ECE 2300
Recitation Class 1

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Pre-class

- 3 exams (25% each for mid, 30% for final)
- 6 in-class quizzes (20% in total)
- Homework not graded, recommended to finish

- RC class format: slides? Written notes?

1.1 Review on terminologies

- Vector: Both magnitude & direction.

  (force, velocity. acceleration)

- Scalar: Only size or magnitude.

  (speed, temperature, mass)

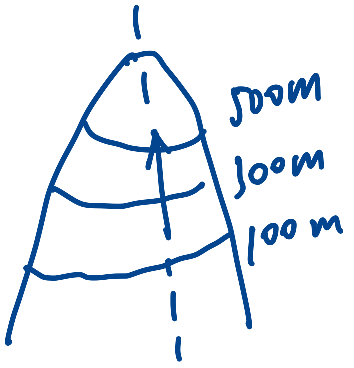
1.1 Review on terminologies

- Del: Operator used on both scalar & vector function.

  ∇

  $\vec{V}$ $\nabla\vec{V}$

- Gradient: ~ of a scalar-valued differentiable function of several variables



is a vector field that describes the direction / rate of fastest increase

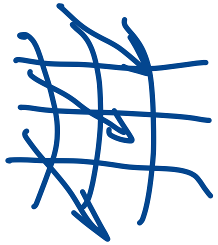
contour

1.1 Review on terminologies

$\nabla \cdot \vec{v}$

- Divergence: operator on vector field $\Rightarrow$ produce a scalar field

giving the quantity of the vector field's source at each point.



water pass through the net.

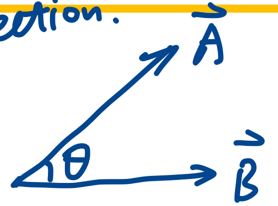
- Curl: operator on vector field $\Rightarrow$ produce a scalar field

$\nabla \times \vec{v}$ giving the infinitesimal circulation of the vector field.

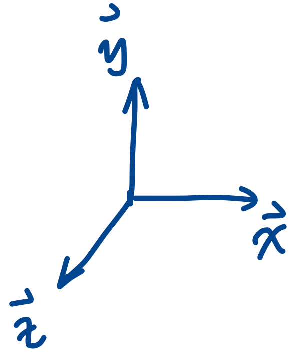
- https://www.bilibili.com/video/BV19s41157Z4/?spm_id_from=333.999.0.0&vd_source=f0a8eb85590d4c542e941ee727d8d28c

1.2 Vector Manipulation

- Dot Products: $\vec{A}\cdot\vec{B} = AB\cos\theta \Rightarrow$ projection.

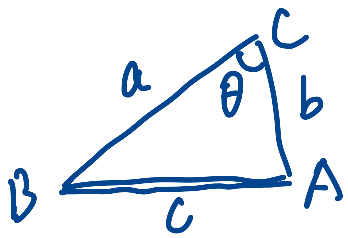


- commutative: $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
- distributive: $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$
- NOT ASSOCIATIVE: $\vec{A}\cdot(\vec{B}\cdot\vec{C}) \neq (\vec{A}\cdot\vec{B})\cdot\vec{C}$



$(\vec{x}\cdot\vec{y})\cdot\vec{z} \neq \vec{x}\cdot(\vec{y}\cdot\vec{z})$

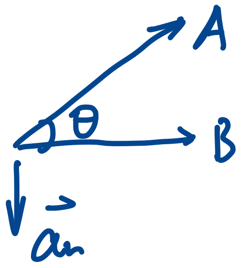
A, B, C in a triangle:



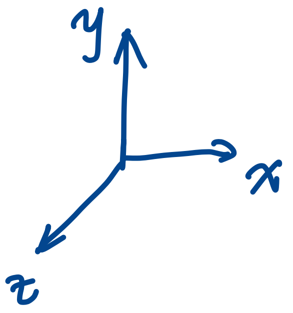
$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

1.2 Vector Manipulation

- Cross Products: $\vec{A} \times \vec{B} = |AB \sin\theta|\, \vec{a}_n \Rightarrow$ area. "right hand rule"



