Columbia University
IEOR 4732: Computational Methods in Derivatives Pricing

Case Study 1 (Due on Sunday Feb 16, 2014)

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Problem 1:

The characteristic function of the log of stock price in Black-Scholes framework as shown on Page 11 is given by:

$$\begin{aligned}\mathbb{E}(e^{iu\ln S_t}) &= \mathbb{E}(e^{ius_t}) \\ &= \exp\left(i(s_0+(r-q-\frac{\sigma^2}{2})t)u-\frac{1}{2}\sigma^2u^2t\right)\end{aligned}$$

For the following parameters: spot price, $S_0 = \$1800$; maturity, $T = 0.5$ year; volatility, $\sigma = 0.30$; risk-free interest rate, $r = 0.25\%$, continuous dividend rate, $q = 2.03\%$ and strike range of $K = 900, 1100, 1300, 1500$ price European put options via the following techniques:

(a) Fast Fourier transform (FFT): consider $\eta = \Delta\nu = 0.25$, $\alpha = -2, -5, -10, -20$, $N = 2^n$ for $n = 8, 10, 12, 14$, and $\beta = \ln K - \frac{\lambda N}{2}$ (look at Section 2.1.4 on Page 43 for implementation of fast Fourier transform).

(b) Fractional fast Fourier transform (FrFFT): consider $\eta = \Delta\nu = 0.25$, $\lambda = \Delta k = 0.1$, $\alpha = -2, -5, -10, -20$, $N = 2^n$ for $n = 7, 8, 9, 10$, and $\beta = \ln K - \frac{\lambda N}{2}$ (look at Section 2.2.2 on Page 52 for implementation of fractional fast Fourier transform).

(c) Fourier-cosine (COS) method: consider values $[-2, 2]$, $[-5, 5]$, $[-10, 10]$, $[-20, 20]$ for the interval $[a, b]$ and find the sensitivity of your results to the choice of $[a, b]$ (look at Section 2.3.2 on Page 47 for implementation of COS method).

Compare and conclude.