Workshop on Hard Computational Problems: Representations, Algorithms and Applications

特殊二部图及其算法

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报告提纲

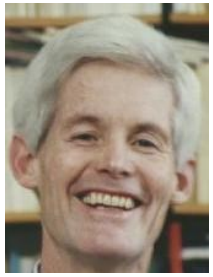
- 为什么研究特殊二部图？
  - NP问题、NP完全问题、如何对付NP完全问题
  - 有哪些特殊二部图？有哪些NP完全问题可以研究？
- 已经取得的结果
  - 树凸二部图（星/梳/三岔凸二部图）、圈凸二部图
  - 反馈顶点集、独立支配集、联通支配集、.....
- 未来的研究问题
  - 组合刻画、判定算法、算法问题、随机二部图、应用

NP问题

- P = 确定型多项式时间
  - 求出问题的解很容易
  - 很容易 = 多项式时间 （P = Polynomial-time）
- NP = 非确定型多项式时间
  - 验证问题的解很容易（ N = Nondeterministic）
  - 可以穷举求解（最坏情况下花费指数时间）
- P v NP 问题： P=NP？ （能避免穷举吗？）

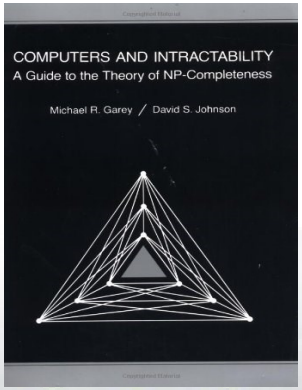
NP完全问题

- NP问题中最难的一类
  - NP中的任何问题都可以归约到NP完全问题上
  - 只要有一个NP完全问题有多项式时间算法，则所有NP问题都有多项式时间算法



- 1970年，Cook-Levin定理
- 1972年，Karp的21个NP完全问题

- 1979年，Garey和Johnson的书



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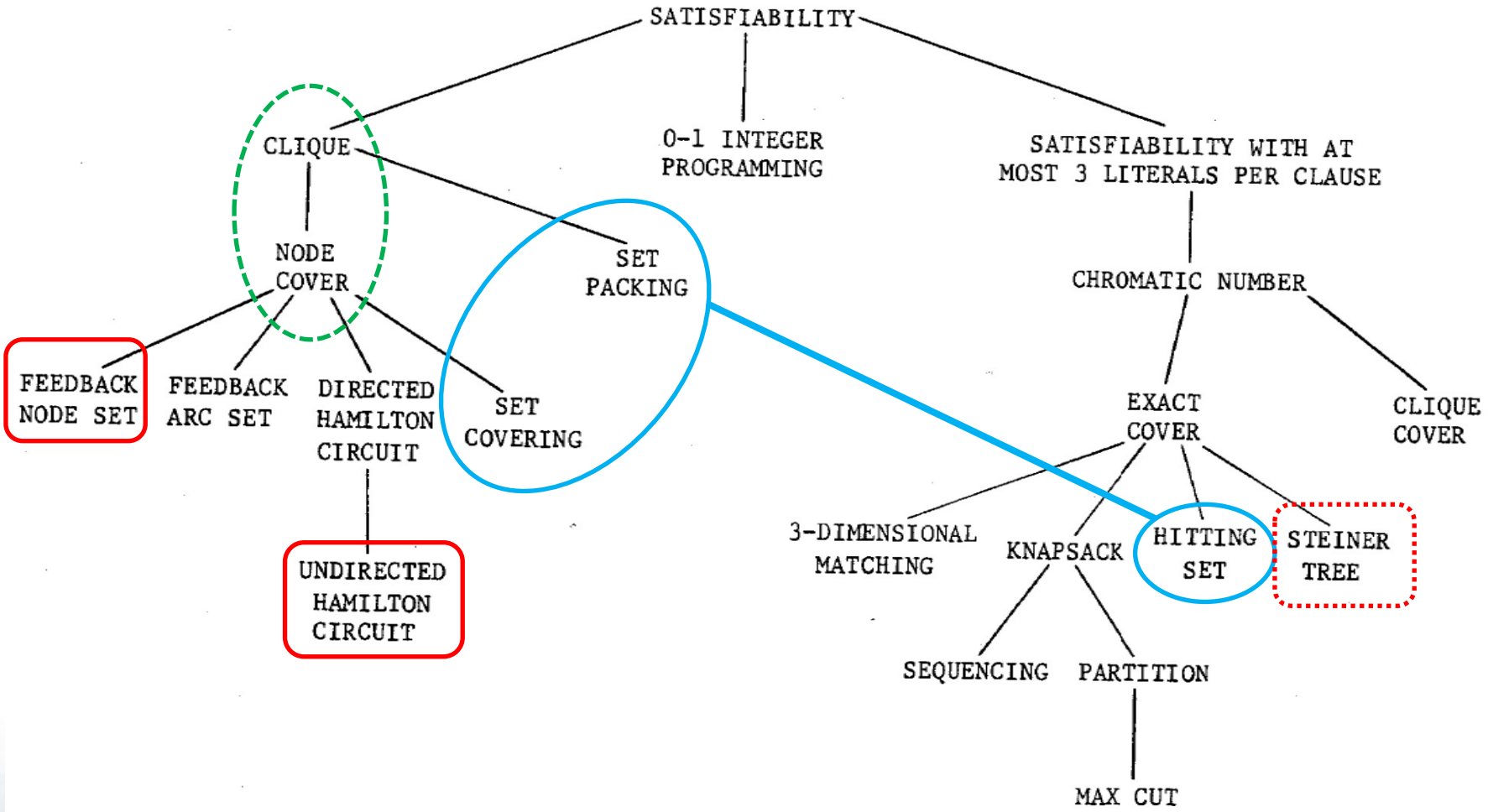
Karp的21个NP完全问题



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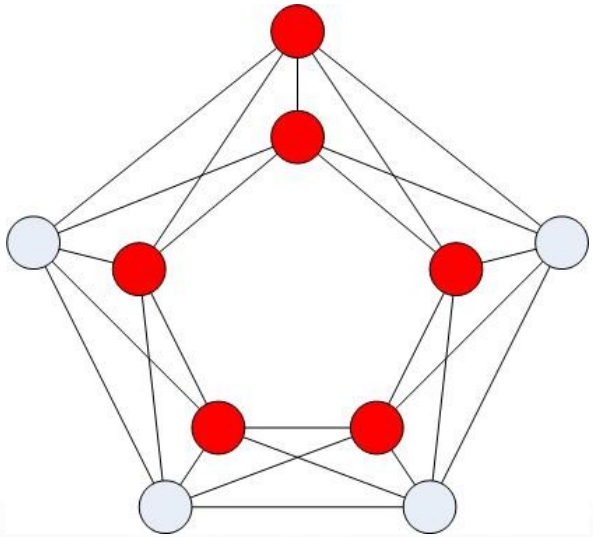
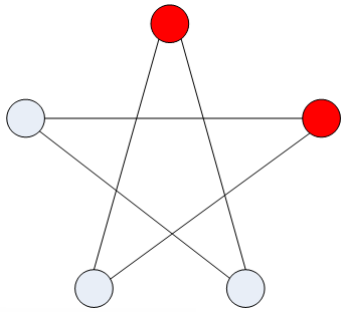
顶点反馈集

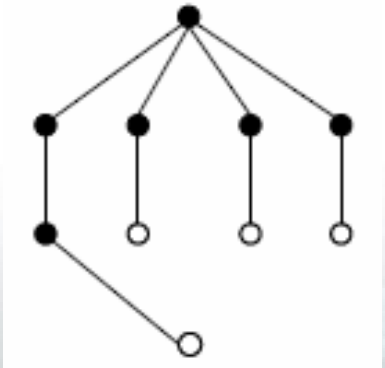
- 在无向图中删除一组顶点，使得剩余的图没有回路
- 应用：
  - 死锁预防与解除
  - 约束满足问题的后门变元
  - 生物信息学
  - 信息安全



独立/连通支配集

- 支配集：一组顶点，使得剩余的顶点都至少与该组顶点中的一个相邻
  - 独立集：一组彼此不相邻的顶点
  - 联通：有通路相连
- 独立支配集
  - 应用：聚类算法第一阶段
- 联通支配集
  - 应用：自组网路由



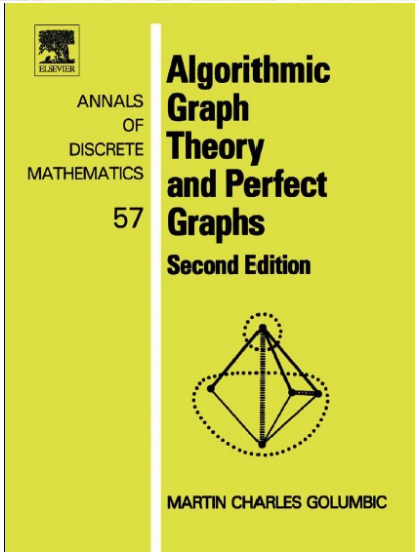


对付NP完全问题

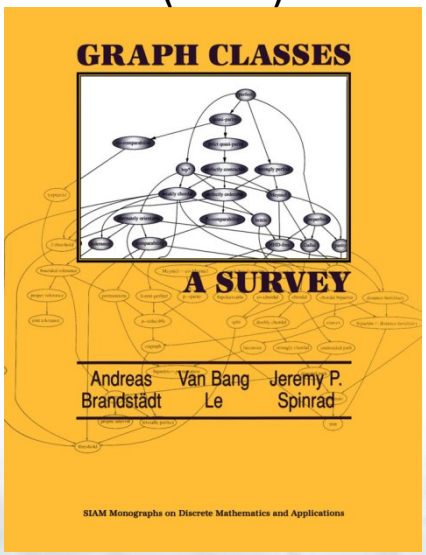
- 限制（Restriction）：二分定理（要么NP完全、要么P）
- 近似（Approximation）：PCP定理（难近似性）
- 参数化（Parameterized）：W层次、核（固定参数易解性）
- 指数时间精确算法（Exponential-time Exact Algorithm）：

  指数时间假设（Exponential Time Hypothesis，ETH）
- 平均（Average-case）：相变现象（随机实例）
- 启发式方法（Heuristics）：算法竞赛
- 量子（Quantum）：相对化结果（负面证据）

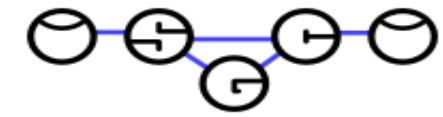
特殊图类:结构与算法



(1980)



(1999)
Almost **200** classes



Information System on Graph Classes and their Inclusions

- ISGCI home
- The Java application
- All classes
- References
- Smallgraphs
- About ISGCI
- Screenshots
- News
- FAQ
- Contact ✉
- Impressum

Find class

What is ISGCI?

