Lecture 7

VE 311 Analog Circuits

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Recap of Last Lecture

- Op-amps

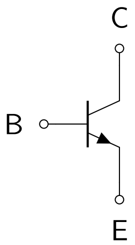
Topic to be covered

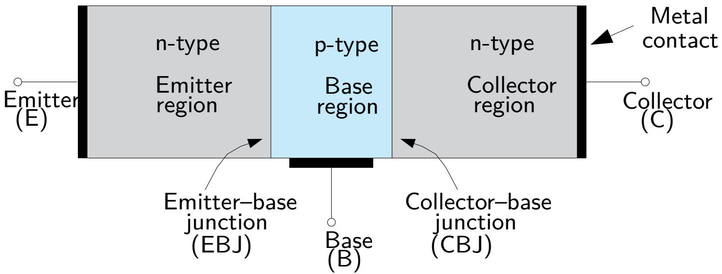
- Op-amps
- BJT

Bipolar Junction Transistors (BJTs)

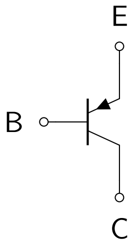
- These three–terminal devices are very useful for amplification, since an input signal can be applied across one pair of terminals and the output signal can be taken from another pair of terminals.
- bipolar junction transistor (BJTs) are still widely used in many analog circuits, including amplifiers, filters, mixers, etc.
- These are very fast devices, but also consume more power. This is the main reason CMOS has become the mainstream technology for digital circuits.

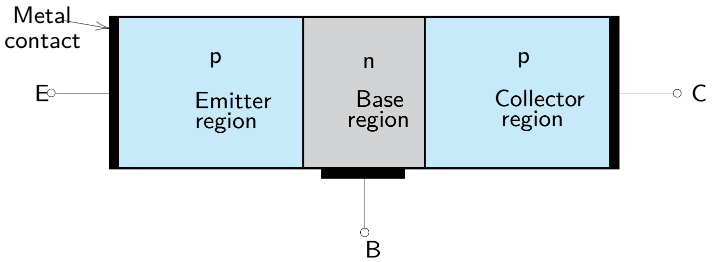
npn BJT Transistors





pnp BJT Transistors



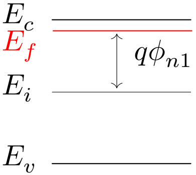


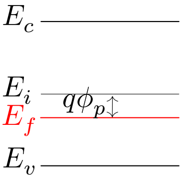
npn & pnp BJT Transistors

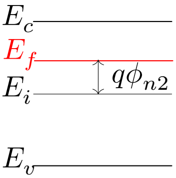
- There are three terminals: emitter, collector, and base
- There are two pn junctions, the base-emitter junction, and the base-collector junction, like two diodes that are connected back to back.
- However, the operation of the transistor is very different than just two back to back connected diodes.

BJT (Before Contact) $N_{d1} \gg N_a$

Emitter n - type	Base p - type	Collector n - type







$$n \cong N_{d1} = n_i e^{\frac{q\phi_{n1}}{kT}} \quad (1)$$

$$p \cong \frac{n_i^2}{N_{d1}} = n_i e^{\frac{-q\phi_{n1}}{kT}} \quad (2)$$

$$p \cong N_a = n_i e^{\frac{q\phi_p}{kT}} \quad (3)$$

$$n \cong \frac{n_i^2}{N_a} = n_i e^{\frac{-q\phi_p}{kT}} \quad (4)$$

$$n \cong N_{d2} = n_i e^{\frac{q\phi_{n2}}{kT}} \quad (5)$$

$$p \cong \frac{n_i^2}{N_{d2}} = n_i e^{\frac{-q\phi_{n2}}{kT}} \quad (6)$$

$V_{BE} > 0$ and $V_{CB} = 0$ (W_B long)

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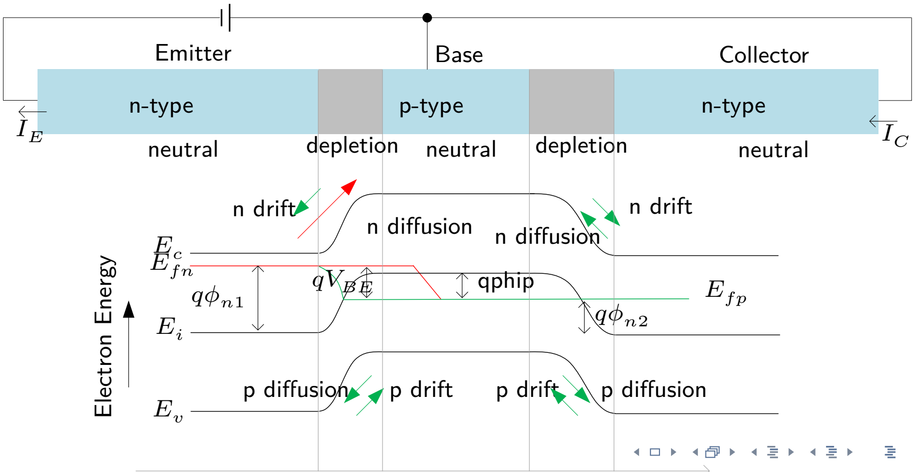
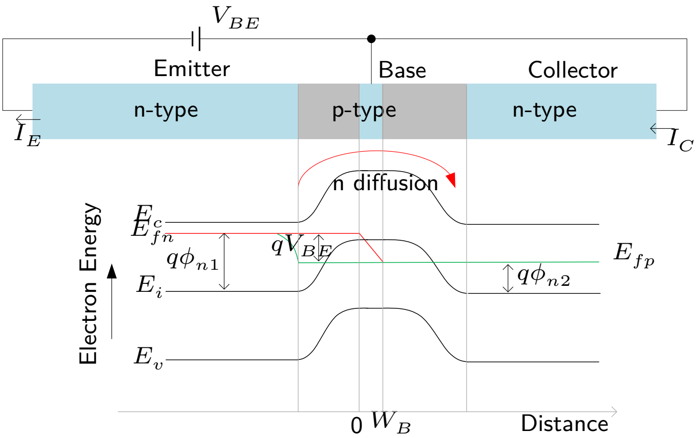


$V_{BE} > 0$ and $V_{CB} = 0$ ($N_{d1} \gg N_a$, W_B very short)



$$I_B \cong 0 \tag{7}$$

$$I_C \cong I_E \tag{8}$$

$$I_E = I_C + I_B \tag{9}$$

$$\frac{I_C}{I_E} = \alpha \tag{10}$$

$$\frac{I_C}{I_B} = \beta \tag{11}$$

$V_{BE} > 0$ and $V_{CB} = 0$ ($N_{d1} \gg N_a$, W_B Narrow)

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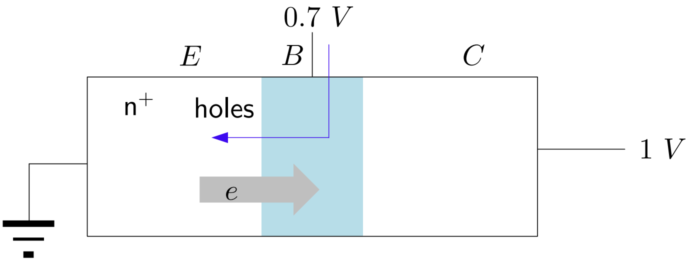
- The n (electron) diffusion is much larger than the p (hole) diffusion at the Base-Emitter junction.
- Nearly all the n (electron) diffusion from the Base-Emitter junction pass through the Base, enter into the depletion region of the Base-Collector junction, and are swept to the Collector side by the built-in electric field.

