VE311 Electronic Circuit Homework 5

Due: **June 19th**

Note:

1) *Please use A4 size paper or page.*
2) *Please clearly state out your final result for each question.*
3) *Please attach the screenshot of Pspice simulation result if necessary.*

Question 1. Common Gate (Easy)

In the common gate stage amplifier, the internal resistance of I is $1k\Omega$, what is the output resistance when $I_{REF} = 0.01mA$ and $0.1mA$ respectively? (Neglect body effect)

Parameter for NMOS: $V_{THN} = 0.7V$, $K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$

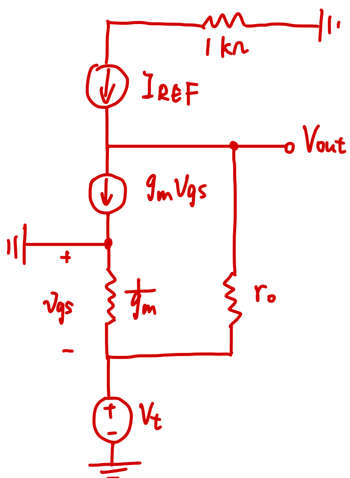
Parameter for PMOS: $V_{THP} = -0.7V$, $K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$

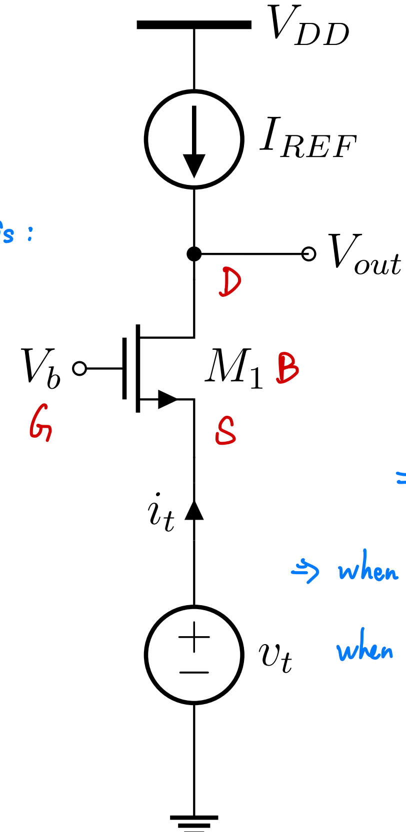
All the size of transistor is $W = 20\mu m, L = 1\mu m$

Transconductance $g_m = \sqrt{2K_n' \frac{W}{L} I_D}$

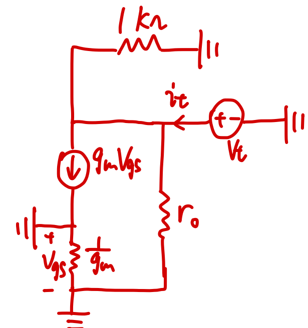
Output Resistance $r_o = \frac{1}{\lambda I_D}$

Then we do the small signal analysis:





To find the output resistance, we just need to analyze the following circuit:



$\Rightarrow R_{out} = 1k\Omega \parallel r_o$

$\Rightarrow$ when $I_D = 0.1mA$, $R_{out} = \frac{1}{\lambda \cdot 0.1\times10^{-3}} \parallel (1k) = 996.02\Omega$

when $I_D = 0.01mA$, $R_{out} = \frac{1}{\lambda \cdot 0.01\times10^{-3}} \parallel (1k) = 999.60\Omega$

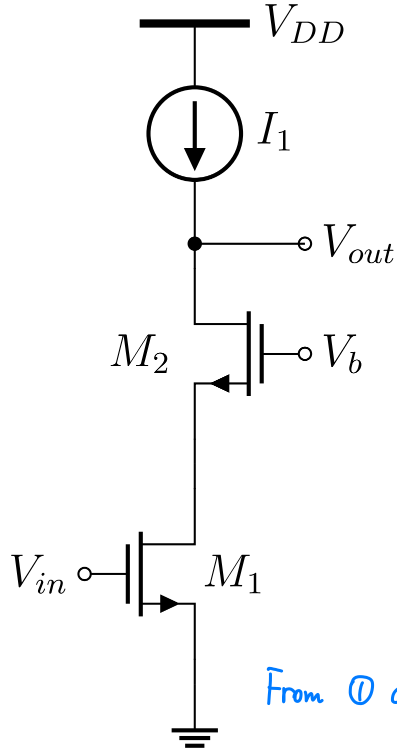
Question 2. Common Gate Common Source (Medium)

