① Poisson's Equation

$$\nabla^2 V = -\frac{\rho}{\varepsilon}$$

- Cartesian System:

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

- Cylindrical System:

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial z^2}$$

- Spherical System:

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \varphi^2}$$

② Laplace's Equation

Simple medium with no free charge : $\nabla^2 V = 0$

Problem involving conductors:

a) use Laplace equation to obtain 电势 V

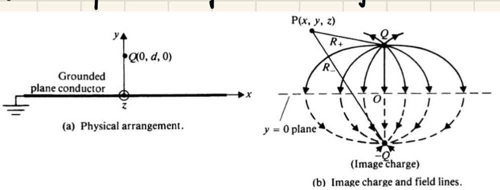
b) use $E = -\nabla V$ to work out E

c) use $\rho_s = \varepsilon E$ to get charge density on surface

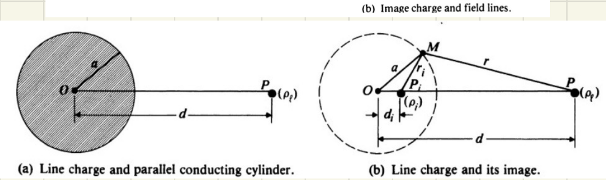
③ Uniqueness of Electrostatistic Solutions

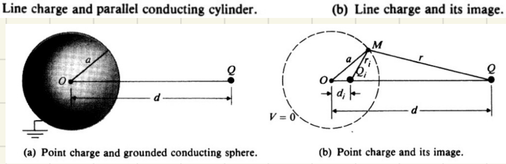
Solution of Poisson's Equation or Laplace's Equation that satisfies the given boundary condition

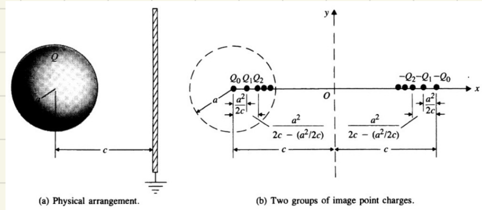
④



(a) Physical arrangement. (b) Image charge and field lines.


(a) Line charge and parallel conducting cylinder. (b) Line charge and its image.


(a) Point charge and grounded conducting sphere. (b) Point charge and its image.


(a) Physical arrangement. (b) Two groups of image point charges.

⑤ Boundary Condition Value in 直角

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \quad \text{直角坐标系}$$

$$0 = \frac{1}{X(x)}\frac{d^2X(x)}{dx^2} + \frac{1}{Y(y)}\frac{d^2Y(y)}{dy^2} + \frac{1}{Z(z)}\frac{d^2Z(z)}{dz^2}$$

$$\left\{\begin{aligned} &\frac{1}{X(x)}\cdot\frac{d^2X(x)}{dx^2} = -k_x^2 \\ &\frac{1}{Y(y)}\cdot\frac{d^2Y(y)}{dy^2} = -k_y^2 \\ &\frac{1}{Z(z)}\cdot\frac{d^2Z(z)}{dz^2} = -k_z^2 \end{aligned}\right. \quad k_x^2 + k_y^2 + k_z^2 = 0$$

k_x^2	k_x	$X(x)$	e form $X(x)$
0	0	$A_0x + B_0$	
+	k	$A_1\sin kx + B_1\cos kx$	$C_1e^{jkx} + D_1e^{-jkx}$
−	jk	$A_2\sinh kx + B_2\cosh x$	$C_2e^{kx} + D_2e^{-kx}$

If V is independent of x, we can see $X(x) = 0$

If $V \to 0$ or ∞ or $X \to \infty$, k_x^2 is negative

⑥ Boundary value in 圆柱型

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

assuming V is independent of Z.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} = 0$$

$$V(r,\phi) = R(r)\,\Phi(n)$$

$$\frac{d^2\Phi(\phi)}{d\phi^2} + k^2\Phi(n) = 0$$

$$\Phi(\phi) = A_\phi \sin n\phi + B_\phi \cos n\phi$$

$$\frac{r}{R(r)}\cdot\frac{d}{dr}\left[r\frac{dR(r)}{dr}\right]=k^2$$

$$\Rightarrow r^2\frac{d^2R(r)}{dr^2}+r\frac{dR(r)}{dr}-n^2R(r)=0$$

general sol: $R(r)=A_r\cdot r^n+B_r\cdot r^{-n}$

$$V_n(r,\phi)=r^n(A_n\sin n\phi+B_n\cos n\phi)+$$

$(n\neq 0)$ $\quad r^{-n}(A_n'\sin n\phi+B_n'\cos n\phi)$

$$\frac{d}{dr}\left[r\frac{dR(r)}{dr}\right]=0$$

$$V(r)=C_1\ln r+C_2$$

⑦ Boundary value in 球体

$$\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right)+\frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)=0$$

$$\frac{1}{\Gamma(R)}\frac{d}{dR}\left[R^2\frac{d\Gamma(R)}{dR}\right]+\frac{1}{\Theta(\theta)\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta(\theta)}{d\theta}\right)=0$$

$$\frac{1}{\Gamma(R)}\cdot\frac{d}{dR}\left[R^2\frac{d\Gamma(R)}{dR}\right]=k^2$$

n	$P_n(\cos\theta)$
0	1
1	$\cos\theta$
2	$\frac{1}{2}(3\cos^2\theta-1)$
3	$\frac{1}{2}(5\cos^3\theta-3\cos\theta)$

