Lecture 8

VE 311 Analog Circuits

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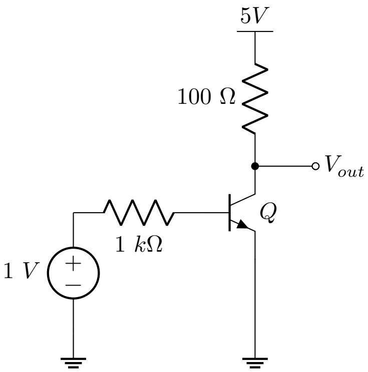
Recap of Last Lecture

- Reviewed Op-amps
- BJT

Topics to Be Covered

- BJT

DC Condition Calculation



Given:

$$\beta = 100, v_{BE,on} = 0.6V \tag{1}$$

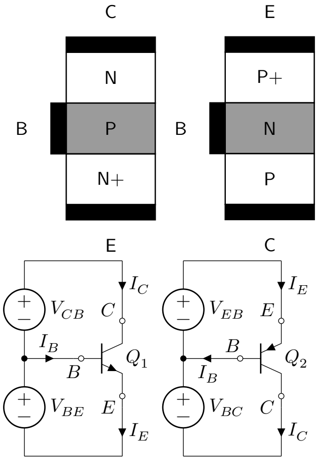
What is v_O? Assume FAR

$$i_B = \frac{1 - 0.6V}{R_B} = 0.4mA \tag{2}$$

$$i_C = \beta i_B = 100 \cdot 0.4\ mA = 40\ mA \tag{3}$$

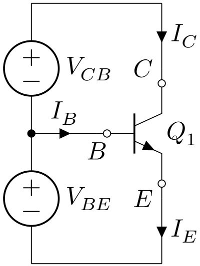
$$v_O = 5V - 100\ \Omega \cdot 40mA = 1V \tag{4}$$

pnp Transistor



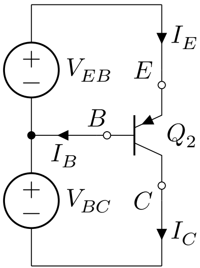
- For pnp, the basics of the operation are the same, except now instead of e^- crossing the base, h^+ cross the base.
- Because holes move a lot slower than e^-, there would be more recombination and fewer of them make it to the collector. Thus, the current gain is much lower for pnp than for npn.
- Note that all polarities are now different, for example for FAR operation, the BE junction has to be forward biased which means that the base-emitter voltage should be $-0.7V$, so the BE junction is forward biased.

pnp Transistor



Applied Voltages	B-E junction Bias (PNP)	B-C junction Bias (PNP)	Mode(NPN)
E $<$ B $<$ C	Reverse	Forward	Reverse-active
E $<$ B $>$ C	Reverse	Reverse	Cut-off
E $>$ B $<$ C	Forward	Forward	Saturation
E $>$ B $>$ C	Forward	Reverse	Forward-active

pnp Transistor

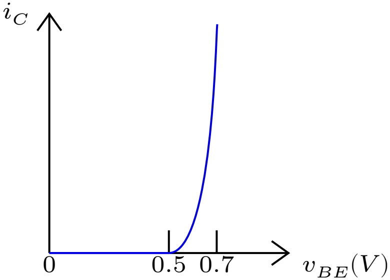


Applied Voltages	B-E junction Bias (NPN)	B-C junction Bias (NPN)	Mode(NPN)
E $<$ B $<$ C	Forward	Reverse	Forward-active
E $<$ B $>$ C	Forward	Forward	Saturation
E $>$ B $<$ C	Reverse	Reverse	Cut-off
E $>$ B $>$ C	Reverse	Forward	Reverse-active

Large Signal Model: T-Model

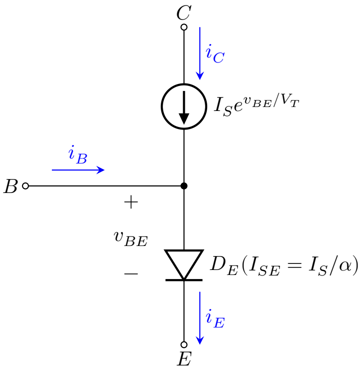
- We can now easily establish large signal models for the transistor that describe the current-voltage relation ship

