VE311 RC1

Jiaying Li

University of Michigan-Shanghai Jiao Tong University Joint Institute

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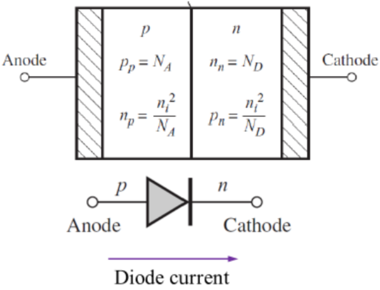
Overview

PN Junction Diode



N_A, N_D: doping densities
n_i: intrinsic carrier concentration

hole in p:

$$p_p = N_A \quad (1)$$

electron in p:

$$n_p = \frac{n_i^2}{N_A} \quad (2)$$

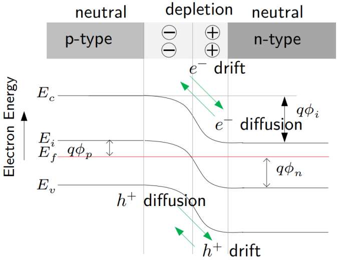
electron in n:

$$n_n = N_D \quad (3)$$

hole in n:

$$p_n = \frac{n_i^2}{N_D} \quad (4)$$

PN Junction Diode



built-in potential:

$$V_{bi} = V_T \ln(\frac{N_A N_D}{n_i^2}) \quad (5)$$

- Some electrons/holes in the neutral regions with sufficient energy continuously diffuse to the opposite sides. $\rightarrow$ Formation of diffusion
- Some electrons/holes wandering into the in the depletion region get swept by the built-in electric field. $\rightarrow$ Formation of drift current
- Diffusion current cancels drift current. **No net current flowing.**

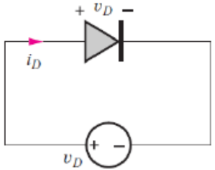
Diode Equation

- Forward Bias: Diode is on. Current flows from p to n.
- Reverse Bias: Diode is off. Current doesn't flow, but saturation current exists.

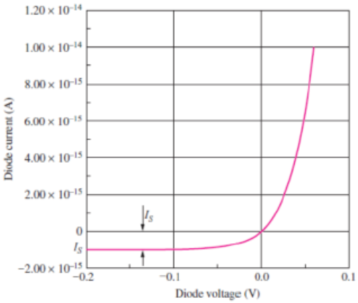
$$I_D = I_S\left(e^{\frac{qV_D}{KT}} - 1\right) = I_S\left(e^{\frac{V_D}{V_T}} - 1\right) \tag{6}$$

Note:

- k = Boltzmann's constant, 8.62 × 105 eV/K
- $\frac{kT}{q} = 0.026(0.025875)\text{V}$ at 300K when calculation.



IV Characteristics of Diode



Non-linear, or rectifying, behavior

- **Non-linear** behavior
- turn-on voltage$> V_{bi}$

Exercise 1

(23FA Mid/1) Choose one description that is not right.

(1) At equilibrium, a PN diode current is zero.

(2) Under a forward bias, a PN diode current increases exponentially.

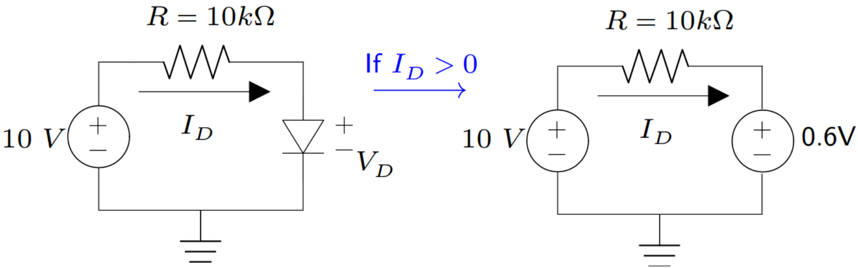
(3) Under a forward bias, a PN diode current is from P to N.

(4) Under a moderate reverse bias (before the breakdown), a PN diode current is almost zero.

