VE311
Electronic Circuits
RC 3

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Razavi 3

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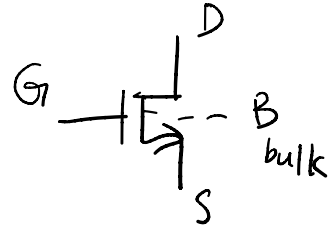
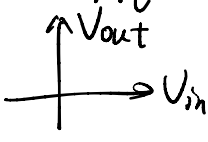


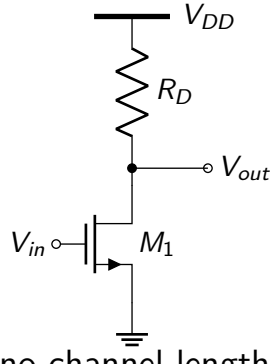
CS with Resistive Load

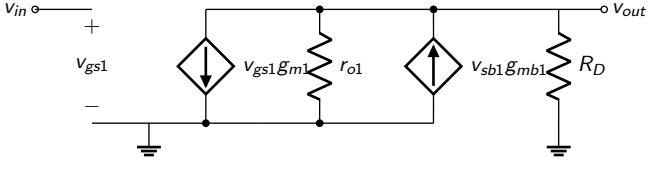
放大器 → 放大，gain

Voltage gain: A_v

transfer curve:





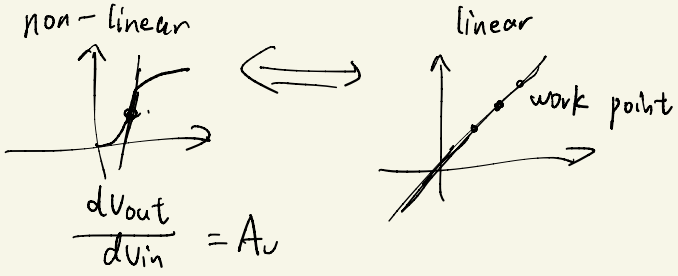


If no channel-length and body effect:

$$A_v = \frac{v_{out}}{v_{in}} = -g_{m1}R_D \tag{1}$$

No body effect:

$$A_v = -g_{m1}(R_D \parallel r_{o1}) \tag{2}$$



Common-Source

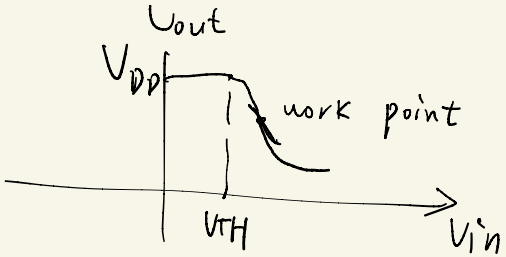
S-ground

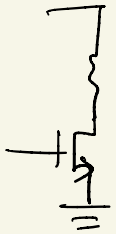
g- input

d- output

pins

- resistive
- diode-connect
- current source





saturation: gain

$$A_V = \frac{\partial V_{out}}{\partial V_{in}} = -\mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH}) R_D$$

$$= -g_m R_D$$

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2 (1 + \lambda V_{out})$$

$$A_V = \frac{\partial V_{out}}{\partial V_{in}} = -R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2 \frac{\partial V_{out}}{\partial V_{in}}$$

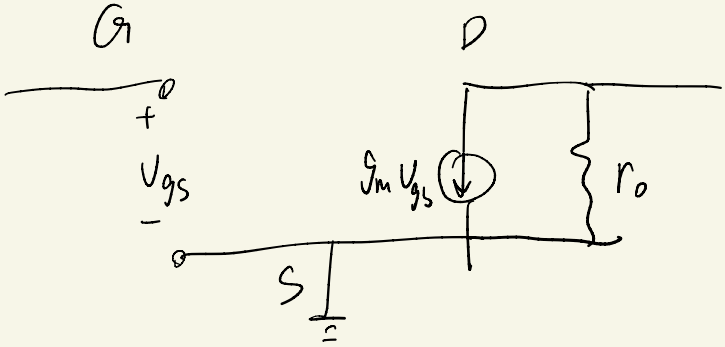
$$- R_D \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})(1 + \lambda V_{out})$$

$$\Rightarrow A_V = -R_D g_m - R_D I_D \lambda A_V$$

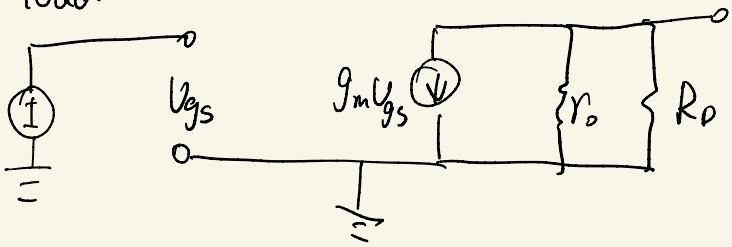
$$r_O = \frac{1}{\lambda I_D} \Rightarrow A_V = \frac{-g_m R_D}{1 + R_D \lambda I_D}$$

$$= -g_m \frac{R_D r_O}{R_D + r_O}$$

$$= -g_m (R_D \,//\, r_O)$$



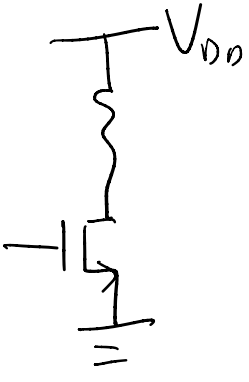
Resistive load:



Example 1

In the common source stage amplifier, $V_{DD} = 5V, R_D = 10k\Omega$, change the input voltage and let the current pass through M1 be 0.1mA. What is the gain of the amplifier? (Neglect body effect)
Parameter for NMOS: $V_{THN} = 0.7V$,
$K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$
Parameter for PMOS: $V_{THP} = -0.7V$,
$K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$
All the size of transistor is $W = 20\mu m, L = 1\mu m$

Example 1



$V_{in} > V_{TH} \rightarrow$ saturation;

assume saturation., neglect channel-length effect

$$I = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS1} - V_{TH})^2$$

$$= \frac{1}{2} \times 110\times10^{-6} \times 20 \times (V_{GS} - 0.7)^2 = 0.1\,mA$$

$$\Rightarrow V_{GS} = 1.0015$$

$$\Rightarrow V_{DS} = V_{DD} - R_D I = 5 \times 10\times10^{3} \times 0.1\times10^{-3} = 4$$

Example 1

☆ saturation

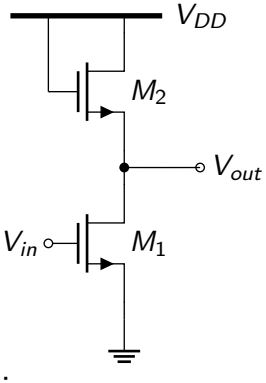
$$\Rightarrow g_m = \sqrt{2 K_n \frac{W}{L} I_D} = 663.3\ \mu A/V$$

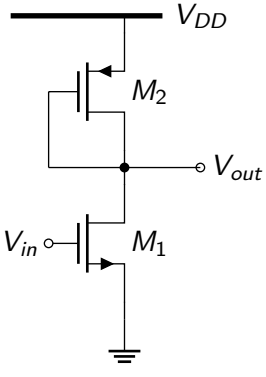
$$r_o = \frac{1}{\lambda I_D} = 250\ k\Omega$$

$$A_V = -g_m (r_o \parallel R_D) = \underline{-6.378}\ V/V$$

inverse amplifier

CS with Diode-connected Load





NMOS:

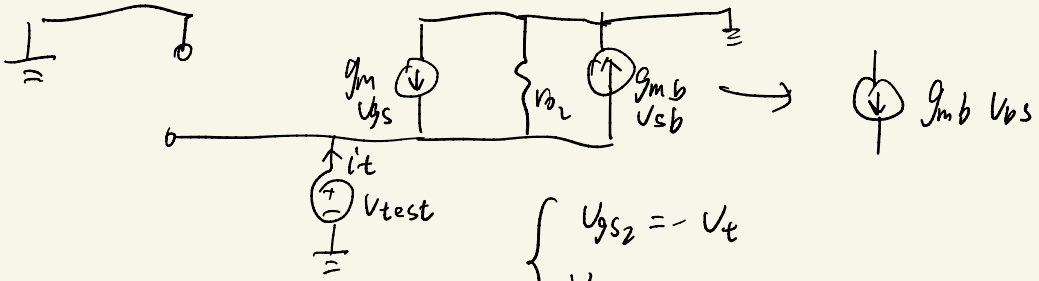
$$A_{\mathrm{v}} = -\sqrt{\frac{(W/L)_1}{(W/L)_2}}\frac{1}{1+\eta} \qquad \eta = g_{\mathrm{mb2}}/g_{\mathrm{m2}} \tag{3}$$

PMOS:

$$A_{\mathrm{v}} = -\sqrt{\frac{\mu_{\mathrm{n}}(W/L)_1}{\mu_{\mathrm{p}}(W/L)_2}} \tag{4}$$

G – D connect : always saturation

small-signal



$$\begin{cases} v_{gs_2} = -v_t \\ v_{bs_2} = -v_t \\ -g_m v_{gs_2} + \dfrac{v_t}{r_{o2}} - g_{mb} v_{bs_2} = i_t \end{cases}$$

$$\Rightarrow \frac{v_t}{i_t} = \frac{1}{g_{m2} + g_{mb2} + \frac{1}{r_{o2}}} = \frac{1}{g_{m2} + g_{mb2}} /\!/ r_{o2}$$

Connect M_1:

$$\frac{v_t}{i_t} = \frac{1}{g_{m2} + g_{mb2}} /\!/ r_{o2} /\!/ r_{o1}$$

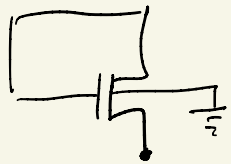
$$A_v = -g_{m1} \cdot \frac{1}{g_{m2} + g_{mb2}} /\!/ r_{o2} /\!/ r_{o1}$$

