RC2: Static Electric Fields

1 Electrostatics in Free Space

1.1 Basic Concepts

Electrostatics:

i. electric charges are **at rest(not moving)**;

ii electric field **do not change with time**.

Static electric charges (source) in free space → electric field

1.2 Electric field intensity vector

$$\mathbf{E} = \lim_{q \to 0} \frac{\mathbf{F}}{q} \quad (\mathbf{V}/\mathrm{m})$$

1.3 Fundamental Postulates of Electrostatics not solenoidal

dielectric is not vacancy

- Differential form:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (divergence)$$

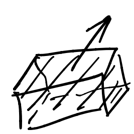
$\varepsilon_r \varepsilon_0 = \varepsilon$

$$\nabla \times \mathbf{E} = 0 \quad (curl) \rightarrow \text{irrotational}$$

  where ρ is the volume charge density of free charges (C/m^3), ϵ_0 is the permittivity of free space, a universal constant.

- Integral form:

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{Q}{\epsilon_0} \rightarrow$$



$$\oint_C \mathbf{E} \cdot d\ell = 0$$

  where Q is the total charge contained in volume V bounded by surface S. Also, the scalar line integral of the static electric field intensity around any closed path vanishes.

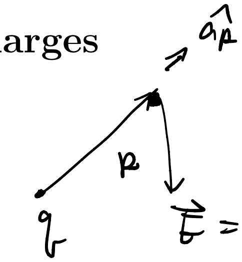
E is **not solenoidal** (unless $\rho = 0$), but **irrotational (conservative)**

2 Coulomb's Law

2.1 Electric Field due to a System of Discrete Charges

- **a single point charge (charge on the origin):**

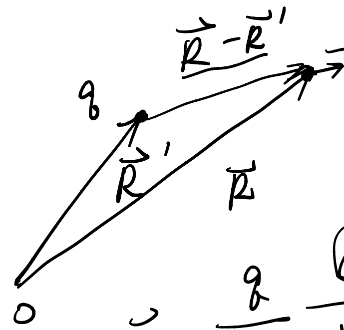
$$\mathbf{E} = \mathbf{a}_R E_R = \mathbf{a}_R \frac{q}{4\pi\epsilon_0 R^2} \quad (\mathbf{V}/\mathrm{m})$$



$\vec{E} = \hat{a}_R \frac{q}{4\pi\varepsilon R^2}$

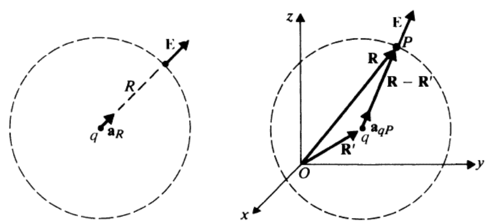
- **a single point charge (charge is not on the origin):**

$$\mathbf{E}_p = \frac{q\left(\mathbf{R}-\mathbf{R}'\right)}{4\pi\epsilon_0 \left|\mathbf{R}-\mathbf{R}'\right|^3} \quad (\mathbf{V}/\mathrm{m})$$



$\vec{R}-\vec{R}'$ → $\frac{\vec{R}-\vec{R}'}{|\vec{R}-\vec{R}'|}$

$\vec{E} = \frac{q}{4\pi\varepsilon(\vec{R}-\vec{R}')^2}$

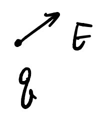


(a) Point charge at the origin. (b) Point charge not at the origin.

FIGURE 3–2
Electric field iFIGUREdue to a point charge.

When a point charge q_2 is placed in the field of another point charge q_1 at the origin, a force F_{12} is experienced by q_2 due to the electric field intensity $\boldsymbol{E_{12}}$ of q_1 at q_2. Then we have:

$$\boldsymbol{F_{12}} = q_2\boldsymbol{E_{12}} = \boldsymbol{a_R}\frac{q_1 q_2}{4\pi\epsilon_0 R^2}$$

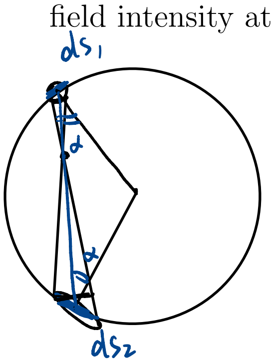


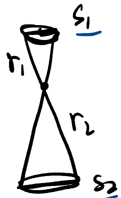
$F = qE$

- **several point charges:**

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\sum_{k=1}^{n}\frac{q_k\left(\mathbf{R}-\mathbf{R}'_k\right)}{\left|\mathbf{R}-\mathbf{R}'_k\right|^3}$$

Ex.1:
A total charge Q is uniformly put on a thin spherical shell of radius b. Determine the electric field intensity at an arbitrary point inside the shell.