ISGCI is an encyclopaedia of graphclasses with an accompanying java application that helps you to research what's known about particular graph classes. You can:

- check the relation between graph classes and get a witness for the result
- draw clear inclusion diagrams
- colour these diagrams according to the complexity of selected problems
- find the P/NP boundary for a problem
- save your diagrams as Postscript, GraphML or SVG files
- find references on classes, inclusions and algorithms

Database contents

1600 classes
217586 inclusions
26190 complexities
46905 bounds
updated 2016-07-07

Classic classes

Meyniel
P_4-bipartite

Classes by definition

All classes
Chords & chordality

http://www.graphclasses.org/index.html

More than **1600** classes

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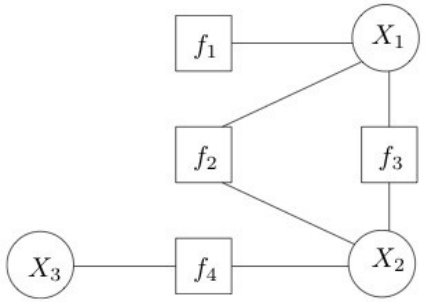
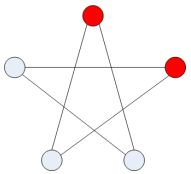
从布尔公式到图



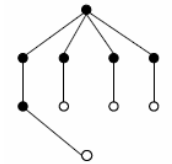
- 特殊图：树、平面图、二部图、……
- 二部图
  - 理论上有好结果：König定理、Hell-Nesetril、……
  - 应用上有广泛用处：因子图 (factor graph)
  - 数学上有丰富结构：
    - 凸二部图(convex bipartite graph)
    - 树凸二部图(tree convex bipartite graph)
    - 圈凸二部图(circular convex bipartite graph)
    - 弦二部图(chordal bipartite graph)

凸二部图

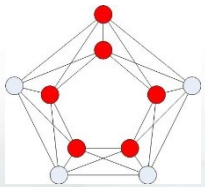
- G=(A,B,E), (A,<), ∀b, N(b)={a|aEb}是区间
  - Grover, F.: Maximum matching in a convex bipartite graph. Nav. Res. Logist. Q. 14, 313–316 (1967).



Independent Domination

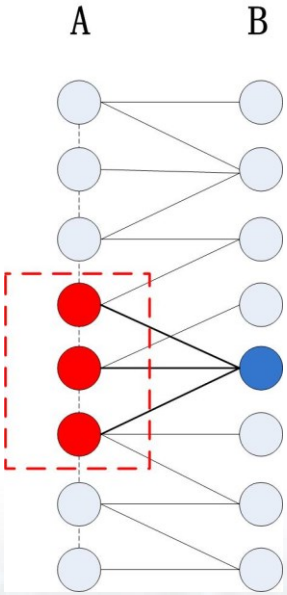


Connected Domination



Feedback vertex set

P



圈凸二部图

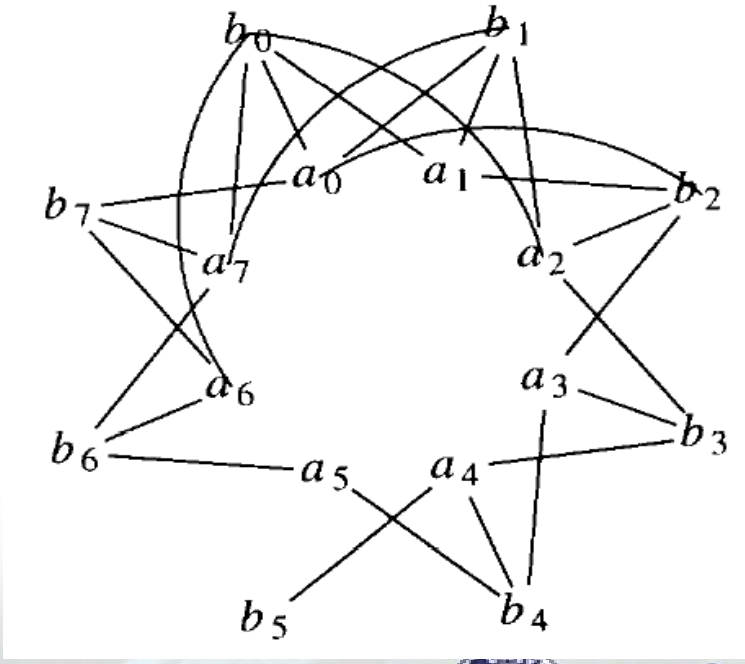
- Liang, Y.D., Blum, N.: Circular convex bipartite graphs: Maximum matching and Hamiltonian circuits. Inf. Process. Lett. 56, 215–219 (1995).

Independent Domination

Connected Domination

Feedback vertex set

P



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树凸二部图

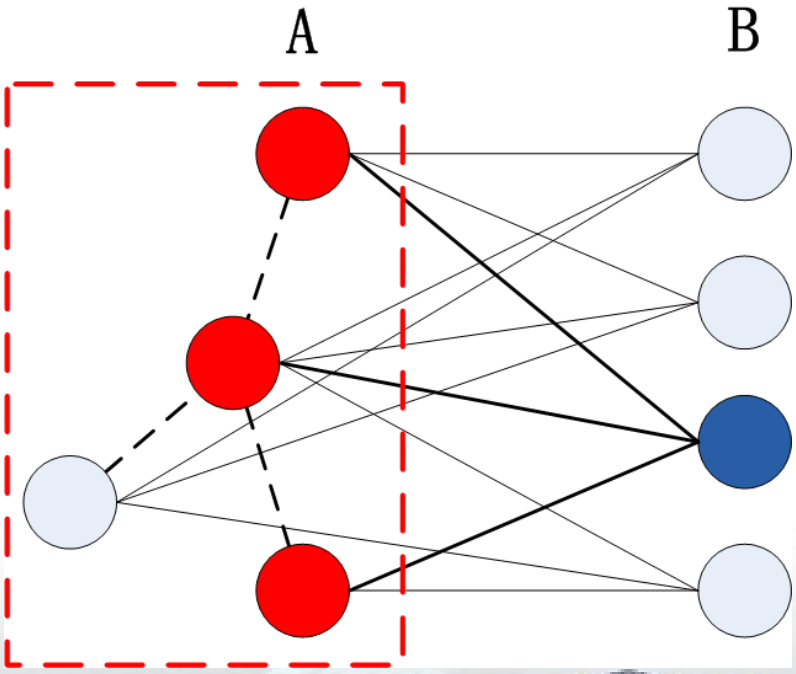
- Jiang, W., Liu, T., Ren, T., Xu, K.: Two Hardness Results on Feedback Vertex Sets. FAW-AAIM, 233-243, (2011).

Independent Domination

Connected Domination

Feedback vertex set

NPC



星凸二部图

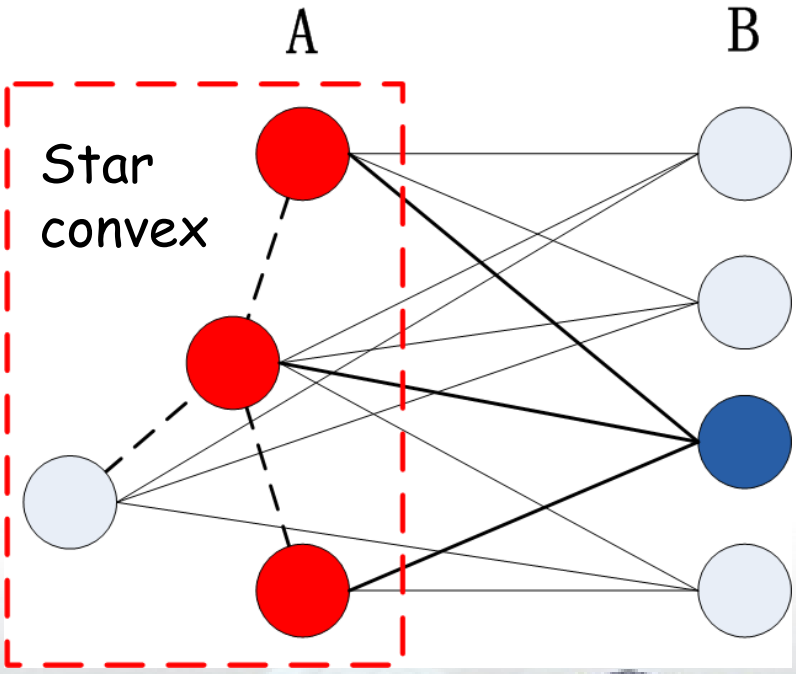
- Jiang, W., Liu, T., Ren, T., Xu, K.: Two Hardness Results on Feedback Vertex Sets. FAW-AAIM, 233-243, (2011).