Find the intrinsic gain A_v and output impedance R_{out} for the amplifier when $I_1 = 0.01$ and $0.1mA$ respectively. (Neglect body effect)

Parameter for NMOS: $V_{THN} = 0.7V$, $K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$

Parameter for PMOS: $V_{THP} = -0.7V$, $K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$

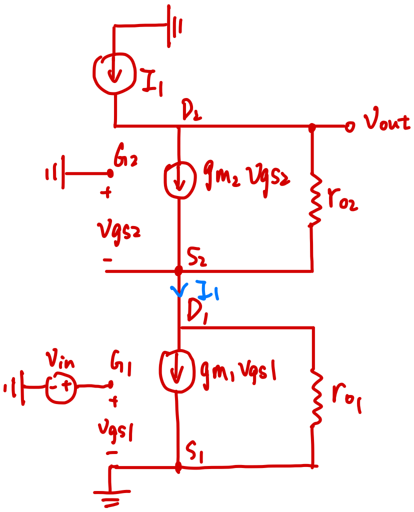
All the size of transistor is $W = 20\mu m, L = 1\mu m$



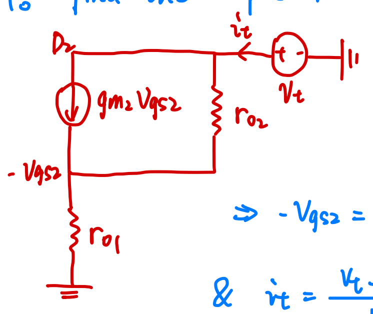
Tranductance $g_m = \sqrt{2k_n' \frac{W}{L} I_D}$

Output Resistance $r_o = \frac{1}{\lambda I_D}$

Small signal analysis :



To find the output resistance :



$\Rightarrow -V_{gs2} = i_t r_{o1}$

& $i_t = \frac{V_t + V_{gs2}}{r_{o2}} + g_{m2} V_{gs2}$

$\Rightarrow \frac{V_t}{i_t} = R_{out} = r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2}$ ①

To get A_v, we need to know $\frac{\partial V_{out}}{\partial V_{in}}$

we can use the AC circuit to obtain the relationship between V_{out} and V_{in}

$$\begin{cases} V_{out} = -V_{gs2} + r_{o2}(I_1 - g_{m2} V_{gs2}) \\ 0 = -V_{gs2} - (I_1 - g_{m1} V_{gs1}) \cdot r_{o1} \\ V_{gs1} = V_{in} \end{cases}$$

$$\Rightarrow v_{out} = V_{in} \cdot -(r_{o1} g_{m1} + r_{o2} r_{o1} g_{m1} g_{m2}) + (r_{o1} + r_{o2} + r_{o1} r_{o2} g_{m2}) I_1$$

$$\Rightarrow A_v = \frac{\partial V_{out}}{\partial V_{in}} = -r_{o1} g_{m1} - r_{o1} r_{o2} g_{m1} g_{m2} \quad ②$$

From ① and ②, we can know that

when $I_1 = 0.01mA$, $r_{o1} = r_{o2} = \frac{1}{\lambda I_1} = 2.5 \times 10^6 \Omega$

$g_{m1} = g_{m2} = \sqrt{2k_n' \frac{W}{L} I_1} = 2.098 \times 10^{-4}$ $1/\Omega$

$\Rightarrow R_{out} = 1.32 \times 10^9 \Omega$. $A_v = -2.76 \times 10^5$

when $I_1 = 0.1mA$, $r_{o1} = r_{o2} = 2.5 \times 10^5 \Omega$

$g_{m1} = g_{m2} = \sqrt{2k_n' \frac{W}{L} I_1} = 6.633 \times 10^{-4}$ $1/\Omega$

$\Rightarrow R_{out} = 4.196 \times 10^7 \Omega$, $A_v = -2.77 \times 10^4$

Question 3. Common Gate Common Source (Medium)

Find the intrinsic gain A_v and output impedance R_{out} for the amplifier when $I_1 = 0.01$ and $0.1mA$ respectively. (Neglect body effect)

Parameter for NMOS: $V_{THN} = 0.7V$, $K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$

Parameter for PMOS: $V_{THP} = -0.7V$, $K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$