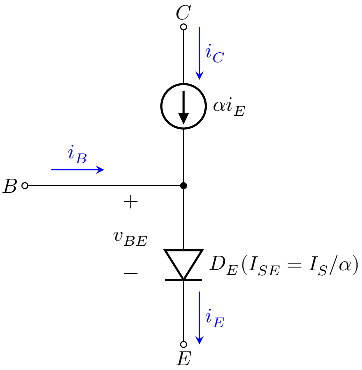
$$i_C \approx I_S e^{v_{BE}/V_T} \tag{5}$$



Large Signal Model: T-Model

- The base is shared between the input port (B-E) and the output port (B-C)

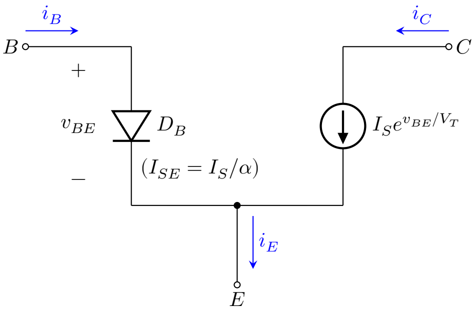


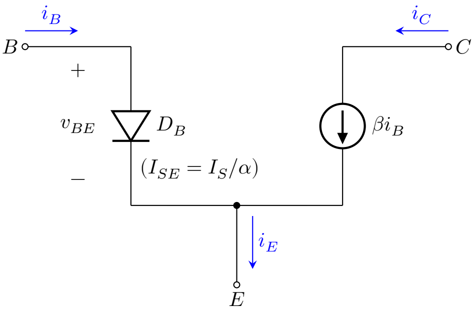


Hybrid-π Model

- In hybrid-π model, where the collector current is shown as a voltage controlled current source (VCCS).
- The hybrid-π model is more useful for analyzing the common-emitter amplifier. Notice that the emitter in this model is shared between the input port (B-E), and the output port (C-E).

Hybrid-π Model





Small Signal

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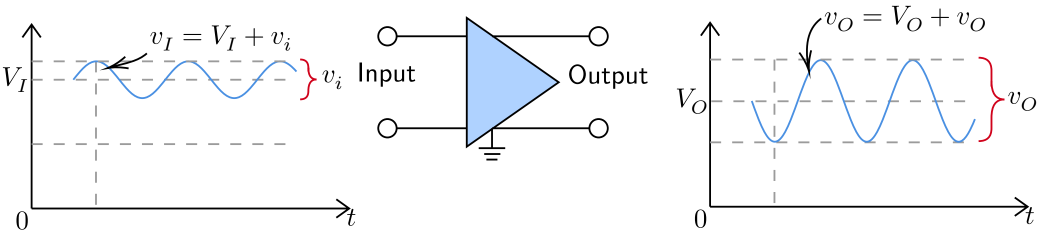
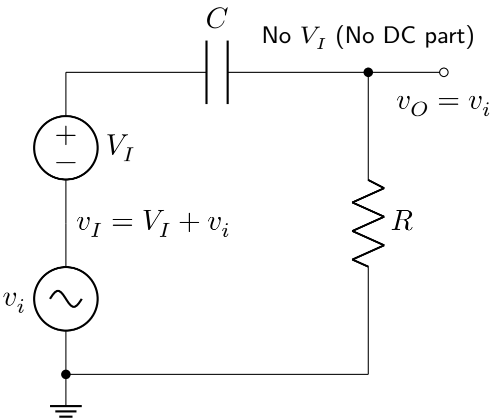


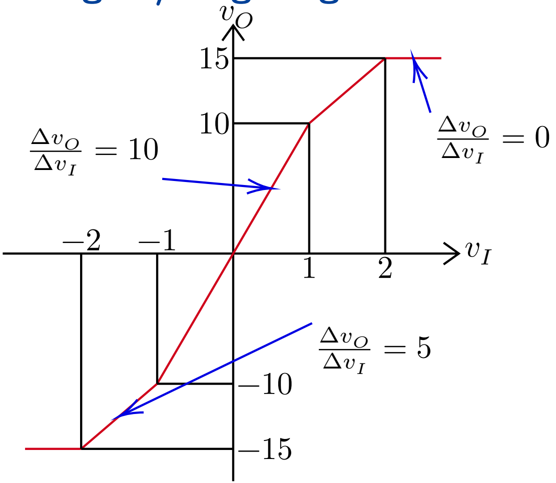
- Amplifiers refer to circuits that amplify AC (time varying) signals; there is generally no interest in the DC component of a signal.
- Circuits that change the DC value of the input to produce an output with a different DC value are generally referred to as level shifters, or DC-DC converters.

