Lecture 7

9/30

Order Parameter (φ) — Ginzburg - Landau Theory

Find minimum in $G(T)$ in relation to φ

$G(\varphi) = G(-\varphi)$

Expand G in order parameter we only need $\boxed{\text{even powers.}}$

$$G = G_0 + \frac{1}{2}a(T-T_c)\varphi^2 + \frac{1}{4}b\varphi^4 + \frac{1}{6}c\varphi^6 + \frac{1}{8}d\varphi^8 + \dots$$

Higher order term φ^n, where $n>4$ doesn't matter.

a, b are T-independent

$$\frac{\partial G}{\partial \varphi} = a(T-T_c)\varphi + b\varphi^3 = 0$$

$\varphi = 0$ | $b\varphi^2 + a(T-T_c) = 0$

$$\varphi = \pm \left(\frac{-a(T-T_c)}{b} \right)^{1/2}$$

G- disorder max ($T > T_c$) — G - ordered minimum ($T < T_c$)

$\varphi \sim t^{1/2}$; $t = T_c - T$

$$G = G_0 + \frac{1}{2}a(T-T_c)\varphi^2 + \frac{1}{4}b\varphi^4 = G_0 + \Delta H - T\Delta S$$

$\Delta H = -\frac{1}{2}aT_c\varphi^2 + \frac{1}{4}b\varphi^4$

$-\cancel{T}\Delta S = \frac{1}{2}a\cancel{T}\varphi^2$

$\Delta S = -\frac{1}{2}a\varphi^2$

$$\Delta C = T\frac{\partial \Delta S}{\partial T} = \begin{cases} 0 & \text{for } T > T_c \\ \frac{a^2}{2b}T & T < T_c \end{cases}$$

$$G = G_0 + \frac{1}{2} a (T - T_0) \varphi^2 - \frac{1}{4} b \varphi^2 + \frac{1}{6} c \varphi^6$$

$$\frac{\partial G}{\partial \varphi} = a (T - T_0) \varphi - b \varphi^3 + c \varphi^5 = 0$$

$$\varphi = 0 \qquad \text{max}$$

$$\varphi^2 = \frac{b \pm \sqrt{b^2 - 4ac(T - T_0)}}{2c} \qquad (1)$$

$$G - G_0 = 0 = \frac{1}{2} a (T - T_0) \varphi^2 - \frac{1}{4} b \varphi^4 + \frac{1}{6} c \varphi^6 = 0 \qquad (2)$$

From (1) & (2) $\rightarrow (\Delta\varphi)^2 = \frac{3b}{4c}$

$$a(T_c - T_0) - b\varphi^2 + c\varphi^4 = 0$$

$$a(T_c - T_0) - b \frac{3b}{4c} + c \left(\frac{3b}{4c}\right)^2 = 0$$

$$T_c = T_0 + \frac{3b^2}{16ac}$$

$$\Delta H\big|_{T_c} = -\frac{1}{2} a T_0 \varphi^2 - \frac{1}{4} b \varphi^4 + \frac{1}{6} c \varphi^6$$

$$= -\frac{3abT_c}{8c}$$

$b > 0$ 2nd order

$b < 0$ 1st order

$b \sim 0$ tricritical point

Tricritical :

$$G - G_0 = \frac{1}{2} a (T - T_0) \varphi^2 + \frac{1}{6} c \varphi^6$$

$$\frac{\partial G}{\partial \varphi}\bigg|_{T_c} = 0 = a(T - T_0)\varphi^4 \quad c\varphi^5$$

$$\varphi^4 = -\frac{a(T - T_0)}{c}$$

Enthalpy for 2nd & 4th order

$$\Delta H = -\tfrac{1}{2} a T_c \varphi^2 + \tfrac{1}{4} b \varphi^4 \qquad \varphi^2 = \frac{a(T_c - T)}{b}$$

$$\Delta H = -\frac{1}{2} a T_c \left(\frac{a(T_c - T)}{b} \right) + \frac{1}{4} b \left(\frac{a(T_c - T)}{b} \right)^2$$

$$= -\frac{a^2}{2b} T_c (T_c - T) + \frac{a^2}{4b} (T_c - T)^2$$

$$= -\frac{a^2}{2b} T_c^2 + \cancel{\frac{a^2}{2b} T_c T} + \frac{a^2}{4b} T_c^2 - \cancel{\frac{a^2}{2b} T_c T} + \frac{a^2}{4b} T^2$$

$$= \frac{a^2}{b} \left(\frac{T^2}{4} - \frac{T_c^2}{4} \right) = \frac{a^2}{4b} (T^2 - T_c^2)$$

