4.5**: If the mechanism $P_{Z|X}$ is ε-DP, then PPR $P_{(Z_i)_i, K|X}$ with $\alpha > 1$ is $2\alpha\varepsilon$-DP.
- **Thm 4.8**: If $P_{Z|X}$ is (ε, δ)-DP, then PPR $P_{(Z_i)_i, K|X}$ is $(\alpha\varepsilon + \tilde{\varepsilon}, 2(\delta + \tilde{\delta}))$-DP, for $\alpha > 1$, $\tilde{\varepsilon} \in (0, 1]$ and $\tilde{\delta} \in (0, 1/3]$ such that

$$\alpha \le e^{-4.2} \tilde{\delta} \tilde{\varepsilon}^2 / (-\ln \tilde{\delta}) + 1.$$

Exactness

- The output Z of PPR follows the conditional distribution $P_{Z|X}$ exactly.

Communication Efficiency

- **Thm 4.3**: For PPR with $\alpha > 1$, message K satisfies

$$\mathbb{E}[\log_2 K] \le D_{\mathsf{KL}}\left(P(\cdot|x) \| Q(\cdot)\right) + \log_2(3.56) / \min\left((\alpha - 1)/2, 1\right).$$

  - K can be encoded by a prefix-free code with expected length $\approx D_{\mathsf{KL}}(P(\cdot|x) \| Q(\cdot))$ bits.
  - If $X \sim P_X$ is random, take $Q = P_Z$ and the expected length $\approx I(X; Z)$.
- **Corollary 4.4**: For $P_{Z|X}$ with ε-local DP, the compression size

$$\le \ell + \log_2(\ell + 1) + 2 \text{ (bits),}$$

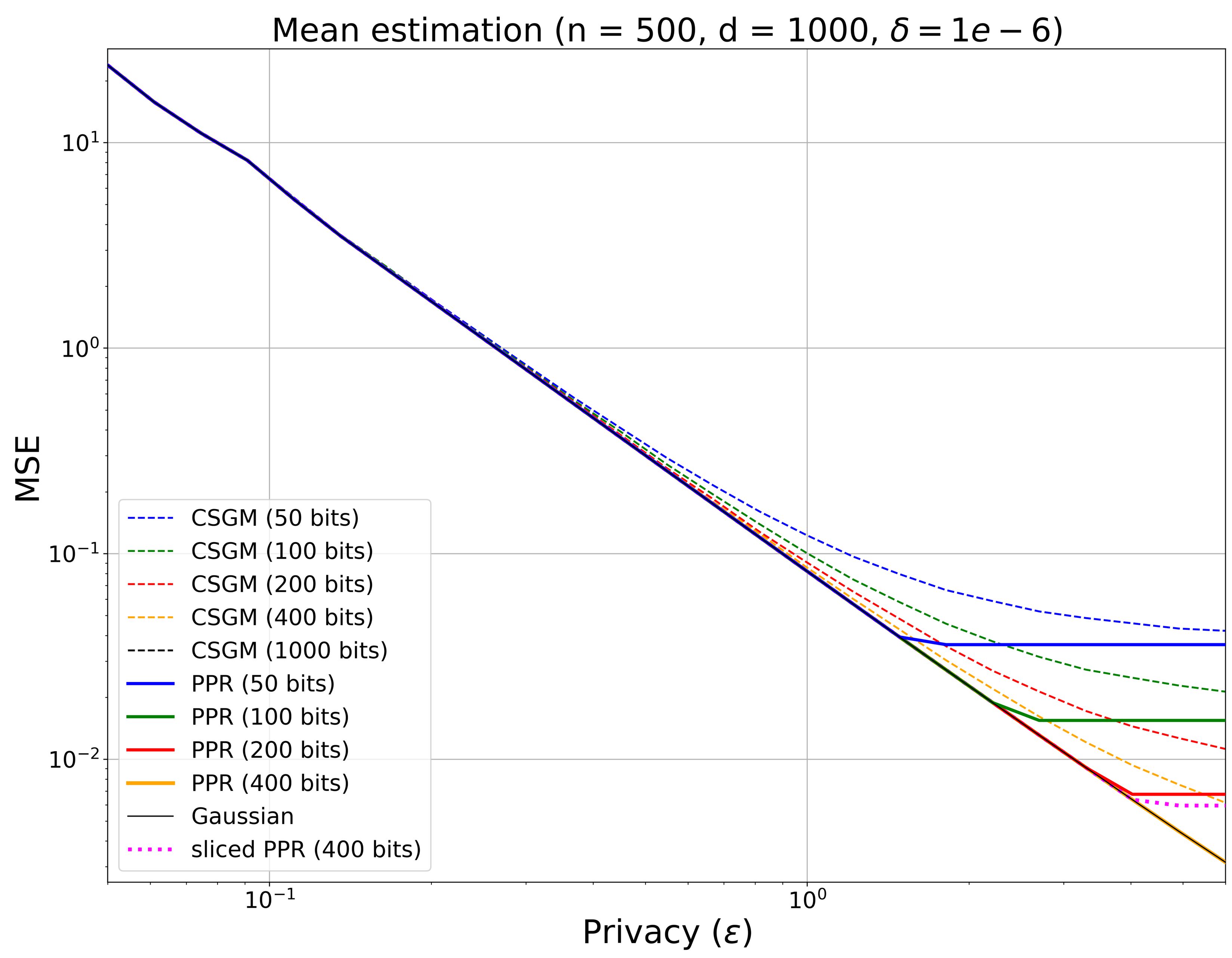
where $\ell \triangleq \varepsilon \log_2 e + \log_2(3.56) / \min\left((\alpha - 1)/2, 1\right)$.

