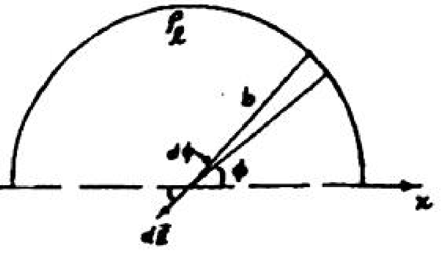
HW2-1

<u>-6</u>



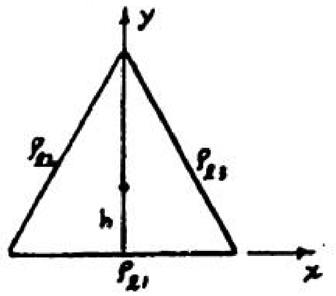
$$dE_y = -\frac{\rho_\ell\,(b\,d\phi)}{4\pi\epsilon_0 b^2}\sin\phi,$$

$$\bar{E} = \bar{a}_y E_y = -\bar{a}_y \frac{\rho_\ell}{4\pi\epsilon_0 b}\int_0^{\pi}\sin\phi\, d\phi$$

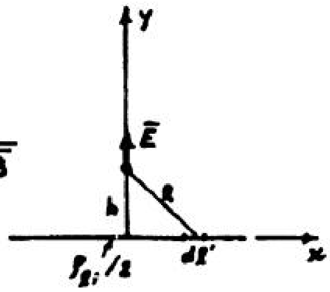
$$= -\bar{a}_y \frac{\rho_\ell}{2\pi\epsilon_0 b}.$$

HW2-2

<u>-7</u>



$$h = \frac{L}{2\sqrt{3}}$$



$\bar{E}$ at the center of triangle would be zero if all three line charges were of the same charge density. The problem is equivalent to that of a single line charge of density $\rho_{\ell 1}/2$. By symmetry, there will only be a y-component.

$$\bar{E} = \bar{a}_y E_y = \bar{a}_y \int_{-L/2}^{L/2} \frac{(\rho_{\ell 1}/2)\,d\ell'}{4\pi\epsilon_0 R^2}\left(\frac{h}{R}\right) = \bar{a}_y \int_{-L/2}^{L/2} \frac{\rho_{\ell 1}\, h\, d\ell'}{8\pi\epsilon_0 (h^2+\ell'^2)^{3/2}}$$

$$= \bar{a}_y \frac{3\rho_{\ell 1}}{4\pi\epsilon_0 L} = \bar{a}_y \frac{3\rho_{\ell 1}}{2\pi\epsilon_0 L}.$$

HW2-3

<u>P.3-10</u> Cylindrical symmetry: $\bar{E} = \bar{a}_r E_r$. Apply Gauss's law.

a) $r < a$, $E_r = 0$; $a < r < b$, $E_r = a\rho_{sa}/\epsilon_0 r$;

$r > b$, $E_r = (a\rho_{sa} + b\rho_{sb})/\epsilon_0 r$.

b) $b/a = -\rho_{sa}/\rho_{sb}$.

HW2-4

a) $x = 2y^2, \quad dx = 4y\,dy$

$$\int \vec{E}\cdot d\vec{l} = \int (\vec{a}_x y + \vec{a}_y x)(\vec{a}_x dx + \vec{a}_y dy) = \int y\,dx + x\,dy = \int y \cdot 4y\,dy + 2y^2 dy$$

$$= \int_1^2 6y^2 dy = 14$$

$$W = -q\int \vec{E}\cdot d\vec{l} = -(-2\times10^{-6})\times 14 = 2.8\times10^{-5}\ \mathrm{J}$$

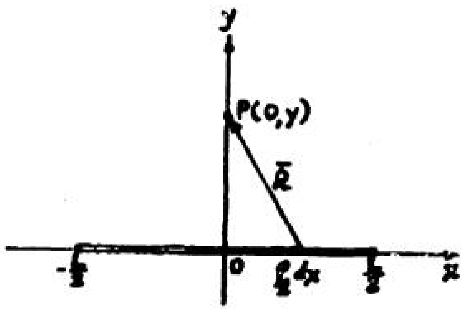
b) $x = 6y - 4, \quad dx = 6\,dy$

$$\int \vec{E}\cdot d\vec{l} = \int y\,dx + x\,dy = \int y\cdot 6\,dy + (6y-4)\,dy = \int_1^2 (12y-4)\,dy = 14$$

$$W = -q\int \vec{E}\cdot d\vec{l} = -(-2\times10^{-6})\times 14 = 2.8\times10^{-5}\ \mathrm{J}$$

HW2-5

-13



a) $$V = 2\int_0^{L/2} \frac{\rho_l\,dx}{4\pi\epsilon_0 R}$$

$$= \frac{\rho_l}{2\pi\epsilon_0}\int_0^{L/2}\frac{dx}{\sqrt{x^2+y^2}}$$

$$= \frac{\rho_l}{2\pi\epsilon_0}\left\{\ln\left[\sqrt{\left(\tfrac{L}{2}\right)^2+y^2} - \tfrac{L}{2}\right] - \ln y\right\}.$$

b) From Coulomb's law:

$$\bar{E} = \bar{a}_y E_y = 2\int_0^{L/2} \bar{a}_y \frac{\rho_l\,y\,dx}{4\pi\epsilon_0 R^3} = \bar{a}_y \frac{\rho_l}{2\pi\epsilon_0 y}\,\frac{L/2}{\sqrt{(L/2)^2+y^2}}.$$

c) $E = -\bar{\nabla} V$ gives the same answer.

HW2-6

P.3-15 Assume the circular tube sits on the xy-plane with its axis coinciding with the z-axis. The surface charge on the tube wall is $\rho_s = Q/2\pi bh$. First find the potential along the axis at z due to a circular line charge of density ρ_ℓ situated at z'.

$$V = \oint \frac{\rho_\ell\, d\ell}{4\pi\epsilon_0 R} = \int_0^{2\pi} \frac{\rho_\ell\, b\, d\phi}{4\pi\epsilon_0\sqrt{b^2+(z-z')^2}} = \frac{\rho_\ell\, b}{2\epsilon_0\sqrt{b^2+(z-z')^2}}.$$

a) The expression above is the contribution dV due to a circular line charge of density $\rho_\ell = \rho_s\, dz'$.

$$dV = \frac{\rho_s\, b\, dz'}{2\epsilon_0\sqrt{b^2+(z-z')^2}}.$$

At a point outside the tube:

$$V = \int_{z'=0}^{z'=h} dV = \frac{b\rho_s}{2\epsilon_0} \ln \frac{z+\sqrt{b^2+z^2}}{(z-h)+\sqrt{b^2+(z-h)^2}}$$

$$\bar{E} = -\bar{a}_z \frac{dV}{dz} = \bar{a}_z \frac{b\rho_s}{2\epsilon_0}\left[\frac{1}{\sqrt{b^2+(z-h)^2}} - \frac{1}{\sqrt{b^2+z^2}}\right].$$

b) Same expressions are obtained for V and $\bar{E}$ at a point inside the tube.