(5) A PN diode shows a rectifying behavior, i.e. nonlinear I-V curve, because a drift current is dominant over a diffusion current under a forward bias.

Diode Simplification

- Ideal Diode Model: **Short circuit** for Forward Bias and **Open Circuit** for Reverse Bias
- Constant Voltage Drop Model: **Replace Diode with a voltage source** for Forward Bias and **Open Circuit** for Reverse Bias.

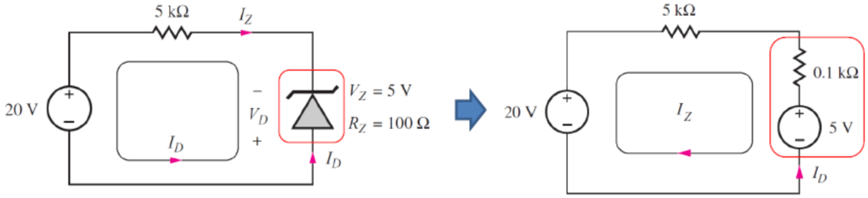


Diode Simplification

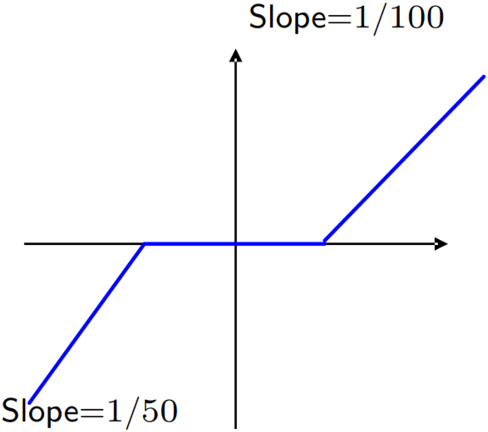
1. Identify the model for simplification.
2. Assume the voltage applied on the diode
3. Transform the diode to source and resistor according to the assumption.
4. Validate that the assumption is correct or not.

Diodes (Zener) in Breakdown Region

Method: Replace the Diode with a "Reverse" voltage source and a resistance.

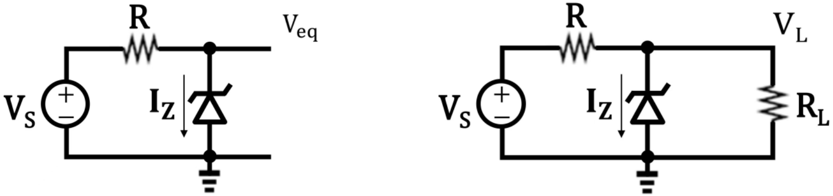


When $V_D < V_{break}$, voltage source is V_{break}, slope is $\frac{1}{R_Z}$.



Voltage Regulator

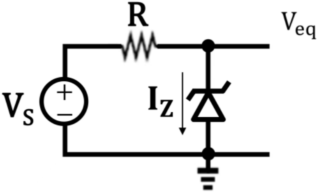
Assume the zener diode operates in reverse breakdown region.

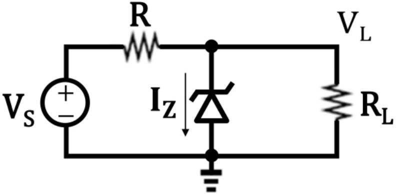


$$V_{eq} = V_{break} + \frac{V_s - V_{break}}{R + R_Z} R_Z \tag{7}$$

$$V_L = V_{eq} \frac{R_L}{(R||R_Z) + R_L} \tag{8}$$

Voltage Regulator

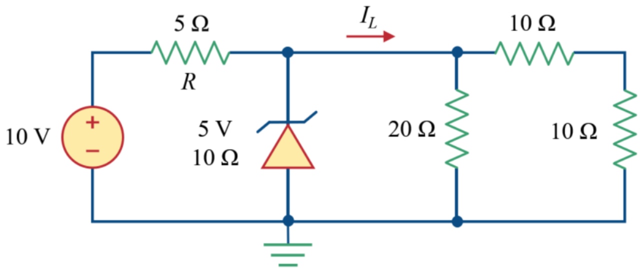




$$\text{Line Regulation} = \frac{dV_L}{dV_S} = \frac{R_Z}{R + R_Z}\,(R_L = \infty) \tag{9}$$

