VE311 Electronic Circuit Homework 5

Due: **June 19th**

Note:

1) *Please use A4 size paper or page.*
2) *Please clearly state out your final result for each question.*
3) *Please attach the screenshot of Pspice simulation result if necessary.*

Question 1. Common Gate (Easy)

In the common gate stage amplifier, the internal resistance of I is $1k\Omega$, what is the output resistance when $I_{REF} = 0.01mA$ and $0.1mA$ respectively? (Neglect body effect)

Parameter for NMOS: $V_{THN} = 0.7V$, $K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$

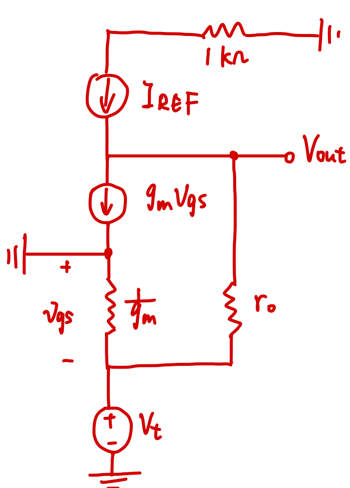
Parameter for PMOS: $V_{THP} = -0.7V$, $K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$

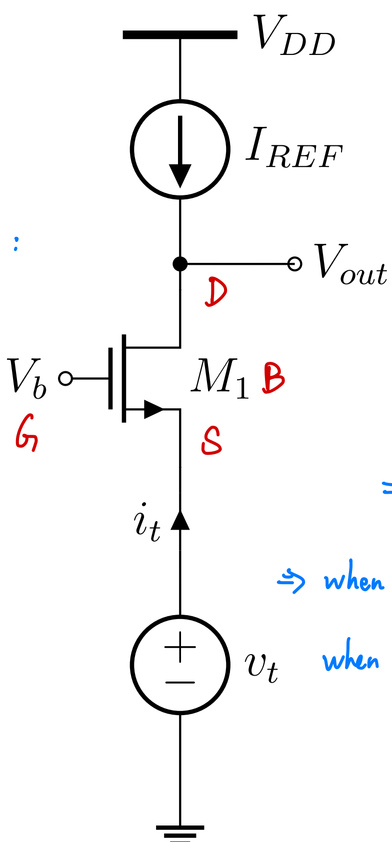
All the size of transistor is $W = 20\mu m, L = 1\mu m$

Transconductance $g_m = \sqrt{2K_n' \frac{W}{L} I_D}$

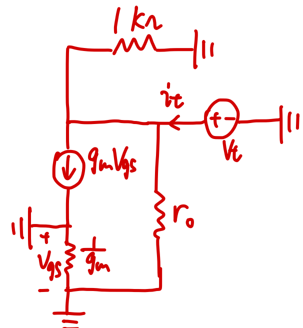
Output Resistance $r_o = \frac{1}{\lambda I_D}$

Then we do the small signal analysis :





To find the output resistance, we just need to analyze the following circuit :



$\Rightarrow R_{out} = 1k\Omega \parallel r_o$

$\Rightarrow$ when $I_D = 0.1mA$, $R_{out} = \frac{1}{\lambda \cdot 0.1 \times 10^{-3}} \parallel (1k) = 996.02\Omega$

when $I_D = 0.01mA$, $R_{out} = \frac{1}{\lambda \cdot 0.01 \times 10^{-3}} \parallel (1k) = 999.60\Omega$

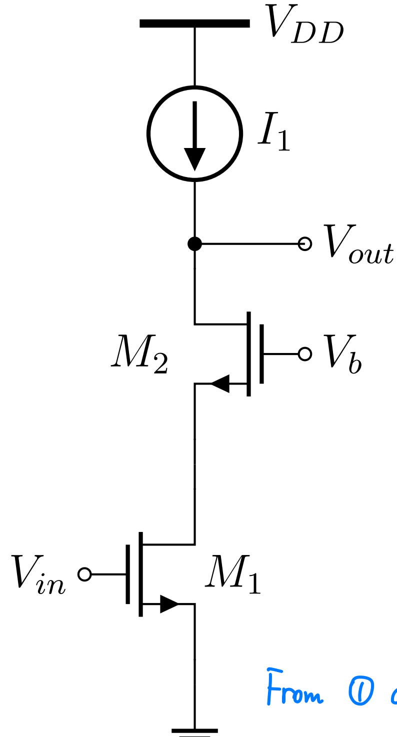
Question 2. Common Gate Common Source (Medium)

Find the intrinsic gain A_v and output impedance R_{out} for the amplifier when $I_1 = 0.01$ and $0.1mA$ respectively. (Neglect body effect)

Parameter for NMOS: $V_{THN} = 0.7V$, $K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$

Parameter for PMOS: $V_{THP} = -0.7V$, $K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$

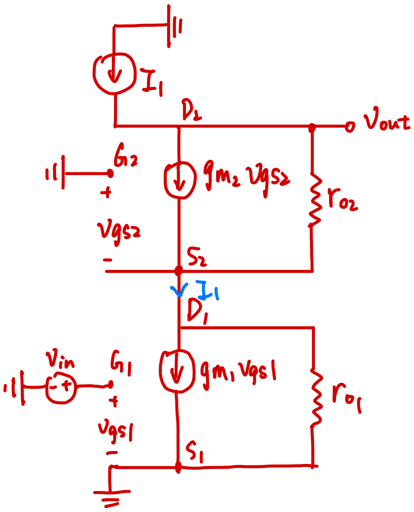
All the size of transistor is $W = 20\mu m, L = 1\mu m$



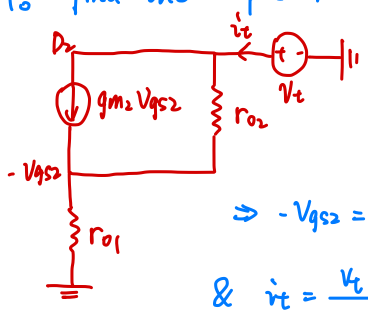
Tranductance $g_m = \sqrt{2k_n' \frac{W}{L} I_D}$

Output Resistance $r_o = \frac{1}{\lambda I_D}$

Small signal analysis :



To find the output resistance :



$\Rightarrow -V_{gs2} = i_t r_{o1}$

& $i_t = \frac{V_t + V_{gs2}}{r_{o2}} + g_{m2} V_{gs2}$

$\Rightarrow \frac{V_t}{i_t} = R_{out} = r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2}$ ①

To get A_v, we need to know $\frac{\partial V_{out}}{\partial V_{in}}$

we can use the AC circuit to obtain the relationship between V_{out} and V_{in}

$$\begin{cases} V_{out} = -V_{gs2} + r_{o2}(I_1 - g_{m2} V_{gs2}) \\ 0 = -V_{gs2} - (I_1 - g_{m1} V_{gs1}) \cdot r_{o1} \\ V_{gs1} = V_{in} \end{cases}$$

$\Rightarrow V_{out} = V_{in} \cdot -(r_{o1} g_{m1} + r_{o2} r_{o1} g_{m1} g_{m2}) + (r_{o1} + r_{o2} + r_{o1} r_{o2} g_{m2}) I_1$

$\Rightarrow A_v = \frac{\partial V_{out}}{\partial V_{in}} = -r_{o1} g_{m1} - r_{o1} r_{o2} g_{m1} g_{m2}$ ②

From ① and ②, we can know that

when $I_1 = 0.01mA$, $r_{o1} = r_{o2} = \frac{1}{\lambda I_1} = 2.5 \times 10^6 \Omega$

$g_{m1} = g_{m2} = \sqrt{2k_n' \frac{W}{L} I_1} = 2.098 \times 10^{-4}\ 1/\Omega$

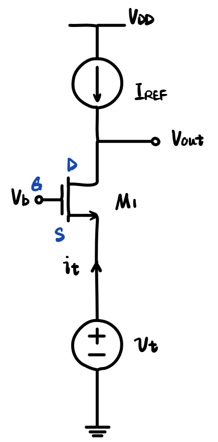
$\Rightarrow R_{out} = 1.32 \times 10^9 \Omega$. $A_v = -2.76 \times 10^5$

when $I_1 = 0.1mA$, $r_{o1} = r_{o2} = 2.5 \times 10^5 \Omega$

$g_{m1} = g_{m2} = \sqrt{2k_n' \frac{W}{L} I_1} = 6.633 \times 10^{-4}\ 1/\Omega$

