exercise1-secant (Score: 14.0 / 14.0)

Lab 2

1. 提交作業之前，建議可以先點選上方工具列的**Kernel**，再選擇**Restart & Run All**，檢查一下是否程式跑起來都沒有問題，最後記得儲存。
2. 請先填上下方的姓名(name)及學號(stduent_id)再開始作答，例如：

```
name = "我的名字"
student_id= "B06201000"
```

3. 四個求根演算法的實作可以參考lab-2 (https://yuanyuyuan.github.io/itcm/lab-2.html)，裡面有教學影片也有範例程式可以套用。
4. **Deadline: 10/9(Wed.)**

In [1]:

```python
name = "陳彥宇"
student_id = "B05303134"
```

Exercise 1 - Secant

Use the secant method to find roots of

$$f(x) = cosh(x) + cos(x) - c, \text{for } c = 1, 2, 3,$$

Import libraries

In [2]:

```python
import matplotlib.pyplot as plt
import numpy as np
```

1. Define a function $g(c)(x) = f(x) = cosh(x) + cos(x) - c$ with parameter $c = 1, 2, 3$.

In [3]:

(Top)

```python
def g(c):
    assert c == 1 or c == 2 or c == 3
    def f(x):
        # Hint: return ...
        # ===== 請實做程式 =====
        return np.cosh(x) + np.cos(x) - c
        # ====================
    return f
```

Pass the following assertion.

In [4]:

cell-b59c94b754b1fc9e (Top)

```python
assert g(1)(0) == np.cosh(0) + np.cos(0) - 1
### BEGIN HIDDEN TESTS
assert g(2)(0) == np.cosh(0) + np.cos(0) - 2
assert g(3)(0) == np.cosh(0) + np.cos(0) - 3
### END HIDDEN TESTS
```

2. Implement the algorithm

In [5]:

(Top)

```python
def secant(
    func,
    interval,
    max_iterations=5,
    tolerance=1e-7,
    report_history=False,
):
    '''Approximate solution of f(x)=0 on interval [a,b] by the secant method.

    Parameters
    ----------
    func : function
        The target function.
    interval: list
        The initial interval to search
    max_iterations : (positive) integer
        One of the termination conditions. The amount of iterations allowed.
    tolerance: float
        One of the termination conditions. Error tolerance.
    report_history: bool
        Whether to return history.

    Returns
    -------
    result: float
        Approximation of the root.
    history: dict
        Return history of the solving process if report_history is True.
    '''

    # ===== 請實做程式 =====
    # Ensure the initial interval is valid
    a, b = interval
    assert func(a) * func(b) < 0, 'This initial interval does not satisfied the prerequisites!'

    ################################
    ### Answer the code here
    # Set the initial condition
    num_iterations = 0
    a_next, b_next = a, b
    ### End answer
    ################################

    # history of solving process
    if report_history:
        history = {'estimation': [], 'x_error': [], 'y_error': []}

    while True:
        ################################
        ### Answer the code here
        # Find the next point
        d_x = -func(a_next)*(b_next-a_next)/(func(b_next)-func(a_next))
        c = a_next+d_x
        ### End answer
        ################################

        # Evaluate the error
        x_error = abs(d_x)
```

```
        x_error = abs(d_x)
        y_error = abs(func(c))

        if report_history:
            history['estimation'].append(c)
            history['x_error'].append(x_error)
            history['y_error'].append(y_error)

        # Satisfy the criterion and stop
        if x_error < tolerance or y_error < tolerance:
            print('Found solution after', num_iterations,'iterations.')
            return (c, history) if report_history else c

        ################################
        ### Answer the code here
        # Check the number of iterations
        if num_iterations < max_iterations:

            num_iterations += 1

            # Find the next interval
            value_of_func_c = func(c)
            if func(a_next) * value_of_func_c < 0:
                a_next = a_next
                b_next = c
            elif value_of_func_c * func(b_next) < 0:
                a_next = c
                b_next = b_next
        ### End answer
        ################################

            else:
                return (c, history) if report_history else c

        # Satisfy the criterion and stop
        else:
            print('Terminate since reached the maximum iterations.')
            return (c, history) if report_history else c

    # ===================
```

Test your implementation with the assertion below.