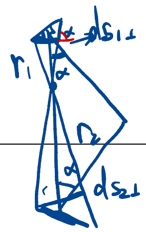


$\frac{S_1}{S_2} = \frac{r_1^2}{r_2^2}$

$ds_1 : E_1 = \frac{1}{4\pi\varepsilon}\frac{\sigma\, ds_1}{r_1^2}$

$ds_2 : E_2 = \frac{1}{4\pi\varepsilon}\frac{\sigma\, ds_2}{r_2^2}$

$E = E_1 - E_2 = 0$



$\frac{ds_{11}}{ds_1} = \cos\alpha$

$\frac{ds_{21}}{ds_2} = \cos\alpha$

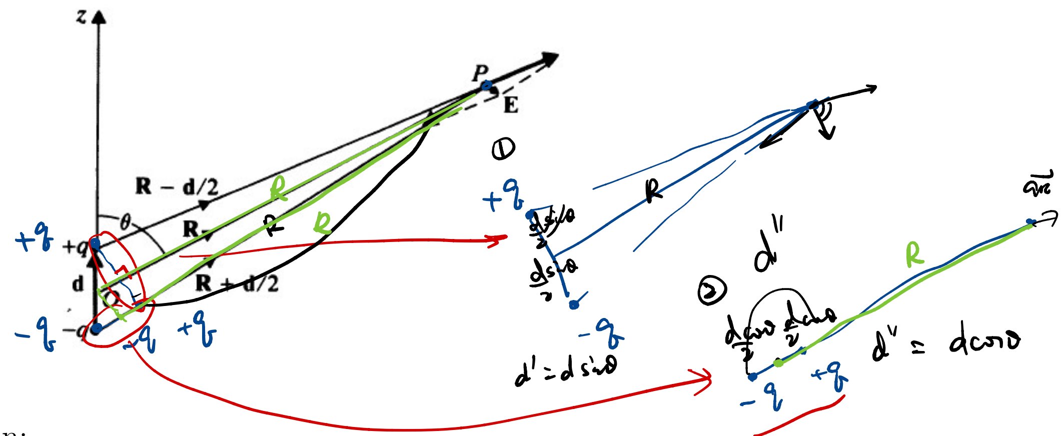
$\frac{ds_{11}}{ds_{21}} = \frac{r_1^2}{r_2^2}$

$\Rightarrow \frac{ds_1}{ds_2} = \frac{r_1^2}{r_2^2}$

2.2 Electric Dipole

- **Electric Field**

$\vec{p}$



general expression:

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0}\left\{\frac{\mathbf{R}-\frac{\mathbf{d}}{2}}{\left|\mathbf{R}-\frac{\mathbf{d}}{2}\right|^3} - \frac{\mathbf{R}+\frac{\mathbf{d}}{2}}{\left|\mathbf{R}+\frac{\mathbf{d}}{2}\right|^3}\right\}$$

if $d << R$:

$$\mathbf{E} \cong \frac{q}{4\pi\epsilon_0 R^3}\left[3\frac{\mathbf{R}\cdot\mathbf{d}}{R^2}\mathbf{R} - \mathbf{d}\right]$$

$$\vec{E}_\theta = \frac{1}{4\pi\varepsilon}\frac{q}{(R^2+(\frac{d'}{2})^2)}\cdot\frac{\frac{d'}{2}}{\sqrt{R^2+(\frac{d'}{2})^2}} \times 2\,\hat{a}_\theta = \frac{q}{4\pi\varepsilon}\frac{d'}{(R^2+(\frac{d'}{2})^2)^{\frac{3}{2}}} = \frac{qd'}{4\pi\varepsilon_0 R^3}\left(1+\cancel{(\frac{d'}{2R})^2}\right)^{-\frac{3}{2}} = \frac{qd\sin\theta}{4\pi\varepsilon_0 R^3}\hat{a}_\theta$$

