Lecture 9

VE 311 Analog Circuits

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Recap of Last Lecture

- Large and small signal
- BJT circuits

$$I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right) \tag{1}$$

$$r_\pi = \frac{\beta}{g_m} \tag{2}$$

$$g_m \cong \frac{I_C}{kT/q} \tag{3}$$

$$r_o \cong \frac{V_A}{I_C} \tag{4}$$

Topics to Be Covered

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- BJT circuits
- MOSFET

Example 3: $I_S = 1e{-}15, \beta = \infty, V_{AF} = \infty, V_{ON} = 1V$

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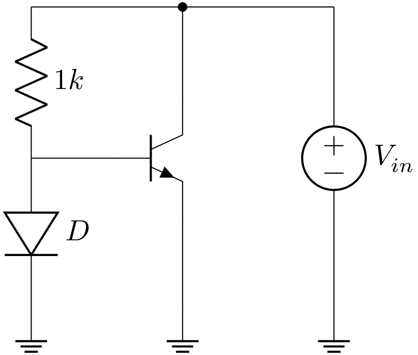
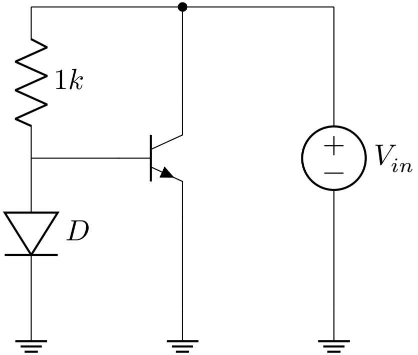
- Find the output if input $= 2 + 0.001\sin(2\pi 100t)$



Example 3

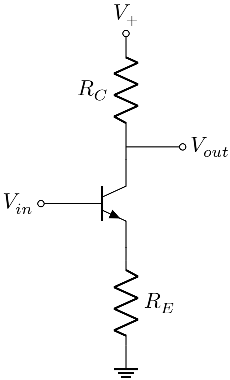
Example 4: $I_S = 1e{-}15, \beta = \infty, V_{AF} = 100$

- Find the output if input $= 2 + 0.001\sin(2\pi 100 t)$



Example 4

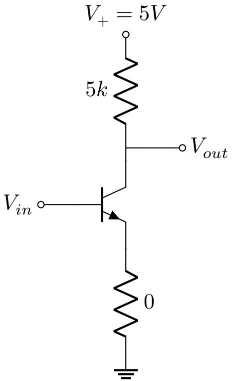
Characteristics of CE Amplifiers



	Definition	Expression	
		With emitter degeneration	Without emitter degeneration
Current gain	$A_i \triangleq \frac{i_{\text{out}}}{i_{\text{in}}}$	β	β
Voltage gain	$A_v \triangleq \frac{v_{\text{out}}}{v_{\text{in}}}$	$-\frac{\beta R_C}{r_\pi+(\beta+1)R_E}$	$-g_m R_C$
Input impedance	$r_{\text{in}} \triangleq \frac{v_{\text{in}}}{i_{\text{in}}}$	$r_\pi + (\beta+1)R_E$	r_π
Output impedance	$r_{\text{out}} \triangleq \frac{v_{\text{out}}}{i_{\text{out}}}$	R_C	R_C

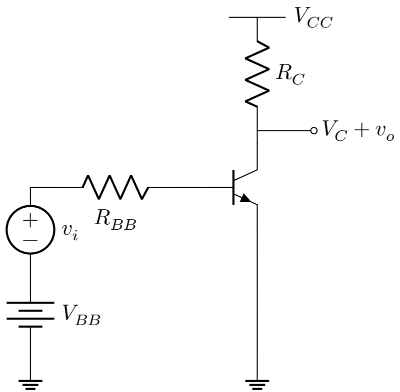
Example 5: $I_s = 5 \times 10^{-16}, \beta = 200, V_{AF} = 100$

- Find the output if input $= 0.75 + 0.001 \sin(2\pi 100 t)$

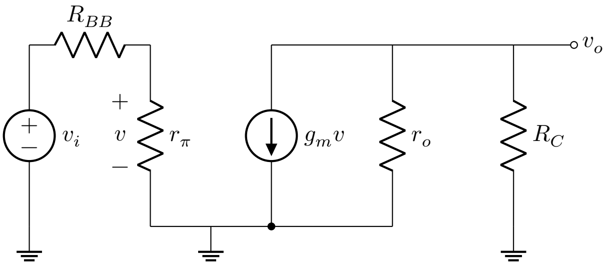


Example 5

CE Amplifier with Base Resistance

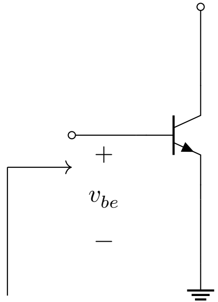


Resistance R_{BB} is added to make the biasing less sensitive to changes in transistor.