Operating Principle of BJT



- Because the BE junction is forward biased, we will turn on this junction.
- Not that because the base is p-doped, it means it has a lot more h^+ than e^-. Because the BE junction is forward biased, the concentration of electrons to increase in the base.

- Holes that are injected into the emitter do not change things too much in the emitter since electrons are majority charge carriers in the emitter and dominate the current in the emitter.
- The BC junction is reverse biased, therefore there are no carriers injected across this junction.

Operating Principle of BJT

- Once electrons are injected into the base, their concentration becomes much higher at the edge of the base-emitter junction, compared to the base-collector junction.
- Because of this concentration gradient, e^- will start diffusing from the emitter side of the base, to the collector side of the base.
  If the base is short, then most of these electrons make it through the base, without recombining with any holes that are in the base.
- When electrons get to the edge of the base-collector junction, they see a very large electric field that pulls then from the base to the collector across this junction.
- Note that e^- are minority charge carriers in the base.

Operating Principle of BJT

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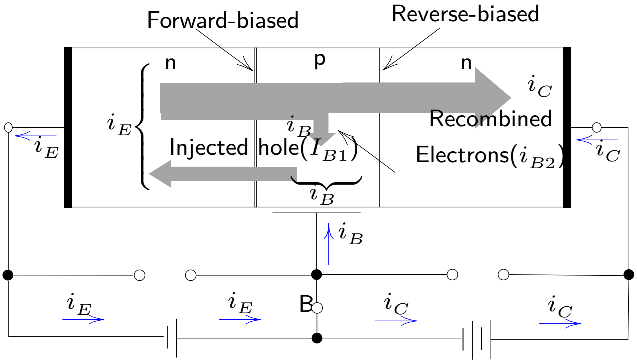
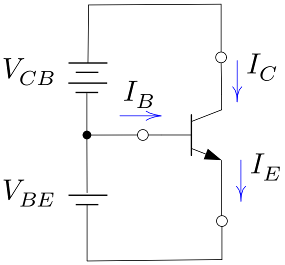




Operating Principle of BJT

- The base current consists of h^+ injected into emitter and h^+ recombined with e^- that do not make across the base.
- The emitter current is h^+ injected from the base, and e^- injected into the base.
- The collector current is mainly e^- collected at the edge of the base .

Collector Current

- I_C is mainly due to electrons that are injected from the BE junction and crossing the base.

$$i_C = I_S e^{v_{BE}/V_T} \tag{12}$$

- Note that the collector current is an exponential function of the base - emitter voltage, that is the current through the collector-emitter terminal changes when the voltage across the base-emitter terminal is modulated.

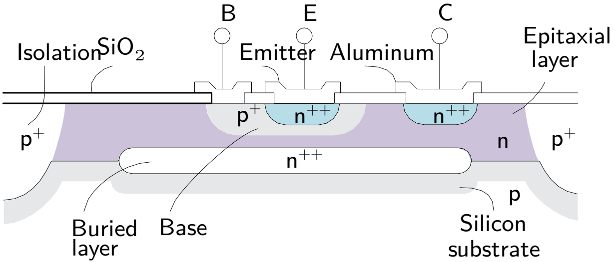
Base Current

- i_B caused (mostly) by holes that are injected from the base into the emitter (although there is also a portion of this current that is due to those recombined with electrons in the base.

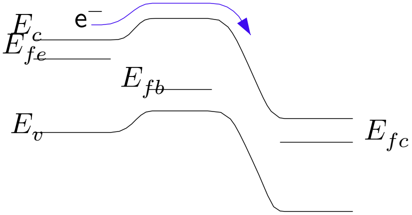
$$i_B = \frac{I_S}{\beta} e^{v_{BE}/V_T} \tag{13}$$

- But $i_B << i_C$ since most of the electrons from the emitter side make it to the collector side of the base.
- Where for a reasonable transistor β is 50-200, so the collector current is much larger than the base current, and the higher the β the better the transistor.
- Clearly, we can see that $i_E = i_B + i_C$

npn Device

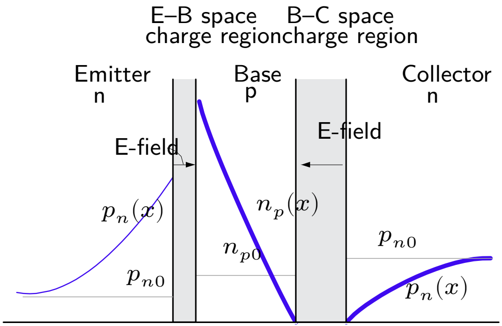


Lateral View of a BJT

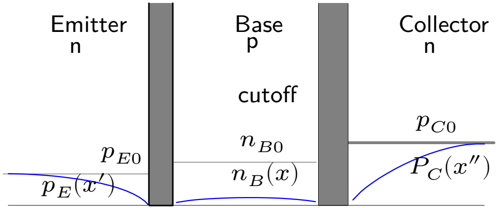


Band Diagram in FAR

Minority Carrier Distribution

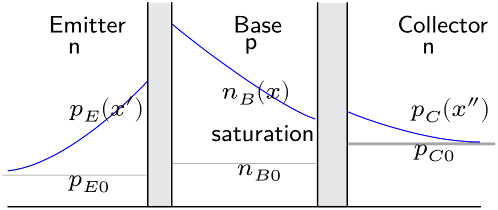


Forward Active Mode

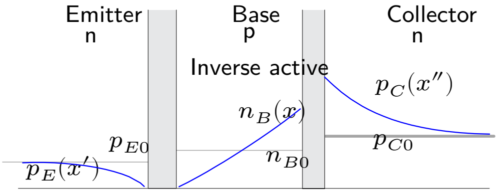


Cutoff Mode

Minority Carrier Distribution



Saturation Mode



Inverse Active Mode

BJT Operation Summary

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$$v_{BE} = v_B - v_E \approx 0.7V \tag{14}$$

Ideal Case:

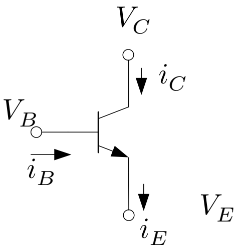
$$v_{BC} = v_B - v_C < 0 \tag{15}$$

Practice:

$$v_{BC} < -0.4 \tag{16}$$

$$i_C = I_S e^{v_{BE}/V_T} \tag{17}$$

- Forward Active Region (FAR)



$$i_B = \frac{i_C}{\beta} \tag{18}$$

BJT Operation Summary

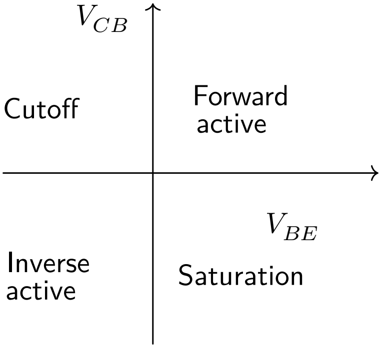
$$i_E = i_C + i_B \tag{20}$$

$$i_E = i_C + \frac{i_C}{\beta} = \frac{i_C(\beta+1)}{\beta} \tag{21}$$

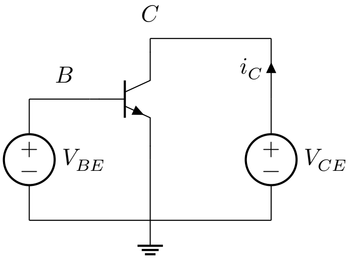
$$i_C = \frac{\beta}{\beta+1} i_E = \alpha i_E \tag{22}$$

