SPM
Bilbao 2018

Optimal Power Diagrams via Function Approximation

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Outline

- Background
- Optimal Power Diagrams
- Optimization Framework
- Results
- Conclusion

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- **Background**
- Optimal Power Diagrams
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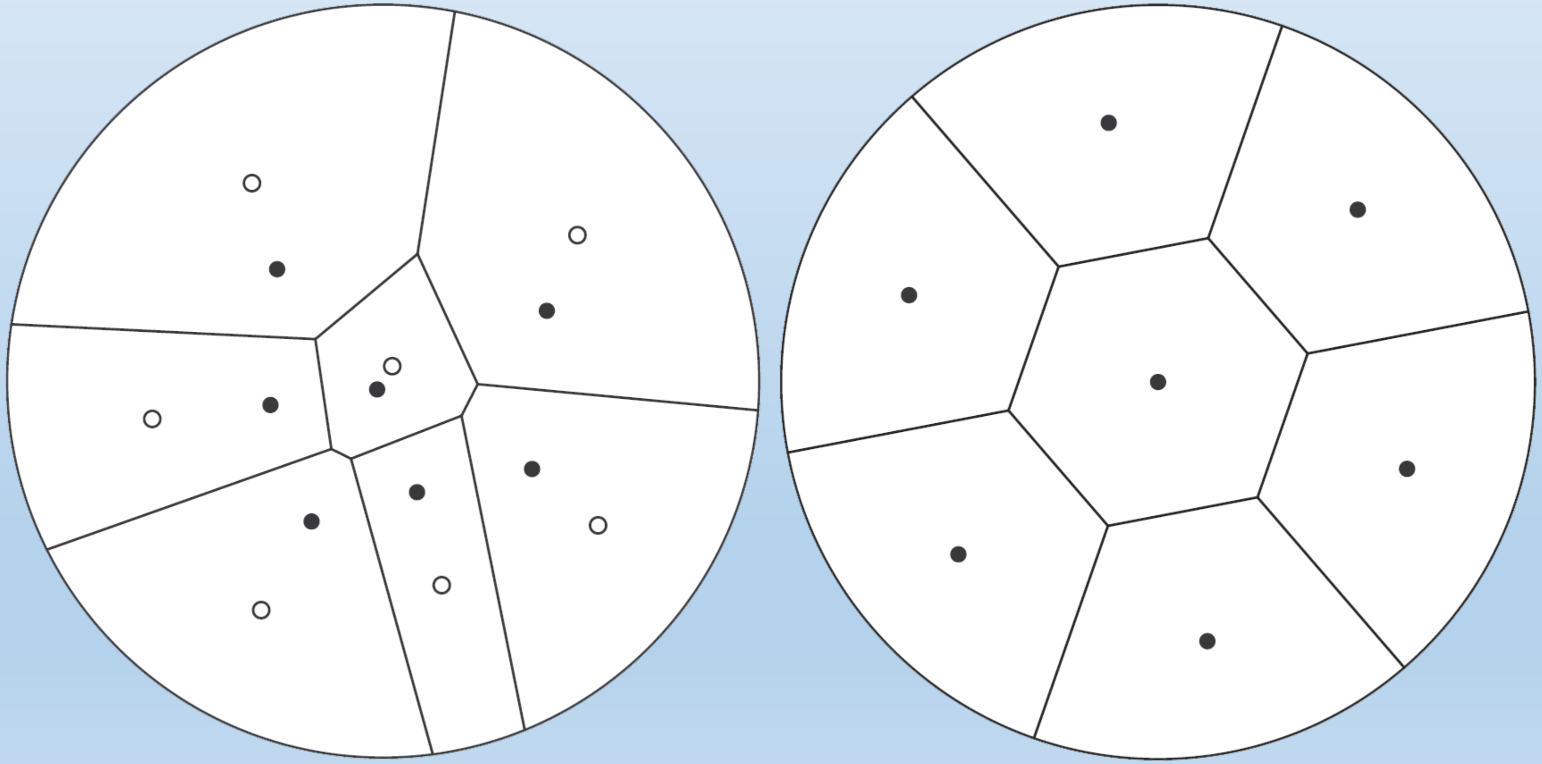
Background

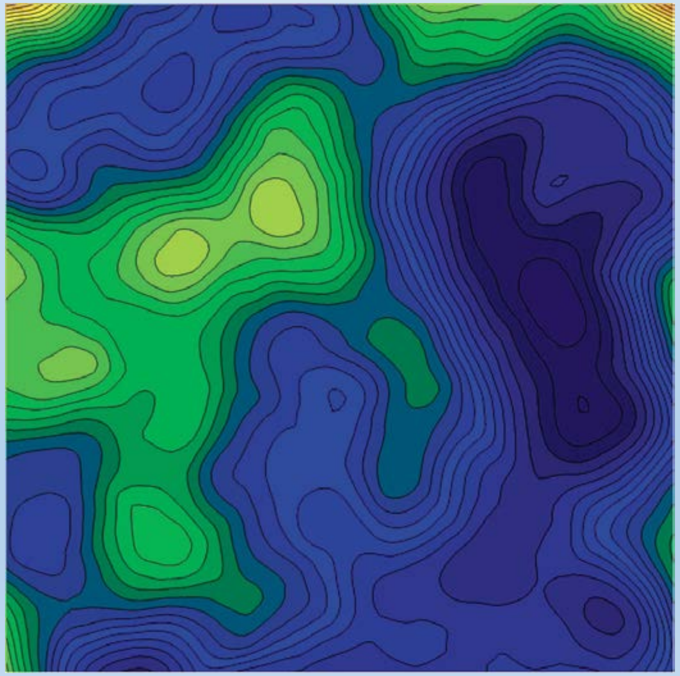
- 1. Centroidal Voronoi Tessellation (CVT) [Qiang Du et.al., SIAM Review 1999]



Background

- 1. Centroidal Voronoi Tessellation (CVT)

$$\mathcal{E}_{CVT}(\mathbf{X},\mathcal{V}) = \sum_{i=1}^{n}\int_{V_i}\left\|\mathbf{x}-\mathbf{x}_i\right\|^2 \mathrm{d}\mathbf{x} = \sum_{i=1}^{n}\int_{V_i}\left(\mathbf{x}^2-\left(2\mathbf{x}_i^T\mathbf{x}-\mathbf{x}_i^2\right)\right)\mathrm{d}\mathbf{x}$$



[Lin Lu et.al., TVCG 2012]

Background

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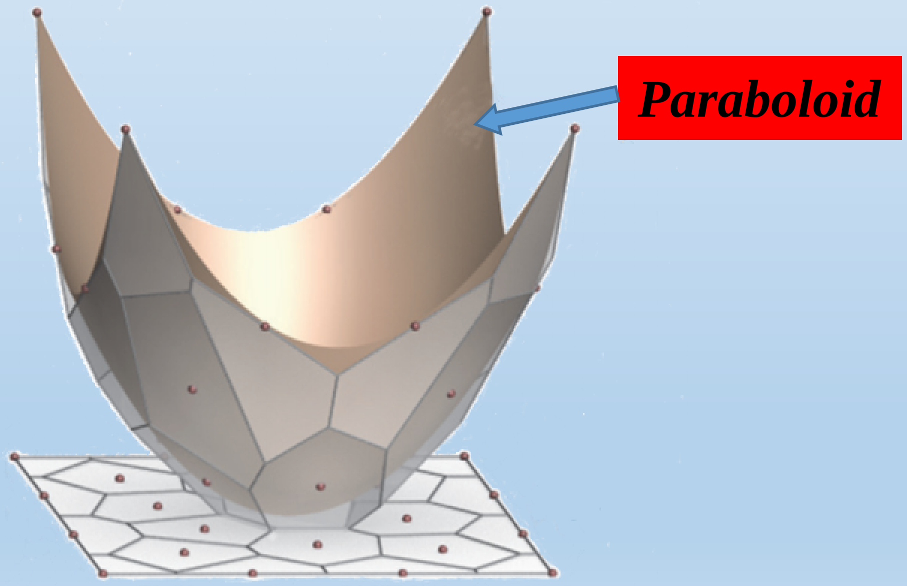

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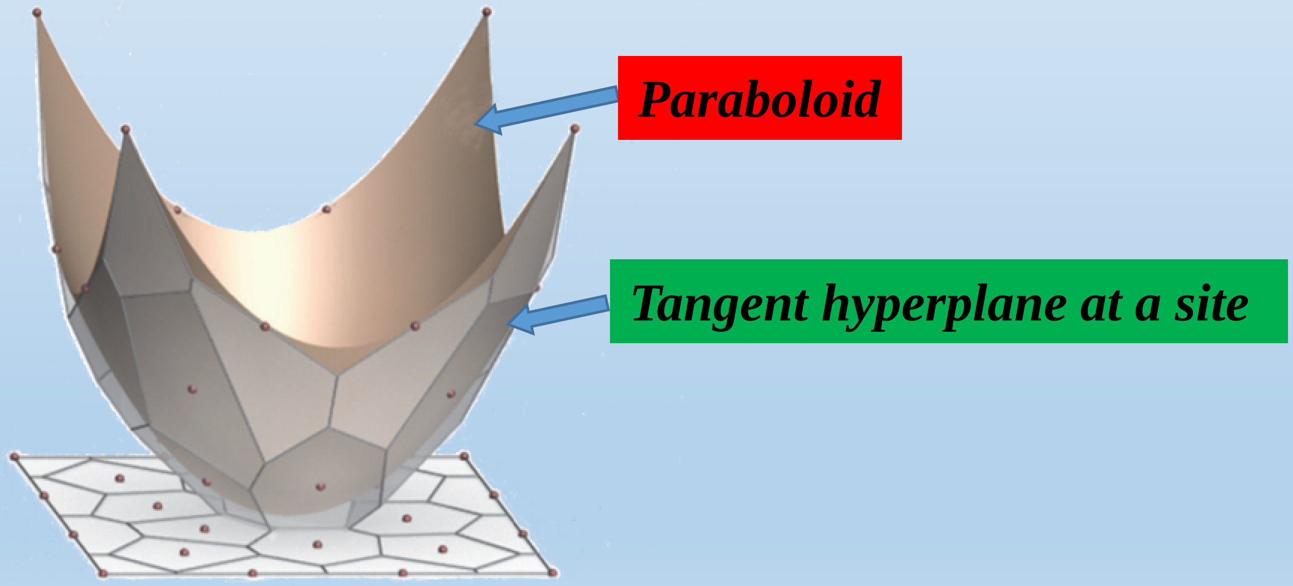
$$\mathcal{E}_{CVT}(\mathbf{X}, \mathcal{V}) = \sum_{i=1}^{n} \int_{V_i} \left\| \mathbf{x} - \mathbf{x}_i \right\|^2 \mathrm{d}\mathbf{x} = \sum_{i=1}^{n} \int_{V_i} \left(\boxed{\mathbf{x}^2} - \boxed{\left(2\mathbf{x}_i^T \mathbf{x} - \mathbf{x}_i^2 \right)} \right) \mathrm{d}\mathbf{x}$$



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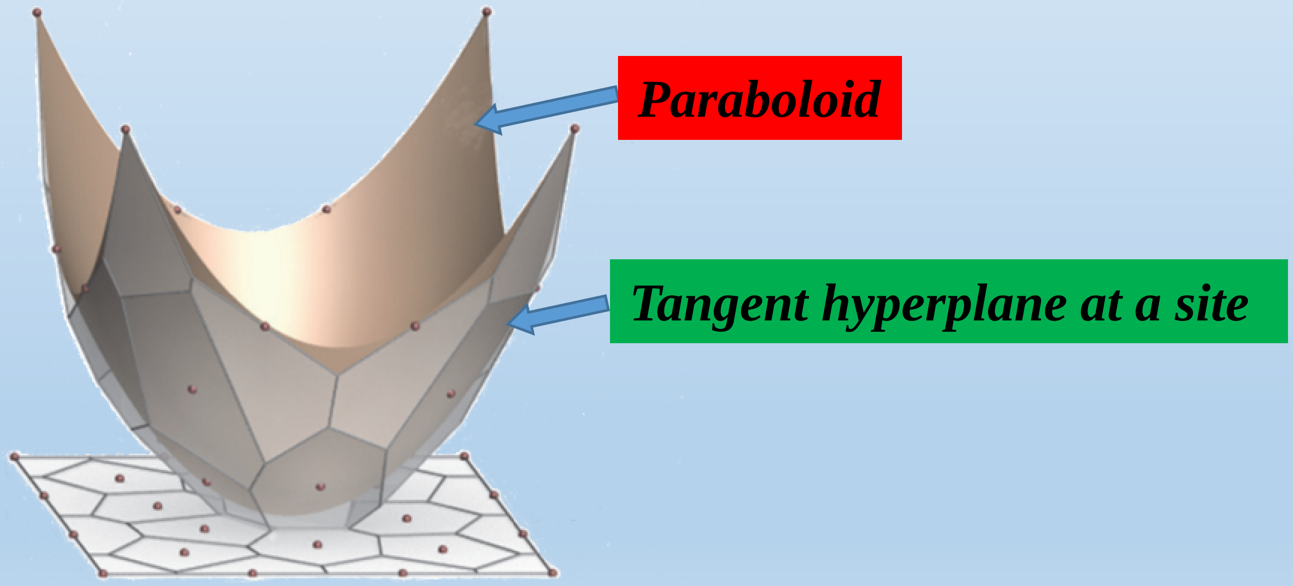

Minimizing $\mathcal{E}_{CVT}$

$\downarrow$

CVT

Background

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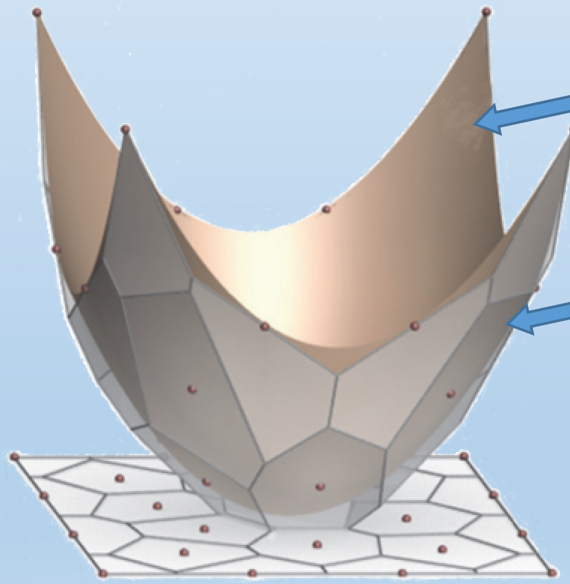


