Heuristics for SAT/MaxSAT

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The SAT Problem

- A set of boolean variables: $X = \{x_1, x_2, ..., x_n\}$.
- Literals: $x_1, \neg x_1, x_2, \neg x_2, ..., x_n, \neg x_n$
- A set of Clauses: $x_1 \vee \neg x_2, x_2 \vee x_3, x_2 \vee \neg x_4, \neg x_1 \vee \neg x_3 \vee x_4, ...$
- A Conjunctive Normal Form (CNF) formula:

$$\varphi = (x_1 \vee \neg x_2) \wedge (x_2 \vee x_3) \wedge (x_2 \vee \neg x_4) \wedge (\neg x_1 \vee \neg x_3 \vee x_4)$$

- The satisfiability problem (SAT): test whether there exists an assignment of truth values to the variables in F under which F evaluates to true.

Importance of SAT

- Theoretical importance
  - the first NP-Complete problem [1971]
- Application everywhere
  - Verification of Win 7
  - Design of Intel Core CPU
  - ...
- Big progress

Solving SAT

Methods for solving SAT

- Complete methods for SAT:
  DPLL $\longrightarrow$ Conflict Driven Clause Learning (CDCL)
- Incomplete methods for SAT: Local search

Local Search for SAT

- Local search starts at some point in the solution space, and moves to adjacent points.
- two assignments are neighbors iff they only differ in one bit.

Algorithm 1: Local Search Framework for SAT

begin
  $s \leftarrow$ a randomly generated truth assignment;
  while *not reach terminal condition* **do**
    if s *satisfies* F **then** return s ;
    pick a variable x;
    $s := s$ with x flipped;
  return "Solution not found";
end

Scoring in Local Search for SAT

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- *make*(x): the number of unsatisfied clauses that would become satisfied by flipping x.
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- $score(x) = make(x) - break(x)$.

Distinguishing Satisfied Clause

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But satisfied clauses are at the same safety level.

Define

A clause is said to be δ-satisfied *if and only if the clause contains* δ *true literals.*

δ-Satisfied Clause

Example:

- Given an assignment $s = \{x_1 = 1, x_2 = 1, x_3 = 0, x_4 = 1, x_5 = 1\}$
- $c_1 = x_1 \vee x_2 \vee \neg x_3 \vee x_4 \vee \neg x_5$
- $c_2 = x_1 \vee \neg x_2 \vee x_3 \vee \neg x_4 \vee \neg x_5$

- c_1 is a 4-satisfied clause, while c_2 has 1-satisfied.
- If variable x_1 is flipped (from 1 to 0), then c_1 is still satisfied, while c_2 is unsatisfied.

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Remark:
1-satisfied clauses are the most dangerous satisfied clauses.

Second Level Scoring

- $make_2(x)$ is the number of 1-satisfied clauses that would become 2-satisfied by flipping x.
- $break_2(x)$ is the number of 2-satisfied clauses that would become 1-satisfied by flipping x.
- $score_2(x) = make_2(x)$ - $break_2(x)$.

Use Second Level Score

Our first try:

- Break ties by $score_2$
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Main Track: Random SAT

Rank	RiG	Solver	#solved	%solved	cum. run-time	median run-time
–	–	Virtual Best Solver (VBS)	558	93.0	72841	39.2
1	1	CCASat	423	70.5	76206	218.8
2	1	SATzilla2012 RAND	321	53.5	80796	714.4
3	2	SATzilla2012 ALL	306	51.0	83273	845.6
–	–	Sparrow2011 (SAT Competition 2011 Gold) (REFERENCE)	303	50.5	76396	876.1
–	–	EagleUP (SAT Competition 2011 Bronze) (REFERENCE)	283	47.2	83787	900.0

Use Second Level Score

More results:

- Combine $score_2$ and $score$, allowing to overwrite priority:
  $cscore(x) = score(x) + score_2(x)/d$
- Hybrid function:
  $hscore(x) = score(x) + score_2(x)/d + age(x)/\beta$

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CSCCSat won 2nd Place Award of Random SAT Track in SAT Competition 2016, and is the best on solving k-SAT with $k > 3$.

Not suitable for short clauses formulas

Theorem

For a random 3-SAT formula $F(n, m)$, under any solution s to the formula, the number of 1-satisfied clauses is more than $m/2$.

In order to satisfy the formula, most clauses should be dangerous!
$\rightarrow$ Should not encourage them to become 2-satisfied.

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Reference:
Shaowei Cai, Kaile Su: Local Search for Boolean Satisfiability with Configuration Checking and Subscore, Artificial Intelligence 2013.

MaxSAT Problems

Variants of MaxSAT

- MaxSAT: find an assignment that satisfies the most clauses.
- Partial MaxSAT: clauses are divided into hard and soft clauses, the goal is to satisfy all hard clauses, and as many soft clauses as possible.
- Weighted Partial MaxSAT

Solving MaxSAT

Popular methods for MaxSAT

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  - Branch and Bound
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Over the past decade, perhaps the most successful approach to solving industrial instances of MaxSAT is the SAT-based approach, which relies on iteratively calling a SAT solver.

Methods for MaxSAT

- For many large industrial instances, because of the NP-hardness and the time constraint, we do not expect to solve them exactly.
- Goal: find a good-quality solution in a reasonable amount of time.
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Local search has shown their great success in random and crafted instances.
However, local search has poor performance on industrial instances. (Maybe just unsuitable...)

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- What is wrong with local search?
  $\leftarrow$ Not consider the relationship among variables.

Apply Unit propagation to MaxSAT

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  - What if we have many different reasoning chains? (Not many decision variables)
  - What if we have chance to correct it?

From UP decimation to local search and back

A novel approach to combine reasoning and local search:

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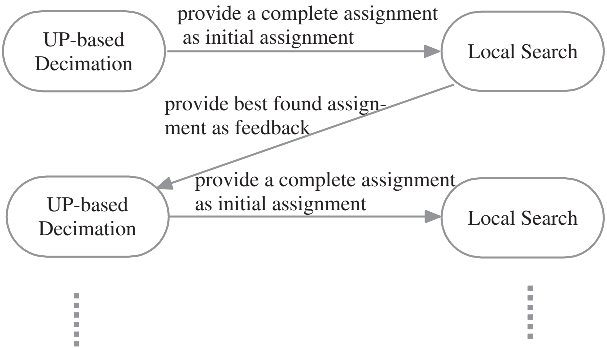

Results

Unweighted Max-SAT - Industrial

Solver	#Ins.	CnC-LS	dsat-wpm3-in-ms	WPM3-2015-in
ial/circuit-debugging-problems	3	2.76(2)	10.50(3)	3.68(3)
sean-safarpour	52	51.05 (45)	43.30(37)	24.18(35)
Total	55	47	40	38

Table: Results on UNSAT benchmarks from SAT Competition 2016.

Benchmark	#inst.	*DeciLS* #win.	*CCLS* #win.	*WPM3-2015-in* #win.	*MiniWalk* #win.	*dsat-wpm3-in-ms* #win.
App_Unknown	103	**80**	8	1	39	1
App_Unsat	109	**99**	22	14	34	12

Extension to (W)PMS

Table: Experimental Results on the MSE2016 PMS benchmarks.

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Still much worse than SAT-based solvers on (W)PMS industrial instances.
DeciDist vs. WPM3-2015-in: 221 vs 533.

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Future works:

- Apply second level score for other combinatorial problems with long clauses.
- Exploit more advanced reasoning in the new approach for Partial MaxSAT.