ECE 2300
Recitation Class 6

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Pre-class

- Quiz 4 this week!
  - After Thursday lecture (June 29th, 8:00 pm – 8:40 pm)
  - Same format as last quiz. Online student need to turn on at least one camara. with no blurry background
  - If you want to take online quiz, notify us beforehand!

- Midterm 2 next week!
  - July 6th, Thursday 7:00 pm – 8:40 pm
  - ~~Location is still under arrangement, will be announced soon~~

check canvas !

Quiz 3 Recap:

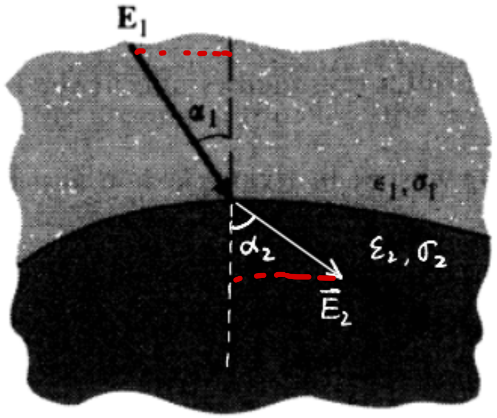
Question 1

Two lossy dielectric media with permittivities and conductivities (ϵ_1, σ_1) and (ϵ_2, σ_2) are in contact. An electric field with a magnitude E_1 is incident from medium 1 upon the interface at an angle α_1 measured from the common normal, as shown in the figure.

a) Find the magnitude and direction of $\mathbf{E}_2$ in medium 2.

b) Find the surface charge density at the interface.

c) Compare the results in parts (a) and (b) with the case in which both media are perfect dielectrics.



a) $E_{1t} = E_{2t} \Rightarrow E_1 \sin\alpha_1 = E_2 \sin\alpha_2$

$J_{1n} = J_{2n} \Rightarrow \sigma_1 E_{1n} = \sigma_2 E_{2n}$

$\Rightarrow \sigma_2 E_2 \cos\alpha_2 = \sigma_1 E_1 \cos\alpha_1$

$\Rightarrow E_2 = E_1 \sqrt{\sin^2\alpha_1 + \left(\frac{\sigma_1}{\sigma_2}\cos\alpha_1\right)^2}$ ①

$\tan\alpha_2 = \frac{\sigma_2}{\sigma_1} \cdot \tan\alpha_1$

$\Rightarrow \alpha_2 = \tan^{-1}\left(\frac{\sigma_2}{\sigma_1}\tan\alpha_1\right)$ ②

b) $D_{2n} - D_{1n} = \rho_s \Rightarrow \varepsilon_2 E_{2n} - \varepsilon_1 E_{1n} = \rho_s$

$\rho_s = \left(\frac{\sigma_1}{\sigma_2}\varepsilon_1 - \varepsilon_2\right)\cdot \underset{\downarrow \atop E_1\cos\alpha_1}{E_{1n}} = \left(\frac{\sigma_1}{\sigma_2}\varepsilon_1 - \varepsilon_2\right)\cdot E_1\cos\alpha_1$

c) if perfect dielectric: $\sigma_1 = \sigma_2 = 0 \Rightarrow$ ① & ② $\Rightarrow \rho_s = 0$

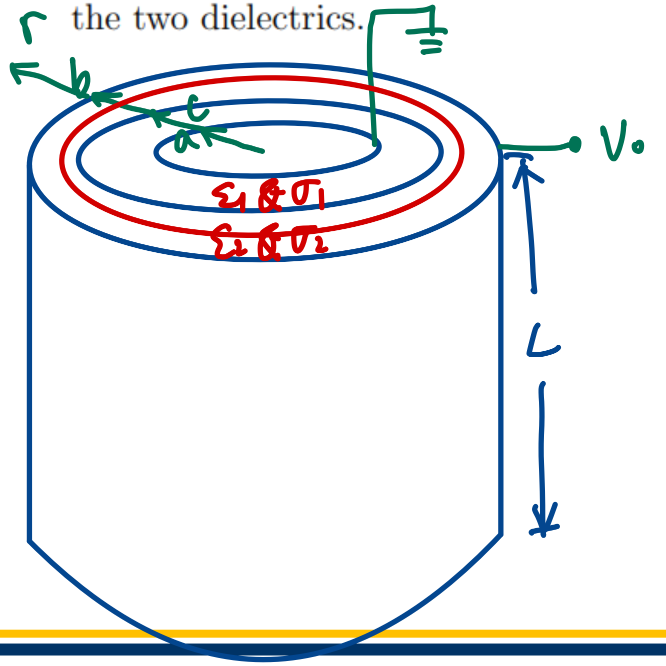
Quiz 3 Recap:

