Computer Programming Language

【Fall, 2020】

Homework 3

Program A： Calculation of an arithmetic series (25%)

Consider the infinite polynomial series

$$A_F(x) = xF_1 + x^2F_2 + x^3F_3 + \cdots + x^nF_n + \cdots$$

where F_n is the nth term in the Fibonacci sequence: 1, 1, 2, 3, 5, 8, … ; that is, $F_n = F_{n-1} + F_{n-2}$, $F_1 = 1$ and $F_2 = 1$. When $x = \frac{1}{2}$, surprisingly we have

$$\begin{aligned} A_F\left(\frac{1}{2}\right) &= \left(\frac{1}{2}\right) \times 1 + \left(\frac{1}{2}\right)^2 \times 1 + \left(\frac{1}{2}\right)^3 \times 2 + \left(\frac{1}{2}\right)^4 \times 3 + \left(\frac{1}{2}\right)^5 \times 5 + \cdots + \left(\frac{1}{2}\right)^n \times F_n + \cdots \\ &= \frac{1}{2} + \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{5}{32} + \cdots \\ &\approx 2 \end{aligned}$$

Write a program to calculate $A_F\left(\frac{1}{2}\right)$ numerically and determine the minimum number of n-terms required to have an estimation error less than 1.0×10^{-3}. Show the $A_F\left(\frac{1}{2}\right)$ values and errors for different n values on the screen. What is the most accurate value of $A_F\left(\frac{1}{2}\right)$ you can obtain?

■ *AUTOLAB Submission Check:*

double answer1; // Store the smallest value of n that makes the error less than 1.0×10^{-3}

Program B： Calculation of prime numbers (25%)

A prime integer number is one that has exactly two different divisors, namely 1 and the number itself. Write, run, and test a C++ program that finds and displays all the prime numbers less than 1000. (*Hint*: For each number from 2 to 1000, find Remainder = Number % n, where n ranges from 2 to sqrt(Number). If n is greater than sqrt(Number), the number is not equally divisible by n. Why? If any Remainder equals 0, the number is not a prime number.)

■ *AUTOLAB Submission Check:*

int answer1; // Store the largest prime number less than 1000 in this global variable

Bonus points (25%): (This is an optional problem)

A pair of positive integer numbers are called twin primes if they are both prime numbers and the difference between them is 2, i.e., they are consecutive odd numbers and they are prime numbers. (3, 5), (5, 7) and (11, 13) are three examples of such pair of twin prime numbers. Write a program to display all the pairs of twin prime numbers that are less than 1000. What is the greatest twin primes you can obtain?

■ *AUTOLAB Submission Check:*

```
int answer1; // First prime number of the largest pair of twin prime number less than 1000
int answer2; // Second prime number of the largest pair of twin prime number less than 1000
```

Program C： Use of selection structure and repetition structure (25%)

A unit fraction contains 1 in the numerator. The decimal representation of the unit fractions with denominators 2 to 10 are given:

1/2 = 0.5
1/3 = 0.(3)
1/4 = 0.25
1/5 = 0.2
1/6 = 0.1(6)
1/7 = 0.(142857)
1/8 = 0.125
1/9 = 0.(1)
1/10 = 0.1

where 0.1(6) means 0.166666..., and has a 1-digit recurring cycle. It can be seen that 1/7 has a 6-digit recurring cycle. Find the value of $d < 5000$ for which $1/d$ contains the longest recurring cycle in its decimal fraction part.

■ *AUTOLAB Submission Check:*

```
int answer1;      // Store the value of d
```

Program D：Rock-Paper-Scissors game (25%)

There is an ancient game played between two participants who simultaneously cast one of three figures by their fingers - Rock, Paper or Scissors. If both cast the same figure, the round is considered a draw. Otherwise the following rules are applied:

Rock beats Scissors (by blunting them)
Scissors beat Paper (by cutting it)
Paper beats Rock (by covering it)

Often the game is played on the staircase. Player who wins the round advances one step. One who reaches the end of the staircase first is the winner of the game.

Please design a computer program for a person to play Rock-Paper-Scissors game against computer. Assume the number of steps of the staircase is 3 and let the computer be your opponent. Play the game as you cast your figure at each round and find who is the winner of this game. Display the result of each round, and finally show who wins the game and report the total number of rounds in the game. Design the user input for Rock, Scissors, and Paper to be 'R', 'S' and 'P', respectively.

■ *AUTOLAB Submission Check:*

```
int answer1;      // Store the total number of rounds in this variable
```

Notes:

1. Please submit your programs (source codes) to the AUTOLAB grading system website (http://140.112.183.225) before **Nov. 5** (3:30PM)

2. Late submission will have a penalty of 10% discount per day of your homework total score toward a maximum of 50% discount. No late submission over five days will be accepted.

3. Criteria of grading include: (1) Program functionality; (2) User interface; (3) Structure of the program; (4) Suitable comments; (5) Programming style; (6) Creativity.