VE311
Electronic Circuits
RC Final

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3 questions

1. Short response
   1. Concept   L13   L16 - 24
   2) Half-circuit
   3) Miller
2. Current Mirror   (Hard)

   ~~Single - Stage~~
3. L19 - 24

1. MOSFET Single Stage Amplifier
   - Common Source Amplifier
   - Common Drain Amplifier (Source Follower)
   - Common Gate Amplifier
   - Cascode Amplifier

Ravazi C3

2. MOSFET Differential Pair Amplifier

C4

3. Current Mirror
   - Current Mirror

C5

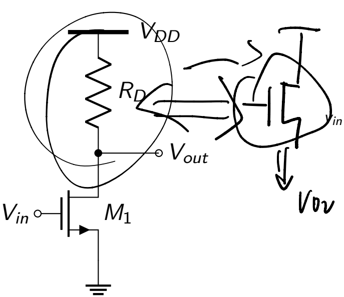
Frequency Response C6

1. MOSFET Single Stage Amplifier
   - Common Source Amplifier
   - Common Drain Amplifier (Source Follower)
   - Common Gate Amplifier
   - Cascode Amplifier

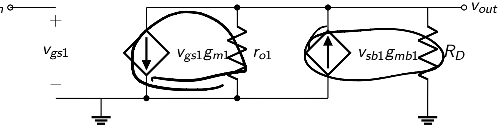
2. MOSFET Differential Pair Amplifier

3. Current Mirror
   - Current Mirror

CS with Resistive Load



Small-singal



$\downarrow g_{mb}v_{bs}$

If no channel-length and body effect:

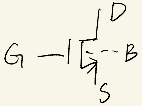
$$A_v = \frac{v_{out}}{v_{in}} = -g_{m1}R_D \tag{1}$$

No body effect:

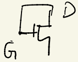
$$A_v = -g_{m1}(R_D \parallel r_{o1}) \tag{2}$$

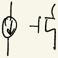
$$A_v = -g_{m1}(r_{o1} \parallel r_{o2})$$

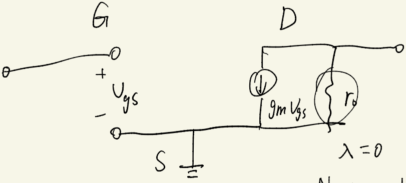
- Common Source
  - Drain (Source Follower)
  - Gate
- Cascode



Resistive Load 

☆ Diode-connected Load 

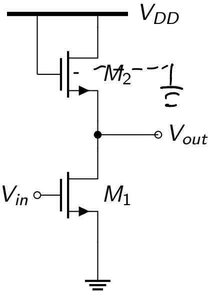
☆ Current Source Load 



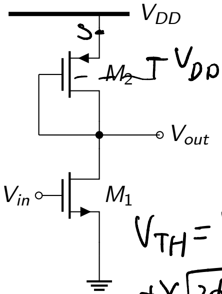
$\lambda = 0$

Neglect channel length ---

CS with Diode-connected Load



$V_{SB} \neq 0$



$\underline{V_{SB} = 0}$

$\frac{1}{1+\eta}$

$V_{TH} = V_{TH0} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right)$

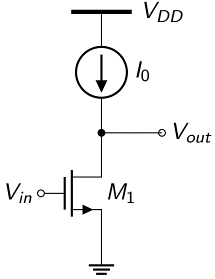
NMOS:

$$A_v = -\sqrt{\frac{(W/L)_1}{(W/L)_2}\frac{1}{1+\eta}} \qquad \eta = g_{mb2}/g_{m2} \tag{3}$$

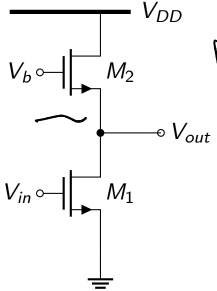
PMOS:

$$A_v = -\sqrt{\frac{\mu_n (W/L)_1}{\mu_p (W/L)_2}} \tag{4}$$

CS with Current Source Load



or



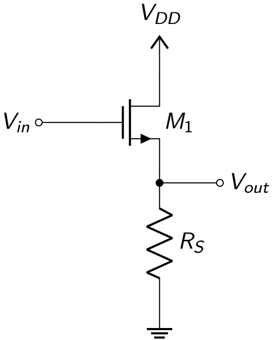
V_{Bias}

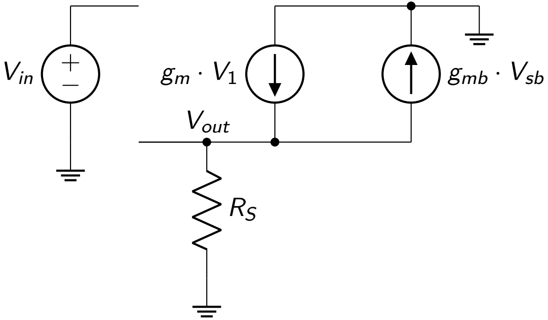
$$\frac{1}{g_{M2}}$$

$$A_v = -g_{m1}(\cancel{r_{o2}} \parallel r_{o1}) \qquad (5)$$

1. MOSFET Single Stage Amplifier
   - Common Source Amplifier
   - Common Drain Amplifier (Source Follower)
   - Common Gate Amplifier
   - Cascode Amplifier
2. MOSFET Differential Pair Amplifier
3. Current Mirror
   - Current Mirror

Source Follower



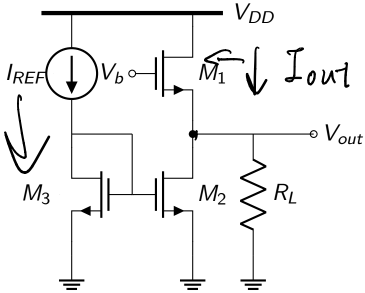


$$A_v = \frac{g_m R_S}{1 + g_m R_S(1+\eta)} = \frac{g_m R_S}{1 + (g_m + g_{mb})R_S} \approx \frac{1}{1+\eta} \qquad (6)$$

Example 1 I

In the source follower with current source load, the current source is ideal. Find the ~~output impedance for~~ the amplifier when $I_0 = 0.01$ and $0.1mA$ respectively. (Neglect body effect)
Parameter for NMOS: $V_{THN} = 0.7V$,
$K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$
Parameter for PMOS: $V_{THP} = -0.7V$,
$K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$
All the size of transistor is $W = 20\mu m, L = 1\mu m$

Example 1



Example 1

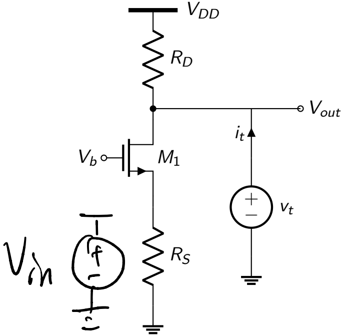
$$g_{m1} = \sqrt{2K_n \frac{W}{L} I_D} = 663.3\ \mu A/V$$

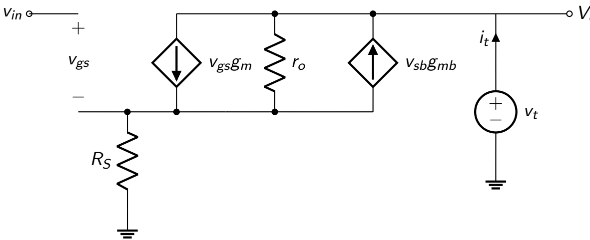
$$r_{o1} = r_{o2} = \frac{1}{\lambda I_D} = 250\ k\Omega$$

$$r_{out} = \frac{1}{g_{m1} + \underbrace{g_{mb1}}_{0} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$$

$$r_{out} \approx \frac{1}{g_{m1}}$$

Common Gate





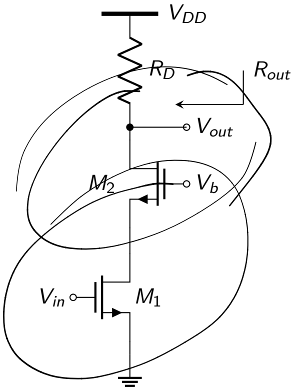
$$\boxed{R_{in}} = \frac{R_D + r_o}{1 + (g_m + g_{mb})r_o} \begin{cases} \text{If } R_D = 0 & R_{in} = r_o \parallel \frac{1}{g_m} \parallel \frac{1}{g_{mb}} \\ \text{If } R_D = \infty & R_{in} = \infty \end{cases} \quad (7)$$

$$R_{out} = [R_S + r_{o1} + (g_{m1} + g_{mb1})r_{o1}R_S] \parallel R_D \quad (8)$$