$$\beta \approx 50 - 200 \ (100) \tag{23}$$

α is less than 1, but ≈ 1



Summary in Forward - Active Mode



$$V_{CE} \geq V_{BE} \tag{24}$$

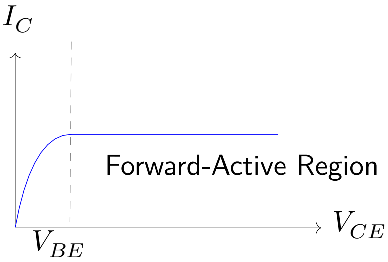
$$I_C = I_S \left(e^{\frac{qV}{BT}} - 1 \right) \tag{25}$$

$$\alpha = \frac{I_C}{I_E} \cong 1 \tag{26}$$

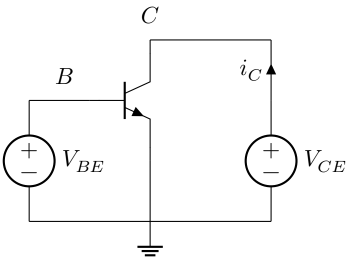
$$\beta = \frac{I_C}{I_B} = \frac{\alpha}{1-\alpha} \tag{27}$$

I_s is a constant in the spice model

I_C vs V_{CE} (Without Early Effect)



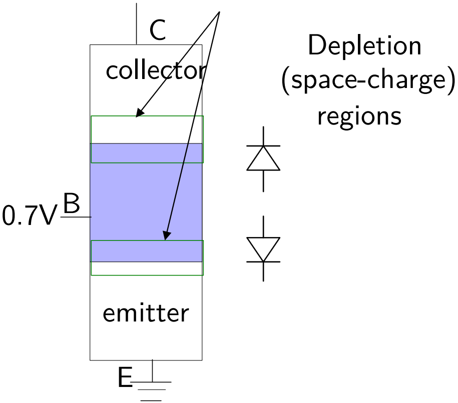
At given $\mathbf{V}_{BE}$, DC sweep $\mathbf{V}_{CE}$

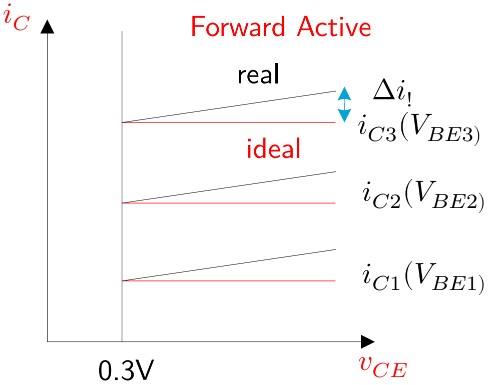


For $V_{CE} \geq V_{BE}$,

$$I_C = I_S \left(e^{\frac{qV_{BE}}{kT}} - 1 \right) \tag{28}$$

Early Effect, Named After Jim Early





Early Effect

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- As the collector voltage changes the width of the depletion region at the BC junction changes, and this causes the effective distance between the edge of the BE junction and BC junction to reduce. Thus smaller base width $\rightarrow$ a higher collector current as there is less recombination.

$$i_C \approx I_s e^{v_{EB}/V_T}(1 + \frac{v_{CE}}{V_A}) \tag{29}$$

- V_A is called the Early voltage. Typically is 50-100 V.

Early Voltage

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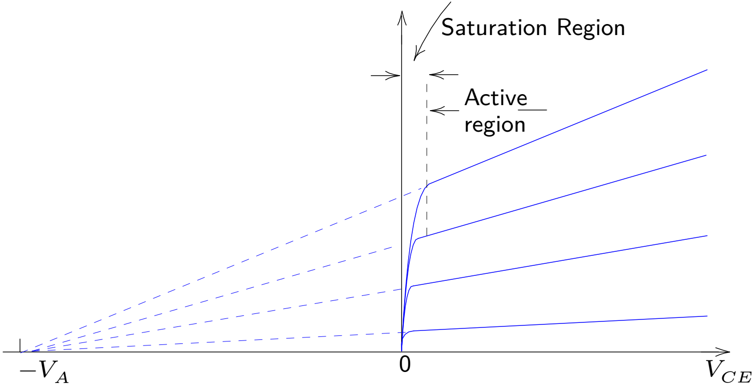
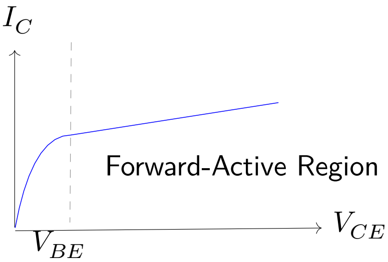
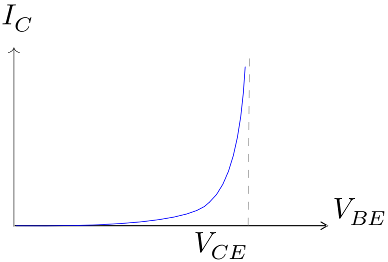


I_C vs V_{CE} (with Early Effect)



At given V_{BE}, DC sweep V_{CE}

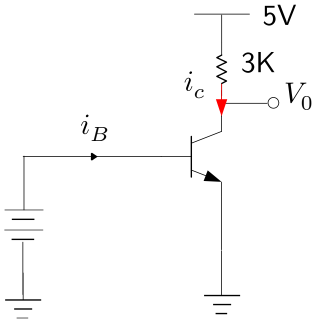


At given V_{CE}, DC sweep V_{BE}

- For $V_{CE} \geq V_{BE}$,

$$I_C = I_S \left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right) \tag{30}$$

Example: Find i_C and i_E



given: $i_B = 10\mu A, \beta = 100$

$$i_C = \beta i_B = 100 \times 10\mu A = 1mA \quad (31)$$

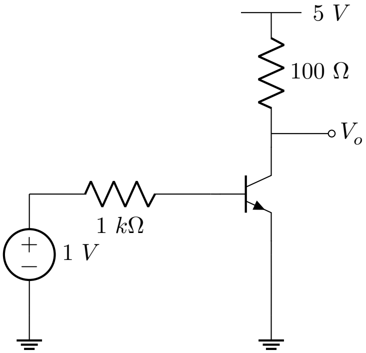
$$v_o = v_C = 5 - 3k\Omega \times 1mA = 2V \quad (32)$$