$T = T_c \Rightarrow \Delta H = 0$

$$\Delta H = -\frac{1}{2} a T_0 \varphi^2 - \frac{1}{4} b \varphi^4 + \frac{1}{6} c \varphi^6$$

$$= -\frac{1}{2} a T_0 \left(\frac{b \pm \sqrt{b^2 - 4ac(T - T_0)}}{2c} \right) - \frac{1}{4} b \left(\frac{b \pm \sqrt{b^2 - 4ac(T - T_0)}}{2c} \right)^2$$
$$+ \frac{c}{6} \left(\frac{b \pm \sqrt{b^2 - 4ac(T - T_0)}}{2c} \right)^3$$

Let $p = \sqrt{b^2 - 4ac(T - T_0)} \Rightarrow \sqrt{b^2 - \cancel{4ac}\left(\cancel{T_0} + \frac{3b^2}{4\cancel{16ac}} - \cancel{T_0}\right)} = \frac{b}{2}$

$$\Delta H \Big|_{T = T_c} = -\frac{a T_0}{2} \left(\frac{b \pm b/2}{2c} \right) - \frac{b}{4} \left(\frac{b \pm b/2}{2c} \right)^2 + \frac{c}{6} \left(\frac{b \pm b/2}{2c} \right)^3$$

$$= -\frac{a T_0}{2} \left(\frac{b \pm b/2}{2c} \right) - \frac{b}{4} \left[\frac{b^2}{4c^2} \pm \frac{b^2}{4c^2} + \frac{b^2}{16c^2} \right] +$$

$$\frac{c}{6(2c)^3} \left[b^3 \pm 3b^2 \left(\frac{b}{2} \right) + 3b \left(\frac{b}{2} \right)^2 \pm \left(\frac{b}{2} \right)^3 \right]$$

$$= -\frac{a T_0}{2} \left(\frac{b}{2c} \pm \frac{b}{4c} \right) - \cancel{\frac{b^3}{16c^2}} \mp \cancel{\frac{b^3}{16c^2}} + \cancel{\frac{b^3}{64c^2}} \pm \cancel{\frac{b^3}{16c^2}} \pm \cancel{\frac{b^3}{32c^2}} \pm \frac{b^3}{\cancel{12}8c^2}$$

$$\varphi^2\Big|_{T=T_c} = \frac{b \pm \sqrt{b^2 - 4ac(T_c - T_0)}}{2c}$$

$$= \frac{b}{2c} \pm \frac{\sqrt{b^2 - \cancel{4ac}(\cancel{T_0} + \frac{3b^2}{4\cancel{16}ac} - \cancel{T_0})}}{2c} = \frac{b}{2c} \pm \frac{\sqrt{b^2/4}}{2c} = \frac{b \pm b/2}{2c}$$

$$\Delta H\Big|_{T=T_c} = -\frac{1}{2} a T_0 \varphi^2 - \frac{1}{4} b \varphi^4 + \frac{1}{6} c \varphi^6$$

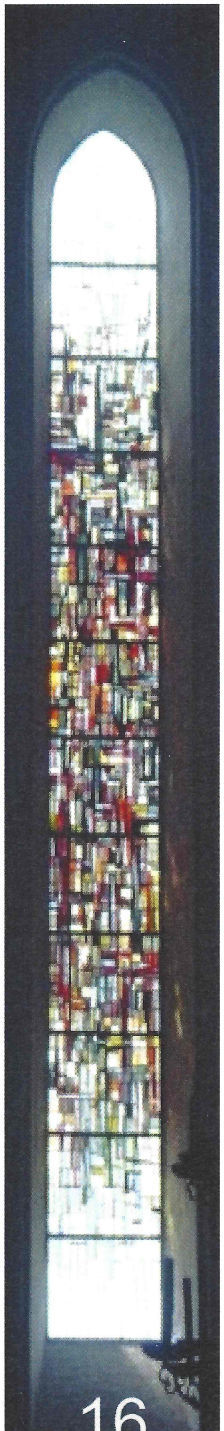
$$= -\frac{aT_0 b}{4c}\left(1 \pm \frac{1}{2}\right) - \frac{b^3}{16c^2}\left(1 \pm \frac{1}{2}\right)^2 + \frac{c}{6}\left(\frac{b \pm b/2}{2c}\right)^3$$

$$= -\frac{aT_0 b}{4c}\left(1 \pm \frac{1}{2}\right) - \frac{b^3}{16c^2}\left(1 \pm \frac{1}{2}\right)^2 + \frac{b^3}{48c^2}\left(1 \pm \frac{1}{2}\right)^3$$