$(1+x)^k \simeq 1+kx$

- **Electric Dipole Moment Definition:**

$$\mathbf{p} = q\mathbf{d}$$

where q is the charge, vector $\mathbf{d}$ goes from $-q$ to $+q$.

$$\mathbf{p} = \mathbf{a}_z p = p\left(\mathbf{a}_R\cos\theta - \mathbf{a}_\theta\sin\theta\right)$$

$$\mathbf{R}\cdot\mathbf{p} = Rp\cos\theta$$

$$\vec{E}_R = \frac{1}{4\pi\varepsilon}\frac{q}{(R-\frac{d''}{2})^2} - \frac{1}{4\pi\varepsilon}\frac{q}{(R+\frac{d''}{2})^2} = \frac{q}{4\pi\varepsilon R^2}\left[\frac{1}{(1-\frac{d''}{2R})^2} - \frac{1}{(1+\frac{d''}{2R})^2}\right] = \frac{q}{4\pi\varepsilon R^2}\left[(1+\frac{d''}{2R}\times 2) - (1-\frac{d''}{2R}\cdot 2)\right] = \frac{2qd''}{4\pi\varepsilon R^3} = \frac{2qd\cos\theta}{4\pi\varepsilon R^3} = \frac{2p\cos\theta}{4\pi\varepsilon_0 R^3}\hat{a}_R$$

- **Electric Field:** (spherical coordinate)

$$\mathbf{E} = \frac{p}{4\pi\epsilon_0 R^3}\left(\mathbf{a}_R 2\cos\theta + \mathbf{a}_\theta\sin\theta\right) \quad (\mathrm{V/m})$$

2.3 Electric Field due to a Continuous Distribution of Charge

- **General Differential Element:**

$$d\mathbf{E} = \mathbf{a}_R\frac{\rho dv'}{4\pi\epsilon_0 R^2}$$

$dq \rightarrow dE$

$\int dE$

$dE = \frac{dq}{4\pi\varepsilon_0 R^2}$

ρdv

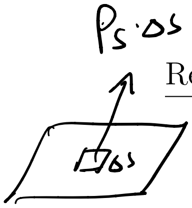
, where dv' is the differential volume element.

- **Line Charge:**

$\Delta l \rightarrow \rho_l \Delta l = \Delta q$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int_{L'}\mathbf{a}_R\frac{\rho_\ell}{R^2}d\ell \quad (\mathbf{V/m})$$

$P(x, y, z)$



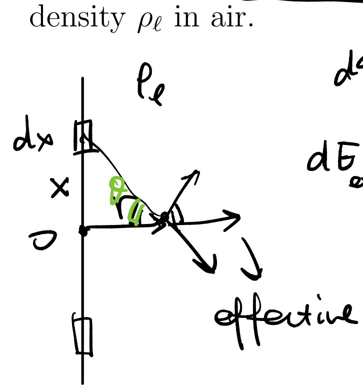
- **Surface Charge:**

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int_{\mathbf{S}'} \mathbf{a}_R \frac{\rho_s}{R^2} ds' \quad (\mathbf{V}/\text{m})$$

- **Volume Charge:**

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int_{V'} \mathbf{a}_R \frac{\rho}{R^2} dv' \quad (\mathbf{V}/\mathbf{m})$$

Ex.2
Determine the electric field intensity of an infinitely long, straight, line charge of a uniform density ρ_ℓ in air.