$$i_B = (\frac{I_S}{\beta})e^{\frac{V_{BE}}{V_T}} \quad (33)$$

$$v_{BE} = 0.71V \quad (34)$$

$$v_{BC} = 0.71 - 2 = -1.29V \quad (35)$$

Example 2: Find V_o



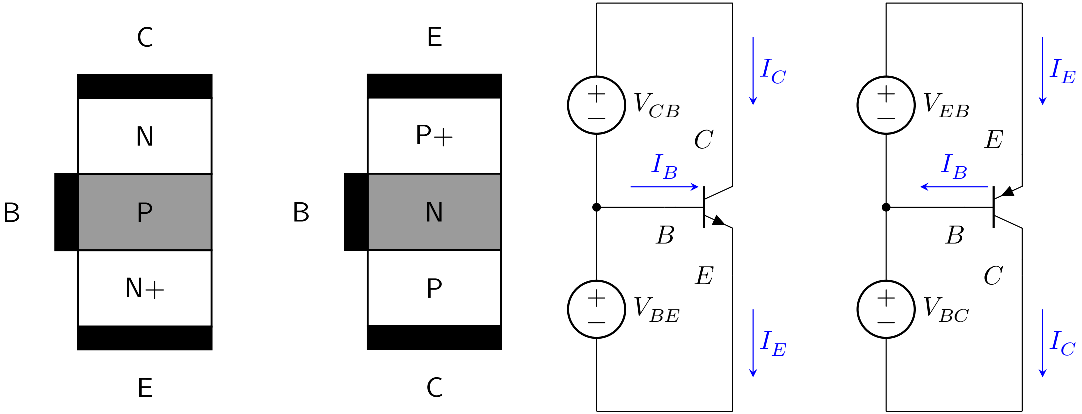
$\beta = 100, v_{BE} = 0.6V$

$$i_B = \frac{1 - 0.6V}{R_B} = 0.4\ mA \tag{36}$$

$$i_C = \beta i_B = 100 \cdot 0.4\ mA = 40\ mA \tag{37}$$

$$V_o = 5\ V - 100\ \Omega \cdot 40mA = 1V \tag{38}$$

pnp Transistor



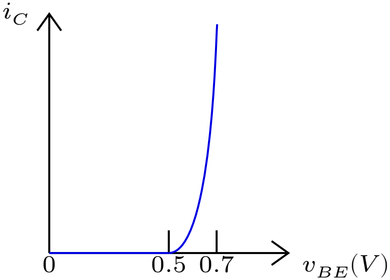
pnp Transistor

- For pnp, the basics of the operation are the same, except now instead of e- crossing the base, h+ cross the base.
- Because holes move a lot slower than e-, there would naturally be more recombination and fewer of them make it to the collector.
- So, the current gain is much lower for pnp transistors than for npn transistors.
- Note that all polarities are now different, for example for FAR operation, the BE junction has to be forward biased which means that the base-emitter voltage should be -0.7 V, or the emitter-base junction should be +0.7V so the BE junction is forward biased.

Large Signal Model: T-Model

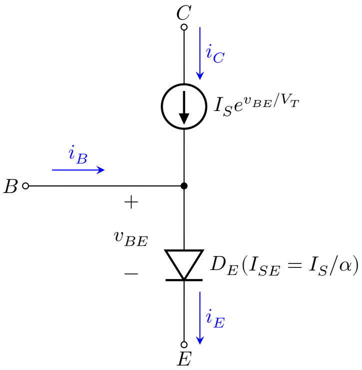
- We can now easily establish large signal models for the transistor that describe the current-voltage relation ship

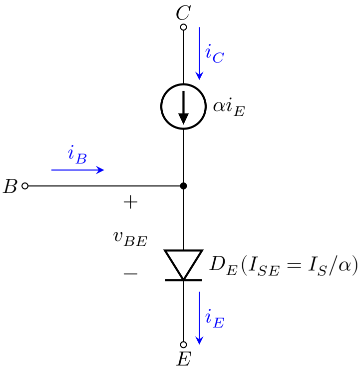
$$i_C \approx I_S e^{v_{BE}/V_T} \tag{39}$$



Large Signal Model: T-Model

- The base is shared between the input port (B-E) and the output port (B-C)

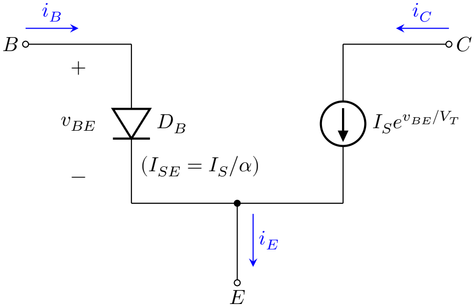


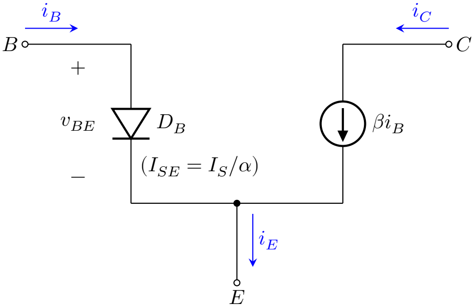


Hybrid-π Model

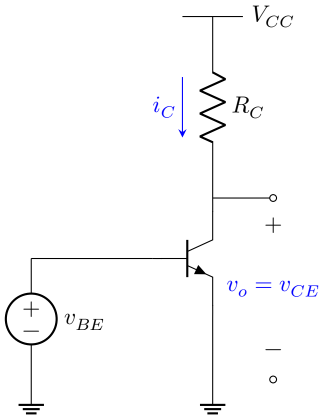
- In hybrid-π model, where the collector current is shown as a voltage controlled current source (VCCS).
- The hybrid-π model is more useful for analyzing the common-emitter amplifier. Notice that the emitter in this model is shared between the input port (B-E), and the output port (C-E).

Hybrid-π Model





Small Signal Gain



$$A_v = \frac{dv_o}{dv_{BE}} \tag{40}$$

$$A_v = -\frac{I_C}{V_T} R_C \tag{41}$$

Small Signal Gain

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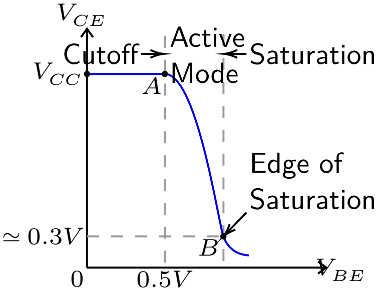
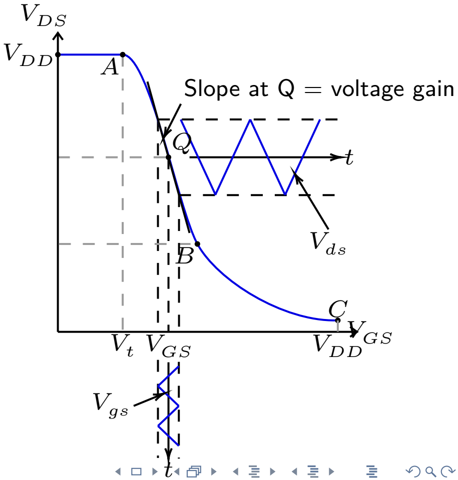
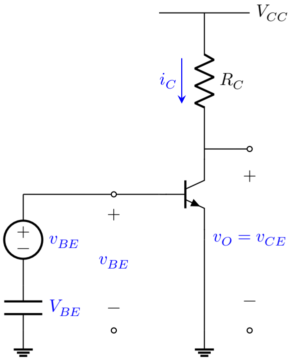




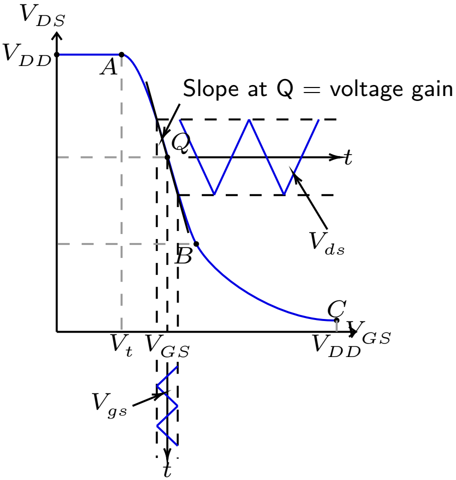
Small Signal Gain



$$v_{CE}(t) = V_{CE} + v_{ce}(t) \tag{42}$$

$$v_{BE}(t) = V_{BE} + v_{be}(t) \tag{43}$$

Quiescent Point (Q)