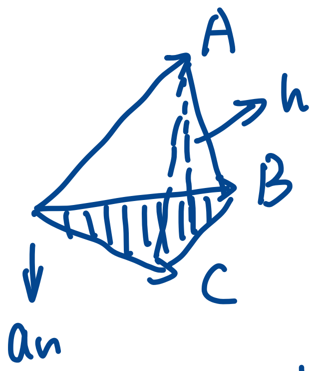
  - result is always perpendicular to both $\vec{A}$ and $\vec{B}$
  - NOT commutative: $\vec{A} \times \vec{B} \neq \underbrace{\vec{B} \times \vec{A}}$ $\quad \vec{a}_n{}' = -\vec{a}_n$
  - Distributive: $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$
  - NOT Associative: $\vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C}$



$$\vec{x} \times \underbrace{(\vec{y} \times \vec{z})}_{\vec{x}} = (\vec{x} \times \vec{y}) \times \vec{z}$$

1.2 Vector Manipulation

- Volume:



$$S_{BC} = |\vec{B} \times \vec{C}|$$

$$h = -\vec{A} \cdot \frac{(\vec{B} \times \vec{C})}{|\vec{B} \times \vec{C}|}$$

$$V = \vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$$

1.2 Vector Manipulation

- BAC-CAB rule (Vector Triple Product):

$$\underbrace{\vec{A}}_{\text{vector}} \times \underbrace{(\vec{B} \times \vec{C})}_{\text{vector}} = \vec{B}\,\underbrace{(\vec{A} \cdot \vec{C})}_{\text{scalar}} - \vec{C}\,\underbrace{(\vec{A} \cdot \vec{B})}_{\text{scalar}}$$

vector

vector

vector

vector

unit vec

$$\vec{A} = A_x \cdot \vec{a}_x + A_y \cdot \vec{a}_y + A_z \cdot \vec{a}_z \Rightarrow \text{hint for proof.}$$

value

1.3 Coordinates

- Basis:

Number of linear independent vectors that we can derive any vector in this coordinate system from.

n = dimension of the space.

$$Sol = a_1 \cdot \vec{n_1} + a_2 \cdot \vec{n_2} + a_3 \cdot \vec{n_3} \cdots$$ solution space.

unit vector

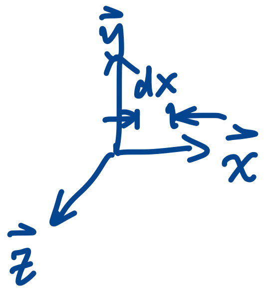
$$\vec{A} = \vec{a}_{u1} \cdot A_{u1} + \vec{a}_{u2} \cdot A_{u2} + \vec{a}_{u3} \cdot A_{u3}$$

scalar

Norm:

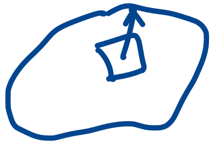
$$|A| = \sqrt{A_{u1}^2 + A_{u2}^2 + A_{u3}^2}$$

- Differential length, area, volume:



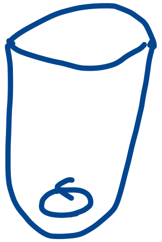
length: $d\vec{l} = \vec{a}_{u_1}(h_1 du_1) + \vec{a}_{u_2}(h_2 du_2) + \vec{a}_{u_3}(h_3 du_3)$

area:



closed surface

direction points outward.



right hand rule.

$d\vec{s} = \vec{a}_n \cdot ds$

$\Downarrow$

perpendicular to the surface.

$ds_1 = \underline{h_2} h_3 du_2 du_3$

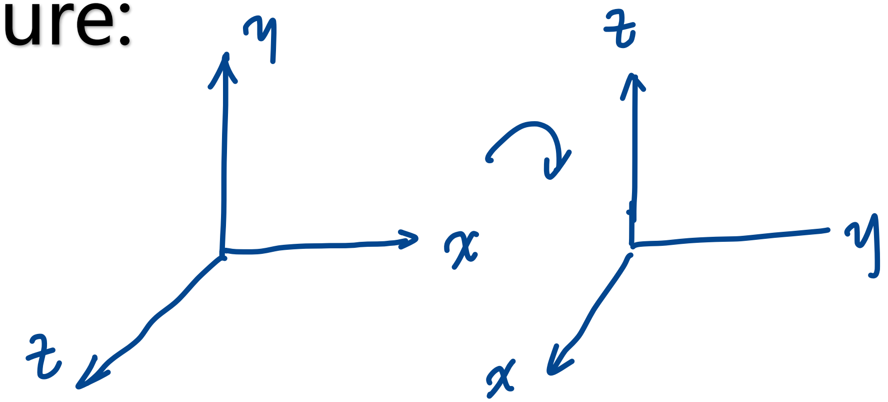
volume: $dv = h_1 h_2 h_3 du_1 du_2 du_3$

