ECE 2300J
Recitation Class 2

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Pre-class

- Quiz this Thursday! (Usually start around 8:00pm)
  - Content: Chap.2
  - Format: 2~3 questions with simple calculation!
  - Online quiz regulations:
    - At least one camara on showing both computer screen and yourself!
    - Have extra 5 mins to submit. No need to rush.
    - I will be supervising the entire process and to help you set up!

Good luck on the first quiz!

2.1 Recap-Useful Vector theorems

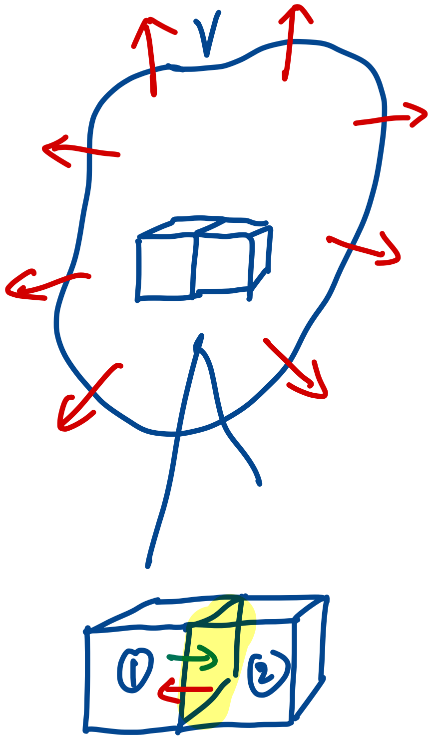
- Divergence Theorem: dimension reduction. 3D $\rightarrow$ 2D

$$\int_V \nabla\cdot\vec{A}\,dv = \oint_S \vec{A}\cdot d\vec{s}$$

$\Downarrow$ integral of divergence.

$\Downarrow$ outward flux

closed surface bounds the volume.



$\Downarrow$

flux cancelled out.

2.1 Recap-Useful Vector theorems

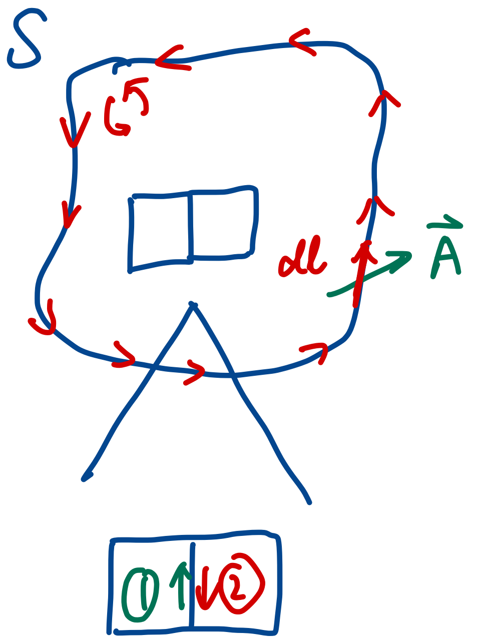
- Stokes Theorem: dimension reduction. 2D → 1D

$$\int_S (\nabla \times \vec{A})\, d\vec{s} = \oint_C \vec{A} \cdot d\vec{l}$$

⇒ dot product.

⇓ integral of curl

⇓ line bounds open surface.



⇓ curl cancelled out.