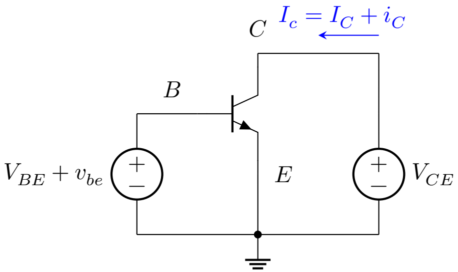


- Operating point for circuit
- Often called “bias” point

$$A_v = \frac{V_o}{v_{be}} \tag{44}$$

$$V_o = V_{CC} - R_C I_S e^{v_{BE}/V_T} \tag{45}$$

Hybrid-π Model (g_m and r_π)



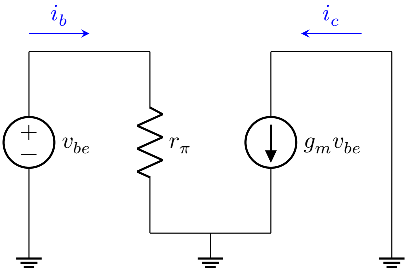
$V_{CE} \geq V_{BE}$
$\Rightarrow$ Forward—Active

$$I_C = I_S \left(e^{\frac{qV_{BE}}{kT}} - 1 \right) \left(1 + \frac{V_{CE}}{V_A} \right) \quad (46)$$

constant

Hybrid-π Model (Derivation of g_m and r_π)

Small-signal circuit:



$$r_\pi = \frac{dV_{BE}}{dI_B} = \frac{1}{\frac{dI_C}{\beta dV_{BE}}} \tag{47}$$

$$= \frac{1}{\frac{g_m}{\beta}} = \frac{\beta}{g_m} \tag{48}$$

$$g_m = \frac{dI_C}{dV_{BE}} \simeq \frac{I_C}{kT/q} \tag{49}$$

Models with the Early Effect Included

- As we see from this relationship, the collector current should change when the collector-emitter voltage changes:

$$i_C \approx I_S e^{v_{EB}/V_T} \left(1 + \frac{v_{CE}}{V_A}\right) \tag{50}$$

- So, this effect is included in the model shown below where:

$$r_o = \frac{V_A}{I_C} = \frac{\Delta v_{CE}}{\Delta i_C} \tag{51}$$

Models with the Early Effect Included

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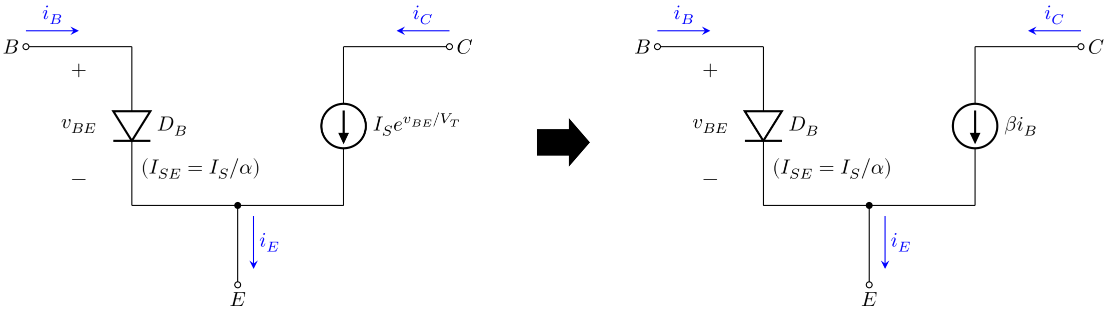

Hybrid-π Model (how to get r_o)

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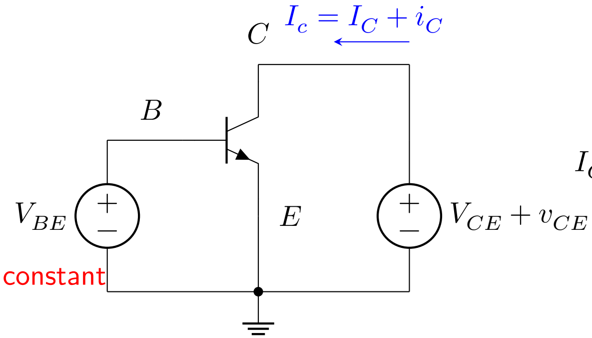
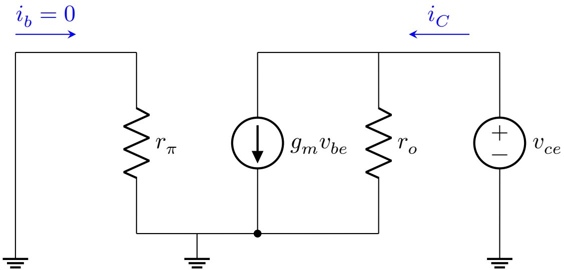


$V_{CE} \geq V_{BE} \Rightarrow$ Forward—Active

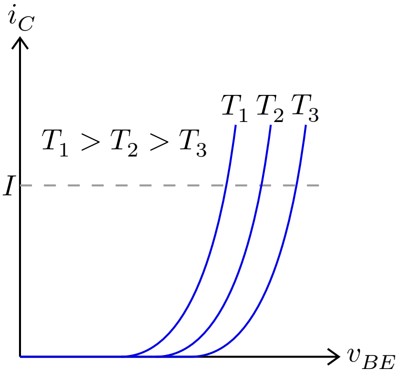
$$I_C = I_S \left(e^{\frac{qV_{BE}}{kT}} - 1 \right) \left(1 + \frac{V_{CE}}{V_A} \right) \tag{52}$$

Hybrid-π Model (Derivation of r_o)



$$r_o = \frac{1}{\frac{dI_C}{dV_{CE}}} \cong \frac{V_A}{I_C} \tag{53}$$

Temperature Variation

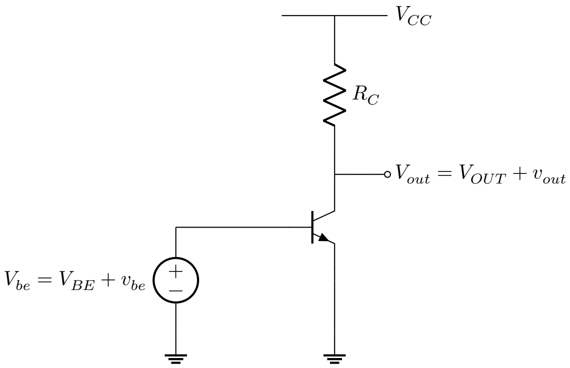


$$i_C = I_S e^{v_{BE}/V_T} \tag{54}$$

- Both I_S and V_T are temperature dependent

Common-Emitter Amplifier

- Sedra 7.1,7.2.2, 7.2.3



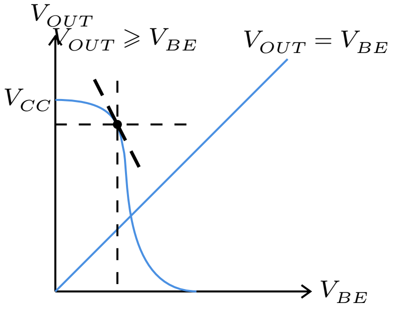
Common-Emitter Amplifier

$$V_{OUT} = V_{CC} - I_C R_C \tag{55}$$

$$= V_{CC} - \frac{AqD_n {n_i}^2}{N_a W_B}\left(e^{\frac{qV_BE}{kT}} - 1\right) R_C \tag{56}$$

