Final RC - part3

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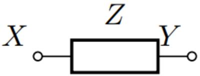
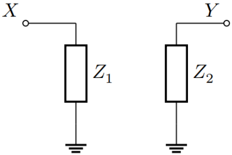
Overview

Miller Effect



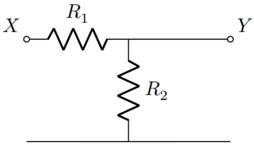


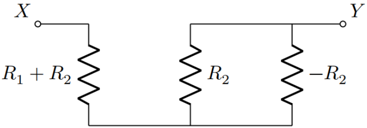
$$A_v = \frac{Y}{X} \tag{1}$$

$$Z_1 = \frac{Z}{1 - A_v} \tag{2}$$

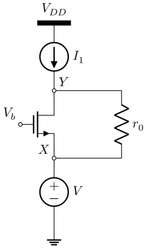
$$Z_2 = \frac{Z}{1 - \frac{1}{A_v}} \tag{3}$$

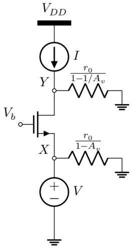
If the impedance Z forms the **ONLY** signal path between X and Y, then Miller conversion is often invalid.





Example 1





$$A_v = \frac{Y}{X} = 1 + \left(g_{m1} + g_{mb1}\right) r_{o1} \tag{4}$$

$$Z_1 = \frac{r_{01}}{1 - \left[1 + \left(g_{m1} + g_{mb1}\right) r_{01}\right]} = \frac{-1}{g_{m1} + g_{mb1}} \tag{5}$$

$$R_{in} = \frac{-1}{g_{m1} + g_{mb1}} \parallel \frac{1}{g_{m1} + g_{mb1}} = \infty \tag{6}$$

Example 1

R_{in} from source in NMOS:

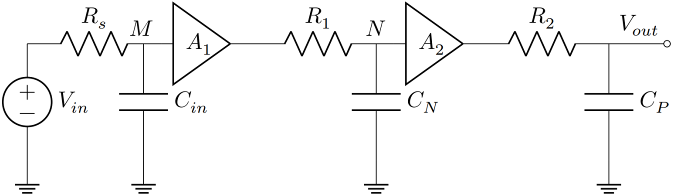
$\lambda = 0$:

$$R_{in} = \frac{1}{g_{m1} + g_{mb1}} \tag{7}$$

$\lambda \neq 0$:

Apply Miller Effect

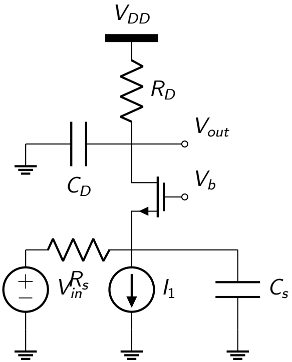
Association of Poles with Nodes



$$\frac{v_{out}}{v_{in}}(\omega) = A_1 \; A_2 \frac{1}{\left(1+\frac{s}{\omega_M}\right)\left(1+\frac{s}{\omega_N}\right)\left(1+\frac{s}{\omega_P}\right)} \tag{8}$$

$$\omega_M = \frac{1}{R_S C_{in}} \quad (9) \qquad \omega_N = \frac{1}{R_1 C_N} \quad (10) \qquad \omega_P = \frac{1}{R_2 C_P} \; (rad/s) \quad (11)$$

Example 2



$$\frac{v_{out}}{v_{in}} = \frac{(g_{m1}+g_{mb1})R_D}{1+(g_{m1}+g_{mb1})R_s}\frac{1}{\left(1+\frac{s}{\omega_X}\right)\left(1+\frac{s}{\omega_Y}\right)} \tag{12}$$

$$\omega_X = \frac{1}{\left(\frac{1}{g_{m1}+g_{mb1}} \parallel R_S\right)C_s}(rad/s) \tag{13}$$

$$\omega_Y = \frac{1}{R_D C_D} \quad (rad/s) \tag{14}$$

Assume the op amp to be ideal except for having a finite differential gain A and $V_{sig} = 1V$. Use Miller's theorem to find R_{in}, V_i, V_o for each of the following values of A : 10, 100, 1000 (**without using knowledge of op-amp circuit analysis**)