CE Amplifier with Base Resistance

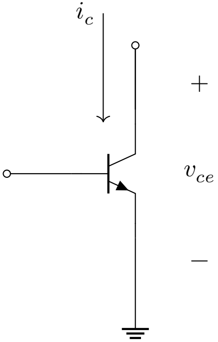
Impedance Looking into Base



$$r_\pi = \Delta v_{BE} / \Delta i_B = \frac{v_{be}}{i_b} \tag{5}$$

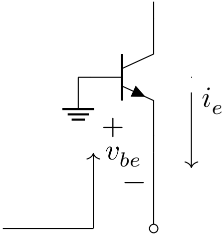
$$= \frac{V_T}{I_B} \approx \frac{\beta}{g_m} \tag{6}$$

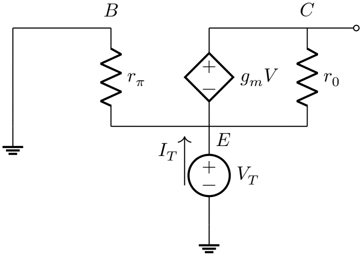
Impedance Looking into Collector



$$r_0 = \frac{\Delta v_{CE}}{\Delta i_C} = \frac{v_{Ce}}{i_C} = \frac{V_A}{I_C} \tag{7}$$

Impedance Looking into Emitter



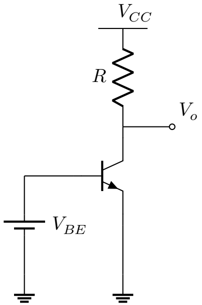


$$r_e = \frac{r_\pi}{(\beta + 1)} \tag{8}$$

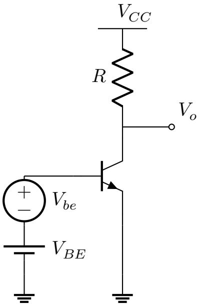
$$= \frac{1}{(\beta+1)}\frac{\beta}{g_m} = \frac{\alpha}{g_m} \cong \frac{1}{g_m} \tag{9}$$

$$r_0 = \frac{\Delta v_{CE}}{\Delta i_C} = \frac{v_{Ce}}{i_C} = \frac{V_A}{I_C} \tag{10}$$

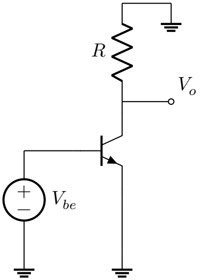
DC Biasing



DC Bias Part only



Complete Circuit

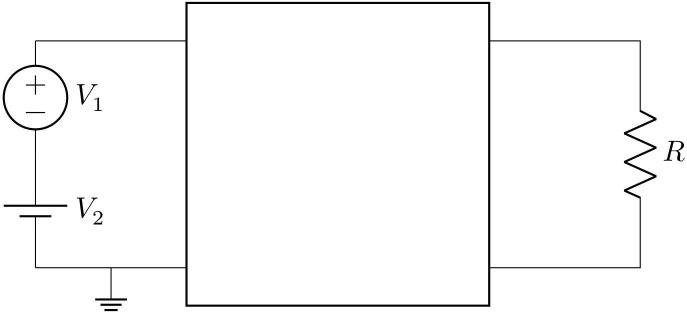


Small-signal only

DC Biasing: How do we ensure transistor is in FAR?

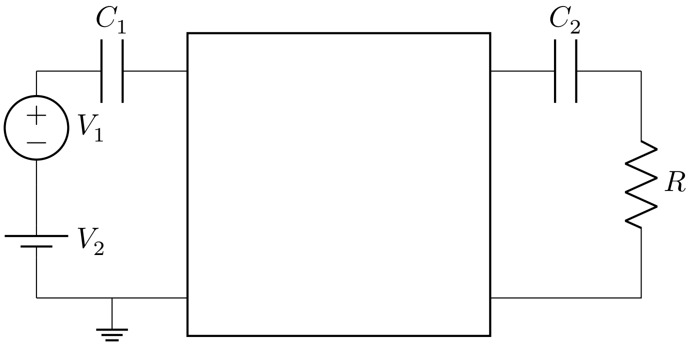
We need to make sure the transistor is always in the FAR, which means the DC base-emitter voltage has to be $> 0.7\ V$ and the collector voltage has to be larger than $0.3\ V$ to keep the $C - E$ junction reverse biased.

DC Coupling



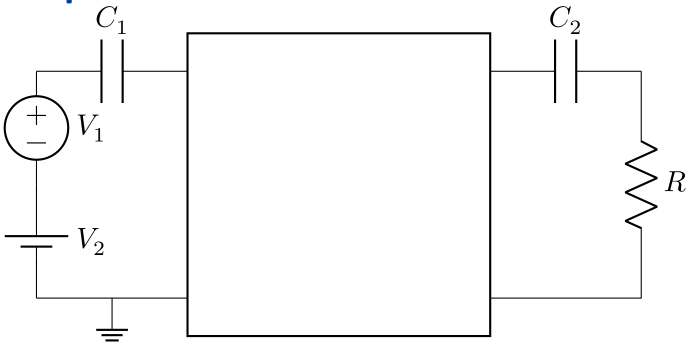
Either rely on the DC component of the input signal to bias the transistor, but this is unreliable and restrictive since we do not know how the input might change, or the input signal might not have any DC component.

AC Coupling



Or, decouple the input and output ports from the core part of the amplifier, and bias the core amplifier internally, so biasing is independent of the input/output blocks.

Biasing: AC-Coupled



Capacitors C_1 and C2 are called decoupling capacitors and are typically selected to have a large value, so for the frequency range of the input signal these capacitors effectively act as short circuits and let the AC signal pass through.