$$A_V = \frac{dV_{OUT}}{dV_{BE}} \tag{57}$$

$$\cong -\frac{I_C}{kT/q} R_C = -g_m R_C \tag{58}$$



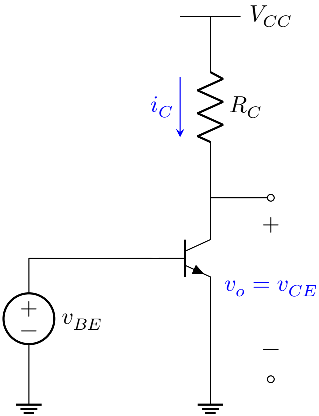
Common-Emitter Amplifier

Gain varies a lot in the FAR region.

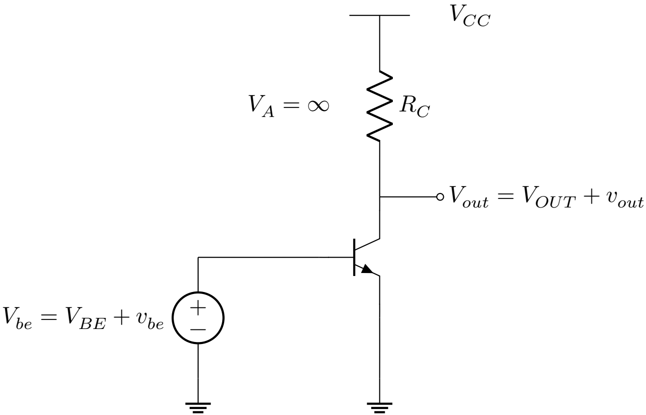
$$A_v = \frac{dv_o}{dv_{BE}} \tag{59}$$

$$A_V = \frac{dV_{OUT}}{dV_{BE}} \tag{60}$$

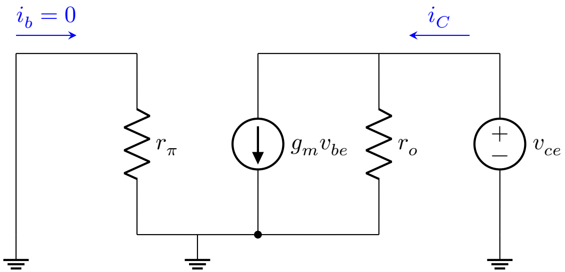
$$\cong -\frac{I_C}{kT/q} R_C = -g_m R_C \tag{61}$$



Common-Emitter Amplifier ($V_A = \infty$)



Common-Emitter Amplifier ($V_A = \infty$)



$$A_v = \frac{v_{out}}{v_{be}} = -g_m \left(R_C \parallel r_O\right) \tag{62}$$

$$= -g_m R_C \quad (\text{since } r_o = \infty) \tag{63}$$

Common-Emitter Amplifier (V_A = finite)

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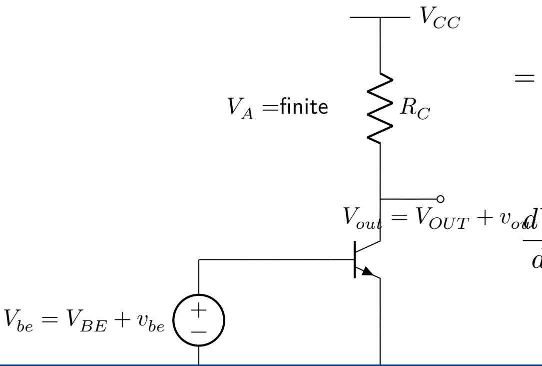
$$V_{OUT} = V_{CC} - I_C R_C \quad (64)$$

$$= V_{CC} - I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{OUT}}{V_A}\right) R_C \quad (65)$$

$$\frac{dV_{OUT}}{dV_{BE}} = -\frac{q}{kT} I_S e^{\frac{qV_{BE}}{kT}}\left(1 + \frac{V_{OUT}}{V_A}\right) R_C \quad (66)$$

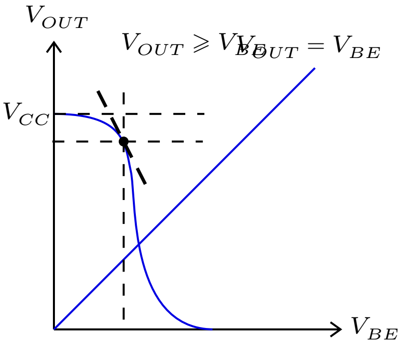
$$I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right) \frac{1}{V_A} \frac{dV_{OUT}}{dV_{BE}} R_C \quad (67)$$



Common-Emitter Amplifier (V_A = finite)

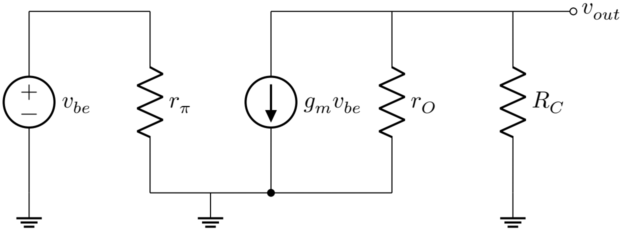
$$A_V = \frac{dV_{OUT}}{dV_{BE}} \tag{69}$$

$$\cong g_m(R_C \parallel r_O) \tag{70}$$



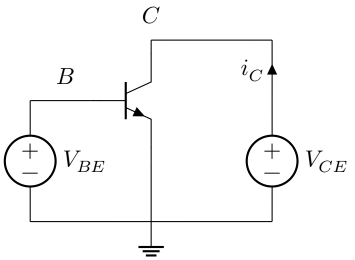
Common-Emitter Amplifier (V_A = finite)

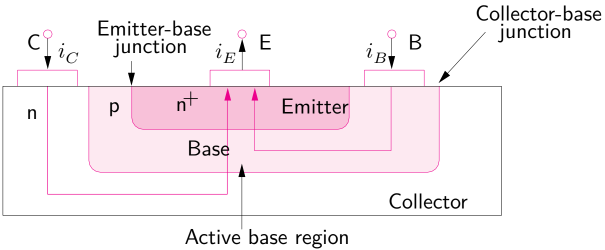
- Small-Signal Analysis



$$A_v = \frac{v_{out}}{v_{be}} = -g_m(R_C \parallel r_O) \quad (71)$$

NPN BJT Cross Section





NPN Pspice Model ($I_S = 1^{-18}$, $B_F = 100$, $V_{AF} = 100$)

$$I_C = IS\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{VAF}\right) \quad (72)$$

$$BF = \frac{I_C}{I_B} \quad (73)$$

$$I_E = I_C + I_B \quad (74)$$

$$gm \cong \frac{I_C}{kT/q} \quad (75)$$

$$r_\pi = \frac{BF}{gm} \quad (76)$$

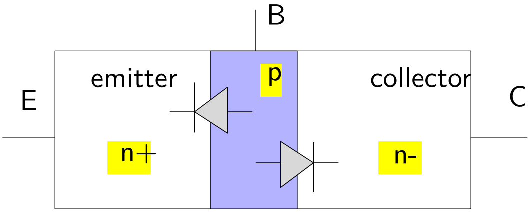
$$r_o \cong \frac{V_{AF}}{I_C} \quad (77)$$

Regions of Operation

- Forward Active Region, where the BE junction is forward biased and the BC junction is reverse biased.
- Notice that since the emitter is more highly doped (it is n+) compared to the collector (which is more lightly doped, n-), we would expect the turn- on voltage for the BE junction to be higher than that of the BC junction.