$$R^2\frac{d^2\Gamma(R)}{dR^2}+2R\frac{d\Gamma(R)}{dR}-k^2\Gamma(R)=0$$

$$\Gamma_n(R)=A_nR^n+B_nR^{-(n+1)}$$

$$\frac{1}{\Theta(\theta)\sin\theta}\frac{d}{d\theta}\left[\sin\theta\frac{d\Theta(\theta)}{d\theta}\right]=-k^2$$

$$\frac{d}{d\theta}\left[\sin\theta\frac{d\Theta(\theta)}{d\theta}\right]+n(n+1)\Theta(\theta)\sin\theta=0$$

$$\Theta(\theta)=P_n(\cos\theta)$$

$$V_n(R,\theta)=\left[A_nR^n+B_nR^{-(n+1)}\right]P_n(\cos\theta)$$

⑧ Boundary Conditions

$$\begin{cases}\nabla\cdot J=0\\ \nabla\times\frac{J}{\sigma}=0\end{cases}\qquad\begin{cases}\oint_s J\cdot ds=0\\ \oint_c\frac{1}{\sigma}J\,dl=0\end{cases}$$

Normal: $J_{1n}=J_{2n}$ $\quad$ Tangential: $\frac{J_{1t}}{J_{2t}}=\frac{\sigma_1}{\sigma_2}$

$$\rho_s=\left(\varepsilon_1\frac{\sigma_2}{\sigma_1}-\varepsilon_2\right)E_{2n}=\left(\varepsilon_1-\varepsilon_2\frac{\sigma_1}{\sigma_2}\right)E_{1n}$$

$$\rho_s=\varepsilon_1E_{1n}=D_{1n}$$

⑨ Power Dissipation

Joule's Law

$$P=\int_V\vec{E}\cdot\vec{J}\,dv$$

$$P=\int_L E\,dl\int_s J\,ds$$

⑩ 洛仑兹力

$$\vec{F}=q(\vec{E}+\vec{u}\times\vec{B})$$

⑪ Fundamental Postulates

$$\nabla\cdot B=0,\ \oint_s\vec{B}\cdot ds=0$$

$$\nabla\times B=0,\ \oint_c\vec{B}\cdot dl=\mu_0 I$$

$$\nabla\cdot J=\frac{\nabla\cdot(\nabla\cdot\vec{B})}{\mu_0}=0$$

$$\nabla\cdot J=\frac{\partial\rho}{\partial t}=0$$

⑫ Vector Magnetic Potential

$$\vec{B}=\nabla\times\vec{A}$$

$$\Phi=\int_s\vec{B}\cdot d\vec{s}=\oint_c\vec{A}\,dl$$

Assume $\nabla\cdot\vec{A}=0$

$$\nabla\times\vec{B}=-\nabla^2A$$

$$\nabla^2A=-\mu_0J$$

$$A=\frac{\mu_0}{4\pi}\int_{V'}\frac{J}{R}dv'$$

$$J\,dv'=JS\,dl'=I\,dl'$$

$$A=\frac{\mu_0 I}{4\pi}\oint_{c'}\frac{dl'}{R}$$

$$B=\frac{\mu_0 I}{4\pi}\oint_{c'}\frac{dl'\times a_R}{R^2}$$

$$=\oint_{c'}dB$$

$$dB=\frac{\mu_0 I}{4\pi}\left(\frac{d\vec{l}'\times\vec{R}}{R^3}\right)$$

⑬ Magnetic Dipole

$$m=I\int ds$$

$$A_{dip}(R)=\frac{\mu_0\vec{m}\times\vec{a}_R}{4\pi R^2}$$

$$=\frac{\mu_0 m\sin\theta}{4\pi R^2}\vec{a}_\phi$$

$$B_{dip}(R)=\nabla\times\vec{A}$$

$$=\frac{\mu_0 m}{4\pi R^3}(2\cos\theta\,\hat{a}_r+\sin\theta\,\hat{a}_\theta)$$

$$B_{dip}(R)=\frac{\mu_0}{4\pi R^2}\left[3(\vec{m}\cdot\hat{a}_r)\hat{a}_r-\vec{m}\right]$$

⑭ 磁场能

$$\vec{B}=-\mu_0\nabla V_m$$

$$V_{m2}-V_{m1}=-\int_{P_1}^{P_2}\frac{1}{\mu_0}\vec{B}\,dl$$

$$V_m=\frac{1}{4\pi}\int_{V'}\frac{\rho_m}{R}dv'$$

$$V_m=\frac{\vec{m}\cdot\hat{a}_R}{4\pi R^2}$$

⑮ Magnetization

$$M=\lim_{\Delta V\to 0}\frac{\sum_{k=1}^{n\Delta v}m_k}{\Delta V}$$

$$\vec{J}_m=\nabla\times\vec{M}$$

$$\vec{J}_{ms}=\vec{M}\times\hat{a}_n$$

$$\vec{A}=\frac{\mu_0}{4\pi}\int_{V'}\frac{\nabla'\times\vec{M}}{R}dv'+\frac{\mu_0}{4\pi}\oint_{S'}\frac{\vec{M}\times\vec{a}_n'}{R}ds'$$

⑯

$$\rho_{ms}=\vec{M}\cdot\hat{a}_n$$

$$\rho_m=-\nabla\cdot\vec{M}$$