$\Rightarrow R_{out} = 4.196 \times 10^7 \Omega$, $A_v = -2.77 \times 10^4$

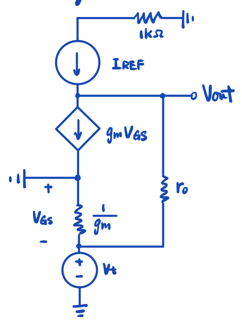
Question 1. Common Gate



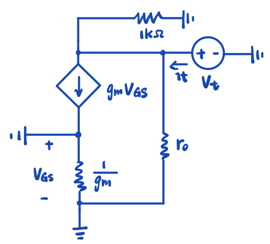
Transconductance $g_m = \sqrt{2 k_n' \frac{W}{L} I_D}$

Output Resistance $r_o = \frac{1}{\lambda I_D}$

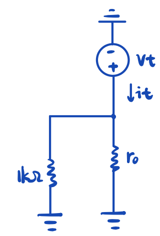
Small Signal Model:



To find the resistance looking from output:



$\Leftrightarrow$

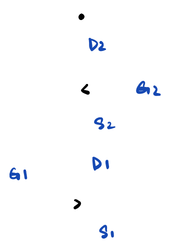


$\Rightarrow R_{out} = 1k\Omega \,//\, r_o$

① $I_{REF} = 0.1\,mA = I_D$ $\Rightarrow R_{out} = \frac{1}{\lambda I_D} \,//\, 1k\Omega = \frac{1}{0.04 \times 0.1 \times 10^{-3}} \,//\, 10^3 = \boxed{996.02\ \Omega}$

② $I_{REF} = 0.01\,mA = I_D$ $\Rightarrow R_{out} = \frac{1}{\lambda I_D} \,//\, 1k\Omega = \frac{1}{0.04 \times 0.01 \times 10^{-3}} \,//\, 10^3 = \boxed{999.60\ \Omega}$

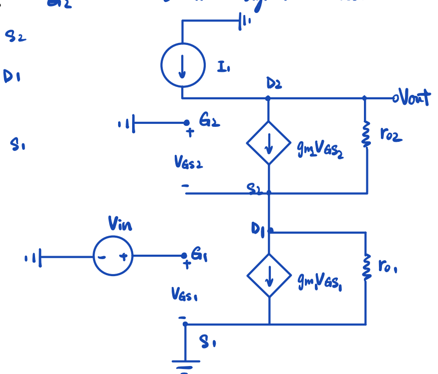
Question 2. Common Gate Common Source



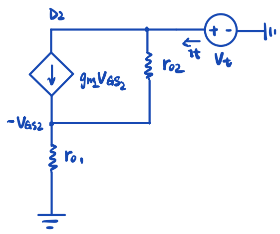
Transconductance $g_m = \sqrt{2 k_n' \frac{W}{L} I_D}$

Output Resistance $r_o = \frac{1}{\lambda I_D}$

Small Signal Model:



To find the resistance looking from output:



$$\begin{cases} -V_{GS2} = i_t r_{o1} \\ i_t = \frac{V_t + V_{GS2}}{r_{o2}} + g_{m2} V_{GS2} \end{cases} \Rightarrow i_t = \frac{V_t - i_t r_{o1}}{r_{o2}} - g_{m2} i_t r_{o1}$$

$\Rightarrow V_t = i_t r_{o2} + i_t r_{o1} + g_{m2} i_t r_{o1} r_{o2}$

$\Rightarrow R_{out} = \frac{V_t}{i_t} = r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2}$

For A_v:

$$\begin{cases} V_{out} = -V_{GS2} + r_{o2}(I_1 - g_{m2} V_{GS2}) \\ -V_{GS2} = r_{o1}(I_1 - g_{m1} V_{GS1}) = r_{o1}(I_1 - g_{m1} V_{in}) \end{cases}$$

$\Rightarrow V_{out} = r_{o1} I_1 - r_{o1} g_{m1} V_{in} + r_{o2} I_1 + r_{o1} r_{o2} g_{m2} (I_1 - g_{m1} V_{in})$

$\Rightarrow A_v = \frac{\partial V_{out}}{\partial V_{in}} = -r_{o1} g_{m1} - r_{o1} r_{o2} g_{m1} g_{m2}$

① $I_1 = 0.01\ mA$

$r_{o1} = r_{o2} = \frac{1}{\lambda I_1} = 2.5 \times 10^6\ \Omega$

$g_{m1} = g_{m2} = \sqrt{2 k_n' \frac{W}{L} I_D}$

$= \sqrt{2 \times 110 \times 10^{-6} \times 20 \times 0.01 \times 10^{-3}} = 2.098 \times 10^{-4}\ A/V$

$R_{out} = r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2} = 2.5 \times 10^6 \times 2 + (2.5 \times 10^6)^2 \times 2.098 \times 10^{-4} = \boxed{1.32 \times 10^9\ \Omega}$

$A_v = -r_{o1} g_{m1} - r_{o1} r_{o2} g_{m1} g_{m2}$

$= -2.5 \times 10^6 \times 2.098 \times 10^{-4} - (2.5 \times 10^6 \times 2.098 \times 10^{-4})^2 = \boxed{-2.76 \times 10^5}$

① $I_1 = 0.1\,mA$

$r_{o1} = r_{o2} = \frac{1}{\lambda I_1} = 2.5 \times 10^5\ \Omega$

$g_{m1} = g_{m2} = \sqrt{2 k_n' \frac{W}{L} I_D}$

$= \sqrt{2 \times 110 \times 10^{-6} \times 20 \times 0.1 \times 10^{-3}} = 6.633 \times 10^{-4}$

$R_{out} = r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2} = 2.5 \times 10^5 \times 2 + (2.5 \times 10^5)^2 \times 6.633 \times 10^{-4} = \boxed{4.196 \times 10^7\ \Omega}$

$A_v = -r_{o1} g_{m1} - r_{o1} r_{o2} g_{m1} g_{m2}$

$= -2.5 \times 10^5 \times 6.633 \times 10^{-4} - (2.5 \times 10^5 \times 6.633 \times 10^{-4})^2 = \boxed{-2.77 \times 10^4}$

Question 3. Common Gate Common Source (Medium)

Find the intrinsic gain A_v and output impedance R_{out} for the amplifier when $I_1 = 0.01$ and $0.1mA$ respectively. (Neglect body effect)

Parameter for NMOS: $V_{THN} = 0.7V$, $K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$

Parameter for PMOS: $V_{THP} = -0.7V$, $K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$

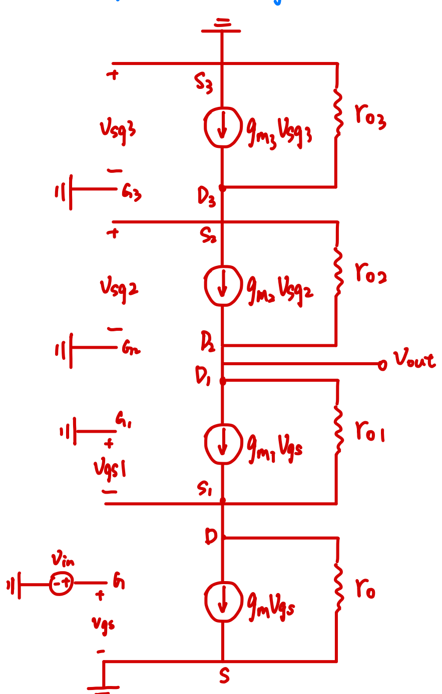
All the size of transistor is $W = 20\mu m, L = 1\mu m$

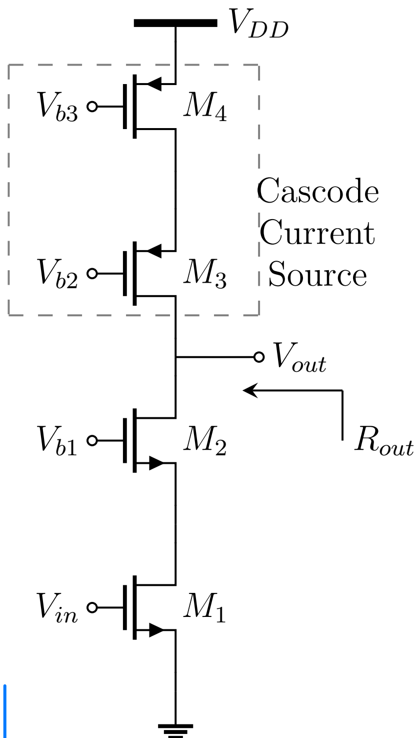
Tranductance $g_{mN} = \sqrt{2k_n' \frac{W}{L} I_D}$