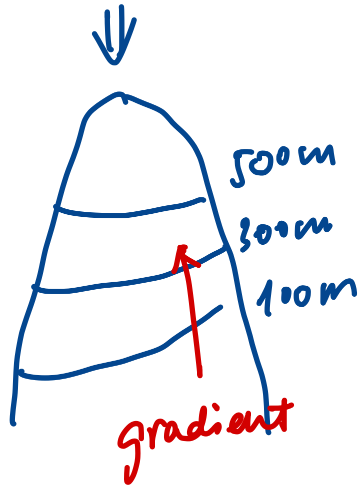
2.1 Recap-Useful Vector theorems

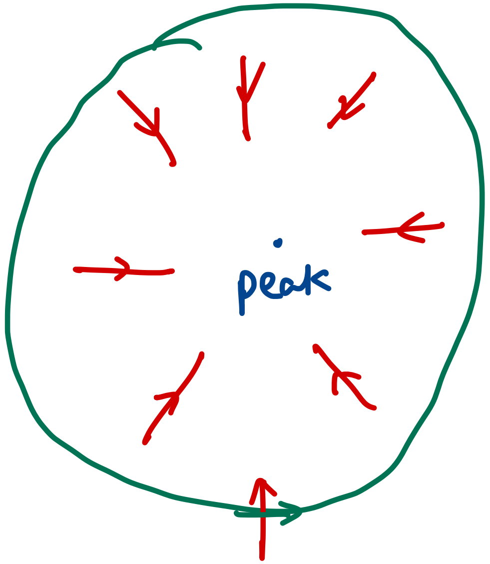
- Null identities:

1. $\nabla \times (\nabla V) = 0$

$\Downarrow$

vector field.





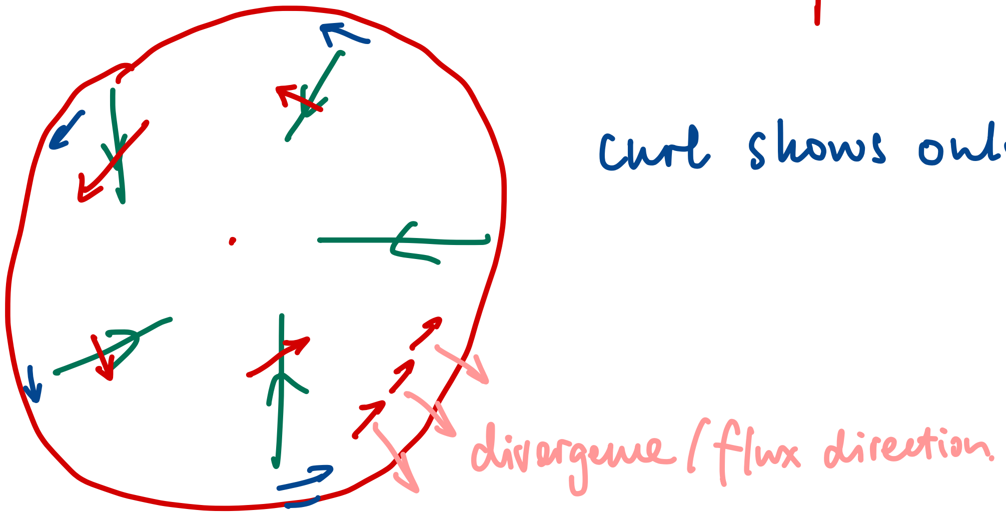
gradient

$\Rightarrow$ shows where the value changes the most.

2. $\nabla \cdot (\nabla \times V) = 0$

$\Downarrow$

vector field.

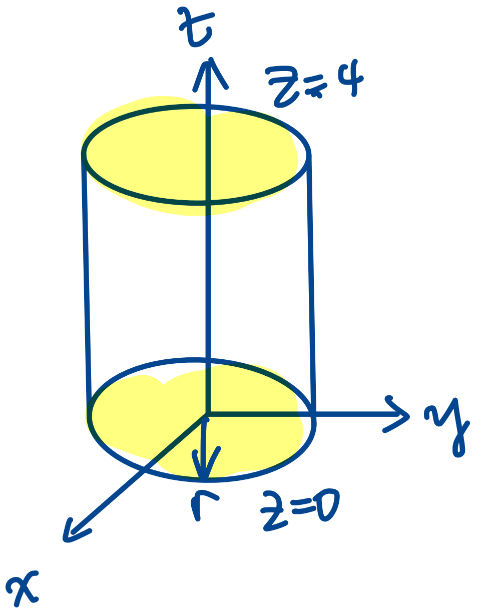


curl shows only rotation.

