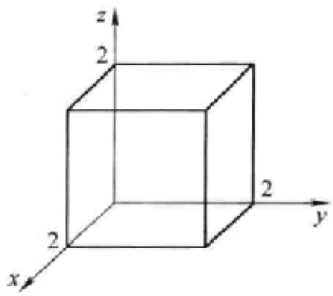
RC3

1 Quiz 1 Recap

Question 1

(a) Use the cube of side length 2 in the following picture and function $\boldsymbol{v} = (xy)\hat{\boldsymbol{x}} + (2yz)\hat{\boldsymbol{y}} + (3xz)\hat{\boldsymbol{z}}$ to verify the divergence theorem.



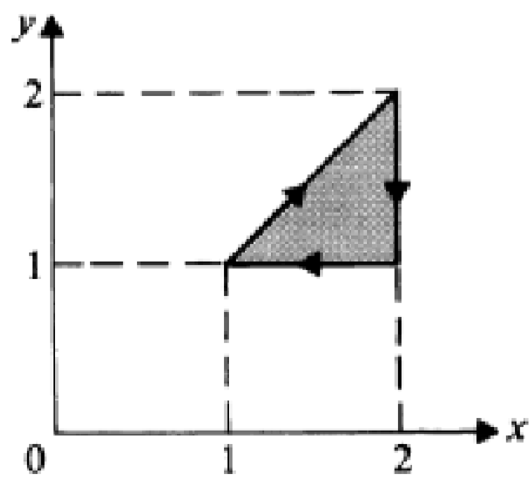
Question 2

Assume the vector function $\mathbf{A} = \mathbf{a}_x 3x^2y^3 - \mathbf{a}_y x^3y^2$.

(a) Find $\oint \mathbf{A} \cdot d\ell$ around the triangular contour shown in the following figure.

(b) Evaluate $\int (\nabla \times \mathbf{A}) \cdot d\mathbf{s}$ over the triangular area.

(c) Can $\mathbf{A}$ be expressed as the gradient of a scalar? Explain.



2 Gauss's Law and Application

2.1 Definition

The total outward flux of the E-field over any closed surface in free space is equal to **the total charge enclosed in the surface** divided by ϵ_0. (Note that we can choose arbitrary surface S for our convenience.)

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{Q}{\epsilon_0}$$

2.2 Application

- **Conditions for Maxwell's Integral Equations:**
  There is **a high degree of symmetry** in the charge distribution or in the electrical field (i.e., spherically symmetric, planar, line charge, etc.

2.3 Example

- Determine the electric field intensity of an infinitely long, straight, line charge of a uniform density ρ_ℓ.

- Determine the electric field intensity of an infinite planar charge with a uniform surface charge density ρ_s.

2.4 Several Useful Models (paste on your ctpp!)

Note: The charge distribution should be **uniform**.

different models	E (magnitude)
infinitely long, line charge	$E = \frac{\rho_\ell}{2\pi r\epsilon_0}$
infinite planar charge	$E = \frac{\rho_s}{2\epsilon_0}$
uniform spherical surface charge with radius R	$\begin{cases} E = 0 \ \ (r < R) \\ E = \frac{Q}{4\pi\epsilon_0 r^2} \ \ (r > R) \end{cases}$
uniform sphere charge with radius R	$\begin{cases} E = \frac{Qr}{4\pi\epsilon_0 R^3} \ \ (r < R) \\ E = \frac{Q}{4\pi\epsilon_0 r^2} \ \ (r > R) \end{cases}$
infinitely long, cylindrical charge with radius R	$\begin{cases} E = \frac{\rho_v r}{2\epsilon_0} \ \ (r < R) \\ E = \frac{\rho_v R^2}{2r\epsilon_0} \ \ (r > R) \end{cases}$

3 Electric Potential

- **Expression:**

$$\mathbf{E} = -\nabla V$$

the reason for the negative sign: consistent with the convention that in going against the **E** field, the electric potential V increases.