DC Bias of 0.7V BE Junction

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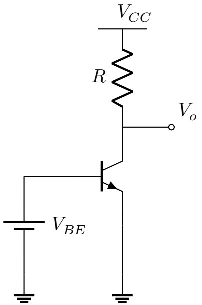

- We need a constant bias voltage source to provide the 0.7V we need across the BE junction.
- We can use another power supply, in addition to V_{CC}, but this is costly.
- So we can just use the V_{CC} supply voltage and use a resistive voltage divider to generate the voltage V_{Bias} we need.

DC Bias of 0.7V BE Junction

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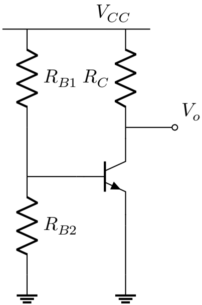
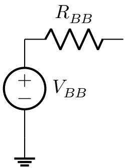
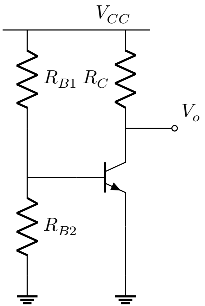


Only if I_B is assumed to be small

$$V_{\text{Bias}} = V_{CC} \cdot \frac{R_{B2}}{R_{B1} + R_{B2}} \tag{11}$$

The Thevenin Equivalent





$$R_{BB} = R_{B1} \parallel R_{B2} = \frac{R_{B1}R_{B2}}{R_{B1} + R_{B2}} \tag{12}$$

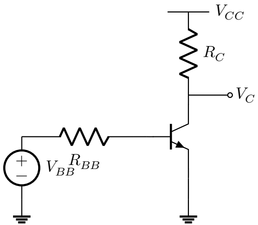
$$V_{BB} = V_{CC} \cdot \frac{R_{B2}}{R_{B1} + R_{B2}} \tag{13}$$

The Thevenin Equivalent

$$V_{BB} = \frac{R_{B2}V_{CC}}{R_{B1}+R_{B2}} \quad (14)$$

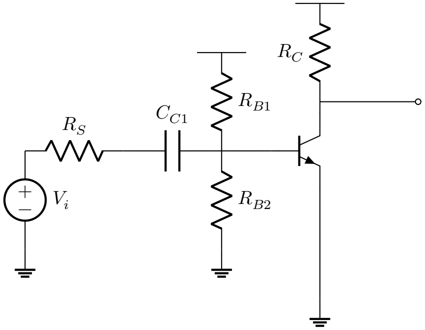
$$R_{BB} = \frac{R_{B1}R_{B2}}{R_{B1}+R_{B2}} \quad (15)$$

$$I_C = \beta I_B = \beta\frac{V_{BB}-V_{BE}}{R_{BB}} \quad (16)$$



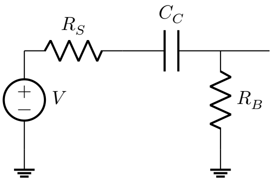
- this current depends on both β and V_{BE}.
- if you replace one transistor with another which has a different set of parameters, the collector current change.
- We know that if the collector current changes, all transistor SS parameters change, which is not good.

Coupling Capacitor



What is v_{be}/v_i ?

How Much of V_i Gets to Base?



$$R_B = r_\pi \mid\mid R_{B1} \mid\mid R_{B2} \tag{17}$$

$$v_{be} = \frac{R_B}{R_S + R_B + 1/\left(sC_C\right)} v_i \tag{18}$$

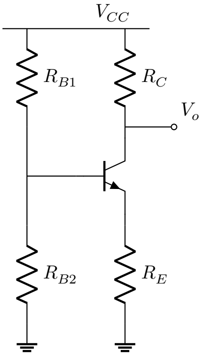
- Pole at $-\frac{1}{C_C(R_S+R_B)}$
- Zero at zero

How Much of V_i Gets to Base?

High frequency value

Note the output voltage can be derived from the rest of the SS model, which we have not shown here.

Problem: Very Sensitive to V_B

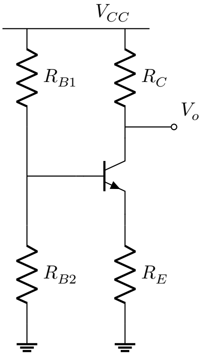


Note that the collector current is an exponential function of the base-emitter voltage and the slightest change (like 26 mV) of this voltage will cause the collector current to change by 10 times!

$$V_{BE} = V_{CC}\frac{R_{B2}}{R_{B1}+R_{B2}} \tag{19}$$

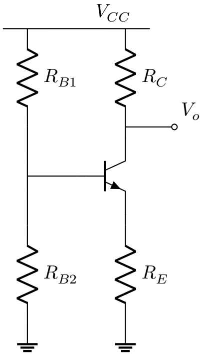
$$I_C \approx I_S e^{V_{BE}/V_T} \tag{20}$$

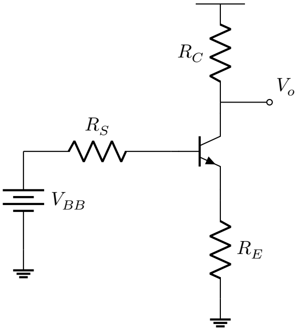
A Better Biasing Scheme: Emitter-Degeneration Resistance



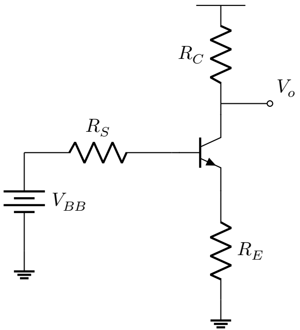
- This will raise the emitter voltage, and thus the base voltage, and will make the collector current less dependent on transistor parameters.