Independent Domination

Connected Domination

Feedback vertex set

NPC



梳凸二部图

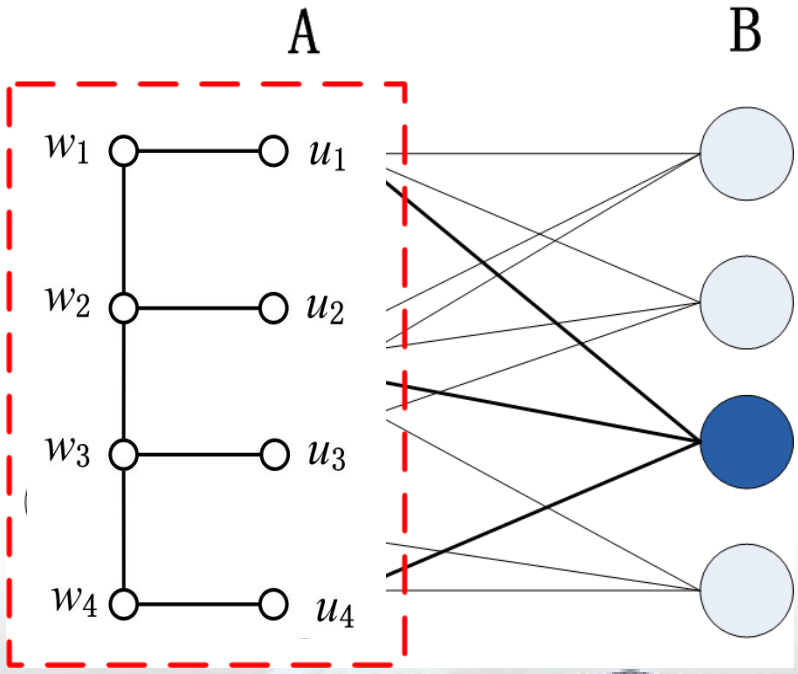
- Jiang, W., Liu, T., Wang, C., Xu, K.: Feedback Vertex Sets on Restricted Bipartite Graphs. *Theoretical Computer Science*, 507:41-51 (2013)

Independent Domination

Connected Domination

Feedback vertex set

NPC



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三岔凸二部图

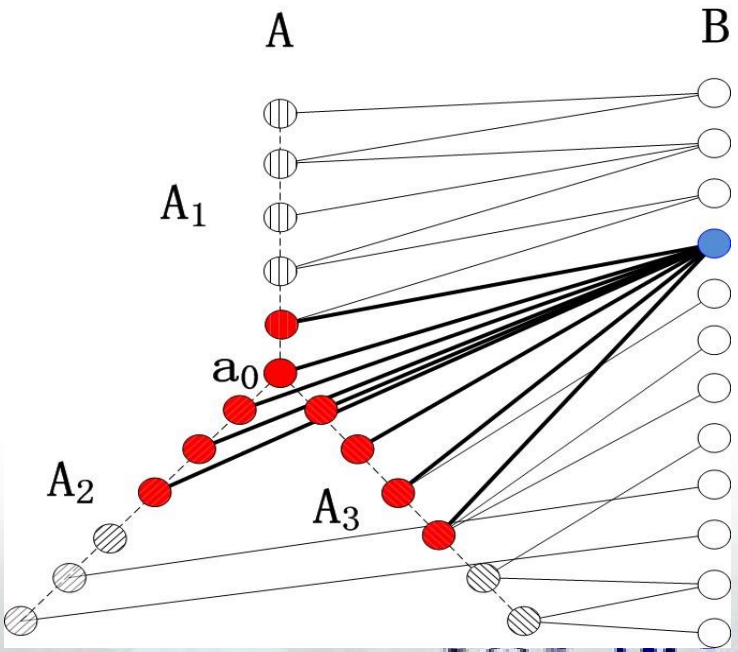
- Jiang, W., Liu, T., Xu, K.: Tractable Feedback Vertex Sets in Restricted Bipartite Graphs. *COCOA*, 424-434, (2011)
  - 三岔图是有一个公共端点的三条路径.

Independent Domination

Connected Domination

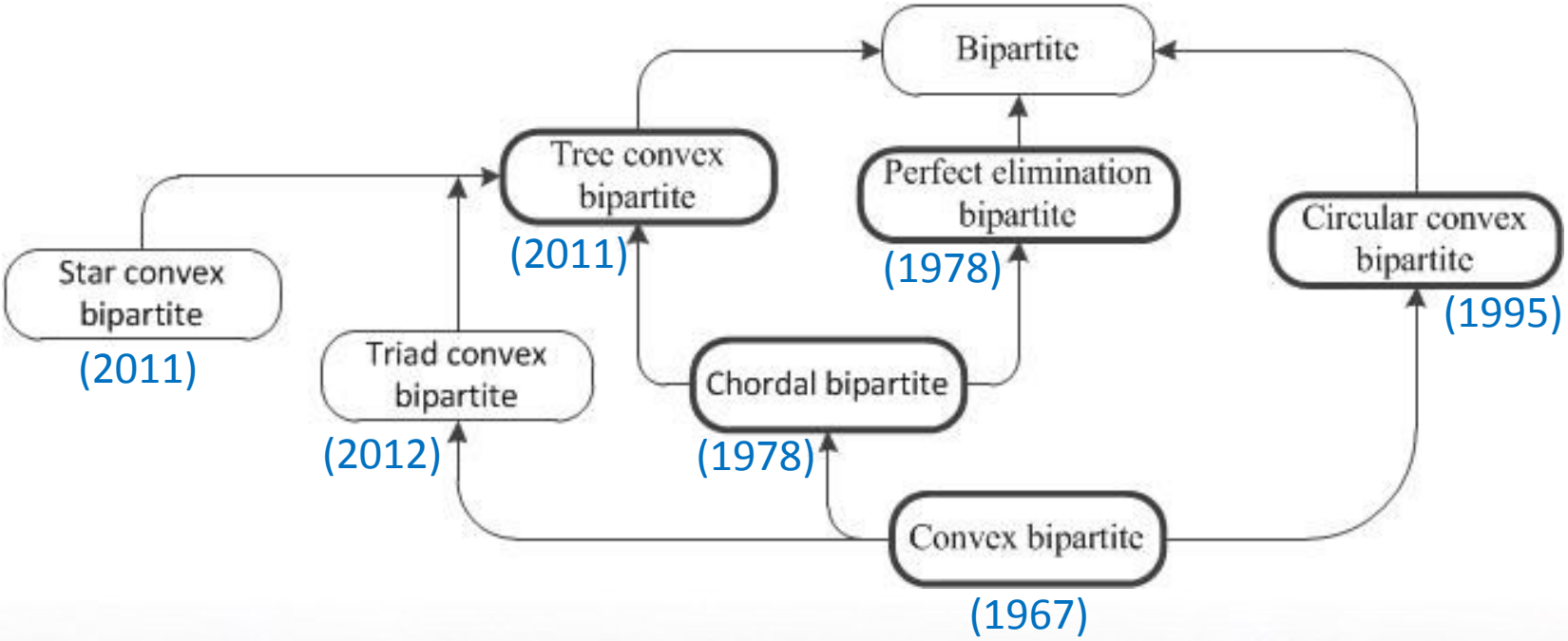
Feedback vertex set

P



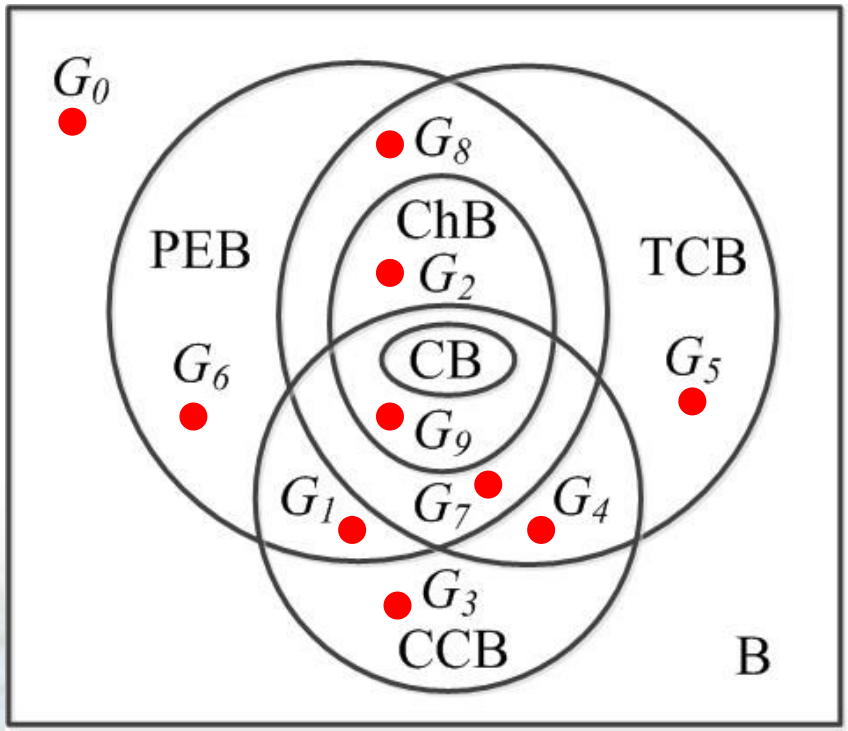
Restricted Bipartite Graphs

- Inclusion



Restricted Bipartite Graphs

- Separation
  - Liu, T.: Restricted bipartite graphs: comparison and hardness results, AAIM 2014, 241-252.