AC-Coupled or DC-Coupled

- AC-coupled, means only AC signals will pass through to the input of the next stage.
- DC coupled means there is no capacitor separating one part of a circuit from another part when the two are connected.

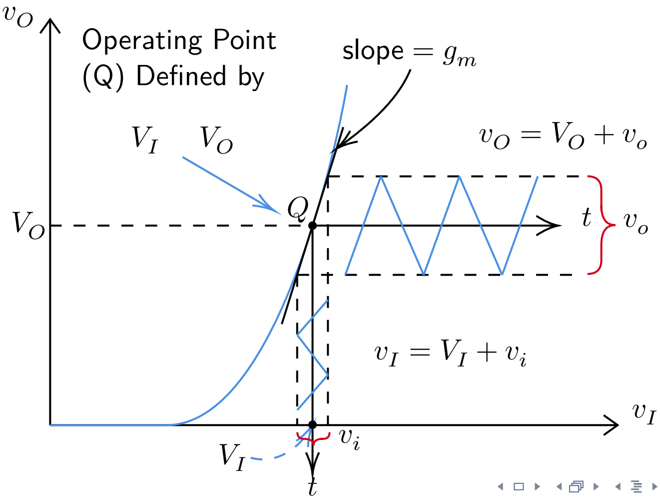


Small-Signal/Large-Signal

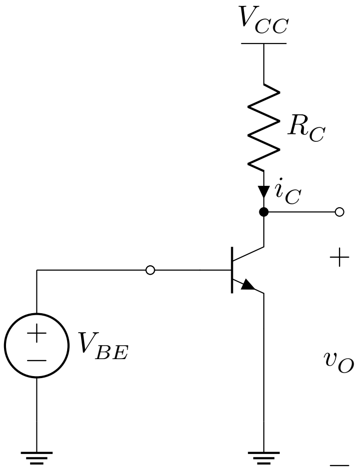


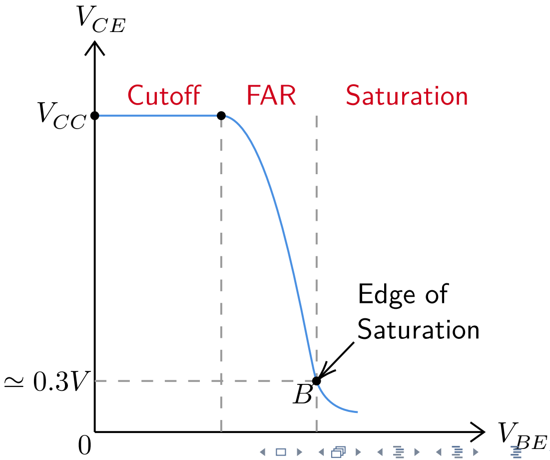
- A circuit with linear behavior over a specific range of signals, could be used as a linear circuit, when/if the range of signals applied to it is limited to where response is linear

Small-Signal/Large-Signal



BJT CE Gain


V_{CC}
R_C
i_C
+
V_{BE}
v_O
−


V_{CE}
Cutoff
FAR
Saturation
V_{CC}
Edge of
Saturation
$\simeq 0.3V$
B
0
V_{BE}

Small Signal Gain

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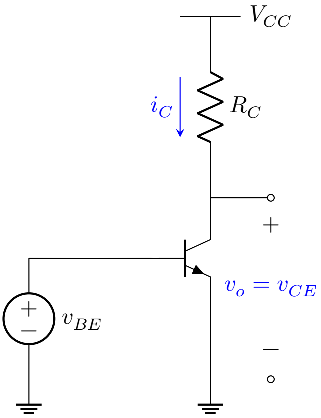

$$A_v = \frac{dv_o}{dv_{BE}} \tag{6}$$

$$A_v = -\frac{I_C}{V_T} R_C \tag{7}$$

Small Signal Gain

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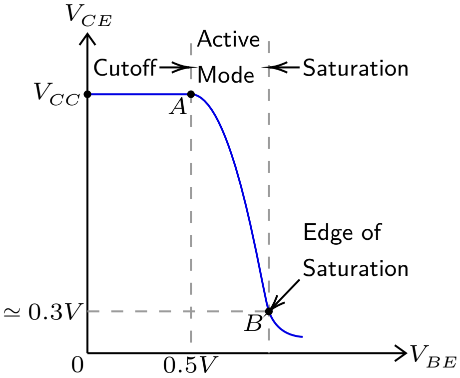