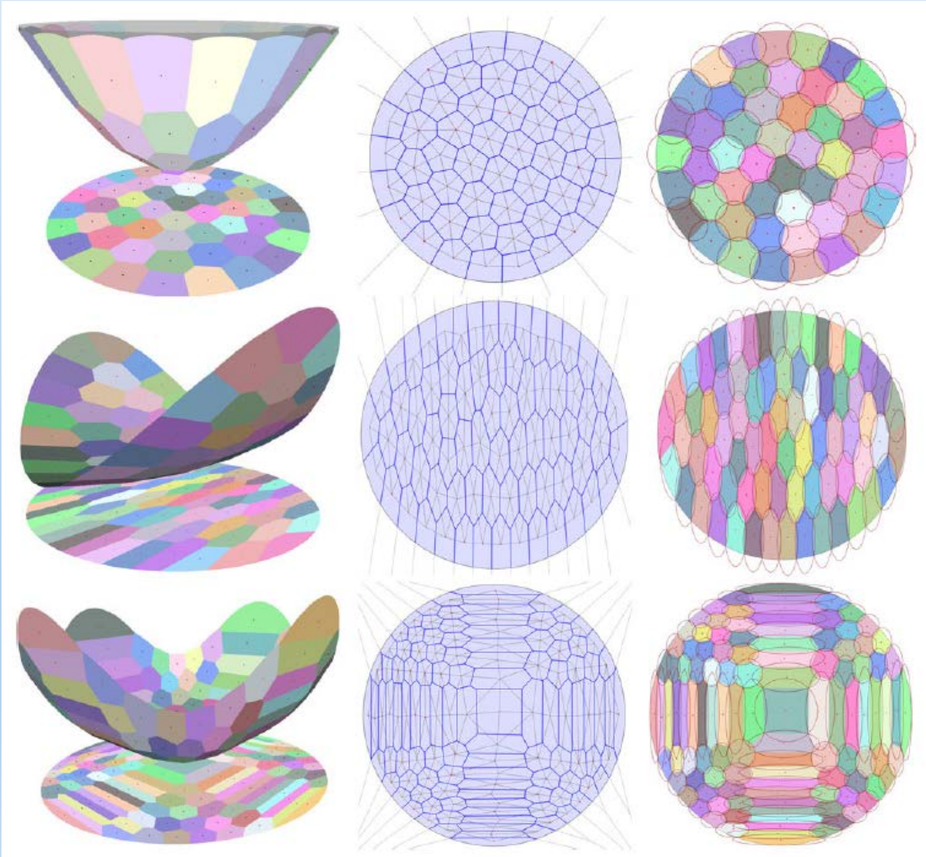
Minimizing $\mathcal{E}_{CVT}$

$\downarrow$ Lloyd & L-BFGS

CVT

Background

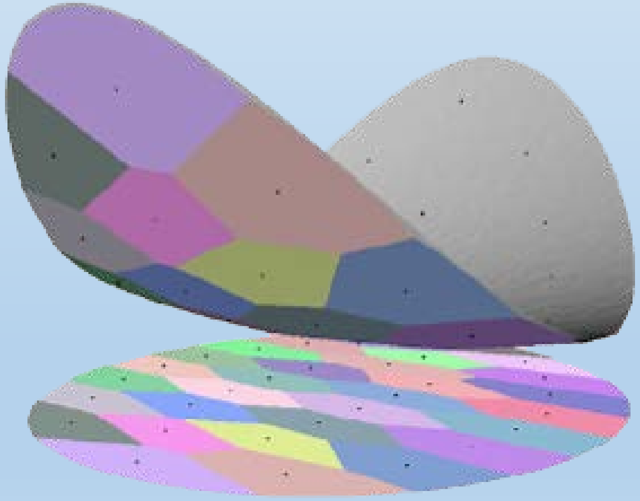
- 2. Optimal Voronoi Tessellation (OVT) [Max Budninskiy et.al., ACM TOG 2016]



Background

- 2. Optimal Voronoi Tessellation (OVT)

$$\mathcal{E}_{OVT}(\mathbf{X},\mathcal{V}) = \| f - f_T \|_{L^1} = \sum_{i=1}^{n} \int_{V_i} (f(\mathbf{x}) - T_i(\mathbf{x}))\mathrm{d}\mathbf{x},$$



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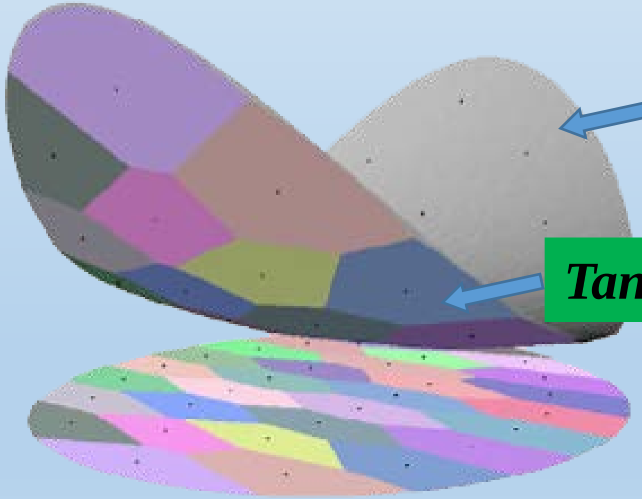

Convex function

Tangent hyperplane at a site

$$T_i(\mathbf{x}) = \nabla f(\mathbf{x}_i) \cdot (\mathbf{x} - \mathbf{x}_i) + f(\mathbf{x}_i)$$

Background

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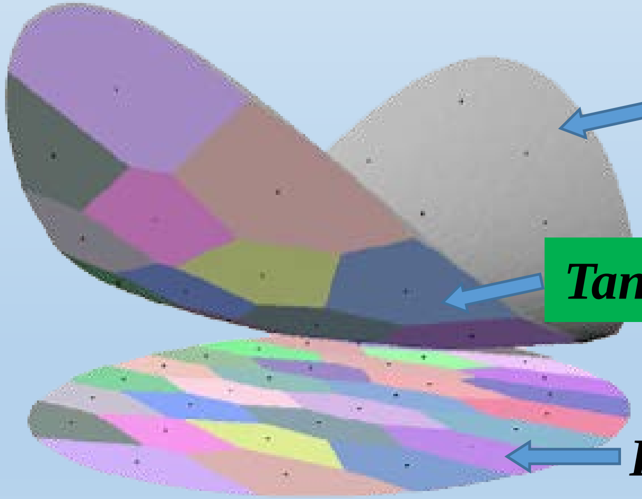

$$\left\{ \begin{aligned} \mathbf{p}_i &= \frac{1}{2} \nabla f(\mathbf{x}_i) \\ w_i &= \frac{1}{4} \left| \nabla f(\mathbf{x}_i) \right|^2 + f(\mathbf{x}_i) - \nabla f(\mathbf{x}_i) \cdot \mathbf{x}_i \end{aligned} \right.$$

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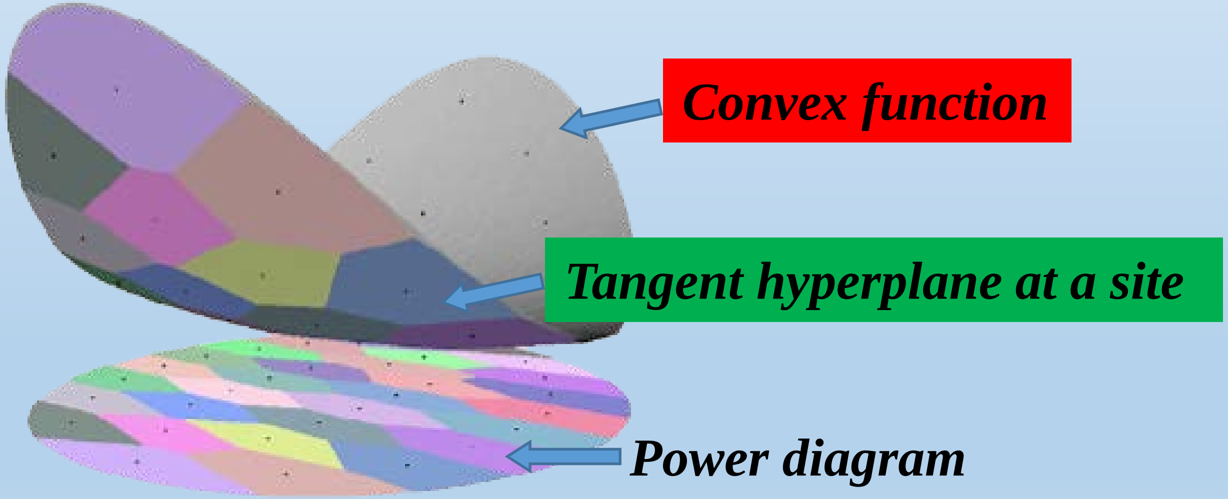

Minimizing $\mathcal{E}_{OVT}$

↓ Lloyd & L-BFGS

OVT

Background

- 2. Optimal Voronoi Tessellation (OVT)

$$f\left(\mathbf{x}\right) = \mathbf{x}^2, \mathcal{E}_{OVT}\left(\mathbf{X}, \mathcal{V}\right) \equiv \mathcal{E}_{CVT}\left(\mathbf{X}, \mathcal{V}\right)$$

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- Background
- **Optimal Power Diagrams**
- Optimization Framework
- Results
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Optimal Power Diagrams

- 1. Energy function formulation

Given $f\left(\mathbf{x}\right), \mathbf{x} \in \Omega$

and $\mathcal{V} = \left\{V_i, i = 1, \ldots, n \,|\, V_i \in \Omega; \forall i \neq j, V_i \bigcap V_j = \varnothing; \bigcup V_i = \Omega\right\}$

construct $P_i\left(\mathbf{x}\right) \approx f\left(\mathbf{x}\right), \mathbf{x} \in V_i, i = 1, \ldots, n$

Optimal Power Diagrams

- 1. Energy function formulation

$$\mathcal{E}_{OVT}(\mathbf{X}, \mathcal{V}) = \| f - f_T \|_{L^1} = \sum_{i=1}^{n} \int_{V_i} (f(\mathbf{x}) - \boxed{T_i(\mathbf{x})}) \mathrm{d}\mathbf{x},$$

Tangent hyperplane

Optimal Power Diagrams

- 1. Energy function formulation

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Tangent hyperplane

Ours $$\mathcal{E}(\mathcal{V}, \{P_i(\mathbf{x})\}_{i=1}^{n}) = \| f - f_P \|_{L^1} = \sum_{i=1}^{n} \int_{V_i} |f(\mathbf{x}) - P_i(\mathbf{x})| \mathrm{d}\mathbf{x},$$

Optimal Power Diagrams

- 1. Energy function formulation

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Best fitting hyperplane

Optimal Power Diagrams

- 1. Energy function formulation

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Optimal Power Diagrams

- 1. Energy function formulation

NP-hard

$$\mathcal{E}(\mathcal{V},\{P_i(\mathbf{x})\}_{i=1}^{n}) = \| f - f_P \|_{L^2} = \sum_{i=1}^{n} \int_{V_i} \left(f(\mathbf{x}) - P_i(\mathbf{x}) \right)^2 \mathrm{d}\mathbf{x},$$

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① Restrict $\mathcal{V}$ to power diagram, determined by $(\mathbf{X}, W) = \left(\{\mathbf{x}_i\}_{i=1}^{n}, \{w_i\}_{i=1}^{n} \right)$

Optimal Power Diagrams

- 1. Energy function formulation

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② When $\mathcal{V}$ is fixed, best-fit hyperplane on each cell is determined, denoted by $P_i^*\left(\mathbf{x}\right)$

Optimal Power Diagrams

- 1. Energy function formulation

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Optimal Power Diagrams

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$$\mathcal{E}_{OPD}(\mathbf{X}, W) = \| f - f_P \|_{L^2} = \sum_{i=1}^{n} \int_{V_i} \left(f(\mathbf{x}) - P_i^*(\mathbf{x}) \right)^2 \mathrm{d}\mathbf{x},$$

Optimal Power Diagrams

- 2. Derivatives

$$\mathcal{E}_{OPD}(\mathbf{X},W) = \| f - f_P \|_{L^2} = \sum_{i=1}^{n} \int_{V_i} \left(f(\mathbf{x}) - P_i^*(\mathbf{x}) \right)^2 \mathrm{d}\mathbf{x},$$

Optimal Power Diagrams

- 2. Derivatives

$$\mathcal{E}_{OPD}(\mathbf{X},W) = \| f - f_P \|_{L^2} = \sum_{i=1}^{n} \int_{V_i} \left(f(\mathbf{x}) - P_i^*(\mathbf{x}) \right)^2 \mathrm{d}\mathbf{x},$$

$$\frac{\partial \mathcal{E}_{OPD}(\mathbf{X},W)}{\partial \mathbf{x}_i} = \sum_{j \in J_i} \int_{V_{ij}} \left(\left| f(\mathbf{x}) - P_i^*(\mathbf{x}) \right|^2 - \left| f(\mathbf{x}) - P_j^*(\mathbf{x}) \right|^2 \right) \frac{\mathbf{x} - \mathbf{x}_i}{\left| \mathbf{x}_j - \mathbf{x}_i \right|} \mathrm{d}s$$

$$\frac{\partial \mathcal{E}_{OPD}(\mathbf{X},W)}{\partial w_i} = \sum_{j \in J_i} \int_{V_{ij}} \left(\left| f(\mathbf{x}) - P_i^*(\mathbf{x}) \right|^2 - \left| f(\mathbf{x}) - P_j^*(\mathbf{x}) \right|^2 \right) \frac{1}{2\left| \mathbf{x}_j - \mathbf{x}_i \right|} \mathrm{d}s$$

where J_i is the indexes of sites with cells adjacent to V_i ,
$V_{ij} = \partial V_i \bigcap \partial V_j$ is the common boundary of V_i and V_j

Optimal Power Diagrams

- 3. Goal

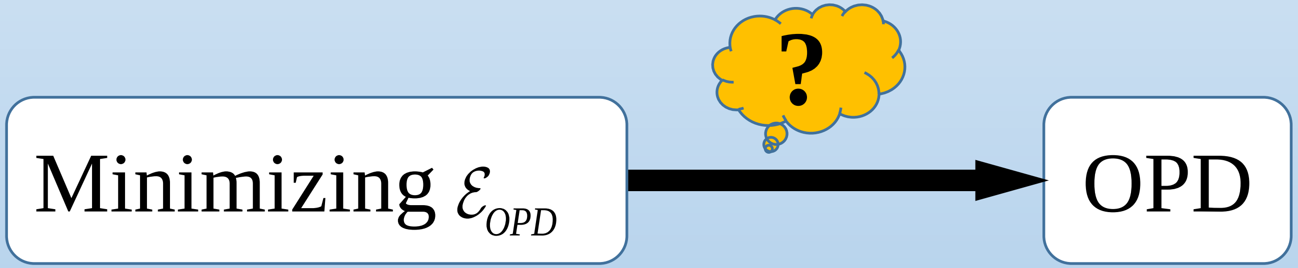
$$\mathcal{E}_{OPD}(\mathbf{X},W)=\| f - f_P \|_{L^2} = \sum_{i=1}^{n} \int_{V_i} \left(f(\mathbf{x}) - P_i^*(\mathbf{x}) \right)^2 \mathrm{d}\mathbf{x},$$

Minimizing $\mathcal{E}_{OPD}$ $\longrightarrow$ OPD

Optimal Power Diagrams

- 3. Goal

$$\mathcal{E}_{OPD}(\mathbf{X},W) = \| f - f_P \|_{L^2} = \sum_{i=1}^{n} \int_{V_i} \left(f(\mathbf{x}) - P_i^*(\mathbf{x}) \right)^2 \mathrm{d}\mathbf{x},$$



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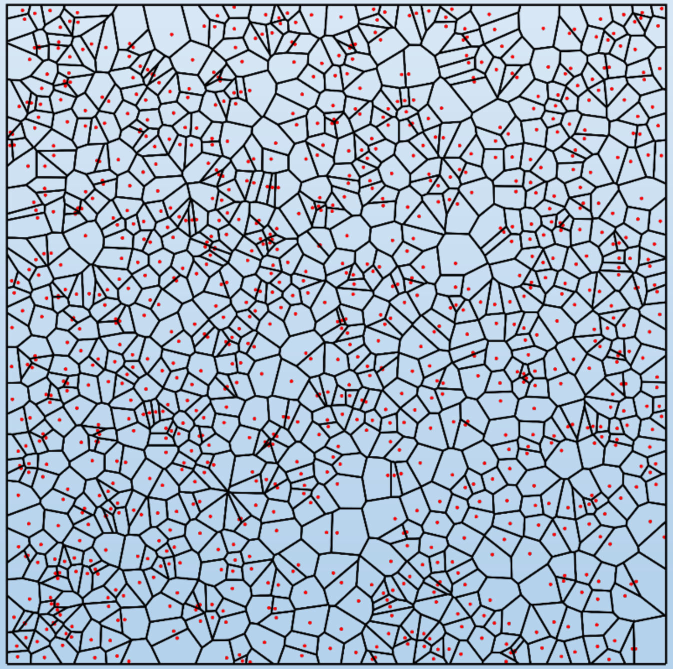
Optimization Framework