$\downarrow \downarrow \downarrow$

metric coefficient.

1.3.1 Cartesian Coordinates

- Figure:



$(u_1, u_2, u_3) = (x, y, z)$

- Right hand rule:

$\vec{a}_x \times \vec{a}_y = \vec{a}_z$

$\vec{A} = \vec{a}_x \cdot A_x + \vec{a}_y \cdot A_y + \vec{a}_z \cdot A_z$

veltor scalar.

1.3.1 Cartesian Coordinates

- Dot/Cross product:

$$\vec{A} = \vec{a}_x \cdot A_x + \vec{a}_y \cdot A_y + \vec{a}_z \cdot A_z$$

$$\vec{B} = \vec{a}_x \cdot B_x + \vec{a}_y \cdot B_y + \vec{a}_z \cdot B_z$$

$$\vec{A} \cdot \vec{B} = A_x \cdot B_x + A_y \cdot B_y + A_z \cdot B_z .$$

$$\vec{A} \times \vec{B} = \begin{array}{ccc} + & - & + \end{array}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \vec{a}_x (A_y B_z - B_y A_z) - \vec{a}_y (A_x B_z - B_x A_z) + \vec{a}_z (A_x B_y - B_x A_y)$$

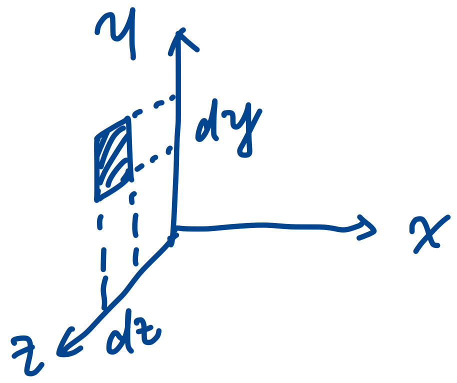
1.3.1 Cartesian Coordinates

$h_1 = h_2 = h_3 = 1.$

- Differential length:

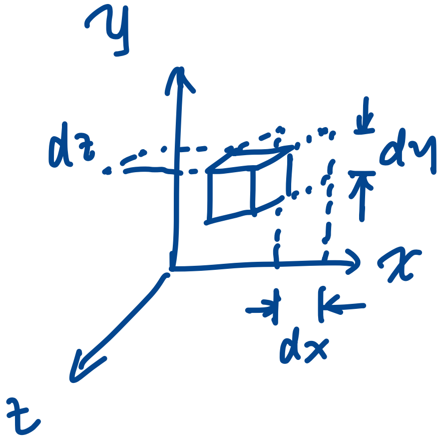
$\vec{dl} = \vec{a}_x dx + \vec{a}_y dy + \vec{a}_z dz$

- Differential area:



$ds_x = dy dz$

- Differential volume:



$dv = dx \cdot dy \cdot dz$

Ex1. Vector

in Cartesian Sys.

