1. Prove that an angle inscribed in a semicircle is a right angle using vector analysis.

2. A field is expressed in spherical coordinates by $\mathbf{E} = \mathbf{a_R}(25/R^2)$.

   a) Find $|\mathbf{E}|$ and E_x at the point $P(-3, 4, -5)$.

   b) Find the angle that $\mathbf{E}$ makes with the vector $\mathbf{B} = \mathbf{a_x}2 - \mathbf{a_y}2 + \mathbf{a_z}$ at point P.

3. Given a vector function $\mathbf{E} = \mathbf{a_x}y + \mathbf{a_y}x$, evaluate the scalar line integral $\int \mathbf{E} \cdot dl$ from $P_1(2, 1, -1)$ to $P_2(8, 2, -1)$

   a) along the parabola $x = 2y^2$,

   b) along the straight line joining the two points.

   Is this $\mathbf{E}$ a conservative field?

4. Find the divergence of the following radial vector fields:

   a) $f_1(\mathbf{R}) = \mathbf{a_R}R^n$,

   b) $f_2(\mathbf{R}) = \mathbf{a_R}\frac{k}{R^2}$.

5. For vector function $\mathbf{A} = \mathbf{a_r}r^2 + \mathbf{a_z}2z$, verify the divergence theorem for the circular cylindrical region enclosed by $r = 5$, $z = 0$, and $z = 4$.

6. For two differentiable vector functions $\mathbf{E}$ and $\mathbf{H}$, prove that

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = \mathbf{H} \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \mathbf{H}).$$

7. Use the definition

$$(\nabla \times \mathbf{A})_u = \mathbf{a_u} \cdot (\nabla \times \mathbf{A}) = \lim_{\Delta s_u \to 0} \frac{1}{\Delta s_u}\left(\oint_{C_u} \mathbf{A} \cdot dl\right)$$

   to derive the expression of the $\mathbf{a_R}$ component of $\nabla \times \mathbf{A}$ in spherical coordinates for a vector field $\mathbf{A} = \mathbf{a_R}A_R + \mathbf{a}_\theta A_\theta + \mathbf{a}_\phi A_\phi$.

8. Given a vector function $\mathbf{F} = \mathbf{a_x}(x + c_1 z) + \mathbf{a_y}(c_2 x - 3z) + \mathbf{a_z}(x + c_3 y + c_4 z)$.

   a) Determine the constants c_1, c_2, and c_3 if $\mathbf{F}$ is irrotational.

   b) Determine the constant c_4 if $\mathbf{F}$ is also solenoidal.

   c) Determine the scalar potential function V whose negative gradient equals $\mathbf{F}$.