A Better Biasing Scheme





Find the Emitter Current:



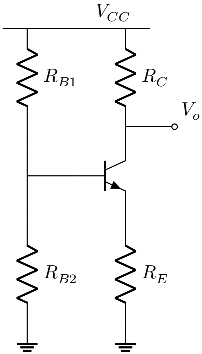
$$I_E = (\beta + 1) I_B \tag{21}$$

$$V_{BB} - R_{BB} I_B - V_{BE} - R_E I_E = 0 \tag{22}$$

$$V_{BB} - R_{BB} \frac{I_E}{\beta + 1} - V_{BE} - R_E I_E = 0 \tag{23}$$

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_{BB}}{\beta+1}} \tag{24}$$

A Better Biasing Scheme



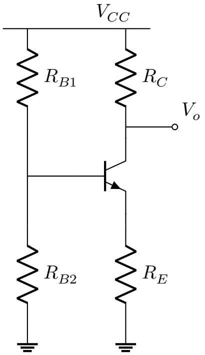
$$I_E = \frac{V_{BB} - V_{BE}}{R_E + R_{BB}/(\beta + 1)} \tag{25}$$

$$V_{BB} \gg V_{BE} \tag{26}$$

$$R_E >> (R_{BB}/(\beta + 1)) \tag{27}$$

- Then I_E would be determined by V_{BB} and R_E.

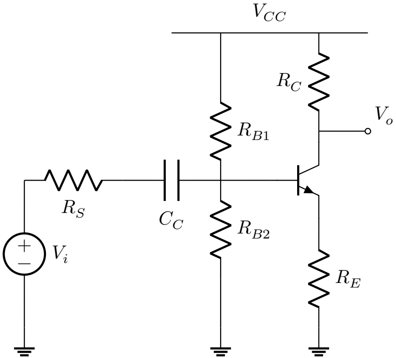
A Better Biasing Scheme



This works because of a feedback loop which stabilizes I_E.

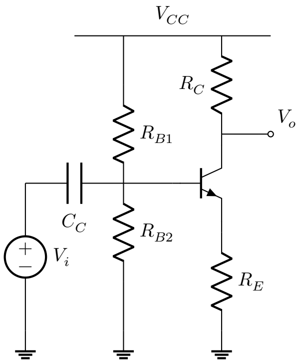
- If I_E becomes too large
- V_E increases
- Which reduces V_{BE}
- Which reduces I_E

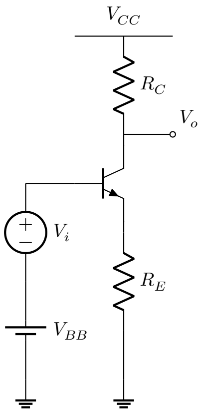
Use the Biasing in CE Amplifier



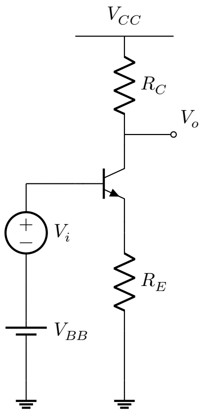
- Stable biasing independent of signal source or transistor parameters.
- But the resistor in the emitter reduces small-signal gain.
- This is called "emitter degeneration".

Assume $R_S = 0$





Small Signal Gain

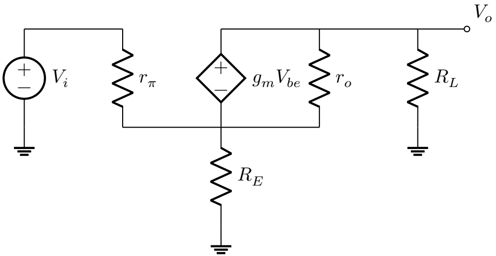


$$g_m v_\pi + \frac{v_o}{R_C} = 0 \tag{28}$$

$$\frac{v_o}{v_\pi} = -g_m R_C \tag{29}$$

$$v_\pi = v_i - v_e \tag{30}$$

Small Signal Gain



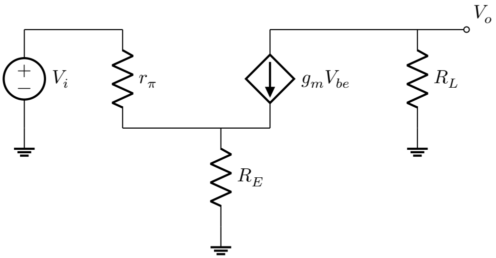
$$v_e = \left(g_m v_\pi + \frac{v_\pi}{r_\pi} \right) R_E \tag{31}$$

$$v_\pi = v_i - \left(g_m v_\pi + \frac{v_\pi}{r_\pi} \right) R_E \tag{32}$$

$$v_\pi = v_i \left(\frac{r_\pi}{r_\pi + R_E + g_m r_\pi R_E} \right) \tag{33}$$

ignore r_0 , since it is usually very large.