$g_{mp} = \sqrt{2k_p' \frac{W}{L} I_D}$

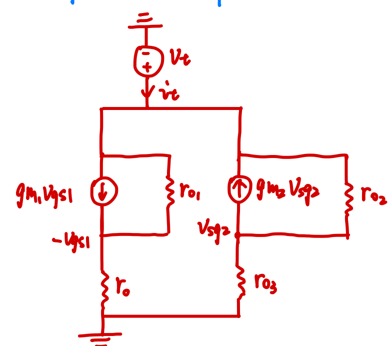
Output Resistance $r_o = \frac{1}{\lambda I_D}$

Small signal analysis:





To find the output resistance:



$$\Rightarrow \begin{cases} \frac{V_t + V_{gs1}}{r_{o1}} + g_{m_1} V_{gs1} = \frac{-V_{gs1}}{r_o} \\ \frac{V_t - V_{sg2}}{r_{o2}} - g_{m_2} V_{sg2} = \frac{V_{sg2}}{r_{o3}} \\ i_t = \frac{-V_{gs1}}{r_o} + \frac{V_{sg2}}{r_{o3}} \end{cases}$$

$$\Rightarrow R_{out} = \frac{V_t}{i_t} = \frac{1}{\frac{1}{r_o + r_{o1} + g_{m_1} r_o r_{o1}} + \frac{1}{r_{o2} + r_{o3} + g_{m_2} r_{o2} r_{o3}}} \quad (1)$$

To get A_V, we need to know $\frac{\partial V_{out}}{\partial V_{in}}$, we can use the AC circuit to obtain the relationship between V_{out} and V_{in}

$$g_m V_{in} + \frac{-V_{gs1}}{r_o} = g_{m_1} V_{gs1} + \frac{V_{out} + V_{gs1}}{r_{o1}} = g_{m_2} V_{sg2} + \frac{V_{sg2} - V_{out}}{r_{o2}} = \frac{0 - V_{sg2}}{r_{o3}}$$

$$\Rightarrow A_V = \frac{\partial V_{out}}{\partial V_{in}} = \frac{g_m}{\left(\frac{1}{r_o} + \frac{1}{r_{o1}} + g_{m_1}\right)\left(\frac{g_{m_2} + \frac{1}{r_{o2}}}{(g_{m_1} + \frac{1}{r_{o1}})(g_{m_2} + \frac{1}{r_{o2}} + \frac{1}{r_{o3}}) r_{o2}} - \frac{\frac{1}{r_{o1}} + \frac{1}{r_{o2}}}{g_{m_1} + \frac{1}{r_{o1}}}\right) + \frac{1}{r_{o1}}} \quad (2)$$

From (1) and (2), we can know that when $I_1 = 0.01mA$,

$r_{o2} = r_{o3} = 2 \times 10^6 \Omega$, $r_o = r_{o1} = 2.5 \times 10^6 \Omega$, $g_{m_2} = g_{m_3} = \sqrt{2} \times 10^{-4}\ 1/\Omega$, $g_{m_1} = g_m = \sqrt{4.4} \times 10^{-4}\ 1/\Omega$

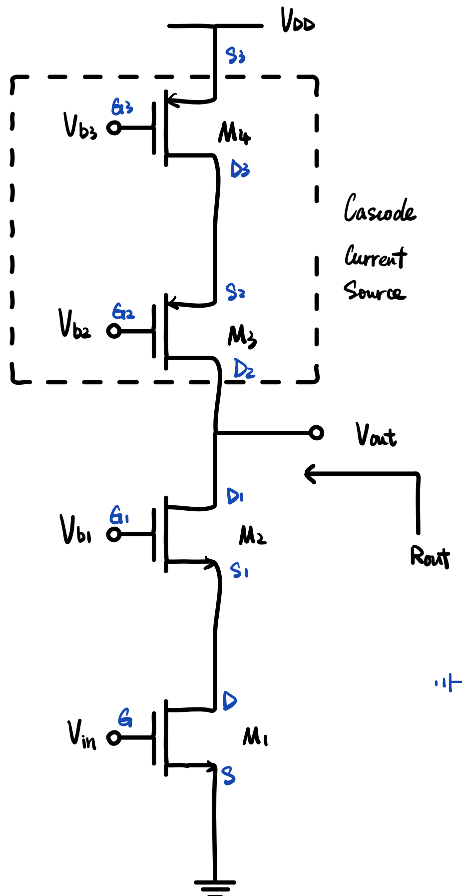
$\Rightarrow R_{out} = 3.976 \times 10^8 \Omega$, $A_V = -8.3 \times 10^4$

From (1) and (2), we can know that when $I_1 = 0.1mA$,

$r_{o2} = r_{o3} = 2 \times 10^5 \Omega$, $r_o = r_{o1} = 2.5 \times 10^5 \Omega$, $g_{m_2} = g_{m_3} = \sqrt{20} \times 10^{-4}\ 1/\Omega$, $g_{m_1} = g_m = \sqrt{44} \times 10^{-4}\ 1/\Omega$

$\Rightarrow R_{out} = 1.273 \times 10^7 \Omega$, $A_V = -8.4 \times 10^3$

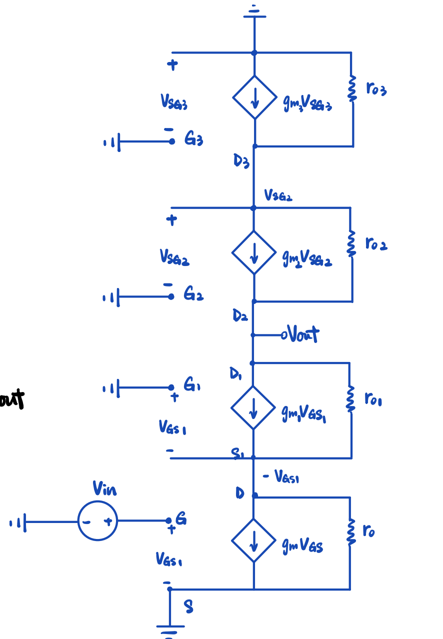
Question 3. Common Gate Common Source



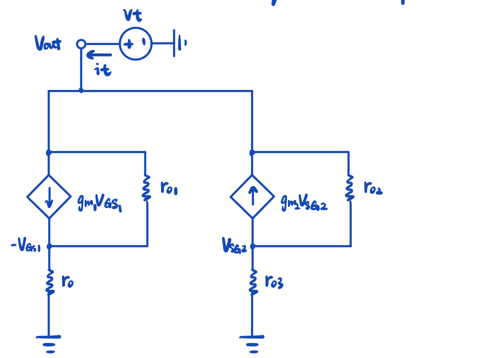
Transconductance $g_m = \sqrt{2 k_n' \frac{W}{L} I_D}$ or $\sqrt{2 k_p' \frac{W}{L} I_D}$

Output Resistance $r_o = \frac{1}{\lambda I_D}$

Draw the small signal model:



To find the resistance looking from output:



$$\begin{cases} \frac{V_t + V_{GS1}}{r_{o1}} + g_{m1} V_{GS1} = \frac{-V_{GS1}}{r_o} \Rightarrow -\frac{V_t}{r_{o1}} = \left(\frac{1}{r_o} + \frac{1}{r_{o1}} + g_{m1}\right) V_{GS1} \\ \frac{V_t - V_{SG2}}{r_{o2}} - g_{m2} V_{SG2} = \frac{V_{SG2}}{r_{o3}} \Rightarrow \frac{V_t}{r_{o2}} = \left(\frac{1}{r_{o2}} + \frac{1}{r_{o3}} + g_{m2}\right) V_{SG2} \\ \frac{-V_{GS1}}{r_o} + \frac{V_{SG2}}{r_{o3}} = i_t \end{cases}$$

$$\Rightarrow i_t = \frac{V_t}{r_{o1} r_o} \frac{1}{\frac{1}{r_o} + \frac{1}{r_{o1}} + g_{m1}} + \frac{V_t}{r_{o2} r_{o3}} \frac{1}{\frac{1}{r_{o2}} + \frac{1}{r_{o3}} + g_{m2}}$$

$$\Rightarrow R_{out} = \frac{V_t}{i_t} = \frac{1}{\frac{1}{r_{o1} + r_o + g_{m1} r_{o1} r_o} + \frac{1}{r_{o3} + r_{o2} + g_{m2} r_{o2} r_{o3}}}$$