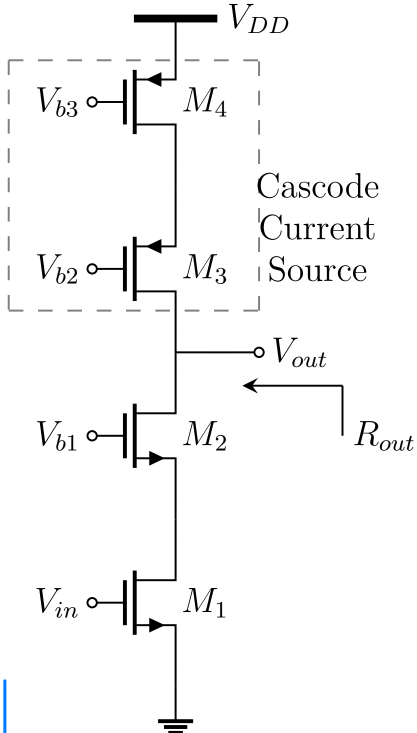
All the size of transistor is $W = 20\mu m, L = 1\mu m$

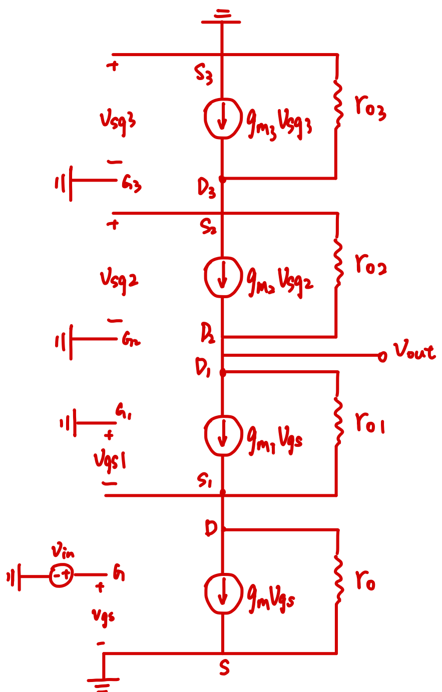
Tranductance $g_{mN} = \sqrt{2k_n' \frac{W}{L} I_D}$

$g_{mp} = \sqrt{2k_p' \frac{W}{L} I_D}$

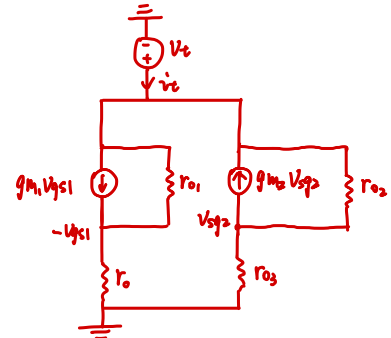
Output Resistance $r_o = \frac{1}{\lambda I_D}$

Small signal analysis :





To find the output resistance :



$$\Rightarrow \begin{cases} \frac{V_t + V_{gs1}}{r_{o1}} + g_{m1} V_{gs1} = \frac{-V_{gs1}}{r_o} \\ \frac{V_t - V_{sg2}}{r_{o2}} - g_{m2} V_{sg2} = \frac{V_{sg2}}{r_{o3}} \\ i_t = \frac{-V_{gs1}}{r_o} + \frac{V_{sg2}}{r_{o3}} \end{cases}$$

$$\Rightarrow R_{out} = \frac{V_t}{i_t} = \frac{1}{\frac{1}{r_o + r_{o1} + g_{m1} r_o r_{o1}} + \frac{1}{r_{o2} + r_{o3} + g_{m2} r_{o2} r_{o3}}} \quad ①$$

To get A_v, we need to know $\frac{\partial V_{out}}{\partial V_{in}}$, we can use the AC circuit to obtain the relationship between V_{out} and V_{in}

$$g_m V_{in} + \frac{-V_{gs1}}{r_o} = g_{m1} V_{gs1} + \frac{V_{out} + V_{gs1}}{r_{o1}} = g_{m2} V_{sg2} + \frac{V_{sg2} - V_{out}}{r_{o2}} = \frac{0 - V_{sg2}}{r_{o3}}$$

$$\Rightarrow A_v = \frac{\partial V_{out}}{\partial V_{in}} = \frac{g_m}{\left(\frac{1}{r_o} + \frac{1}{r_{o1}} + g_{m1}\right)\left(\frac{g_{m2} + \frac{1}{r_{o2}}}{(g_{m1} + \frac{1}{r_{o1}})(g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o3}}) r_{o2}} - \frac{\frac{1}{r_{o1}} + \frac{1}{r_{o2}}}{g_{m1} + \frac{1}{r_{o1}}}\right) + \frac{1}{r_{o1}}} \quad ②$$