$dq = \rho_\ell dx$

$$dE_{eff} = \frac{1}{4\pi\varepsilon}\frac{\rho_\ell dx}{x^2+r^2}\cdot\frac{r}{\sqrt{x^2+r^2}} = \frac{\rho_\ell r}{4\pi\varepsilon}(x^2+r^2)^{-\frac{3}{2}}dx$$

$$\int (x^2+r^2)^{-\frac{3}{2}}dx = \frac{1}{r^2}\left(\frac{x^2}{r^2}+1\right)^{-\frac{3}{2}} d\left(\frac{x}{r}\right) \to m = \frac{x}{r}$$

$$= \frac{1}{r^2}\int_{-\infty}^{\infty}(m^2+1)^{-\frac{3}{2}}dm$$

$\tan\theta$

$(1+x^2)^k$

$x = \tan\theta$ $\quad 1+\tan^2\theta = \frac{1}{\cos^2\theta}$

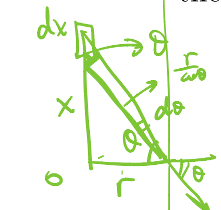
$x = \sinh\theta$ $\quad 1+\sinh^2\theta = \cosh^2\theta$

$\sinh\theta = \frac{e^\theta+e^{-\theta}}{2}$

$\cosh\theta = \frac{e^\theta-e^{-\theta}}{2}$

Ex.3
A line charge of uniform density ρ_l in free space forms a semicircle of radius b. Determine the magnitude and direction of the electric field intensity at the center of the semicircle.

Ex.2 continue



$$dx = \frac{r}{\cos\theta}\cdot d\theta \,/\cos\theta = r\frac{d\theta}{\cos^2\theta}$$

$$\frac{1}{4\pi\varepsilon}\frac{\rho_s\cdot r\,d\theta}{\cos^2\theta}\cdot\cos\theta\Big/\left(\frac{r}{\cos\theta}\right)^2 = \frac{\rho_s r}{4\pi\varepsilon}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\theta\, d\theta = \frac{\rho_s r}{4\pi\varepsilon}\cdot 2 = \frac{\rho_s r}{2\pi\varepsilon}$$

Ex.4 (An interesting Question)
How can you use a solid ball to create a uniform field somewhere? (you may)

- **Surface Charge:**

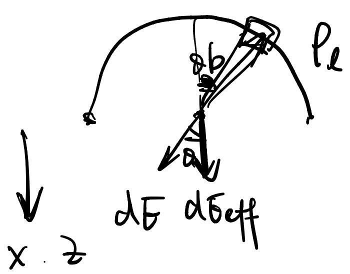
$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int_{\mathbf{S'}} \mathbf{a}_R \frac{\rho_s}{R^2} ds' \quad (\mathbf{V}/\mathrm{m})$$

- **Volume Charge:**

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int_{V'} \mathbf{a}_R \frac{\rho}{R^2} dv' \quad (\mathbf{V}/\mathbf{m})$$

Ex.2
Determine the electric field intensity of an infinitely long, straight, line charge of a uniform density ρ_ℓ in air.

Ex.3
A line charge of uniform density ρ_l in free space forms a semicircle of radius b. Determine the magnitude and direction of the electric field intensity at the center of the semicircle.

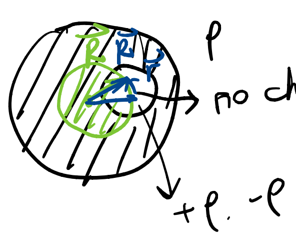


$$d\vec{E}_{eff} = \frac{dq}{4\pi\varepsilon b^2}\cdot\cos\theta \qquad dq = b\,d\theta\cdot\rho_l$$

$$dE_{eff} = \frac{b\rho_l\,d\theta\cdot\cos\theta}{4\pi\varepsilon b^2} = \frac{\rho_l\cos\theta\,d\theta}{4\pi\varepsilon b}$$

$$\vec{E}_{eff} = \frac{\rho_l}{4\pi\varepsilon b}\sin\theta\Big|_{-1}^{1} = \frac{\rho_l}{2\pi\varepsilon b}\hat{x}$$

Ex.4 (An interesting Question)
How can you use a solid ball to create a uniform field somewhere? (you may)