Cascode

Common Gate + Common Source

级联

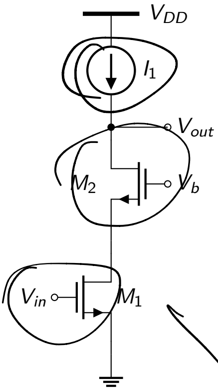


$$G_m = -g_{m1}\frac{r_{o1}}{r_{o1} + \left(r_{o2} \parallel \frac{1}{g_{m2}+g_{mb2}}\right)} \quad (9)$$

$$R_{out} = [r_{o1}+r_{o2}+(g_{m2}+g_{mb2})r_{o2}r_{o1}] \parallel R_D \quad (10)$$

Cascode

$A_v = G_m R_{out}$



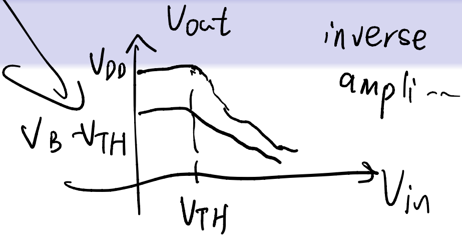
$$G_m = -g_{m1} \frac{r_{o1}}{r_{o1} + \left(r_{o2} \parallel \frac{1}{g_{m2}+g_{mb2}}\right)} \tag{11}$$

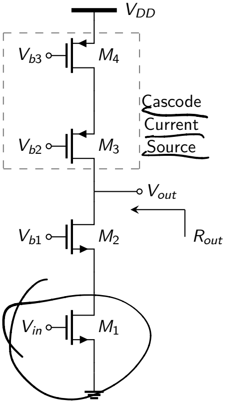
$$R_{out} = r_{o1} + r_{o2} + (g_{m2} + g_{mb2}) r_{o2} r_{o1} \tag{12}$$

$$A_v = G_m R_{out} \tag{13}$$

G_m g_m

Cascode

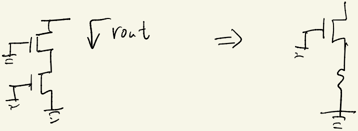




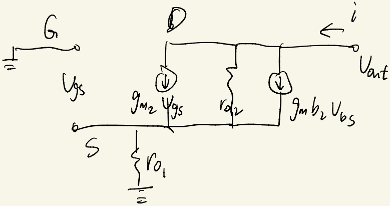
$$G_m = -g_{m1}\frac{r_{o1}}{r_{o1} + (r_{o2} \parallel \frac{1}{g_{m2}g_{mb2}})} \quad (14)$$

$$R_{out} = [r_{o1} + r_{o2} + (g_{m2} + g_{mb2})r_{o2}r_{o1}] \parallel [r_{o3} + r_{o4} + (g_{m3} + g_{mb3})r_{o3}r_{o4}] \quad (15)$$

$$A_v = G_m R_{out} \quad (16)$$



⇒



$$\Rightarrow r_{o2}\left[i-(g_{m2}+g_{mb2})V_2\right]+i\,r_{o1}=V_{out}$$

$$r_{out}=\frac{V_{out}}{i}=\left[1+(g_{m2}+g_{mb2})r_{o2}\right]r_{o1}+r_{o2}$$

Example 5

hw 5

In the cascode amplifier , what is the gain when $I_{REF} = 0.01mA$ and $0.1mA$ respectively? (Neglect body effect)
Parameter for NMOS: $V_{THN} = 0.7V$,
$K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$
Parameter for PMOS: $V_{THP} = -0.7V$,
$K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$
All the size of transistor is $W = 20\mu m, L = 1\mu m$

$g_{m1} = g_{m2}$

$r_{o1} = r_{o2}$

$A_v = -g_{m1}\left[(g_{m2} + g_{mb2})\, r_{o2} r_{o1}\right]$

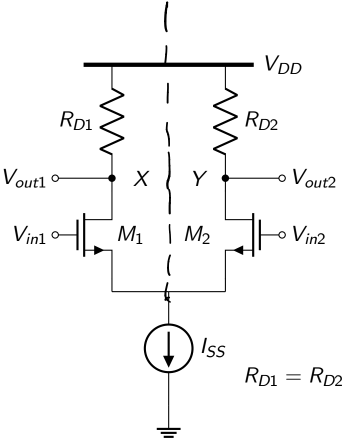
$= 27498$ ..