Small Signal Gain



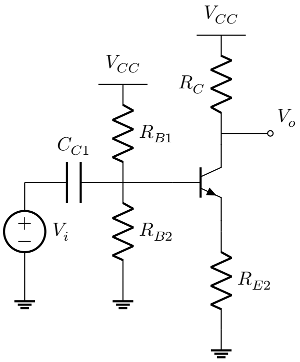
$$v_o = -g_m R_C v_\pi \quad (34)$$

$$v_\pi = v_i \frac{r_\pi}{r_\pi + R_E + g_m r_\pi R_E} \quad (35)$$

$$\frac{v_o}{v_i} = \frac{-g_m r_\pi R_C}{r_\pi + R_E + g_m r_\pi R_E} \quad (36)$$

$$\frac{v_o}{v_i} \approx \frac{-g_m R_C}{1 + g_m R_E} \quad (37)$$

CE With Degeneration



$$\frac{v_o}{v_i} \approx \frac{-g_m R_C}{1 + g_m R_E} \tag{38}$$

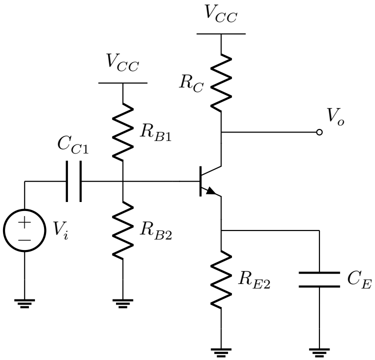
the gain has reduced by $1 + g_m R_E$

- If $g_m R_E \gg 1$ then the gain is:

$$\frac{v_o}{v_i} \approx \frac{-R_C}{R_E} \tag{39}$$

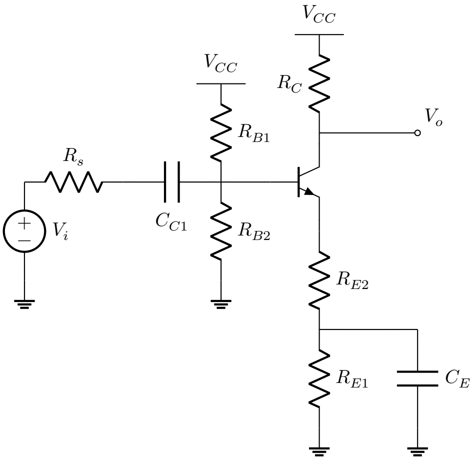
- This means the gain is independent of the BJT transistor

High Gain with Stable Bias



- Since the only function of RE is to make the bias more stable, we only need it in the DC circuit.
- We can "bypass" the effect of this resistor for the AC small- signal circuit but bypassing it with a large capacitor C_E.

Split R_E for Even More Flexibility



Now can set gain and bias separately:

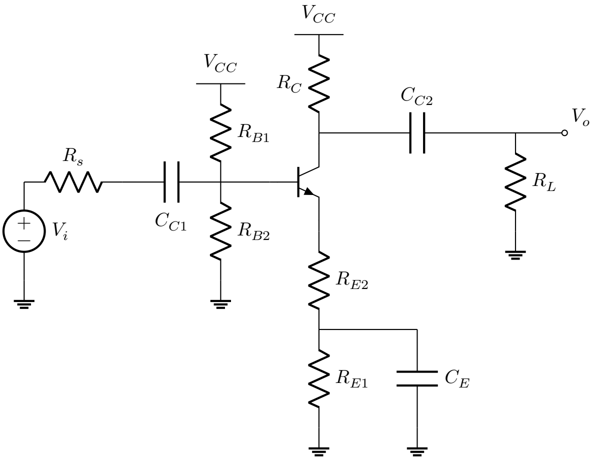
$$\omega \gg 1/R_{E1}C_E \tag{40}$$

$$\frac{v_o}{v_i} \approx \frac{-g_m R_C}{1 + g_m R_{E2}} \tag{41}$$

Robust biasing

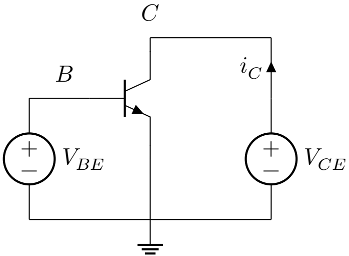
$$I_E = \frac{V_{BB} - V_{BE}}{R_{E1} + R_{E2} + R_B/(\beta + 1)} \tag{42}$$

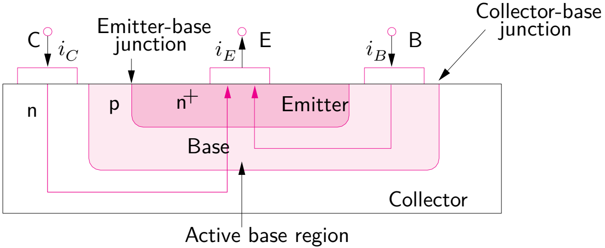
Capacitively Coupled R_L



- Now add a coupling capacitor between the amplifier output and the load.
- This will mean the load resistor will not affect the DC bias of the BJT.