For A_v: $g_m V_{in} + \frac{-V_{GS1}}{r_o} = g_{m1} V_{GS1} + \frac{V_{out} + V_{GS1}}{r_{o1}} = g_{m2} V_{SG2} + \frac{V_{SG2} - V_{out}}{r_{o2}} = \frac{0 - V_{SG2}}{r_{o3}}$

$$\Rightarrow V_{out} = \left(g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o3}}\right) r_{o2} V_{SG2}$$

$$\Rightarrow \left(g_{m1} + \frac{1}{r_{o1}}\right) V_{GS1} = \left(g_{m2} + \frac{1}{r_{o2}}\right) V_{SG2} - \left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}}\right) V_{out}$$

$$\Rightarrow g_m V_{in} = \left(\frac{1}{r_o} + \frac{1}{r_{o1}} + g_{m1}\right) V_{GS1} + \frac{1}{r_{o1}} V_{out} = \left(\frac{1}{r_o} + \frac{1}{r_{o1}} + g_{m1}\right)\left(\frac{g_{m2} + \frac{1}{r_{o2}}}{g_{m1} + \frac{1}{r_{o1}}} V_{SG2} - \frac{\frac{1}{r_{o1}} + \frac{1}{r_{o2}}}{g_{m1} + \frac{1}{r_{o1}}} V_{out}\right) + \frac{1}{r_{o1}} V_{out}$$

$$= \left(\frac{1}{r_o} + \frac{1}{r_{o1}} + g_{m1}\right)\left(\frac{g_{m2} + \frac{1}{r_{o2}}}{g_{m1} + \frac{1}{r_{o1}}} \frac{1}{\left(g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o3}}\right) r_{o2}} - \frac{\frac{1}{r_{o1}} + \frac{1}{r_{o2}}}{g_{m1} + \frac{1}{r_{o1}}}\right) V_{out} + \frac{1}{r_{o1}} V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{g_m}{\left(\frac{1}{r_o} + \frac{1}{r_{o1}} + g_{m1}\right)\left(\frac{g_{m2} + \frac{1}{r_{o2}}}{g_{m1} + \frac{1}{r_{o1}}} \frac{1}{\left(g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o3}}\right) r_{o2}} - \frac{\frac{1}{r_{o1}} + \frac{1}{r_{o2}}}{g_{m1} + \frac{1}{r_{o1}}}\right) + \frac{1}{r_{o1}}}$$

① $I = 0.01\ mA$

$r_{o2} = r_{o3} = \frac{1}{\lambda_p I} = \frac{1}{0.05 \times 10^{-5}} = 2 \times 10^6\ \Omega$

$r_o = r_{o1} = \frac{1}{\lambda_n I} = \frac{1}{0.04 \times 10^{-5}} = 2.5 \times 10^6\ \Omega$

$g_{m2} = g_{m3} = \sqrt{2 k_p' \frac{W}{L} I_D} = \sqrt{2 \times 50 \times 10^{-6} \times 20 \times 10^{-5}} = \sqrt{2} \times 10^{-4}\ A/V$

$g_m = g_{m1} = \sqrt{2 k_n' \frac{W}{L} I_D} = \sqrt{2 \times 110 \times 10^{-6} \times 20 \times 10^{-5}} = \sqrt{4.4} \times 10^{-4}\ A/V$

$\Rightarrow$ $\boxed{R_{out} = \frac{1}{\frac{1}{2r_o + g_m r_o^2} + \frac{1}{2r_{o2} + g_{m2} r_{o2}^2}} = 3.976 \times 10^8\ \Omega \quad A_v = -8.3 \times 10^4}$

② $I = 0.1\ mA$

$r_{o2} = r_{o3} = \frac{1}{\lambda_p I} = \frac{1}{0.05 \times 10^{-4}} = 2 \times 10^5\ \Omega$

$r_o = r_{o1} = \frac{1}{\lambda_n I} = \frac{1}{0.04 \times 10^{-4}} = 2.5 \times 10^5\ \Omega$

$g_{m2} = g_{m3} = \sqrt{2 k_p' \frac{W}{L} I_D} = \sqrt{2 \times 50 \times 10^{-6} \times 20 \times 10^{-4}} = \sqrt{20} \times 10^{-4}\ A/V$

$g_m = g_{m1} = \sqrt{2 k_n' \frac{W}{L} I_D} = \sqrt{2 \times 110 \times 10^{-6} \times 20 \times 10^{-4}} = \sqrt{44} \times 10^{-4}\ A/V$

$\Rightarrow$ $\boxed{R_{out} = 1.274 \times 10^7\ \Omega \quad A_v = -8.4 \times 10^3}$

Question 4. Differential Pair (Hard)

Find the intrinsic gain A_v for the amplifier when $I_{SS} = 0.02$ and $0.2mA$ respectively. (Neglect body effect)

Parameter for NMOS: $V_{THN} = 0.7V$, $K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$

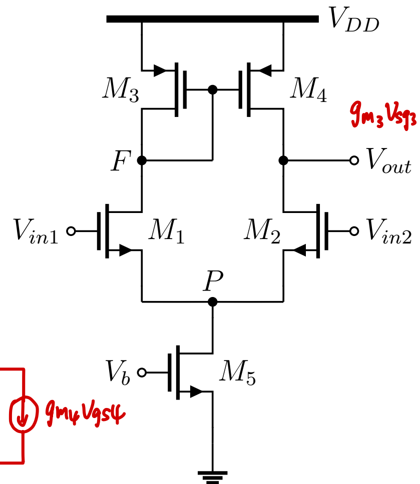
Parameter for PMOS: $V_{THP} = -0.7V$, $K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$

All the size of transistor is $W = 20\mu m, L = 1\mu m$

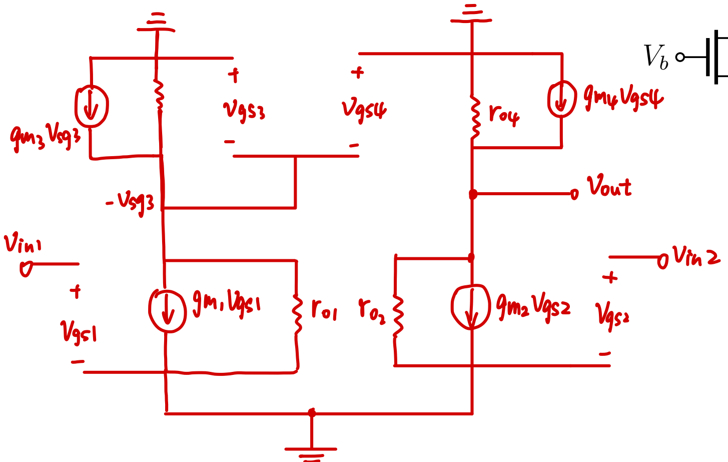
Tranductance $g_{mN} = \sqrt{2k_n' \frac{W}{L} I_D}$

$g_{mp} = \sqrt{2k_p' \frac{W}{L} I_D}$

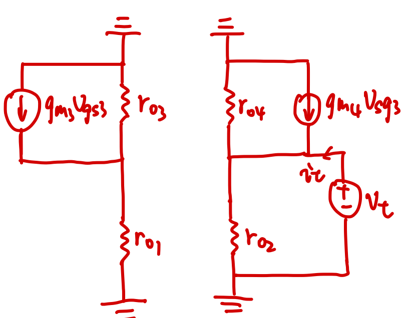
Output Resistance $r_o = \frac{1}{\lambda I_D}$



Small signal analysis :



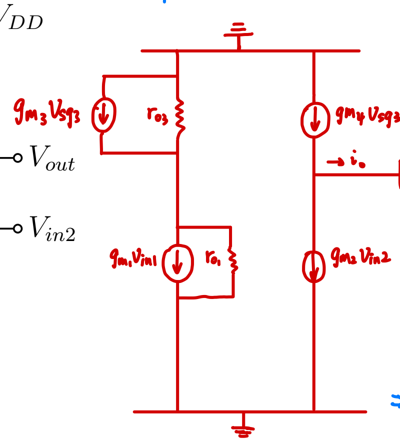
To find the output resistance :



$$\Rightarrow \begin{cases} g_{m3}V_{sg3} + \dfrac{V_{sg3}}{r_{o3}} = \dfrac{-V_{sg3}}{r_{o1}} \\ i_t + \dfrac{-V_t}{r_{o4}} + g_{m4}V_{sg} = \dfrac{V_t}{r_{o2}} \end{cases}$$

$$\Rightarrow R_{out} = \frac{V_t}{i_t} = \frac{r_{o2}r_{o4}}{r_{o2}+r_{o4}}$$

To find G_m :



$$\begin{cases} g_{m3}V_{sg3} + \dfrac{V_{sg3}}{r_{o3}} = g_{m1}V_{in1} + \dfrac{-V_{sg3}}{r_{o1}} \\ i_o = g_{m4}V_{sg3} - g_{m2}V_{in2} \end{cases}$$

$$\Rightarrow G_m = \frac{i_o}{V_{in1}-V_{in2}} = g_{m1}$$

$$\Rightarrow A_v = G_m R_{out} = g_{m1}\frac{r_{o2}r_{o4}}{r_{o2}+r_{o4}} \quad ①$$

