

# Lecture 6

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$$N = 2 \left( \frac{L}{2\pi} \right)^3 \frac{4\pi}{3} K_F^3$$

# of modes per unit  $k$ -space

$$n = \frac{N}{V} = \frac{K_F^3}{3\pi^2}$$

$$\begin{aligned} K_F &= (3\pi^2 n)^{1/3} \\ &= \left( 3\pi^2 \cdot \frac{3}{4\pi} \right)^{1/3} \frac{1}{r_s} \\ &= \left( \frac{9\pi}{4} \right)^{1/3} \frac{1}{r_s} \end{aligned}$$

(Review)

Go through: Chapter 3, lattice & Brillouin Zone, empty lattice, Bragg's reflection

$$g(\epsilon) d\epsilon = \frac{1}{V} \times \# \text{ of } 1e^- \text{ levels between } \epsilon \text{ \& } \epsilon + d\epsilon$$

$N(\epsilon)$  &  $D(\epsilon) \rightarrow$  Density of states:

$$g(\epsilon) d\epsilon = \frac{1}{V} D(\epsilon) d\epsilon$$

$$T=0 \rightarrow n = \int_{-\infty}^{\infty} g(\epsilon) d\epsilon \Rightarrow dn = g(\epsilon) d\epsilon$$

$g(\epsilon) = \frac{dn}{d\epsilon}$

$$\bar{x} = \int_{-\infty}^{\infty} g(\epsilon) x d\epsilon$$

$$U = \int_{-\infty}^{\infty} g(\epsilon) \epsilon d\epsilon$$

$$T \neq 0$$

$$n = \int_{-\infty}^{\infty} g(\epsilon) f_{FD}(\epsilon) d\epsilon$$

$$U = \int_{-\infty}^{\infty} g(\epsilon) \epsilon f_{FD}(\epsilon) d\epsilon$$

$$\begin{aligned} g(\epsilon) &= \frac{dn}{d\epsilon} = \frac{dn}{dk} \cdot \frac{dk}{d\epsilon} \\ &= \frac{k^2}{2\pi^2} \left( \frac{2m}{\hbar^2} \right)^{1/2} \epsilon^{-1/2} \\ &= \frac{3}{2} \frac{n}{\epsilon_F} \left( \frac{\epsilon}{\epsilon_F} \right)^{1/2} \end{aligned}$$

At low T:

$$U = U_0 + \frac{\pi}{6} (k_B T)^2 g(\epsilon_F)$$

$$C_V = \frac{\partial U}{\partial T} = \frac{\pi k_B^2}{3} T g(\epsilon_F) = \gamma T$$

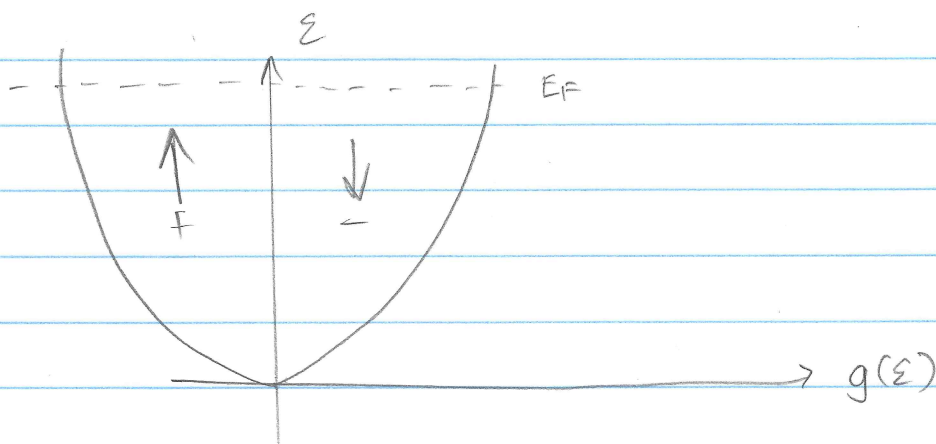
$$\gamma = \frac{\pi k_B^2}{3} g(\epsilon_F) \rightarrow \text{Sommerfeld Coefficient}$$

$$C_V = \gamma T + C_{\text{mag}} + C_{\text{lattice}} + \dots$$

$$C_{\text{latt}} = \beta T^3 \rightarrow \text{Debye model at low } T \quad T \ll \frac{\Theta_D}{50}$$

$$\text{For most material: } C_V = \gamma T + \beta T^3 \Rightarrow \Delta S = C \ln(T_F - T_i)$$

Pauli Paramagnetism → Comparing temperature to the Fermi temperature



$$g_{\pm}(\epsilon) = \frac{g(\epsilon)}{2}$$

$$U_+ - U_- = 2\mu_B H$$

$$\chi_{CW} = \frac{1}{3} \frac{n \mu_B^2}{k_B T}$$

$$\chi \sim \left( \frac{n T}{T_F} \right) \cdot \mu_B^2 \sim \frac{n \mu_B^2}{k_B T_F}$$

$$g_{\pm}(\epsilon) = \frac{1}{2} g(\epsilon \mp \mu_B H)$$

$$\chi = \mu_B^2 g(\epsilon_F) = \mu_B^2 \frac{3}{2} \frac{n}{k_B T_F}$$