NPN BJT Cross Section





NPN Pspice Model ($I_S = 1^{-18}$, $B_F = 100$, $V_{AF} = 100$)

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$$I_C = IS\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{VAF}\right) \quad (43)$$

$$BF = \frac{I_C}{I_B} \quad (44)$$

$$I_E = I_C + I_B \quad (45)$$

$$gm \cong \frac{I_C}{kT/q} \quad (46)$$

$$r_\pi = \frac{BF}{gm} \quad (47)$$

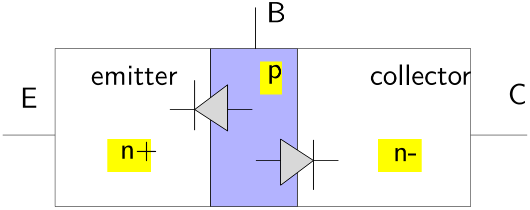
$$r_o \cong \frac{V_{AF}}{I_C} \quad (48)$$

Regions of Operation

- Forward Active Region, where the BE junction is forward biased and the BC junction is reverse biased.
- Notice that since the emitter is more highly doped (it is n+) compared to the collector (which is more lightly doped, n-), we would expect the turn- on voltage for the BE junction to be higher than that of the BC junction.

- Typically:

$$V_{BE}\text{ (ON)} \quad 0.7\text{ V}$$
$$V_{BC}\text{ (ON)} \quad 0.5\text{V}$$

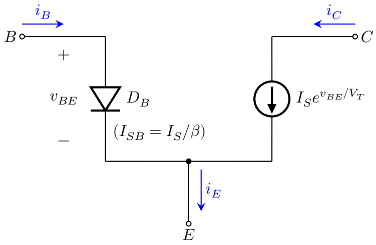


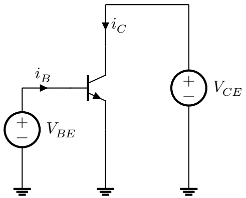
Forward Active Region (FAR)

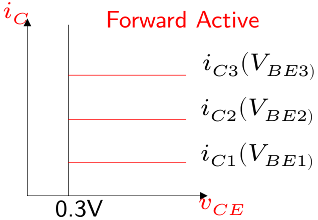
- In FAR, the BE junction is forward biased (meaning V_{BE} (ON) $\sim$ 0.7 V) and the BC junction is reverse biased, meaning V_{BC} (ON) $<$ 0.4V.
- Remember that we indicated that: $i_C \approx I_S e^{v_{BE}/V_T}$
- This shows that the collector current depends on the voltage applied across the base-emitter junction.
- It also shows that the collector current is independent of the collector-base voltage, which as we will shortly see is in fact not quite the case.
- But for now let' s assume this relationship is correct.

Forward Active Current

- Forward active model: the collector current does not change as a function of CE voltage (V_{CE}).







Models for pnp Transistors

- For pnp transistor, the relationship is the same except all polarities are reversed. The BE junction is forward biased when the emitter voltage is larger than the base voltage by about 0.7V and the collector current flows out of the collector and emitter current flows into the emitter.

$$i_C \approx I_S e^{v_{EB}/V_T} \tag{49}$$

Models for pnp Transistors

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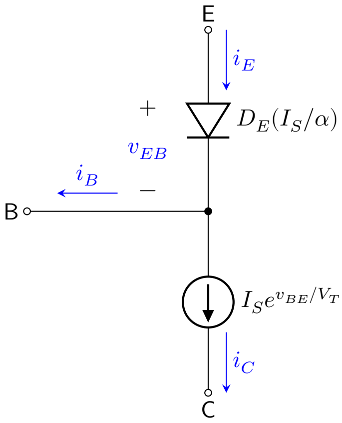
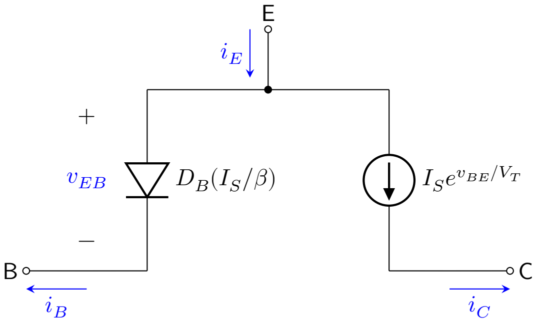
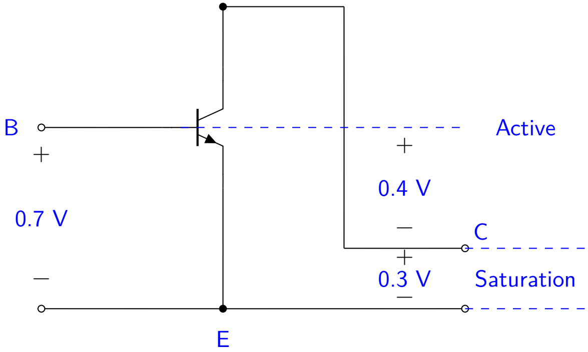




Graphical Depiction of FAR



- Forward Active
- $V_{CB} \geq -0.4V$
- $V_{CE} \geq 0.3V$
- Otherwise get "Saturation"

V_{BE} (ON) 0.7 V
V_{BC} (ON) 0.5 V

Saturation Region

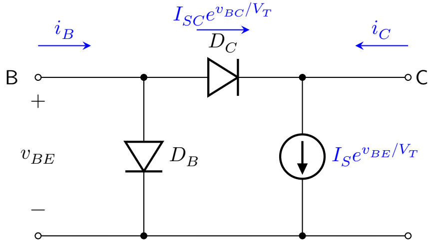
- What happens if the collector base junction begins to be forward biased?
- This means the collector voltage gets smaller until the base voltage become larger than it by about 0.4V.
- With BC junction forward-biased, most of the base current will simply be injected into the collector, as shown by the model below.
- As the BC voltage goes beyond 0.4V, the collector current will become smaller since the current coming from the diode DC will simply go down the voltage controlled current source.

