Lecture 21

EECS 311 Analog Circuits

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Recap of Last Lecture

- Review of 216

Topic to be covered

- Frequency Response, Bode plot Amplitude and Phase Estimation

Representation of Poles and Zeros

The angle of the transfer function is the sum of the angles of the vectors associated with the zeros minus the sum of the angles of the vectors associated with the poles

Representation of Poles and Zeros

- A second-order system has a pair of complex conjugate poles at $s = -2 \pm j3$ and a single zero at the origin of the s-plane. Find the transfer function and use the pole-zero plot to evaluate the transfer function at $s = 0 + j5$.

$$H(s) = K\frac{s}{(s-(-2+j3))(s-(-2-j3))} = K\frac{s}{s^2+4s+13} \tag{1}$$

Representation of Poles and Zeros

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$$|H(s)| = K\frac{\sqrt{(0-5)^2}}{\sqrt{(0-(-2))^2+(5-3)^2}\sqrt{(0-(-2))^2+(5-(-3))^2}} = K\frac{5}{4\sqrt{34}} \tag{2}$$

$$\angle H(s) = \tan^{-1}(5/0) - \tan^{-1}(2/2) - \tan^{-1}(8/2) = -31^\circ \tag{3}$$

Frequency Responses

- y substituting $j\omega$ for s

$$H(j\omega) = K\frac{(j\omega - z_1)(j\omega - z_2)\ldots(j\omega - z_{m-1})(j\omega - z_m)}{(j\omega - p_1)(j\omega - p_2)\ldots(j\omega - p_{n-1})(j\omega - p_n)} \tag{4}$$

$$|j\omega - p_i| = \sqrt{\sigma_i^2 + (\omega - \omega_i)^2} \tag{5}$$

$$\angle(s - p_i) = \tan^{-1}\left(\frac{\omega - \omega_i}{-\sigma_i}\right) \tag{6}$$

Frequency Responses Example

- Sketch of Amp/phase from Complex plane

Frequency Responses Example

- Sketch of Amp/phase from Complex plane

Constructing Magnitude Bode Plot

- Each pole or zero contributes a change in the slope of the asymptotic plot of ± 20 dB/decade above its break frequency. A complex conjugate pole or zero pair gives a total change in the slope of ± 40 dB/decade.

Constructing Phase Bode Plot

- For lefthand plane
- Pole
  - Slope changes by -20 dB/decade
  - Phase decreases by 90°
- Zero
  - Slope changes by 20 dB/decade
  - Phase increases by 90°

Constructing Magnitude Bode Plot

$$H(s) = \frac{1}{s+1} \Rightarrow H(j\omega) = \frac{1}{j\omega + 1} \tag{7}$$

$$\angle H(j\omega) = -\tan^{-1}(\omega) \tag{8}$$

$$\angle H(j\omega) = \begin{cases} 0^\circ & \omega \ll 1 \\ -90^\circ & \omega \gg 1 \end{cases} \tag{9}$$

- The behavior for a zero is similar. The phase increases by 90° and passes though the midpoint of 45° at the break point

Constructing Phase Bode Plot

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We can extend the results for simple repeated poles and zeroes as before using the more general function

$$H(s) = (s+a)^{\pm r} \Rightarrow H(j\omega) = (j\omega + a)^{\pm r} \tag{10}$$

$$\angle H(j\omega) = \begin{cases} 0^\circ & \omega \ll a \\ \pm r 90^\circ & \omega \gg a \end{cases} \quad \angle H(ja) = \pm r 45^\circ \tag{11}$$

Unstable (right half plane) poles and zeros have opposite behavior

Zero @ Origin

$$H(s) = s \tag{12}$$

$$|H(j\omega)| = \omega \Rightarrow \omega = 1 \Rightarrow \log|H(j\omega)| = 0dB \tag{13}$$

- Slope 20 dB/decade

Pole @ Origin

$$H(s) = \frac{1}{s} \text{ and } s = j\omega \tag{14}$$

$$\angle H(j\omega) = -90^\circ \text{ Phase always at } -90^\circ \tag{15}$$

$$|H(j\omega)| = \frac{1}{\omega} \Rightarrow \omega = 1 \Rightarrow \log(1) = 0dB \tag{16}$$