From ①, we can know that when $I_{ss} = 0.02\ mA$

$\Rightarrow r_{o2} = 2.5\times10^6\ \Omega$, $r_{o4} = 2\times10^6\ \Omega$, $g_{m1} = \sqrt{44}\times10^{-4}\ 1/\Omega$

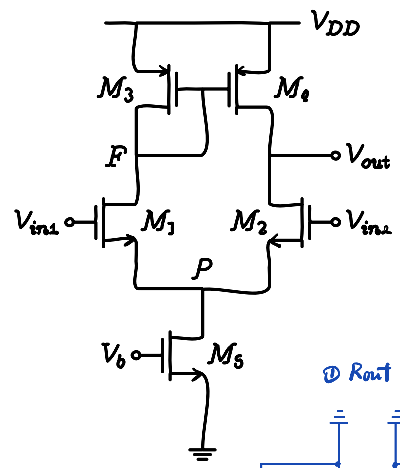
$\Rightarrow A_v = 233.07$

And when $I_{ss} = 0.2\ mA$

$\Rightarrow r_{o2} = 2.5\times10^5\ \Omega$, $r_{o4} = 2\times10^5\ \Omega$, $g_{m1} = \sqrt{44}\times10^{-4}\ 1/\Omega$

$\Rightarrow A_v = 73.70$

Question 4 Differential Pair

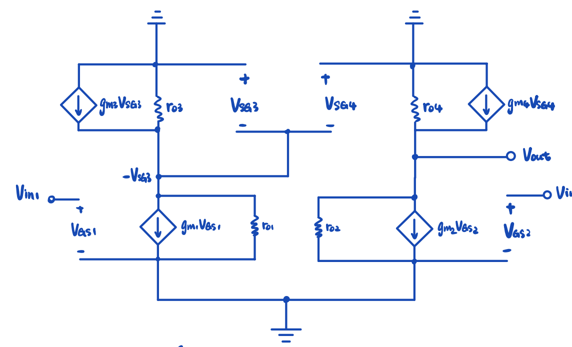


Transconductance $g_m = \sqrt{2k_n' \frac{W}{L} I_D}$

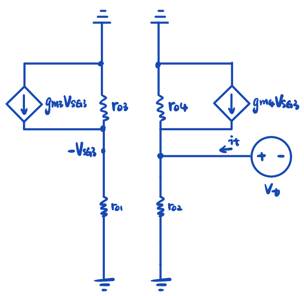
or $\sqrt{2k_p' \frac{W}{L} I_D}$

Output Resistance $r_o = \dfrac{1}{\lambda I_D}$

Draw small signal model:



① Rout

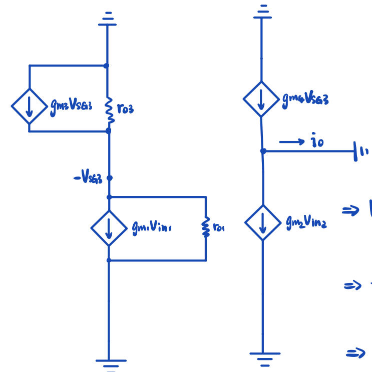


$$\begin{cases} g_{m3}V_{SG3} + \dfrac{V_{SG3}}{r_{o3}} = \dfrac{-V_{SG3}}{r_{o1}} \\ i_t + \dfrac{-V_t}{r_{o4}} + g_{m4}V_{SG3} = \dfrac{V_t}{r_{o2}} \end{cases}$$

$\Rightarrow V_{SG3} = 0$

$\Rightarrow R_{out} = \dfrac{V_t}{i_t} = \dfrac{r_{o2}\, r_{o4}}{r_{o4} + r_{o2}}$

② Gm



$$\begin{cases} g_{m3}V_{SG3} + \dfrac{V_{SG3}}{r_{o3}} = g_{m1}V_{in1} + \dfrac{-V_{SG3}}{r_{o1}} \\ i_o = g_{m4}V_{SG3} - g_{m2}V_{in2} \end{cases}$$

$\Rightarrow V_{SG3} = \dfrac{g_{m1}V_{in1}}{g_{m3} + \frac{1}{r_{o3}} + \frac{1}{r_{o1}}} \approx \dfrac{g_{m1}}{g_{m3}} V_{in1}$

$\Rightarrow i_o = g_{m1,2}\,(V_{in1} - V_{in2})$

$\Rightarrow G_m = \dfrac{i_o}{V_{in} - V_{in2}} = g_{m1,2}$

$\Rightarrow A_v = G_m R_{out} = g_{m1,2} \dfrac{r_{o1,2}\, r_{o3,4}}{r_{o1,2} + r_{o3,4}}$

① $I_{SS} = 0.02\,mA$

$r_{o1,2} = \dfrac{1}{\lambda_n I} = \dfrac{1}{0.04 \times 10^{-5}} = 2.5 \times 10^6\,\Omega$

$r_{o3,4} = \dfrac{1}{\lambda_p I} = \dfrac{1}{0.05 \times 10^{-5}} = 2 \times 10^6\,\Omega$

$g_{m1,2} = \sqrt{2k_n' \frac{W}{L} I_D} = \sqrt{2 \times 110 \times 10^{-6} \times 20 \times 10^{-5}} = \sqrt{4.4} \times 10^{-4}\,A/V$

$\Rightarrow \boxed{A_v = 233.07}$

② $I_{SS} = 0.2\,mA$

$r_{o1,2} = \dfrac{1}{\lambda_n I} = \dfrac{1}{0.04 \times 10^{-4}} = 2.5 \times 10^5\,\Omega$

$r_{o3,4} = \dfrac{1}{\lambda_p I} = \dfrac{1}{0.05 \times 10^{-4}} = 2 \times 10^5\,\Omega$

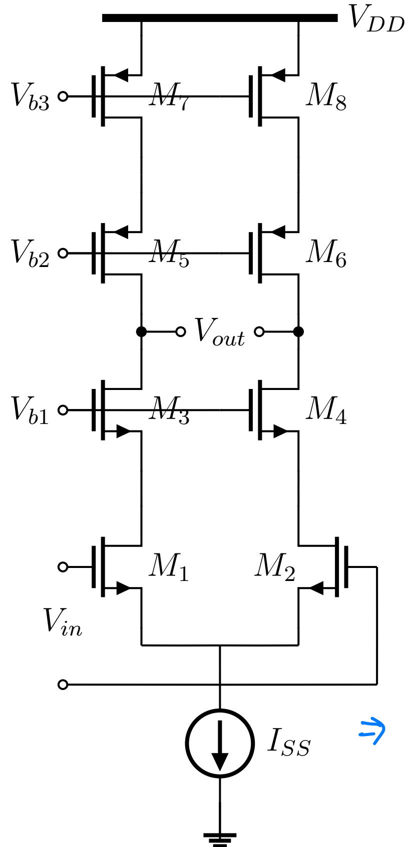
$g_{m1,2} = \sqrt{2k_n' \frac{W}{L} I_D} = \sqrt{2 \times 110 \times 10^{-6} \times 20 \times 10^{-4}} = \sqrt{44} \times 10^{-4}\,A/V$

$\Rightarrow \boxed{A_v = 73.70}$

Question 5. Differential Pair (Hard)

Find the intrinsic gain A_v for the amplifier when $I_{SS} = 0.02$ and $0.2mA$ respectively. (Neglect body effect)

Parameter for NMOS: $V_{THN} = 0.7V$, $K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$
Parameter for PMOS: $V_{THP} = -0.7V$, $K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$
All the size of transistor is $W = 20\mu m, L = 1\mu m$