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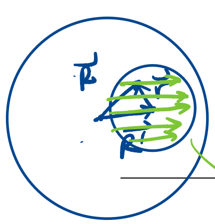
$$\vec{E}_\rho = \frac{1}{4\pi\varepsilon}\frac{\frac{4}{3}\pi|\vec{R}|^3\cdot\rho}{|\vec{R}|^2}\frac{\vec{R}}{|\vec{R}|} = \frac{\rho}{3\varepsilon}\vec{R}$$



$$\vec{E}_{-\rho} = \frac{1}{4\pi\varepsilon}\frac{\frac{4}{3}\pi|\vec{r}|^3\cdot(-\rho)}{|\vec{r}|^2}\cdot\frac{\vec{r}}{|\vec{r}|} = -\frac{\rho}{3\varepsilon}\vec{r}$$

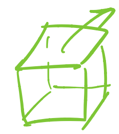
$$\vec{E} = \vec{E}_\rho + \vec{E}_{-\rho} = \frac{\rho}{3\varepsilon}(\vec{R}-\vec{r}) = \frac{\rho}{3\varepsilon}\vec{R}'$$

3 Gauss's Law and Application

3.1 Definition

The total outward flux of the E-field over any closed surface in free space is equal to **the total charge enclosed in the surface** divided by ϵ_0. (Note that we can choose arbitrary surface S for our convenience.)

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{Q}{\epsilon_0}$$

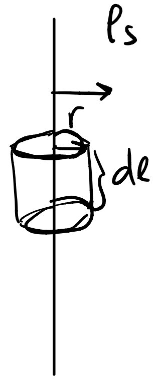


3.2 Application

$\vec{E} \cdot s$

- **Conditions for Maxwell's Integral Equations:**
  There is **a high degree of** **symmetry** in the charge distribution or in the electrical field (i.e., spherically symmetric, planar, line charge, etc.

Ex.5
Determine the electric field intensity of an infinitely long, straight, line charge of a uniform density ρ_s in air.

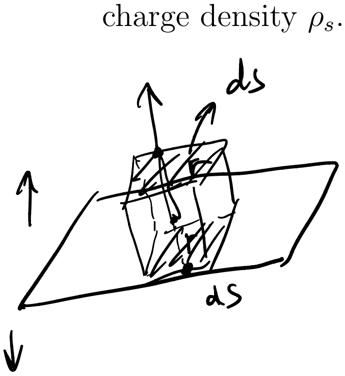


$$|\vec{E_r}| \cdot 2\pi r \cdot \cancel{dl} = \frac{\rho_s \cdot \cancel{dl}}{\varepsilon_0}$$

$$\Rightarrow |\vec{E_r}| = \frac{\rho_s}{2\pi\varepsilon_0 r}$$

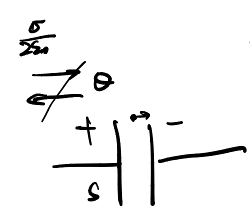
$$\vec{E_r} = \frac{\rho_s}{2\pi\varepsilon_0 r}\hat{r}$$

Ex.6
Determine the electric field intensity of an infinite planar charge with a uniform surface charge density ρ_s.



$$|\vec{E_r}| \cdot 2\cancel{ds} = \frac{\rho_s \cdot \cancel{ds}}{\varepsilon_0}$$

$$\Rightarrow |\vec{E_r}| = \frac{\rho_s}{2\varepsilon_0}$$



$\sigma = \frac{Q}{S}$

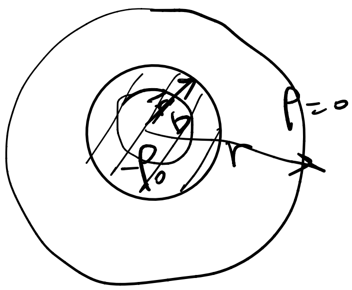
$E_+ = \frac{\sigma}{2\varepsilon_0}$

$E_- = \frac{\sigma}{2\varepsilon_0}$

$E = E_+ + E_- = \frac{\sigma}{\varepsilon_0}$

Ex.7

Determine the **E** field caused by a spherical cloud of electrons with a volume charge density $\rho = -\rho_o$ for $0 \le R \le b$ (both ρ_o and b are positive) and $\rho = 0$ for $R > b$.



