Columbia University
IEOR 4721: Computational Methods in Derivatives Pricing

Case Study 4 (Due date: Saturday May 10th, 2014)

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1 Parameter estimation of Heston via Extended Kalman Filter

The goal is to set up parameter estimation of the Heston stochastic volatility model via the extended Kalman filter. In Heston stochastic volatility, the underlying process follows the following SDE

$$\begin{aligned} dS_t &= \mu S_t dt + \sqrt{v_t} S_t dW_t^1 \\ dv_t &= \kappa(\theta - v_t)dt + \lambda\sqrt{v_t} dW_t^2 \end{aligned}$$

where the two Brownian components W_t^1 and W_t^2 are correlated with rate ρ under physical measure. Define $y_t = \ln(S_t)$ and using Itô's lemma we can write it as

$$\begin{aligned} dy_t &= (\mu - \frac{1}{2}v_t)dt + \sqrt{v_t} dW_t^1 \\ dv_t &= \kappa(\theta - v_t)dt + \sigma\sqrt{v_t} dW_t^2 \end{aligned}$$

For Heston, we assume that the state equation is

$$\begin{aligned} x_k = f(x_{k-1}, u_k) &= \begin{pmatrix} y_k \\ v_k \end{pmatrix} \\ &= \begin{pmatrix} y_{k-1} + (\mu - \frac{1}{2}v_{k-1})\Delta t + \sqrt{v_{k-1}}\sqrt{\Delta t} Z_k^1 \\ v_{k-1} + \kappa(\theta - v_{k-1})\Delta t + \lambda\sqrt{v_{k-1}}\sqrt{\Delta t} Z_k^2 \end{pmatrix} \end{aligned}$$

with system noise

$$u_k = \begin{pmatrix} Z_k^1 \\ Z_k^2 \end{pmatrix}$$

and the covariance matrix

$$Q_k = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

It is easy to see that in the extended Kalman filter, F_k and U_k for Heston stochastic volatility are

$$F_k = \begin{pmatrix} 1 & -\frac{1}{2}\Delta t \\ 0 & 1 - \kappa\Delta t \end{pmatrix}$$

and

$$U_k = \begin{pmatrix} \sqrt{v_{k-1}}\sqrt{\Delta t} & 0 \\ 0 & \lambda\sqrt{v_{k-1}}\sqrt{\Delta t} \end{pmatrix}$$

We assume measurement equation $y_k = \ln(S_k)$, which implies $H_k = (1\ 0)$ and $V_k = (0\ 0)$. For a given set of parameters $\Theta = \{\mu, \kappa, \theta, \lambda, \rho, v_0\}$, we would minimize the following summation as our objective function to obtain the optimal parameter set for the model

$$\sum_{i=1}^{N} \left(\ln(A_k) + \frac{e_k^2}{A_k} \right)$$

where

$$e_k = y_k - h(\hat{x}_{k|k-1}, 0)$$

and

$$A_k = H_k P_{k|k-1} H_k^\top + V_k R_k V_k^\top$$

where $h(\hat{x}_{k|k-1}, 0) = y_{k-1} + (\mu - \frac{1}{2}v_{k-1})\Delta t$. Here θ does not come into the objective function explicitly but implicitly comes in the filtering.

Use the given data to estimate the parameter set.