Question 2

A d-c voltage V_0 is applied across a cylindrical capacitor of length L. The radii of the inner and outer conductors are a and b, respectively. The space between the conductors is filled with two different lossy dielectrics having, respectively, permittivity ϵ_1 and conductivity σ_1 in the region $a < r < c$, and permittivity ϵ_2 and conductivity σ_2 in the region $c < r < b$. Determine

a) the current density in each region,

b) the surface charge densities on the inner and outer conductors and at the interface between the two dielectrics.



a) $V_0 \xrightarrow{R} I \xrightarrow{1/S} J$

$$dR = \frac{l}{\sigma S}$$

$l = dr \qquad \sigma = \sigma_1 \text{ or } \sigma_2$

$S = 2\pi r \cdot L$

$$R = \int dR = \int \frac{dr}{2\pi r \cdot L \cdot \sigma}$$

For $a<r<c$: $R_1 = \int_a^c dR = \dfrac{\ln(c/a)}{2\pi L \sigma_1}$

For $c<r<b$: $R_2 = \int_c^b dR = \dfrac{\ln(b/c)}{2\pi L \sigma_2}$

$$I = \frac{V_0}{R_{sum}} = \frac{V_0}{R_1+R_2}$$

$$= \frac{2\pi \sigma_1 \sigma_2 L V_0}{\sigma_1 \ln(b/c) + \sigma_2 \ln(c/a)}$$

$$J = I/S = \frac{I}{2\pi r L}$$

$$= \frac{\sigma_1 \sigma_2 V_0}{r[\sigma_1 \ln b/c + \sigma_2 \ln c/a]}$$

* both applies to $a<r<c$ & $c<r<b$.

Quiz 3 Recap:

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a) the current density in each region,

b) the surface charge densities on the inner and outer conductors and at the interface between the two dielectrics.

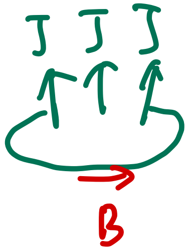
b) $$\rho_{sa} = \varepsilon_1 E_1 \Big|_{r=a} = \frac{\varepsilon_1 \sigma_2 V_0}{a\left[\sigma_1 \ln(b/c) + \sigma_2 \ln(c/a)\right]}$$

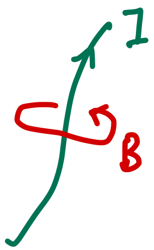
$$\rho_{sb} = -\varepsilon_2 E_2 \Big|_{r=b} = -\frac{\varepsilon_2 \sigma_1 V_0}{b\left[\sigma_1 \ln(b/c) + \sigma_2 \ln(c/a)\right]}$$

$$\rho_{sc} = -(\varepsilon_1 E_1 - \varepsilon_2 E_2)\Big|_{r=c} = \frac{(\varepsilon_2 \sigma_1 - \varepsilon_1 \sigma_2) V_0}{c\left[\sigma_1 \ln(b/c) + \sigma_2 \ln(c/a)\right]}$$

6.1 Fundamental Postulates

Differential form:	integral form:
$\nabla\cdot B = 0$	$\oint_s B\cdot ds = 0$
$\nabla\times B = \mu_0 J$	$\oint_c B\cdot dl = \mu_0 I$ $\Rightarrow$ Ampere's circuital law.





Right hand rule.

* Chap 3:

$\nabla\cdot E = \frac{\rho}{\varepsilon_0}$

$\nabla\times E = 0$

μ_0: permeability of free space.

$\mu_0 = 4\pi * 10^{-7}\ H/m$

Null-identity: div. of a curl. is zero

$\nabla\cdot(\nabla\times B) = 0 \Rightarrow \nabla\cdot J = \frac{\nabla\cdot(\nabla\times B)}{\mu_0} = 0$

($\nabla\times B \Downarrow \mu_0 J$)

* consistant with: $\nabla\cdot J = \frac{\partial\rho}{\partial t} = 0$ for steady current.

6.2 Ampere's Circuit Law

al

$\nabla \times B = \mu_0 J$

current density.

$\int_S (\nabla \times B)\, ds = \mu_0 \int_S J \cdot ds$

$\Downarrow \qquad \Downarrow$

$\oint_C B \cdot dl = \mu_0 \cdot I$

C: contour bounds the surface

I: total current flows through surface.

* Chap 3:

$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$.

$\Downarrow$ integrat

Gauss's Law.

Ampere's Law: the circulation of magnetic flux density (B) in free space around any closed path is equal to μ_0 times the total current flowing through the surface bounded by the path.

6.3 Vector Magnetic Potential

As: $\nabla \cdot B = 0 \Rightarrow B$ is solenoidal

express B: $B = \nabla \times A$

$\Downarrow$

A: vector magnetic potential

Magnetic flux: $\Phi = \int_S B \cdot ds = \int_C A \cdot dl$

stoke's Law.