Ex.1 Theorems application

$h_1 = 1 \quad h_2 = r \quad h_3 = 1$

- (HW1-5) For vector function $\mathbf{A} = \mathbf{a_r} r^2 + \mathbf{a_z} 2z$, verify the divergence theorem for the circular cylindrical region enclosed by $r = 5$, $z = 0$, and $z = 4$.



$$\int_V \nabla\cdot\vec{A}\, dv = \oint_S \vec{A}\cdot d\vec{s}$$

Left: $\nabla\cdot\vec{A} = \frac{1}{h_1h_2h_3}\left(\frac{\partial}{\partial r}\cdot h_2h_3A_r + \frac{\partial}{\partial\theta}h_1h_3A_\theta + \frac{\partial}{\partial z}h_1h_2A_z\right)$

$$= \frac{1}{r}\left(\frac{\partial}{\partial r}\cdot r^3 + 0 + \frac{\partial}{\partial z}\cdot r\cdot 2z\right) = 3r+2$$

metrix coeffi

$$\int_V \nabla\cdot\vec{A}\cdot dv = \int_0^4\int_0^{2\pi}\int_0^5 (3r+2)\cdot r\, dr\, d\theta\, dz = 1200\pi.$$

Ex.1 Theorems application Cont.

right: Top surface: $\int \vec{A} \cdot d\vec{s} = \int 8 \cdot ds = 8 \cdot (\pi \cdot 5^2) = 200\pi$.

$\Downarrow$ 8 25

unit vector for surface is a_z

$\vec{A} \cdot d\vec{s} = 2z\, \vec{a_z} \cdot d\vec{s} = 2z\big|_{z=4} = 8$

Bottom surface: $\int \vec{A} \cdot d\vec{s} = 0$

$\Downarrow$

$\vec{A} \cdot d\vec{s} = -2z\, \vec{a_z} \cdot d\vec{s} = 2z\big|_{z=0} = 0$

Side: $\vec{A} = \vec{a_r} \cdot 25 + \vec{a_z} \cdot 2z \qquad d\vec{s} = \vec{a_r} \cdot ds$

$\int \vec{A} \cdot d\vec{s} = 25 \cdot \int ds = 25 \cdot (2\pi \times 5 \times 4) = 1000\pi$

Adding up:

$1000\pi + 200\pi + 0 = 1200\pi$

2.2 Electrostatics

- Key Requirements:

1. electric charges are stationary.

2. electric field is not changing with time.

- Field density:

$$E = \lim_{q \to 0} \frac{F}{q} \quad (V/m)$$

$\Rightarrow$ potential

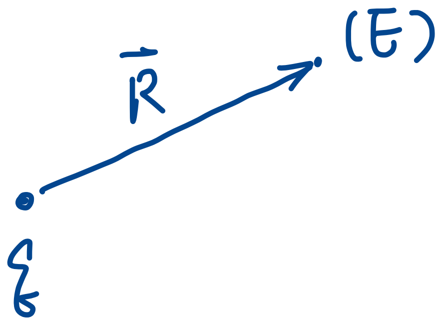
$\Downarrow$ meter.

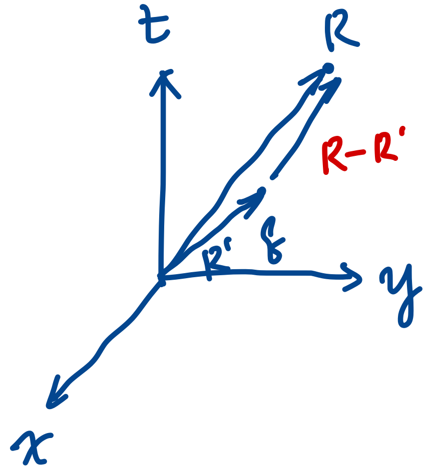
2.2 Electrostatics

- Strength-Colomb’ s Law

$$E = \bar{a}_R \frac{q}{4\pi\varepsilon_0 R^2}$$

ε_0: constant of electrostatistic





Single point charge:

$$E_p = \frac{q \cdot (R-R')}{4\pi\varepsilon_0 |R-R'|^3}$$

2.2.1 Maxwell’ s Description

- Gauss’ s Law:

Differential form:

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

ρ → density of free charges (C/m^3)

ε_0 → constant of electrostatics.

permitivity of free space.