In [6]:

cell-4d88293f2527c82d (Top)

```
root = secant(lambda x: x**2 - x - 1, [1.0, 2.0], max_iterations=100, tolerance=1e-7, report_history=False)
assert abs(root - ((1 + np.sqrt(5)) / 2)) < 1e-7
```

```
Found solution after 8 iterations.
```

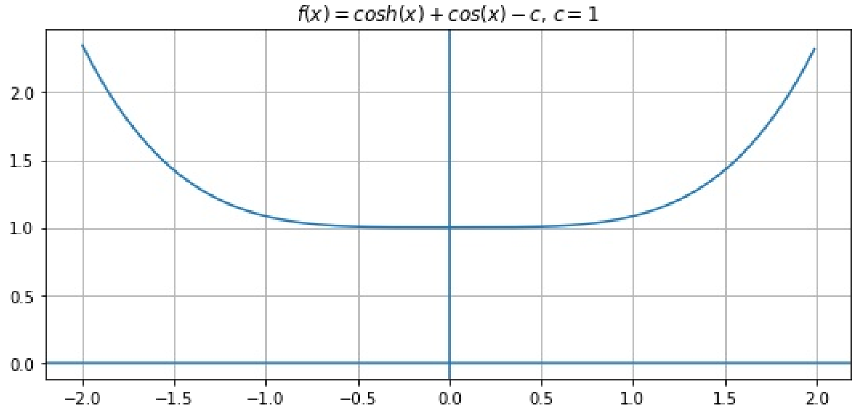
3. Answer the following questions under the case $c = 1$.

Plot the function to find an interval that contains the zero of f if possible.

In [7]:

(Top)

```
c = 1
f = g(c)

# Hint: search_range = np.arange(左端點, 右端點, 點與點之間距),
# e.g. search_range = np.arange(0.0, 1.0, 0.01)
# ===== 請實做程式 =====
search_range = np.arange(-2.0, 2.0, 0.01)
# ===================

fig, ax = plt.subplots(figsize=(9, 4))
ax.plot(search_range, f(search_range))
ax.set_title(r'$f(x)=cosh(x)+cos(x)-c$, $c=$%d' % c)
ax.grid(True)
ax.axhline(y=0)
ax.axvline(x=0)
plt.show()
```



According to the figure above, estimate the zero of f.

For example,

```
root = 3        # 單根
root = -2, 1    # 多根
root = None     # 無解
```

In [8]:

(Top)

```
# Hint: root = ?
# ===== 請實做程式 =====
root = None
# ===================
```

In [9]:

cell-d872c7c57f11c968 (Top)

```
print('My estimation of root:', root)
### BEGIN HIDDEN TESTS
if root == None:
    print('Right answer!')
else:
    raise AssertionError('Wrong answer!')
### END HIDDEN TESTS
```

```
My estimation of root: None
Right answer!
```

Try to find the zero with a tolerance of 10^{-10}.I f it works, plot the error and estimation of each step. Otherwise, state the reason why the method failed on this case.

(Top)

According to the figure as above showing that the minimal value of the function is strictly greater than 0 , this function does not have zero in real number and thus our method faild to find the zero.

---

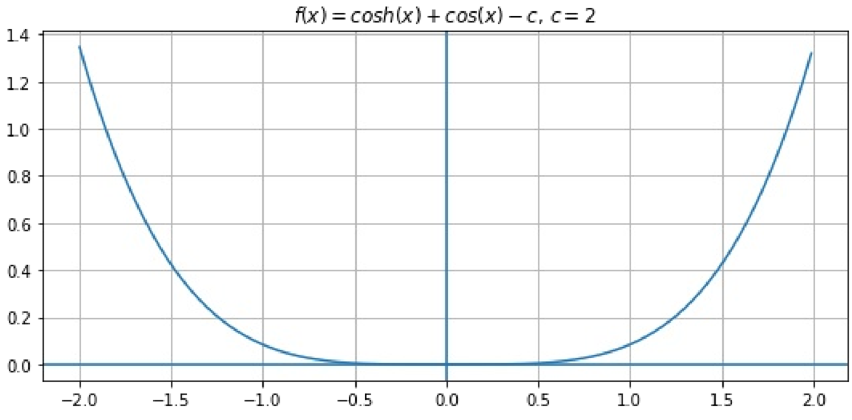
4. Answer the following questions under the case $c = 2$.

Plot the function to find an interval that contains the zero of f if possible.