Since: $\nabla \times B = \mu_0 J \Rightarrow \nabla \times \nabla \times A = \mu_0 J$

trying to do laplacian transformation:

$$\mu_0 J = \nabla(\nabla \cdot A) - \nabla^2 A$$

$\downarrow$

$\nabla \cdot A$ made zero for simplicity.

$\Rightarrow -\nabla^2 A = \mu_0 J \Rightarrow \nabla^2 A = -\mu_0 J$

To find solution:

$$A = \frac{\mu_0}{4\pi} \int_{V'} \frac{J}{R} dv'$$

For a thin wire with cross-sec. area S.

$dv = S \cdot dl'$

if current flow goes fully along wire:

$J \cdot dv' = J \cdot S \cdot dl' = I \cdot dl'$

plug in:

$$A = \frac{\mu_0 I}{4\pi} \int_C \cdot \frac{dl'}{R}$$

6.3 Vector Magnetic Potential

Based on form & property of diff.

$$B = \frac{\mu_0 I}{4\pi} \int_{C'} \frac{dl' \times a_R}{R^2}$$

Biot - savart Law.

Also written as:

$$B = \oint_C dB$$

$$dB = \frac{\mu_0 I}{4\pi} \left(\frac{dl' \times a_R}{R^2} \right)$$

$$= \frac{\mu_0 I}{4\pi} \left(\frac{dl' \times R}{R^3} \right)$$

Biot-Savart Law is more complicated to apply than Ampere's circuital Law.

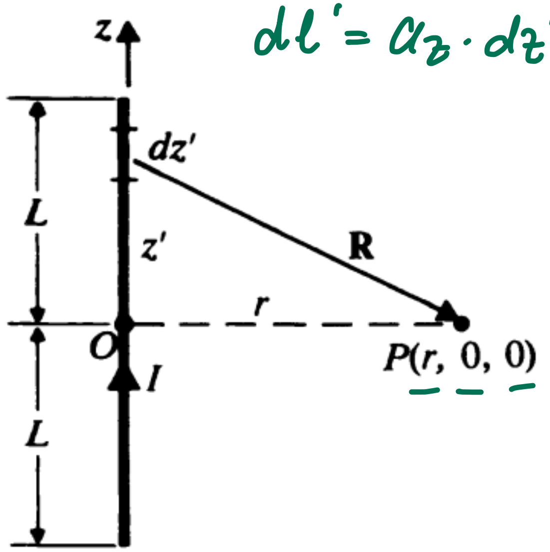
Ampere's circuital Law cannot be used to determine B from I in a circuit, if a closed path cannot be found.

Ex.1 Vector magnetic potential

A direct current I flows in a straight wire of length $2L$. Find the magnetic flux density $\boldsymbol{B}$ at a point located at a distance r from the wire in the bisecting plane:

1) • by determining the vector magnetic potential $\boldsymbol{A}$ first,

2) • by applying Biot-Savart law.



$dl' = a_z \cdot dz'$

1) find B from $\nabla \times A$

$R = \sqrt{z'^2 + r^2}$ → equation of A

$$A = a_z \frac{\mu_0 I}{4\pi} \int_{-L}^{L} \frac{dz'}{\sqrt{z'^2 + r^2}}$$

$$= a_z \frac{\mu_0 I}{4\pi} \left[\ln\left(z' + \sqrt{z'^2 + r^2}\right) \right] \Bigg|_{-L}^{L}$$

$$= a_z \frac{\mu_0 I}{4\pi} \ln \frac{\sqrt{L^2 + r^2} + L}{\sqrt{L^2 + r^2} - L}$$

Therefore: $B = \nabla \times A$

$$= \nabla \times (a_z \cdot A_z)$$

$$= a_r \frac{1}{r} \frac{\partial A_z}{\partial \phi} - a_\phi \frac{\partial A_z}{\partial r}$$

$\frac{\partial A_z}{\partial \phi} = 0$ according to symmetry.

$$B = -a_\phi \frac{\partial}{\partial r} \left[\frac{\mu_0 I}{4\pi} \ln \frac{\sqrt{L^2 + r^2} + L}{\sqrt{L^2 + r^2} - L} \right] = a_\phi \frac{\mu I L}{2\pi r \sqrt{L^2 + r^2}}$$

