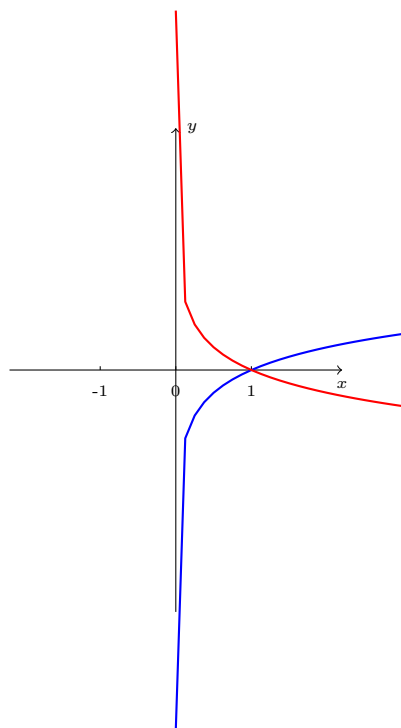


Problem 1

Logarithmic function : $f(x) = \log_b X$



BLUE LINE : $f(x) = \log_{10} X$ **REDLINE** : $f(x) = \log_{0.5} X$

DOMAIN: $b: (0,1) \cup (1, +\infty)$ $X : (0, +\infty)$

CO-DOMAIN: \mathbb{R}

CHARACTERISTICS:

Fixed point: Function image is always over fixed point (1,0).

Monotonicity: when $a > 1$, it is a monotonic increasing function in the domain of definition.

Parity: Non-odd and Non-even Functions

Periodicity: not a periodic function

Symmetry: None

Null point: $X=1$

Problem 2

1. Problem description

Develop a Java system to calculate the result for the Logarithmic function : $f(x) = \log_b X$.

2. Requirements

a .When the system starts, the console should display the function name and allow the user to select the logarithmic function.

-Type attribute: Functional

b . The primary requirement to the function is to have only two number value as input to the function.

-Type attribute: Design Constraints

c . In case any other form of input is given, the program should prompt an effective error message to the user.

-Type attribute: Functional

d . The function accepts only a real number as its input argument. Hence, it is the responsibility of the program/function to change the illegal input to the desired input needed for it to work efficiently.

-Type attribute: Design Constraints

e .If the base is valid, the system should ask the user to input the value for variable and set it.

-Type attribute: Functional

f.If the variable is valid, the system should calculate the logarithm of in base without relying on java built-in functions, and store the result.

-Type attribute: Functional

g .After the calculation completes, the system should display the result on the console.

-Type attribute: Functional

h. The calculation result shall be accurate to 6 decimal places.

-Type attribute: Performance

3. Constraints

There are few constraints that need to be followed:

a . Apart from the functions related to input, output and arithmetic, use of any built-in functions provided in Java is prohibited.

b . The domain of $f(x)$ is $b : (0,1) \cup (1, +\infty)$ $X : (0, +\infty)$

4. Assumptions

a . We assume that the user interface will be text-based, depending on console input and output.

b . User gives input for both X and a value.

c . The 'Java system' refers to the scientific calculator

d . Users may enter illegal characters such as letters or non-real numbers..

5. References

a .Shapiro, J. F., Shapiro, J. F. (1979). Mathematical programming: structures and algorithms (No. 04; QA402. 5, S4.). New York: Wiley.

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c .Olver, F. W., Lozier, D. W., Boisvert, R. F., Clark, C. W. (Eds.). (2010). NIST handbook of mathematical functions hardback and CD-ROM. Cambridge university press.

Problem 3

RECURSIVE ALGORITHM - LOGARITHMIC FUNCTION – $mylog(nk, x, y, N)$

```
1  x = (q - 1) / (q + 1)
2  z = (w - 1) / (w + 1)
3  nk = 2 * N + 1
4  y = 1.0 / nk
5  if (N = 0)
6    res = 2.0 * x * y
7  else
8    nk = nk - 2
9    y = 1.0 / nk + x * x * y
10   mylog(nk,x,y,N-1)
11 end if
12 return res
13 mylog(nk, z, y, N) / mylog(nk, x, y, N)
```

ITERATIVE ALGORITHM - LOGARITHMIC FUNCTION – $mylog(a)$

```
1  x = (a - 1) / (a + 1)
2  nk = 2 * N + 1
3  y = 1.0 / nk
4  for k = N to 0
5    nk = nk - 2
6    y = 1.0 / nk + x * x * y
7  end for
8  return 2.0 * x * y
9  mylog(w) / mylog(q)
```

Advantages and Disadvantages

RECURSIVE ALGORITHM:

Advantages:

1. Recursion is easy to understand, and it is easy to read. For code, using recursion is much clear than loop.
2. Recursion has higher maintainability than loop.

Disadvantages:

1. When using recursion, it needs system continuously allocates memory space, it has a bad effect on efficiency.
2. Recursion could lead the problem of memory overflow, when the input number is very large, the program may have an error.

ITERATIVE ALGORITHM:

Advantages:

1. Iterative could avoid memory overflow of input. The value of input is unrestricted.
2. Iterative needs less time to execute. Besides that, it also use less memory .

Disadvantages:

1. The structure pf iterative is more complex than recursion. It is weak in readability, because of its complex code structure.

CONCLUSION

I used Taylor's expansion as the basic idea. Because Taylor's expansion is especially for $\ln(X)$, so I also used the formula of change of base of logarithms, so that I could calculate the logarithm with any number as the base.

After comparing with advantages and disadvantages of two algorithm, I decide to use loop, because when I loop, the input is unlimited in domain, but recursion is not. Besides, the loop has better efficiency on executing and it is important to users. As a result, the loop is more suitable for this function.

Taylor's expansion: $\ln(1+X) = x - (x^2/2) + (x^3/3) - \dots$ for $|x| \leq 1$

Formula of change of base of logarithms : $\log_b X = \log_a X / \log_a b$