- 1. Observation

$$f\left(x,y\right)=100x^2+y^2,-1\le x,y\le 1$$

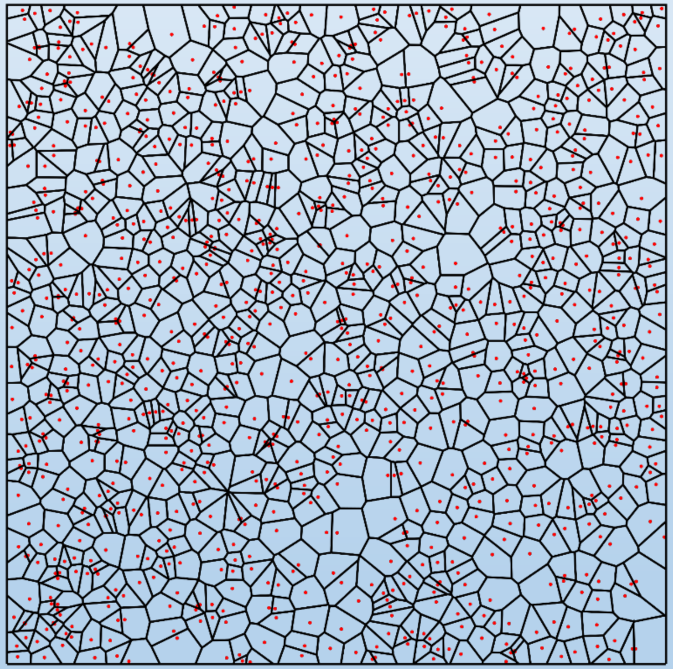


Random initialization

Optimization Framework

- 1. Observation

$$f(x,y) = 100x^2 + y^2, -1 \le x, y \le 1$$



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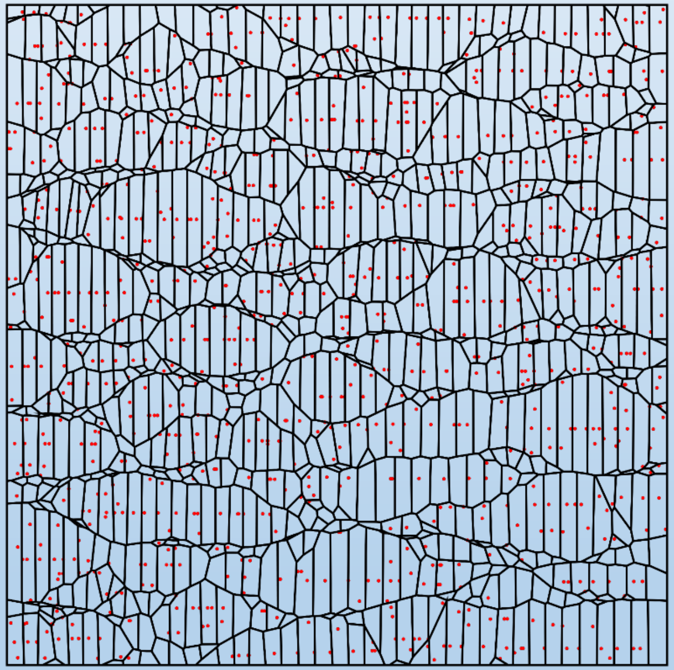

Resulting tessellation after optimizing $(\mathbf{X}, W)$ simultaneously

Optimization Framework

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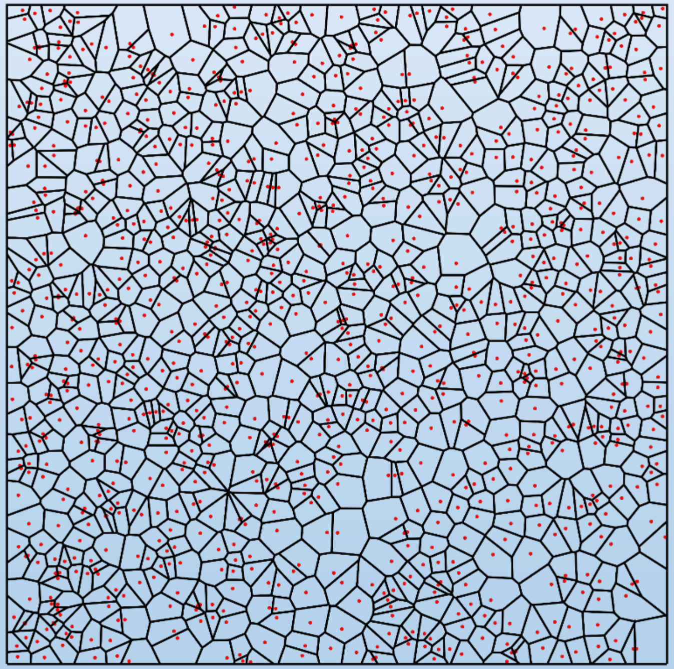

Random initialization

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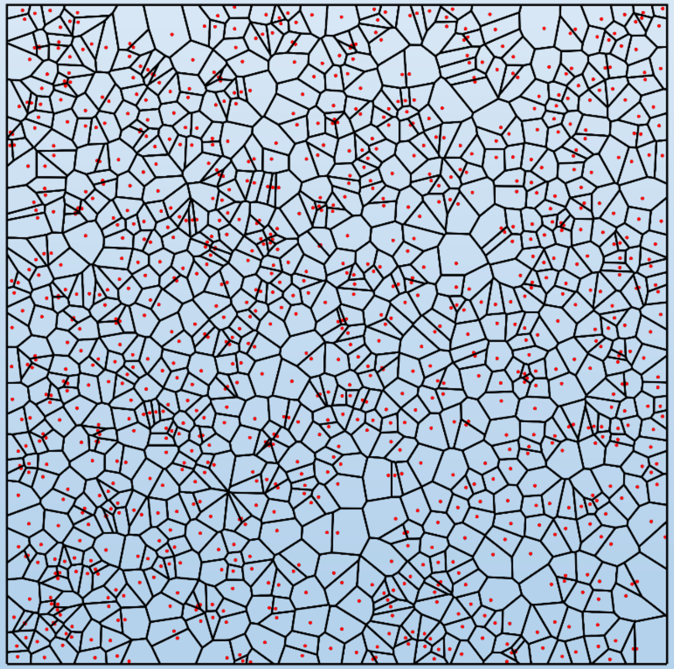

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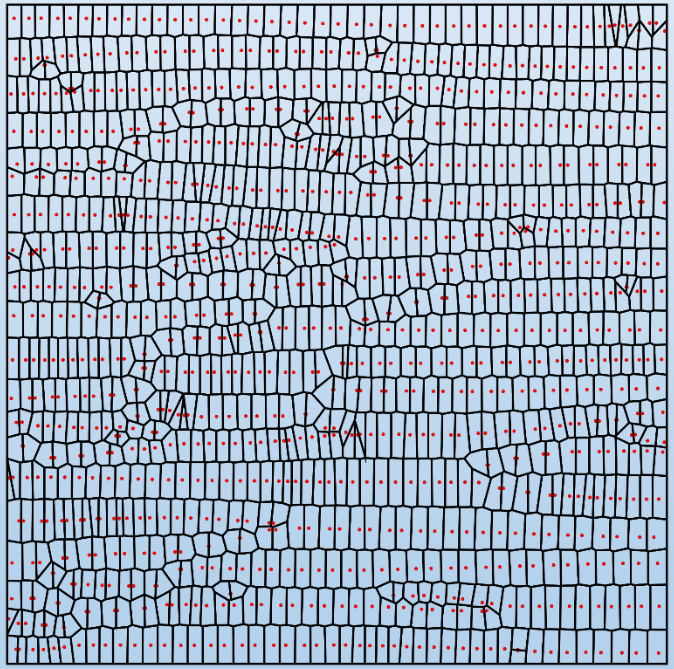

Only position optimization

Optimization Framework

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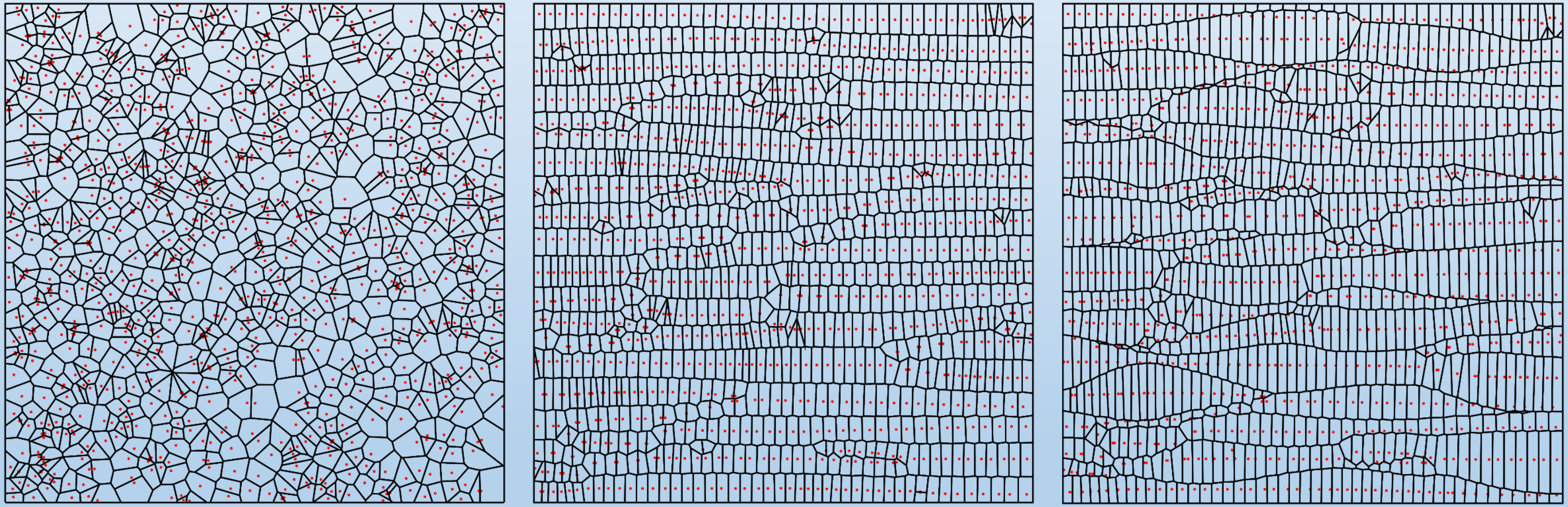

Random initialization | Only position optimization | Position-weight optimization Based on middle tessellation

Optimization Framework

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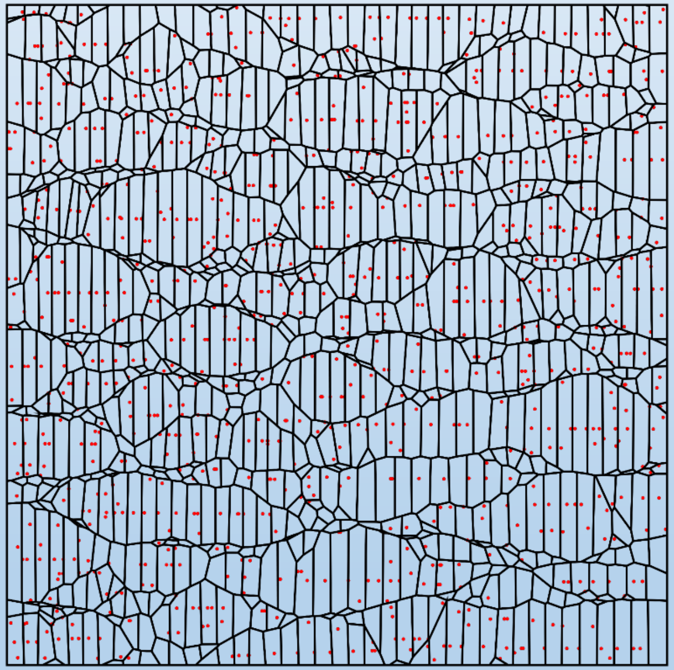
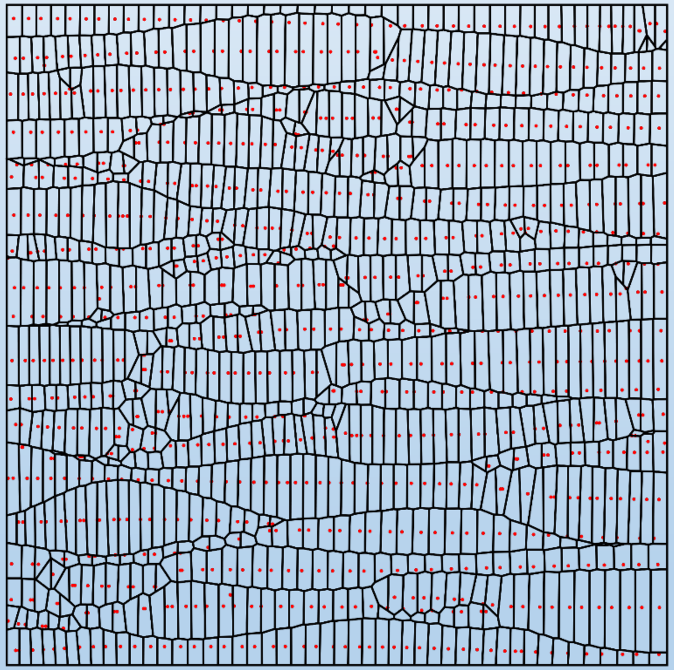


Resulting tessellation with
only position-weight optimization



Resulting tessellation with
firstly position optimization
then position-weight optimization