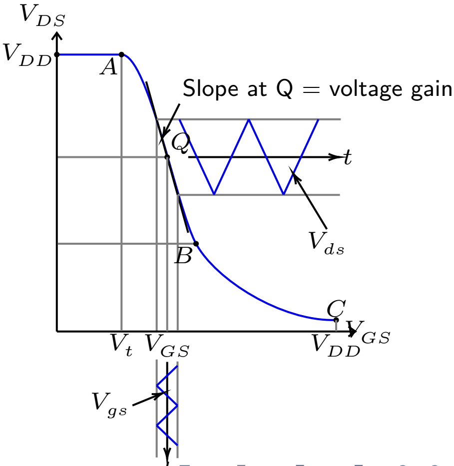

Small Signal Gain

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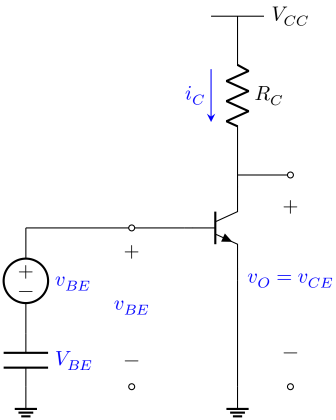
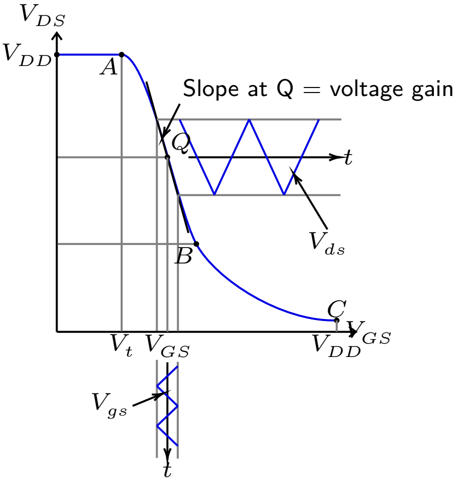


$$v_{CE}(t) = V_{CE} + v_{ce}(t) \tag{8}$$

$$v_{BE}(t) = V_{BE} + v_{be}(t) \tag{9}$$

Quiescent Point (Q)

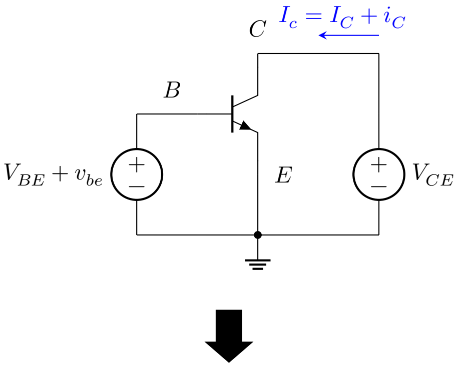


- Operating point for circuit
- Often called "bias" point

$$A_v = \frac{V_o}{v_{be}} \tag{10}$$

$$V_o = V_{CC} - R_C I_S e^{v_{BE}/V_T} \tag{11}$$

Hybrid-π Model (g_m and r_π)



$V_{CE} \geq V_{BE} \Rightarrow$ Forward—Active

$$I_C = I_S \left(e^{\frac{qV_{BE}}{kT}} - 1 \right) \left(1 + \frac{V_{CE}}{V_A} \right) \qquad (12)$$

Hybrid-π Model (Derivation of g_m and r_π)

