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1

$X \sim Bin(n,p)$

p $\hat{p} = \dfrac{X}{n}$ $E_p[\hat{p}] = \dfrac{E_p[X]}{n} = \dfrac{np}{n} = p$

1.1. $\hat{\theta}$

$$E_\theta[\hat{\theta}(X)] = \theta, \quad \delta\theta$$

$\hat{p} = \dfrac{X}{n}$ UMVU

1.2 (UMVU). $\hat{\theta}^*$

$$Var_\theta[\hat{\theta}^*] = \frac{1}{I_n(\theta)}, \quad \delta\theta$$

$\hat{\theta}* \ UMVU$

$Var_\theta\left[\hat{p}\right] = \dfrac{1}{I_n(\theta)}$

$Var[\hat{p}] = \dfrac{Var[X]}{n^2} = \dfrac{p(1-p)}{n}$

$I_n(\theta)$

1.3 ( ). $\ell'(\theta, X)$ $\ell(\theta, X)$ θ

$$I_n(\theta) = E_\theta\left[(\ell'(\theta, X)^2)\right]$$

$f(x,p) = p^x(1-p)^{n-x}\dbinom{n}{x}$ p

$$\begin{aligned}\ell'(p,x) &= \frac{\partial}{\partial p}\left(x\log p + (n-x)\log(1-p) + \log\binom{n}{x}\right)\\ &= \frac{x}{p} + \frac{-(n-x)}{1-p} = \frac{x-np}{p(1-p)}\end{aligned}$$

$$I(p) = E[\ell'(p,X)^2] = \frac{E[(X-np)^2]}{(p(1-p))^2} = \frac{np(1-p)}{(p(1-p))^2} = \frac{n}{p(1-p)}$$

$\dfrac{1}{Var_p[\hat{p}]}$ $\hat{p}$ $UMVU$

$E[(X-np)^2]$

2

μ $\bar{X}$ UMVU

μ

$$\ell(\mu, x) = -\frac{(x-\mu)^2}{2\sigma^2} - \frac{1}{2}\log(2\pi\sigma^2)$$

μ $\ell'(\mu, x) = \dfrac{x-\mu}{\sigma^2}$

$$I(\mu) = \frac{E[(X-\mu)^2]}{\sigma^4} = \frac{1}{\sigma^2}$$

$\bar{X}$ UMVU

$E[(X-\mu)^2]$