HW3-1

a) $\rho_{ps} = \bar{P} \cdot \bar{a}_n = P_0 \frac{L}{2}$ on all six faces of the cube.

$\rho_p = -\bar{\nabla} \cdot \bar{P} = -3P_0$.

b) $Q_s = 6L^2 \rho_{ps} = 3P_0 L^3$, $Q_v = L^3 \rho_p = -3P_0 L^3$.

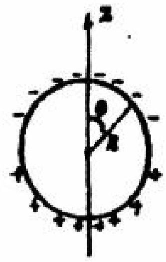
Total bound charge $= Q_s + Q_v = 0$.

HW3-2

3-19 Assume $\bar{P} = \bar{a}_z P$. Surface charge denity $\rho_{ps} = \bar{P} \cdot \bar{a}_n$

$= (\bar{a}_z P) \cdot (-\bar{a}_R)$

$= -P \cos\theta$.



The z-component of the electric field intensity due to a ring of ρ_{ps} contained in width $R d\theta$ at θ is

$$dE_z = \frac{P\cos\theta}{4\pi\epsilon_0 R^2}(2\pi R\sin\theta)(R d\theta)\cos\theta$$
$$= \frac{P}{2\epsilon_0}\cos^2\theta \sin\theta \, d\theta .$$

At the center of the cavity : $\bar{E} = \bar{a}_z E_z = \bar{a}_z \frac{P}{2\epsilon_0}\int_0^{\pi} \cos^2\theta \sin\theta \, d\theta = \frac{\bar{P}}{3\epsilon_0}$.

HW3-3

·21 At the z=0 plane : $\bar{E}_1 = \bar{a}_x 2y - \bar{a}_y 3x + \bar{a}_z 5$.

$\bar{E}_{1t}(z=0) = \bar{E}_{2t}(z=0) = \bar{a}_x 2y - \bar{a}_y 3x$.

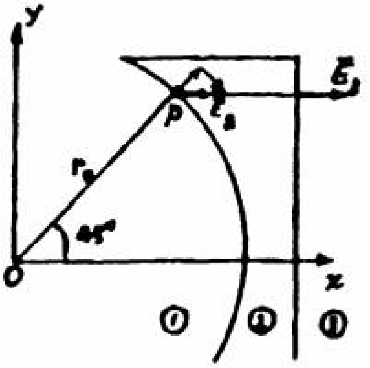
$\bar{D}_{1n}(z=0) = \bar{D}_{2n}(z=0) \longrightarrow 2\bar{E}_{1n}(z=0) = 3\bar{E}_{2n}(z=0)$

$\longrightarrow \bar{E}_{2n}(z=0) = \frac{2}{3}(\bar{a}_z 5) = \bar{a}_z \frac{10}{3}$.

$\therefore \bar{E}_2(z=0) = \bar{a}_x 2y - \bar{a}_y 3x + \bar{a}_z \frac{10}{3}$,

$\bar{D}_2(z=0) = (\bar{a}_x 6y - \bar{a}_y 9x + \bar{a}_z 10)\epsilon_0$.

HW3-4



Assume $\bar{E}_2 = \bar{a}_r E_{2r} + \bar{a}_\phi E_{2\phi}$

B.C.: $\bar{a}_n \times \bar{E}_1 = \bar{a}_n \times \bar{E}_2 \longrightarrow E_{2\phi} = -$

For $\bar{E}_3$, and hence $\bar{E}_2$, to be parallel to the x-axis,

$$E_{2\phi} = -E_{2r} \longrightarrow E_{2r} = 3.$$

B.C.: $\bar{a}_n \cdot \bar{D}_1 = \bar{a}_n \cdot \bar{D}_2 \longrightarrow 5 = 3$

$$\therefore \epsilon_{r2} = 5/3.$$

HW3-5

$$\bar{D} = \bar{a}_r \frac{\rho_\ell}{2\pi r}. \quad \bar{E}_1 = \bar{a}_r \frac{\rho_\ell}{2\pi\epsilon_0\epsilon_{r1} r}, \ r_i < r < b; \ \bar{E}_2 = \bar{a}_r \frac{\rho_\ell}{2\pi\epsilon_0\epsilon_{r2} r}, \ b < r < r_o$$

$$V = -\int_{r_o}^{r_i} \bar{E} \cdot d\bar{r} = \frac{\rho_\ell}{2\pi\epsilon_0}\left[\frac{1}{\epsilon_{r1}} \ln\left(\frac{b}{r_i}\right) + \frac{1}{\epsilon_{r2}} \ln\left(\frac{r_o}{b}\right)\right],$$

$$C = \frac{\rho_\ell}{V} = \frac{2\pi\epsilon_0}{\frac{1}{\epsilon_{r1}} \ln\left(\frac{b}{r_i}\right) + \frac{1}{\epsilon_{r2}} \ln\left(\frac{r_o}{b}\right)} \quad (F/m).$$

HW3-6

Two conductors at potentials V_1 and V_2 carrying charges $+Q$ and $-Q$:

$$W_e = \frac{1}{2} V_1 \int_{S_1} \rho_s \, ds + \frac{1}{2} V_2 \int_{S_2} \rho_s \, ds = \frac{1}{2} Q (V_1 - V_2)$$

$$= \frac{1}{2} C V^2, \quad V = V_1 - V_2.$$