From ① and ②, we can know that when $I_1 = 0.01mA$,

$r_{o2} = r_{o3} = 2 \times 10^6 \Omega$, $r_o = r_{o1} = 2.5 \times 10^6 \Omega$, $g_{m2} = g_{m3} = \sqrt{2} \times 10^{-4}\ 1/\Omega$, $g_{m1} = g_m = \sqrt{4.4} \times 10^{-4}\ 1/\Omega$

$\Rightarrow R_{out} = 3.976 \times 10^8 \Omega$, $A_v = -8.3 \times 10^4$

From ① and ②, we can know that when $I_1 = 0.1mA$,

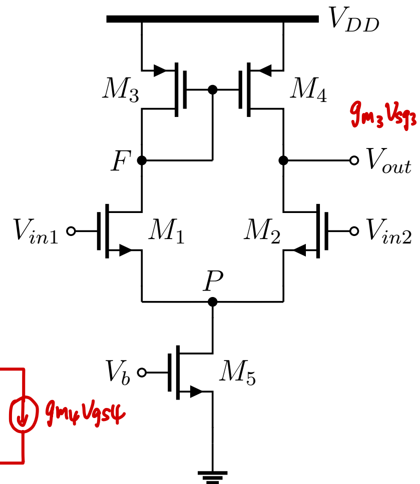
$r_{o2} = r_{o3} = 2 \times 10^5 \Omega$, $r_o = r_{o1} = 2.5 \times 10^5 \Omega$, $g_{m2} = g_{m3} = \sqrt{20} \times 10^{-4}\ 1/\Omega$, $g_{m1} = g_m = \sqrt{44} \times 10^{-4}\ 1/\Omega$

$\Rightarrow R_{out} = 1.273 \times 10^7 \Omega$, $A_v = -8.4 \times 10^3$

Question 4. Differential Pair (Hard)

Find the intrinsic gain A_v for the amplifier when $I_{SS} = 0.02$ and $0.2mA$ respectively. (Neglect body effect)

Parameter for NMOS: $V_{THN} = 0.7V$, $K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$
Parameter for PMOS: $V_{THP} = -0.7V$, $K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$
All the size of transistor is $W = 20\mu m, L = 1\mu m$

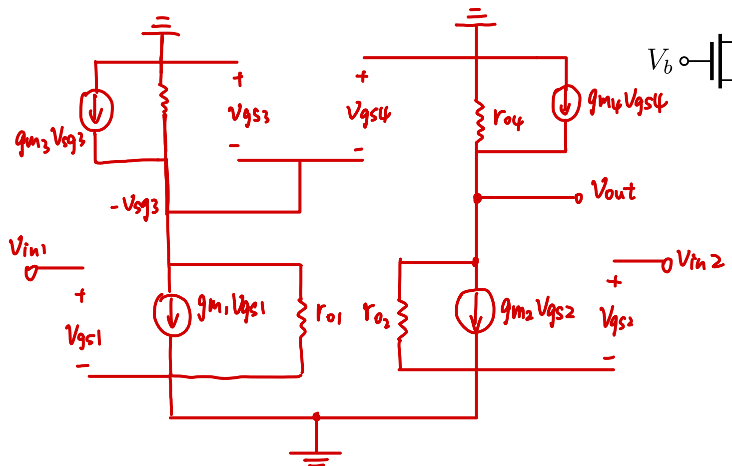


Tranductance $g_{mN} = \sqrt{2k_n' \frac{W}{L} I_D}$

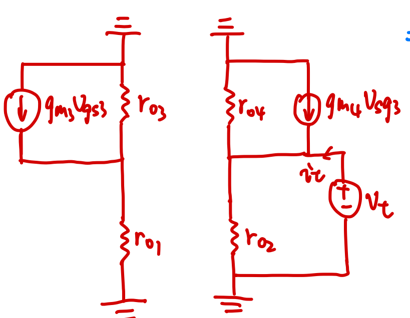
$g_{mp} = \sqrt{2k_p' \frac{W}{L} I_D}$

Output Resistance $r_o = \frac{1}{\lambda I_D}$

Small signal analysis :