NOT SOLENOIDAL

(unless $\rho = 0$)

Intergral form

$$\oint_S E \cdot ds = \frac{Q}{\varepsilon_0}$$

Q → total charge bounded by the closed surface.

2.2.1 Maxwell’s Description

- Conservativeness:

Differential form:

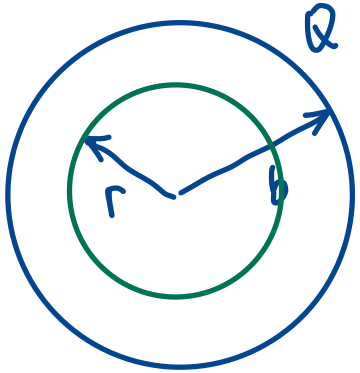
$$\nabla \times E = 0 \quad \Rightarrow \quad \text{irrotational.}$$

Integral form:

$$\oint_C E \cdot dl = 0 \quad \Rightarrow \quad \text{conservative.}$$

Ex.2 Electrostatics

A total charge Q is put on a thin spherical shell of radius b. Determine the electric field intensity at an arbitrary point inside the shell



Gauss's Law: **Step 1:** select Gaussian Surface: Green Spherical Surface

Step 2: Check the charge bounded by the Surface

$Q = 0$

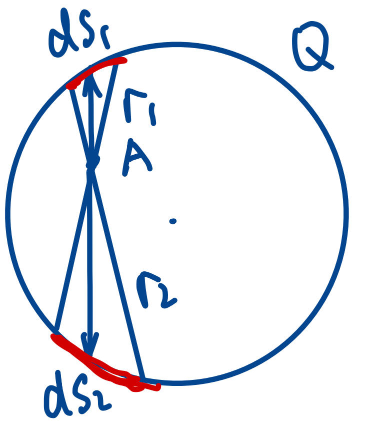
Step 3: calculate.

$$\oint \vec{E} \cdot ds = \frac{Q_{enclosed} \nearrow 0}{\varepsilon_0}$$

$$\vec{E} = 0$$

Ex.2 Electrostatics Cont.

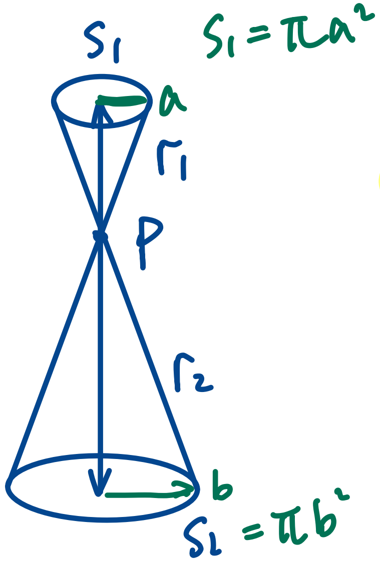
A total charge Q is put on a thin spherical shell of radius b. Determine the electric field intensity at an arbitrary point inside the shell



density of free charge.

⇑

$$dE = \frac{\rho_s}{4\pi\varepsilon_0}\left(\frac{dS_1}{r_1^2} - \frac{dS_2}{r_2^2}\right) \quad \text{in which } \rho_s = \frac{Q}{4\pi b^2}$$



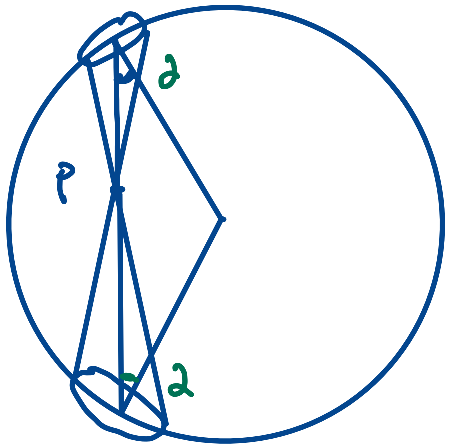
$$\frac{S_1}{S_2} = \frac{\pi r_1^2}{\pi r_2^2}$$

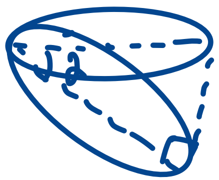
$$\frac{a}{b} = \frac{r_1}{r_2}$$

$$\frac{a^2}{b^2} = \frac{r_1^2}{r_2^2}$$

Ex.2 Electrostatics Cont.