Since $\frac{1}{g_{m2} + g_{mb2}} \ll r_{o1}, r_{o2}$

$$A_v \approx -\frac{g_{m1}}{g_{m2} + g_{mb2}} = -\frac{g_{m1}}{g_{m2}} \cdot \frac{1}{1+\eta} = -\sqrt{\frac{(W/L)_1}{(W/L)_2}} \cdot \frac{1}{1+\eta}$$

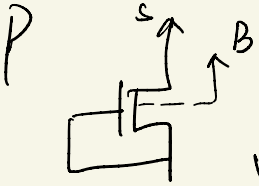
PMOS: $r_{out} = \frac{1}{g_{m2}} /\!/ r_{o2} /\!/ r_{o1}$ (no g_{mb2}!)

N



$V_{SB} \neq 0$

P

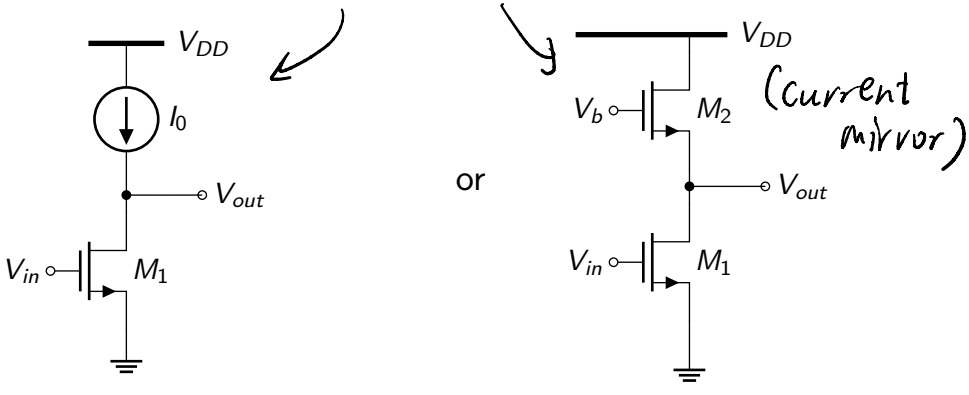


$V_{SB} = 0$

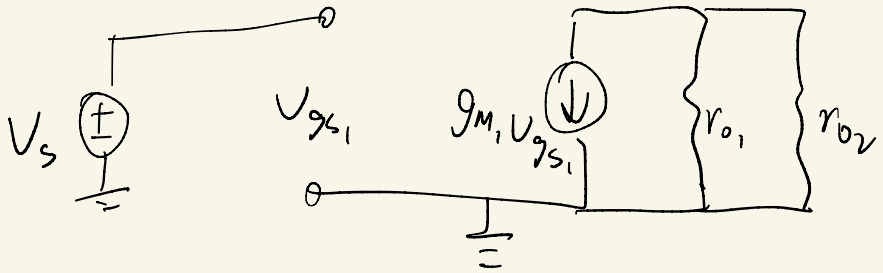
$$A_V = -g_{m_1} r_{out_p} = -\frac{g_{m_1}}{g_{m_2}} = -\sqrt{\frac{\mu_n (W/L)_1}{\mu_p (W/L)_2}}$$

CS with Current Source Load

how to realize



$$A_v = -g_{m1}(r_{o2} \parallel r_{o1}) \tag{5}$$


V_s
V_{gs_1}
$g_{m_1} V_{gs_1}$
r_{o_1}
r_{o_2}

Example 2

In the common source stage amplifier, the current source is ideal. Find the intrinsic gain A_v for the amplifier when $I_0 = 0.01$ and $0.1mA$ respectively. (Neglect body effect)
Parameter for NMOS: $V_{THN} = 0.7V$,
$K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$
Parameter for PMOS: $V_{THP} = -0.7V$,
$K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$
All the size of transistor is $W = 20\mu m, L = 1\mu m$

Example 2

0.1 mA

$$g_m = \sqrt{2 K_n \frac{W}{L} I_{D1}} = 663.3 \ \mu A/V$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.04 \times 0.1 \times 10^{-3}} = 250 \ k\Omega$$