Saturation Region

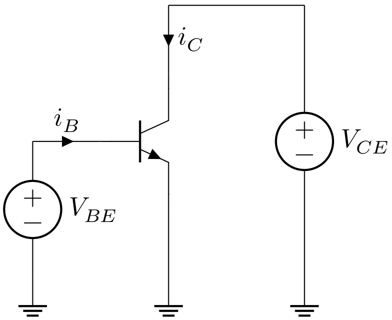
- This means the current gain Beta of the transistor will begin to go down as collector current decreases.
- When the base collector voltage becomes equal to the BE voltage, collector current becomes zero as the model shows.
- With BC junction forward-biased, most of the base current will simply be injected into the collector, as shown by the model below.
- The IV relationships for the transistor in Saturation can then be shown graphically.

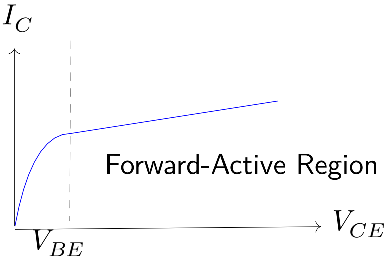
Saturation Region

Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward

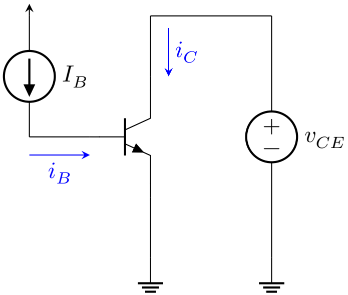


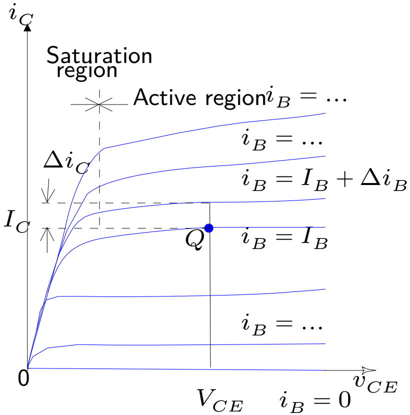
Saturation Current





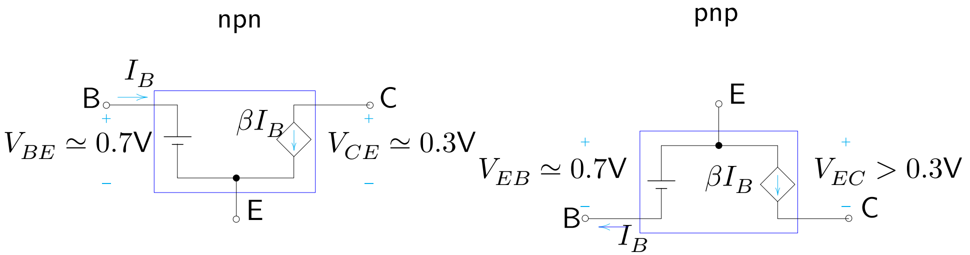
Saturation Region: Closer Look





Simplified Models for Two Regions

Active
EBJ : Forward Biased
CBJ : Reverse Biased



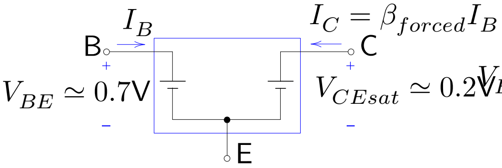
Simplified Models for Two Regions

Saturation

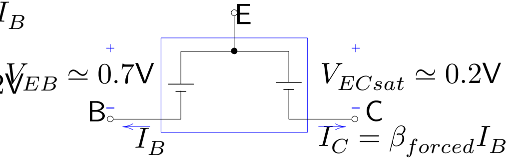
EBJ : Reverse Biased

CBJ : Forward Biased

npn



pnp



Summary

$$v_{BE} = v_B - v_E \approx 0.7 \text{ V} \qquad (50)$$

$$v_{BC} = v_B - v_C < 0 \qquad (51)$$

$$i_C = I_S e^{v_{BE}/v_T} \qquad (52)$$

Note: For the pnp transistor, replace v_{BE} with v_{EB}

$$i_B = \frac{i_C}{\beta} = \left(\frac{I_S}{\beta}\right) e^{v_{BE}/V_T} \qquad (53)$$

$$i_E = \frac{i_C}{\alpha} = \left(\frac{I_S}{\alpha}\right) e^{v_{BE}/V_T} \qquad (54)$$

Summary

$$i_C = \alpha i_E \tag{55}$$

$$i_C = \beta i_B \tag{56}$$

$$i_B = (1-\alpha) i_E = \frac{i_E}{\beta + 1} \tag{57}$$

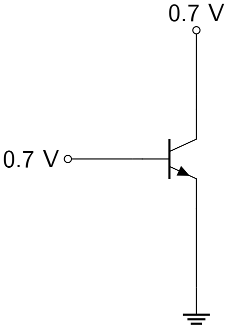
$$i_E = (\beta + 1) i_B \tag{58}$$

$$\beta = \frac{\alpha}{1-\alpha} \tag{59}$$

$$\alpha = \frac{\beta}{\beta + 1} \tag{60}$$

$$V_T = \text{thermal voltage} = \frac{kT}{q} \simeq 25 \text{ mV at room temperature}$$