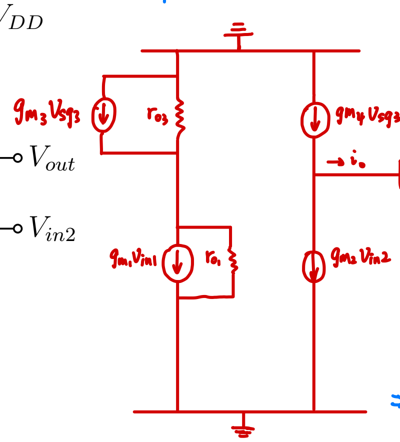
To find the output resistance :



$$\Rightarrow \begin{cases} g_{m3} V_{sg3} + \frac{V_{sg3}}{r_{o3}} = \frac{-V_{sg3}}{r_{o1}} \\ i_t + \frac{-V_t}{r_{o4}} + g_{m4} V_{sg} = \frac{V_t}{r_{o2}} \end{cases}$$

$$\Rightarrow R_{out} = \frac{V_t}{i_t} = \frac{r_{o2} r_{o4}}{r_{o2}+r_{o4}}$$

To find G_m :



$$\begin{cases} g_{m3} V_{sg3} + \frac{V_{sg3}}{r_{o3}} = g_{m1}V_{in1} + \frac{-V_{sg3}}{r_{o1}} \\ i_o = g_{m4}V_{sg3} - g_{m2}V_{in2} \end{cases}$$

$$\Rightarrow G_m = \frac{i_o}{V_{in1}-V_{in2}} = g_{m1}$$

$$\Rightarrow A_v = G_m R_{out} = g_{m1}\frac{r_{o2} r_{o4}}{r_{o2}+r_{o4}} \quad ①$$

From ①, we can know that when $I_{ss} = 0.02\ mA$

$\Rightarrow r_{o2} = 2.5\times10^6\ \Omega$, $r_{o4} = 2\times10^6\ \Omega$, $g_{m1} = \sqrt{44}\times10^{-4}\ 1/\Omega$

$\Rightarrow A_v = 233.07$

And when $I_{ss} = 0.2\ mA$

$\Rightarrow r_{o2} = 2.5\times10^5\ \Omega$, $r_{o4} = 2\times10^5\ \Omega$, $g_{m1} = \sqrt{44}\times10^{-4}\ 1/\Omega$

$\Rightarrow A_v = 73.70$

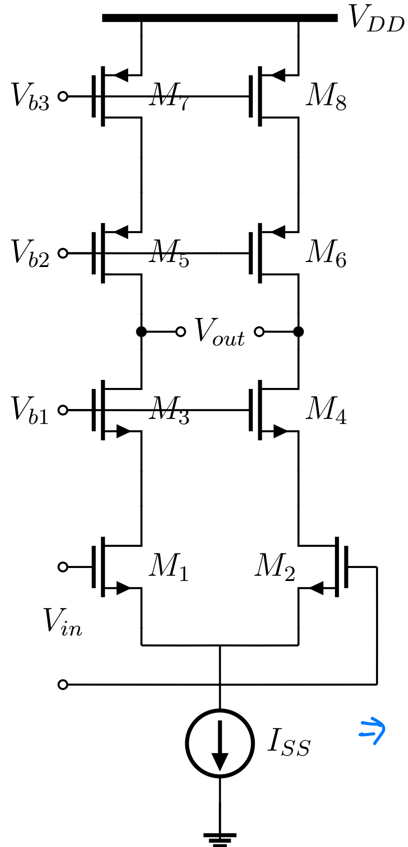
Question 5. Differential Pair (Hard)

Find the intrinsic gain A_v for the amplifier when $I_{SS} = 0.02$ and $0.2mA$ respectively. (Neglect body effect)

Parameter for NMOS: $V_{THN} = 0.7V$, $K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$

Parameter for PMOS: $V_{THP} = -0.7V$, $K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$