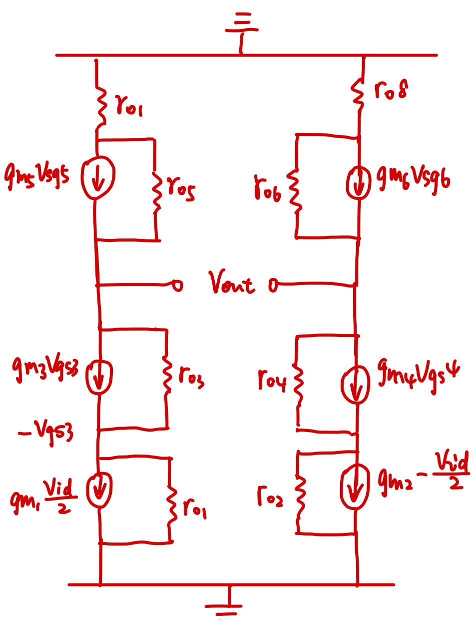


Tranductance $g_{mN} = \sqrt{2k_n' \frac{W}{L} I_D}$

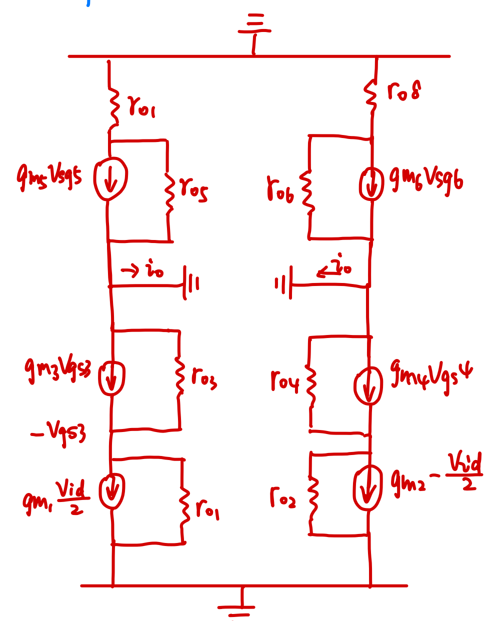
$g_{mp} = \sqrt{2k_p' \frac{W}{L} I_D}$

Output Resistance $r_o = \frac{1}{\lambda I_D}$

Small signal analysis :



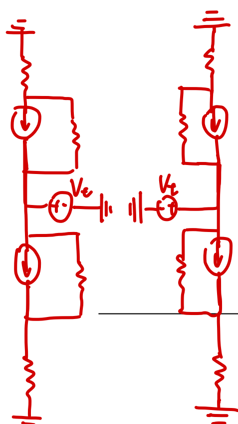
To find G_m :



$$\Rightarrow \begin{cases} \dfrac{-V_{sg5}}{r_o} = \dfrac{V_{sg5}}{r_o} + g_{mp} V_{sg5} \\ \dfrac{V_{gs3}}{r_o} + g_{mN} V_{gs3} = g_{mN}\dfrac{V_{id}}{2} + \dfrac{-V_{gs3}}{r_o} \end{cases}$$

$$\Rightarrow G_m = \frac{r_o g_{mN}^2}{-r_o g_{mN} - 1} = -g_{mN} \text{ (round to } -g_{mN})$$

To find Rout :



$$\begin{cases} \dfrac{V_{sg5}}{r_o} = \dfrac{V_t - V_{sg5}}{r_o} - g_{mp} V_{sg5} \\ \dfrac{V_{gs3} + V_t}{r_o} + g_{mN} V_{gs3} = \dfrac{-V_{gs3}}{r_o} \\ i_t = \dfrac{V_{gs5}}{r_o} + \dfrac{-V_{gs3}}{r_o} \end{cases}$$

$$\Rightarrow R_{out} = \frac{(2+g_{mp}r_o)(2+g_{mN}r_o)r_o^2}{(2+g_{mp}r_o+2+g_{mN}r_o)\cdot r_o}$$

$$= \frac{(2+g_{mp}r_o)(2+g_{mN}r_o)r_o}{4+g_{mp}r_o+g_{mN}r_o}$$

$$\Rightarrow A_v = R_{out} \cdot G_m = -g_{mN} \cdot \frac{(2+g_{mp}r_o)(2+g_{mN}r_o)r_o}{4+g_{mp}r_o+g_{mN}r_o} \quad ①$$

From ①. we can know that when $I_{ss} = 0.02\ mA$

$r_{o2} = 2.5\times10^6\ \Omega$, $r_{o4} = 2\times10^6\ \Omega$, $g_{mN} = \sqrt{44}\times10^{-4}\ 1/\Omega$. $g_{mp} = \sqrt{2}\times10^{-4}\ 1/\Omega$

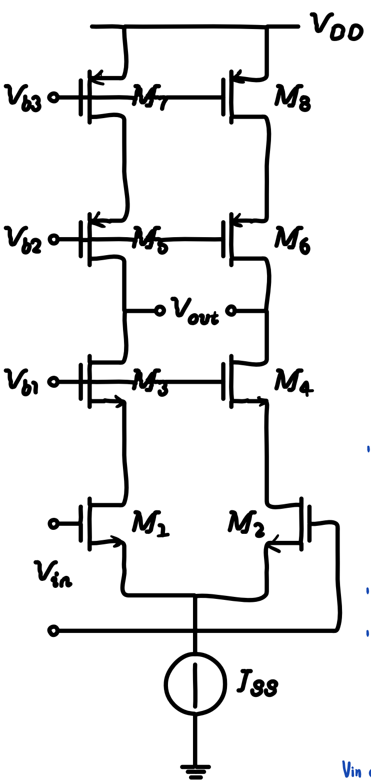
$\Rightarrow A_v = -8.34\times10^4$

And when $I_{ss} = 0.2mA$

$r_{o2} = 2.5\times10^5\ \Omega$, $r_{o4} = 2\times10^5\ \Omega$, $g_{mN} = \sqrt{44}\times10^{-4}\ 1/\Omega$. $g_{mp} = \sqrt{50}\times10^{-6}\ 1/\Omega$

$\Rightarrow A_v = -8.449\times10^3$

Question 5

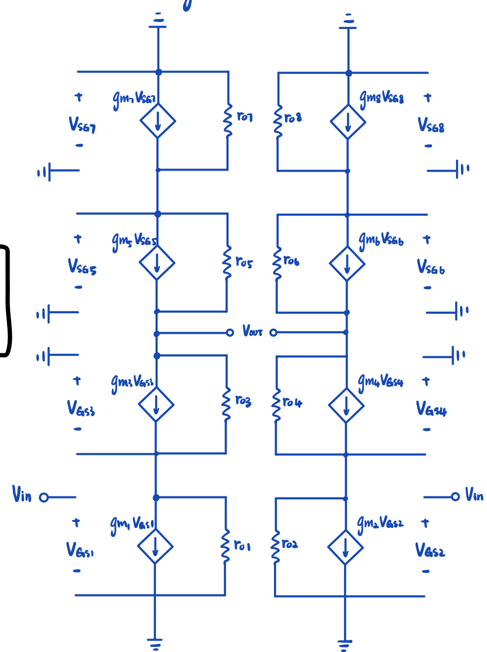


Transconductance $g_m = \sqrt{2 k_n' \frac{W}{L} I_D}$

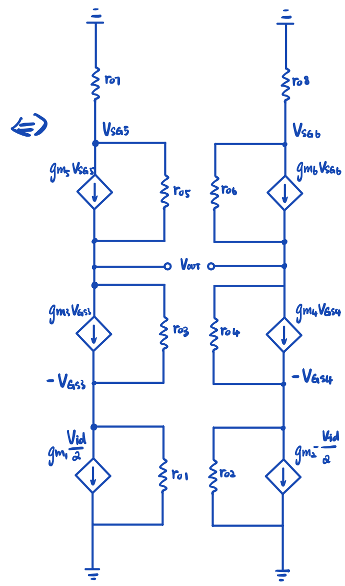
or $\sqrt{2 k_p' \frac{W}{L} I_D}$

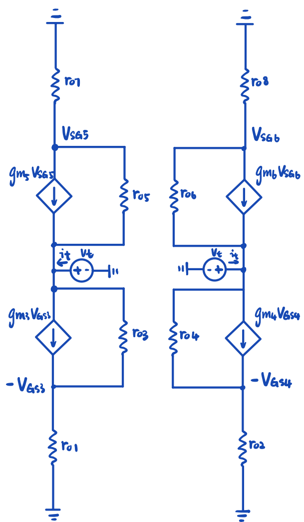
Output Resistance $r_o = \frac{1}{\lambda I_D}$

Small Signal Model:



⇔





① Rout $\begin{cases} r_{o5,6,7,8} = r_{op} \\ r_{o1,2,3,4} = r_{on} \end{cases}$ $\begin{cases} g_{m1,2,3,4} = g_{mn} \\ g_{m5,6,7,8} = g_{mp} \end{cases}$