$$A_v = -g_m r_o = -165.8 \ V/V$$

0.01 mA

$$g_m = 209.76 \qquad r_o = 2500 \ k$$

$$A_v = -524.4 \ V/V$$

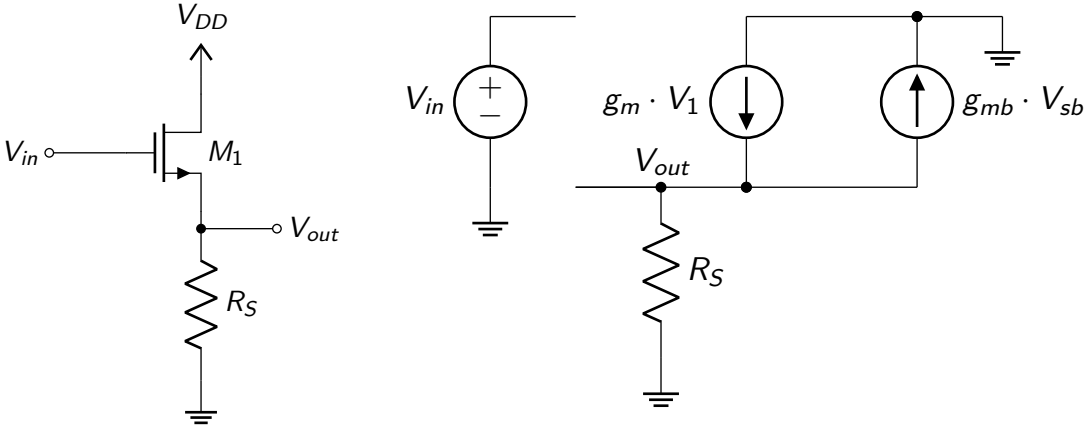
Swing :

① medium

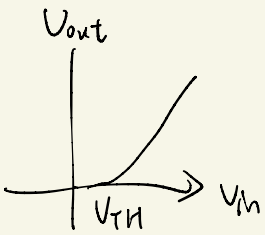
② short $V_{GS_1} - V_{TH} \leq V_{out} \leq V_{DD} - V_{GS_2}$

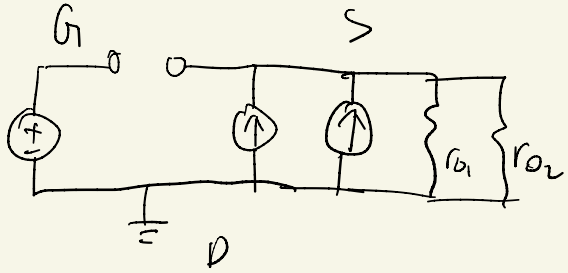
③ high $V_{GS} - V_{TH} \leq V_{out} \leq V_B + |V_{TH}|$

Source Follower



$$A_v = \frac{g_m R_S}{1 + g_m R_S(1+\eta)} = \frac{g_m R_S}{1 + (g_m + g_{mb})R_S} \approx \frac{1}{1+\eta} \qquad (6)$$





$$g_m V_{in} = g_{m1} V_{out} + g_{mb1} V_{out} + \frac{V_{out}}{r_{o1} // r_{o2}}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{g_{m1}}{g_{m1} + g_{mb1} + \frac{1}{r_{o1} // r_{o2}}}$$

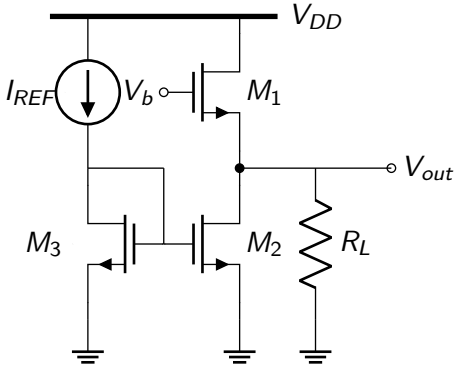
neglect channel-length $\quad A_V = \frac{g_{m1}}{g_{m1} + g_{mb1}} = \frac{1}{1+\eta}$

Example 3

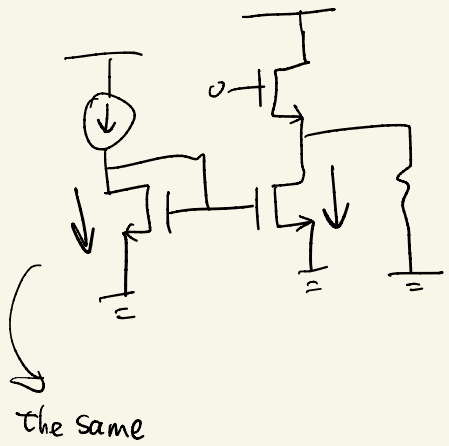
In the source follower with current source load, the current source is ideal. Find the output impedance for the amplifier when $I_0 = 0.01$ and $0.1mA$ respectively. (Neglect body effect)
Parameter for NMOS: $V_{THN} = 0.7V$,
$K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$
Parameter for PMOS: $V_{THP} = -0.7V$,
$K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$
All the size of transistor is $W = 20\mu m, L = 1\mu m$