A total charge Q is put on a thin spherical shell of radius b. Determine the electric field intensity at an arbitrary point inside the shell





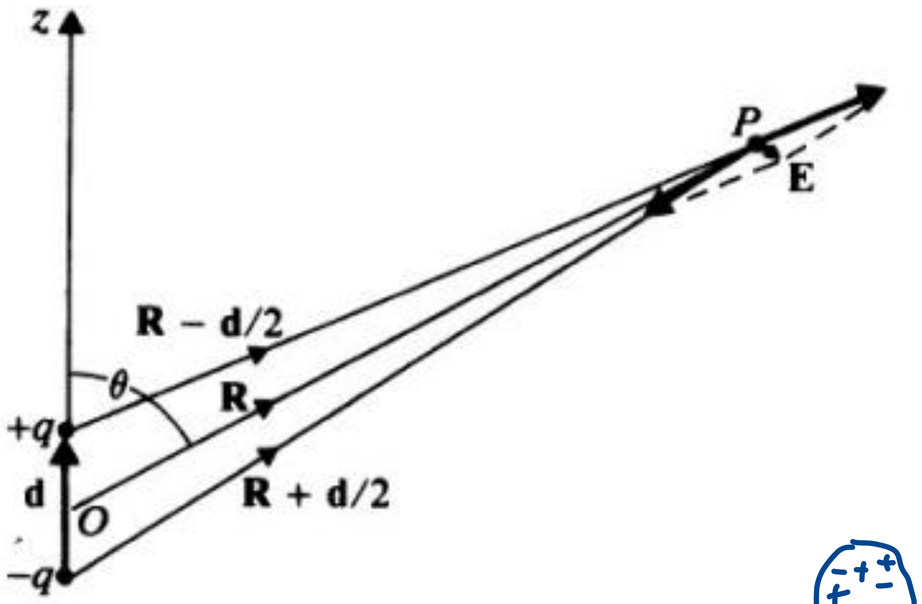
$$\frac{dS_1}{S_1'} = \cos\alpha \quad \Rightarrow \quad \frac{dS_2}{S_2'} = \cos\alpha$$

$$\frac{dS_1}{r_1^2} = \frac{S_1' \cdot \cos\alpha}{r_1^2} \qquad \frac{dS_2}{r_2^2} = \frac{S_2' \cdot \cos\alpha}{r_2^2}$$

$$\frac{S_1'}{S_2'} = \frac{r_1^2}{r_2^2}$$

$$\Rightarrow \frac{dS_1}{r_1^2} = \frac{dS_2}{r_2^2} \quad \Rightarrow \quad dE = 0 \quad \Rightarrow \int dE = 0.$$

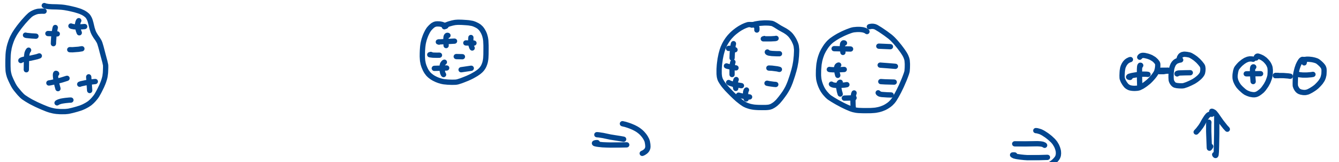
2.2.2 Dipole



- Definition:

A pair of equal and opposite electric charges seperated by a small distance.

- E.g.: London force (weakest molecular force)



seperately ⇒ closely ⇒ simplified model of London force.

