CSC412 / CSC2506 Sample Problems for Midterm

1. Let $p(k)$ be a one-dimensional discrete distribution that we wish to approximate, with support on non-negative integers. One way to fit an approximating distribution $q(k)$ is to minimize the Kullback-Leibler divergence:

$$KL(p||q) = \sum_{k=0}^{\infty} p(k) log \frac{p(k)}{q(k)}$$

   Show that when $q(k)$ is a Poisson distribution,

$$q(k) = \frac{\lambda^k \exp(-\lambda)}{k!}$$

   this KL divergence is minimized by setting λ to the mean of $p(k)$.

2. Recall that the definition of an exponential family model is:

$$f(x|\eta) = h(x)g(\eta)\exp(\eta^\top T(x))$$

   where:

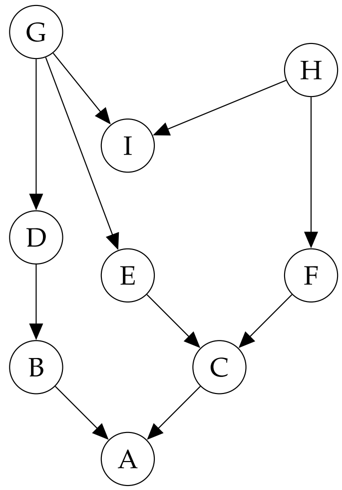
$$\begin{array}{rl} \eta & \text{are the parameters} \\ T(x) & \text{are the sufficient statistics} \\ h(x) & \text{is the base measure} \\ g(\eta) & \text{is the normalizing constant} \end{array}$$

   Consider the univariate Gaussian, with mean μ and precision $\lambda = \frac{1}{\sigma^2}$:

$$p(D|\mu, \lambda) = \prod_{i=1}^{N} (\frac{\lambda}{2\pi})^{\frac{1}{2}} \exp(-\frac{1}{2}(x_i - \mu)^2)$$

   What are η and $T(x)$ for this distribution when represented in exponential family form?

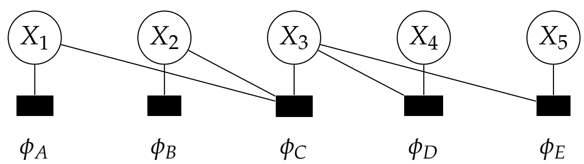
3. Consider the following directed graphical model:



(a) List all variables that are independent of A given evidence on B.
(b) Write down the factorized normalized joint distribution that this graphical model represents.

4. Murphy 20.1

5. Consider the Factor Graph:



(a) Write down the normalized joint distribution $P(X_1, X_2, X_3, X_4, X_5)$ in terms of the potentials.
(b) Write down any conditional independence relationships given by the graph.