$(R_L = \infty)$

Example 3



Current mirror → replicate the I_{REF}



the same

$$g_{m_1} = \sqrt{2K_n \frac{W}{L} I_D}$$

$$= \sqrt{2 \times 110 \times 10^{-6} \times 20 \times 0.1 \times 10^{-3}}$$

$$= 663.3\ \mu A/V$$

$$r_{o_1} = r_{o_2} = \frac{1}{\lambda I_D} = 250k\Omega$$

$$\Rightarrow r_{out} = \frac{1}{g_{m_2} + \underbrace{g_{mb_1}}_{0} + \frac{1}{r_{o_2}} + \frac{1}{r_{o_1}}}$$

$$= 1689.6\Omega$$

$$r_{out} \approx \frac{1}{g_{m_1}} = 1507.6\Omega$$

$I = 0.01mA$

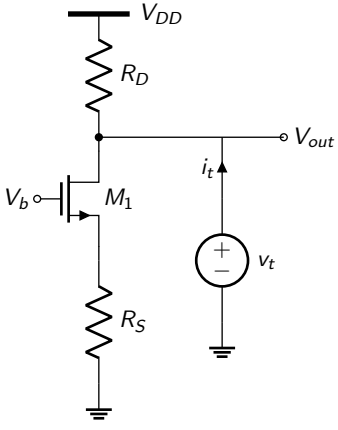
$$\Rightarrow r_{out} = 4749.2\Omega$$

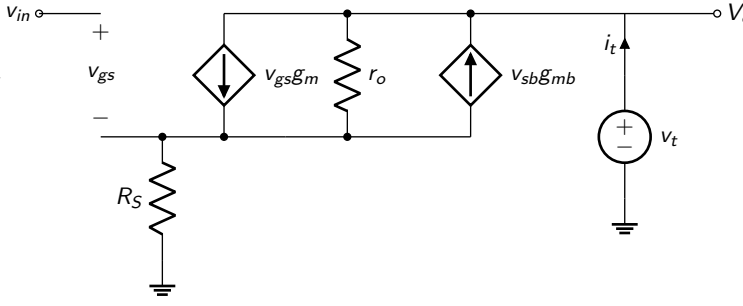
$$\frac{1}{g_m} = 4767\Omega$$

$I \downarrow$ $r_{out} \uparrow$

Example 3

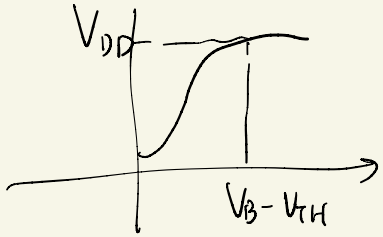
Common Gate

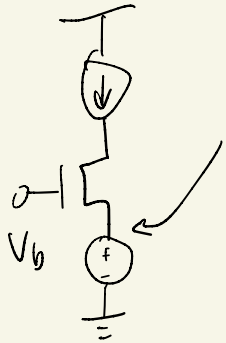




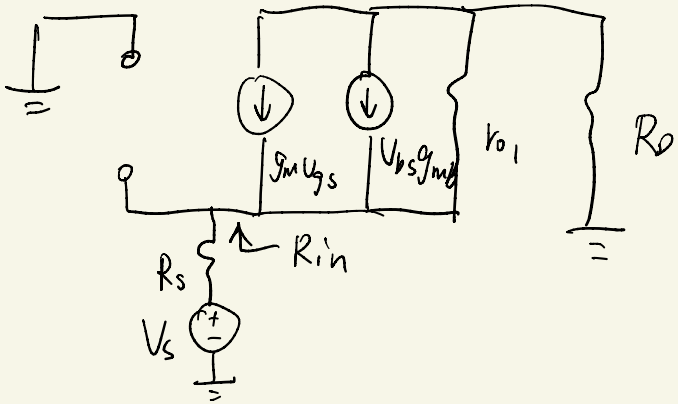
$$R_{in} = \frac{R_D + r_o}{1 + (g_m + g_{mb})r_o} \begin{cases} \text{If } R_D = 0 & R_{in} = r_o \parallel \frac{1}{g_m} \parallel \frac{1}{g_{mb}} \\ \text{If } R_D = \infty & R_{in} = \infty \end{cases} \quad (7)$$

$$R_{out} = [R_S + r_{o1} + (g_{m1} + g_{mb1})r_{o1}R_S] \parallel R_D \quad (8)$$





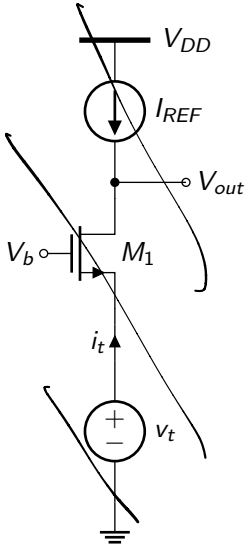
Internal resistance: ?