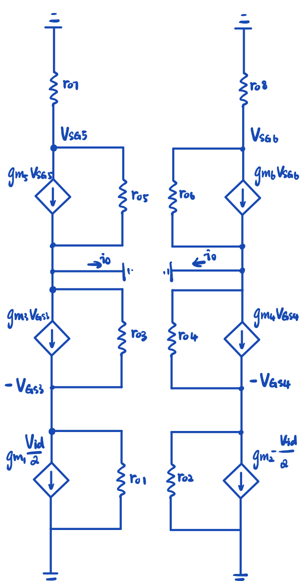
Each Half:

$$\frac{V_{SG5}}{r_{op}} = \frac{V_t - V_{SG5}}{r_{op}} - g_{mp}V_{SG5} \Rightarrow \frac{V_{SG5}}{r_{op}} = \frac{V_t}{(2+g_{mp}r_{op})r_{op}}$$

$$\frac{V_{GS3}+V_t}{r_{on}} + g_{mn}V_{GS3} = \frac{-V_{GS3}}{r_{on}} \Rightarrow \frac{-V_{GS3}}{r_{on}} = \frac{V_t}{(2+g_{mn}r_{on})r_{on}}$$

$$i_t = \frac{V_{SG5}}{r_{op}} + \frac{-V_{GS3}}{r_{on}} = \frac{V_t}{(2+g_{mp}r_{op})r_{op}} + \frac{V_t}{(2+g_{mn}r_{on})r_{on}}$$

$$\Rightarrow R_{out} = \frac{(2+g_{mp}r_{op})r_{op} \cdot (2+g_{mn}r_{on})r_{on}}{(2+g_{mp}r_{op})r_{op} + (2+g_{mn}r_{on})r_{on}}$$



② Gm, for each half:

$$\frac{-V_{SG5}}{r_{op}} = \frac{V_{SG5}}{r_{op}} + g_{mp}V_{SG5} \Rightarrow V_{SG5} = 0$$

$$\frac{V_{GS3}}{r_{on}} + g_{mn}V_{GS3} = g_{mn}\frac{V_{id}}{2} + \frac{-V_{GS3}}{r_{on}}$$

$$\Rightarrow -V_{GS3} = \frac{i_o}{\frac{1}{r_{on}} + g_{mn}} \approx \frac{i_o}{g_{mn}}$$

$$g_{mn}\frac{V_{id}}{2} + \frac{i_o}{r_{on}g_{mn}} = -i_o$$

$$\Rightarrow G_m = \frac{i_o}{\frac{1}{2}V_{in}} = \frac{r_{on}g_{mn}^2}{-r_{on}g_{mn} - 1} \approx -g_{mn}$$

$$\Rightarrow A_v = G_m \cdot R_{out}$$

① $I_{SS} = 0.02$ mA

$$r_{on} = \frac{1}{\lambda_n I} = \frac{1}{0.04 \times 10^{-5}} = 2.5 \times 10^6\ \Omega$$

$$r_{op} = \frac{1}{\lambda_p I} = \frac{1}{0.05 \times 10^{-5}} = 2 \times 10^6\ \Omega$$

$$g_{mn} = \sqrt{2 k_n' \frac{W}{L} I_D}$$

$$= \sqrt{2 \times 110 \times 10^{-6} \times 20 \times 10^{-5}} = \sqrt{4.4} \times 10^{-4}\ A/V$$

$$g_{mp} = \sqrt{2 k_p' \frac{W}{L} I_D} = \sqrt{2 \times 50 \times 10^{-6} \times 20 \times 10^{-5}} = \sqrt{2} \times 10^{-4}\ A/V$$

$$\Rightarrow \boxed{A_v = -8.340 \times 10^4}$$

② $I_{SS} = 0.2$ mA

$$r_{on} = \frac{1}{\lambda_n I} = \frac{1}{0.04 \times 10^{-4}} = 2.5 \times 10^5\ \Omega$$

$$r_{op} = \frac{1}{\lambda_p I} = \frac{1}{0.05 \times 10^{-4}} = 2 \times 10^5\ \Omega$$

$$g_{mn} = \sqrt{2 k_n' \frac{W}{L} I_D}$$

$$= \sqrt{2 \times 110 \times 10^{-6} \times 20 \times 10^{-4}} = \sqrt{44} \times 10^{-4}\ A/V$$

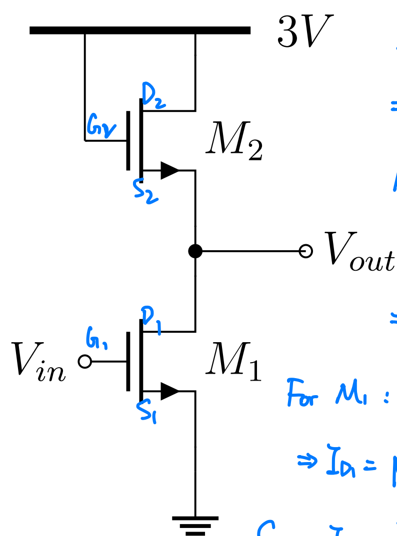
$$g_{mp} = \sqrt{2 k_p' \frac{W}{L} I_D} = \sqrt{2 \times 50 \times 10^{-6} \times 20 \times 10^{-4}} = \sqrt{20} \times 10^{-4}\ A/V$$

$$\Rightarrow \boxed{A_v = -8.449 \times 10^3}$$

Question 6. Common Source with Diode-connected Load (Lunatic)

Consider the circuit of Fig. 2 with $(W/L)_1 = 50/0.5$ and $(W/L)_2 = 10/0.5$. Assume that $\lambda = \gamma = 0$. Then 3 V for V_{DD}, 0.7 V for V_{TH1}, 0.45 $\mathrm{V}^{\frac{1}{2}}$ for γ, and 0.9 V for $2\Phi_F$

1) At what input voltage is M_1 at the edge of the triode region? What is the small-signal gain under this condition?
   线性区到饱和区的转换点
2) What input voltage drives M_1 into the triode region by 50mV ? What is the small-signal gain under this condition?
   比转换点还大50mV，即在饱和区中



1) Since M_1 is at the edge of the triode region:

$V_{DS_1} = V_{GS_1} - V_{TH_1}$

$V_{out} = V_{in} - 0.7$

Then we analyze M_2:

Since $V_{GS_2} = V_{DS_2} \Rightarrow V_{GS_2} - V_{TH_2} < V_{DS_2}$

$\Rightarrow M_2$ at saturation region

$\Rightarrow I_{D_1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH_1})^2$

$= I_{D_2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} + V_{TH_1} - V_{TH_2})^2$

$\Rightarrow 5(V_{in} - 0.7)^2 = (3 - V_{in})^2 \Rightarrow V_{in} = 1.41V$

Meanwhile $5(V_{in} - 0.7)^2 = (2.3 - V_{out})^2 \Rightarrow V_{out} = -\sqrt{5}V_{in} + C$

$\Rightarrow A_v = \frac{\partial V_{out}}{\partial V_{in}} = -\sqrt{5}$ (C is constant)

2) 50mV into triode

$\Rightarrow V_{out} = \underbrace{1.41 - 0.7}_{\text{from 1)}} - 0.05 = 0.66V$

Now for M_2: $V_{GS} = V_{DD} - V_{out} = 2.34V > V_{TH}$

$\Rightarrow$ saturation region

$\Rightarrow I_{D_2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{GS} - V_{TH})^2$

For M_1: triode region

$\Rightarrow I_{D_1} = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[(V_{in} - V_{TH})\cdot V_{out} - \frac{1}{2}V_{out}^2\right]$

Since $I_{D_1} = I_{D_2} \Rightarrow \frac{1}{2}(2.34 - 0.7)^2 = 5\left[(V_{in} - 0.7)\cdot 0.66 - \frac{1}{2}\cdot 0.66^2\right]$

Then we can get that $V_{in} = 1.4375\,V$

Meanwhile. $\frac{1}{2}(3 - V_{out} - 0.7)^2 = 5\left[(V_{in} - 0.7)\cdot V_{out} - \frac{1}{2}V_{out}^2\right]$

$$\Rightarrow V_{in} = \frac{(2.3 - V_{out})^2}{10\,V_{out}} + \frac{1}{2}V_{out}$$

$$\Rightarrow A_v = \frac{\partial V_{out}}{\partial V_{in}} = -1.93$$

Question 6

1) Edge of triode $\Rightarrow$ $V_{in} - V_{Th} = V_{out}$ $\wedge$ $V_{in} > V_{Th} = 0.7V$

M_2 : $V_{GS} = V_{DS}$ $\Rightarrow$ $V_{GS} - V_{Th} < V_{DS}$ $\Leftrightarrow$ Saturation Region as long as $V_{GS} > V_{Th}$

$\Rightarrow I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{Th})^2 = I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} + V_{Th} - V_{Th})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in})^2$

$\Rightarrow 5(V_{in} - 0.7)^2 = (3 - V_{in})^2$ $\Rightarrow$ $\boxed{V_{in} = 1.41\,V}$

Meanwhile $5(V_{in} - 0.7)^2 = (2.3 - V_{out})^2$ $\Rightarrow$ $V_{out} = 2.3 - \sqrt{5}\,V_{in} + \sqrt{5} \times 0.7$