2.2.2 Dipole

- Field:
  - Vector Form: $E = \frac{q}{4\pi\varepsilon_0}\left\{\frac{R-\frac{d}{2}}{|R-\frac{d}{2}|^3} - \frac{R+\frac{d}{2}}{|R+\frac{d}{2}|^3}\right\}$

    if $d \ll R$ $\quad E \simeq \frac{q}{4\pi\varepsilon_0}\left[3\frac{R\cdot d}{R^2}\cdot R - d\right]$

  - Spherical coordination:

    $$E = \frac{p}{4\pi\varepsilon_0 R^2}(a_R 2\cos\theta + a_\theta \sin\theta)$$

- Moment: $\underset{\text{moment}}{p} = \underset{\text{charge}}{q}\cdot d \Rightarrow$ distance.

  $R\cdot p = Rp\cos\theta$

  $p = a_z\cdot p = p(a_R\cdot\cos\theta - a_\theta\cdot\sin\theta)$.

2.2.3 Continuous Distributed Charges

- Differentiated element:

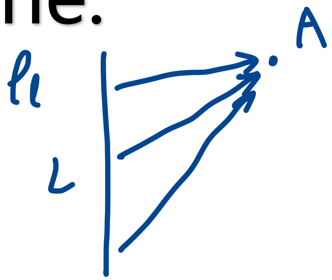
$$dQ = \rho \cdot dv$$

$$dE = a_R \frac{dQ}{4\pi\varepsilon_0 R^2} = a_R \frac{\rho \cdot dv}{4\pi\varepsilon_0 R^2}$$

dv : differential volume element.

2.2.3 Continuous Distributed Charges

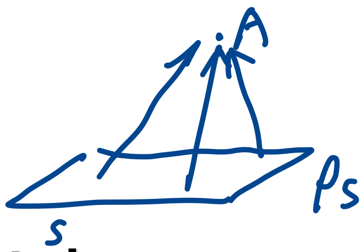
- Line:



$$E = \frac{1}{4\pi\varepsilon_0}\int_{L'} a_R \frac{\rho_l}{R^2}\, dl$$

line integral

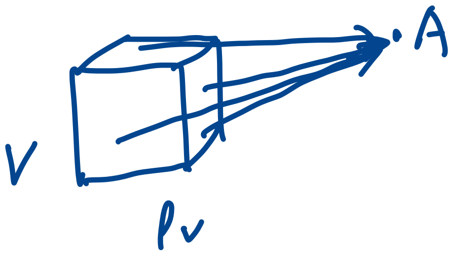
- Surface:



$$E = \frac{1}{4\pi\varepsilon_0}\int_{S'} a_R \frac{\rho_s}{R^2}\, ds$$

surface integral

- Volume:



$$E = \frac{1}{4\pi\varepsilon_0}\int_{V'} a_R \frac{\rho_v}{R^2}\cdot dv$$

volume integral.

2.2.4 Application of Gauss’ s Law

- When to use?

High degree of symmetry in charge distribution or electric field.

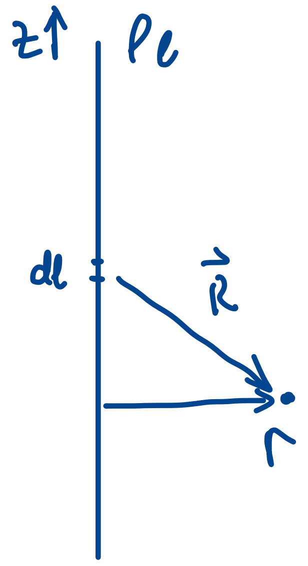
$$\int_S \vec{E} \cdot d\vec{s} = \frac{Q}{\varepsilon_0}$$

- Example:

Determine the electric field intensity of an infinitely long, straight, line charge of a uniform density ρ_ℓ in air.

Ex.3 Method 1 – Integration

Determine the electric field intensity of an infinitely long, straight, line charge of a uniform density ρ_ℓ in air.