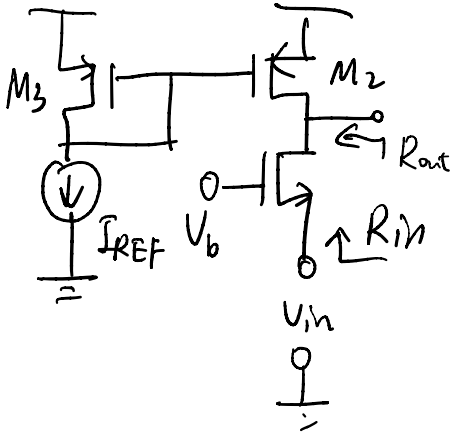


Example 4

In the common gate stage amplifier , what is the input resistance when $I_{REF} = 0.01mA$ and $0.1mA$ respectively? (Neglect body effect)
Parameter for NMOS: $V_{THN} = 0.7V$,
$K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$
Parameter for PMOS: $V_{THP} = -0.7V$,
$K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$
All the size of transistor is $W = 20\mu m, L = 1\mu m$

Example 4





Example 4

$0.1\,mA$

$$g_m = \sqrt{2K_n \frac{W}{L} I_D} = 663.3\,\mu A/V$$

$$r_{o1} = r_{o2} = 250\,k\Omega$$

$$r_{o2} = 200\,k\Omega$$

$$= \frac{1}{0.5\times10^{-3}\times0.1}$$

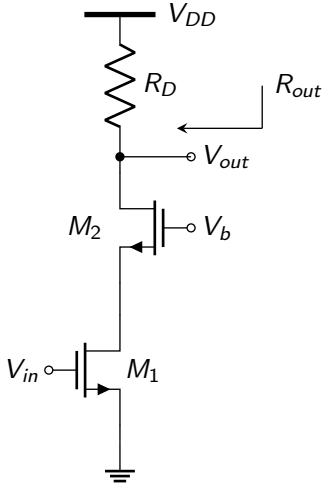
$$r_{in} = \frac{R_D + r_o}{r_{o1}(g_{m1}+g_{mb1})}(+1)$$

$$r_{o1}(g_{m1}+g_{mb1}) \gg 1 \quad \rightarrow \quad \approx \frac{200\times10^3 + 250\times10^3}{663.3\times10^{-6}\times250\times10^3}$$

$$\approx 2713.7\,\Omega$$

$$\frac{2}{g_{m1}} = 3015.2\,\Omega$$

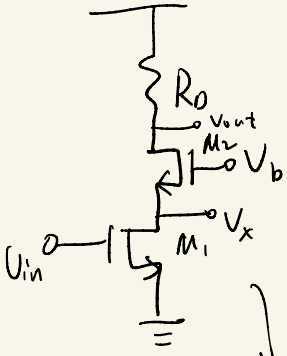
Cascode



$$G_m = -g_{m1}\frac{r_{o1}}{r_{o1} + \left(r_{o2} \parallel \frac{1}{g_{m2}+g_{mb2}}\right)} \quad (9)$$

$$R_{out} = [r_{o1}+r_{o2}+(g_{m2}+g_{mb2})r_{o2}r_{o1}] \parallel R_D \quad (10)$$

Common Gate
+ Common Source → Cascode



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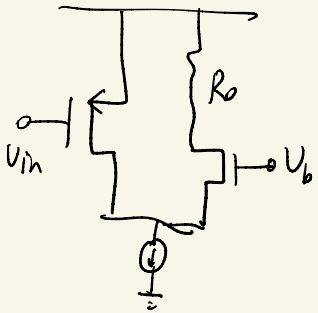
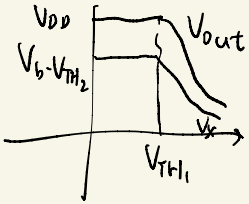
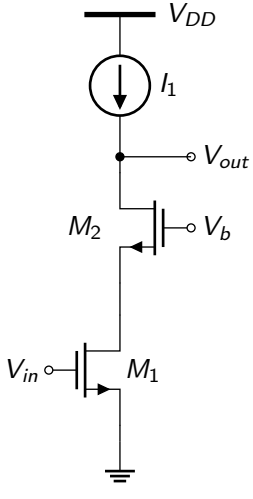




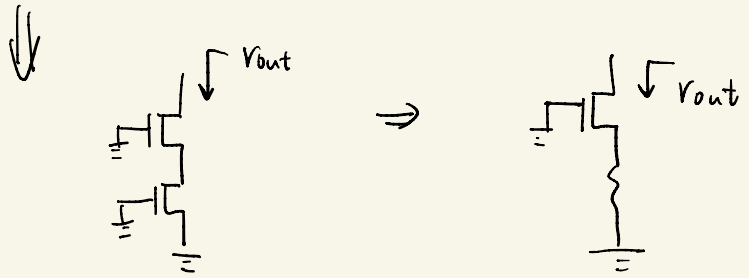
Cascode

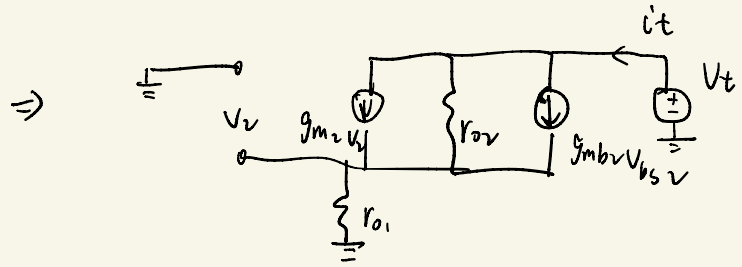


$$G_m = -g_{m1} \frac{r_{o1}}{r_{o1} + \left(r_{o2} \parallel \frac{1}{g_{m2}+g_{mb2}} \right)} \tag{11}$$