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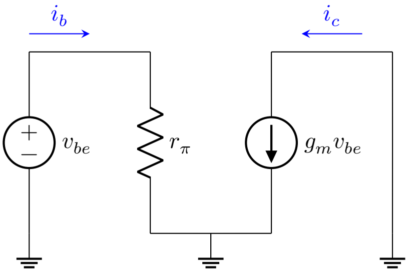
Small-signal circuit:



$$r_\pi = \frac{dV_{BE}}{dI_B} = \frac{1}{\frac{dI_C}{\beta dV_{BE}}} \tag{13}$$

$$= \frac{1}{\frac{g_m}{\beta}} = \frac{\beta}{g_m} \tag{14}$$

$$g_m = \frac{dI_C}{dV_{BE}} \cong \frac{I_C}{kT/q} \tag{15}$$

Models with the Early Effect Included

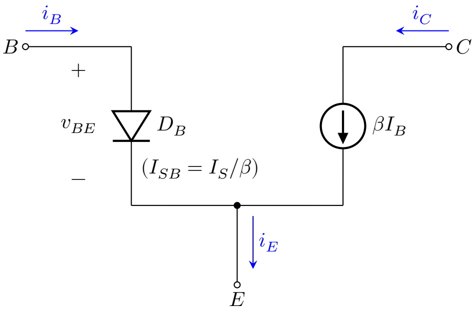
- As we see from this relationship, the collector current should change when the collector-emitter voltage changes:

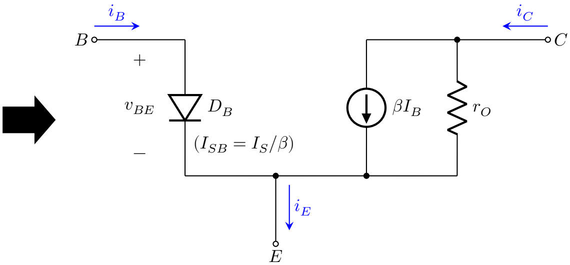
$$i_C \approx I_S e^{v_{EB}/V_T}\left(1 + \frac{v_{CE}}{V_A}\right) \tag{16}$$

- So, this effect is included in the model shown below where:

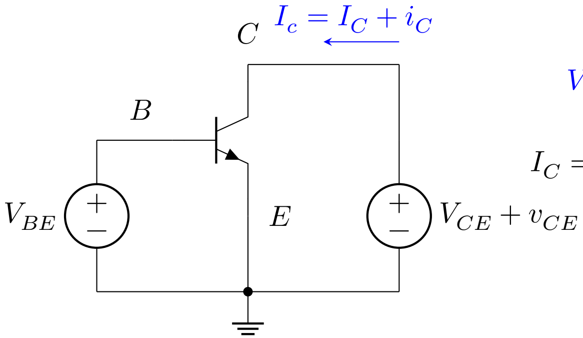
$$r_o = \frac{V_A}{I_C} = \frac{\Delta v_{CE}}{\Delta i_C} \tag{17}$$

Models with the Early Effect Included





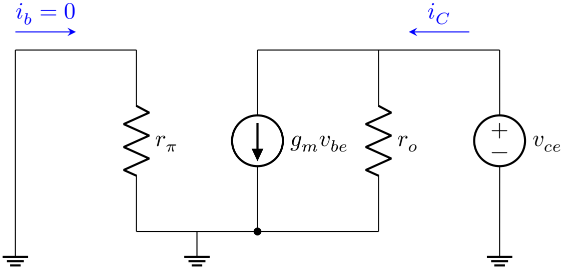
Hybrid-π Model (how to get r_o)



$V_{CE} \geq V_{BE} \Rightarrow$ Forward—Active

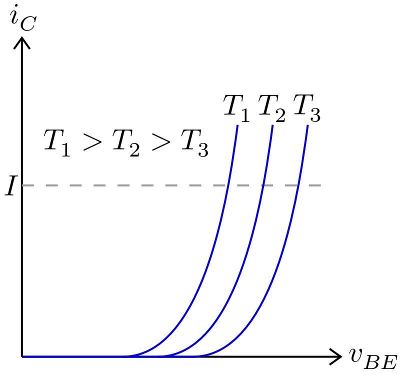
$$I_C = I_S \left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right) \tag{18}$$