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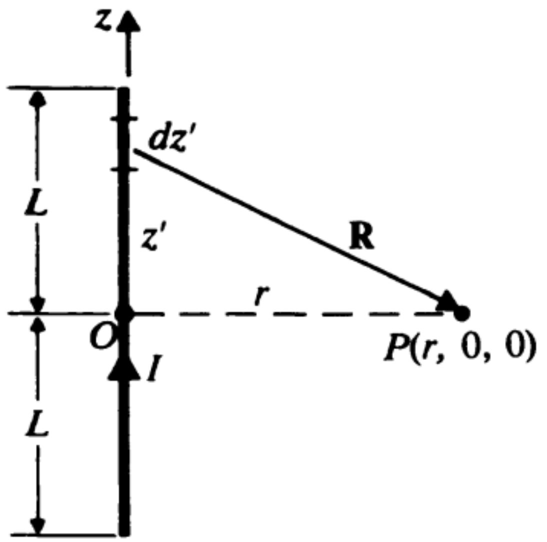


2) $R = a_r r - a_z z'$

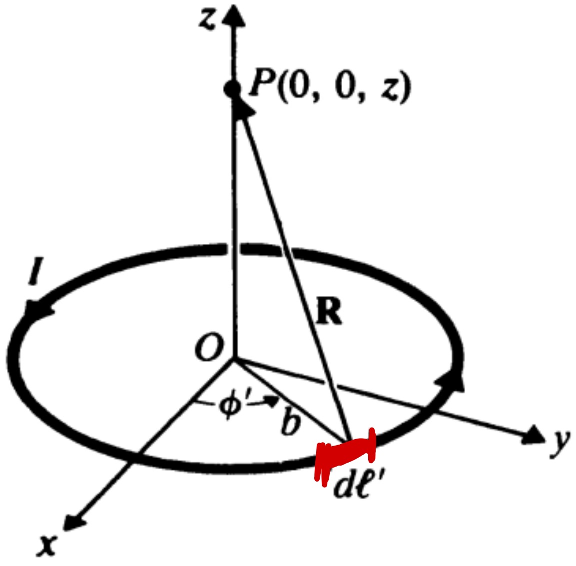
$dl \times R = a_z dz' \times (a_r r - a_z z')$

$= a_\phi r \, dz'$

$B = \int dB = a_\phi \frac{\mu_0 I}{4\pi} \int_{-L}^{L} \frac{r\,dz'}{(z'^2 + r^2)^{3/2}}$

$= a_\phi \frac{\mu_0 I L}{2\pi r \sqrt{L^2 + r^2}}$

Ex.2 Vector magnetic potential



Find the magnetic flux density at a point on the axis of a circular loop of radius b that carries a direct current I

$$d\ell = a_\phi b\, d\phi'$$

$$R = a_z z - a_r b$$

$$R = (z^2 + b^2)^{1/2}$$

$$d\ell' \times R = a_\phi b\, d\phi' \times (a_z z - a_r b)$$

$$= a_r b z\, d\phi + a_z b^2 d\phi'$$

$$B = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} a_z \frac{b^2 d\phi}{(z^2+b^2)^{3/2}} = a_z \frac{\mu_0 I b^2}{2(z^2+b^2)^{3/2}}$$

6.4 Scalar Magnetic Potential

if a region is current free:

$$J = 0;$$

we will have:

$$\nabla \times B = \mu_0 J = 0$$

B can be expressed as the gradient of a scalar field.

conventional.

$$B = -\mu_0 \nabla V_m$$

$\Downarrow$

scalar magnetic potential

Between two points: P_1 & P_2:

$$V_{m2} - V_{m1} = -\int_{P_1}^{P_2} \frac{1}{\mu_0} B \cdot dl$$

Not exist.

virtual element for understanding

if there were magnetic charges of volume density ρ_m. we would find V_m:

$$V_m = \frac{1}{4\pi} \int_{V'} \frac{\rho_m}{R} dv'$$

For a magnetic bar: $+q_m$, $-q_m$. seperated by d.

$$V_m = \frac{\vec{m} \cdot a_R}{4\pi R^2}$$

6.4 Scalar Magnetic Potential

Analogous to electric potential

more similar to displacement vector. $\Uparrow$ Ⓓ

$$\boxed{\mathbf{E} = -\nabla V}$$

$$\mathbf{B} = -\mu_0 \nabla V_m,$$

$$V_2 - V_1 = -\int_{P_1}^{P_2} \mathbf{E} \cdot d\ell \qquad \text{(V)}.$$

$$V_{m2} - V_{m1} = -\int_{P_1}^{P_2} \frac{1}{\mu_0} \mathbf{B} \cdot d\ell.$$

E $\downarrow$ $1/(4\pi\varepsilon_0)$

B/μ_0 $\downarrow$ $(\mu_0/(4\pi))/\mu_0$

$$V = \frac{1}{4\pi\epsilon_0} \int_{V'} \frac{\rho}{R}\, dv' \qquad \text{(V)}.$$

$$V_m = \frac{1}{4\pi} \int_{V'} \frac{\rho_m}{R}\, dv' \qquad \text{(A)}.$$

No μ_0 in V_m

If there ***were*** magnetic charges

Thank You

Credit to Deng Naihao for this slides & information