$\vec{A} = (1 \quad 2 \quad -3)$

$\vec{B} = (0 \quad -4 \quad 1)$

$\vec{C} = (5 \quad 0 \quad -2)$

$5+6$

$(\vec{A} \times \vec{B}) \times \vec{C}$

$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

$= -\underbrace{\vec{C} \times (\vec{A} \times \vec{B})}$

BAC-CAB

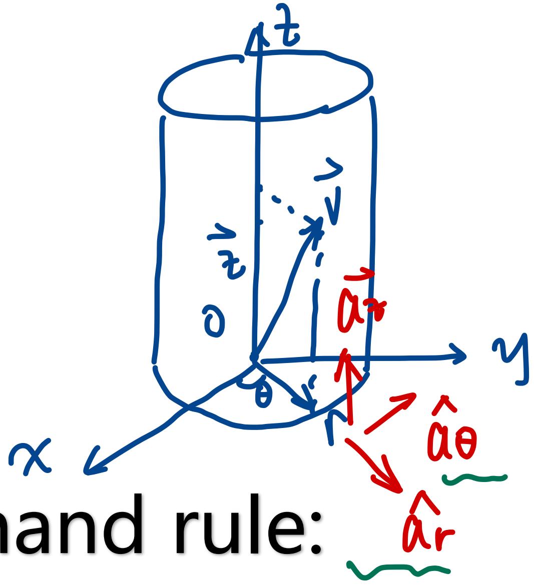
$= -\left(\vec{A}(\vec{C} \cdot \vec{B}) - \vec{B}(\vec{C} \cdot \vec{A})\right)$

$= -\left((1 \quad 2 \quad -3) \cdot (-2) - (0 \quad -4 \quad 1)(11)\right)$

$= -(-2 \quad -4 \quad 6) + (0 \quad -44 \quad 11)$

1.3.2 Cylindrical Coordinates

- Figure:



rotating according to θ

- Right hand rule:

$$\vec{a}_r \times \vec{a}_\theta = \vec{a}_z$$

1.3.2 Cylindrical Coordinates

- Differential length:

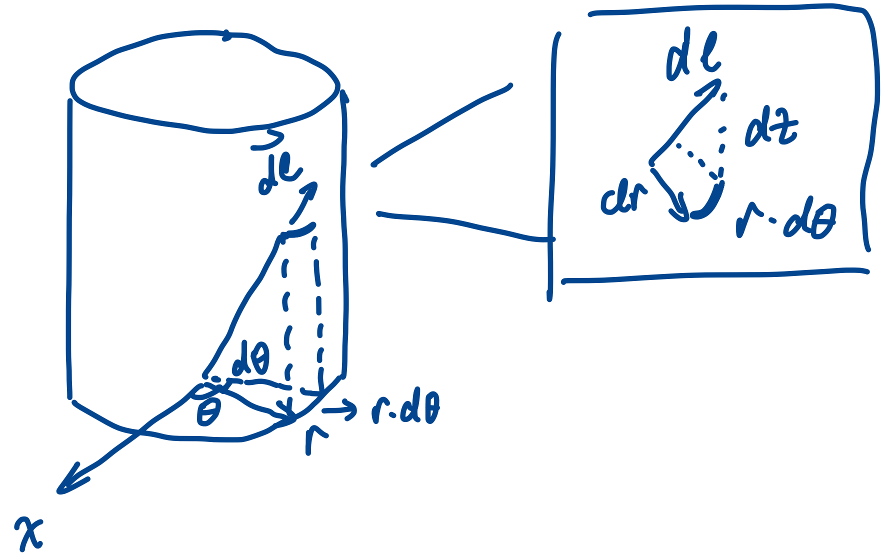
$$d\vec{l} = \vec{a}_r dr + \vec{a}_\theta r\, d\theta + \vec{a}_z\, dz$$

- Differential area:

$$ds_r = r\, d\theta \cdot dz$$

- Differential volume:

$$dv = r\, dr\, d\theta\, dz$$



metric coefficients: ratio of unit length.

$$h_1 = 1 = h_3 \qquad h_2 = r$$

1.3.2 Cylindrical Coordinates

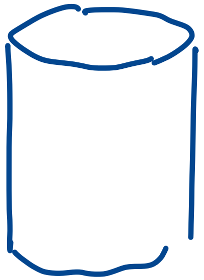
- Transformation ~~vector~~ metrix.:

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_z \end{bmatrix}$$

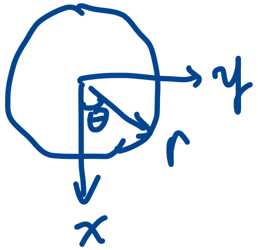
$$\begin{bmatrix} A_r \\ A_\theta \\ A_z \end{bmatrix} = \underline{\underline{M}} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