① $0 \le r \le b$

$$E(r) \cdot \cancel{4\pi} r^{\cancel{2}} = \frac{-\rho_0 \cdot \frac{4}{3}\cancel{\pi} r^{\cancel{3}}}{\varepsilon_0}$$

$$\Rightarrow \overrightarrow{E(r)} = \frac{-\rho_0 r}{3\varepsilon_0} \hat{r}$$

② $r > b$

$$E(r) \cdot \cancel{4\pi} r^2 = \frac{-\rho_0 \cdot \frac{\cancel{4}}{3}\cancel{\pi} b^3}{\varepsilon_0}$$

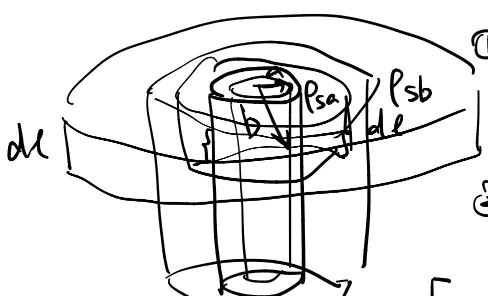
$$\Rightarrow \overrightarrow{E(r)} = \frac{-\rho_0 b^3}{3\varepsilon_0 r^2} \hat{r}$$

Ex.8

Two infinite long coaxial cylindrical surfaces, $r = a$ and $r = b (b > a)$, carry surface charge densities ρ_{sa}, ρ_{sb}, respectively.

a) Determine $\boldsymbol{E}$ everywhere.

b) What must be the relation between a and b in order that $\boldsymbol{E}$ vanishes for $r > b$?



① $r < a$

$E = 0$

② $a \le r \le b$

$$E(r) \cdot \cancel{2\pi} r \cdot \cancel{dl} = \frac{\rho_{sa} \cdot \cancel{2\pi} a \cdot \cancel{dl}}{\varepsilon_0} \Rightarrow \overrightarrow{E(r)} = \frac{\rho_{sa} \cdot a}{\varepsilon_0 r} \hat{r}$$

③ $r > b$

$$E(r) \cdot \cancel{2\pi} r \cdot \cancel{dl} = \frac{\rho_{sa} \cdot \cancel{2\pi} a \cdot \cancel{dl} + \rho_{sb} \cdot \cancel{2\pi} b \, \cancel{dl}}{\varepsilon_0}$$

$$\Rightarrow \overrightarrow{E(r)} = \frac{\rho_{sa} \cdot a + \rho_{sb} \cdot b}{\varepsilon_0 r} \hat{r}$$

(b) $\frac{\rho_{sa} \cdot a + \rho_{sb} \cdot b}{\varepsilon_0 r} = 0 \quad \forall r \qquad \rho_{sa} \cdot a + \rho_{sb} \cdot b = 0$

$$\Rightarrow \frac{\rho_{sa}}{\rho_{sb}} = -\frac{b}{a}$$

3.3 Several Useful Models (Paste on your CTPP!)

Note: The charge distribution should be **uniform**.

different models	E(magnitude)
infinitely long, line charge	$E = \frac{\rho_\ell}{2\pi r\epsilon_0}$
infinite planar charge	$E = \frac{\rho_s}{2\epsilon_0}$
uniform spherical surface charge with radius R	$\begin{cases} E = 0 (r < R) \\ E = \frac{Q}{4\pi r^2\epsilon_0}(r > R) \end{cases}$
uniform sphere charge with radius R	$\begin{cases} E = \frac{Qr}{4\pi R^3}(r < R) \\ E = \frac{Q}{4\pi r^2\epsilon_0}(r > R) \end{cases}$
infinitely long, cylindrical charge with radius R	$\begin{cases} E = \frac{\rho_v r}{2\epsilon_0}(r < R) \\ E = \frac{\rho_v R^2}{2r\epsilon_0}(r > R) \end{cases}$

4 Electric Potential

- **Expression:**

$V_{ba} = -\int_a^b E \cdot d\ell$

$a \xrightarrow{E} b$

$$\mathbf{E} = -\nabla V$$

$\nabla V = -E$

the reason for the negative sign: consistent with the convention that in going against the **E** field, the electric potential V increases.

