SU2023 ECE2300J Quiz 4

Question 1

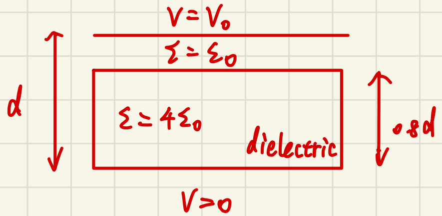
The upper and lower conducting plates of a large parallel-plate capacitor are separated by a distance d and maintained at potentials V_0 and 0, respectively. A dielectric slab of dielectric constant 4.0 and uniform thickness of $\cancel{d}$ $0.8d$ is placed over the lower plate. Assuming negligible fringing effect, determine

a) the potential and electric field distribution in the dielectric slab,

b) the potential and electric field distribution in the air space between the dielectric slab and the upper plate,

c) the surface charge densities on the upper and lower plates.

d) Compare the results in part b) with those without the dielectric slab.



Since C_{air} and $C_{dielectric}$ is in series

$\Rightarrow C_{eq}$ $\Leftrightarrow$ $C_{air} \longrightarrow C_{ca}$ (for short)

$C_{dielectric} \longrightarrow C_{cd}$

& $Q_{ca} = Q_{cd} \Leftrightarrow C_{ca} V_{ca} = C_{cd} V_{cd}$ ①

It's obvious that $V_0 = V_{ca} + V_{cd}$ ②

For parallel capacitor : $C = \dfrac{\varepsilon_0 \varepsilon_r A}{d}$ ③

From ① ③ : $\dfrac{\varepsilon_0 A}{0.2d} V_{ca} = \dfrac{4\varepsilon_0 A}{0.8d} V_{cd} \Rightarrow V_{ca} = V_{cd}$

Then From ② : $V_{ca} = V_{cd} = \frac{1}{2} V_0$

(a)

$$E_{cd} = \frac{V_{cd}}{0.8d} = \frac{5}{8}\frac{V_0}{d}$$

(b)

$$E_{ca} = \frac{V_{ca}}{0.2d} = \frac{5}{2}\frac{V_0}{d}$$

(c)

$$C_{eq} = \frac{1}{\frac{1}{C_{ca}} + \frac{1}{C_{cd}}} = \frac{\frac{\varepsilon_0 A}{0.2d} \cdot \frac{4\varepsilon_0 A}{0.8d}}{\frac{\varepsilon_0 A}{0.2d} + \frac{4\varepsilon_0 A}{0.8d}} = \frac{1}{2} \cdot \frac{\varepsilon_0 A}{0.2d} = \frac{5\varepsilon_0 A}{2d}$$

$\Rightarrow$ on upper plate : $\rho_u = \dfrac{C_{eq} \cdot V_0}{A} = \dfrac{5\varepsilon_0 V_0}{2d}$

$$\rho_d = \frac{-C_{eq} \cdot V_0}{A} = \frac{-5\varepsilon_0 V_0}{2d}$$

(d)

Since $\frac{5}{2}\frac{V_0}{d} > \frac{5}{8}\frac{V_0}{d}$, voltage and electric field are higher for air space than dielectric

Question 2

In the presence of a dielectric which is a linear and isotropic medium, we can reduce the Eqs.

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \times \mathbf{E} = 0$$

to a differential equation for the electric potential.

a) Show this differential equation and identify the condition(s) in which it reduces to Poisson's equation.

$$\nabla^2 V = -\frac{\rho}{\epsilon}$$

b) Suppose we do not have a homogeneous medium, but still isotropic, so ϵ is a scalar. In general, $\epsilon = \epsilon(x, y, z)$. Suppose we reduce to the 1D example so $\epsilon = \epsilon(x)$. How is Poisson's equations modified?

(a)

From equation 1, we know $D = \varepsilon E$, where ε is the permittivity of the medium

Substituting into equation 2:

$\nabla \cdot (\varepsilon E) = \rho$

$\Leftrightarrow \varepsilon(\nabla \cdot E) + E \cdot (\nabla \varepsilon) = \rho$ Assume ε is a constant (homogeneous medium)

$\nabla \cdot E = \frac{\rho}{\varepsilon}$

We know that $E = -\nabla V$ where V is the electrical potential

$\Rightarrow \nabla \cdot (-\nabla V) = \frac{\rho}{\varepsilon}$

$\Leftrightarrow \nabla^2 V = \frac{-\rho}{\varepsilon}$ ← equation for the electric potential in linear and isotropic medium

$= 0$ when $\rho = 0$ (It reduces to Poisson's equation when $\rho = 0$)

(b) (directly use it)

If the medium is non-homogeneous but isotropic: $\varepsilon = \varepsilon(x)$

$\frac{\partial^2 V}{\partial x^2} = -\frac{\rho}{\varepsilon(x)}$ (reduce the problem to one-dimension)

↓

Since one-dimension, we don't need to use ∇