$$R_{out} = r_{o1} + r_{o2} + (g_{m2} + g_{mb2}) r_{o2} r_{o1} \tag{12}$$

$$A_v = G_m R_{out} \tag{13}$$





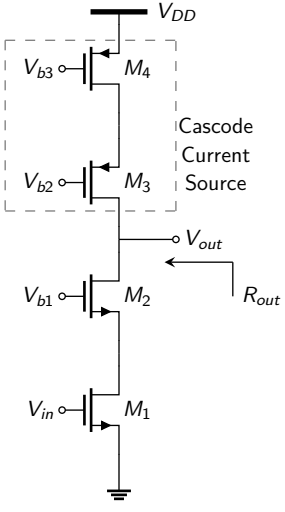
$$\Rightarrow \quad r_{o2}\left[i_t - (g_{m2} + g_{mb2})\, V_2 \right] + i_1\, r_{o1} = V_t$$

$$r_{out} = \frac{V_t}{i_t} = \left[1 + (g_{m2} + g_{mb2})\, r_{o2}\right] r_{o1} + r_{o2}$$

$r_{o2}(g_{m2} + g_{mb2}) >> 1 \quad \Rightarrow \quad r_{out} \approx (g_{m2} + g_{mb2})\, r_{o1}\, r_{o2} + r_{o2}$

$$A_v \approx -g_{m1}\left[(g_{m2} + g_{mb2})\, r_{o2} r_{o1}\right]$$

Cascode



$$G_m = -g_{m1}\frac{r_{o1}}{r_{o1} + (r_{o2} \parallel \frac{1}{g_{m2}g_{mb2}})} \quad (14)$$

$$R_{out} = [r_{o1} + r_{o2} + (g_{m2} + g_{mb2})r_{o2}r_{o1}] \parallel [r_{o3} + r_{o4} + (g_{m3} + g_{mb3})r_{o3}r_{o4}] \quad (15)$$

$$A_v = G_m R_{out} \quad (16)$$

$$r_{out} \approx (g_{m2} r_{o2} r_{o1}) \,/\!/\, (g_{m3} r_{o3} r_{o4})$$

$$A_V \approx -g_{m1} [(g_{m2} (r_{o2} r_{o1}) \,/\!/\, (r_{o3} r_{o4} g_{m3})]$$

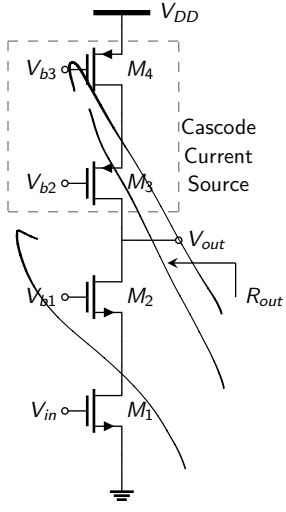
$$\Rightarrow g_m^2 r_o^2 >> g_m r_o \quad \text{(Common Source)}$$

Example 5

gain

In the cascode amplifier , what is the ~~input resistance~~ when $I_{REF} = 0.01mA$ and $0.1mA$ respectively? (Neglect body effect)
Parameter for NMOS: $V_{THN} = 0.7V$,
$K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$
Parameter for PMOS: $V_{THP} = -0.7V$,
$K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$
All the size of transistor is $W = 20\mu m, L = 1\mu m$

Example 5


V_{DD}
V_{b3}
M_4
Cascode
Current
Source
V_{b2}
M_3
V_{out}
R_{out}
V_{b1}
M_2
V_{in}
M_1

Example 5

$$g_{m1} = g_{m2} = \sqrt{2 K_n \frac{W}{L} I_D}$$

$$= 668.3$$

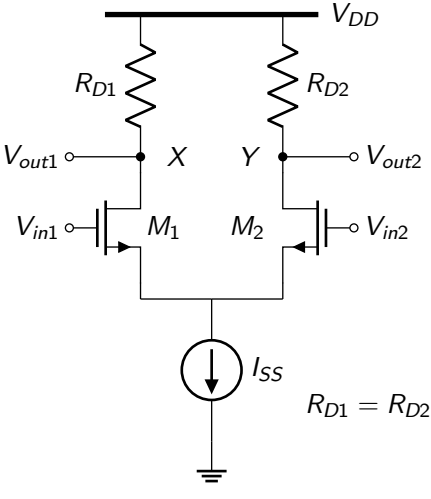
$$r_{o1} = r_{o2} = \frac{1}{\lambda I_D}$$