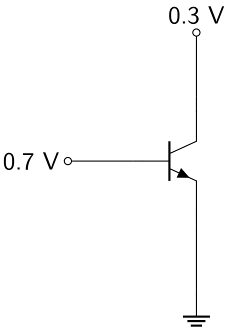
Condition for Forward Active



- Is this Forward Active?
- Yes

Condition for Forward Active

Not really on yet

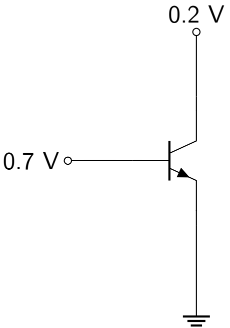


Forward

- Is this Forward Active?
- Yes
- $V_{BC} = 0.4V$

How Far Can We Go?

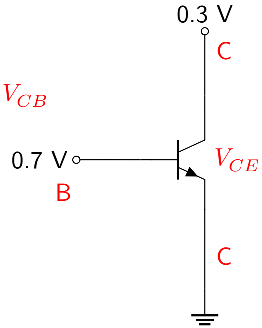
$v_{CB} = -0.5V$



Forward

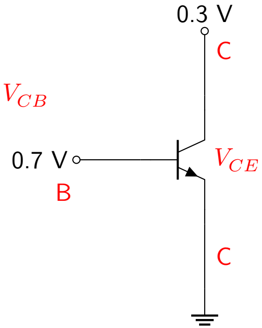
- Is this Forward Active?
- No

Forward Active Constraints



- $V_{BE} \approx 0.7V$
- $V_{BC} \leq 0.4V$
- $V_{CB} \geq -0.4V$

Forward Active vs “Saturation”



- Forward Active
- $V_{CB} \geq -0.4V$
- $V_{CE} \geq 0.3V$

If we are not in forward active, we are in saturation region.

Graphical Depiction

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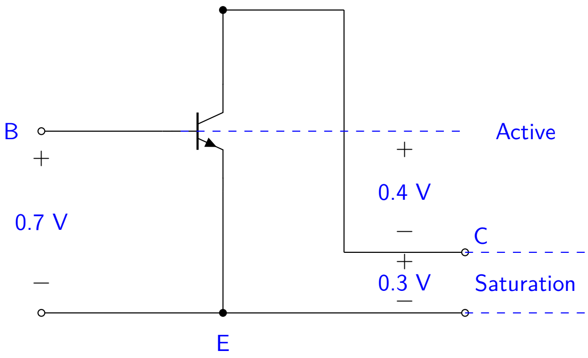
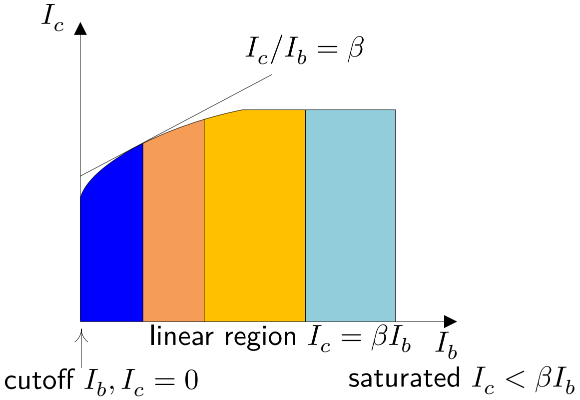


- Forward Active
- $V_{CB} \geq -0.4V$
- $V_{CE} \geq 0.3V$

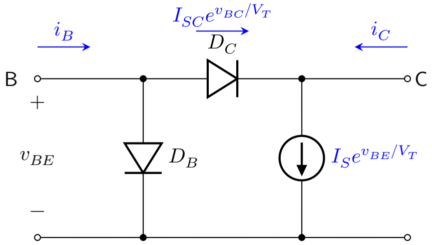
Otherwise get "Saturation"

Saturation



I_c and I_b for the Different Operating Modes of a BJT

Saturation



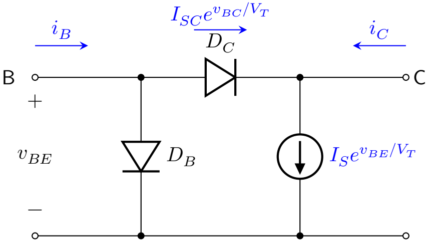
- BC diode starts conducting
- Less current for i_c (D_C is forward biased)
- Collect current falls

$$i_C = I_S e^{v_{BE}/V_T} - I_{SC} e^{V_{BC}/V_T} \quad (61)$$

- Base current rises

$$i_B = I_S/\beta e^{v_{BE}/V_T} + I_{SC} e^{v_{BC}/V_T} \quad (62)$$

Saturation



$$\beta_{forced} = \frac{i_C}{i_B}\Big|_{saturation} \leq \beta \tag{63}$$

$$= \beta_{\text{forced}} = \beta \frac{e^{V_{CE\,\text{sat}}/V_T} - I_{SC}/I_S}{e^{V_{CE\,\text{sat}}/V_T} + \beta I_{SC}/I_S} \tag{64}$$