- **Electric Potential Difference:**

$$V_2 - V_1 = -\int_{P_1}^{P_2} \mathbf{E} \cdot d\ell$$

- **Electric Potential due to a Charge Distribution**

$$V = \frac{q}{4\pi\epsilon_0 R}$$

i. **Line Charge:**

$$V = \frac{1}{4\pi\epsilon_0}\int_{L'} \frac{\rho_\ell}{R} d\ell' \quad (V)$$

ii. **Surface Charge:**

$$V = \frac{1}{4\pi\epsilon_0}\int_{\mathbf{S}'} \frac{\rho_s}{R} ds' \quad (V)$$

iii. **Volume Charge:**

$$V = \frac{1}{4\pi\epsilon_0} \int_{V'} \frac{\rho}{R} dv' \quad (V)$$

- **Example:**
  Obtain a formula for the electric field intensity and potential on the axis of a circular disk of radius b that carries a uniform surface charge ρ_s.

3.1 Exercise

- (HW2-5) A finite line charge of length L carrying uniform line charge density ρ_l is coincident with the x-axis.

  a) Determine V in the plane bisecting the line charge.

  b) Determine $\mathbf{E}$ from ρ_l directly by applying Coulomb's law.

  c) Check the answer in part (b) with $-\nabla V$.

- (HW2-6) A charge Q is distributed uniformly over the wall of a circular tube of radius b and height h. Determine V and $\mathbf{E}$ on its axis

  a) at a point outside the tube, then

  b) at a point inside the tube.

4 Conductors and Dielectrics in Static Electric Field

- Conductors:
  - electrons migrate easily.
  - charges reach the surface and conductor redistribute themselves in a way that both the charge and the field vanish.
  - **static state conditions**:
    * inside the conductor:

      $$\rho = 0,\ \mathbf{E} = 0$$

      where $\rho = 0$ represents no charge in the interior
    * on the conductor surface (boundary conditions)

      $$E_t = 0,\ E_n = \frac{\rho_s}{\epsilon_0}$$
  - electric field intensity tends to be higher at a point near the surface of a charged conductor with a larger curvature
- Dielectrics (Insulators):
  - electrons are confined to their orbits.
  - external electric field polarizes a dielectric material and create electric dipoles. The induced electric dipoles will modify the electric field both inside and outside the dielectric material, as shown in Fig 1.

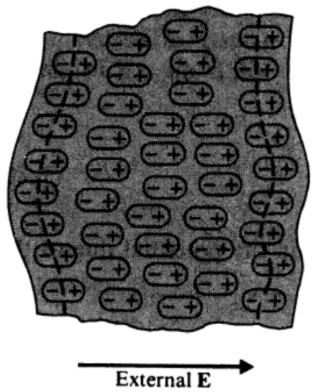


Figure 1: A cross section of a polarized dielectric medium

- **polarization charge densities/ bound-charge densities:**
  - **polarization vector, P**:

$$\mathbf{P} = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{n\Delta v} \mathbf{p_k}}{\Delta v}$$

    where the numerator represents the vector sum of the induced dipole moment contained in a very small volume Δv.
  - charge distribution on surface density:

$$\rho_{ps} = \mathbf{P} \cdot \mathbf{a_n}$$

  - volume charge distribution density:

$$\rho_p = -\nabla \cdot \mathbf{P}$$

    .

4.1 Exercise

- (HW3-1) The polarization in a dielectric cube of side L centered at the origin is given by $\mathbf{P} = P_0(\mathbf{a_x}x + \mathbf{a_y}y + \mathbf{a_z}z)$.
  a) Determine the surface and volume bound-charge densities.
  b) Show that the total bound charge is zero.

- (HW3-2) Determine the electric field intensity at the center of a small spherical cavity cut out of a large block of dielectric in which a polarization $\mathbf{P}$ exists.