1.

assume $\vec{k} = |k|\,\hat{a}_k$ and $\hat{a}_k = \hat{a}_z$

$$|k| = w\sqrt{\mu\varepsilon}$$

and $\vec{E} = \varepsilon_0 \exp(-j\vec{k}\cdot\vec{R})\,\hat{a}_n$

$$\vec{R} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$$

so $\vec{k}\cdot\vec{R} = z|k|$

and $\vec{H} = H_0 e^{-j\vec{k}\cdot\vec{R}}\,\hat{a}_y$

where $H_0 = \frac{E_0}{\eta} = E_0\sqrt{\frac{\varepsilon}{\mu}} \Rightarrow E_0 = \eta(H_0) = \sqrt{\frac{\mu}{\varepsilon}}\,H_0$

$$\vec{k}\times\vec{E} = |k|\hat{a}_k \times |E|\hat{a}_E = w\mu H_0 e^{-j\vec{k}\cdot\vec{R}}\,\hat{a}_y$$

$\Rightarrow \vec{k}\times\vec{E} = w\mu\vec{H}$ proved

$$\vec{k}\times\vec{H} = w\sqrt{\mu\varepsilon}\;\hat{a}_z \times |H|\,\hat{a}_y$$

$$= -w\varepsilon E_0\, e^{-j\vec{k}\cdot\vec{R}}\,\hat{a}_n$$

$\Rightarrow \vec{k}\times\vec{H} = -w\varepsilon\vec{E}$ proved

$$\vec{k}\cdot\vec{E} = |k|\,\hat{a}_k \cdot |E|\,\hat{a}_E = w\sqrt{\mu\varepsilon}\,(\hat{a}_z\cdot\hat{a}_n)\,|E| = 0$$

$$\vec{k}\cdot\vec{H} = |k|\,\hat{a}_k \cdot |H|\cdot\hat{a}_H = w\sqrt{\mu\varepsilon}\,(\hat{a}_z\cdot\hat{a}_y)\,|H| = 0$$

2.

(1) we have known that $\vec{H} = 4\times10^{-6}\cos(10^7\pi t - \beta y + \frac{\pi}{4})\ \hat{a}_z\ A/m$

and $\beta = w\sqrt{\mu\varepsilon}$, $w = 10^7\pi\ \frac{rad}{sec}$. $\mu_r = 1$, $\varepsilon_r = 1$

$$\Rightarrow \beta = w\sqrt{\mu_0\varepsilon_0} = 10^7\pi\cdot\sqrt{4\pi\times10^{-7}\cdot 8.854\times10^{-12}} = \frac{\pi}{30}\ \frac{rad}{m}$$

$$\vec{H} = 4\times10^{-6}\cos(10^7\pi t - \frac{\pi}{30}y + \frac{\pi}{4})\ a_z\ \frac{A}{m}$$

$t = 3ms$, $H_z = 0$

$$\Rightarrow 4\times10^{-6}\cos(10^7\pi t - \beta y + \frac{\pi}{4}) = 0$$

since $\cos(\frac{\pi}{2}) = 0$

$$\Rightarrow 10^7\pi t - \beta y + \frac{\pi}{4} = \frac{\pi}{2}$$

$$\Rightarrow y = 89992.5\ m$$

(2)

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}$$

$$|\eta| = \frac{|\vec{E}|}{|\vec{H}|}\ ,\ |\vec{E}| = 120\pi\times4\times10^{-6} = 1.507\times10^{-3}\ \frac{V}{m}$$

Also, $\hat{a}_k = \hat{a}_E\cdot\hat{a}_H$ where $\hat{a}_k = \hat{a}_y$ and $\hat{a}_H = \hat{a}_z$

$$\Rightarrow \vec{E} = -1.507\times10^{-3}\cos(10^7\pi t - \frac{\pi}{30}y + \frac{\pi}{4})\ \hat{a}_x\ \frac{V}{m}$$

3. $E(t,z) = a_x 2\cos\left(10^8 t - \frac{z}{\sqrt{3}}\right) - a_y \sin\left(10^8 t - \frac{z}{\sqrt{3}}\right)$ V/m

(1)

$w = 10^8$ rad/s

$\Rightarrow 2\pi f = 10^8$ rad/s

$\Rightarrow f = \frac{10^8}{2\pi} = 1.59 \times 10^7$ Hz

wave number : $k = \frac{1}{\sqrt{3}}$

$\Rightarrow \lambda = \frac{2\pi}{k} = 2\pi\sqrt{3} = 10.88$ m

(b)

$k = w\sqrt{\mu \varepsilon_r} \iff \sqrt{\varepsilon_r} = \frac{k}{2\pi f\sqrt{\mu_0 \varepsilon_0}} = 1.73$

$\Rightarrow \varepsilon_r = 1.73^2 = 3$

(c) left - handed elliptically polarized

(d)

$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{\sqrt{3}}\ \Omega$

$\vec{H} = \frac{1}{\eta}(a_z \times \vec{E}) = \frac{\sqrt{3}}{120\pi}\left[a_x \sin\left(10^8 t - \frac{z}{\sqrt{3}}\right) + a_y \cos\left(10^8 z - \frac{z}{\sqrt{3}}\right)\right]$