M^{-1}

cartesian. metrix cylindrical

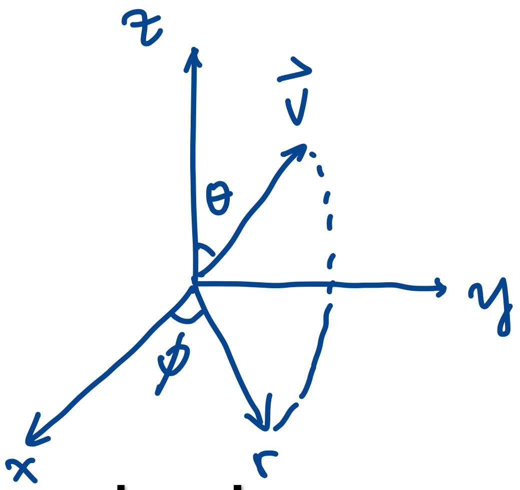


$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$$



1.3.3 Spherical Coordinates

- Figure:



$(u_1\ u_2\ u_3) = (R, \theta, \phi)$

⇓

vector form

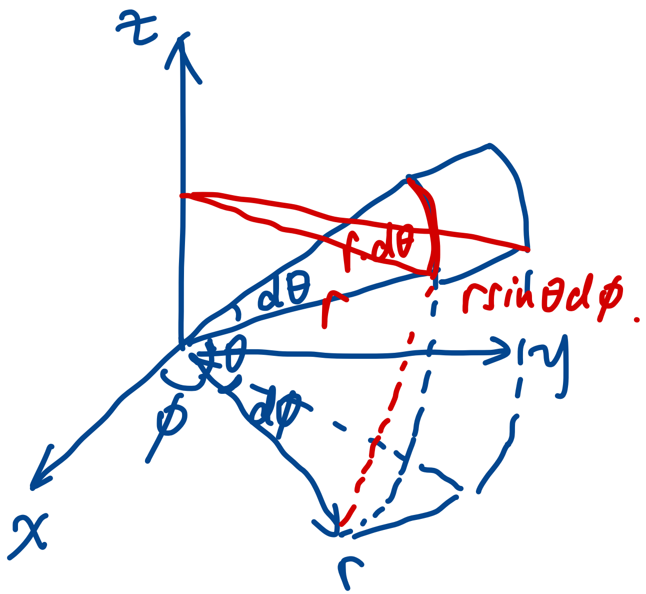
- Right hand rule:

$\vec{a}_R \times \vec{\theta} = \vec{\phi}$

1.3.3 Spherical Coordinates

- Differential length:

$$d\vec{l} = \vec{a}_R\, dR + \vec{a}_\theta\, R\, d\theta + \vec{a}_\phi\, R\sin\theta\, d\phi$$



$h_1 = 1$

$h_2 = R$

$h_3 = R\sin\theta$.

- Differential area:

$$dS_R = R^2 \cdot \sin\theta\, d\theta\, d\phi$$

- Differential volume:

$$dV = R^2 \cdot \sin\theta\, dr\, d\theta\, d\phi$$

1.3.3 Spherical Coordinates

- Transformation ~~vector~~ metrix.:

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & -\sin\phi & \cos\theta\cos\phi \\ \sin\theta\sin\phi & \cos\phi & \cos\theta\sin\phi \\ \cos\theta & 0 & -\sin\theta \end{bmatrix} \begin{bmatrix} A_R \\ A_\theta \\ A_\phi \end{bmatrix}$$

cylindrical $\Rightarrow$ spherical

$\downarrow$ $\uparrow$

cartesian

$$\begin{cases} x = R\sin\theta\cos\phi \\ y = R\sin\theta\sin\phi \\ z = R\cos\theta \end{cases}$$

1.4 Integrals

- Line Integral:

$\int_c F \cdot dl$ $\quad$ $F \Rightarrow$ vector function

$dl \rightarrow$ small displacement vector. ($dl = \hat{x}\,dx + \hat{y}\,dy + \hat{z}\,dz$)

$\oint F \cdot dl$

circle $\Rightarrow$ loop integration.

1.4 Integrals

- Surface Integral:

$\int_S F \cdot ds$ $F \Rightarrow$ vector function

$ds \Rightarrow$ small area (differential area).

