
RC2: Static Electric Fields

1 Electrostatics in Free Space

1.1 Basic Concepts

Electrostatics:

- i. electric charges are **at rest(not moving)**;
- ii electric field **do not change with time**.

Static electric charges (source) in free space \rightarrow electric field

1.2 Electric field intensity

$$\mathbf{E} = \lim_{q \rightarrow 0} \frac{\mathbf{F}}{q} \quad (\text{V/m})$$

1.3 Fundamental Postulates of Electrostatics

- Differential form:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (\text{divergence})$$

$$\nabla \times \mathbf{E} = 0 \quad (\text{curl})$$

where ρ is the volume charge density of free charges (C/m^3), ϵ_0 is the permittivity of free space, a universal constant.

- Integral form:

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{Q}{\epsilon_0}$$

$$\oint_C \mathbf{E} \cdot d\boldsymbol{\ell} = 0$$

where Q is the total charge contained in volume V bounded by surface S . Also, the scalar line integral of the static electric field intensity around any closed path vanishes.

\mathbf{E} is **not solenoidal** (unless $\rho = 0$), but **irrotational (conservative)**

2 Coulomb's Law

2.1 Electric Field due to a System of Discrete Charges

- a single point charge (charge on the origin):

$$\mathbf{E} = \mathbf{a}_R E_R = \mathbf{a}_R \frac{q}{4\pi\epsilon_0 R^2} \quad (\text{V/m})$$

- a single point charge (charge is not on the origin):

$$\mathbf{E}_p = \frac{q(\mathbf{R} - \mathbf{R}')}{4\pi\epsilon_0 |\mathbf{R} - \mathbf{R}'|^3} \quad (\text{V/m})$$

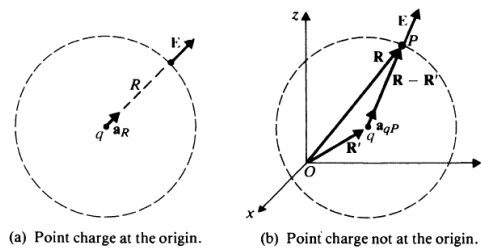


FIGURE 3-2
Electric field due to a point charge.

When a point charge q_2 is placed in the field of another point charge q_1 at the origin, a force F_{12} is experienced by q_2 due to the electric field intensity \mathbf{E}_{12} of q_1 at q_2 . Then we have:

$$\mathbf{F}_{12} = q_2 \mathbf{E}_{12} = \mathbf{a}_R \frac{q_1 q_2}{4\pi\epsilon_0 R^2}$$

- several point charges:

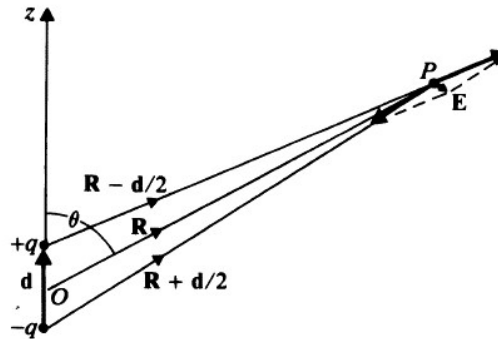
$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^n \frac{q_k (\mathbf{R} - \mathbf{R}'_k)}{|\mathbf{R} - \mathbf{R}'_k|^3}$$

Ex.1:

A total charge Q is uniformly put on a thin spherical shell of radius b . Determine the electric field intensity at an arbitrary point inside the shell.

2.2 Electric Dipole

- Electric Field



general expression:

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0} \left\{ \frac{\mathbf{R} - \frac{\mathbf{d}}{2}}{|\mathbf{R} - \frac{\mathbf{d}}{2}|^3} - \frac{\mathbf{R} + \frac{\mathbf{d}}{2}}{|\mathbf{R} + \frac{\mathbf{d}}{2}|^3} \right\}$$

if $d \ll R$:

$$\mathbf{E} \cong \frac{q}{4\pi\epsilon_0 R^3} \left[3 \frac{\mathbf{R} \cdot \mathbf{d}}{R^2} \mathbf{R} - \mathbf{d} \right]$$

- Electric Dipole Moment

Definition:

$$\mathbf{p} = q\mathbf{d}$$

where q is the charge, **vector \mathbf{d}** goes from $-q$ to $+q$.

$$\mathbf{p} = \mathbf{a}_z p = p (\mathbf{a}_R \cos \theta - \mathbf{a}_\theta \sin \theta)$$

$$\mathbf{R} \cdot \mathbf{p} = R p \cos \theta$$

- Electric Field: (spherical coordinate)

$$\mathbf{E} = \frac{p}{4\pi\epsilon_0 R^3} (\mathbf{a}_R 2 \cos \theta + \mathbf{a}_\theta \sin \theta) \quad (\text{V/m})$$

2.3 Electric Field due to a Continuous Distribution of Charge

- General Differential Element:

$$d\mathbf{E} = \mathbf{a}_R \frac{\rho dv'}{4\pi\epsilon_0 R^2}$$

, where dv' is the differential volume element.