Hybrid-π Model (Derivation of r_o)



$$r_o = \frac{1}{\frac{dI_C}{dV_{CE}}} \cong \frac{V_A}{I_C} \tag{19}$$

Temperature Variation

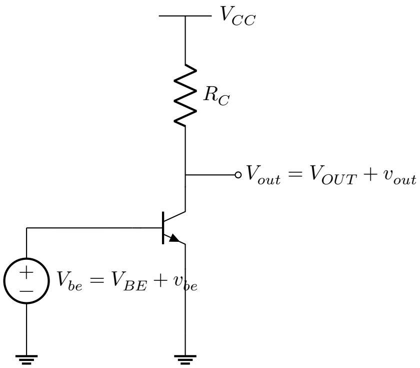


$$i_C = I_S e^{v_{BE}/V_T} \tag{20}$$

- Both I_S and V_T are temperature dependent

Common-Emitter Amplifier

- Sedra 7.1,7.2.2, 7.2.3



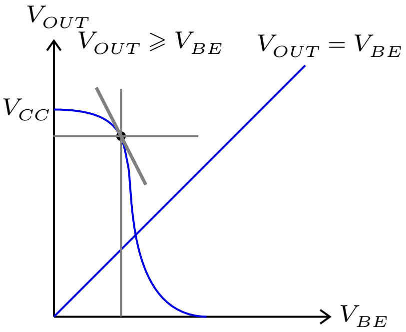
Common-Emitter Amplifier

$$V_{OUT} = V_{CC} - I_C R_C \tag{21}$$

$$= V_{CC} - \frac{AqD_n {n_i}^2}{N_a W_B}\left(e^{\frac{qV_{BE}}{kT}} - 1\right) R_C \tag{22}$$

$$A_V = \frac{dV_{OUT}}{dV_{BE}} \tag{23}$$

$$\cong -\frac{I_C}{kT/q} R_C = -g_m R_C \tag{24}$$



Common-Emitter Amplifier

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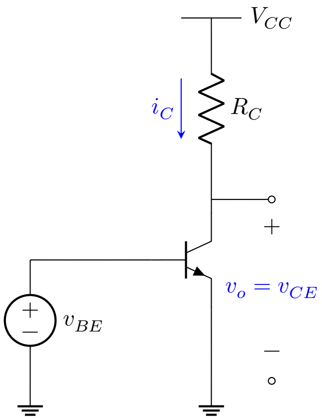
Gain varies a lot in the FAR region.

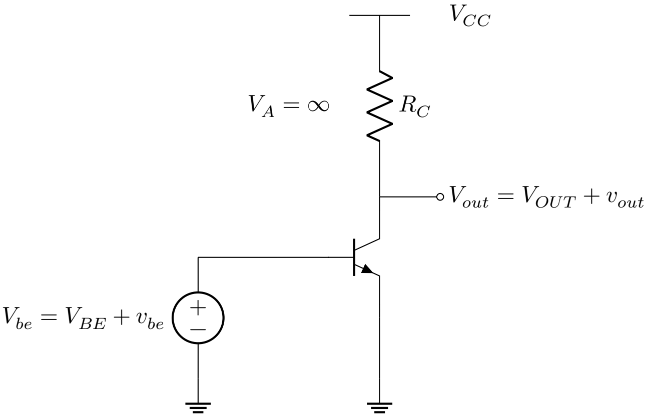
$$A_v = \frac{dv_o}{dv_{BE}} \tag{25}$$

$$A_V = \frac{dV_{OUT}}{dV_{BE}} \tag{26}$$

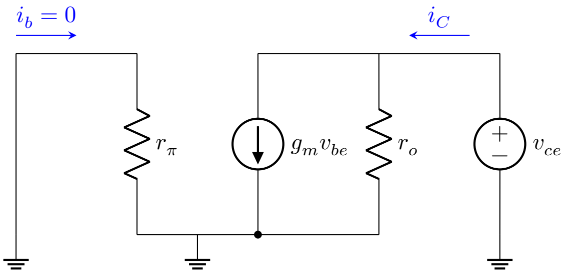
$$\cong -\frac{I_C}{kT/q}R_C = -g_m R_C \tag{27}$$



Common-Emitter Amplifier ($V_A = \infty$)