- Typically:

$$V_{BE}\ (\text{ON}) \quad 0.7\ \text{V}$$
$$V_{BC}\ (\text{ON}) \quad 0.5\text{V}$$



Forward Active Region (FAR)

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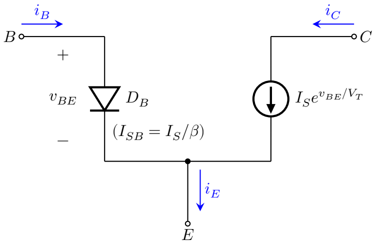
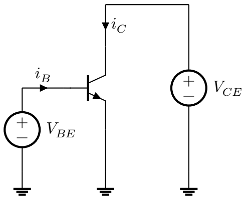
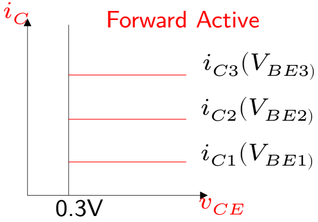
- In FAR, the BE junction is forward biased (meaning V_{BE} (ON) $\sim$ 0.7 V) and the BC junction is reverse biased, meaning V_{BC} (ON) $<$ 0.4V.
- Remember that we indicated that: $i_C \approx I_S e^{v_{BE}/V_T}$
- This shows that the collector current depends on the voltage applied across the base-emitter junction.
- It also shows that the collector current is independent of the collector-base voltage, which as we will shortly see is in fact not quite the case.
- But for now let' s assume this relationship is correct.

Forward Active Current

- Forward active model: the collector current does not change as a function of CE voltage (V_{CE}).





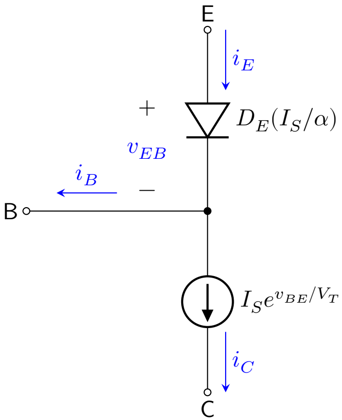


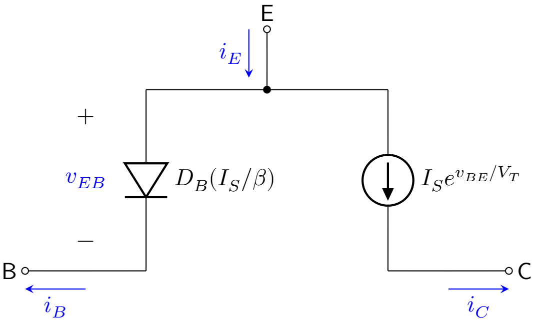
Models for pnp Transistors

- For pnp transistor, the relationship is the same except all polarities are reversed. The BE junction is forward biased when the emitter voltage is larger than the base voltage by about 0.7V and the collector current flows out of the collector and emitter current flows into the emitter.

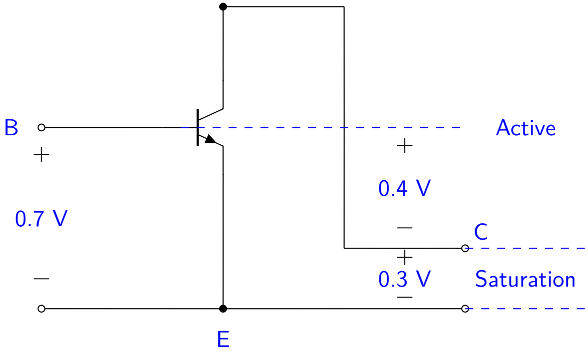
$$i_C \approx I_S e^{v_{EB}/V_T} \tag{78}$$

Models for pnp Transistors





Graphical Depiction of FAR



- Forward Active
- $V_{CB} \geq -0.4V$
- $V_{CE} \geq 0.3V$
- Otherwise get "Saturation"

V_{BE} (ON)	0.7 V
V_{BC} (ON)	0.5 V

Saturation Region

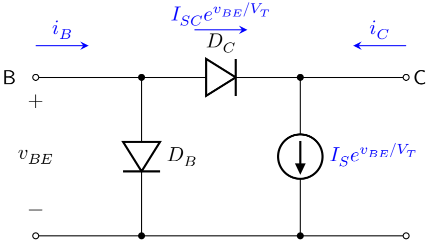
- What happens if the collector base junction begins to be forward biased?
- This means the collector voltage gets smaller until the base voltage become larger than it by about 0.4V.
- With BC junction forward-biased, most of the base current will simply be injected into the collector, as shown by the model below.
- As the BC voltage goes beyond 0.4V, the collector current will become smaller since the current coming from the diode DC will simply go down the voltage controlled current source.

Saturation Region

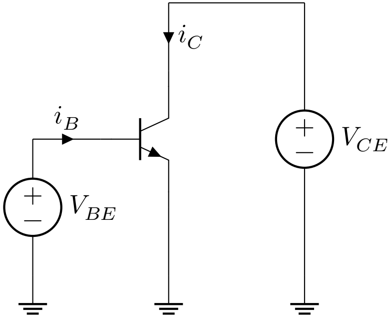
- This means the current gain Beta of the transistor will begin to go down as collector current decreases.
- When the base collector voltage becomes equal to the BE voltage, collector current becomes zero as the model shows.
- With BC junction forward-biased, most of the base current will simply be injected into the collector, as shown by the model below.
- The IV relationships for the transistor in Saturation can then be shown graphically.