Optimization Framework

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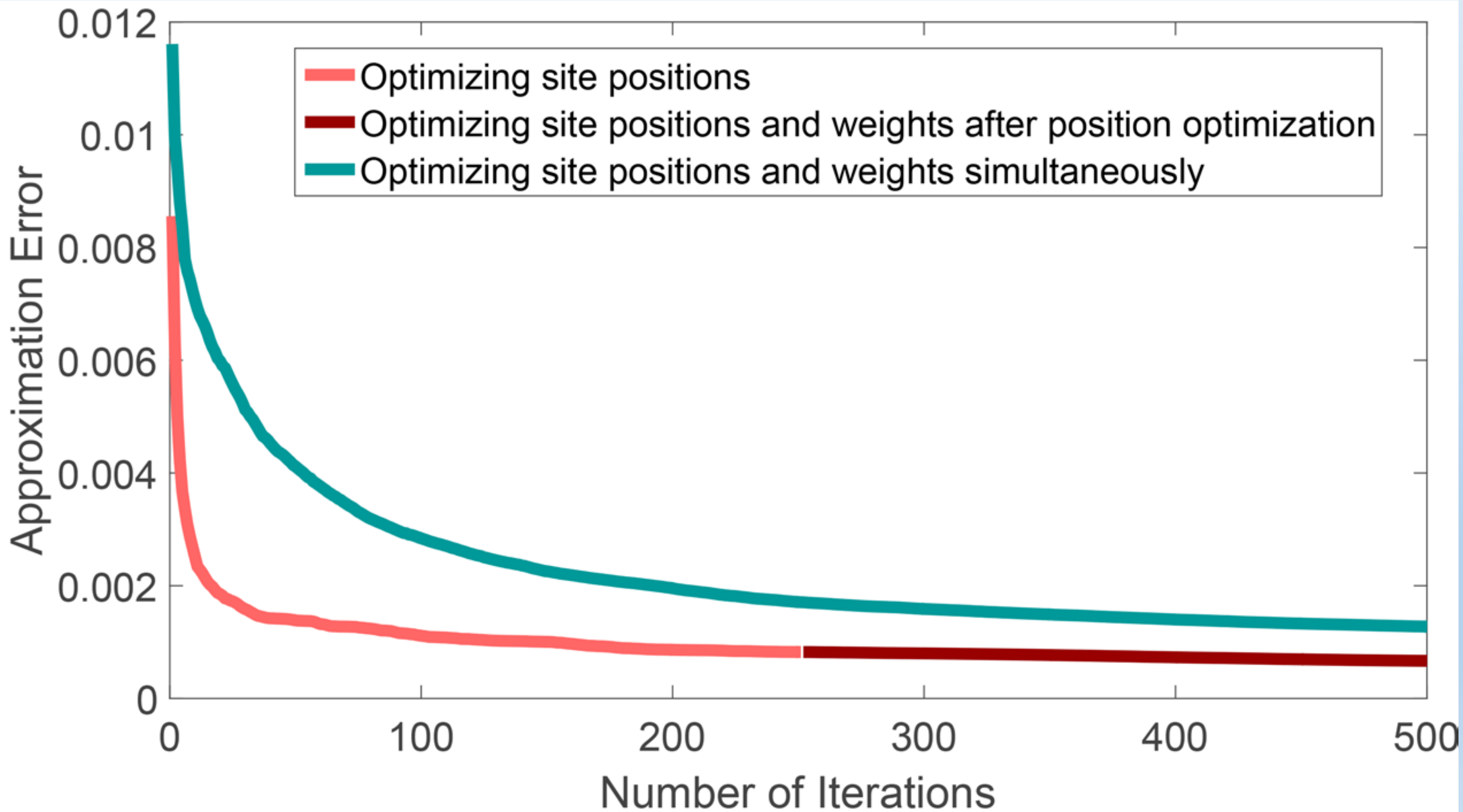
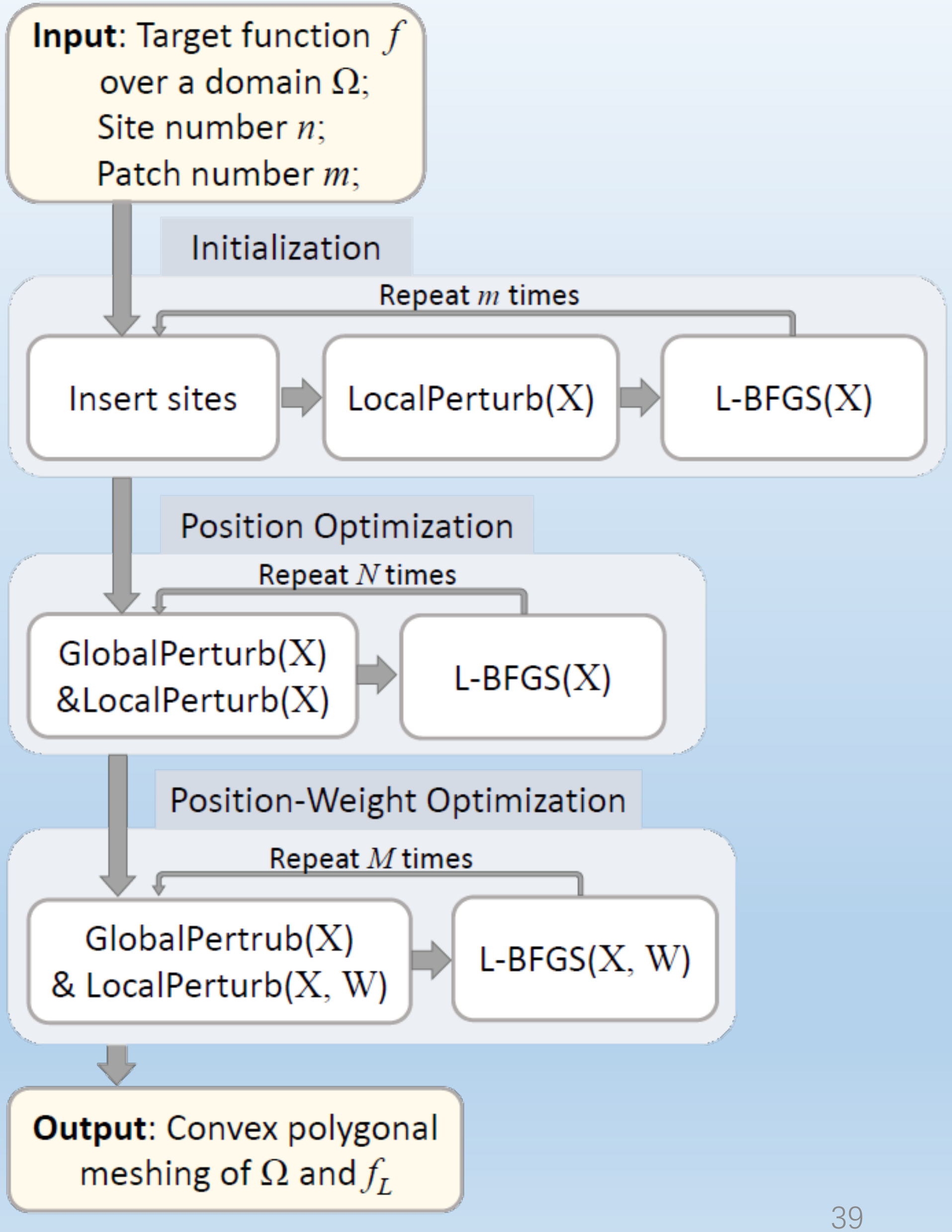


Optimization Framework

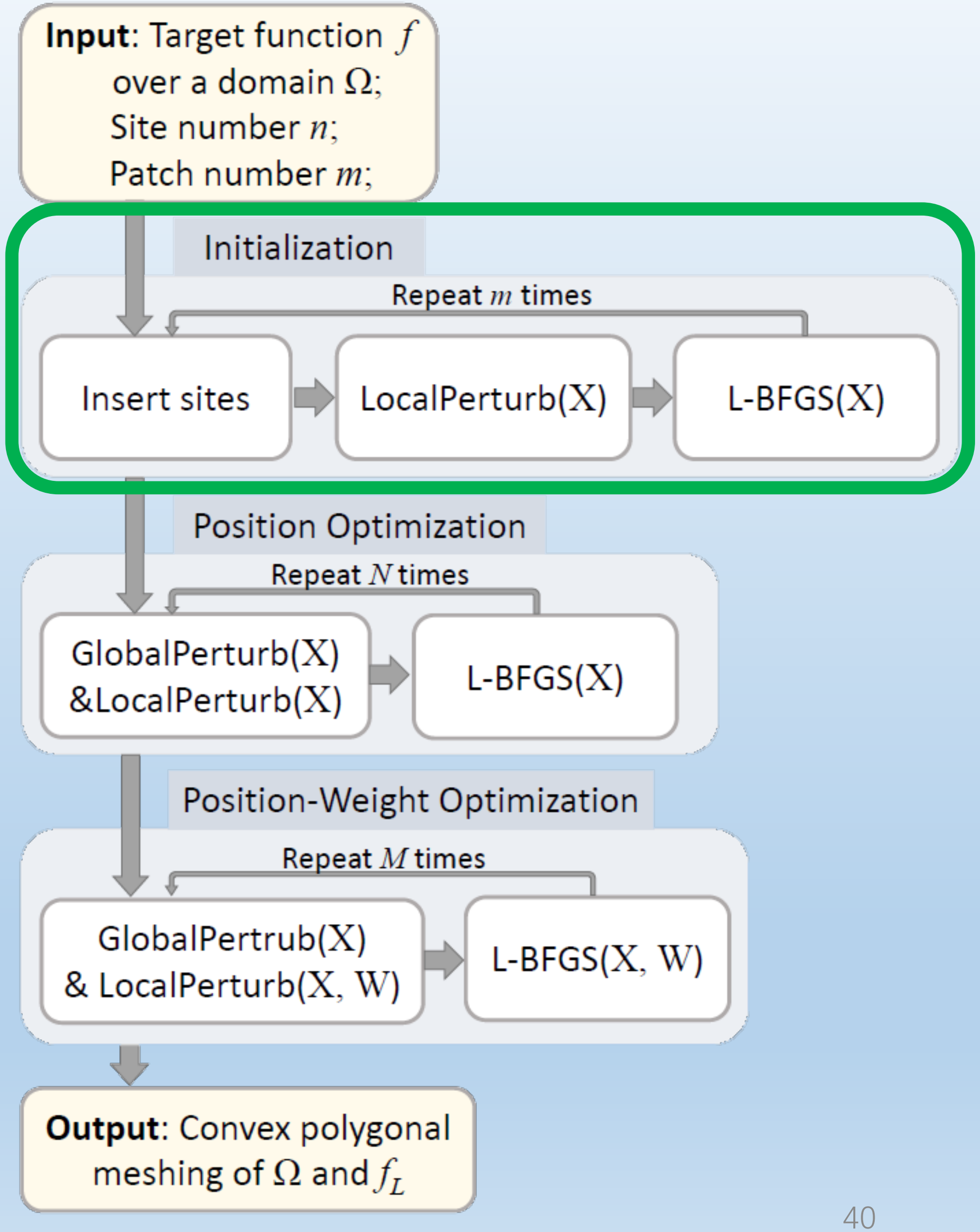
- 2. Overview



Optimization Framework

- 2. Overview

(1) Initialization:
Insert and optimize site positions



Optimization Framework

- 2. Overview

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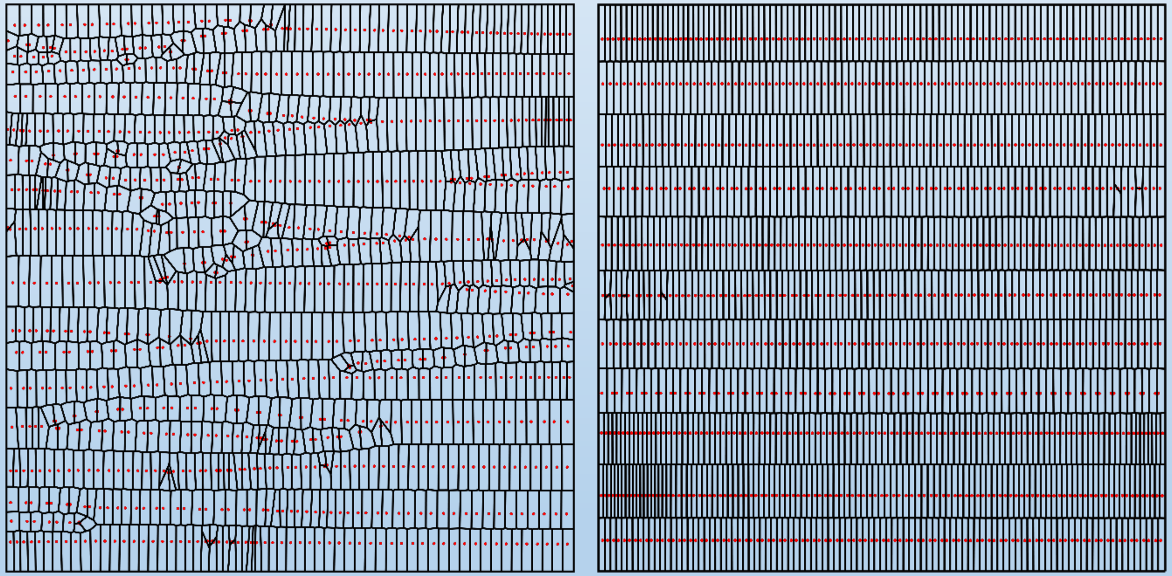
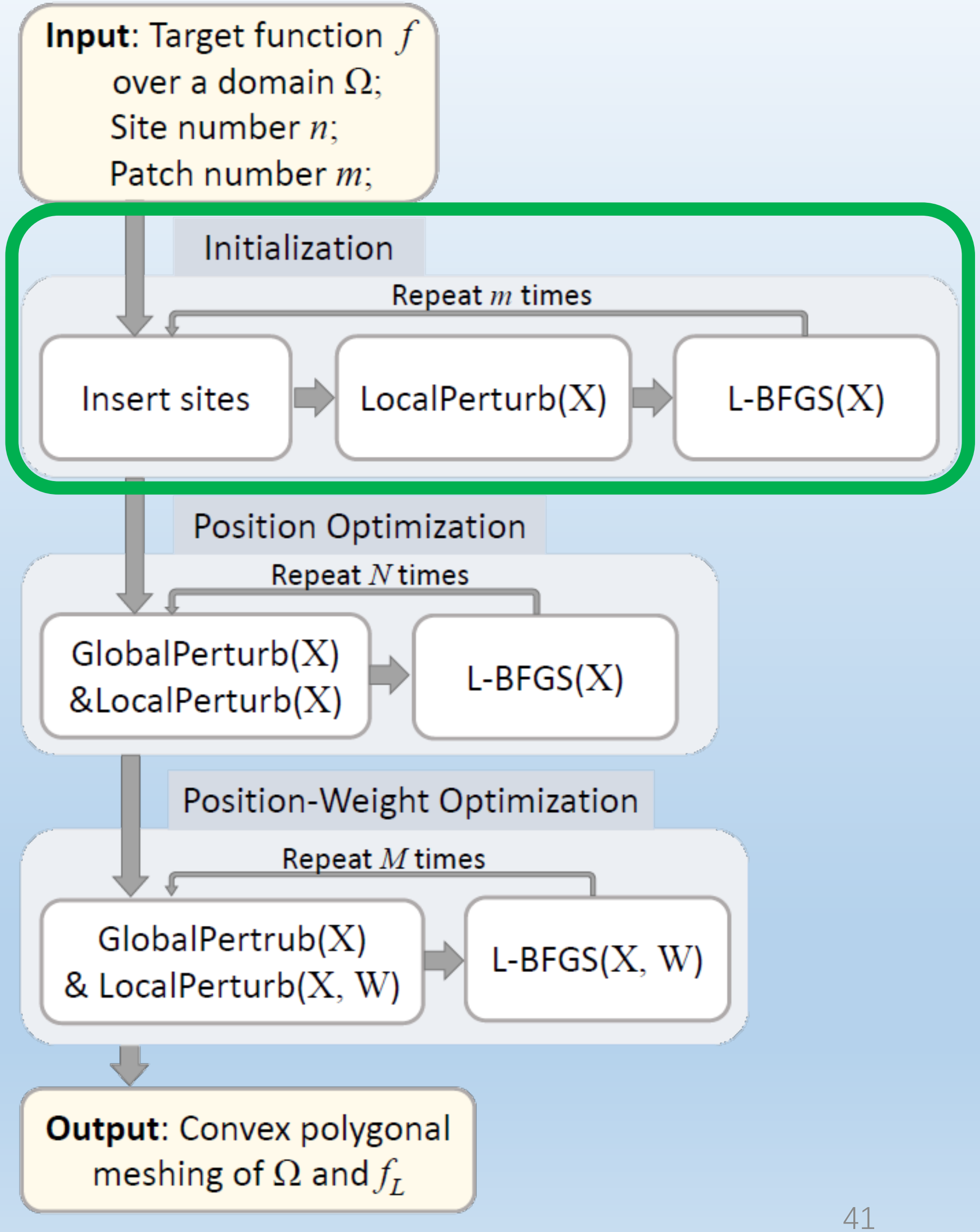


