AERO 4630 - Aerospace Structural Dynamics

Project 5

Assigned: Saturday, April 4 2020
Due: Wednesday April 22 2020 at 17:00, uploaded on Canvas

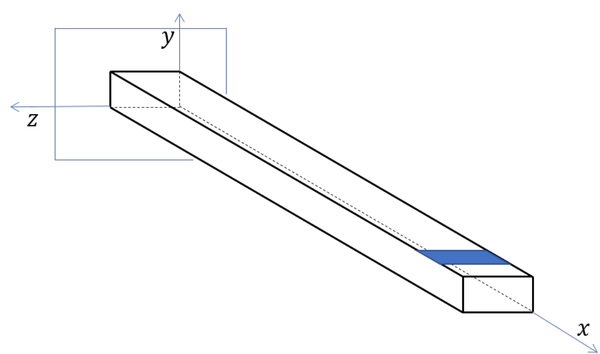
Office Hours: Davis 335, Wednesdays 1300-1400 hrs

Instructions

1. You are to submit a single **.zip** file containing a) your code as .py file, and b) a PDF containing your solution with appropriate plots.
2. Practice good coding guidelines with appropriate comments and meaningful variable names. Good coding practice will be rewarded with bonus points.
3. Plagiarism of any sort will not be tolerated. You code might be checked against plagiarism software (like MOSS). Any instance of plagiarism will be dealt in accordance to AU policy.

Problem 1: Newmark's beta method

Let's compare the Newmark β method with our previous approach using the familiar steel beam vibration example in Project 2. Consider a steel ($E = 200GPa, \nu = 0.3$) rod of rectangular cross section area. The dimensions are



length $L = 50mm$, $W = 5mm$ and $H = 5mm$. Just like last time, the face $x = 0$ is clamped, and a traction (force/area) is applied on a patch in the negative y direction. We have a constant traction of $T = 20kN/m^2$ applied uniformly on a patch located at $0.95L \leq x \leq L$, $y = W$ and $0 \leq z \leq H$.
As a consequence the beam will bend. We are interested in vibrations of the beam once the traction is removed.

(1a) First let's use the Newmark's β method in conjunction with generalized alpha method to solve this problem. Use the mesh $20 \times 6 \times 6$. Plot the vertical displacement of the point $(L, W/2, H/2)$ as a function of time. Run the simulation for at least 5 cycles and obtain the frequency of vibration. *Note: your displacement should not decay significantly. Choose your timestep wisely*

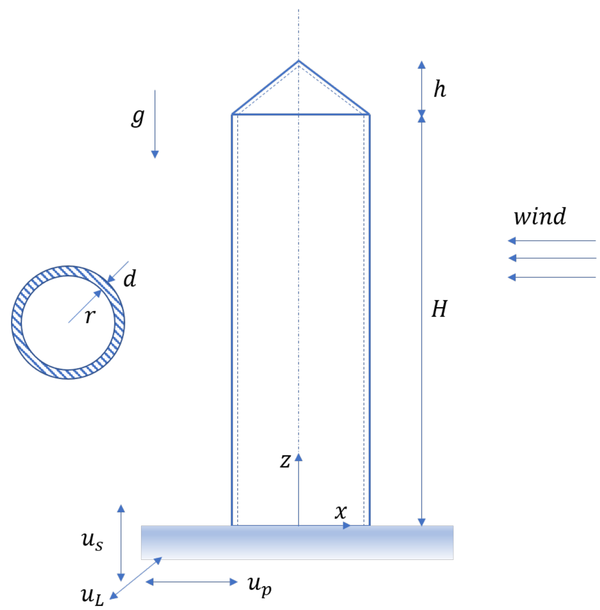
(1b) Now use the time step above and repeat your analysis using the 'naive' approximation we used in Project 2.

$$\ddot{u}(t+\Delta t) = \frac{u(t+\Delta t) - 2u(t) + u(t-\Delta t)}{(\Delta t)^2} \tag{1b.1}$$

Repeat the problem for the time step and mesh used in part (a). Plot the vertical displacement of the point $(L, W/2, H/2)$ as a function of time. Has your system damped?

Problem 2: Analysis of a rocket

Let's say you are tasked with understanding stresses on a rocket's body under wind and earthquake loading. You decide to a simple analysis as a starting point. Let's draw the schematic of the problem as below. The rocket is



approximated as a hollow cylinder (height $H = 100m$) with a hollow cone (height $h = 3m$) on top. The radius of the cylinder is $r = 10$ and the wall thickness is $d = 1$. The bottom end is attached to a platform which, unfortunately, is not vibration isolating. You have to account for the wind blowing from right to left, and gravity acting vertically. Rockets are usually very tall, so you have to account for the fact that wind speed is different with altitude. A rough approximation of wind speed v_{wind} at a height z from ground is given by

$$v_{wind}(z) = v_{ref} \frac{\ln(z/z_0)}{\ln(z_{ref}/z_0)} \tag{2.1}$$

where $v_{ref} = 5$ miles/hr, $z_{ref} = 10$ ft and the roughness length $z_0 = 0.0024$ m. As a consequence of the wind, a drag force will act on the rocket surface in the direction of the wind. You approximate the drag traction (force/area) t_d acting on the surface of the rocket as

$$t_d = \frac{1}{2} C_D \, \rho \, v_{wind}^2 \tag{2.2}$$

where $C_D = 0.5$ and density $\rho \approx 1.2$ kg/m^3 is approximated to be constant. Next, you have to account for earthquakes. Earthquakes primarily consist of P waves, S waves and surface (mainly Rayleigh and Love) waves. These are described with amplitudes, frequencies and speed of propagation. Here we ignore Rayleigh surface waves, and focus on Love waves. The motion of the platform due to P, S, and Love waves are in x, z and y directions. You choose to approximate the motion of the platform from these waves as

$$u_s = A_s \cos \omega_s t \tag{2.3}$$
$$u_p = A_p \cos \omega_p t \tag{2.4}$$
$$u_L = A_L \cos \omega_L t \tag{2.5}$$

where ω_p, ω_s and ω_L can be computed from measured frequencies of P, S and Love waves as $1Hz$, $1Hz$ and $0.1Hz$. Additionally the amplitude is measured as $A_p = 1in$, $A_s = 0.8in$ and $A_L = 4in$.

(2a) We are interested in obtaining the response of the rocket under these kinds of loading. First, let's turn off the earthquake loading and just apply the wind load. This is a static force (not varying with time). Let's look at the bottom surface of the rocket that is in contact with the platform. Plot the components (psuedocolor plot) of the stress normal and tangential to this surface.

(2b) Let's look at the cross section at height $z = H/2$. Once again, plot the components of stress normal and tangential to this cross section. Compute the net force acting in the x direction on this cross section. *Hint: look at weighted variable sum option in Query option of VisIT*.

(2c) Now let's turn on the platform displacement as a result of earthquake. The rocket will vibrate as a result. Plot the x, y and z displacement of the tip of the cone for at least **one cycle of Love wave loading**. Choose your timestep wisely.

Assume that the rocket shell is made of AL 7075 ($E = 72GPa$, $\nu = 0.33$, $\rho = 2810kg/m^3$).