All the size of transistor is $W = 20\mu m, L = 1\mu m$

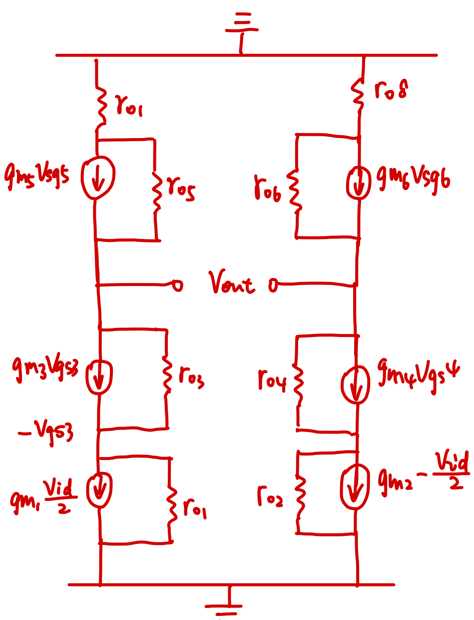


Tranductance $g_{mN} = \sqrt{2k_n' \frac{W}{L} I_D}$

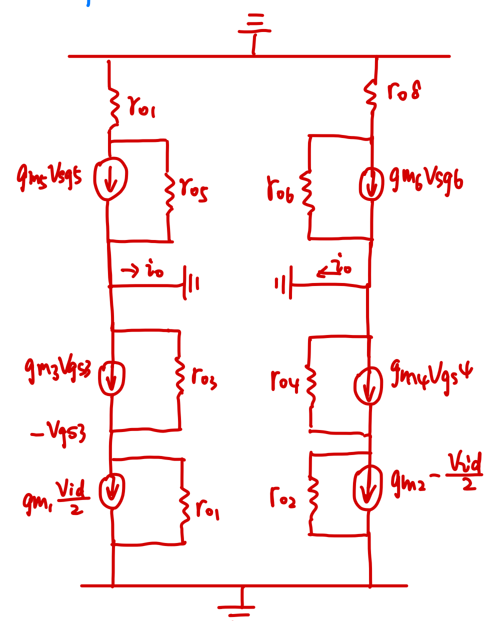
$g_{mp} = \sqrt{2k_p' \frac{W}{L} I_D}$

Output Resistance $r_o = \frac{1}{\lambda I_D}$

Small signal analysis :



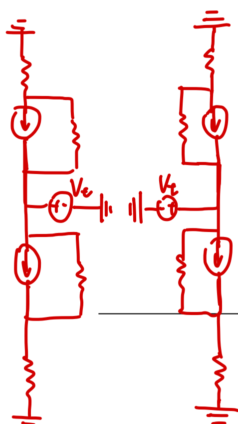
To find G_m :



$$\Rightarrow \begin{cases} \frac{-V_{sg5}}{r_o} = \frac{V_{sg5}}{r_o} + g_{mp} V_{sg5} \\ \frac{V_{gs3}}{r_o} + g_{mN} V_{gs3} = g_{mN}\frac{V_{id}}{2} + \frac{-V_{gs3}}{r_o} \end{cases}$$

$$\Rightarrow G_m = \frac{r_o g_{mN}^2}{-r_o g_{mN} - 1} = -g_{mN} \text{ (round to } -g_{mN}\text{)}$$

To find Rout :



$$\begin{cases} \frac{V_{sg5}}{r_o} = \frac{V_t - V_{sg5}}{r_o} - g_{mp} V_{sg5} \\ \frac{V_{gs3} + V_t}{r_o} + g_{mN} V_{gs3} = \frac{-V_{gs3}}{r_o} \\ i_t = \frac{V_{sg5}}{r_o} + \frac{-V_{gs3}}{r_o} \end{cases}$$

$$\Rightarrow R_{out} = \frac{(2+g_{mp}r_o)(2+g_{mN}r_o)r_o^2}{(2+g_{mp}r_o + 2 + g_{mN}r_o)\cdot r_o}$$

$$= \frac{(2+g_{mp}r_o)(2+g_{mN}r_o)r_o}{4+g_{mp}r_o+g_{mN}r_o}$$

$$\Rightarrow A_v = R_{out} \cdot G_m = -g_{mN} \cdot \frac{(2+g_{mp}r_o)(2+g_{mN}r_o)r_o}{4+g_{mp}r_o+g_{mN}r_o} \quad ①$$

From ①. we can know that when $I_{ss} = 0.02$ mA

$r_{o2} = 2.5\times10^6\Omega$, $r_{o4} = 2\times10^6\Omega$, $g_{mN} = \sqrt{44}\times10^{-4}$ $1/\Omega$. $g_{mp} = \sqrt{2}\times10^{-4}$ $1/\Omega$