Random insertion PCA-based insertion

$$f(x, y) = 100x^2 + y^2$$



Optimization Framework

- 2. Overview

(1) Initialization:
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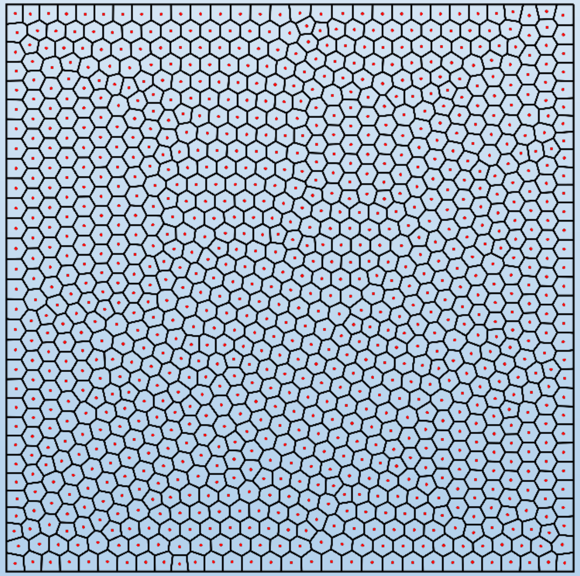
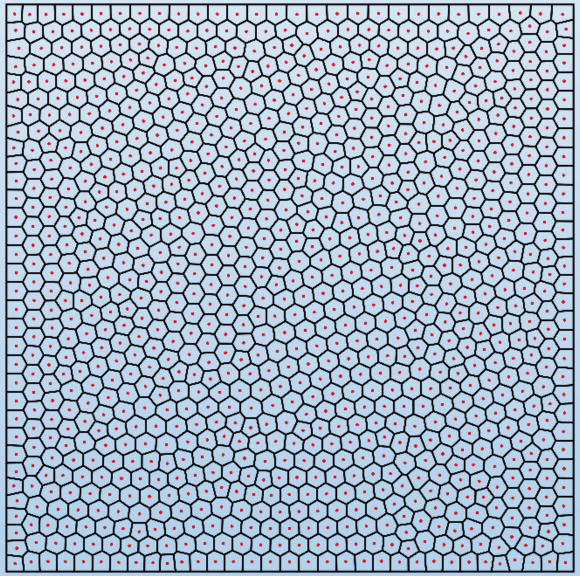
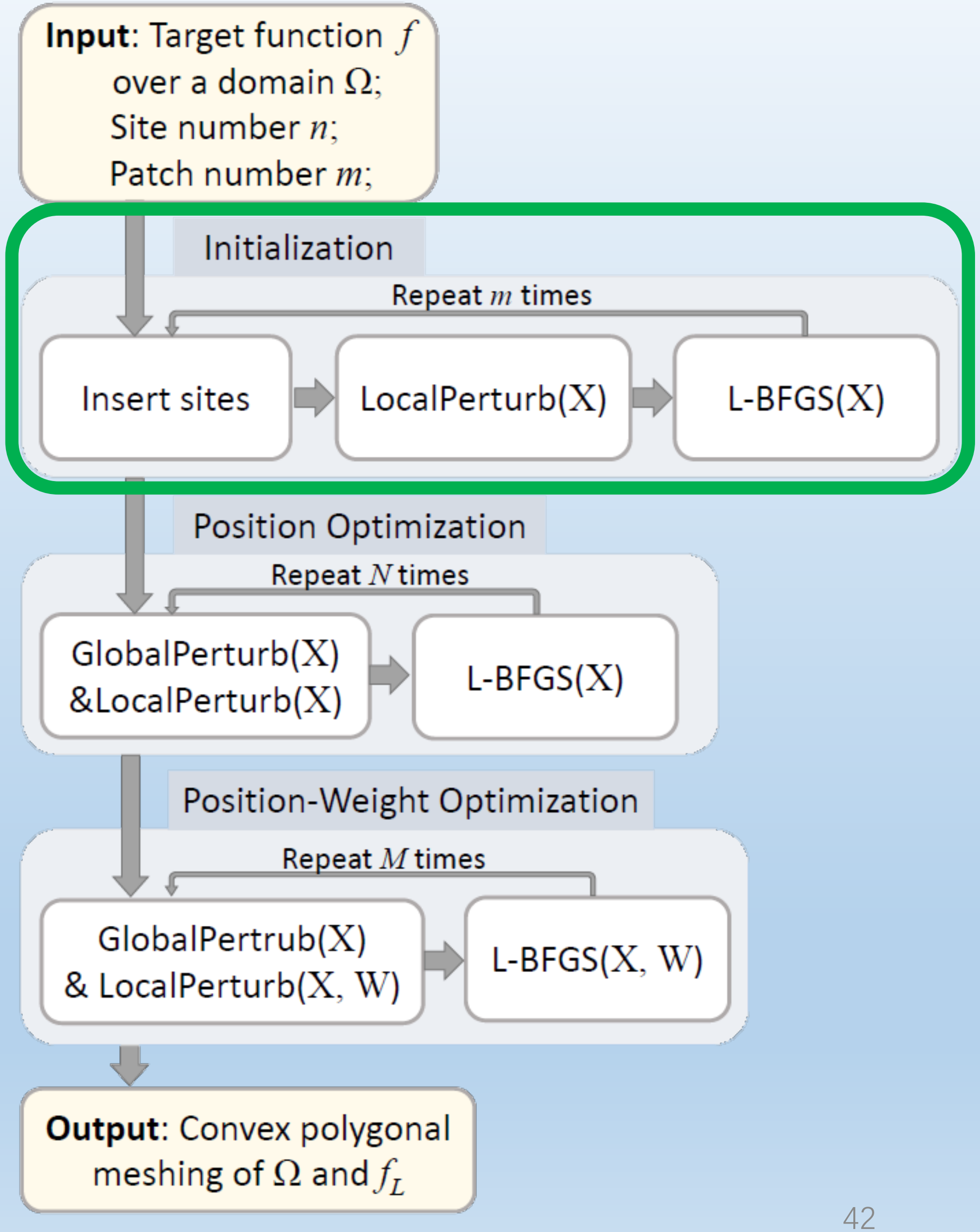


Random insertion



PCA-based insertion

$$f(x,y)=x^2+y^2$$

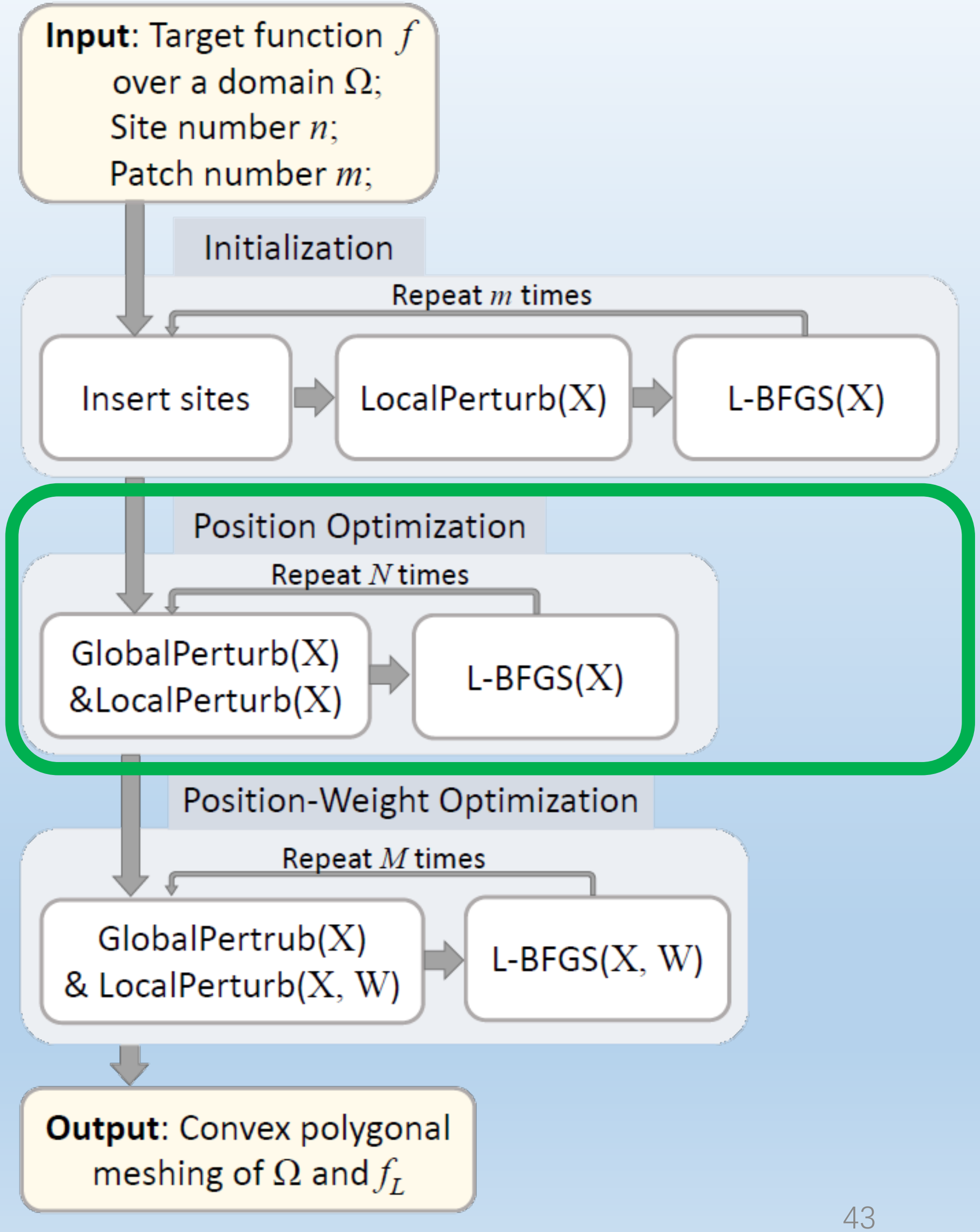


Optimization Framework

- 2. Overview

(1) Initialization:
Insert and optimize site positions

(2) Position optimization:

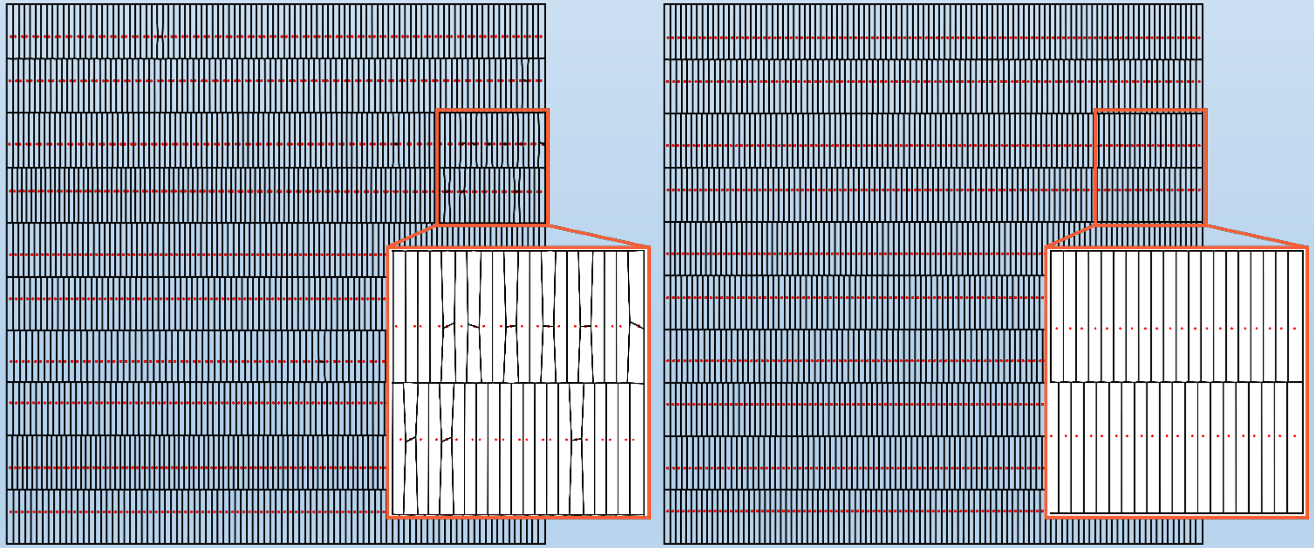


Optimization Framework

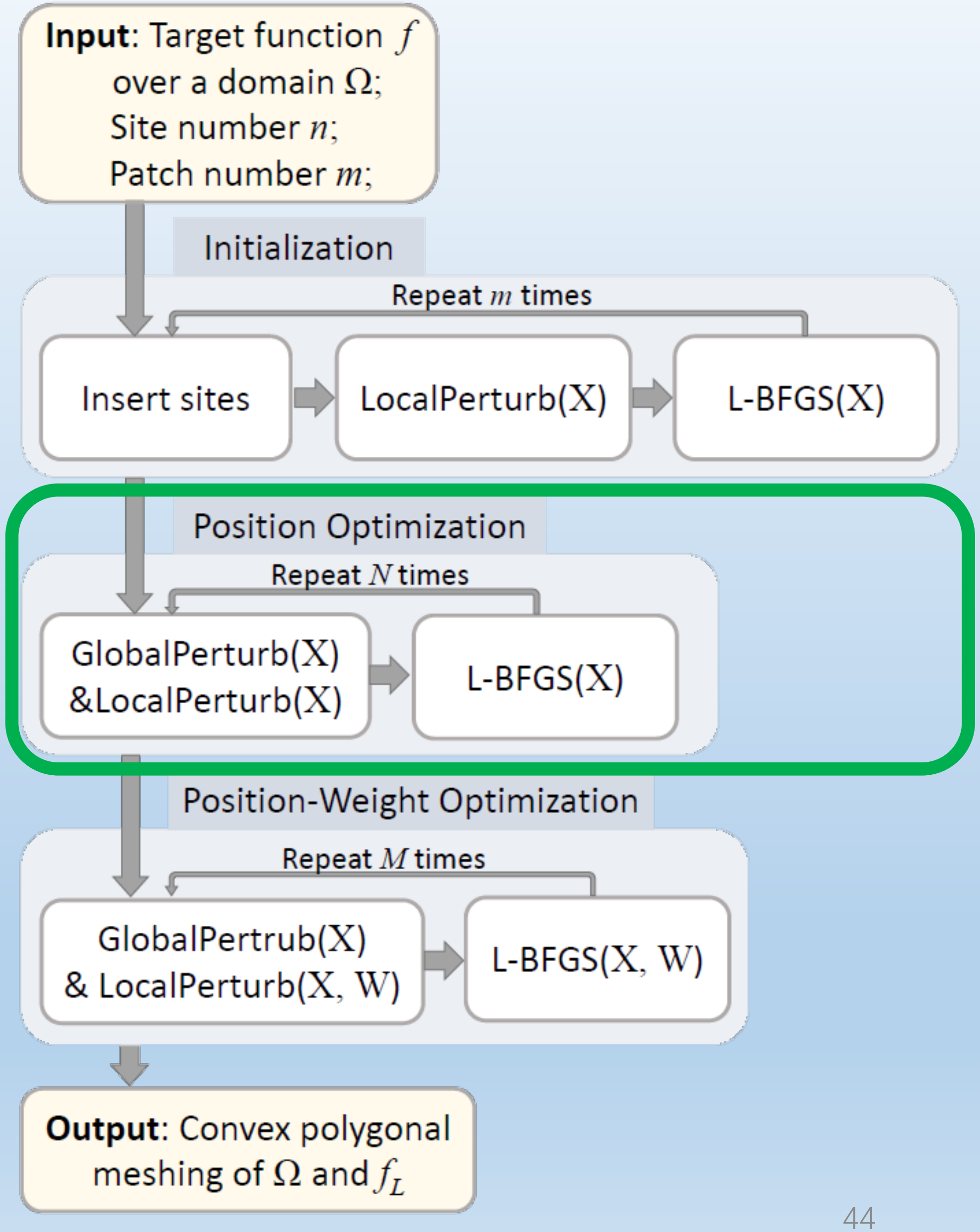
- 2. Overview

(1) Initialization:
Insert and optimize site positions

(2) Position optimization:



Local perturbation   Local&Global perturbation



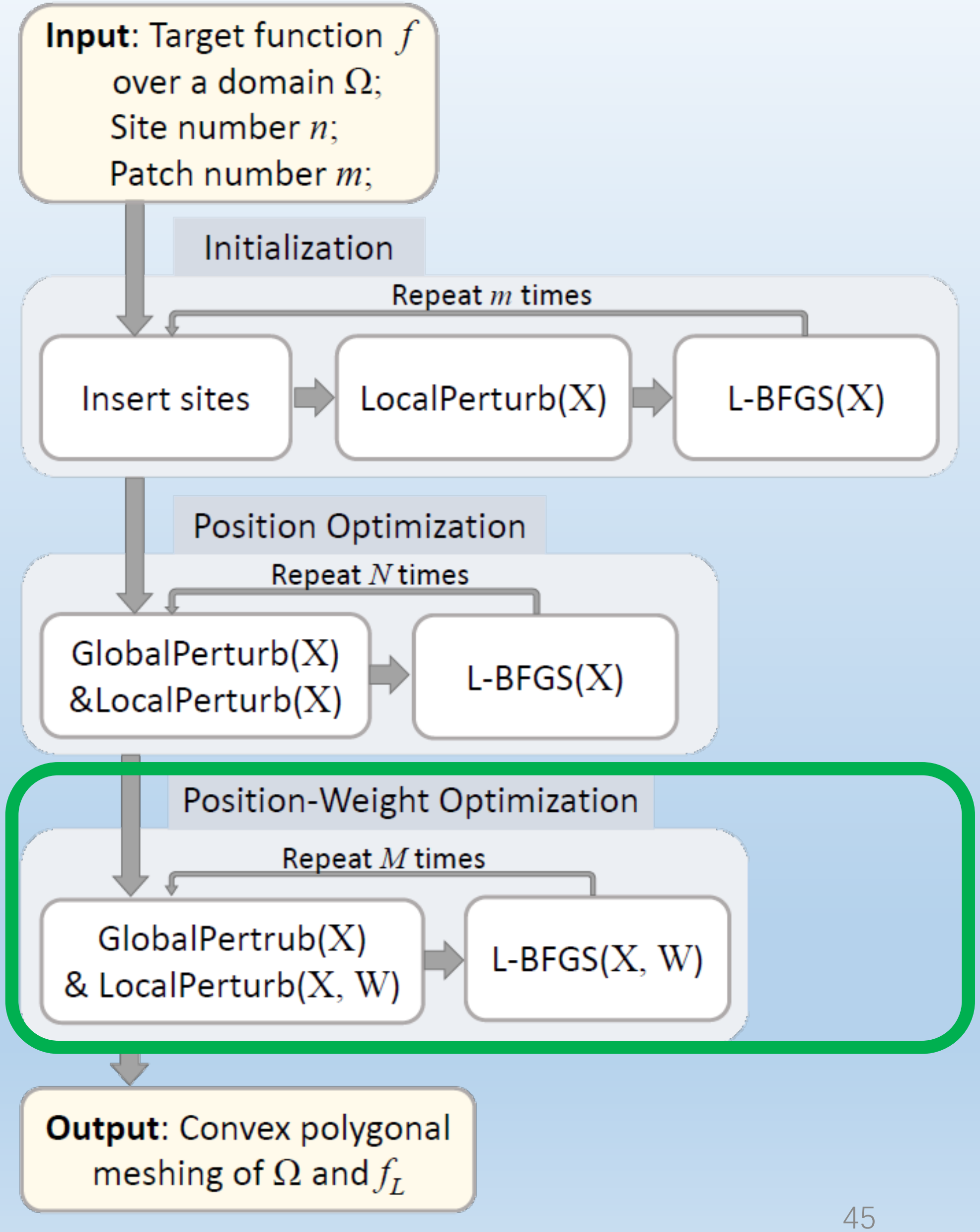
Optimization Framework

- 2. Overview

(1) Initialization:
Insert and optimize site positions

(2) Position optimization:

(3) Position-weight optimization:



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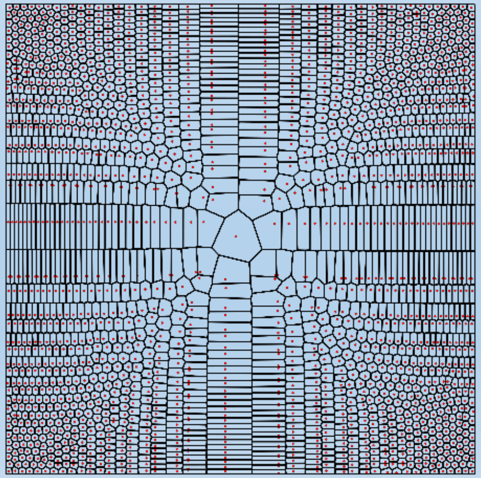
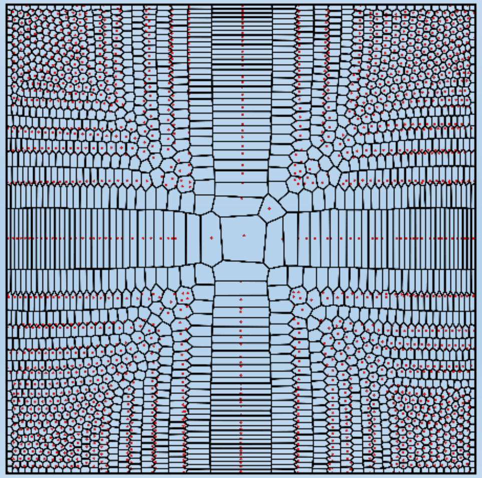
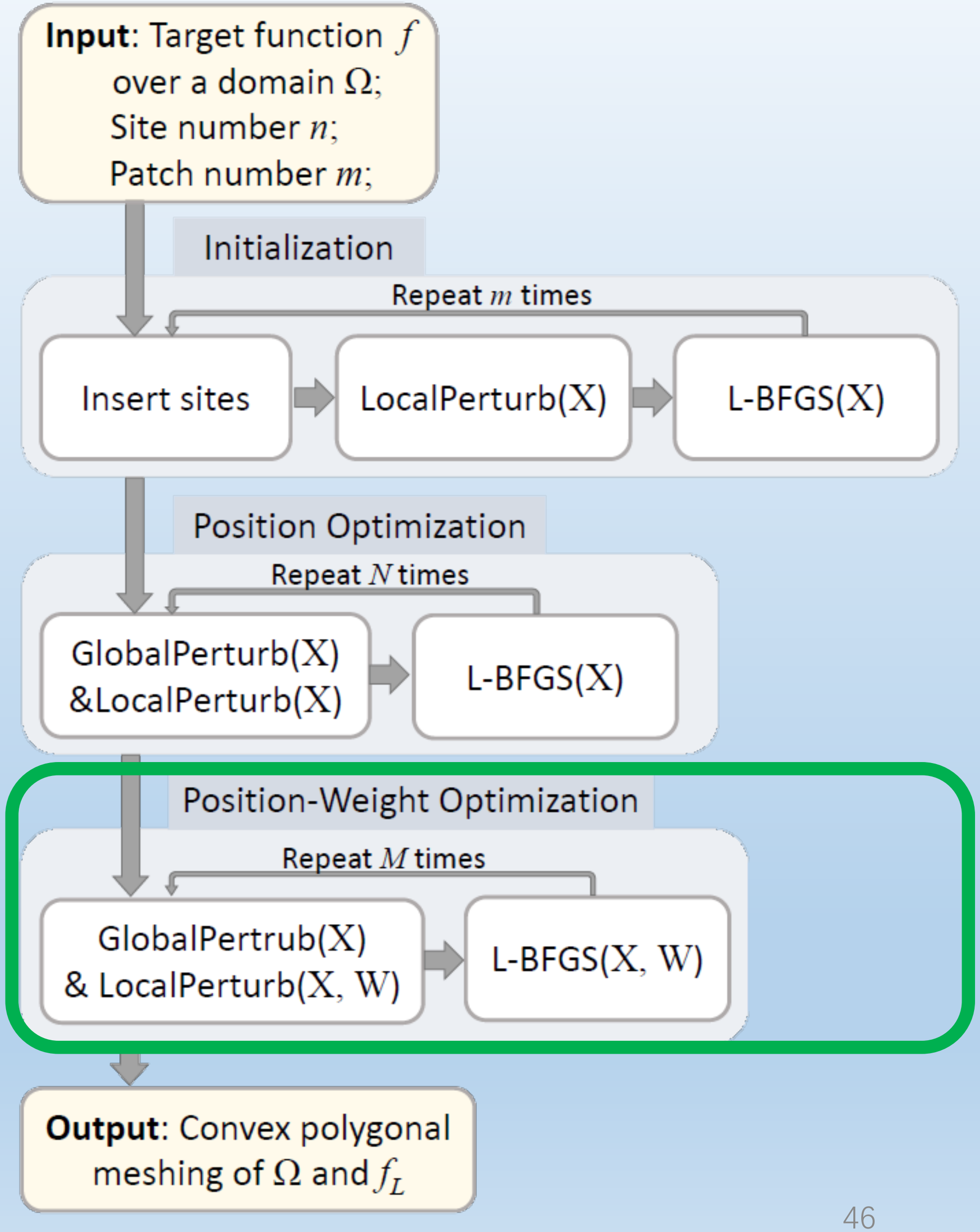




Position optimization result   Position-weight optimization result

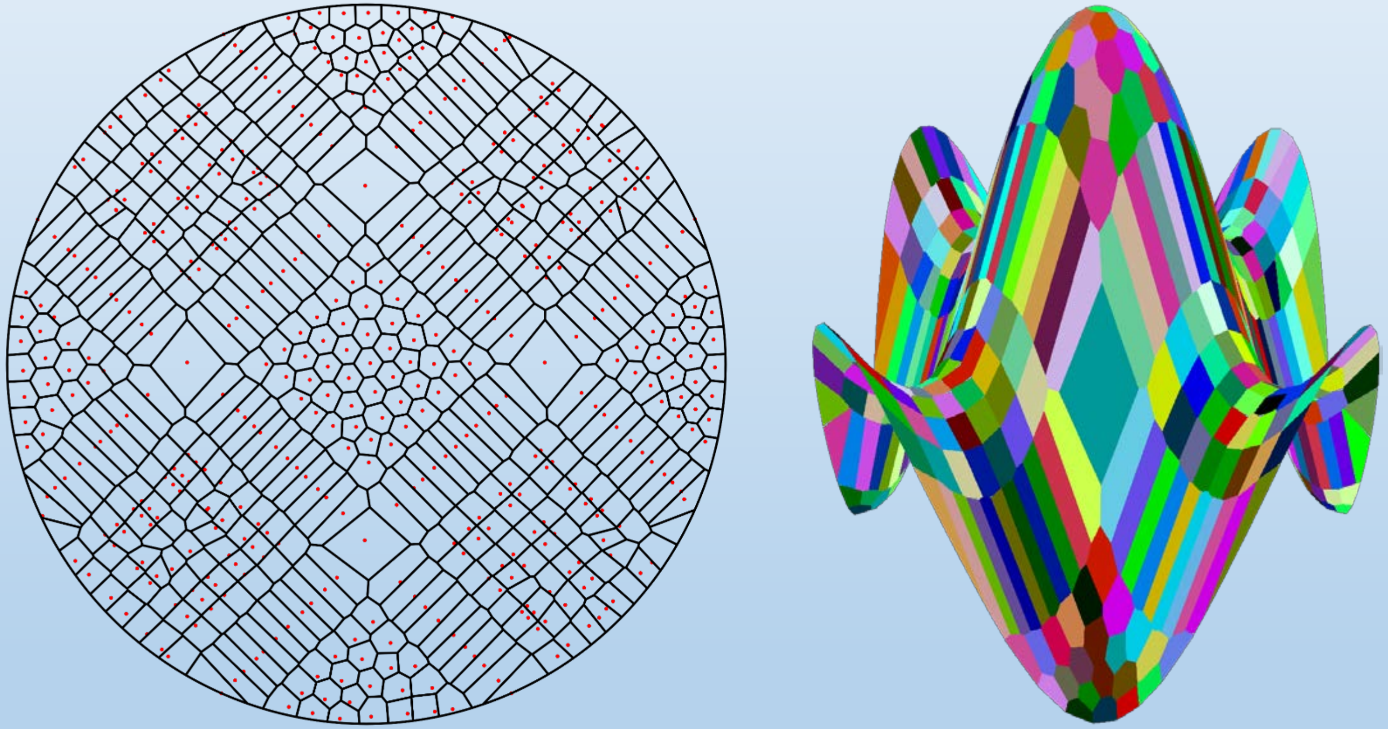


Outline

- Background
- Optimal Power Diagrams
- Optimization Framework
- **Results**
- Conclusion

Results

- 1. Non-convex function approximation



Resulting tessellation (left) and piecewise linear fit (right) of a non-convex target function $f(x, y) = \sin(\pi(x+0.5))\cos(\pi y), x^2 + y^2 \leq 1$ with 500 sites

Results

- 2. Density control

$$\mathcal{E}_{OPD}(\mathbf{X},W) = \| f - f_P \|_{L^2} = \sum_{i=1}^{n} \int_{V_i} \boxed{\rho(\mathbf{x})} \left(f(\mathbf{x}) - P_i^*(\mathbf{x}) \right)^2 \mathrm{d}\mathbf{x},$$

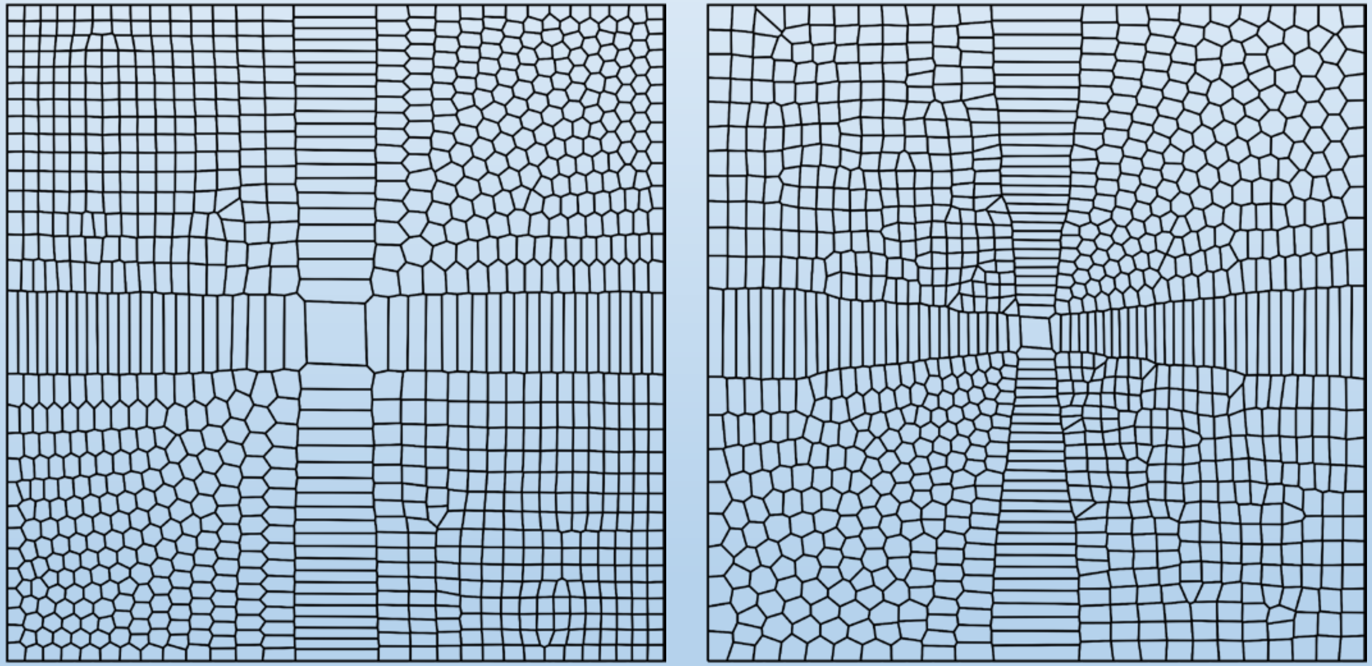
$$\frac{\partial \mathcal{E}_{OPD}(\mathbf{X},W)}{\partial \mathbf{x}_i} = \sum_{j \in J_i} \int_{V_{ij}} \rho(\mathbf{x}) \left(\left| f(\mathbf{x}) - P_i^*(\mathbf{x}) \right|^2 - \left| f(\mathbf{x}) - P_j^*(\mathbf{x}) \right|^2 \right) \frac{\mathbf{x} - \mathbf{x}_i}{\left| \mathbf{x}_j - \mathbf{x}_i \right|} \mathrm{d}s$$

$$\frac{\partial \mathcal{E}_{OPD}(\mathbf{X},W)}{\partial w_i} = \sum_{j \in J_i} \int_{V_{ij}} \rho(\mathbf{x}) \left(\left| f(\mathbf{x}) - P_i^*(\mathbf{x}) \right|^2 - \left| f(\mathbf{x}) - P_j^*(\mathbf{x}) \right|^2 \right) \frac{1}{2\left| \mathbf{x}_j - \mathbf{x}_i \right|} \mathrm{d}s$$