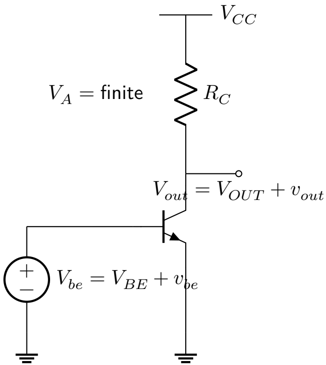
Common-Emitter Amplifier ($V_A = \infty$)



$$A_v = \frac{v_{out}}{v_{be}} = -g_m \left(R_C \parallel r_O\right) \tag{28}$$

$$= -g_m R_C \quad (\text{since } r_o = \infty) \tag{29}$$

Common-Emitter Amplifier (V_A = finite)

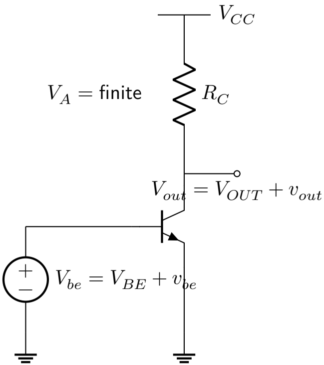


$$V_{OUT} = V_{CC} - I_C R_C \tag{30}$$

$$= V_{CC} - I_S \left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{OUT}}{V_A}\right) R_C \tag{31}$$

$$\frac{dV_{OUT}}{dV_{BE}} = -\frac{q}{kT} I_S e^{\frac{qV_{BE}}{kT}} \left(1 + \frac{V_{OUT}}{V_A}\right) R_C \tag{32}$$

Common-Emitter Amplifier (V_A = finite)



$$-I_S\left(e^{\frac{qV_{BE}}{kT}}-1\right)\frac{1}{V_A}\frac{dV_{OUT}}{dV_{BE}}R_C \tag{33}$$

$$\cong -g_m R_C - \frac{1}{r_o}\frac{dV_{OUT}}{dV_{BE}}R_C \tag{34}$$

Common-Emitter Amplifier (V_A = finite)

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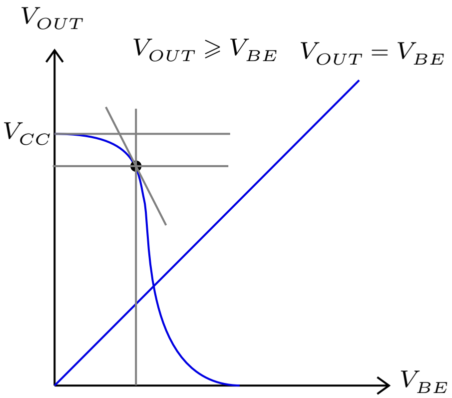
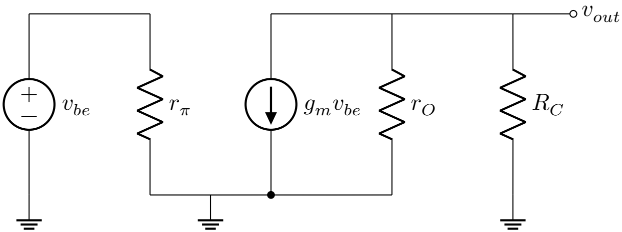
$$A_V = \frac{dV_{OUT}}{dV_{BE}} \tag{35}$$

$$\cong g_m(R_C \parallel r_O) \tag{36}$$



Common-Emitter Amplifier (V_A = finite)

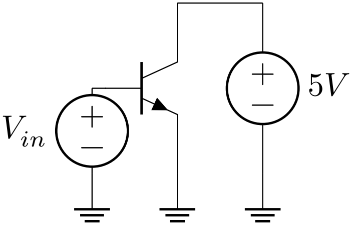
- Small-Signal Analysis



$$A_v = \frac{v_{out}}{v_{be}} = -g_m(R_C \parallel r_O) \tag{37}$$

Example 1: $I_S = 1e{-}15\ \beta = 100\ V_{AF} = 50$

- What is current?
- What is small signal gain?
- What is the output if input $= 2 + 0.001\sin(2 \cdot \pi \cdot 100t)$

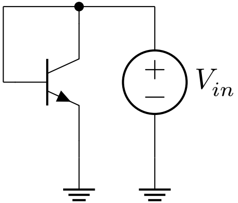


$$I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right) \quad (38)$$

Example 1

Example 2: $I_S = 10^{-15}, \beta = 100, V_{AF} = 50$

Find the output if input $= 2 + 0.001\sin(2\pi 100 t)$



Example 2