$\Rightarrow A_v = -8.34\times10^4$

And when $I_{ss} = 0.2$mA

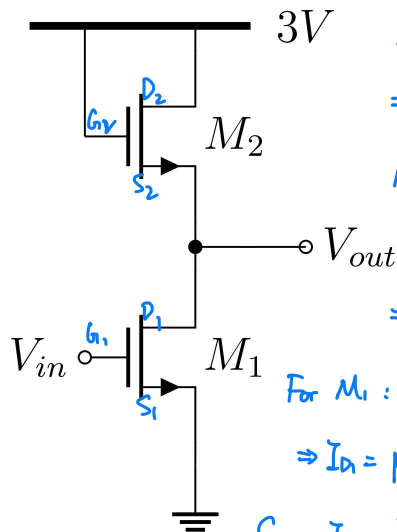
$r_{o2} = 2.5\times10^5\Omega$, $r_{o4} = 2\times10^5\Omega$. $g_{mN} = \sqrt{44}\times10^{-4}$ $1/\Omega$. $g_{mp} = \sqrt{20}\times10^{-6}$ $1/\Omega$

$\Rightarrow A_v = -8.449\times10^3$

Question 6. Common Source with Diode-connected Load (Lunatic)

Consider the circuit of Fig. 2 with $(W/L)_1 = 50/0.5$ and $(W/L)_2 = 10/0.5$. Assume that $\lambda = \gamma = 0$. Then 3 V for V_{DD}, 0.7 V for V_{TH1}, 0.45 $\text{V}^{\frac{1}{2}}$ for γ, and 0.9 V for $2\Phi_F$

1) At what input voltage is M_1 at the edge of the triode region? What is the small-signal gain under this condition?
2) What input voltage drives M_1 into the triode region by 50mV ? What is the small-signal gain under this condition?



1) Since M_1 is at the edge of the triode region :

$V_{DS_1} = V_{GS_1} - V_{TH_1}$

$V_{out} = V_{in} - 0.7$

Then we analyze M_2 :

Since $V_{GS_2} = V_{DS_2} \Rightarrow V_{GS_2} - V_{TH_2} < V_{DS_2}$

$\Rightarrow M_2$ at saturation region

$\Rightarrow I_{D_1} = \frac{1}{2}\mu_n C_{ox} (\frac{W}{L})_1 (V_{in} - V_{TH_1})^2$

$= I_{D_2} = \frac{1}{2}\mu_n C_{ox} (\frac{W}{L})_2 (V_{DD} - V_{in} + V_{TH_2} - V_{TH_2})^2$

$\Rightarrow 5(V_{in} - 0.7)^2 = (3 - V_{in})^2 \Rightarrow V_{in} = 1.41V$

Meanwhile $5(V_{in} - 0.7)^2 = (2.3 - V_{out})^2 \Rightarrow V_{out} = -\sqrt{5} V_{in} + C$

$\Rightarrow A_v = \frac{\partial V_{out}}{\partial V_{in}} = -\sqrt{5}$ (C is constant)

2) 50mV into triode

$\Rightarrow V_{out} = \underbrace{1.41 - 0.7}_{\text{from 1)}} - 0.05 = 0.66V$

Now for M_2 : $V_{GS} = V_{DD} - V_{out} = 2.34V > V_{TH}$

$\Rightarrow$ saturation region

$\Rightarrow I_{D_2} = \frac{1}{2}\mu_n C_{ox} (\frac{W}{L})_2 (V_{GS} - V_{TH})^2$

For M_1 : triode region

$\Rightarrow I_{D_1} = \mu_n C_{ox} (\frac{W}{L})_1 [(V_{in} - V_{TH}) \cdot V_{out} - \frac{1}{2} V_{out}^2]$

Since $I_{D_1} = I_{D_2} \Rightarrow \frac{1}{2}(2.34 - 0.7)^2 = 5[(V_{in} - 0.7) \cdot 0.66 - \frac{1}{2} \cdot 0.66^2]$

Then we can get that $V_{in} = 1.4375V$

Meanwhile . $\frac{1}{2}(3 - V_{out} - 0.7)^2 = 5[(V_{in} - 0.7) \cdot V_{out} - \frac{1}{2} V_{out}^2]$