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_{L'} \hat{a}_R \frac{\rho_\ell}{R^2}\cdot dl'$$

$$= \frac{1}{4\pi\varepsilon_0}\int_{L'} \hat{a}_R \frac{\rho_\ell}{R^3}\cdot \vec{R}\cdot dl'$$

$$\Rightarrow d\vec{E} = \frac{\overbrace{\rho_\ell\cdot dl}^{\Rightarrow dQ}}{4\pi\varepsilon_0}\cdot\frac{\hat{a}_r\cdot r - \hat{a}_z\cdot z}{(r^2+z^2)^{3/2}}$$

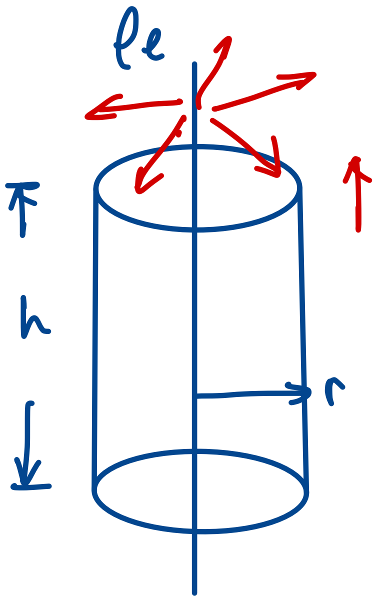
$$= \hat{a}_r \frac{\rho_\ell\cdot r\cdot dl}{4\pi\varepsilon_0(r^2+z^2)^{3/2}} + \hat{a}_z \cancel{\frac{-\rho_\ell\cdot z\cdot dl}{4\pi\varepsilon_0(r^2+z^2)^{3/2}}}$$

odd function.
namely. integration from $-\infty$ to ∞ gives zero!
$\Uparrow$

$$\vec{E} = \int d\vec{E} = \frac{\hat{a}_r\rho_\ell\cdot r}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{dl}{(r^2+z^2)^{3/2}} = \hat{a}_r\frac{\rho_\ell}{2\pi\varepsilon_0 r}.$$

Ex.3 Method 2 – Gauss’ s Law

Determine the electric field intensity of an infinitely long, straight, line charge of a uniform density ρ_ℓ in air. Step 1: Gaussian Surface. $\Rightarrow$ cylindrical surface.



No up & down part

$$\int_S E\cdot ds = \overset{=0}{\int_{top-surface} E\cdot ds} + \overset{=0}{\int_{bottom-surface} E\cdot ds}$$

$$+ \int_{side} E\cdot ds.$$

$$= E(r)\cdot 2\pi r\cdot h.$$

$$E(r)\cdot 2\pi r\cdot h = \frac{Q}{\varepsilon_0} = \frac{\rho_\ell \cdot h}{\varepsilon_0}$$

$$E(r) = \frac{\rho_\ell \cdot \cancel{h}}{2\pi r\cdot \varepsilon_0 \cancel{h}} = \frac{\rho_\ell}{2\pi r\cdot \varepsilon_0}\cdot \hat{a}_r$$

2.2.4 Application of Gauss' s Law

- Some Important Results:

Symmetrical & uniform.

different models	E(magnitude)
infinitely long, line charge	$E = \frac{\rho_\ell}{2\pi r \epsilon_0}$
infinite planar charge	$E = \frac{\rho_s}{2\epsilon_0}$
uniform spherical surface charge with radius R	$\begin{cases} E = 0 (r < R) \\ E = \frac{Q}{4\pi r^2 \epsilon_0} (r > R) \end{cases}$
uniform sphere charge with radius R	$\begin{cases} E = \frac{Qr}{4\pi R^3} (r < R) \\ E = \frac{Q}{4\pi r^2 \epsilon_0} (r > R) \end{cases}$
infinitely long, cylindrical charge with radius R	$\begin{cases} E = \frac{\rho_v r}{2\epsilon_0} (r < R) \\ E = \frac{\rho_v R^2}{2 r \epsilon_0} (r > R) \end{cases}$

$Q=0$

$Q=Q_0.$

$Q = Q_0 \cdot \frac{r^3}{R^3}$

$Q=Q_0$

Thank You

Credit to Deng Naihao for this slides & information