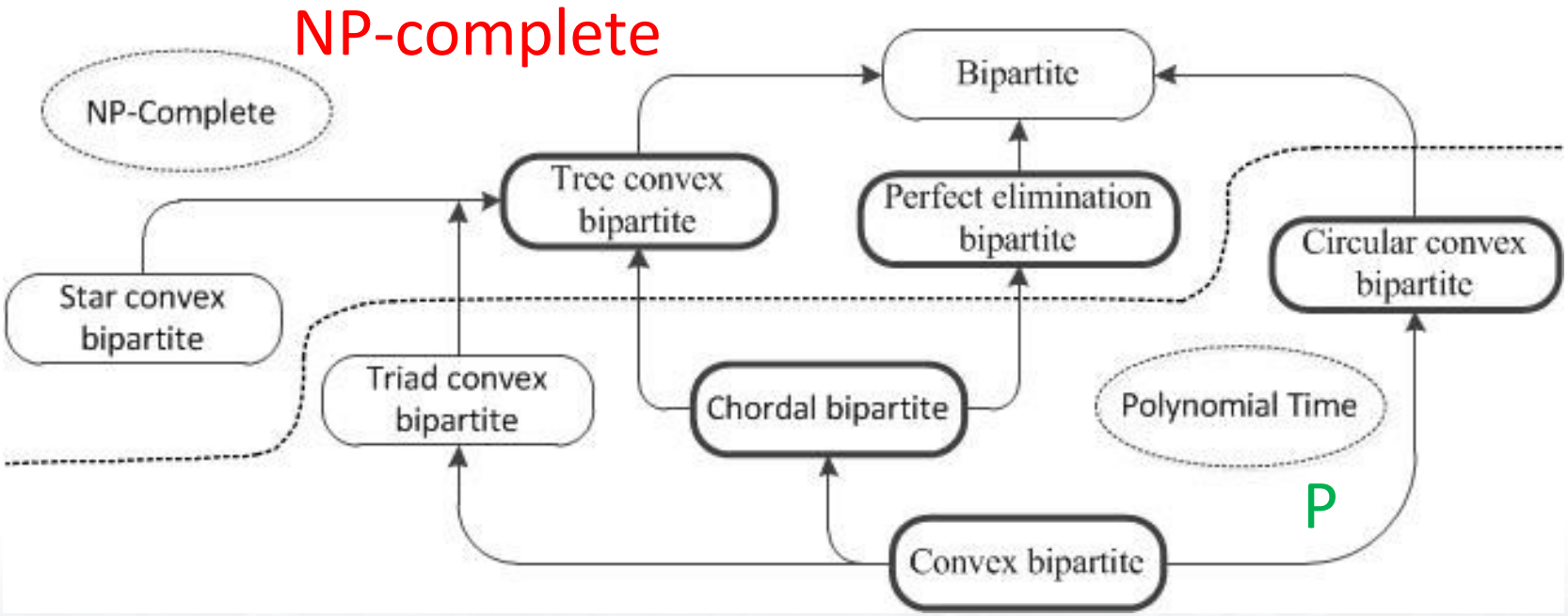


PEB: Perfect Elimination Bipartite
CCB: Circular Convex Bipartite
TCB: Tree Convex Bipartite
ChB: Chordal Bipartite
CB: Convex Bipartite

B: Bipartite

Computational Complexity

- Feedback vertex sets, independent/connected dominating sets, tree-width, hamiltonicity, etc.

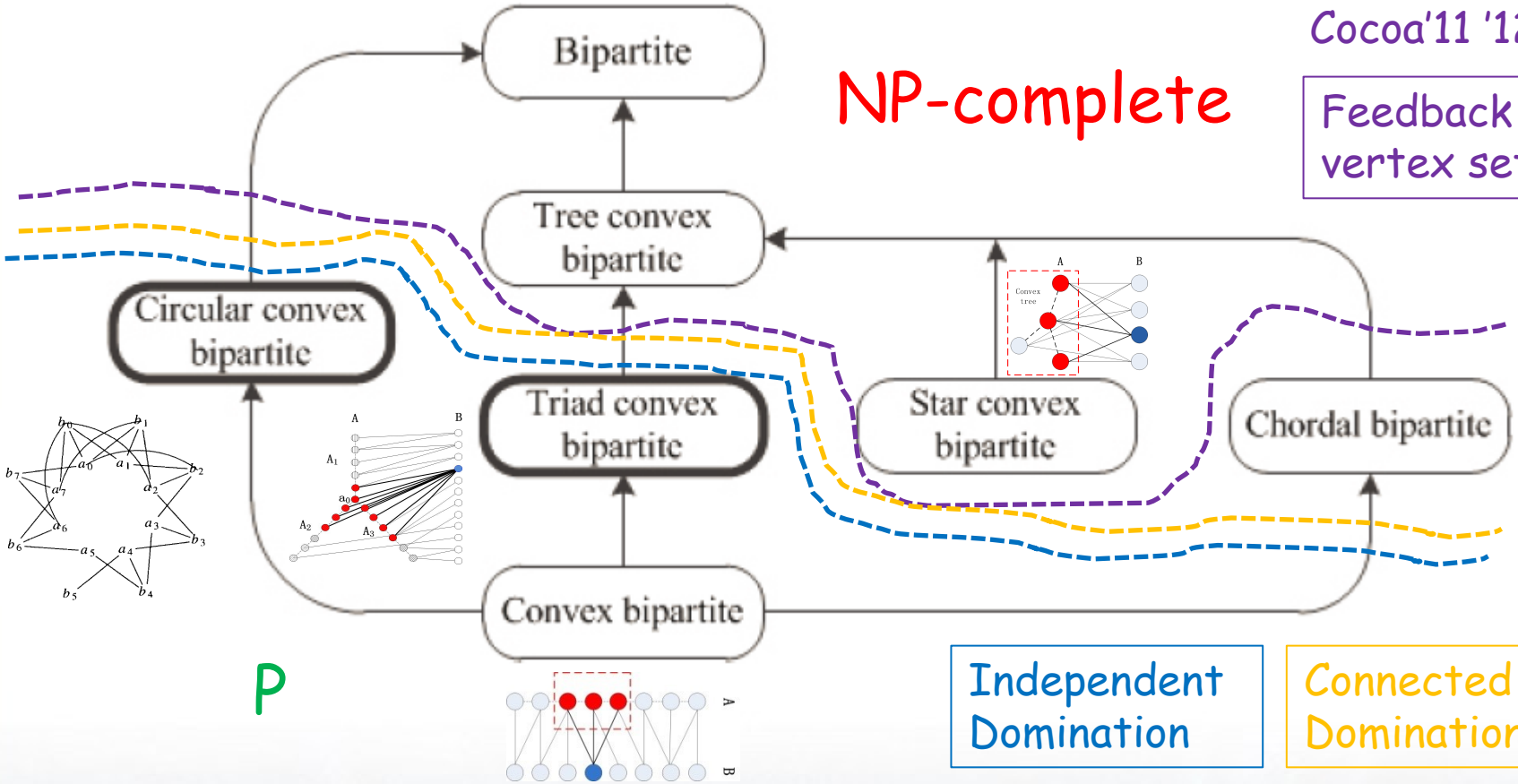


P与NPC的边界

Faw-aaim'11
Cocoa'11 '12 '13

NP-complete

Feedback vertex set



P

Independent Domination

Connected Domination

Faw-aaim'12 '13

Cocoon'13

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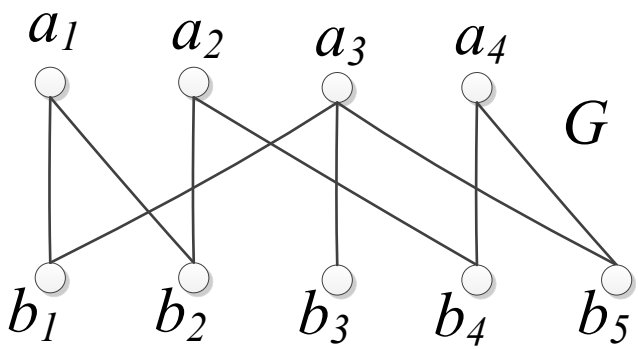
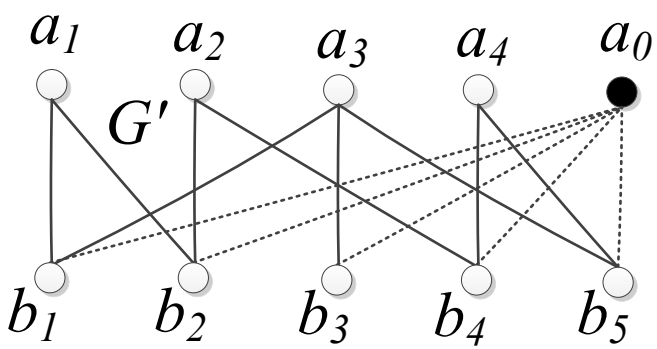
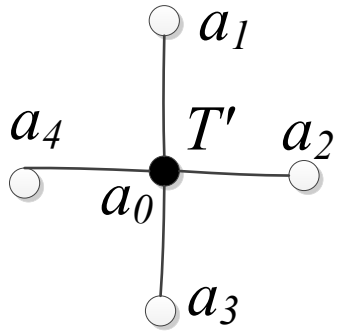
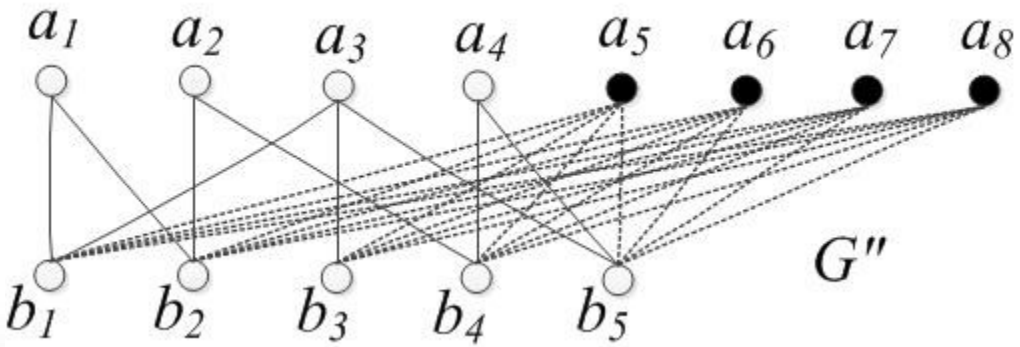
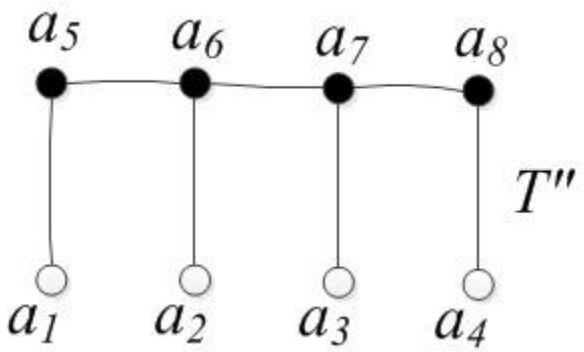
二部图归约到星/梳凸二部图