Results

- 2. Density control

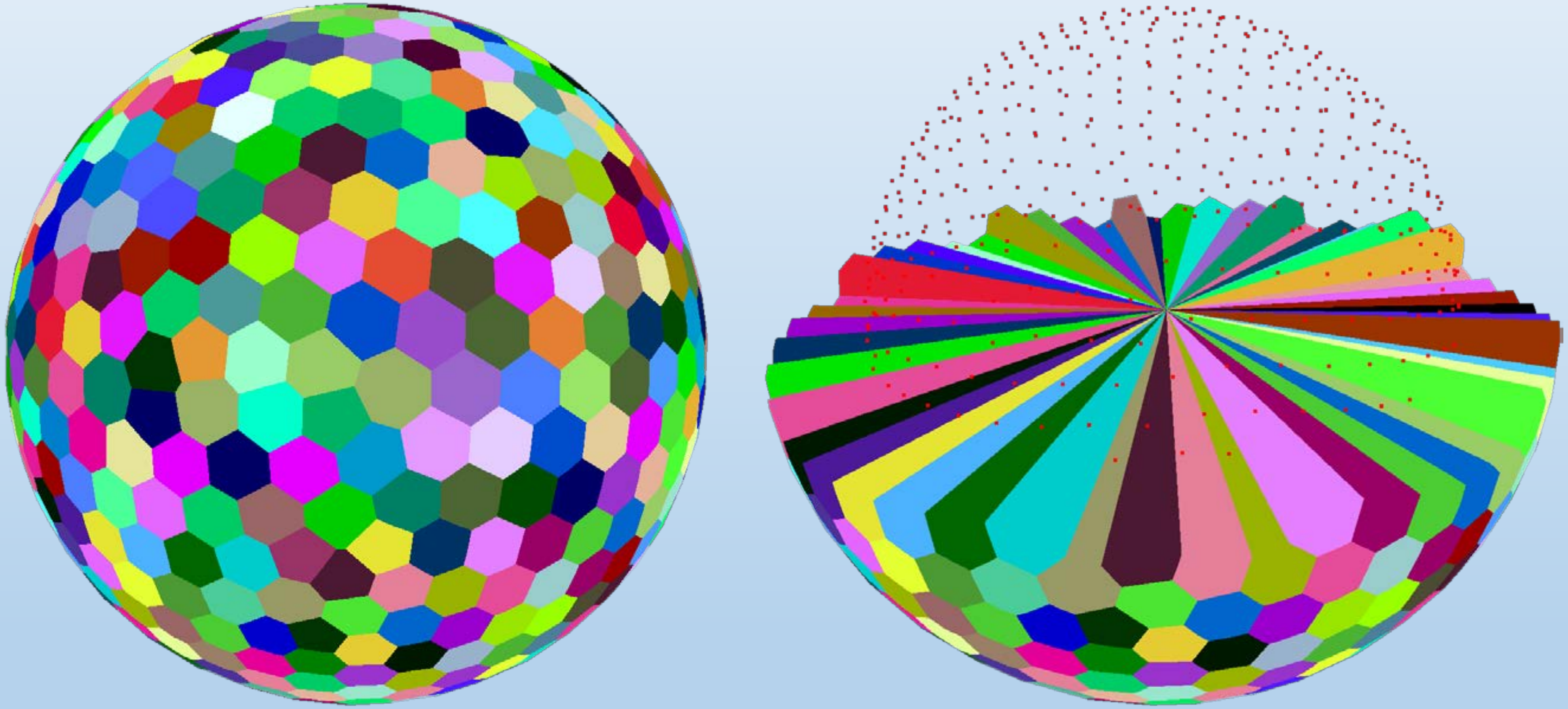
$$\rho(x,y)=1.0 \qquad \rho(x,y)=1.0\Big/\left(\left(x^2+y^2\right)^2+0.001\right)$$



Resulting tessellations for a non-convex target function $f(x,y)=x^3+y^3, -1\le x,y\le 1$ with a constant density (left) and a non-uniform density function (right)

Results

- 3. 3D results



Tessellation of sphere for a non-smooth target function $f\left(x,y,z\right)=\sqrt{x^2+y^2+z^2}$ with 800 sites

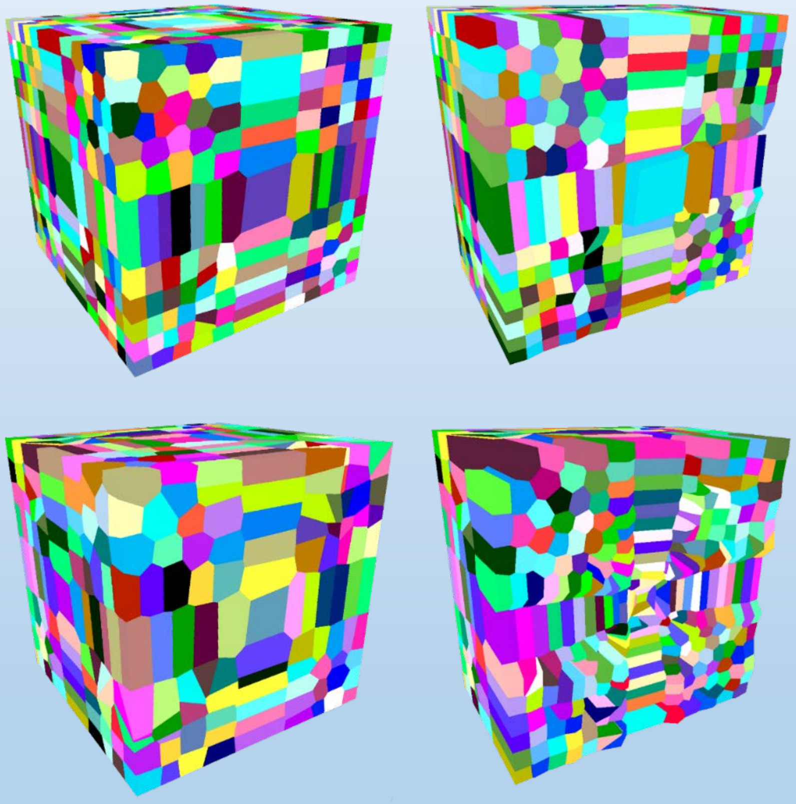
Results

- 3. 3D results

3D optimal power diagrams with increasing anisotropy
Left: isotropic, middle: 2:1:1, right: 8:1:1

Results

- 3. 3D results



$$\rho(x,y,z)=1.0$$

$$\rho(x,y,z)=1.0\Big/\left(\left(x^2+y^2+z^2\right)^2+0.001\right)$$

Exterior and cutaway views of tessellations for a non-convex target function with a constant density (top) and non-uniform density (bottom)

$$f(x,y,z)=x^3+y^3+z^3,-1\le x,y,z\le 1$$

Results

- 4. Comparisons with OVT

Three measures of tessellations [Max Budninsky et.al, ACM TOG 2016]:

(1) Hessian variation: $\max_{\mathbf{x},\mathbf{y}\in V_i} \| \mathit{Hess}[f](\mathbf{x}) - \mathit{Hess}[f](\mathbf{y}) \|_F$

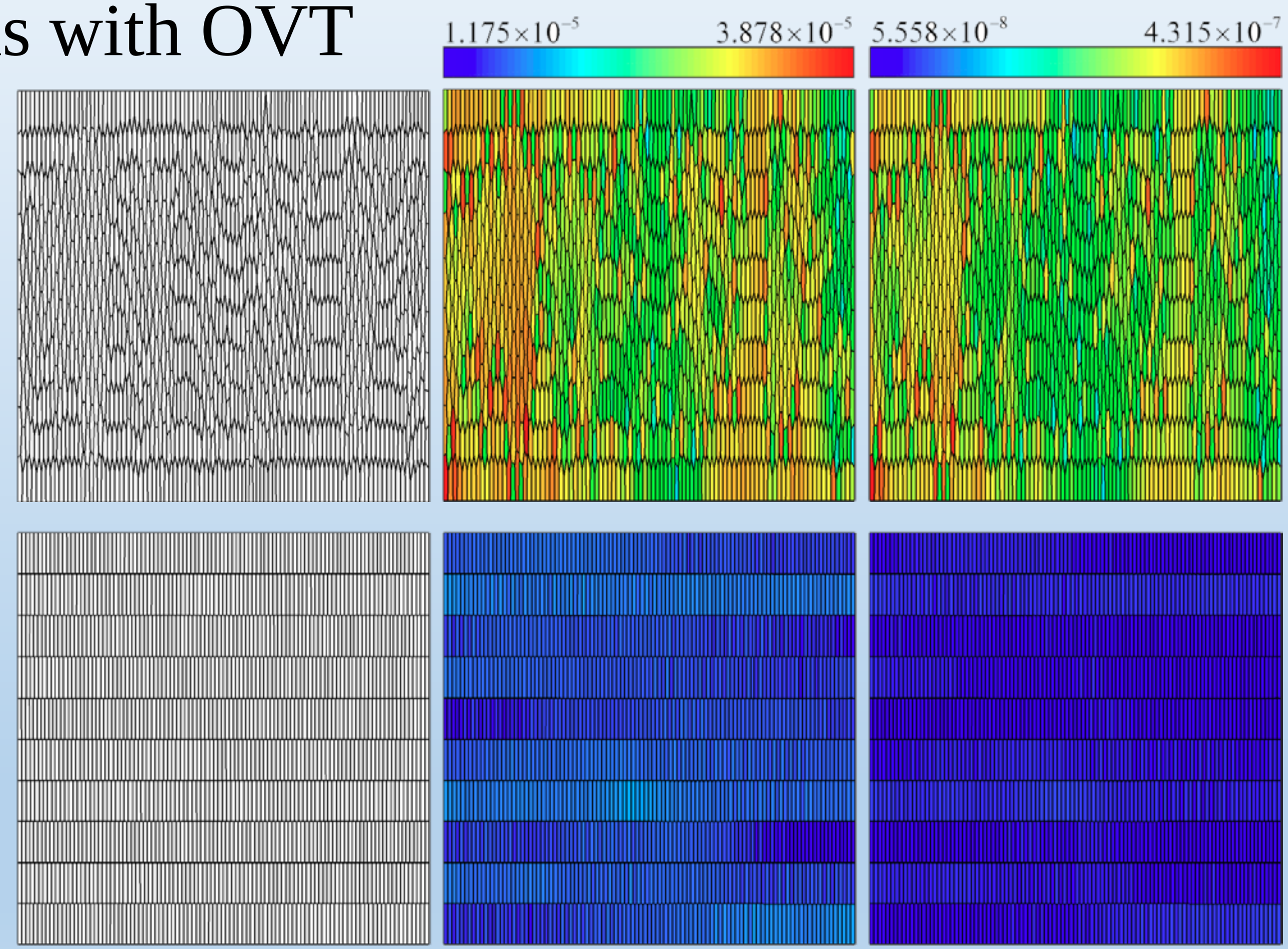
(2) Shape ratio: $\max_{\mathbf{x},\mathbf{y}\in V_i} \left[\sqrt{(\mathbf{x}-\mathbf{y})^t \,\overline{H}_{V_i}\, (\mathbf{x}-\mathbf{y})} \right] \left[|V_i| \sqrt{\det \overline{H}_{V_i}} \right]^{-\frac{1}{d}}$

(3) Modified area: $\left(\overline{\rho}_{V_i}^{d} \det \overline{H}_{V_i} \right)^{\frac{1}{d+2}} |V_i|$

Results

$$f(x,y)=100x^2+y^2, -1 \le x,y \le 1$$

- 4. Comparisons with OVT



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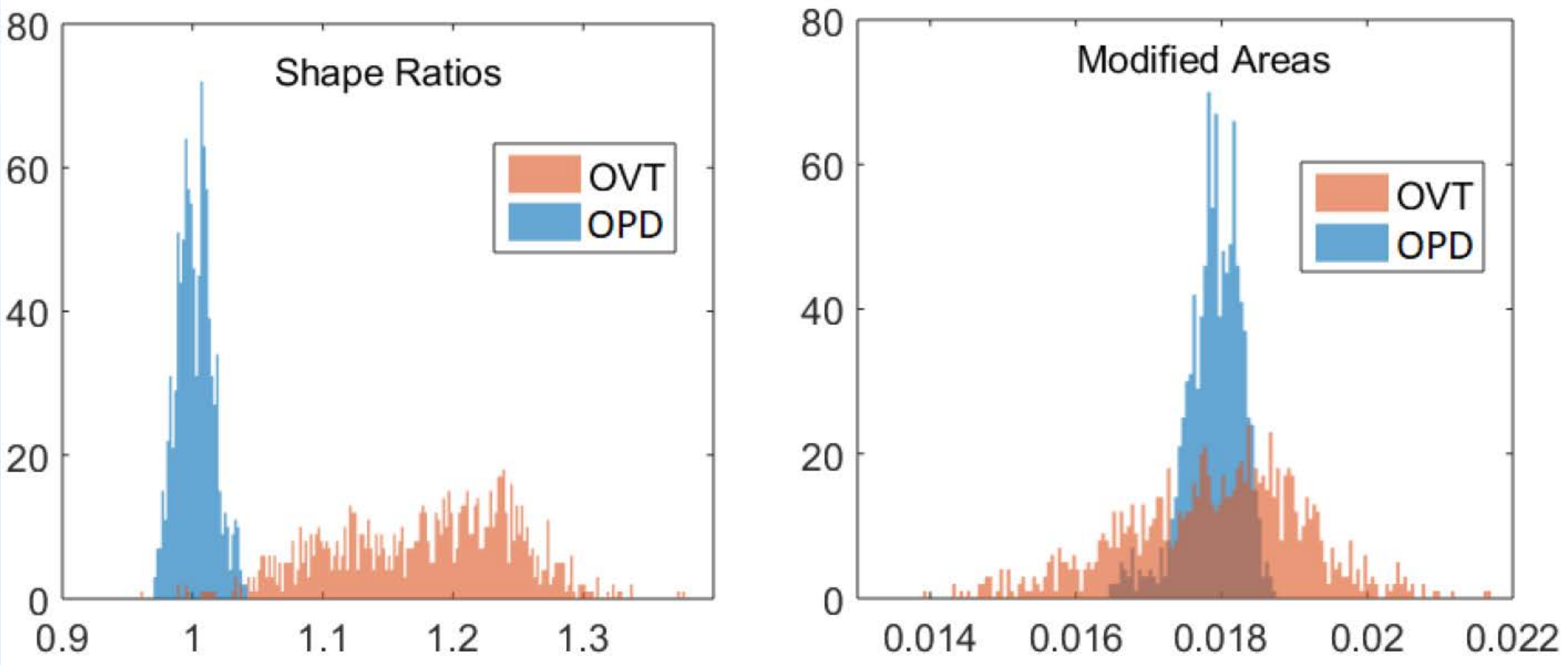
$$f(x,y)=100x^2+y^2, -1\le x,y\le 1$$