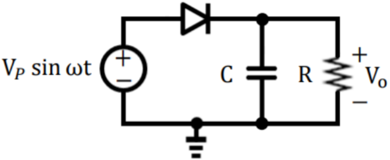
$$\text{Load Regulation} = \frac{dV_L}{dI_L} = R \parallel R_Z \tag{10}$$

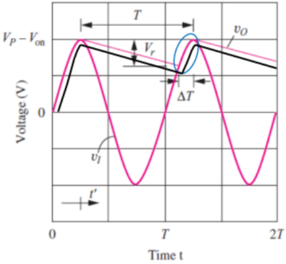
(23FA Mid/6) For the diode circuit below:
(1) Calculate the value I_L. [10 points]
(2) Find the maximum value of R required for reverse breakdown to happen. [5 points]



Half-Wave Rectifier

The basic rectifier circuit converts an AC voltage to a pulsating DC voltage.





$V_p \sin \omega t > V_O$
Diode ON

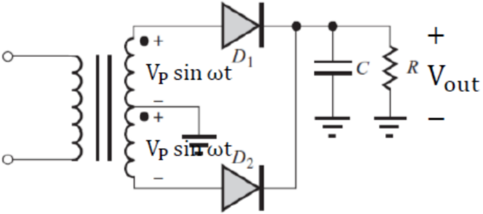
Half-Wave Rectifier

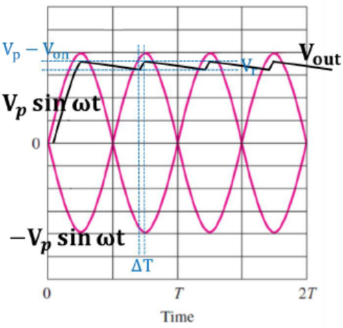
CTPP Time!

1. $V_{dc} = V_s - V_{on}$
2. $I_{dc} = \frac{V_{dc}}{R}$
3. Ripple Voltage: $V_r = V_{dc}(1 - e^{-\frac{T-\Delta T}{RC}}) \approx V_{dc}\frac{T}{RC}$
4. Conduction Angle: $\theta_c = \sqrt{\frac{2V_r}{V_S}}$
5. Conduction Interval: $\Delta T = \frac{\theta_c}{\omega}$
6. Diode Current: $I_{peak} = \frac{2I_{dc}T}{\Delta T}$
7. Surge Current: $I_{surge} = C\frac{dV}{dt} = \omega C V_S(t = 0)$
8. Peak Inverse Voltage (PIV): $PIV = 2V_s - V_{on}$

Full-Wave Rectifier

Full-wave rectifier circuits cut the capacitor discharge time in half and offer the advantage of requiring only one-half the filter capacitance to achieve a given ripple voltage





Full-Wave Rectifier

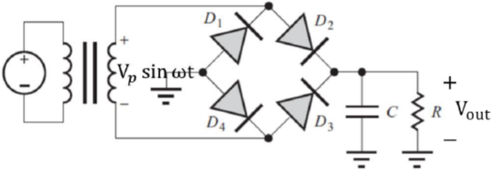
CTPP Time!

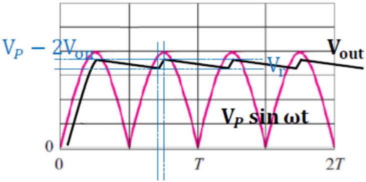
Just replace T with $\frac{T}{2}$ in the Half-Wave rectifier.