$Z_1 = \frac{Z}{1+A}$

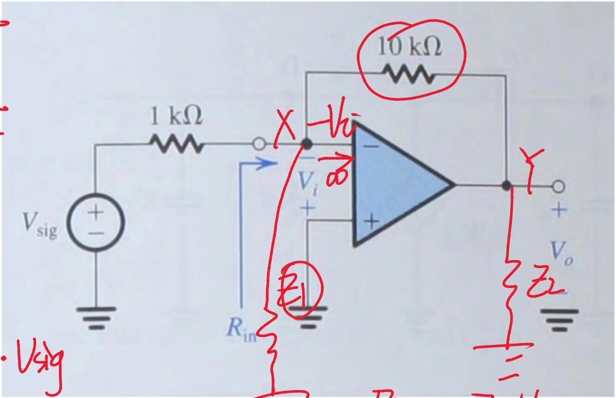
$Z_2 = \frac{Z}{1+\frac{1}{A}}$

$Z = 10k\Omega$

$A_v = -A$

$-V_i = \frac{Z_1}{1k+Z_1} \cdot V_{sig}$

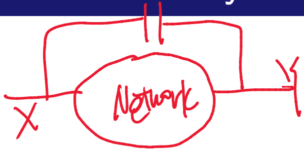
$V_o = AV_i$



$R_{in} = Z_1 \| \infty = Z_1$

Overview

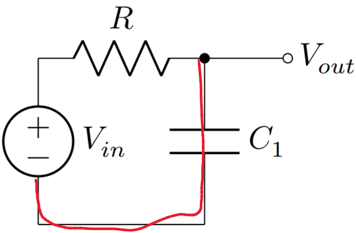
First Order Systems



$$H(s) = \frac{H^0 + H^1 \tau S}{1 + \tau S} \tag{15}$$

- Time constant: (1) For capacitor: $\tau = RC$; (2) For inductor: $\tau = \frac{L}{R}$
- To find the time constant, remove the cap/ind nulling all the sources and find the resistance.
- To find H^0, use low frequency gain(cap cut off and ind shorted).
- To find H^1, use high frequency gain(cap shorted and ind cut off).

Example 1



$$H^0 = 1 \quad \text{cut off } C_1 \tag{16}$$

$$\tau = RC_1 \tag{17}$$

$$H^1 = 0 \quad \text{short } C_1 \tag{18}$$

$$H(s) = \frac{1}{1 + RCS} \tag{19}$$

$$H(s) = \frac{H^0 + H^1 \tau s}{1 + \tau s}$$

$$= \frac{1 + 0}{1 + RCs}$$

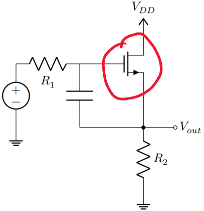
Example 2

$V_{gs} = V_t$

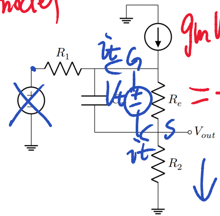
$R_2 (g_m V_t - i_t) + V_t = R_1 \cdot i_t$

$\frac{V_t}{i_t} = R$

$\tau = R C_\pi$



t-model



$g_m V_{gs}$

$= \frac{1}{g_m}$

$\downarrow g_m V_{gs}$

$\downarrow g_m V_{gs} - i_t$

$V_x = V_{in}$

$$H^0 = \frac{R_2}{R_2 + R_e} \quad \text{cut off } C_\pi \tag{20}$$

$$H^1 = \frac{R_2}{R_2 + R_1} \quad \text{short } C_\pi \tag{21}$$

$$R = \frac{R_1 + R_2}{1 + g_m R_2} \tag{22}$$

Example 2

$$H(s) = \frac{R_2}{R_2 + R_e} \cdot \frac{1 + \frac{R_e + R_2}{R_1 + R_2} \cdot \frac{R_1 + R_2}{1 + g_m R_2} C_\pi S}{1 + R_\pi^0 C_\pi S} \tag{23}$$

$$= \frac{R_2}{R_2 + R_e} \cdot \frac{1 + R_e C_\pi S}{1 + R_\pi^0 C_\pi S} \tag{24}$$