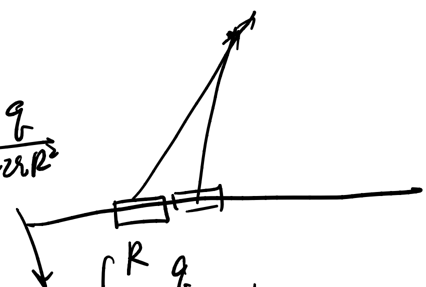
- **Electric Potential Difference:**

$$V_2 - V_1 = -\int_{P_1}^{P_2} \mathbf{E} \cdot d\ell$$

- **Electric Potential due to a Charge Distribution**

$E = \frac{q}{4\pi\varepsilon R^2}$

$$V = \frac{q}{4\pi\epsilon_0 R}$$



$-\int_{-\infty}^{R} \frac{q}{4\pi\varepsilon r^2} dr = \frac{q}{4\pi\varepsilon r} = V$

i. **Line Charge:**

$$V = \frac{1}{4\pi\epsilon_0}\int_{L'} \frac{\rho_\ell}{R} d\ell' \quad (V)$$

ii. **Surface Charge:**

$$V = \frac{1}{4\pi\epsilon_0}\int_{S'} \frac{\rho_s}{R} ds' \quad (V)$$

– **Volume Charge:**

$$V = \frac{1}{4\pi\epsilon_0}\int_{V'} \frac{\rho}{R} dv' \quad (V)$$

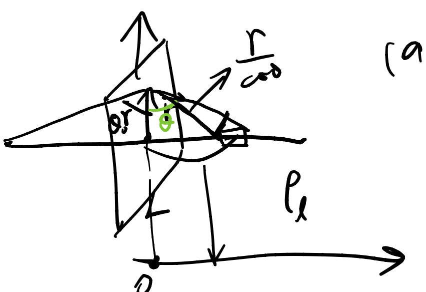
Ex.9

A finite line charge of length L carrying uniform line charge density ρ_l is coincident with the x-axis.

a) Determine V in the plane bisecting the line charge.

b) Determine $\boldsymbol{E}$ on the bisecting plane from ρ_l directly by applying Coulomb's law.

c) Check the answer in part (b) with $-\nabla V$.

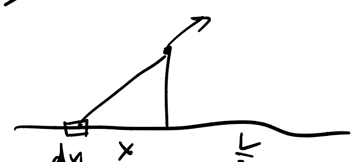


$$r\tan\theta \qquad \tan\theta_0 = \frac{L/2}{r} = \frac{L}{2r} \qquad \theta_0 = \tan^{-1}\frac{L}{2r}$$

(a) $$dl = \frac{r}{\cos\theta}\cdot d\theta \cdot \frac{1}{\cos\theta} = \frac{r\,d\theta}{\cos^2\theta} \checkmark$$

$$dV = \frac{1}{4\pi\varepsilon}\frac{\rho_l\cdot dl}{(r/\cos\theta)} = \frac{\rho_l}{4\pi\varepsilon}\frac{\frac{r\,d\theta}{\cos^2\theta}}{r/\cos\theta} = \frac{\rho_l}{4\pi\varepsilon}\int_{-\theta_0}^{\theta_0}\frac{d\theta}{\cos\theta}$$

both ok



$$dV = \frac{1}{4\pi\varepsilon}\frac{dx\cdot\rho_l}{\sqrt{x^2+r^2}}$$

$$V = \frac{\rho_l}{4\pi\varepsilon}\int_{-\frac{L}{2}}^{\frac{L}{2}}\frac{dx}{\sqrt{x^2+r^2}} \qquad \int_{-\frac{L}{2}}^{\frac{L}{2}}\frac{\rho_l}{4\pi\varepsilon r}\frac{dx}{\sqrt{1+(\frac{x}{r})^2}} = \frac{\rho_l}{4\pi\varepsilon}\int_{-\frac{L}{2r}}^{\frac{L}{2r}}\frac{d(\frac{x}{r})}{\sqrt{1+(\frac{x}{r})^2}}$$

$$\frac{x}{r} = m$$

$$V = \frac{\rho_l}{4\pi\varepsilon}\int_{-\frac{L}{2r}}^{\frac{L}{2r}}\frac{dm}{\sqrt{1+m^2}} = \qquad \frac{d\sinh\theta}{\cosh\theta} = \frac{\cosh\theta\,d\theta}{\cosh\theta}$$

