HW4-1

Use subscripts d and a to denote dielectric and air regions respectively. $\nabla^2 V = 0$ in both regions.

$$V_d = c_1 y + c_2,\quad \bar{E}_d = -\bar{a}_y c_1,\quad \bar{D}_d = -\bar{a}_y \epsilon_0 \epsilon_r c_1.$$

$$V_a = c_3 y + c_4,\quad \bar{E}_a = -\bar{a}_y c_3,\quad \bar{D}_a = -\bar{a}_y \epsilon_0 c_3.$$

B.C: At $y=0$, $V_d = 0$; at $y=d$, $V_a = V_0$;

at $y = 0.8d$: $V_d = V_a$, $\bar{D}_d = \bar{D}_a$.

Solving: $c_1 = \frac{V_0}{(0.8+0.2\epsilon_r)d}$, $c_2 = 0$, $c_3 = \frac{\epsilon_r V_0}{(0.8+0.2\epsilon_r)d}$, $c_4 = \frac{(1-\epsilon_r)V_0}{1+0.25\epsilon_r}$.

a) $V_d = \frac{5yV_0}{(4+\epsilon_r)d}$, $\quad \bar{E}_d = -\bar{a}_y \frac{5V_0}{(4+\epsilon_r)d}$.

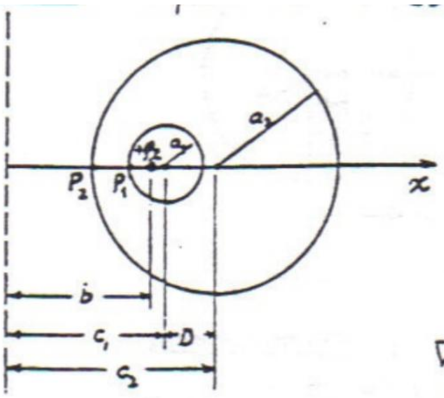
b) $V_a = \frac{5\epsilon_r y - 4(\epsilon_r - 1)d}{(4+\epsilon_r)d} V_0$, $\quad \bar{E}_a = -\bar{a}_y \frac{5\epsilon_r V_0}{(4+\epsilon_r)d}$.

c) $(\rho_s)_{y=d} = -(D_a)_{y=d} = \frac{5\epsilon_0 \epsilon_r V_0}{(4+\epsilon_r)d}$.

$(\rho_s)_{y=0} = (D_d)_{y=0} = -\frac{5\epsilon_0\epsilon_r V_0}{(4+\epsilon_r)d}$.

HW4-4

Eq. (4-61): $c_1 = \frac{1}{2D}(a_2^2 - a_1^2 - D^2)$; Eq. (4-62): $c_2 = \frac{1}{2D}(a_2^2 - a_1^2 + D^2)$.



Eq. (4-55): $b^2 = c_1^2 - a_1^2$;

Eq. (4-56): $b^2 = c_2^2 - a_2^2$.

a) $V = \frac{\rho_\ell}{2\pi\epsilon_0} \ln \frac{r_2}{r_1}$ — r_2: Distance to $-\rho_\ell$; r_1: Distance to $+\rho_\ell$.

At P_1: $r_2 = b + (c_1 - a_1)$, $r_1 = b - (c_1 - a_1)$.

At P_2: $r_2 = b + (c_2 - a_2)$, $r_1 = b - (c_2 - a_2)$.

$$V_1 - V_2 = \frac{\rho_\ell}{2\pi\epsilon_0} \ln\left[\frac{b+(c_1-a_1)}{b-(c_1-a_1)}\,\frac{b-(c_2-a_2)}{b+(c_2-a_2)}\right].$$

Expressing b, c_1 & c_2 in terms of D, a_1 & a_2 and simplifying: $V_1 - V_2 = \frac{\rho_\ell}{2\pi\epsilon_0}\ln\left\{\left(\frac{a_1^2+a_2^2-D^2}{2a_1a_2}\right) + \left[\left(\frac{a_1^2+a_2^2-D^2}{2a_1a_2}\right)^2 - 1\right]^{1/2}\right\}$

$$C' = \frac{\rho_\ell}{V_1 - V_2} = \frac{2\pi\epsilon_0}{\ln\left\{\left(\frac{a_1^2+a_2^2-D^2}{2a_1a_2}\right) + \left[\left(\frac{a_1^2+a_2^2-D^2}{2a_1a_2}\right)^2 - 1\right]^{1/2}\right\}} = \frac{2\pi\epsilon_0}{\cosh^{-1}\left(\frac{a_1^2+a_2^2-D^2}{2a_1a_2}\right)} \quad (F/m).$$

b) Force per unit length $F' = \frac{\rho_\ell^2}{2\pi\epsilon_0(4b^2)} = \frac{D^2\rho_\ell^2}{2\pi\epsilon_0\left[(a_1^2+a_2^2-D^2)^2 - 4a_1^2D^2\right]}$ (N/m).

HW4-5

4-17 Required boundary conditions at $x=0$: $V_1 = V_2$, and $\epsilon_1 \frac{\partial V_1}{\partial x} = \epsilon_2 \frac{\partial V_2}{\partial x}$.

From Fig. 4-23 and the hypotheses in parts a) and b:

$$V_1 = \frac{Q}{4\pi\epsilon_1\sqrt{(x-d)^2+y^2+z^2}} - \frac{Q_1}{4\pi\epsilon_1\sqrt{(x+d)^2+y^2+z^2}},$$

$$V_2 = \frac{Q+Q_2}{4\pi\epsilon_2\sqrt{(d-x)^2+y^2+z^2}}.$$

In order to satisfy the b.c.'s at $x=0$, we require

$$\frac{Q-Q_1}{\epsilon_1} = \frac{Q+Q_2}{\epsilon_2} \quad \text{and} \quad Q+Q_1 = Q+Q_2 \longrightarrow Q_1 = Q_2 = \frac{\epsilon_2-\epsilon_1}{\epsilon_2+\epsilon_1}Q.$$

HW4-6

Solution: $V(\phi) = A_0\phi + B_0$.

a) B.C. 1: $V(0) = 0 \longrightarrow B_0 = 0$.
B.C. 2: $V(\alpha) = V_0 = A_0\alpha \longrightarrow A_0 = \frac{V_0}{\alpha}$.

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \therefore V(\phi) = \frac{V_0}{\alpha}\phi, \quad 0 \le \phi \le \alpha.$$

b) B.C. 1: $V(\alpha) = V_0 = A_1\alpha + B_1$
B.C. 2: $V(2\pi) = 0 = 2\pi A_1 + B$

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \longrightarrow A_1 = -\frac{V_0}{2\pi-\alpha}, \quad B_1 = \frac{2\pi V_0}{2\pi-\alpha}.$$

$$\therefore V(\phi) = \frac{V_0}{2\pi-\alpha}(2\pi-\phi), \quad \alpha \le \phi \le 2\pi.$$