10 29

2024-10-29

1 n-1

1.1

G Helmert

$$G=\begin{bmatrix} \frac{1}{\sqrt{n}} & \frac{1}{\sqrt{n}} & \frac{1}{\sqrt{n}} & \cdots & \frac{1}{\sqrt{n}} \\ \frac{-1}{\sqrt{1\cdot 2}} & \frac{1}{\sqrt{1\cdot 2}} & 0 & \cdots & 0 \\ \frac{-1}{\sqrt{2\cdot 3}} & \frac{-1}{\sqrt{2\cdot 3}} & \frac{2}{\sqrt{2\cdot 3}} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{-1}{\sqrt{(n-1)\cdot n}} & \frac{-1}{\sqrt{(n-1)\cdot n}} & \frac{-1}{\sqrt{(n-1)\cdot n}} & \cdots & \frac{n-1}{\sqrt{(n-1)\cdot n}} \end{bmatrix}$$

( )

$s^2=\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2$ $V^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$

$$\sum_{i=1}^{n}(X_i-\bar{X})^2=\sum_{i=1}^{n}X_i^2-n\bar{X}^2=\sum_{i=1}^{n}Y_i^2-Y_1^2=\sum_{i=2}^{n}Y_i^2$$

Y $\sum_{i=1}^{n}(X_i-\bar{X})^2$
$n-1$ $ns^2,(n-1)V^2$ $n-1$

1.2

G $(\frac{1}{\sqrt{n}}, \dots, \frac{1}{\sqrt{n}})$

g_1 $g_1, \dots, g_n$ R^n $g_2, \dots, g_n$ G

G G $\frac{(1,-1,0,\dots,0)}{\sqrt{2}}$ $\frac{(1,1,-2,0,\dots,0)}{\sqrt{6}}$ k

$$\frac{\left(\overbrace{1, \dots, 1}^{k-1}, -k+1, 0, \dots, 0\right)}{\sqrt{k(k-1)}}$$

G G G Helmert

G

$$\sum_{i=1}^{n} X_i^2 = X^\top X = (GX)^\top GX = Y^\top Y = \sum_{i=1}^{n} Y_i^2$$

$X = (X_1, \dots, X_n)^\top$ $X^\top X = (X_1, \dots, X_n)(X_1, \dots, X_n)^\top = \sum_{i=1}^{n} X_i^2$

$X^\top X = (GX)^\top GX$ G $GG^\top = I_n$ Y

G $Y_1 = \sqrt{n}\bar{X}$ $(\frac{1}{\sqrt{n}}, \dots, \frac{1}{\sqrt{n}})$

$$\sum_{i=1}^{n} (X_i - \bar{X})^2 = \sum_{i=1}^{n} X_i^2 - n\bar{X}^2 = \sum_{i=1}^{n} Y_i^2 - Y_1^2 = \sum_{i=2}^{n} Y_i^2$$

$\sum_{i=1}^{n} (X_i - \bar{X})^2 = \sum_{i=1}^{n} X_i^2 - n\bar{X}^2$ n n

$ns^2 = \sum_{i=1}^{n} (X_i - \bar{X})^2 = \sum_{i=2}^{n} Y_i^2$ $n-1$ (Y
n-1 Y_i n-1)

$Y_2, \dots, Y_n$ $Y_1 = \sqrt{n}\bar{X}$ $ns^2 = \sum_{i=2}^{n} Y_i^2$ $\bar{X}$ $\bar{X}$ s^2

$\sigma^2 \neq 1$ $\frac{ns^2}{\sigma^2} = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{\sigma^2}$ $n-1$