1. $V_{dc} = V_s - V_{on}$
2. $I_{dc} = \frac{V_{dc}}{R}$
3. Ripple Voltage: $V_r = V_{dc}(1 - e^{-\frac{T/2-\Delta T}{RC}}) \approx V_{dc}\frac{T}{2RC}$
4. Conduction Angle: $\theta_c = \sqrt{\frac{2V_r}{V_S}}$
5. Conduction Interval: $\Delta T = \frac{\theta_c}{\omega}$
6. Diode Current: $I_{peak} = \frac{I_{dc}T}{\Delta T}$
7. Surge Current: $I_{surge} = C\frac{dV}{dt} = \omega C V_S(t=0)$
8. Peak Inverse Voltage (PIV): $PIV = 2V_s - V_{on}$

Full-Wave Bridge Rectifier

- For $v_I > 0$, D_2 and D_4 will be on and D_1 and D_3 will be off.
- For $v_I < 0$, D_2 and D_4 will be off and D_1 and D_3 will be on.





Full-Wave Bridge Rectifier

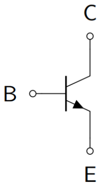
CTPP Time!

Just replace V_{on} with $2V_{on}$ in the Full-Wave rectifier and change PIV.

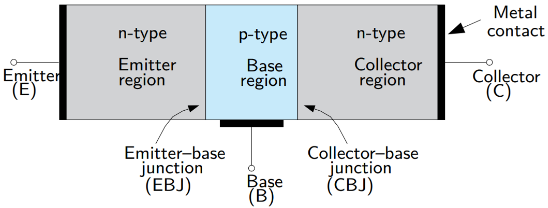
1. $V_{dc} = V_s - 2V_{on}$
2. $I_{dc} = \frac{V_{dc}}{R}$
3. Ripple Voltage: $V_r = V_{dc}(1 - e^{-\frac{T/2-\Delta T}{RC}}) \approx V_{dc}\frac{T}{2RC}$
4. Conduction Angle: $\theta_c = \sqrt{\frac{2V_r}{V_S}}$
5. Conduction Interval: $\Delta T = \frac{\theta_c}{\omega}$
6. Diode Current: $I_{peak} = \frac{I_{dc}T}{\Delta T}$
7. Surge Current: $I_{surge} = C\frac{dV}{dt} = \omega CV_S(t=0)$
8. Peak Inverse Voltage (PIV): $PIV = V_s - V_{on}$

Overview

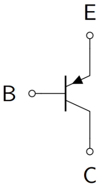
npn and pnp



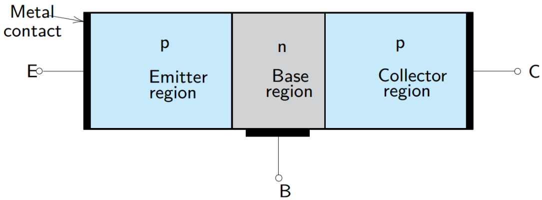
C
B
E



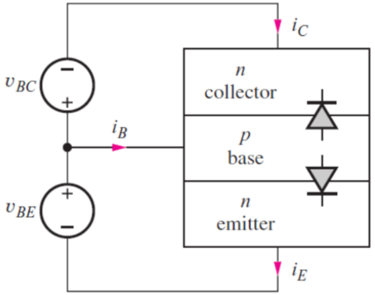
n-type
Emitter region
p-type
Base region
n-type
Collector region
Metal contact
Emitter (E)
Collector (C)
Emitter–base junction (EBJ)
Base (B)
Collector–base junction (CBJ)



E
B
C

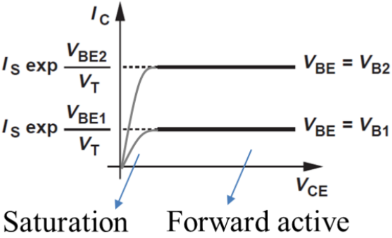


Metal contact
E
p
Emitter region
n
Base region
p
Collector region
C
B



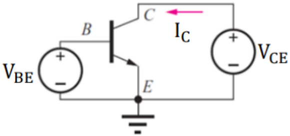
- EB diode is forward biased ($V_{BE} > 0$ or $V_{EB} < 0$)
- BC diode is reversed biased ($V_{BC} < 0$ or $V_{CB} > 0$)

Saturation v.s. Forward Active



- $V_{CE} < V_{BE}$, then $V_{CB} < 0$ or $V_{BC} > 0$, meaning that BC diode is forward biased. Both diodes are on, which is called as saturation.
- $V_{CE} > V_{BE}$, then $V_{CB} > 0$ or $V_{BC} < 0$, meaning that BC diode is reverse biased. $\rightarrow$ Forward Active

