# 10 29

2024-10-29

## n-1

**0.1** (Line)**.** *The equation of any straight line, called a linear equation, can be written as:*

$$y = mx + b$$

See **??**.

$G$ Helmert

$$G = \begin{bmatrix} \frac{1}{\sqrt{n}} & \frac{1}{\sqrt{n}} & \frac{1}{\sqrt{n}} & \cdots & \frac{1}{\sqrt{n}} \\ \frac{1}{\sqrt{1\cdot 2}} & \frac{-1}{\sqrt{1\cdot 2}} & 0 & \cdots & 0 \\ \frac{1}{\sqrt{2\cdot 3}} & \frac{1}{\sqrt{2\cdot 3}} & \frac{-2}{\sqrt{2\cdot 3}} & \cdots & 0 \\ \cdots & \cdots & \cdots & \ddots & \cdots \\ \frac{1}{\sqrt{(n-1)\cdot n}} & \frac{1}{\sqrt{(n-1)\cdot n}} & \frac{1}{\sqrt{(n-1)\cdot n}} & \cdots & \frac{1-n}{\sqrt{(n-1)\cdot n}} \end{bmatrix}$$

( )

$s^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$ $V^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$

$$\sum_{i=1}^{n}(X_i-\bar{X})^2=\sum_{i=1}^{n}X_i^2-n\bar{X}^2=\sum_{i=1}^{n}Y_i^2-Y_1^2=\sum_{i=2}^{n}Y_i^2$$

$Y$ $Y_i$ $\sum_{i=1}^{n}(X_i-\bar{X})^2$ $n-1$
$ns^2,(n-1)V^2$ $n-1$

$G$ $(\frac{1}{\sqrt{n}},\dots,\frac{1}{\sqrt{n}})$

$g_1$ $g_1,\dots,g_n$ $R^n$ $g_2,\dots,g_n$ $G$

$G$ $G$ $\frac{(1,-1,0,\dots,0)}{\sqrt{2}}$ $\frac{(1,1,-2,0,\dots,0)}{\sqrt{6}}$ $k$

$$\frac{\left(\overbrace{1,\dots,1}^{k-1},-k+1,0,\dots,0\right)}{\sqrt{k(k-1)}}$$

$G$ $G$ $G$ Helmert

$G$

$$\sum_{i=1}^{n}X_i^2=X^\top X=(GX)^\top GX=Y^\top Y=\sum_{i=1}^{n}Y_i^2$$

1

$X=(X_1,\dots,X_n)^\top$ $X^\top X=(X_1,\dots,X_n)(X_1,\dots,X_n)^\top=\sum_{i=1}^{n}X_i^2$

2

$X^\top X=(GX)^\top GX$ $G$ $GG^\top=I_n$ $Y$

$G$ $Y_1=\sqrt{n}\bar{X}$ $(\frac{1}{\sqrt{n}},\dots,\frac{1}{\sqrt{n}})$

$$\sum_{i=1}^{n}(X_i-\bar{X})^2=\sum_{i=1}^{n}X_i^2-n\bar{X}^2=\sum_{i=1}^{n}Y_i^2-Y_1^2=\sum_{i=2}^{n}Y_i^2$$

1

$\sum_{i=1}^{n}(X_i-\bar{X})^2=\sum_{i=1}^{n}X_i^2-n\bar{X}^2$

2

$Y_1 = \sqrt{n}\bar{X}$ Helmert $GX = G(X_1, X_2, ..., X_n)^\top = \begin{bmatrix} \frac{1}{\sqrt{n}} & \frac{1}{\sqrt{n}} & \frac{1}{\sqrt{n}} & ... & \frac{1}{\sqrt{n}} \\ \frac{1}{\sqrt{1\cdot 2}} & \frac{-1}{\sqrt{1\cdot 2}} & 0 & ... & 0 \\ \frac{1}{\sqrt{2\cdot 3}} & \frac{1}{\sqrt{2\cdot 3}} & \frac{-2}{\sqrt{2\cdot 3}} & ... & 0 \\ ... & ... & ... & \ddots & ... \\ \frac{1}{\sqrt{(n-1)\cdot n}} & \frac{1}{\sqrt{(n-1)\cdot n}} & \frac{1}{\sqrt{(n-1)\cdot n}} & ... & \frac{1-n}{\sqrt{(n-1)\cdot n}} \end{bmatrix}$

$ns^2 = \sum_{i=1}^n (X_i - \bar{X})^2 = \sum_{i=2}^n Y_i^2$ $n-1$ (Y
$n-1$ $Y_i$ $n-1$ )

$Y_1, Y_2, ..., Y_n$ $Y_2, ..., Y_n$ $Y_1 = \sqrt{n}\bar{X}$ $ns^2 = \sum_{i=2}^n Y_i^2$ $\bar{X}$ $\bar{X}$ $s^2$

$\sigma^2 \neq 1$ $\frac{ns^2}{\sigma^2} = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{\sigma^2}$ $n-1$

$Y_1 = \sqrt{n}\bar{X}$ $Y_1$ $\bar{X}$

$\bar{X} \sim N(0, \frac{1}{n})$ $\mu = 0, \sigma^2 = 1$ $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

$\sigma^2 \neq 1$

$X_1, X_2, ..., X_n$ $\mu = 0, \sigma^2 = 1$

$\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{\sigma^2} = \sum_{i=1}^n (\frac{X_i - \bar{X}}{\sigma})^2 = \sum_{i=1}^n (\frac{X_i - \mu}{\sigma} - \frac{\mu - \bar{X}}{\sigma})^2 = \sum_{i=1}^n (Z_i - \bar{Z})^2$

$\sigma^2$