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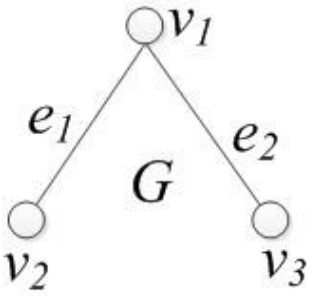
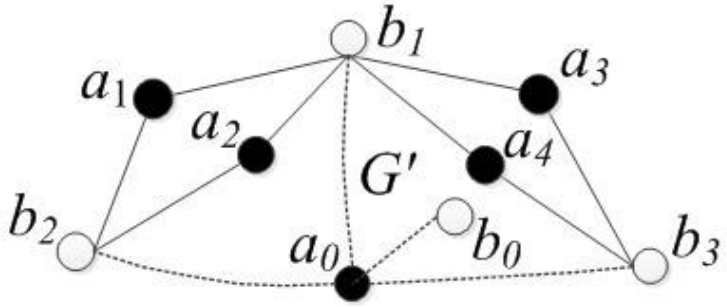
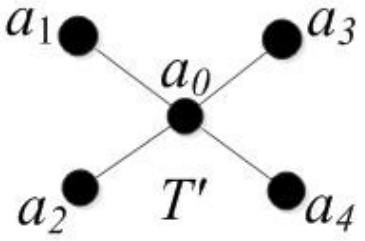
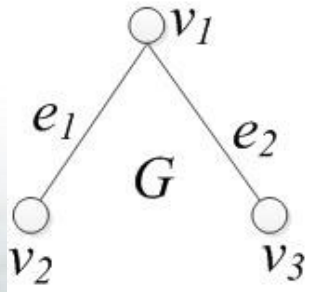
顶点覆盖归约到独立支配集

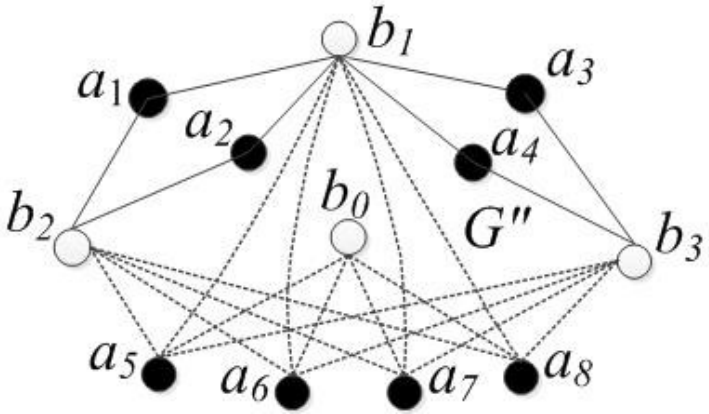
- G’是星凸二部图；G”是梳凸二部图
- G有k大小顶点覆盖 ⇔ G’有k+1大小独立支配集
- G有k大小顶点覆盖 ⇔ G”有k+1大小独立支配集

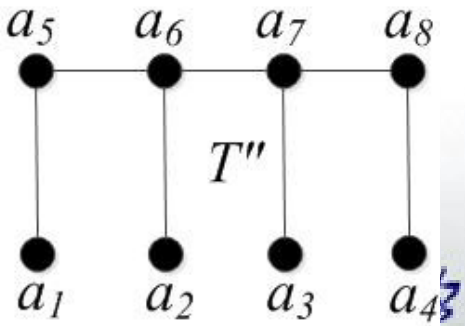






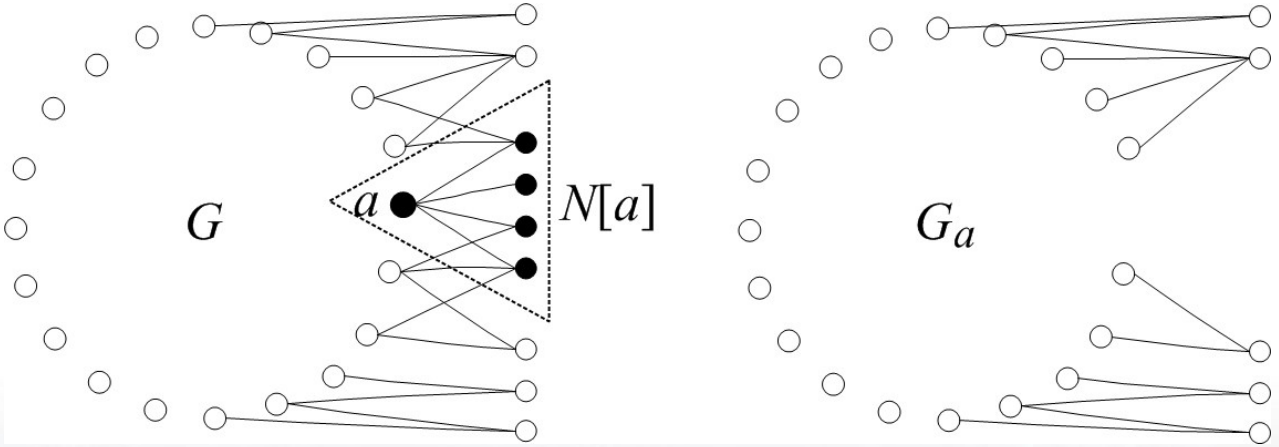






圈凸二部图归约到凸二部图

- 以独立支配集为例
  - D是独立支配集 $\Rightarrow \forall a,\ a\in D$ 或 $N(a)\cap D\neq\varnothing$
  - $S = \{B\} \cup \{D_a \mid D_a$是$G_a$的最小独立支配集, $a\in A\}$
  - S 多项式时间可计算且包含G的最小独立支配集



圈凸二部图上顶点反馈集

- 求最小顶点反馈集 = 求最大无回集(MCFS)
- $S = S_1 \cup S_2 \cup S_3 \cup S_4$

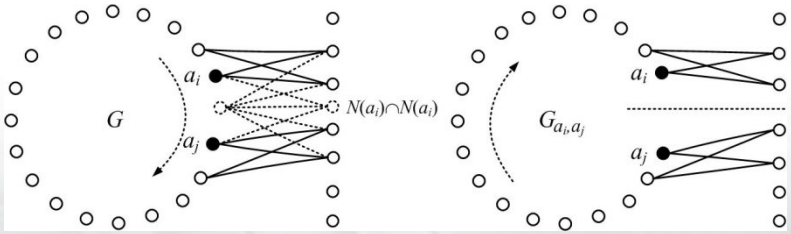
$$\mathcal{S}_1 = \{\{a\} \cup B \mid a \in A\}.$$

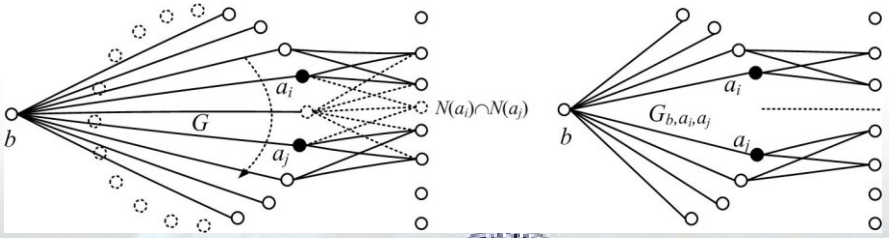
$$\mathcal{S}_2 = \{\{a', a'', b\} \cup B \setminus (N(a') \cap N(a'')) \mid a', a'' \in A, b \in N(a') \cap N(a'')\} \cup \{\{a', a''\} \cup B \mid a', a'' \in A \text{ and } N(a') \cap N(a'') = \emptyset\}.$$

$$\mathcal{S}_3 = \{F_{a_i,a_j} \mid a_i, a_j \in A \text{ and } F_{a_i,a_j} \text{ is a MCFS of } G_{a_i,a_j}\}.$$

$$\mathcal{S}_4 = \{F_{b,a_i,a_j} \mid b \in B, a_i, a_j \in N(b) \text{ and } F_{b,a_i,a_j} \text{ is a MCFS of } G_{b,a_i,a_j}\}.$$

– S 多项式时间可计算且包含G的最大无回集





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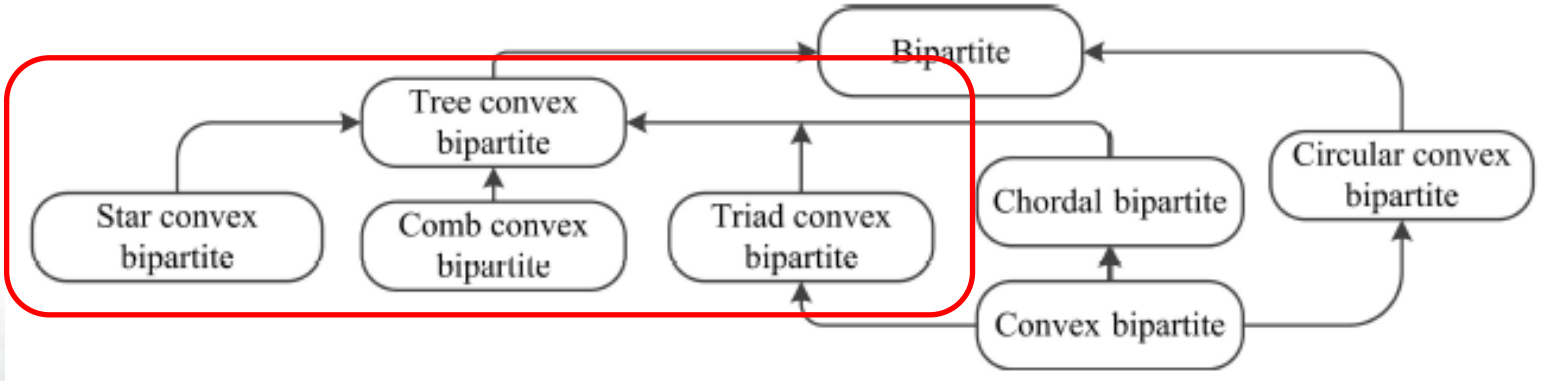
已有的结果