Summary:



$$V_{CE} \geq V_{BE}$$
$$\Rightarrow \text{Forward} - \text{Active}$$

Ideal case

$$I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)$$
$$\alpha = \frac{I_C}{I_E} \cong 1$$
$$\beta = \frac{I_C}{I_B} = \frac{\alpha}{1-\alpha}$$

$\Rightarrow$

$$I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)$$
$$\alpha = \frac{I_C}{I_E} = 1$$
$$\beta = \frac{I_C}{I_B} = \infty$$

Don't forget this: $I_E = I_C + I_B$

Early effect

Early effect: V_A is called the Early voltage

$$I_C = I_S \left(e^{\frac{qV_{BE}}{kT}} - 1 \right) \left(1 + \frac{V_{CE}}{V_A} \right) \tag{11}$$

or

$$I_C = I_S \left(e^{\frac{qV_{BE}}{kT}} \right) \left(1 + \frac{V_{CE}}{V_A} \right) \tag{12}$$

Small Signal

$V_{out} = V_{OUT} + v_{out}$

V_{out}: DC+AC, V_{OUT} : DC, v_{out}: AC

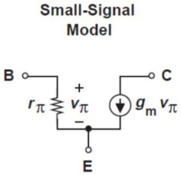
It is important to bear in mind that small-signal analysis deals with only (small) changes in voltages and currents in a circuit around their quiescent values. Thus, all constant sources, i.e., **voltage and current sources that do not vary with time (independent sources), must be set to zero** for small-signal analysis.

Small Signal

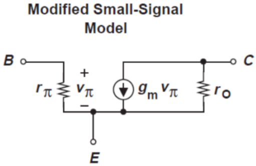
How to get a small signal circuit:

- DC voltage source: short circuit
- DC current source: open circuit
- Capacitor: short circuit (short in AC, open in DC)

Small Signal



Without Early Effect

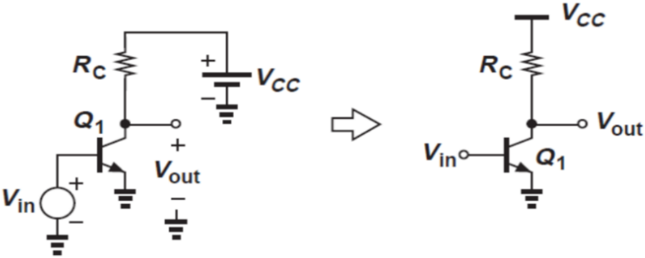


With Early Effect

$$r_\pi = \frac{dV_{BE}}{dI_B} = \frac{1}{\frac{dI_C}{\beta dV_{BE}}} = \frac{1}{\frac{g_m}{\beta}} = \frac{\beta}{g_m} \tag{13}$$

$$g_m = \frac{dI_C}{dV_{BE}} \cong \frac{I_C}{kT/q} \tag{14}$$

$$r_o = \frac{1}{\frac{dI_C}{dV_{CE}}} \cong \frac{V_A}{I_C} \tag{15}$$



Attention:

- V_{out} is not connected in the circuit! There is no current to it.
- V_{in} is connected. There is current from it.

Tips: See V_{out} as a voltmeter, V_{in} as a voltage source.

R_{out}

How to calculate R_{out}?

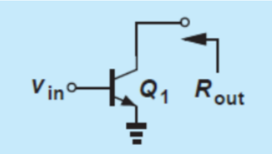
1. Draw small signal model.
2. Connect port with V_{in} to ground. Connect a test voltage v_t on port with V_{out}.
3. Derive i_t.
4. Calculate $R_{out} = \frac{v_t}{i_t}$

How to calcaulate R_{in}?