$\oint F \cdot ds$

circle $\Rightarrow$ closed surface.

1.4 Integrals

- Volume Integral:

$\int_V F \cdot dv$

$F \Rightarrow$ vector function

$dv \Rightarrow$ differential volume.

e.g. Cartesian

$$\int_V F\,dv = \int_V (F_x\hat{x} + F_y\hat{y} + F_z\hat{z})\,dv$$

$$= \hat{x}\int_V F_x\,dv + \hat{y}\int_V F_y\,dv + \hat{z}\cdot\int_V F_z\,dv.$$

1.5 Language used in Maxwell Equations

- Gradient:

∇V : V is a scalar function.

$$\vec{a}_{u1}\frac{\partial V}{h_1\partial u_1}+\vec{a}_{u2}\frac{\partial V}{h_2\partial u_2}+\vec{a}_{u3}\frac{\partial V}{h_3\partial u_3}$$

$\Rightarrow$ cartesian Sys. $h_1=h_2=h_3=1$

$$\Rightarrow \nabla V=\left(\frac{\partial V}{\partial x},\frac{\partial V}{\partial y},\frac{\partial V}{\partial z}\right) \qquad \nabla=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right) \text{ operator}$$

$\Downarrow$

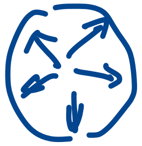
used as a vector $\Rightarrow$ though it is not

1.5 Language used in Maxwell Equations

- Divergence:

$$\operatorname{div}\vec{A} = \lim_{\Delta V \to 0} \frac{\oint_S \vec{A}\, d\vec{s}}{\Delta V}$$

net outward



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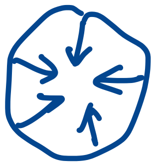
net inward



sink.

$$\operatorname{div}\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{z} = \nabla \cdot A = \left(\frac{\partial}{\partial x}\ \frac{\partial}{\partial y}\ \frac{\partial}{\partial z}\right)\cdot\left(A_x\ A_y\ A_z\right)$$

1.5 Language used in Maxwell Equations

- Curl:

$$\mathrm{curl}\,A = \nabla \times \vec{A} = \lim_{\Delta S \to 0} \frac{1}{\underset{\text{surface}}{\Delta S}} \left[\vec{a}_n \underbrace{\oint_C \vec{A} \cdot d\vec{l}}_{\text{loop integral}} \right]_{max}$$

$$\mathrm{curl}\,A = \nabla \times \vec{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} \vec{a}_{u_1} h_1 & \vec{a}_{u_2} h_2 & \vec{a}_{u_3} h_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix}$$

cartesian $\Rightarrow$ $\nabla \times \vec{A} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$

1.5 Language used in Maxwell Equations

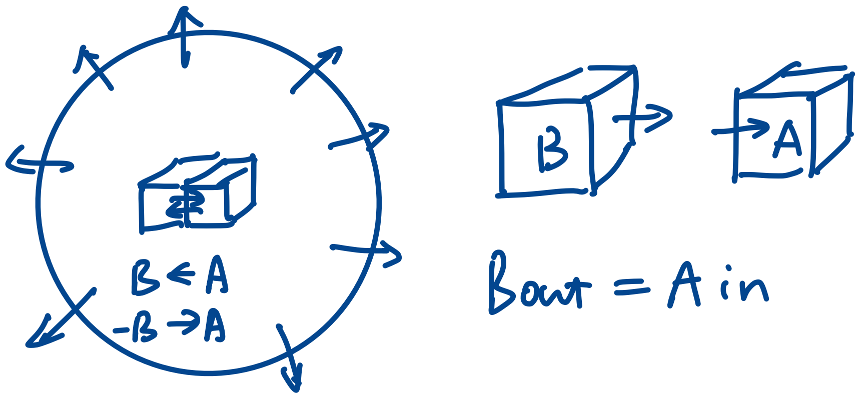
- Curl:

1.6 Useful Vector theorems

- Divergence Theorem: 3D ⇒ 2D

$$\int_V \nabla \cdot \vec{A}\, dv = \oint_S \vec{A} \cdot d\vec{s}$$

volumn integration ⇒ surface integration.