Graph classes	FVS	DS	CDS	IDS	PDS	TDS	HC	HP	TW
Bipartite	N[29]	N[4]	N[26]	N[8]	N[2]	N[26]	N[17]	N[17]	N[1, 14]
Star conv. b.	N[9, 10]	N[⋆]	N[⋆]	N[27]	N[⋆]	N[⋆]	N[⋆]	N[⋆]	N[⋆]
Comb conv. b.	N[28, 10]	N[⋆]	N[⋆]	N[27]	N[⋆]	N[⋆]	N[⋆]	N[⋆]	O
Chordal b.	P[15]	N[24]	N[24]	N[3]	N[25]	P[3]	N[23]	N[23]	P[14]
Triad conv. b.	P[11, 10]	O	P[21]	P[27, 20]	O	O	O	O	O
Circular conv. b.	P[22]	O	P[21]	P[20]	O	O	P[18]	O	O
Convex b.	P[19]	P[3]	P[3]	P[3]	P[3, 7]	P[3]	P[18]	O	P[14]

(N: $\mathcal{NP}$-complete, P: Polynomial time, O: Open, ⋆: This paper)



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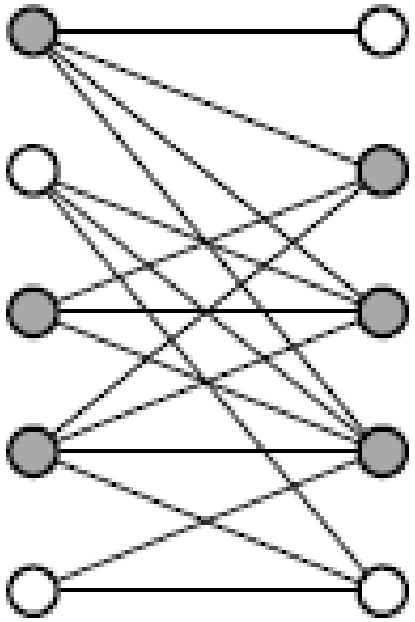
A Recent Result

- FAW 2014, 2017-6-30, Chengdu
- Maximum Edge Biclique

Maximum Edge Bicliques

- Biclique is an induced complete bipartite subgraph G[R,S]
- Max |R|×|S| (instead of |R|+|S|)

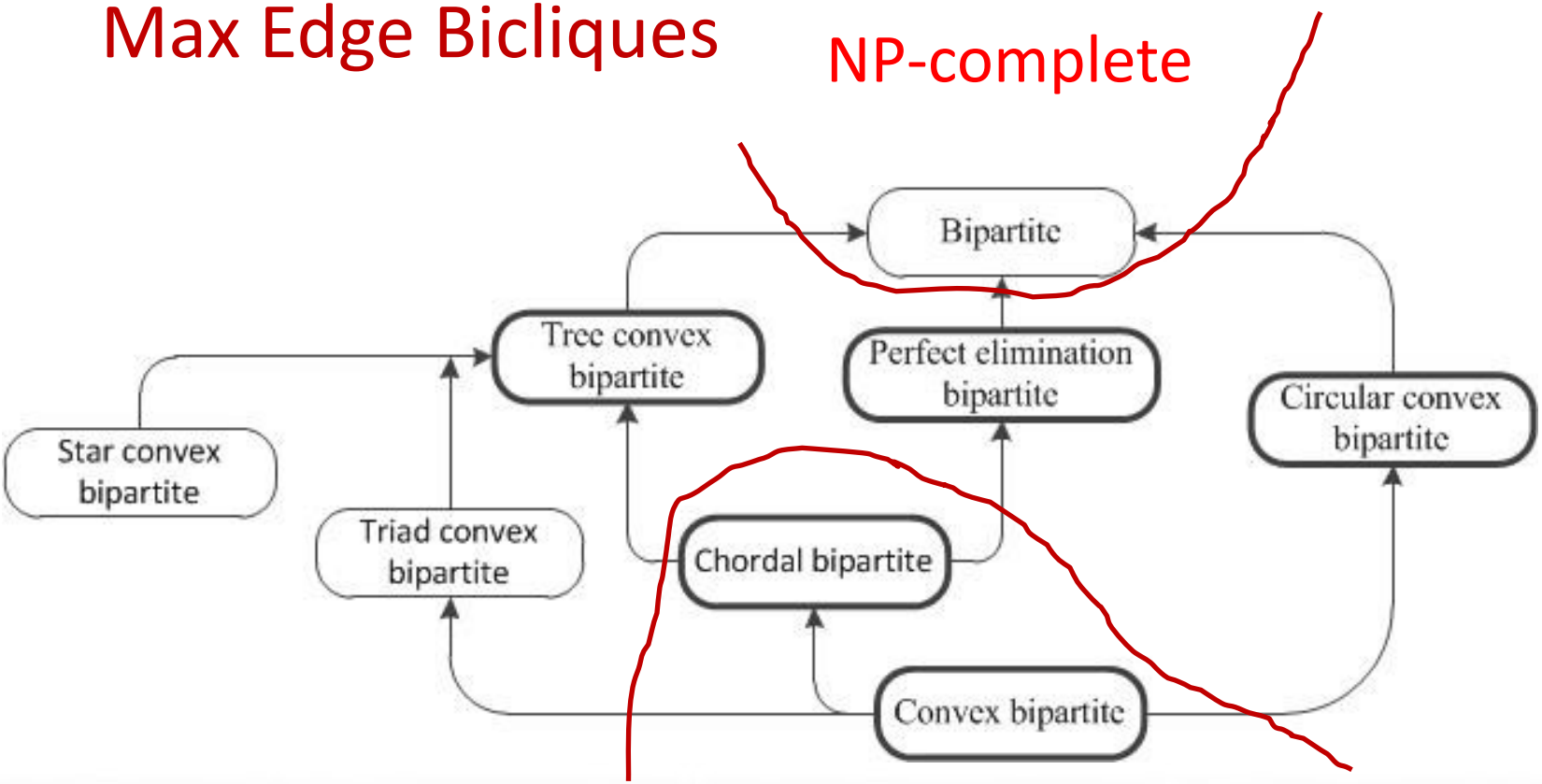


- Dawande, M., Keskinocak, P., Swaminathan, J.M., Tayur, S.: On bipartite and multipartite clique problems. J. Algorithms 41, 388–403 (2001)