1. Draw small signal model.
2. Connect a test voltage v_t on port with V_{in}. Do nothing on output port.
3. Derive i_t.
4. Calculate $R_{in} = \frac{v_t}{i_t}$

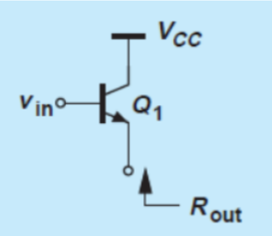
Exercise 3

Assuming that the BJT operates in the forward active region, determine the impedance seen into the collector of Q1. Please **consider the Early effect**.



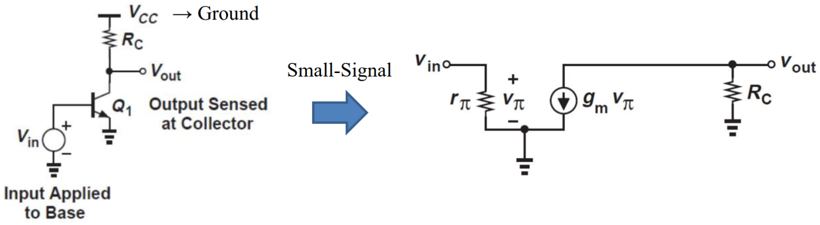
Exercise 4

Assuming that the BJT operates in the forward active region, determine the impedance seen at the emitter. Please **neglect the Early effect** for simplicity.



Overview

CE Amplifier (Without Early Effect)



$$A_v = \frac{V_{out}}{V_{in}} = -g_m R_C \tag{16}$$

$$R_{in} = r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C} \tag{17}$$

$$R_{out} = R_C \tag{18}$$

CE Amplifier (Without Early Effect)

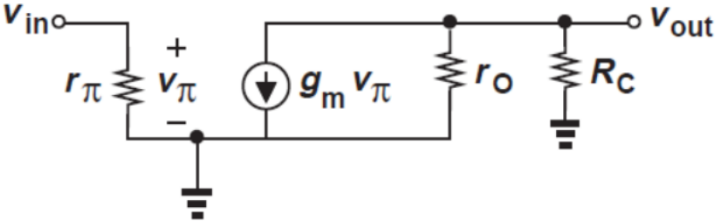
Important properties of the CE stage:

1. The small-signal gain is negative, i.e. $V_{BE} \uparrow$ and $I_C \uparrow$ lower V_{out}.
2. The voltage gain of the stage is limited by the supply voltage.

$$|A_v| = g_m R_C = \frac{I_C R_C}{V_T} \tag{19}$$

$I_C R_C$ indicates the voltage drops at the collector resistor R_C. Because $I_C R_C < V_{CC}$, $|A_v| < \frac{V_{CC}}{V_T}$. Finally, as the BJT is in the active region, we can say that, $|A_v| < \frac{V_{CC} - V_{BE}}{V_T}$.
However, when $R_C \to \infty$, $|A_v| \to \infty$. Contradiction!

CE Amplifier (With Early Effect)



$$A_v = \frac{V_{out}}{V_{in}} = -g_m(R_C \parallel r_O) \tag{20}$$

$$R_{in} = r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C} \tag{21}$$

$$R_{out} = R_C \parallel r_O \tag{22}$$

CE Amplifier (With Early Effect)

If $R_C \to \infty$, $A_v = -g_m r_O$ →intrinsic gain
Because

$$g_m = \frac{I_C}{V_T} \tag{23}$$

and

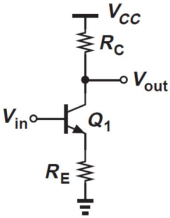
$$r_O = \frac{V_A}{I_C}, \tag{24}$$

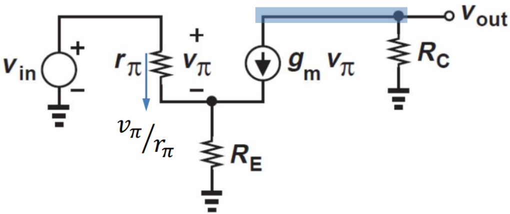
we can get

$$A_v = -\frac{V_A}{V_T} \tag{25}$$

The Early effect limits the voltage gain even if R_C approaches infinity.