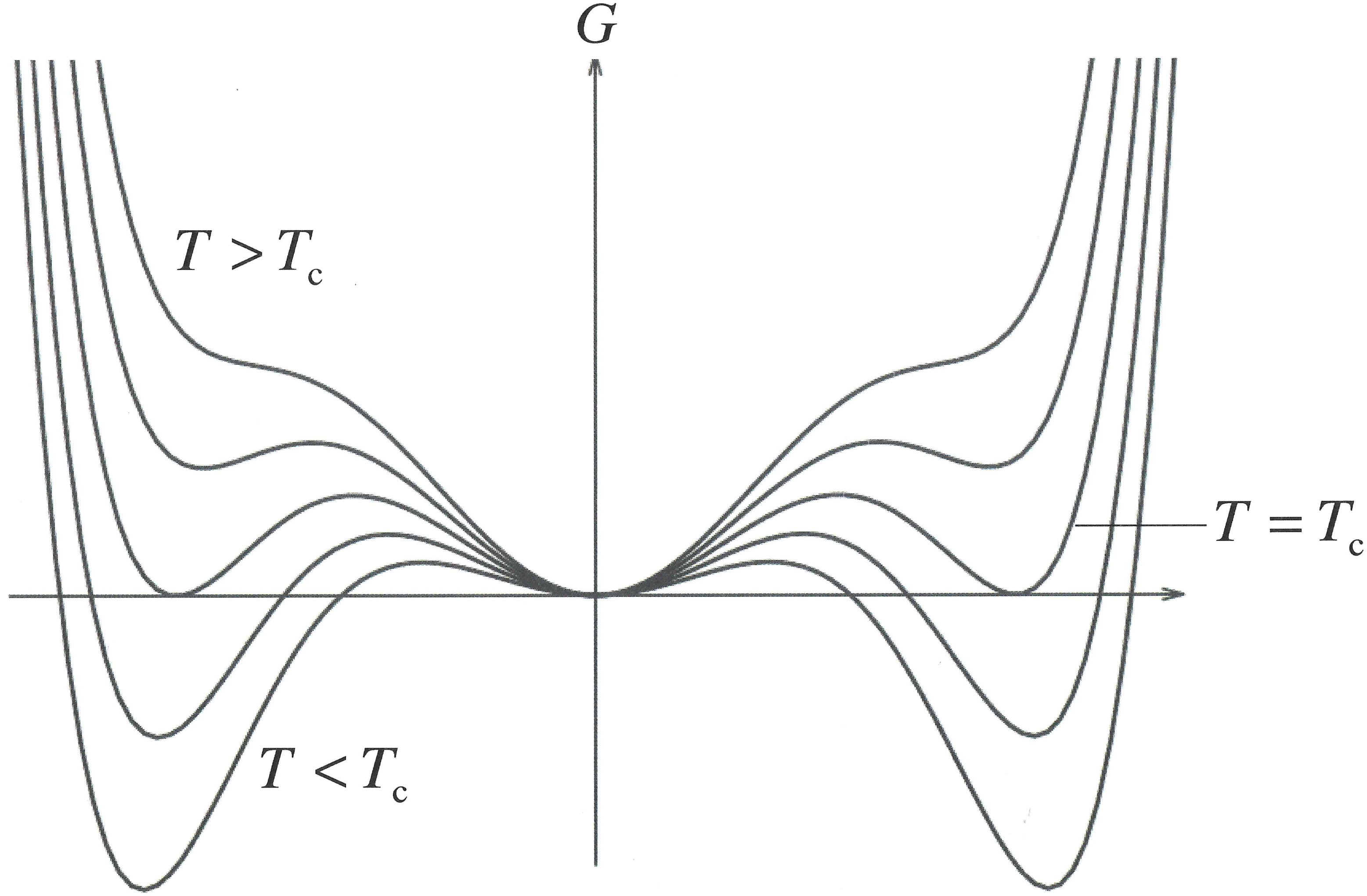
$$= -\frac{aT_0 b}{4c}\left(1 \pm \frac{1}{2}\right) - \frac{b^3}{16c^2}\left(1 \pm 1 + \frac{1}{4}\right) + \frac{b^3}{48c^2}\left(1 \pm \frac{3}{2} + \frac{3}{4} \pm \frac{1}{8}\right)$$

$$= -\frac{ab}{4c}\left(T_c - \frac{3b^2}{16ac}\right)\left(1 \pm \frac{1}{2}\right)$$