Known Results

Max Edge Bicliques

NP-complete

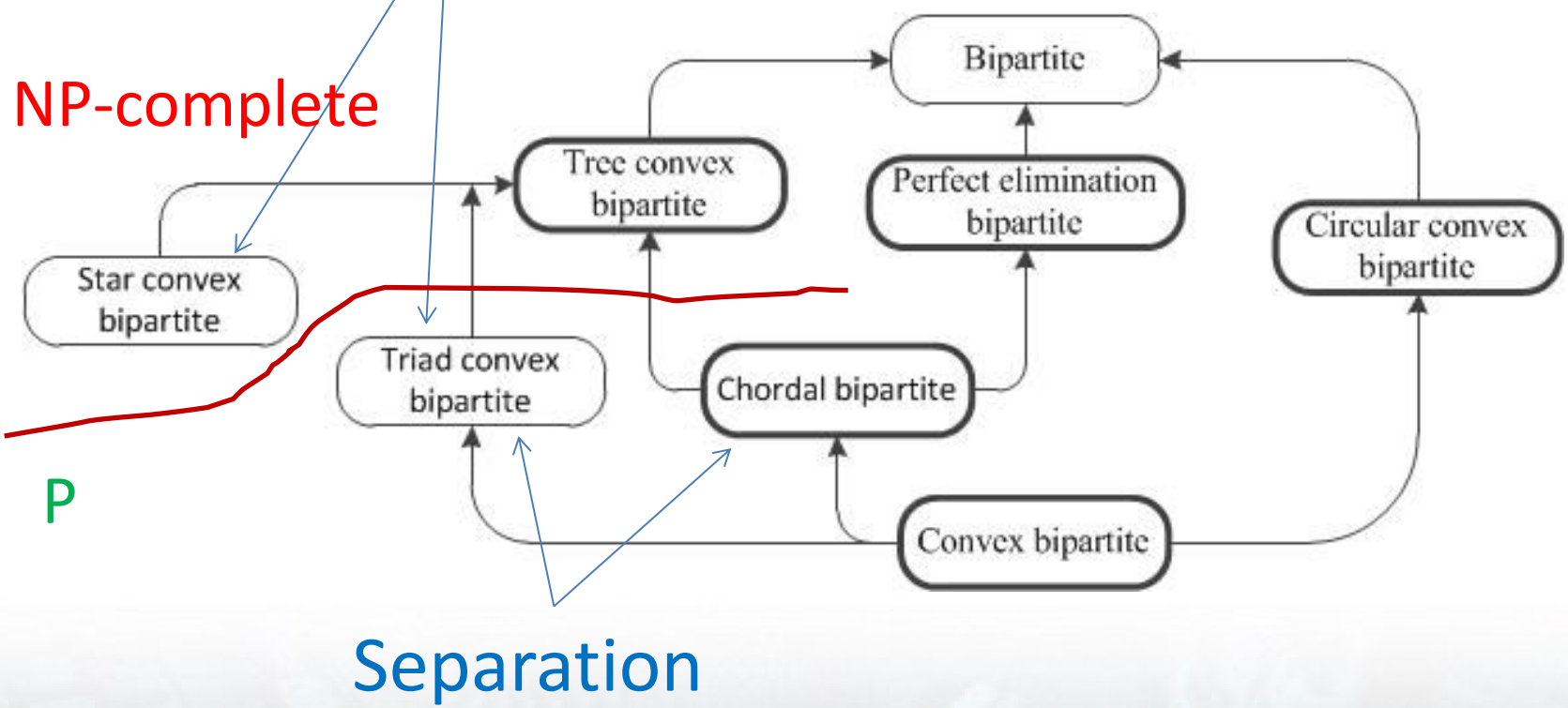


P

Our Results

Max Edge Bicliques

NP-complete



P

Separation

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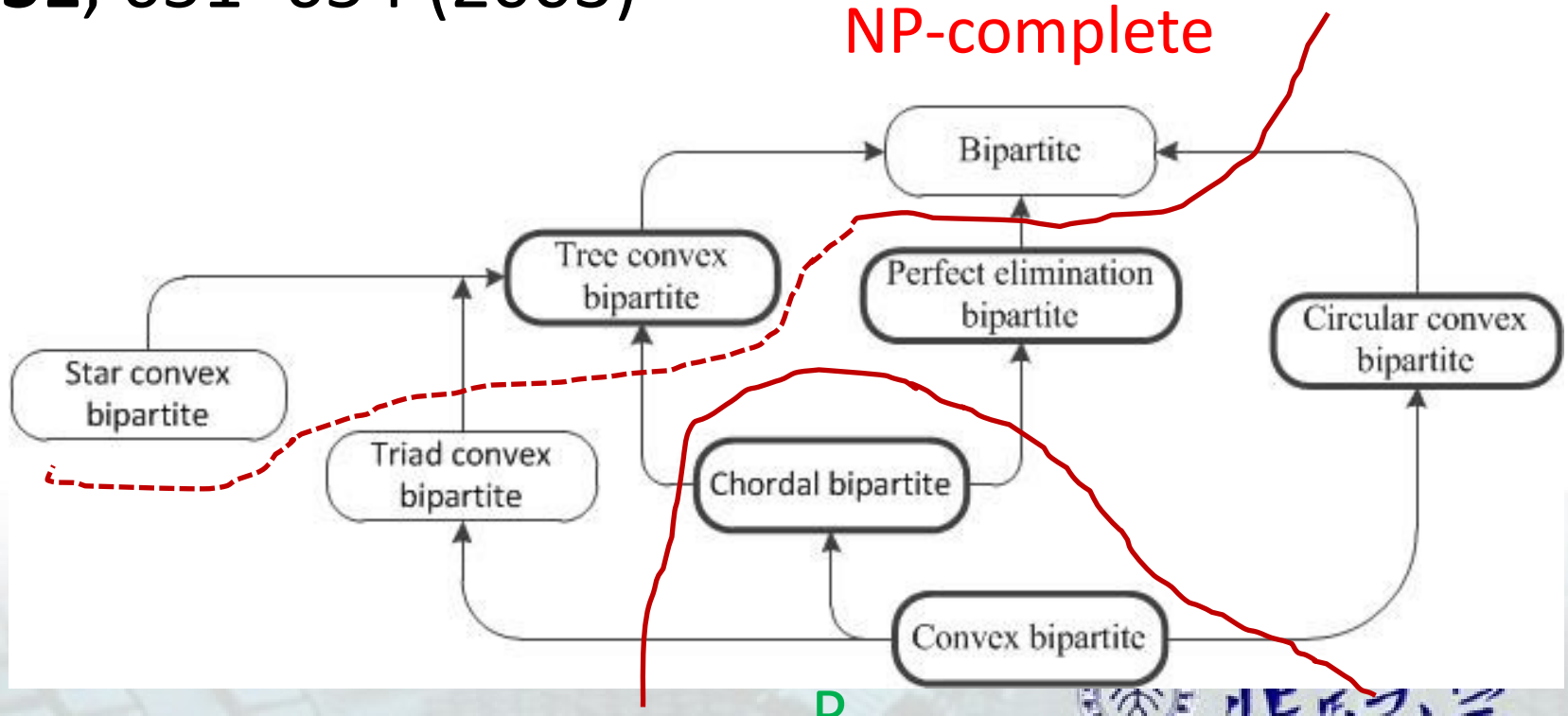
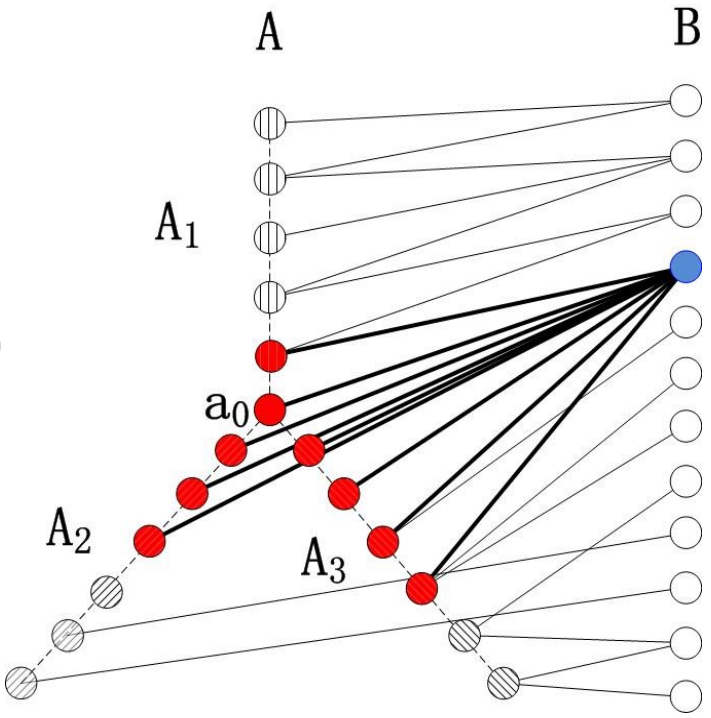
NP-Completeness

- Peeters, R.: The maximum edge biclique problem is NP-complete. Discret. Appl. Math. **131**, 651–654 (2003)



Tractability

- For a biclique G[R,S] in a tree convex bipartite graph, T[R] must be a subtree in T
- For a triad, #(subtrees)=$O(n^3)$
- Enumeration



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An Observation

- For a biclique G[R,S] in a circular convex bipartite graph, T[R] may not be an interval

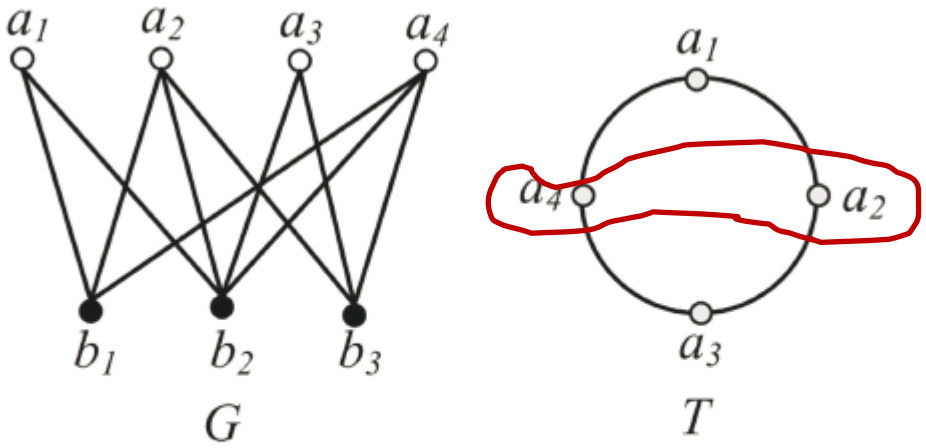

Fig. 1. A circular convex bipartite graph G whose optimal solution does not induce an interval in T.

Separation

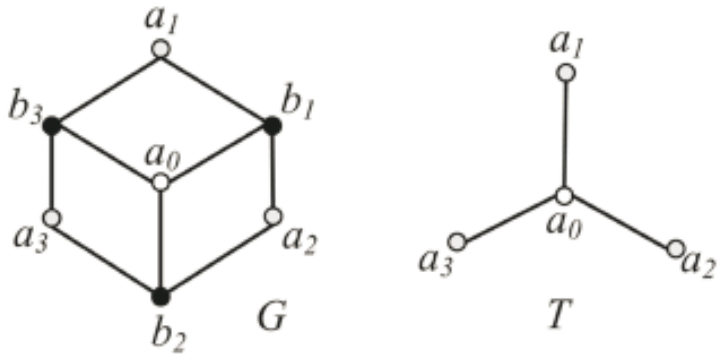

Fig. 2. A triad convex bipartite graph G which is not a chordal bipartite graph.

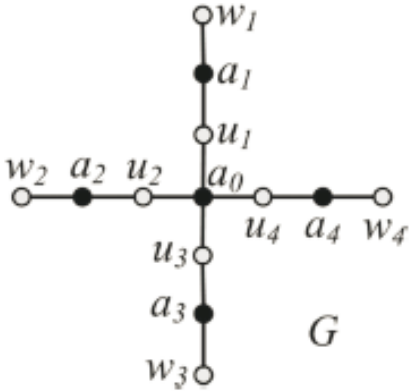

Fig. 3. A chordal bipartite graph G which is not a triad convex bipartite graph.