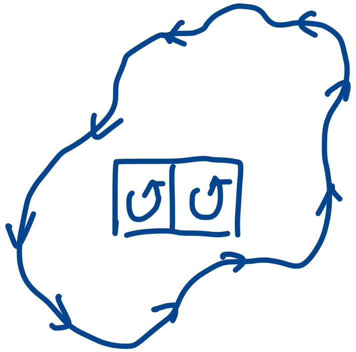
Bout = A in

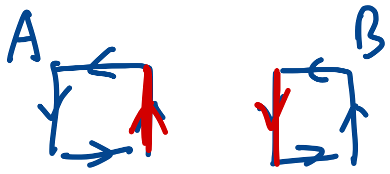
1.6 Useful Vector theorems

- Stokes' s Theorem:

* corresponding theorem of divergence theorem in curl.

$$\int_S (\nabla \times \vec{A}) \, d\vec{s} = \oint_C \vec{A} \cdot d\vec{l}$$





cancelled out.

1.6 Useful Vector theorems

- Null identities:

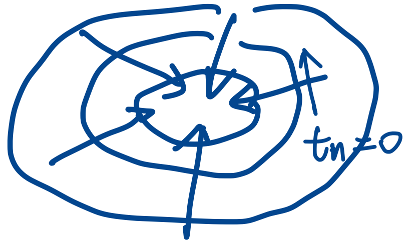
$\nabla \times (\nabla V) = 0$

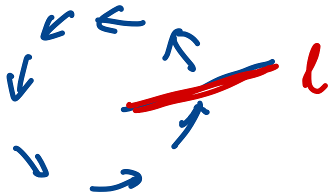
curl of gradient. = 0

$\nabla \cdot (\nabla \times \vec{A}) = 0$

div of curl = 0

For grad. direct $\perp$ contour





curl : how rotational it is.

div: projection $\vec{v} \to l = 0$

1.6 Useful Vector theorems

- Helmholtz Theorem (Fundamental theorem of vector calculus):

Ex2. Theorems application

- (HW1-8) Given a vector function $\mathbf{F} = \mathbf{a_x}(x + c_1 z) + \mathbf{a_y}(c_2 x - 3z) + \mathbf{a_z}(x + c_3 y + c_4 z)$.

  $c_1 = 1$, $c_2 = 0$, c_4; $x - 3y - z$

  a) Determine the constants c_1, c_2, and c_3 if $\mathbf{F}$ is irrotational. curl $= 0$

  b) Determine the constant c_4 if $\mathbf{F}$ is also solenoidal.

  c) Determine the scalar potential function V whose negative gradient equals $\mathbf{F}$.

a)

$$\nabla \times F = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x + c_1 z & c_2 x - 3z & x + c_3 y + c_4 z \end{vmatrix} = \vec{a}_x (c_3 + 3) + \vec{a}_y (c_1 - 1) + \vec{a}_z \cdot c_2 = 0$$

$$\begin{cases} c_3 + 3 = 0 \\ c_1 - 1 = 0 \\ c_2 = 0 \end{cases} \Rightarrow \begin{cases} c_1 = 1 \\ c_2 = 0 \\ c_3 = -3 \end{cases}$$

Ex2. Theorems application

b) $\nabla \cdot \vec{F} = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z} = 1 + 0 + C_4 = 0 \quad \Rightarrow C_4 = -1$

c) Null identity: $\text{curl} = 0 \Rightarrow$ can be formed in a gradient.

$$-\nabla V = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right) = \vec{F} = \vec{a}_x (x+z) + \cdots$$

$$\Rightarrow -V = \frac{1}{2}x^2 + xz + f(y,z)$$

$$\frac{\partial}{\partial y} f(y,z) = -3z \quad \Rightarrow f(y,z) = -3yz + g(z)$$

$$\frac{\partial}{\partial z} g(z) = x - 3y + g'(z) \quad \Rightarrow g(z) = -\frac{1}{2}z^2 + C$$

$$\Rightarrow V = -\frac{1}{2}x^2 - xz + 3yz + \frac{1}{2}z^2 + C$$

Thank You

Credit to Deng Naihao for this slides & information

Also Boyuan Zhang for HW questions