		gain	rout	rin	swing
CS	R	~	$R_o // r_{o1}$	∞	medium
	PMOS	~	$\approx \frac{1}{g_{m2}} // r_{o2} // r_{o1}$	∞	small
	I	~	$r_{o2} // r_{o1}$	∞	high
CD				∞	
CG				$\frac{r_{o1} + r_{o2}}{(g_{m1}+g_{mb1}) r_{o1} + 1}$	
CAS		$-g_{m1}[g_{m2} r_{o2} r_{o1} // g_{m1} r_{o3} r_{o4}]$	$[g_{m2} r_{o2} r_{o1} // g_{m1} r_{o3} r_{o4}]$	∞	

half-circuit

Differential Pair

A_{CM}



$R_{D1} = R_{D2} = R_D$

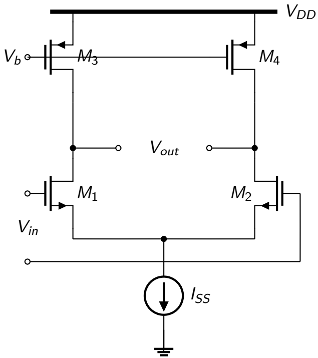
$$A_{DM} = \frac{V_{out1} - V_{out2}}{v_d} = -g_m(R_D \parallel r_o) \tag{17}$$

If the circuit is fully symmetric,

$$A_{CM-DM} = \frac{V_{out1} - V_{out2}}{v_{in,CM}} = 0 \tag{18}$$

$$CMRR = \left| \frac{A_{DM}}{\cancel{A_{CM-DM}}\ A_{CM}} \right| = \infty \tag{19}$$

Differential Pair with MOS Loads

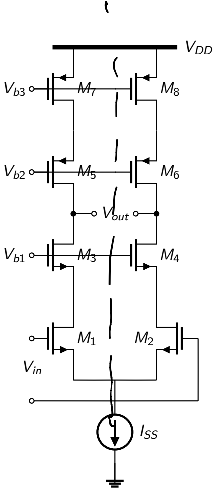


A_{DM} : Differential Response Gain

A_{CM} : Common Response

$$A_{DM} = -g_{m1,2}(r_{o1,2} \parallel r_{o3,4}) \qquad (20)$$

Differential Pair with Cascode Loads

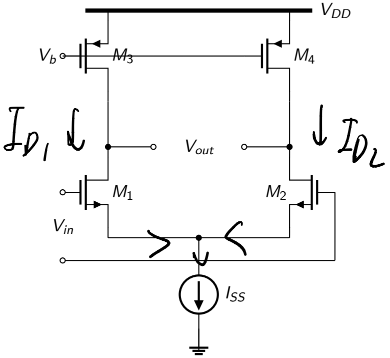


$$A_{DM} \cong -g_{m1,2}[(g_{m3,4} + gmb_{3,4})r_{o3,4}r_{o1,2} \parallel (g_{m5,6} + g_{mb5,6})r_{o5,6}r_{o7,8}] \quad (21)$$

Example 2

In the differential pair , what is the gain when $I_{SS} = 0.02mA$ and $0.2mA$ respectively? (Neglect body effect)
Parameter for NMOS: $V_{THN} = 0.7V$,
$K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$
Parameter for PMOS: $V_{THP} = -0.7V$,
$K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$
All the size of transistor is $W = 20\mu m, L = 1\mu m$

Example 2



KCL

Example 2

$$g_{m1,2} = \sqrt{2K_n W/L \, I_{D1,2}}$$

$r_{o1,2} = 250k\Omega$

$$I_{D1,2} = \frac{1}{2} I_{SS} = 0.1mA$$

$r_{o3,4} = 200k\Omega$

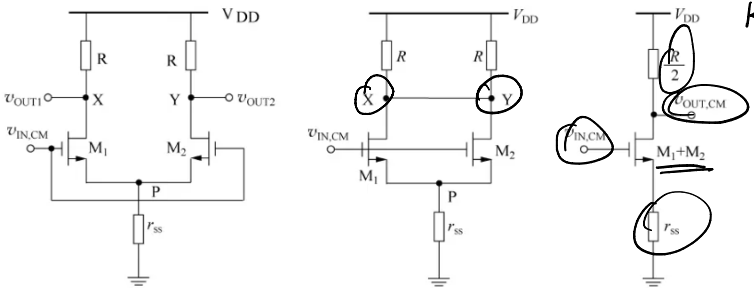
$$A_{vd} = \cdots$$

Common Mode Response

$V_X = V_Y$

Folded

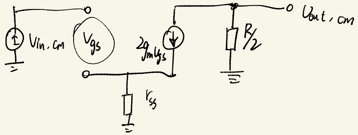
$R // R$



$$\left.\begin{aligned} v_{in,cm} &= v_{gs} + 2g_m v_{gs} \cdot r_{ss} \\ 2g_m v_{gs} + \frac{v_{out,cm}}{R/2} &= 0 \end{aligned}\right\} \text{KCL}$$