$$A_V = -g_{m1} \left[(g_{m2} + g_{mb2}) r_{o2} r_{o1} \right]$$

$$\simeq 27498$$

		gain	r_{out}	r_{in}	swing
CS	R	$-g_m(R_D \parallel r_{o1})$	$R_D \parallel r_{o1}$	∞	medium
	PMOS	$-\sqrt{\frac{\mu_n (W/L)_1}{\mu_p (W/L)_2}}$	$\approx \frac{1}{g_{m2}} \parallel r_{o2} \parallel r_{o1}$	∞	small
	I	$-g_{m1}(r_{o1} \parallel r_{o2})$	$r_{o1} \parallel r_{o2}$	∞	high
CD		$\frac{g_{m1}}{g_{m1}+g_{mb1}+\frac{1}{r_{o1}}+\frac{1}{r_{o2}}}$	$\frac{1}{g_{m1}+g_{mb1}+\frac{1}{r_{o1}}+\frac{1}{r_{o2}}}$	∞	/
CG		$\frac{g_{m1}+g_{mb1}+\frac{1}{r_{o1}}}{\frac{1}{r_{o1}}+\frac{1}{r_{o2}}}$	/	$\frac{r_{o1}+r_{o2}}{(g_{m1}+g_{mb1})r_{o1}+1}$	/
CAS + CAS		$-g_{m1}[g_{m2} r_{o2} r_{o1} \parallel g_{m3} r_{o3} r_{o4}]$	[ ]	∞	/

Differential Pair



$R_{D1} = R_{D2} = R_D$

$$A_{DM} = \frac{V_{out1} - V_{out2}}{v_d} = -g_m(R_D \parallel r_o) \quad (17)$$

If the circuit is fully symmetric,

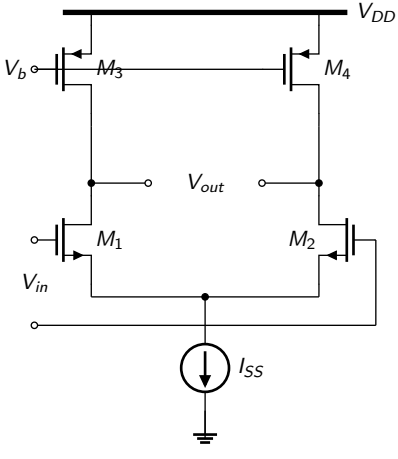
$$A_{CM-DM} = \frac{V_{out1} - V_{out2}}{v_{in,CM}} = 0 \quad (18)$$

$$CMRR = \left| \frac{A_{DM}}{A_{CM-DM}} \right| = \infty \quad (19)$$

Differential Pair: Specific Structure

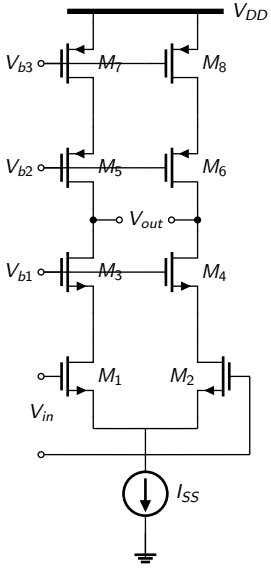
1. Swing ↑
2. linearity ↑
3. noise ↓

Differential Pair with MOS Loads



$$A_{DM} = -g_{m1,2}(r_{o1,2} \parallel r_{o3,4}) \qquad (20)$$

Differential Pair with Cascode Loads



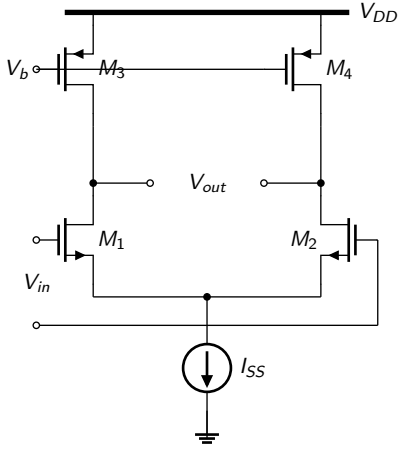
$$A_{DM} \cong -g_{m1,2}[(g_{m3,4} + gmb_{3,4})r_{o3,4}r_{o1,2} \parallel (g_{m5,6} + g_{mb5,6})r_{o5,6}r_{o7,8}] \tag{21}$$

Example 6

gain

In the differential pair , what is the ~~input resistance when~~
$I_{SS} = 0.02mA$ and $0.2mA$ respectively? (Neglect body effect)
Parameter for NMOS: $V_{THN} = 0.7V$,
$K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$
Parameter for PMOS: $V_{THP} = -0.7V$,
$K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$
All the size of transistor is $W = 20\mu m, L = 1\mu m$

Example 6



Example 6

$g_{m1,2} = \sqrt{2K_n \frac{W}{L} I_{D1,2}} = \sqrt{2 \times 110 \times 10^{-6} \times 20 \times 10^{-6}} = 663.3\ \mu A/V$

$I_{D1,2} = \frac{1}{2} I_{SS} = 0.1\ mA$

$r_{o1,2} = 250\ k\Omega \qquad r_{o3,4} = 200\ k\Omega$

$A_{vd} = -663.3 \times 10^{-6} \times (250 \times 10^3 \parallel 200 \times 10^3) = \sim$

END

Thanks