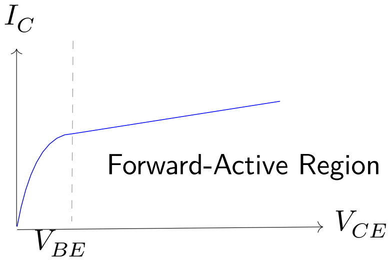
Saturation Region

Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward

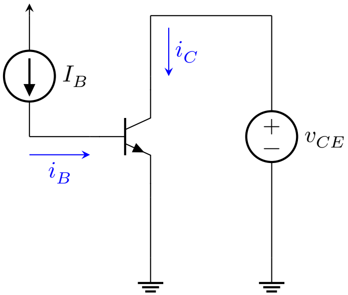


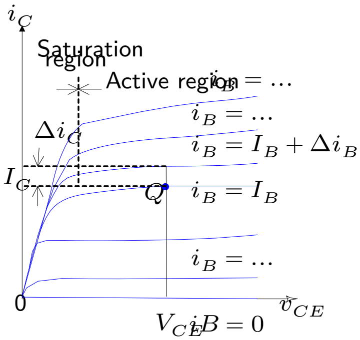
Saturation Current





Saturation Region: Closer Look

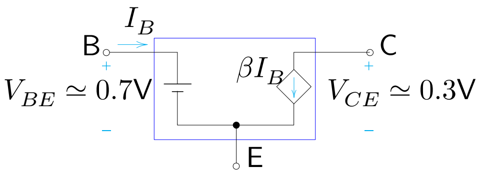




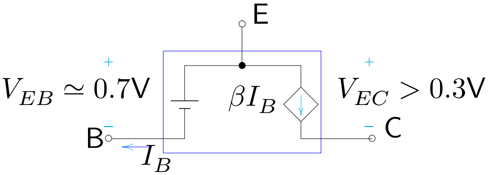
Simplified Models for Two Regions

Active
EBJ : Forward Biased
CBJ : Reverse Biased

npn

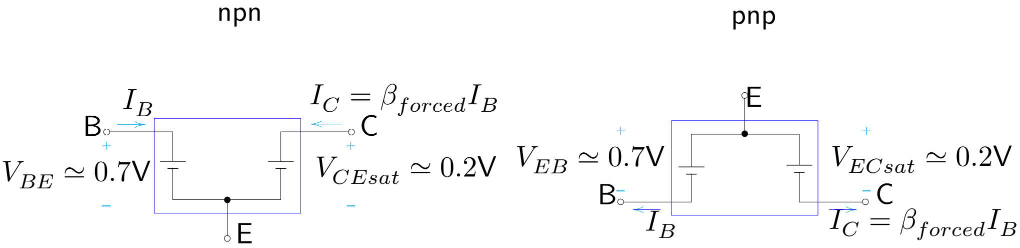


pnp



Simplified Models for Two Regions

Saturation
EBJ : Reverse Biased
CBJ : Forward Biased



Summary

$$v_{BE} = v_B - v_E \approx 0.7 \text{ V} \tag{79}$$

$$v_{BC} = v_B - v_C < 0 \tag{80}$$

$$i_C = I_S e^{v_{BE}/v_T} \tag{81}$$

$$i_B = \frac{i_C}{\beta} = \left(\frac{I_S}{\beta}\right) e^{v_{BE}/V_T} \tag{82}$$

$$i_E = \frac{i_C}{\alpha} = \left(\frac{I_S}{\alpha}\right) e^{v_{BE}/V_T} \tag{83}$$

Note: For the pnp transistor, replace v_{BE} with v_{EB}

Summary

$$i_C = \alpha i_E \tag{84}$$

$$i_C = \beta i_B \tag{85}$$

$$i_B = (1-\alpha) i_E = \frac{i_E}{\beta + 1} \tag{86}$$

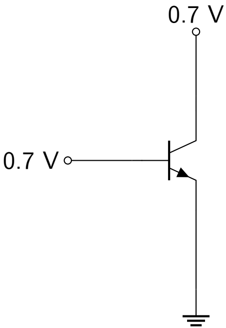
$$i_E = (\beta + 1) i_B \tag{87}$$

$$\beta = \frac{\alpha}{1-\alpha} \tag{88}$$

$$\alpha = \frac{\beta}{\beta + 1} \tag{89}$$

$$V_T = \text{thermal voltage} = \frac{kT}{q} \simeq 25 \text{ mV at room temperature}$$

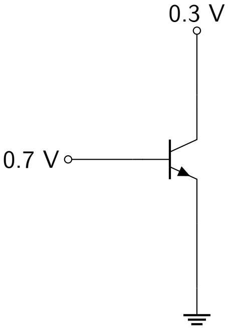
Condition for Forward Active



- Is this Forward Active?
- Yes

Condition for Forward Active

Not really on yet

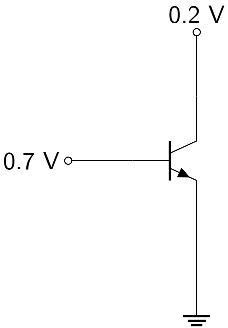


Forward

- Is this Forward Active?
- Yes
- $V_{BC} = 0.4V$

How Far Can We Go?

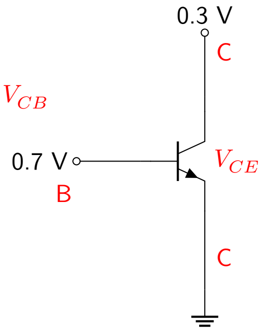
$v_{CB} = -0.5V$



Forward

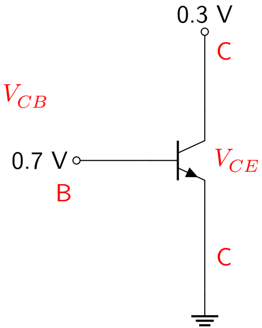
- Is this Forward Active?
- No

Forward Active Constraints



- $V_{BE} \approx 0.7V$
- $V_{BC} \leq 0.4V$
- $V_{CB} \geq -0.4V$

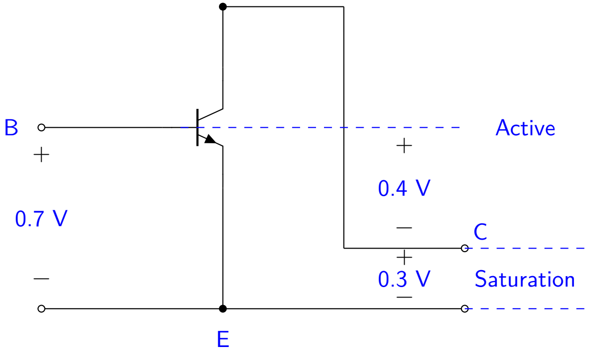
Forward Active vs "Saturation"



- Forward Active
- $V_{CB} \geq -0.4V$
- $V_{CE} \geq 0.3V$

If we are not in forward active, we are in saturation region.

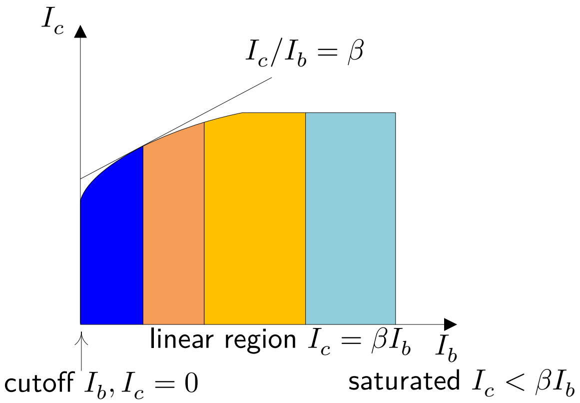
Graphical Depiction



- Forward Active
- $V_{CB} \geq -0.4V$
- $V_{CE} \geq 0.3V$

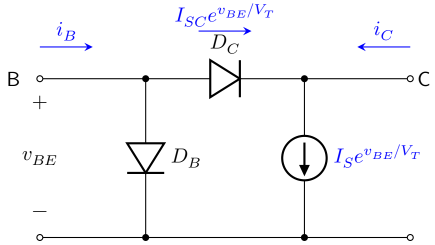
Otherwise get "Saturation"

Saturation



I_c and I_b for the Different Operating Modes of a BJT

Saturation



- BC diode starts conducting
- Less current for i_c (D_C is forward biased)
- Then Collect current falls

$$i_C = I_S e^{v_{BE}/V_T} - I_{SC} e^{V_{BC}/V_T} \tag{90}$$

- Base current rises

$$i_B = I_S/\beta e^{vBE/V_T} + I_S c e^{vBC/V_T} \tag{91}$$