$$\frac{A_{vd}}{A_{vc}}$$

$$\mathrm{CMRR} = \frac{|A_{vd}|}{|A_{vc}|} = 1 + 2g_m r_{ss}$$

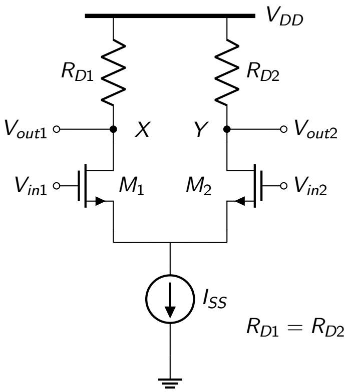


Vin,cm
Vgs
2gmVgs
rss
R/2
Vout,cm

Example 3

In the differential pair , what is the common gain and differential gain and CMRR when $I_{SS} = 0.2mA$? (Neglect body effect, $R = 10k\Omega, V_{DD} = 5V$)
Parameter for NMOS: $V_{THN} = 0.7V$,
$K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$
Parameter for PMOS: $V_{THP} = -0.7V$,
$K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$
All the size of transistor is $W = 20\mu m, L = 1\mu m$

Example 3



$R_{D1} = R_{D2} = R_D$

Example 3

$$|A_{vd}| = g_m R$$

$$|A_{vc}| = \frac{g_m R}{1 + 2 g_m r_{ss}}$$

$$CMRR = \frac{|A_{vd}|}{|A_{vc}|}$$

$$g_m = \sqrt{2 K_n W/L\, I_D}$$

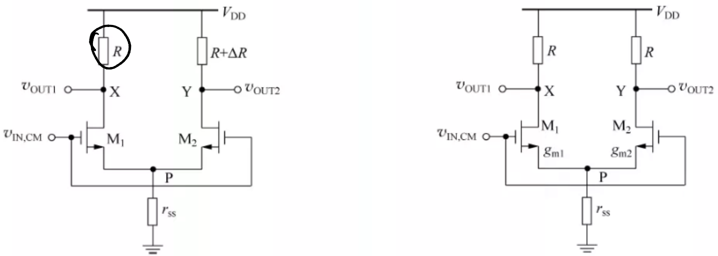
$$I_D = \frac{1}{2} I_{ss} = 0.1\ mA$$

$$r_{ss} = \frac{1}{\lambda I_{ss}}$$

$$= \frac{1}{0.04 \times 0.2 \times 10^{-3}}$$

$$= 125\ k\Omega$$

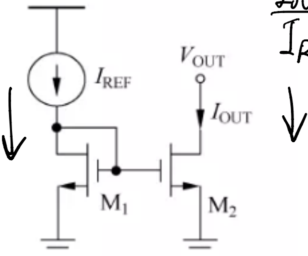
Mismatch 失配



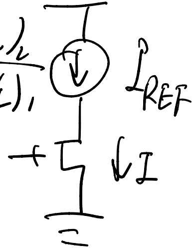
$$A_{\mathrm{cm-dm}} = -\left(\frac{g_{\mathrm{m}}\Delta R + \Delta g_{\mathrm{m}} R}{2g_{\mathrm{m}} r_{\mathrm{ss}} + 1}\right)$$