$$\sinh\theta \qquad 1+\sinh^2\theta = \cosh^2\theta$$

$$\theta = \sinh^{-1}(m)$$

$$\Rightarrow \sinh\theta = \frac{L}{2r} = k$$

$$V = \frac{\rho_l}{4\pi\varepsilon}\sinh^{-1}\theta\Big|_{\sinh^{-1}\frac{-L}{2r}}^{\sinh^{-1}\frac{L}{2r}} = \frac{\rho_l}{2\pi\varepsilon}\sinh^{-1}\left(\frac{L}{2r}\right)$$

$$\sinh\theta = \frac{e^\theta - e^{-\theta}}{2} = k$$

$$\sinh^{-1}\left(\frac{L}{2r}\right) = \theta = \ln\left(\frac{L}{2r}+\sqrt{\left(\frac{L}{2r}\right)^2+1}\right)$$

$$e^\theta - \frac{1}{e^\theta} = 2k$$

$$V = \frac{\rho_s}{2\pi\varepsilon}\sinh^{-1}\left(\frac{L}{2r}\right)$$

$$e^\theta = n$$

$$\text{or } \frac{\rho_s}{2\pi\varepsilon}\ln\left(\frac{L}{2r}+\sqrt{\left(\frac{L}{2r}\right)^2+1}\right)$$

$$n^2+1 = 2kn$$

$$\Rightarrow n^2-2kn+1=0 \Rightarrow n = k \pm \sqrt{k^2+1}$$

Ex.9

A finite line charge of length L carrying uniform line charge density ρ_l is coincident with the x-axis.

a) Determine V in the plane bisecting the line charge.

b) Determine $\boldsymbol{E}$ on the bisecting plane from ρ_l directly by applying Coulomb's law.

c) Check the answer in part (b) with $-\nabla V$.

(b)

$$E = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{4\pi\varepsilon} \frac{\rho_s dx}{(\sqrt{x^2+r^2})^2} \cdot \frac{r}{\sqrt{x^2+r^2}} = \frac{\rho_s r}{4\pi\varepsilon} \frac{dx}{(x^2+r^2)^{\frac{3}{2}}}$$

$$= \frac{\rho_s r}{4\pi\varepsilon r^3} \int_{-\frac{L}{2r}}^{\frac{L}{2r}} \frac{d(x/r)}{(1+(\frac{x}{r})^2)^{\frac{3}{2}}}$$

$$\frac{d \sinh^{-1}(x)}{dx} = \frac{1}{\sqrt{x^2+1}}$$

$$= \frac{\rho_s \frac{L}{r}}{4\pi\varepsilon r\sqrt{(\frac{L}{2r})^2+1}} = \frac{\rho_s L}{4\pi\varepsilon r^2} \frac{1}{\sqrt{(\frac{L}{2r})^2+1}}$$

(c)

$$-\nabla V = -\frac{\rho_s}{2\pi\varepsilon_0} \cdot \frac{1}{\sqrt{(\frac{L}{2r})^2+1}} \cdot \frac{\partial \frac{L}{2r}}{\partial r} = + \frac{\rho_s}{2\pi\varepsilon} \frac{1}{\sqrt{(\frac{L}{2r})^2+1}} \left(+\frac{L}{2r^2}\right)$$

$$= \frac{\rho_s}{4\pi\varepsilon r^2} \frac{L}{\sqrt{(\frac{L}{2r})^2+1}}$$

$E = -\nabla V$ is verified

$$\frac{d \sinh^{-1}(x)}{dx} = \frac{1}{\sqrt{x^2+1}} \rightarrow \text{prove:}$$

$$\sinh^{-1}(x) = t$$

$$\Leftrightarrow \frac{e^t - e^{-t}}{2} = x$$

$$t = \ln(x+\sqrt{x^2+1})$$

$$\frac{dt}{dx} = \frac{1+\frac{2x}{2\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{(1+\frac{x}{\sqrt{x^2+1}})(\sqrt{x^2+1}-x)}{1} = \sqrt{x^2+1} - x + x - \frac{x^2}{\sqrt{x^2+1}}$$

$$= \frac{1}{\sqrt{x^2+1}}$$