Lecture 3

Assuming for $\vec{M}$ & $\vec{H}$ are parallel & free - Heicenberg spins

Ising spins - only 1 fixed direction (up/down)

(31.1) $$M(H) = -\frac{1}{V}\frac{\partial E_0(H)}{\partial H}$$

(31.2) $$M(H,T) = \frac{\sum_n M_n(H)\, e^{-E_n/k_BT}}{\sum_n e^{-E_n/k_BT}}$$, for finite temperature

(31.4) $$M = -\frac{1}{V}\frac{\partial F}{\partial H}$$, F is magnetic Helmholtz Free energy.

(31.6) $$\chi = \frac{\partial M}{\partial H} = -\frac{1}{V}\frac{\partial^2 F}{\partial H^2}$$

Interaction energy with each electron spin $s^i = \frac{1}{2}\sigma_i$:

$$\Delta\mathcal{H} = g_0\,\mu_B\, H S_z \qquad \left(S_z = \sum_i s_z^i \right)$$

$$g_0 = 2\left[1 + \frac{\alpha}{2\pi} + O(\alpha^2) + \dots \right], \quad \alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

$$= 2.0023 \approx 2$$

(31.15)

$$T = \frac{1}{2m}\sum_i \left[\vec{p}_i + \frac{e}{c}\vec{A}(\vec{r}_i) \right]^2 = \frac{1}{2m}\sum_i \left(p_i - \frac{e}{2c}\vec{r}_i \times \vec{H} \right)^2$$

$$\vec{r}_i \times \vec{H} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & 0 \\ 0 & 0 & H \end{vmatrix}$$

$$= \frac{1}{2m}\sum_i \left(\vec{p}_i^{\,2} - \frac{e}{c}\vec{p}_i\cdot(\vec{r}_i\times\vec{H}) + \frac{e^2}{4c^2}(\vec{r}_i\times\vec{H})^2 \right)$$

$$= \frac{1}{2m}\sum_i p_i^2 - \sum_i \frac{e}{2mc}\vec{p}_i\cdot(\vec{r}_i\times\vec{H}) + \sum_i \frac{e^2}{8mc^2}(\vec{r}_i\times\vec{H})^2$$

$$\hbar\vec{L} = \sum_i \vec{r}_i \times \vec{p}_i$$

$$= T_0 + \sum_i \frac{e}{2mc}(\vec{r}_i\times\vec{p}_i)\cdot\vec{H} + \sum_i \frac{e^2}{8mc^2} H^2 (x_i^2 + y_i^2)$$

(31.16) $$T = T_0 + \mu_B \vec{L}\cdot\vec{H} + \frac{e^2}{8mc^2} H^2 \sum_i (x_i^2 + y_i^2)$$

Second-order perturbation theory :

(31.19) $E_n \rightarrow E_n + \Delta E_n \,;\; \Delta E_n = \langle n|\Delta\mathcal{H}|n\rangle + \sum_{n'\neq n} \frac{|\langle n|\Delta\mathcal{H}|n'\rangle|^2}{E_n - E_{n'}}$

(31.20) $\Delta E_n = \mu_B \vec{H}\cdot\langle n|\vec{L}+g_0\vec{S}|n\rangle + \sum_{n'\neq n} \frac{\langle n|\mu_B\vec{H}\cdot(\vec{L}+g_0\vec{S})|n'\rangle|^2}{|E_n - E_n'|}$

$+ \frac{e^2}{8mc^2} H^2 \langle n|\sum_i (x_i^2+y_i^2)|n\rangle$

$\Delta E_n^{(1)} \rightarrow$ local moment / curie term / Langevin
$\Delta E_n^{(2)} \rightarrow$ Van Vleck mixing of different E states
$\Delta E_n^{(3)} \rightarrow$ Lenz's law / Larmor / Langevin diamagnetism

$\rightarrow 1$ (in the order of unity)

$\Delta E_n^{(1)} = \mu_B\vec{H}\cdot\langle n|\cancel{\vec{L}+g_0S}|n\rangle = O(\mu_B H) \sim \frac{\hbar e H}{mc} \sim \hbar\omega_c$

$\Delta E_n^{(3)} \approx \frac{e^2}{mc^2} H^2 a_0^2 = \left(\frac{eH}{mc}\right)^2 \frac{\hbar^2}{\hbar^2} m a_0^2 \quad \left\{ \frac{a_0}{\hbar^2/m a_0} = \frac{a_0}{e^2} \right.$

$= (\hbar\omega_c)\left(\frac{(\hbar\omega_c)}{e^2/a_0}\right) \ll k_B T$

e^2/a_0 is in the order of 27 eV (13.6 eV × 2)

$\Delta E_n^{(2)} \sim \frac{\hbar\omega_c}{\Delta}$ $\quad \Delta = \min|E_n - E_n'|$ is typically about 1eV,
Δ will be large enough to make $\Delta E_n^{(2)}$ small.

CePdSb $\quad$ HW : From the data , 1. predict $\chi = \chi_0 + \frac{C}{T-\theta}$, where C is Curie constant, θ is curie-weiss factor
2. get the p_{eff} $\quad$ 3. χ_{mol}^{Larmor}