- Slope -20 dB/decade

Single Zero @-σ

Single Pole @-σ

Poles

$$A_v = \frac{1}{1 + sRC} \tag{17}$$

$$\omega_p = -\frac{1}{RC} \tag{18}$$

What happens at the poles?

$$A_v\left(w_p\right) = \frac{1}{1 + j\omega_p CR} \tag{19}$$

$$A_v\left(w_p\right) = \frac{1}{1 + j\frac{1}{CR}CR} = \frac{1}{1 + j} \tag{20}$$

$$|A_v\left(w_p\right)| = |\frac{1}{1 + j}| = \frac{1}{\sqrt{2}} \tag{21}$$

Poles

- Gains drops by 3dB at ωp
- This is why we care about 3dB bandwidth, since we can easily estimate that using the location of the lowest pole.

Bode Plot Example

$$H(s) = \frac{s}{\left(1 + \frac{s}{10}\right)\left(1 + \frac{s}{10^4}\right)} \tag{22}$$

Zero @ 0
Zero @ -10
Zero @ -10^4

Bode Plot Example

$$H(s) = \frac{1000}{s+100} = \frac{10}{1+\frac{s}{100}} \tag{23}$$

$$DC \text{ gain } = 10 = 20\log 10 = 20 \text{ dB} \tag{24}$$

Pole at -100.

Finite Op-amp Bandwidth

Remember for a RC circuit

$$V_{out} = \frac{1}{1 + j\omega CR} V_{in} \tag{25}$$

$$\omega_p = -\frac{1}{RC} \Rightarrow \tag{26}$$

$$Av(\omega) = \frac{1}{1 + j\omega/\omega_p} \tag{27}$$

- Ideal : Gain is Infinite
- Gain is finite, bandwidth infinite
- Gain is finite, bandwidth finite
- Assume single pole amplifier
- A0 is the DC gain,

$$A(j\omega) = \frac{Ao}{1 + j\omega/\omega p} \tag{28}$$

Finite Op-amp Bandwidth

$$A(j\omega) = \frac{A_0}{1 + j\omega/\omega p} \tag{29}$$

Typical op-amps have every low 3dB bandwidth

Unity Gain Frequency

ω_t is ω where $|A(j\omega)| = 1$

$$A(j\omega) = \frac{A_0}{1 + j\omega/\omega b} \quad (30)$$

$$1 = \frac{A_0}{\omega_{T/\omega_b}} \Rightarrow \omega_T = A_0\omega_b \quad (31)$$

$$\omega_T = A_0\omega_b \quad (32)$$

Gain bandwidth product (GBW) important parameter

Non-Inverting Amplifier

$$v_n = v_O \frac{R_1}{R_1 + R_2} = v_O \beta \tag{33}$$

$$\beta = \frac{R_1}{R_1 + R_2} \tag{34}$$

$$V_0(\omega) = A(\omega) V_{Id} = A(\omega)\left(V_I - V_n\right) \tag{35}$$

$$= A(\omega)\left(V_I - \beta V_0\right) \tag{36}$$

$$= \frac{A_0}{1 + \omega/\omega_b}\left(V_I - \beta V_0\right) \tag{37}$$

Non-Inverting Amplifier

We pick $\omega_p = -\omega_b$ that frequency looks positive.

$$v_O = \frac{A_0}{1 - \frac{s}{\omega_p}} (v_I - \beta v_O) \tag{38}$$

$$v_O \left(1 - \frac{s}{\omega_p}\right) + A_0 \beta v_O = A_0 v_I \tag{39}$$

$$\frac{V_0}{V_I} = \frac{A_0}{1 + \beta A_0 - s/\omega_p} = \frac{A_0}{1 + \beta A_0} \frac{1}{1 - \frac{S}{\omega_p (1+\beta A_0)}} \tag{40}$$

This tells me the bandwidth of a non-inverting amplifier is $\omega_p (1 + \beta A_0)$