Current Mirror

Replicate



$$\frac{I_{OUT}}{I_{REF}} = \frac{(W/L)_2}{(W/L)_1}$$

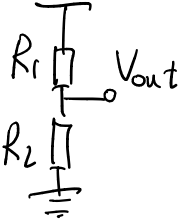


ideal I

$$I_{REF} = \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{THN1})^2$$

$$I_{OUT} = \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{GS} - V_{THN2})^2$$

$$I_{OUT} = \frac{(W/L)_2}{(W/L)_1} I_{REF}$$



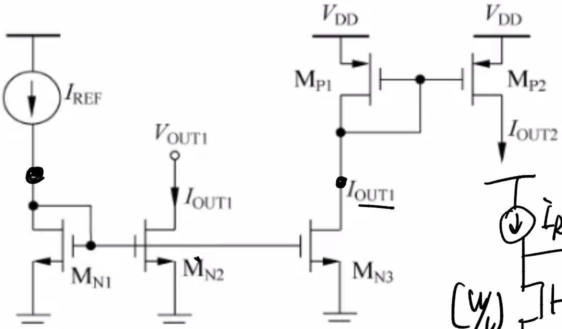
Resistor

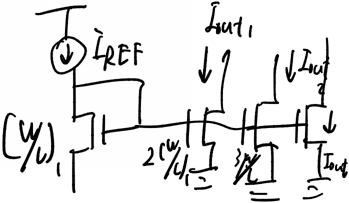
Temperature /

P S

Current Mirror

at most 4 MOSFET





$$I_{OUT1} = \frac{(W/L)_{N2}}{(W/L)_{N1}} I_{REF}$$

$$I_{OUT2} = \frac{(W/L)_{N3}}{(W/L)_{N1}} \frac{(W/L)_{P2}}{(W/L)_{P1}} I_{REF}$$

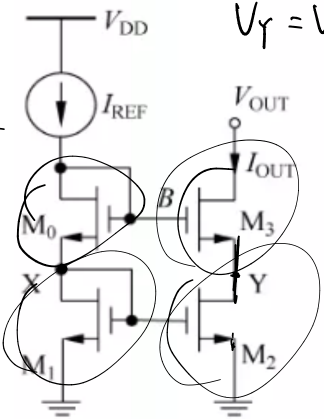
$I_{out1} = 2 I_{REF}$

$I_{out2} = 3 I_{REF}$

Cascode Current Mirror

I_{out}

$V_{out} \geq V_{GS1} + (V_{GS3} - V_{TH})$



$V_Y = V_X$

$r_{out,cas} > r_{out,sim}$

Example 4

In the cascode current mirror , what is the range of output voltage when $I_{REF} = 0.1mA$? (Neglect body effect, channel-length effect)
Parameter for NMOS: $V_{THN} = 0.7V$,
$K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$
Parameter for PMOS: $V_{THP} = -0.7V$,
$K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$
All the size of transistor is $W = 20\mu m, L = 1\mu m$

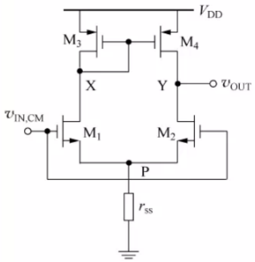
Example 4

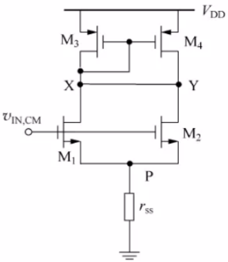
Differential Pair with Current Mirror Load

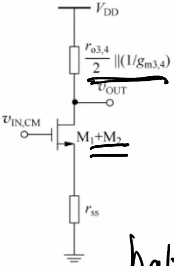
provide

$V_X = V_Y$

$\frac{1}{g_{m3,4}} // \frac{r_{o3,4}}{2}$







half-circuit

Differential Pair with Current Mirror Load

$$A_{\mathrm{vd}} = \frac{v_{\mathsf{out}}}{v_{\mathrm{d}}} = g_{\mathrm{m}} r_{\mathsf{out}} = g_{\mathrm{m}} \left(r_{\mathrm{o}2} \| r_{\mathrm{o}4} \right)$$

$$A_{\mathrm{vc}} \approx -\frac{\frac{1}{2g_{\mathrm{m}3,4}} \| \frac{r_{\mathrm{o}3,4}}{2}}{\frac{1}{2g_{\mathrm{m}1,2}} + r_{\mathrm{ss}}} \approx -\frac{1}{1 + 2g_{\mathrm{m}1,2} r_{\mathrm{ss}}} \frac{g_{\mathrm{m}1,2}}{g_{\mathrm{m}3,4}}$$

$$\mathrm{CMRR} = \frac{|A_{\mathrm{vd}}|}{|A_{\mathrm{vc}}|} = \left(1 + 2g_{\mathrm{m}1,2} r_{\mathrm{ss}}\right) g_{\mathrm{m}3,4} \left(r_{\mathrm{o}2} \| r_{\mathrm{o}4} \right)$$

END

Thanks