Lecture 12

11/3

Drude Model

$$\cancel{\frac{d\vec{p}}{dt}}^{\,0} = -\frac{\vec{p}}{\tau} + \vec{F}$$

$$0 = -\frac{m\vec{v}}{\tau} + (-e\vec{E})$$

$$\vec{v}_{avg} = -\frac{e\tau}{m}\vec{E}$$

$$\vec{J} = ne\vec{v}_{avg} = \frac{ne^2\tau}{m}\vec{E} = \sigma\vec{E} = \frac{\vec{E}}{\rho}$$

$$\tau = \left(\frac{0.22}{\rho_\mu}\right)\left(\frac{r_s}{a_0}\right)^3 \times 10^{-14}\ s$$

ρ_μ is in $\mu\Omega$-cm

For $\tau \to \infty$:

$$\frac{d\vec{p}}{dt} = \vec{F} - \cancel{\frac{\vec{p}(t)}{\tau}}^{\,0}$$

$$m\frac{d\vec{v}_s}{dt} = -e\vec{E} \quad ; \quad \vec{J} = -en_s\vec{v}_s$$

$$m\frac{d}{dt}\left(\frac{+\vec{J}}{en_s}\right) = +e\vec{E}$$

$$\boxed{\vec{E} = \frac{m}{n_s e^2}\frac{d\vec{J}}{dt}}$$

Faraday's law :

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\frac{m}{n_s e^2}\left(\nabla \times \frac{\partial \vec{J}}{\partial t}\right) + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$$

$$\frac{\partial}{\partial t}\left((\nabla \times \vec{J}) + \frac{n_s e^2}{mc}\vec{B}\right) = 0$$

Maxwell Eq (Footnote 30 pg. 738)

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J}$$

$$\boxed{\text{London : } \nabla \times \vec{J} = -\frac{n_s e^2}{mc}\vec{B}}$$

$$\nabla \times \left[\nabla \times \frac{c}{4\pi}\vec{B}\right] = -\frac{n_s e^2}{mc}\vec{B}$$

$$\cancelto{0}{\nabla\left(\nabla \cdot \frac{c}{4\pi}\vec{B}\right)} + \nabla^2\vec{B} = +\frac{n_s e^2}{mc}\vec{B}$$

$$\nabla^2 B - \frac{4\pi n_s e^2}{mc^2}\vec{B} = 0$$

$$\nabla^2 J = \frac{4\pi n_s e^2}{mc^2}\vec{B}^2$$

For 1D $\rightarrow B = B_0 e^{-x/\lambda} \rightarrow \lambda = \left(\frac{4\pi n_s e^2}{mc^2}\right)^{-1/2}$

$$\lambda = 41.9\left(\frac{r_s}{a_0}\right)^{3/2}\left(\frac{n}{n_s}\right)^{1/2} \text{Å}$$

G-L formalism $\quad \Psi(\vec{r})$ is order parameter

if not SC $\Psi = 0$

$$f_s(T) - f_n(T) = a(T)\,|\Psi|^2 + \frac{b(T)}{2}|\Psi|^4 + \dots$$

if mag. field $\neq 0$ add $\frac{\hbar^2}{2m}\left|\frac{1}{i}\nabla^2 + 2eA\Psi\right|^2$

$$f_s(T) - f_n(T) = a_0(T-T_c)\,|\Psi|^2 + \frac{b}{2}|\Psi|^4$$

if $T < T_c$ min at $|\Psi|^2 = \frac{-a_0(T-T_c)}{b}$

$$f_s(T) - f_n(T) = -\frac{H_c^2}{8\pi} = -\frac{a_0^2}{2b}(T-T_c)^2$$

e^- not homogeneous : $f_s(T) - f_n(T) = a_0(T)\,|\Psi|^2 + \frac{b}{2}|\Psi|^4 + \frac{\hbar^2}{2m}|\nabla\Psi|^2$

$$F_s(T) - F_n(T) = \int_V f_s(T) - f_n(T)\, d^3r$$

minimize of F_s w.r.t $\Psi * \Psi^*$:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + a\Psi + b\Psi|\Psi|^2 = 0$$

$$\Psi(x) = \Psi_0 \tanh\left[\frac{x}{\sqrt{2}\,\xi(T)}\right]$$

$$\xi(T) = \left[\frac{\hbar^2}{2ma(T)}\right]^{1/2} \qquad a(T) = a_0(T-T_c)$$

$$\xi(T) = \xi(0)\,t^{-1/2} \quad \text{where} \quad t = \frac{T-T_c}{T_c}$$

$$\xi_0 = \frac{\hbar v_F}{k T_c}$$

near $T_c \rightarrow \xi \propto (1 - T/T_c)^{-1/2}$