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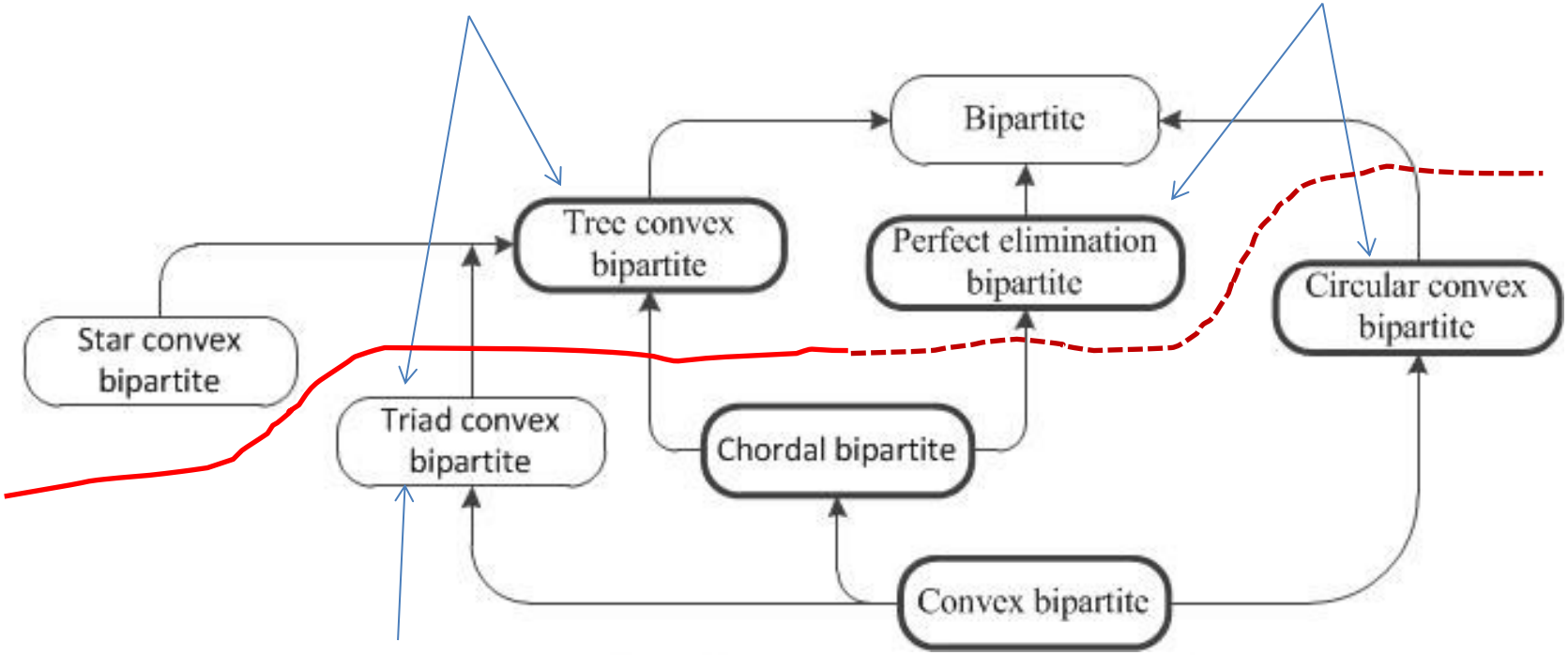
Open Questions

Characterizations

Max Edge Bicliques



Recognization

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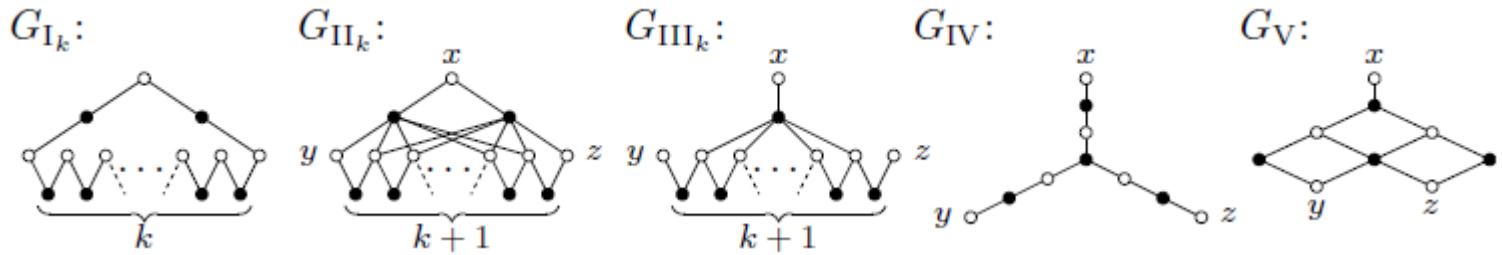
未来的研究问题

- 组合刻画
- 判定算法
- 算法问题
- 随机二部图
- 应用

组合刻画

- **Tucker对凸二部图的组合刻画**
  - **Tucker, A.C.: A structure theorem for the consecutive 1's property. *J. Comb. Theory, Ser. B* 12:153–162 (1972)**
  - 凸二部图 ⇔ 不含以下五种导出子图



- **研究问题：树凸二部图的组合刻画**
  - 三岔/星/梳/圈凸二部图的组合刻画？
  - 弦凸二部图的组合刻画及与树凸二部图的对应关系？
  - 难点：树凸二部图对于删除顶点或导出子图不封闭

判定算法

- (树/圈)凸二部图有线性时间判定算法
  - Booth, K.S., Lueker, S.G.: Testing for the consecutive ones property, interval graphs, and graph planarity using PQ-Tree algorithms. *J. Computer & Sys. Sci.*, 13, 335–379 (1976)
  - Tarjan, R.E., Yannakakis, M.: Simple linear-time algorithms to test chordality of graphs, test acyclicity of hypergraphs, and selectively reduce acyclic hypergraphs. *SIAM J. Computing*, 13(3):566–579 (1984)
  - Bao, F.S., Zhang Y.L.: A review of tree convex sets test. *Computational Intelligence*, 28(3): 358-372 (2012)
- 弦凸二部图的平方时间判定算法
- 研究问题：三岔/星/梳凸二部图的判定算法？

算法问题

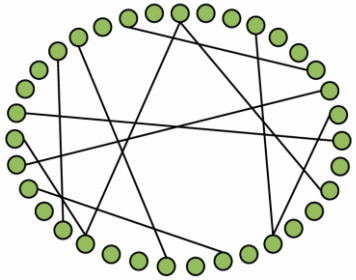
- P问题与NP完全问题的分类
  - 反馈顶点集、独立支配集、联通支配集
  - 哈密顿回路、哈密顿通路、树宽、.....
- 在(树/圈)凸二部图上有何改进？
  - 参数算法
  - 指数时间精确算法
  - 多项式时间近似算法

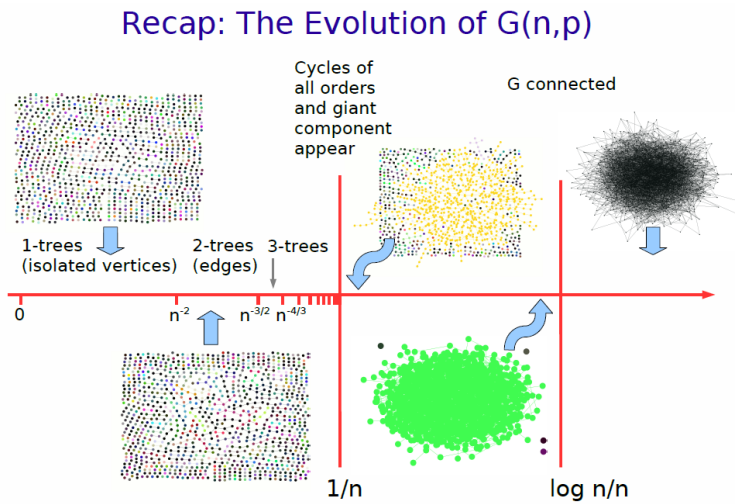
一些后续的工作

- Arti Pandey and B.S. Panda, **Domination in Some Subclasses of Bipartite Graphs,** *CALDAM 2015*, LNCS 8959, 169–180, 2015
- Min-Sheng Lin, Chien-Min Chen, **Counting independent sets in tree convex bipartite graphs**, *Discrete Applied Math*. 218 (2017) 113–122
- Hoang-Oanh Le and Van Bang Le, **Hardness and structural results for half-squares, of restricted tree convex bipartite graphs**, *COCOON 2017*, to appear.

随机二部图

- 随机图： G(n,p), G(n,m), G(n,r), 幂律, ……
  - Erdös-Renyi模型、Configuration模型、相变现象





- 随机二部图： $G(n_1,n_2,p)$, $G(n_1,n_2,m)$, $G(n_1,n_2,r)$等
- 随机树凸二部图：定义、性质、相变？

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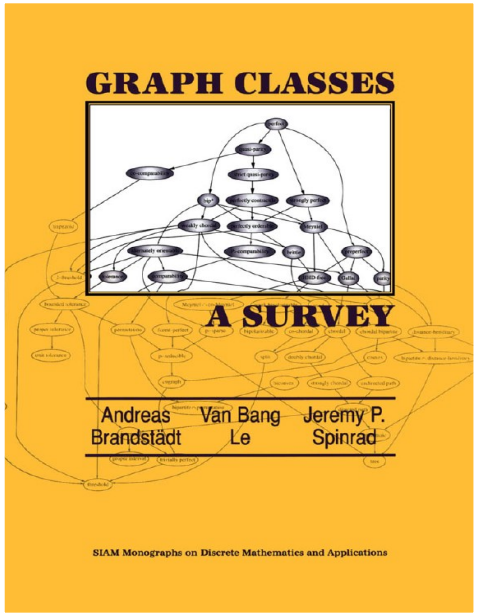
应用

- 凸二部图的应用
  - DNA物理图谱(physical map)

- 研究问题：树凸二部图的应用？
  - 三岔凸二部图在DNA物理图谱上的应用？
  - 其他？

总结

- NP完全问题依然是核心研究问题
  - 限制到特殊实例是研究NP完全问题的途径之一
- 二部图是一个非常好的研究平台
  - 关于树凸二部图还有大量问题可以研究
- 归约是基本的研究工具
  - 把三岔凸/圈凸二部图归约到凸二部图
  - 把一般(二部)图归约到星/梳凸二部图
- 二分定理是值得追求的研究结果



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谢谢

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