In [10]:

(Top)

```
c = 2
f = g(c)

# Hint: search_range = np.arange(左端點, 右端點, 點與點之間距),
# e.g. search_range = np.arange(0.0, 1.0, 0.01)
# ===== 請實做程式 =====
search_range = np.arange(-2.0, 2.0, 0.01)
# ====================

fig, ax = plt.subplots(figsize=(9, 4))
ax.plot(search_range, f(search_range))
ax.set_title(r'$f(x)=cosh(x)+cos(x)-c$, $c=$%d' % c)
ax.grid(True)
ax.axhline(y=0)
ax.axvline(x=0)
plt.show()
```



According to the figure above, estimate the zero of f.

For example,

```
root = 3        # 單根
root = -2, 1    # 多根
root = None     # 無解
```

In [11]:

(Top)

```python
# Hint: root = ?
# ===== 請實做程式 =====
root = 0
# ====================
```

In [12]:

cell-20fddbe6fa4c437b (Top)

```python
print('My estimation of root:', root)

### BEGIN HIDDEN TESTS
assert type(root) is float or int, 'Wrong type!'
### END HIDDEN TESTS
```

```
My estimation of root: 0
```

Try to find the zero with a tolerance of 10^{-10}. If it works, plot the error and estimation of each step.Otherwise, state the reason why the method failed on this case.

(Top)

Although the function has the zero at x=0 , the secant method based on the intermediate value theorem required the interval be alternating sign at the end points. However, the value of the function is always greater or equal to 0 , having no such required interval.

---

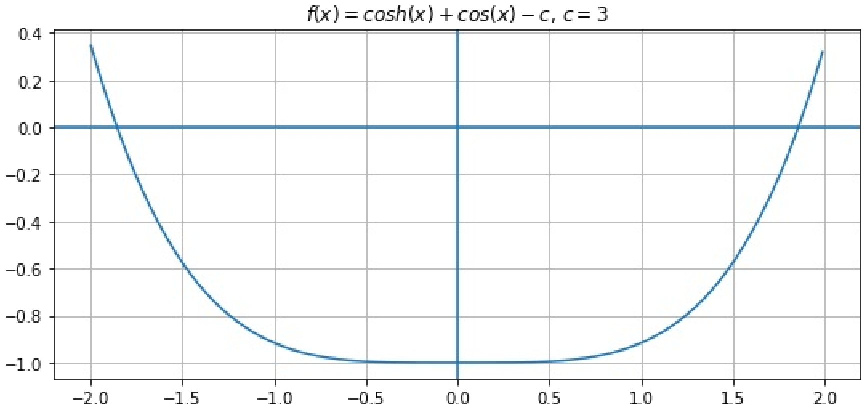
5. Answer the following questions under the case $c = 3$.

Plot the function to find an interval that contains the zeros of f if possible.

In [13]:

(Top)

```
c = 3
f = g(c)

# Hint: search_range = np.arange(左端點, 右端點, 點與點之間距),
# e.g. search_range = np.arange(0.0, 1.0, 0.01)
# ===== 請實做程式 =====
search_range = np.arange(-2.0, 2.0, 0.01)
# ===================

fig, ax = plt.subplots(figsize=(9, 4))
ax.plot(search_range, f(search_range))
ax.set_title(r'$f(x)=cosh(x)+cos(x)-c$, $c=$%d' % c)
ax.grid(True)
ax.axhline(y=0)
ax.axvline(x=0)
plt.show()
```



According to the figure above, estimate the zero of f.

For example,

```
root = 3        # 單根
root = -2, 1    # 多根
root = None     # 無解
```

In [14]:

(Top)

```
# Hint: root = ?
# ===== 請實做程式 =====
root = 1.8579208, -1.8579208
# ===================
```

In [15]:

cell-06ec0b20844075c7 (Top)

```
print('My estimation of root:', root)

### BEGIN HIDDEN TESTS
assert type(root) == tuple, 'Should be multiple roots!'
### END HIDDEN TESTS
```

```
My estimation of root: (1.8579208, -1.8579208)
```

Try to find the zero with a tolerance of 10^{-10}. If it works, plot the error and estimation of each step. Otherwise, state the reason why the method failed on this case.

According to the figure, we first apply the intermediate value theorrem to find the positive zero between 1 and 2.