Distributed Mean Estimation

- Consider there are n users, each with data $X_i \in \mathbb{R}^d$. They use **Gaussian mechanism** and send $Z_i \sim \mathcal{N}(X_i, \frac{\sigma^2}{n} \mathbb{I}_d)$ to server, where $\sigma \ge C\sqrt{2\ln(1.25/\delta)}/\varepsilon$.
- The server estimates the mean by $\hat{\mu}(Z^n) = \frac{1}{n} \sum_i Z_i$.
- Using PPR to compress the Gaussian mechanism:
  1. $\hat{\mu}(Z^n) = \frac{1}{n} \sum_i Z_i$ is unbiased and has (ε, δ)-central DP.
  2. PPR satisfies $(2\alpha\sqrt{n}\varepsilon, 2\delta)$-local DP for $\epsilon < 1/\sqrt{n}$.
  3. The average per-user communication $\le \ell + \log_2(\ell + 1) + 2$ bits,

$$\ell := \frac{d}{2} \log\left(\frac{n\varepsilon^2}{2d\log(1.25/\delta)} + 1 \right) + \frac{\log_2(3.56)}{\min\{(\alpha - 1)/2,\ 1\}}.$$

- Comparing to the scheme in [10]:



References

[1] Kasiviswanathan, Lee, Nissim, Raskhodnikova and Smith, *"What can we learn privately?"* SIAM Journal on Computing, 2011.

[2] Feldman and Talwar, *"Lossless compression of efficient private local randomizers,"* ICML 2021.

[3] Shah, Chen, Balle, Kairouz and Theis, *"Optimal Compression of Locally Differentially Private Mechanisms,"* AISTATS 2022.

[4] Triastcyn, Reisser and Louizos, *"DP-REC: Private & Communication-Efficient Federated Learning,"* arXiv:2111.05454.

[5] Bassily and Smith, *"Local, private, efficient protocols for succinct histograms,"* STOC 2015.

[6] Hegazy, Leluc, Li and Dieuleveut, *"Compression with exact error distribution for federated learning,"* AISTATS 2024.

[7] Shahmiri, Ling and Li, *"Communication-efficient Laplace mechanism for differential privacy via random quantization,"* ICASSP 2024.

[8] Li and El Gamal, *"Strong Functional Representation Lemma and Applications to Coding Theorems,"* IEEE Trans. Inf. Theory, 2018.

[9] Andrés, Bordenabe, Chatzikokolakis and Palamidessi, *"Geo-indistinguishability: Differential privacy for location-based systems,"* CCS 2013.

[10] Chen, Song, Ozgur and Kairouz, *"Privacy Amplification via Compression: Achieving the Optimal Privacy-Accuracy-Communication Trade-off in Distributed Mean Estimation,"* NeurIPS 2023.

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