



Forward pass:

$$z_1(x_1, x_2) = w_1 x_1 + w_2 x_2 + b_1$$

$$h_1(z_1) = \sigma(z_1) = \sigma(w_1 x_1 + w_2 x_2 + b_1)$$

$$z_2 = \dots$$

$$h_2 = \dots$$

$$o_1(h_1, h_2) = w_5 \cdot h_1 + w_6 \cdot h_2 + b_3 \rightarrow E_1 = \frac{1}{2} (t_1 - o_1)^2$$

$$o_2(h_1, h_2) = w_7 \cdot h_1 + w_8 \cdot h_2 + b_4 \rightarrow E_2 = \frac{1}{2} (t_2 - o_2)^2$$

$$E_{\text{total}} = E_1 + E_2$$

Backward pass:

$$\frac{\partial E_{\text{total}}}{\partial w_1} = \frac{\partial E_{\text{total}}}{\partial h_1} \cdot \frac{\partial h_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_1} = \left( -(t_1 - o_1) \cdot w_5 - (t_2 - o_2) \cdot w_7 \right) \cdot \sigma(z_1) \cdot (1 - \sigma(z_1)) \cdot x_1$$

$$1. \frac{\partial E_1}{\partial h_1} = \frac{\partial E_1}{\partial o_1} \cdot \frac{\partial o_1}{\partial h_1} = \frac{1}{2} \cdot 2 (t_1 - o_1) \cdot (-1) \cdot w_5$$

$$2. \frac{\partial E_2}{\partial h_1} = \frac{\partial E_2}{\partial o_2} \cdot \frac{\partial o_2}{\partial h_1} = \frac{1}{2} \cdot 2 (t_2 - o_2) \cdot (-1) \cdot w_7$$

$$3. \frac{\partial h_1}{\partial z_1} = \sigma(z_1) (1 - \sigma(z_1))$$

$$4. \frac{\partial z_1}{\partial w_1} = x_1$$

$$E_{\text{total}} = \frac{1}{2} (t_1 - o_1)^2 + \frac{1}{2} (t_2 - o_2)^2$$

Output

Target

$$o = \begin{bmatrix} o_1 \\ o_2 \end{bmatrix}$$

$$t = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

Error function:

$$E(o, t) = \frac{1}{2} \sum_{i=1}^2 (t_i - o_i)^2 =$$

$$= \frac{1}{2} \underbrace{(t_1 - o_1)^2}_{E_1} + \frac{1}{2} \underbrace{(t_2 - o_2)^2}_{E_2}$$