$\Rightarrow$ $\boxed{A_V = \frac{\partial V_{out}}{\partial V_{in}} = -\sqrt{5}}$

2) At triode edge $V_{out,edge} = 1.41 - 0.7 = 0.71\,V$

$\Rightarrow$ 50mV into triode $V_{out,into} = 0.71 - 0.05 = 0.66\,V$

Now for M_2 : $V_{GS} = V_{DD} - V_{out,into} = 2.34V > V_{Th}$ $\Rightarrow$ Saturation

$\Rightarrow I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{GS} - V_{Th})^2$

For M_1 : Triode Region

$\Rightarrow I_{D1} = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[(V_{in} - V_{Th}) \cdot V_{out,into} - \frac{1}{2} V_{out,into}^2\right]$

Since $I_{D1} = I_{D2}$ $\Rightarrow$ $\frac{1}{2}(2.34 - 0.7)^2 = 5\left[(V_{in} - 0.7) \times 0.66 - \frac{1}{2} \times 0.66^2\right]$ $\Rightarrow$ $\boxed{V_{in} = 1.4375V}$

Meanwhile, $\frac{1}{2}(3 - V_{out} - 0.7)^2 = 5\left[(V_{in} - 0.7) \times V_{out} - \frac{1}{2} V_{out}^2\right]$

$$V_{in} = \frac{(2.3 - V_{out})^2}{10 V_{out}} + \frac{1}{2} V_{out}$$

$$10 V_{out} V_{in} = 2.3^2 - 4.6 V_{out} + 1.5 V_{out}^2$$

$$10 \frac{\partial V_{out}}{\partial V_{in}} + 10 V_{out} = 4.6 \frac{\partial V_{out}}{\partial V_{in}} + 3 V_{out} \frac{\partial V_{out}}{\partial V_{in}}$$

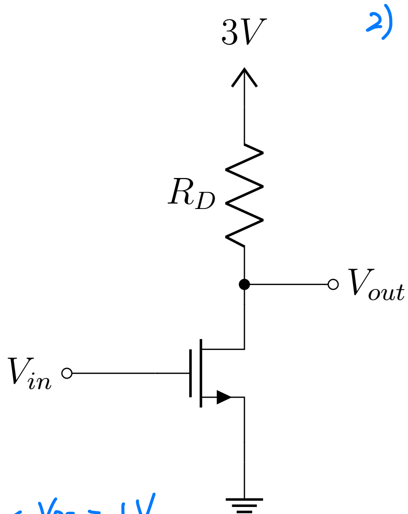
$$\Rightarrow \boxed{A_V = \frac{\partial V_{out}}{\partial V_{in}} = -1.93}$$

Question 7. Common Source with Resistive Load (Hell)

Assume $V_{TH} = 0.7\ V$, $K_n' = 110\ \mu A/V^2$

Suppose the common-source stage of Fig. 1 is to provide an output swing from 1 V to 2.5 V. Assume that $(W/L)_1 = 50/0.5$, $R_D = 2\text{k}\Omega$, and $\lambda = 0$.

1) Calculate the input voltages that yield $V_{\text{out}} = 1$ V and $V_{\text{out}} = 2.5$ V.
2) Calculate the drain current and the transconductance of M_1 for both cases.
3) How much does the small-signal gain, gm R_D, vary as the output goes from 1 V to 2.5 V ? (Variation of small-signal gain can be viewed as nonlinearity.)



1)

① $V_{out} = 1\,V \Rightarrow I_D = \dfrac{V_{DD} - V_{out}}{R_D} = 1\,mA$

Assume MOSFET at saturation :

$I_D = \frac{1}{2} K_n' \frac{W}{L} (V_{GS} - V_{TH})^2$

$= \frac{1}{2} K_n' \frac{W}{L} (V_{in} - 0.7)^2 = 10^{-3}$

$\Rightarrow V_{in} = 1.126\,V \Rightarrow V_{GS} - V_{TH} = 0.426\,V < V_{DS} = 1\,V$

$\Rightarrow$ indeed at saturation $\Rightarrow$ assumption stands

② $V_{out} = 2.5\,V \Rightarrow I_D = \dfrac{V_{DD} - V_{out}}{R_D} = 0.25\,mA$

Assume MOSFET at saturation :

$I_D = \frac{1}{2} K_n' \frac{W}{L} (V_{GS} - V_{TH})^2 = \frac{1}{2} K_n' \frac{W}{L} (V_{in} - 0.7)^2 = 0.25 \times 10^{-3}$

$\Rightarrow V_{in} = 0.9132\,V \Rightarrow V_{GS} - V_{TH} = 0.2132\,V < V_{DS} = 2.5\,V$

$\Rightarrow$ indeed at saturation $\Rightarrow$ assumption stands

2) $g_m = \sqrt{2 K_n' \frac{W}{L} I_D}$

① $V_{out} = 1\,V$, $I_D = 10^{-3}\,A$

$\Rightarrow g_m = \sqrt{2 \times 110 \times 10^{-6} \times 100 \times 10^{-3}}$

$= 4.69 \times 10^{-3}\ 1/\Omega$

② $V_{out} = 2.5\,V$, $I_D = 2.5 \times 10^{-4}\,A$

$\Rightarrow g_m = \sqrt{2 \times 110 \times 10^{-6} \times 100 \times 2.5 \times 10^{-4}}$

$= 2.34 \times 10^{-3}\ 1/\Omega$

3) $R_{out} = r_o \,//\, R_D = R_D$ (since $\lambda = 0$)

$A_v = -g_m R_{out}$

① $V_{out} = 1\,V$, $I_D = 10^{-3}\,A$, $g_m = 4.69 \times 10^{-3}\ 1/\Omega$

$\Rightarrow A_v = -9.38$

② $V_{out} = 2.5\,V$, $I_D = 2.5 \times 10^{-4}\,A$, $g_m = 2.34 \times 10^{-3}\ 1/\Omega$

$\Rightarrow A_v = -4.68$

Question 7 If $V_{Th} = 0.7V$, $K_n' = 110\ \mu A/V^2$

1) ① $V_{out} = 1V \Rightarrow I_D = \frac{V_{DD} - V_{out}}{R_D} = \frac{3-1}{2k} = 1mA$

Assume saturation $\Rightarrow I_D = \frac{1}{2} K_n' \frac{W}{L} (V_{GS} - V_{Th})^2 = \frac{1}{2} \times 110 \times 10^{-6} \times \frac{50}{0.5} (V_{in} - 0.7)^2 = 10^{-3}$

$\Rightarrow \boxed{V_{in} = 1.126\ V} \Rightarrow V_{GS} - V_{Th} = 0.426V < V_{DS} = V_{out} = 1V$ indeed in saturation

② $V_{out} = 2.5V \Rightarrow I_D = \frac{V_{DD} - V_{out}}{R_D} = \frac{3-2.5}{2k} = 0.25mA$

Assume saturation $\Rightarrow I_D = \frac{1}{2} K_n' \frac{W}{L} (V_{GS} - V_{Th})^2 = \frac{1}{2} \times 110 \times 10^{-6} \times \frac{50}{0.5} (V_{in} - 0.7)^2 = 0.25 \times 10^{-3}$

$\Rightarrow \boxed{V_{in} = 0.9132\ V} \Rightarrow V_{GS} - V_{Th} = 0.2132\ V < V_{DS} = V_{out} = 2.5V$ indeed in saturation

② $g_m = \sqrt{2 k_n' \frac{W}{L} I_D}$

① $V_{out} = 1V$, $I_D = 10^{-3} A \Rightarrow g_m = \sqrt{2 \times 110 \times 10^{-6} \times \frac{50}{0.5} \times 10^{-3}} = \boxed{4.69 \times 10^{-3}\ A/V}$

② $V_{out} = 2.5V$, $I_D = 2.5 \times 10^{-4} A \Rightarrow g_m = \sqrt{2 \times 110 \times 10^{-6} \times \frac{50}{0.5} \times 2.5 \times 10^{-4}} = \boxed{2.34 \times 10^{-3} A/V}$

③ $R_{out} = r_o \parallel R_D = \frac{1}{\lambda I_D} \parallel R_D \xrightarrow{\lambda = 0} R_D$

$A_V = -g_m R_{out}$

① $V_{out} = 1V$, $I_D = 10^{-3} A$, $g_m = 4.69 \times 10^{-3}\ A/V$

$\Rightarrow A_V = -g_m R_D = \boxed{-9.38}$

② $V_{out} = 2.5V$, $I_D = 2.5 \times 10^{-4} A$, $g_m = 2.34 \times 10^{-3}\ A/V$

$\Rightarrow A_V = -g_m R_{out} = \boxed{-4.68}$