In [16]:

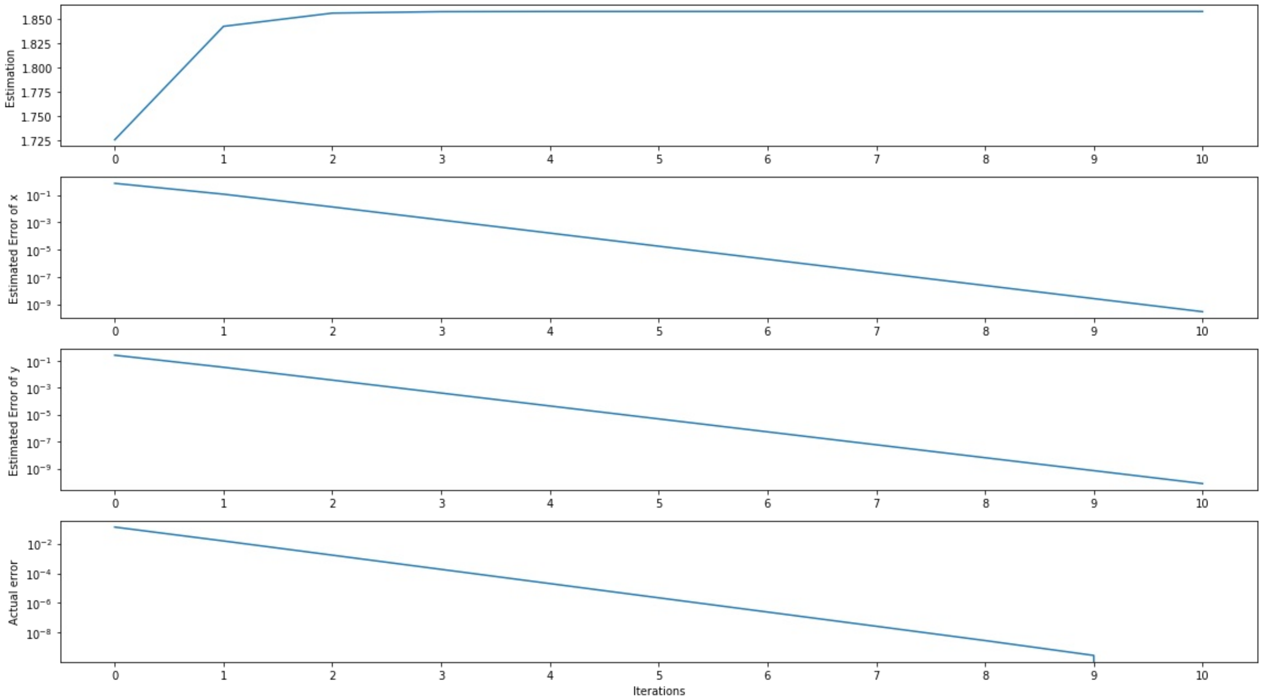
(Top)

```
my_initial_interval = [1.0, 2.0]

solution, history = secant(
    f,
    interval = my_initial_interval,
    max_iterations=100,
    tolerance=1e-10,
    report_history=True,
)

print(solution)
```

```
Found solution after 10 iterations.
1.85792082911445
```

In [17]:

```
fig, axes = plt.subplots(4, 1, figsize=(16, 9))
ax1, ax2, ax3, ax4 = axes

num_iterations = len(history['estimation'])
iterations = range(num_iterations)
for ax in axes:
    ax.set_xticks(iterations)

ax1.plot(iterations, history['estimation'])
ax1.set_ylabel('Estimation')

ax2.plot(iterations, history['x_error'])
ax2.set_ylabel('Estimated Error of x')
ax2.set_yscale('log')

ax3.plot(iterations, history['y_error'])
ax3.set_ylabel('Estimated Error of y')
ax3.set_yscale('log')

actual_error = np.abs(history['estimation']-solution)
ax4.plot(iterations, actual_error)
ax4.set_ylabel('Actual error')
ax4.set_yscale('log')
ax4.set_xlabel('Iterations')

plt.tight_layout()
plt.show()
```



Then, we use the analogous way to find the negative root between -2 and -1.