- 4. Comparisons with OVT



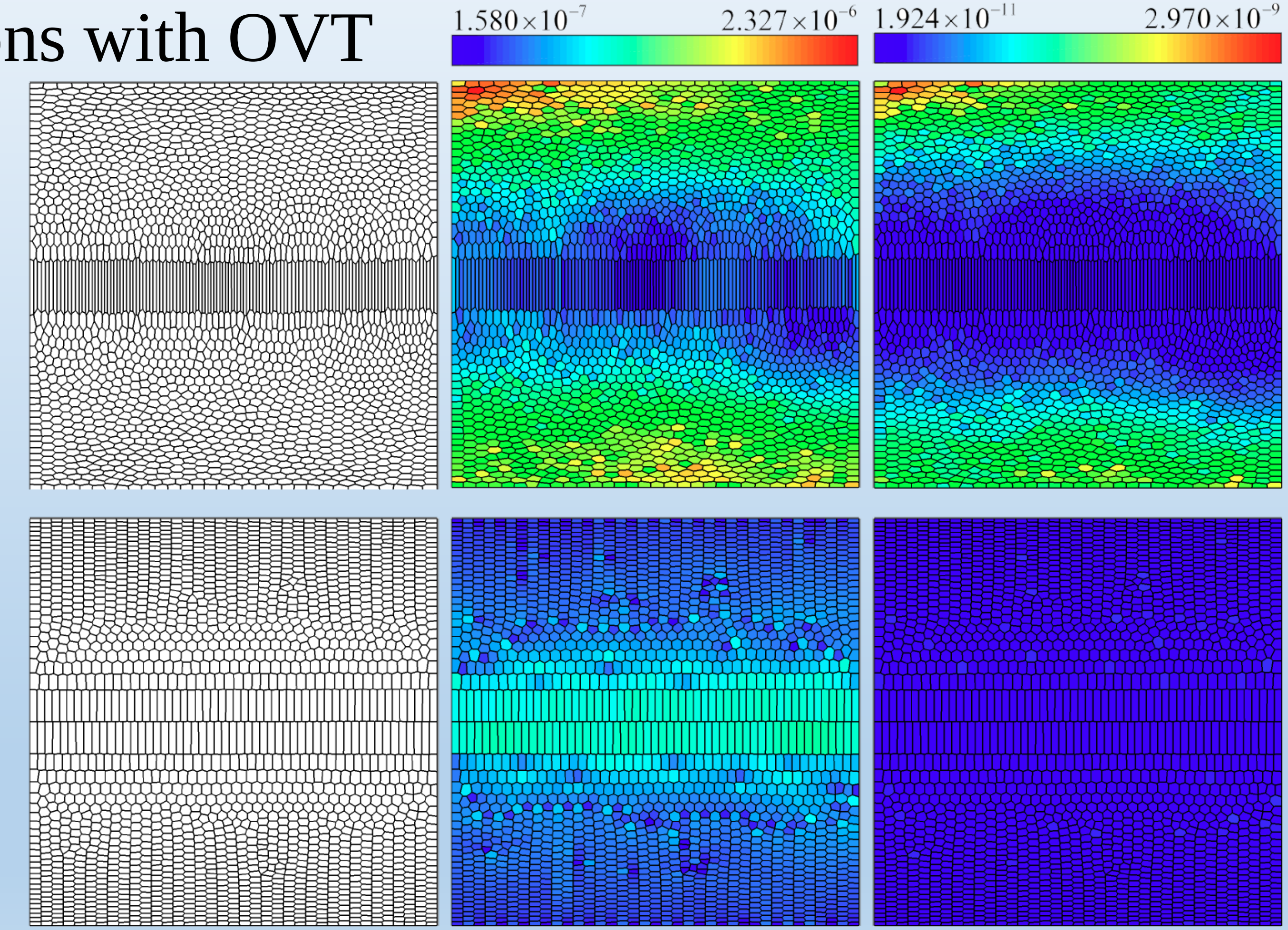
Results

$$f(x,y) = x^2 + 10^{-5}y^2 + y^4, -1 \le x, y \le 1$$

- 4. Comparisons with OVT

OVT

OPD

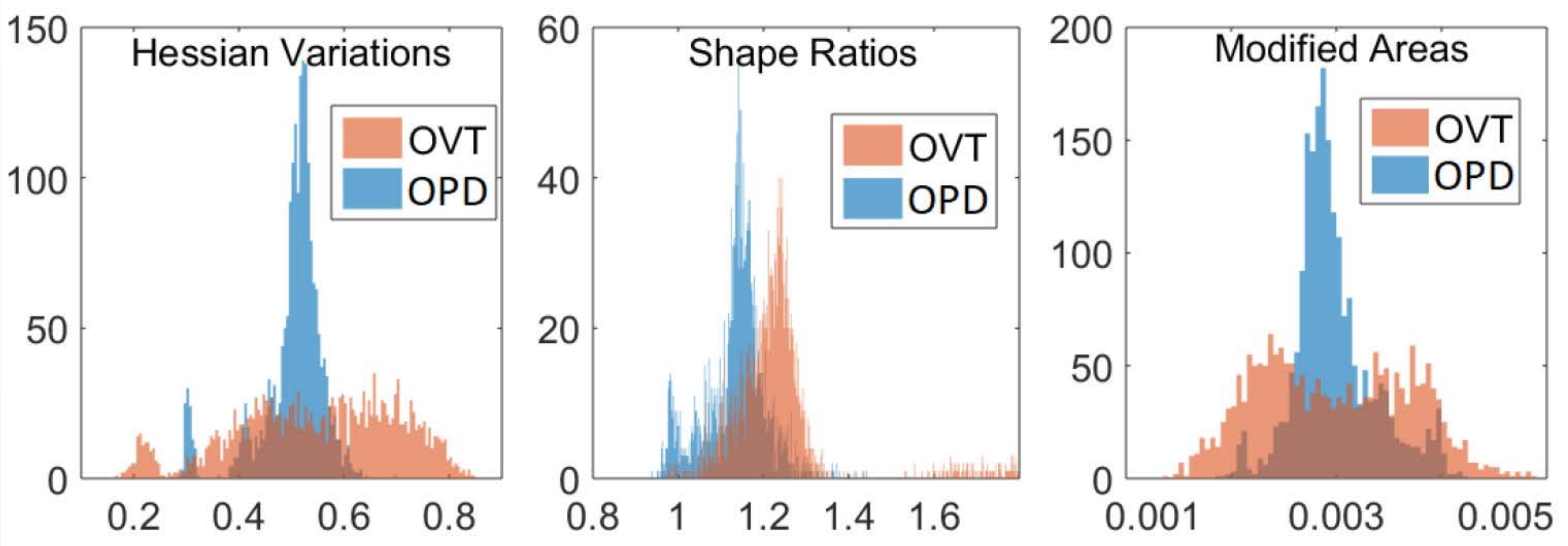


tessellation L^1 error L^2 error

Results

$$f\left(x,y\right)=x^2+10^{-5}y^2+y^4, -1\le x,y\le 1$$

- 4. Comparisons with OVT

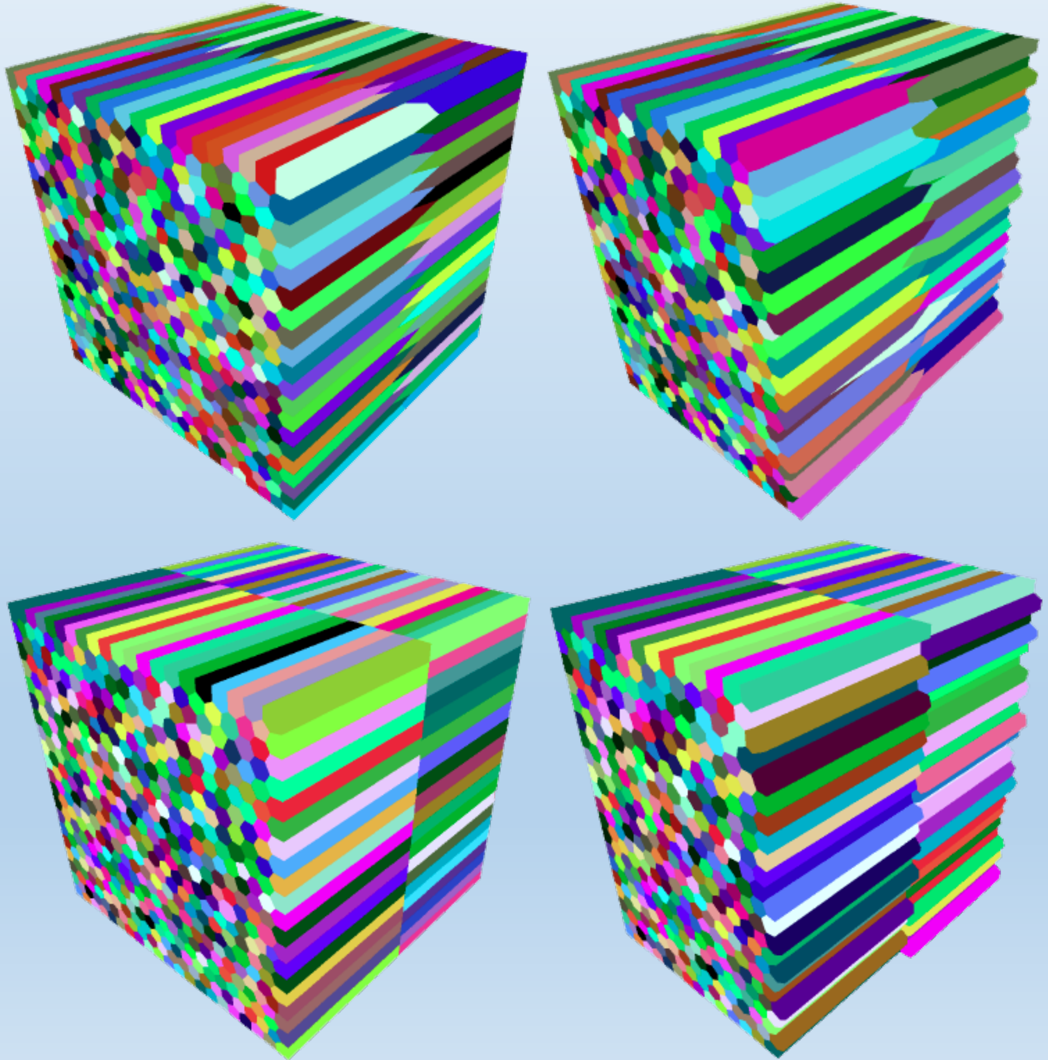


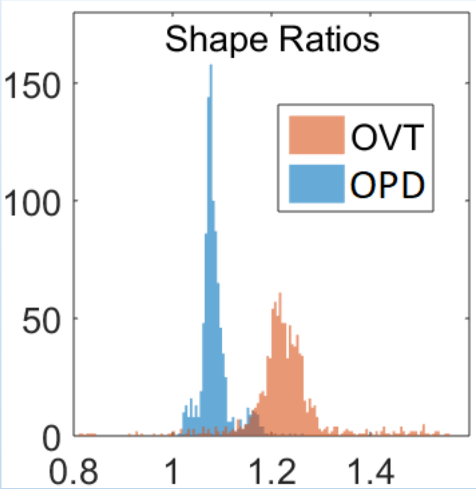
Results

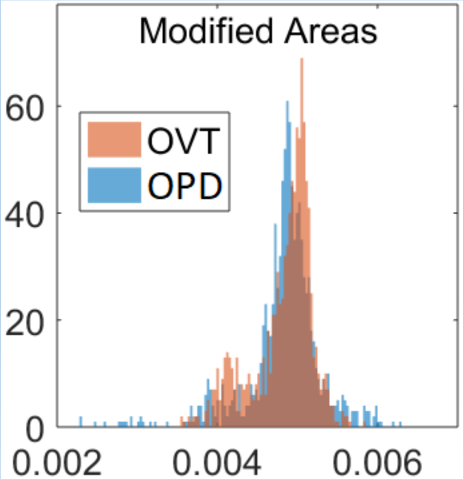
- 4. Comparisons with OVT

OVT

OPD







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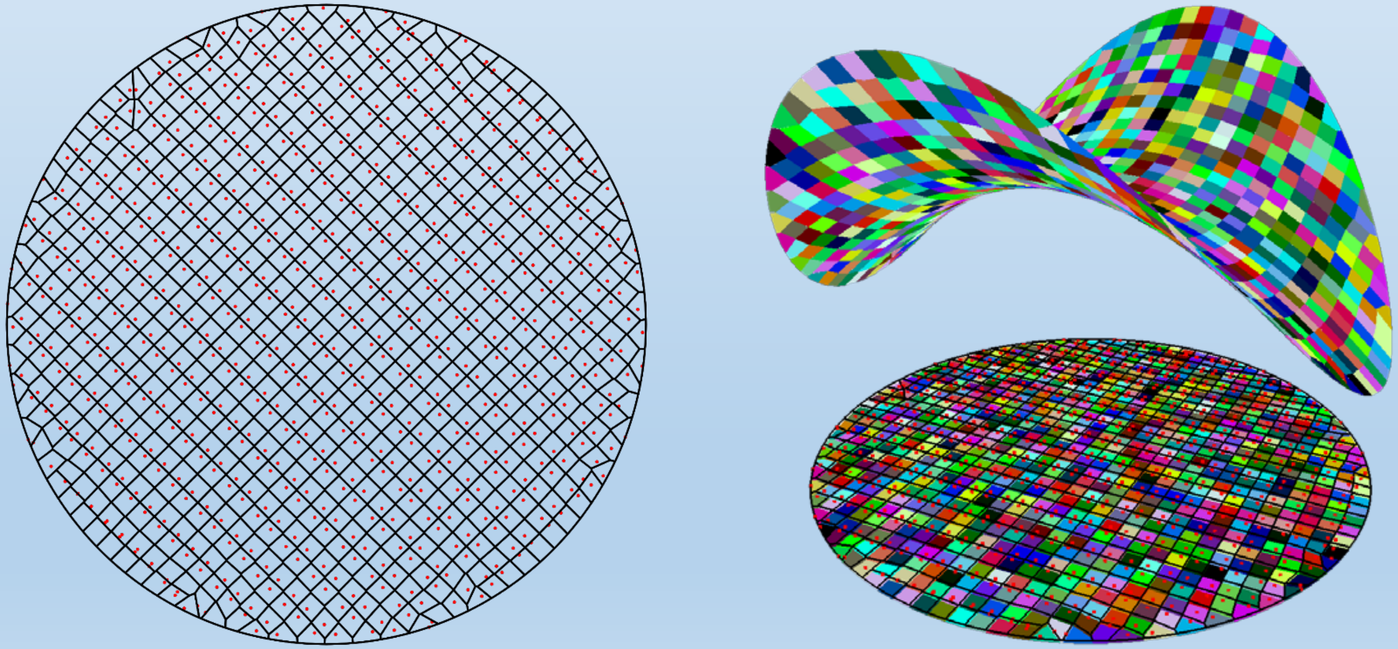
Conclusion

- 1. Contributions

(1) We extend the OVT method for generating anisotropic meshes that the target function is not necessarily convex;



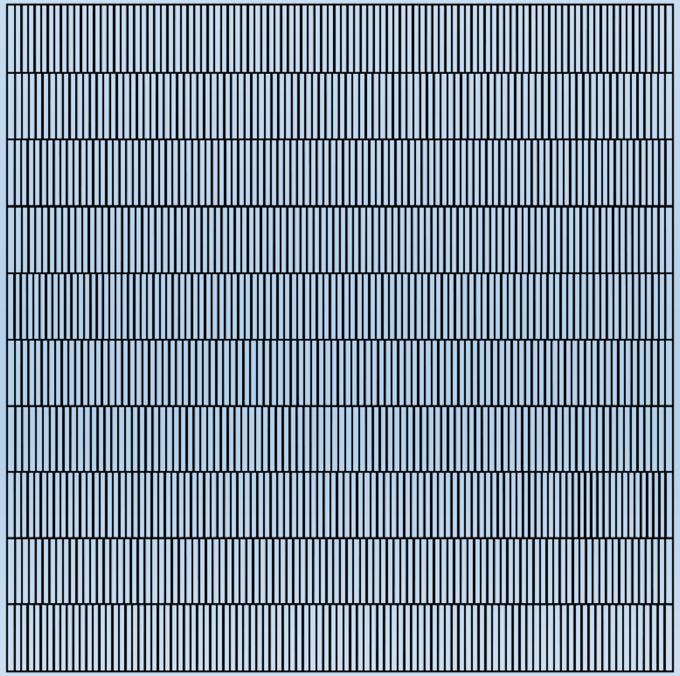
$$f\left(x,y\right)=xy$$

Conclusion

- 1. Contributions

(1) We extend the OVT method for generating anisotropic meshes that the target function is not necessarily convex;

(2) The anisotropy of the resulting power cells conforms to the Hessian of an arbitrarily given function;



$$f(x,y) = 100x^2 + y^2$$

Conclusion

- 1. Contributions

(1) We extend the OVT method for generating anisotropic meshes that the target function is not necessarily convex;

(2) The anisotropy of the resulting power cells conforms to the Hessian of an arbitrarily given function;

(3) A modified Monte Carlo minimization method with a local search strategy is tailored for effective optimization.

Conclusion

- 2. Limitations and future work

(1) High computational cost;

Figure	Site Number	Initialization (second)	Position Optimization (second)	Position-Weight Optimization (second)	Total Time (second)
1(a)	50	0.1	1.6	2.4	4.1
1(b)	200	49	65	72	186
1(c)	800	32	18.8	17.6	68.4
7	2,000	270	213	109	592
8	500	56	27.8	44.8	128.6
9	1,000	57	59.7	43.3	160
10	800	780	246	1,245	2,271
11	500	860	414	283	1,557
12(a)	2,000	1,731	3,615	615	5,961
12(b)	2,000	1,694	3,521	354	5,579
13	1,000	63	67	23	153
14	2,000	98	200	36	334
15	1,000	598	1,028	286	1,912

Conclusion

- 2. Limitations and future work

(2) Cannot directly used for the situation that generating meshes adapted to a given tensor field instead of Hessian matrix;

(3) Integrating more constraints on the geometry shape of cells and apply it to analysis and simulation tasks.

Acknowledgements

- National Natural Science Foundation of China (Nos. 61472332, 61572020, 61728206)
- Natural Science Foundation of Fujian Province of China (No. 2018J01104)
- China Scholarship Council (Nos. 201706315019, 201706315001)
- National Natural Science Foundation of China (No. U1605254)
- PECASE Award N00014-16-1-2254 and NSF CAREER Award OCI-1149591

Thank You