CE Stage With Emitter Degeneration (Without Early Effect)



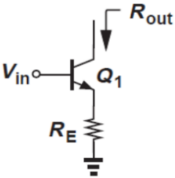


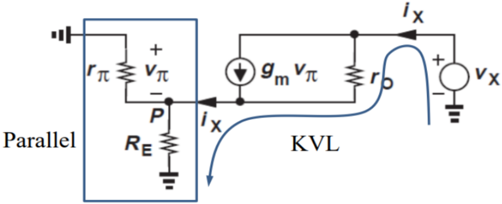
$$A_v = \frac{V_{out}}{V_{in}} = \frac{-g_m R_C}{1 + (\frac{1}{r_\pi} + g_m) R_E} \quad (26)$$

$$R_{in} = r_\pi + (\beta + 1) R_E \quad (27)$$

$$R_{out} = R_C \quad (28)$$

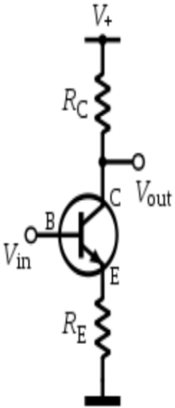
CE Stage With Emitter Degeneration (With Early Effect)





$$R_{out} = r_o + (g_m r_o + 1)(R_E \parallel r_\pi) \approx r_o[1 + g_m(R_E \parallel r_\pi)] \qquad (29)$$

Summary

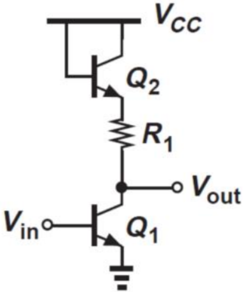


	Definition	Expression	
		With emitter degeneration	Without emitter degeneration; i.e., $R_E = 0$
Current gain	$A_i \triangleq \frac{i_{out}}{i_{in}}$	β	β
Voltage gain	$A_v \triangleq \frac{v_{out}}{v_{in}}$	$-\frac{\beta R_C}{r_\pi + (\beta+1)R_E}$	$-g_m R_C$
Input impedance	$r_{in} \triangleq \frac{v_{in}}{i_{in}}$	$r_\pi + (\beta+1)R_E$	r_π
Output impedance	$r_{out} \triangleq \frac{v_{out}}{i_{out}}$	R_C	R_C

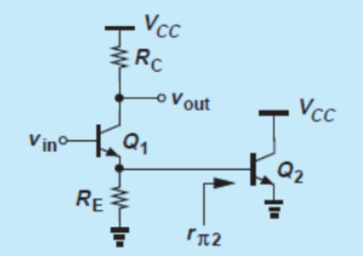
Suggestion:

- Remember the general case
- First identify B, C, E port
- Know how to analyze small signal model
- Understand all the exercise in lecture slides
- Don't ignore some small values too easily! Be careful!

(23FA Mid/5) Using a small-signal model, determine output impedance of the circuit shown below with and without Early effect. [10 points]

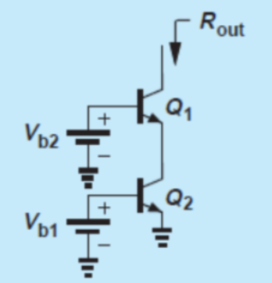


Determine the voltage gain of the stage shown below. No Early effect.



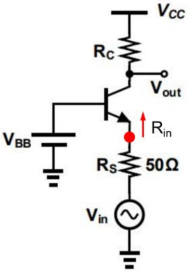
Extra Exercise 1

Determine the output resistance of the circuit below. With Early Effect.



Extra Exercise 2

(23FA Mid/9) The figure below shows a BJT amplifier. Assume $V_{BE} = 0.7$ V, $V_T = 25$ mV, β is very large (to be infinity). No Early effect. [15 points]



(a) Draw the small signal model and derive the gain for the amplifier. [5 points]

(b) Find values of V_{BB} and R_C to have a gain of 20 and a matched resistance of 100 Ω at the input, i.e. $R_{in} = 100\ \Omega$ from the point above R_S. [5 points]
(c) By cascading another BJT circuit to the circuit in part b as shown below, find the total parametric gain of the amplifier. [5 points]