Exercise 2

For the circuit shown below

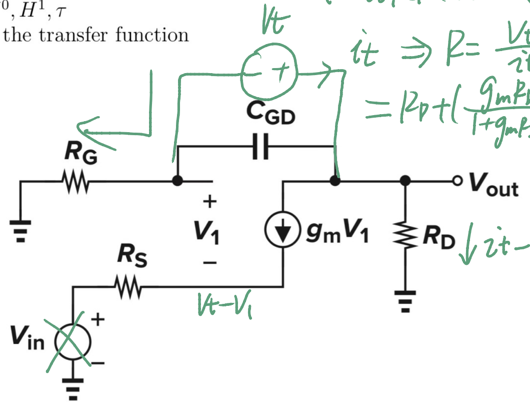
(a) Calculate H^0, H^1, τ

(b) Write down the transfer function

① H^0: cut off C_{GD}

$$\begin{cases} V_{in} + R_S \cdot g_m V_1 = - \\ V_{out} = -R_D g_m V_1 \end{cases}$$

$$\Rightarrow H^0 = \frac{V_{out}}{V_{in}} = \frac{g_m R_D}{R_S g_m + 1}$$



③ τ: $\begin{cases} g_m V_1 R_S = -V_1 + V_t \\ -i_t R_G + V_t = (i_t - g_m V_1) R_D \end{cases}$

$$i_t \Rightarrow R = \frac{V_t}{i_t} = R_D + \left(\frac{g_m R_D}{1 + g_m R_S} + 1\right) R_G$$

$$\tau = R C_{GD}$$

② H^1: short C_{GD}

$$\begin{cases} V_{in} + R_S g_m V_1 = V_{out} - V_1 \\ \frac{V_{out}}{R_G} + \frac{V_{out}}{R_D} + g_m V_1 = 0 \end{cases} \Rightarrow H^1 = \frac{1}{1 + \left(R_S + \frac{1}{g_m}\right)\left(\frac{1}{R_D} + \frac{1}{R_G}\right)}$$

Overview

Nth Order Systems

$$H(s) = \frac{a_0 + a_1 s + a_2 s^2}{1 + b_1 s + b_2 s^2} \quad (25)$$

$$b_1 = \sum_{i=1}^{N} \tau_i^0 \quad (26)$$

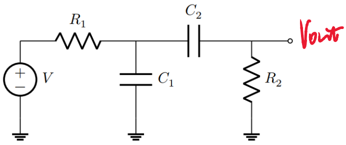
$$a_1 = \sum_{i=1}^{N} \tau_i^0 H^i \quad (27)$$

$$b_2 = \sum_{i}^{i<j<N} \sum_{j}^{0} \tau_j^i \quad (28)$$

$$a_2 = \sum \sum_{j}^{i<j<N} \tau_i^0 \tau_j^i H^{ij} \quad (29)$$

- τ_j^i: the time constant of capacitor j when capacitor i is shorted
- τ_i^0: cut off all other caps

Example



C2's time constant when C1 is shorted

- $\boxed{\tau_2^1} = R_2 C_2$
- $\tau_1^2 = (R_1 \parallel R_2)\, C_1$
- $\tau_1^0 \tau_2^1 = R_1 C_1 R_2 C_2 = \tau_2^0 \tau_1^2$
- $\tau_2^0 \tau_1^2 = (R_1 + R_2)\, C_2 \frac{R_1 R_2}{R_1 + R_2} C_1$
- $\tau_2^0 = (R_1 + R_2)\, C_2$
- $H^1 = 0$ C1 shorted
- $H^2 = \frac{R_2}{R_2 + R_1}$
- $H^{12} = 0$ C1, C2 shorted

Example

- $H(s) = \frac{a_0 + a_1 s + a_2 s^2}{1 + b_1 s + b_2 s^2}$
- $b_2 = \tau_1^0 \tau_2^1 = \tau_2^0 \tau_1^2$
- $a_2 = \tau_1^0 \tau_2^1 H^{12} = \tau_2^0 \tau_1^2 H^{12}$
- $b_1 = \tau_1^0 + \tau_2^0$
- $a_1 = \tau_1^0 H^1 + \tau_2^0 H^2$

$$H(s) = \frac{R_2 C_2 S}{1 + (R_1 C_1 + R_1 C_2 + R_2 C_2) S + R_1 C_1 R_2 C_2 S^2} \tag{30}$$

Tips

If you couldn't understand first order systems and n-th order systems clearly, just treat capacitor as $\frac{1}{sC}$ and inductor as sL, then use KCL and KVL!

Good luck in final exam!