$\Rightarrow V_{in} = \frac{(2.3 - V_{out})^2}{10 V_{out}} + \frac{1}{2} V_{out}$

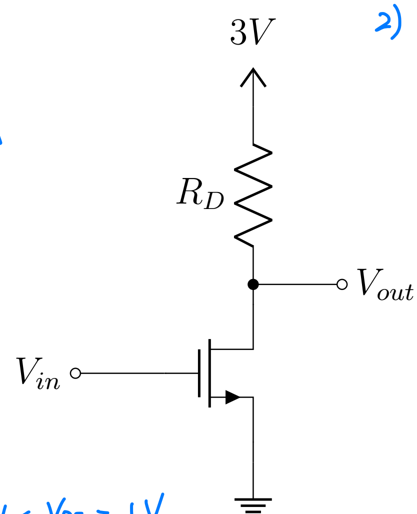
$\Rightarrow A_v = \frac{\partial V_{out}}{\partial V_{in}} = -1.93$

Question 7. Common Source with Resistive Load (Hell)

Assume $V_{TH} = 0.7\text{ V}$, $K_n' = 110\ \mu A/V^2$

Suppose the common-source stage of Fig. 1 is to provide an output swing from 1 V to 2.5 V. Assume that $(W/L)_1 = 50/0.5$, $R_D = 2\text{k}\Omega$, and $\lambda = 0$.

1) Calculate the input voltages that yield $V_{\text{out}} = 1$ V and $V_{\text{out}} = 2.5$ V.
2) Calculate the drain current and the transconductance of M_1 for both cases.
3) How much does the small-signal gain, gm R_D, vary as the output goes from 1 V to 2.5 V ? (Variation of small-signal gain can be viewed as nonlinearity.)



1)

① $V_{out} = 1\text{V} \Rightarrow I_D = \frac{V_{DD} - V_{out}}{R_D} = 1\text{ mA}$

Assume MOSFET at saturation:

$I_D = \frac{1}{2} K_n' \frac{W}{L} (V_{GS} - V_{TH})^2$

$= \frac{1}{2} K_n' \frac{W}{L} (V_{in} - 0.7)^2 = 10^{-3}$

$\Rightarrow V_{in} = 1.126\text{V} \Rightarrow V_{GS} - V_{TH} = 0.426\text{V} < V_{DS} = 1\text{V}$

$\Rightarrow$ indeed at saturation $\Rightarrow$ assumption stands

② $V_{out} = 2.5\text{V} \Rightarrow I_D = \frac{V_{DD} - V_{out}}{R_D} = 0.25\text{ mA}$

Assume MOSFET at saturation:

$I_D = \frac{1}{2} K_n' \frac{W}{L} (V_{GS} - V_{TH})^2 = \frac{1}{2} K_n' \frac{W}{L} (V_{in} - 0.7)^2 = 0.25 \times 10^{-3}$

$\Rightarrow V_{in} = 0.9132\text{V} \Rightarrow V_{GS} - V_{TH} = 0.2132\text{V} < V_{DS} = 2.5\text{V}$

$\Rightarrow$ indeed at saturation $\Rightarrow$ assumption stands

2) $g_m = \sqrt{2 K_n' \frac{W}{L} I_D}$

① $V_{out} = 1\text{V}$, $I_D = 10^{-3}\text{A}$

$\Rightarrow g_m = \sqrt{2 \times 110 \times 10^{-6} \times 100 \times 10^{-3}}$

$= 4.69 \times 10^{-3}\ 1/\Omega$

② $V_{out} = 2.5\text{V}$, $I_D = 2.5 \times 10^{-4}\text{ A}$

$\Rightarrow g_m = \sqrt{2 \times 110 \times 10^{-6} \times 100 \times 2.5 \times 10^{-4}}$

$= 2.34 \times 10^{-3}\ 1/\Omega$

3) $R_{out} = r_o // R_D = R_D$ (since $\lambda = 0$)

$A_v = -g_m R_{out}$

① $V_{out} = 1\text{V}$, $I_D = 10^{-3}\text{A}$, $g_m = 4.69 \times 10^{-3}\ 1/\Omega$

$\Rightarrow A_v = -9.38$

② $V_{out} = 2.5\text{V}$, $I_D = 2.5 \times 10^{-4}\text{A}$, $g_m = 2.34 \times 10^{-3}\ 1/\Omega$

$\Rightarrow A_v = -4.68$