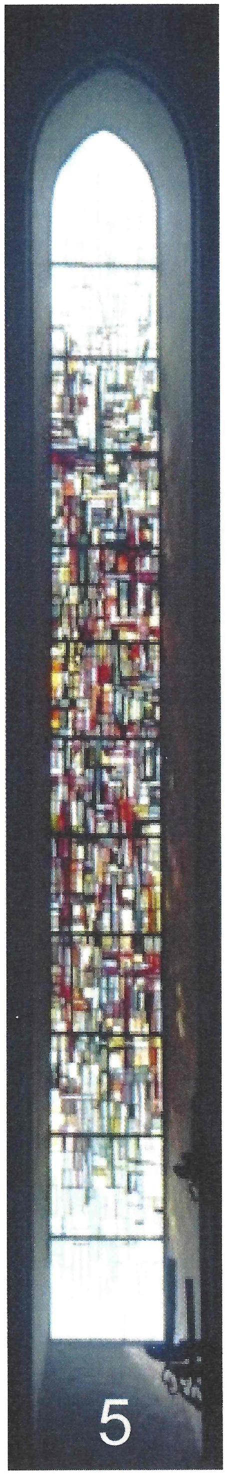
$$= -\frac{abT_c}{4c} + \left(\frac{-\cancel{a}b}{4c}\right)\left(\frac{-3b^2}{16\cancel{a}c}\right)\left(1 \pm \frac{1}{2}\right)$$

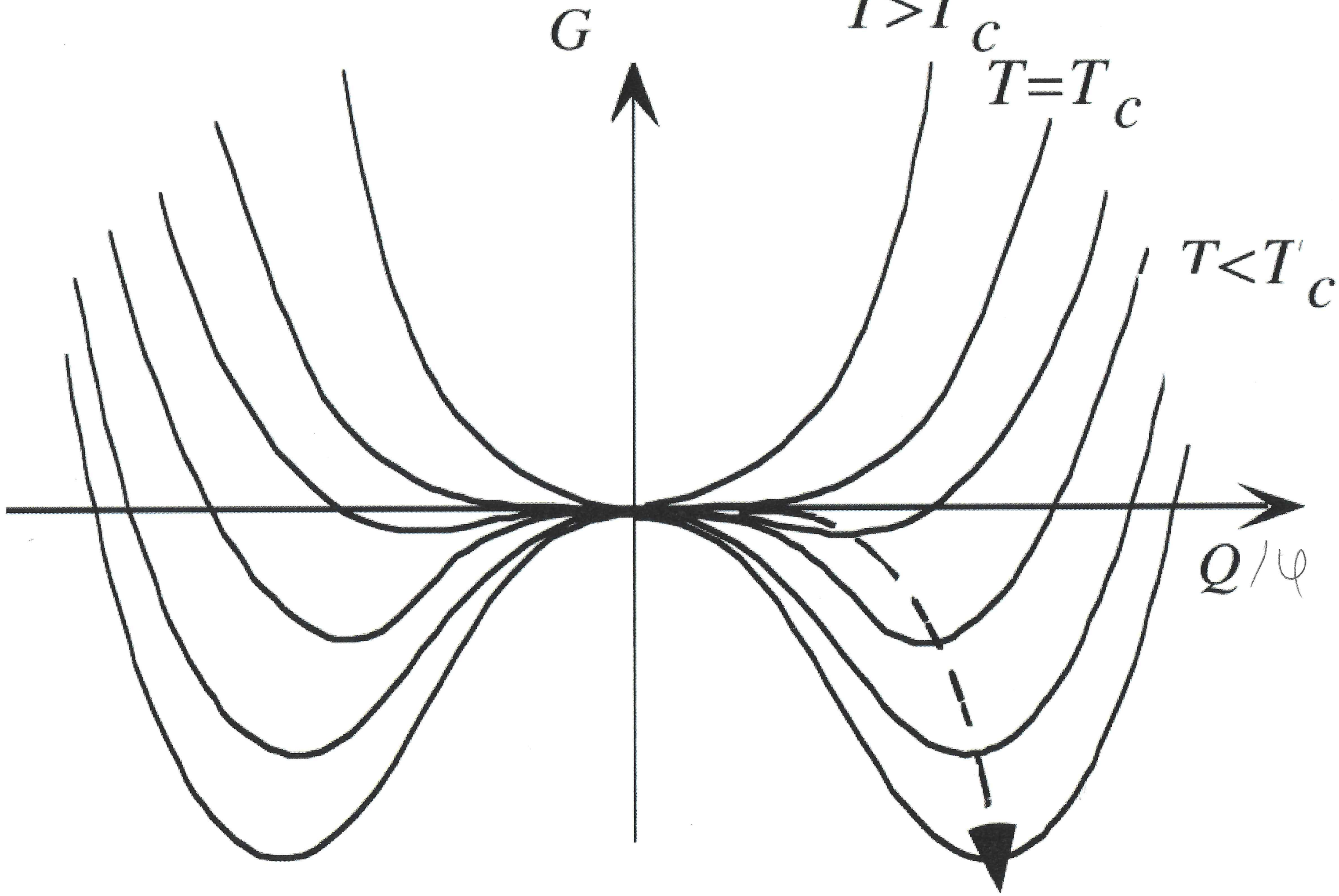


Free energy for a first-order phase transition (-b)



Free energy for a second-order phase transition (+b) no Latent Heat





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