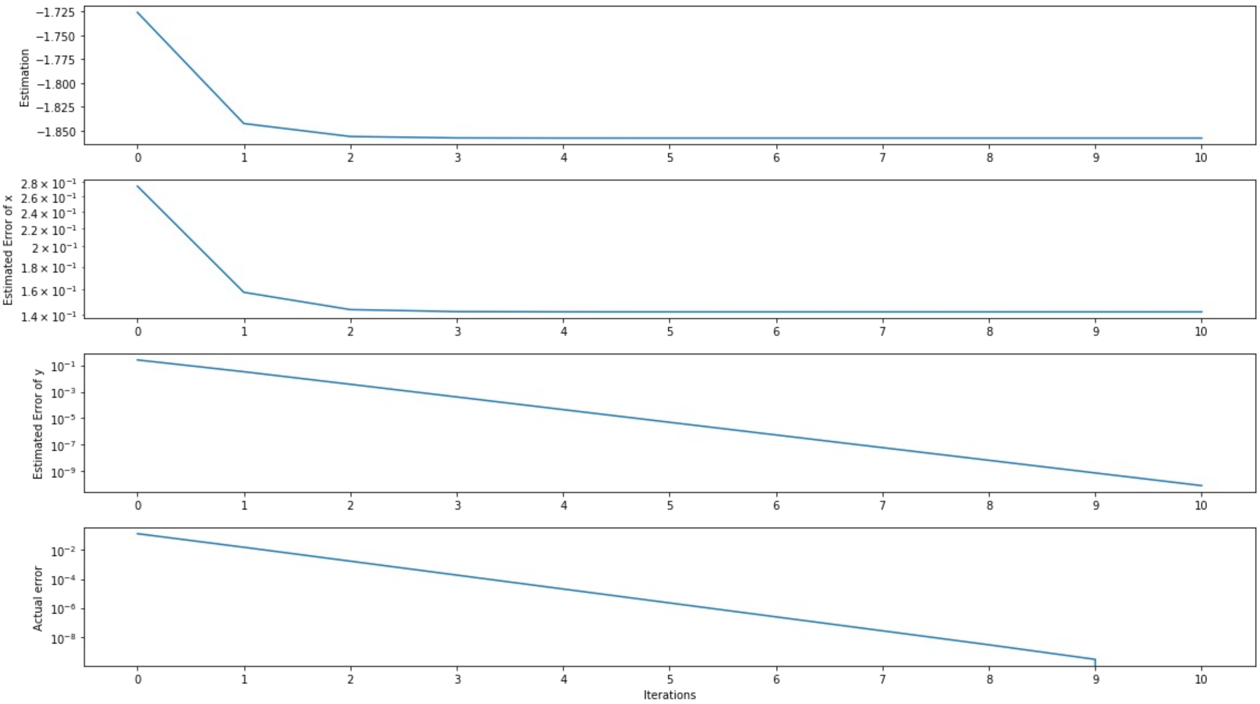
In [18]:

```
my_initial_interval = [-2.0, -1.0]

solution, history = secant(
    f,
    interval = my_initial_interval,
    max_iterations=100,
    tolerance=1e-10,
    report_history=True,
)

print(solution)
```

```
Found solution after 10 iterations.
-1.85792082911445
```

In [19]:

```
fig, axes = plt.subplots(4, 1, figsize=(16, 9))
ax1, ax2, ax3, ax4 = axes

num_iterations = len(history['estimation'])
iterations = range(num_iterations)
for ax in axes:
    ax.set_xticks(iterations)

ax1.plot(iterations, history['estimation'])
ax1.set_ylabel('Estimation')

ax2.plot(iterations, history['x_error'])
ax2.set_ylabel('Estimated Error of x')
ax2.set_yscale('log')

ax3.plot(iterations, history['y_error'])
ax3.set_ylabel('Estimated Error of y')
ax3.set_yscale('log')

actual_error = np.abs(history['estimation']-solution)
ax4.plot(iterations, actual_error)
ax4.set_ylabel('Actual error')
ax4.set_yscale('log')
ax4.set_xlabel('Iterations')

plt.tight_layout()
plt.show()
```



Discussion

For all cases above(c=1,2,3), do the results(e.g. error behaviors, estimations, etc) agree with the theoretical analysis?

(Top)

For $c = 1, 2$, the secant method does not work since the requirement, having zero and satisfied the condition of intermediate value theorem, dose not satisfied, but for $c = 3$, it does work as the Newton mathod though the convergence is superlinear instead of quadratic. Moreover, we do not need to calculate the derivative of the given function in the secant method.