Q1 (a)

$E_{1t} = E_{2t} \Rightarrow E_1 \sin\alpha_1 = E_2 \sin\alpha_2$ ①

$J_{1n} = J_{2n} \Rightarrow \sigma_1 E_1 \cos\alpha_1 = \sigma_2 E_2 \cos\alpha_2$ ②

From ②, $E_2 = \frac{\sigma_1}{\sigma_2} \frac{\cos\alpha_1}{\cos\alpha_2} E_1$ ③

From ①, $\sin\alpha_2 = \frac{E_1 \sin\alpha_1}{E_2}$ ④

since $\sin^2\alpha_2 + \cos^2\alpha_2 = 1$

we can know $\cos^2\alpha_2 = 1 - \sin^2\alpha_2$ ⑤

Squaring ③ $\Rightarrow E_2^2 = \left(\frac{\sigma_1}{\sigma_2} \frac{\cos\alpha_1}{\cos\alpha_2}\right)^2 E_1^2$ ⑥

From ④ and ⑤, we can get $\cos^2\alpha_2 = 1 - \left(\frac{E_1 \sin\alpha_1}{E_2}\right)^2$ ⑦

From ⑦ and ⑥. $E_2^2 = \left(\frac{\sigma_1}{\sigma_2} \cos\alpha_1\right)^2 \frac{E_1^2}{1 - E_1^2 \frac{\sin^2\alpha_1}{E_2^2}}$

Therefore, we can get that $E_2 = E_1 \sqrt{\sin^2\alpha_1 + \left(\frac{\sigma_1}{\sigma_2} \cos\alpha_1\right)^2}$

Direction : $\tan\alpha_2 = \frac{\sigma_2}{\sigma_1} \tan\alpha_1$

(b)

$D_{2n} - D_{1n} = \rho_s \Rightarrow \varepsilon_2 E_{2n} - \varepsilon_1 E_{1n} = \rho_s$

Also, $E_{2n} = E_2 \cos\alpha_2$ and $E_{1n} = E_1 \cos\alpha_1$

$\Rightarrow \rho_s = \varepsilon_2 \frac{\sigma_1}{\sigma_2} E_1 \cos\alpha_1 - \varepsilon_1 E_1 \cos\alpha_1$

$= \left(\frac{\sigma_1}{\sigma_2} \varepsilon_2 - \varepsilon_1\right) \cdot E_1 \cos\alpha_1$

(c)

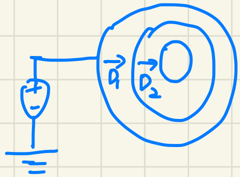
We can conclude that $\sigma_1 = \sigma_2 = 0$

From part (a), $E_2 = E_1$ and $\tan\alpha_2 = \tan\alpha_1$

From part (b), $\rho_s = (\varepsilon_2 - \varepsilon_1) \cdot E_1 \cos\alpha_1$

Q2

(a)



$$\vec{D_1} = \vec{D_2} = \vec{D_{net}} \quad ①$$

$$C_1 = \frac{2\pi L \varepsilon_1}{\ln\frac{a}{c}} \quad ②$$

$$C_2 = \frac{2\pi L \varepsilon_2}{\ln\frac{c}{b}} \quad ③$$

$$\Rightarrow C_{eq} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} = \frac{2\pi L \varepsilon_1 \varepsilon_2}{\varepsilon_1 \ln\frac{c}{b} + \varepsilon_2 \ln\frac{a}{c}} \quad ④$$

$$\Rightarrow Q_{ext} = C_{eq}\cdot V_0 = \frac{2\pi L \varepsilon_1 \varepsilon_2 V_0}{\varepsilon_1 \ln\frac{c}{b} + \varepsilon_2 \ln\frac{a}{c}} \quad ⑤$$

For a cylindrical capacitor:

$$D = \frac{Q}{2\pi L r} \Rightarrow D_{net} = \frac{Q_{ext}}{2\pi L r} = \frac{\varepsilon_1 \varepsilon_2 V_0}{r[\varepsilon_1 \ln\frac{c}{b} + \varepsilon_2 \ln\frac{a}{c}]} \quad ⑥$$

Then we consider $J = \frac{\sigma D}{\varepsilon}$

$\Rightarrow$ in region $r \in (a, c)$, $J_1 = \frac{\sigma_1 D_1}{\varepsilon_1} = \frac{\sigma_1 D_{net}}{\varepsilon_1} \Rightarrow J_1 = \frac{\sigma_1 \varepsilon_2 V_0}{r[\varepsilon_1 \ln\frac{c}{b} + \varepsilon_2 \ln\frac{a}{c}]}$ $(a<r<c)$

and similarly, we can get $J_2 = \frac{\sigma_2 \varepsilon_1 V_0}{r[\varepsilon_1 \ln\frac{c}{b} + \varepsilon_2 \ln\frac{a}{c}]}$ $(c<r<b)$

(b) From ⑤

we can know $\sigma_{inner} = \frac{Q_{ext}}{A_{inner}} = \frac{L\varepsilon_1 \varepsilon_2 V_0}{2a^2[\varepsilon_1 \ln\frac{c}{b} + \varepsilon_2 \ln\frac{a}{c}]}$

$$\sigma_{outer} = \frac{L\varepsilon_1 \varepsilon_2 V_0}{2b^2[\varepsilon_1 \ln\frac{c}{b} + \varepsilon_2 \ln\frac{a}{c}]}$$

since no charge accumulation occurs on the interface between the two electrical insulators, the charge density at the interface is 0