- Line Charge:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int_{L'} \mathbf{a}_R \frac{\rho_\ell}{R^2} d\ell' \quad (\text{V/m})$$

- **Surface Charge:**

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int_{S'} \mathbf{a}_R \frac{\rho_s}{R^2} ds' \quad (\mathbf{V}/\text{m})$$

- **Volume Charge:**

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int_{V'} \mathbf{a}_R \frac{\rho}{R^2} dv' \quad (\mathbf{V}/\text{m})$$

Ex.2

Determine the electric field intensity of an infinitely long, straight, line charge of a uniform density ρ_ℓ in air.

Ex.3

A line charge of uniform density ρ_l in free space forms a semicircle of radius b . Determine the magnitude and direction of the electric field intensity at the center of the semicircle.

Ex.4 (An interesting Question)

How can you use a solid ball to create a uniform field somewhere? (you may)

3 Gauss's Law and Application

3.1 Definition

The total outward flux of the E-field over any closed surface in free space is equal to **the total charge enclosed in the surface** divided by ϵ_0 . (Note that we can choose arbitrary surface S for our convenience.)

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{Q}{\epsilon_0}$$

3.2 Application

- **Conditions for Maxwell's Integral Equations:**

There is **a high degree of symmetry** in the charge distribution or in the electrical field (i.e., spherically symmetric, planar, line charge, etc).

Ex.5

Determine the electric field intensity of an infinitely long, straight, line charge of a uniform density ρ_s in air.

Ex.6

Determine the electric field intensity of an infinite planar charge with a uniform surface charge density ρ_s .

Ex.7

Determine the \mathbf{E} field caused by a spherical cloud of electrons with a volume charge density $\rho = -\rho_o$ for $0 \leq R \leq b$ (both ρ_o and b are positive) and $\rho = 0$ for $R > b$.

Ex.8

Two infinite long coaxial cylindrical surfaces, $r = a$ and $r = b$ ($b > a$), carry surface charge densities ρ_{sa}, ρ_{sb} , respectively.

- a) Determine \mathbf{E} everywhere.
- b) What must be the relation between a and b in order that \mathbf{E} vanishes for $r > b$?

3.3 Several Useful Models (Paste on your CTPP!)

Note: The charge distribution should be **uniform**.

| different models | E(magnitude) |
|---|---|
| infinitely long, line charge | $E = \frac{\rho_\ell}{2\pi r\epsilon_0}$ |
| infinite planar charge | $E = \frac{\rho_s}{2\epsilon_0}$ |
| uniform spherical surface charge with radius R | $\begin{cases} E = 0 (r < R) \\ E = \frac{Q}{4\pi r^2\epsilon_0} (r > R) \end{cases}$ |
| uniform sphere charge with radius R | $\begin{cases} E = \frac{Qr}{4\pi R^3} (r < R) \\ E = \frac{Q}{4\pi r^2\epsilon_0} (r > R) \end{cases}$ |
| infinitely long, cylindrical charge with radius R | $\begin{cases} E = \frac{\rho_v r}{2\epsilon_0} (r < R) \\ E = \frac{\rho_v R^2}{2r\epsilon_0} (r > R) \end{cases}$ |

4 Electric Potential

- **Expression:**

$$\mathbf{E} = -\nabla V$$

the reason for the negative sign: consistent with the convention that in going against the \mathbf{E} field, the electric potential V increases.

- **Electric Potential Difference:**

$$V_2 - V_1 = - \int_{P_1}^{P_2} \mathbf{E} \cdot d\ell$$

- **Electric Potential due to a Charge Distribution**

$$V = \frac{q}{4\pi\epsilon_0 R}$$

- Line Charge:**

$$V = \frac{1}{4\pi\epsilon_0} \int_{L'} \frac{\rho_\ell}{R} d\ell' \quad (V)$$

- Surface Charge:**

$$V = \frac{1}{4\pi\epsilon_0} \int_{S'} \frac{\rho_s}{R} ds' \quad (V)$$

- Volume Charge:**

$$V = \frac{1}{4\pi\epsilon_0} \int_{V'} \frac{\rho}{R} dv' \quad (V)$$

Ex.9

A finite line charge of length L carrying uniform line charge density ρ_l is coincident with the x-axis.

- a) Determine V in the plane bisecting the line charge.
- b) Determine \mathbf{E} on the bisecting plane from ρ_l directly by applying Coulomb's law.
- c) Check the answer in part (b) with $-\nabla V$.