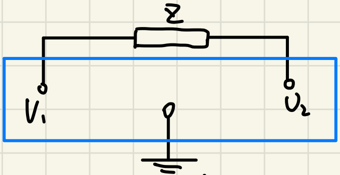
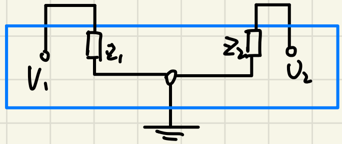
Miller Effect :

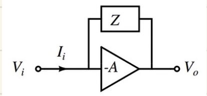


$\Leftrightarrow$



设已经 $A = \frac{V_2}{V_1}$

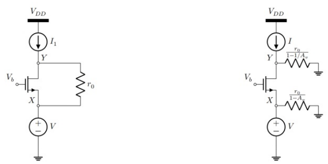
$Z_1 = \frac{Z}{1-A}$, $Z_2 = \frac{Z}{1-\frac{1}{A}}$



$$I_i = \frac{V_i - V_o}{Z} = V_i\left(\frac{1+A}{Z}\right)$$

$$Z_{in} = \frac{V_i}{I_i} = \frac{Z}{1+A}$$

Example 1

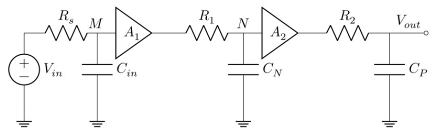


$$A_v = \frac{Y}{X} = 1 + (g_{m1} + g_{mb1})\, r_{o1} \tag{4}$$

$$Z_1 = \frac{r_{01}}{1 - [1 + (g_{m1} + g_{mb1})\, r_{01}]} = \frac{-1}{g_{m1} + g_{mb1}} \tag{5}$$

$$R_{in} = \frac{-1}{g_{m1} + g_{mb1}} \parallel \frac{1}{g_{m1} + g_{mb1}} = \infty \tag{6}$$

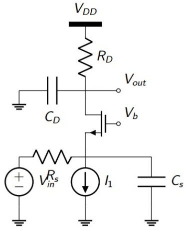
Association of Poles with Nodes



$$\frac{V_{out}}{V_{in}}(\omega) = A_1\, A_2 \frac{1}{\left(1 + \frac{s}{\omega_M}\right)\left(1 + \frac{s}{\omega_N}\right)\left(1 + \frac{s}{\omega_P}\right)} \tag{8}$$

$$\omega_M = \frac{1}{R_S C_{in}} \quad (9) \qquad \omega_N = \frac{1}{R_1 C_N} \quad (10) \qquad \omega_P = \frac{1}{R_2 C_P} \; (rad/s) \quad (11)$$

Example 2



$$\frac{V_{out}}{V_{in}} = \frac{(g_{m1} + g_{mb1})\, R_D}{1 + (g_{m1} + g_{mb1})\, R_s} \frac{1}{\left(1 + \frac{s}{\omega_X}\right)\left(1 + \frac{s}{\omega_Y}\right)} \tag{12}$$

$$\omega_X = \frac{1}{\left(\frac{1}{g_{m1}+g_{mb1}} \parallel R_S\right) C_s} (rad/s) \tag{13}$$

$$\omega_Y = \frac{1}{R_D C_D} \quad (rad/s) \tag{14}$$

Exercise 1

Assume the op amp to be ideal except for having a finite differential gain A and $V_{sig} = 1V$. Use Miller's theorem to find R_{in}, V_i, V_o for each of the following values of A : 10, 100, 1000 (**without using knowledge of op-amp circuit analysis**)

$Z_1 = \frac{Z}{1+A}$, $Z_2 = \frac{Z}{1+\frac{1}{A}}$

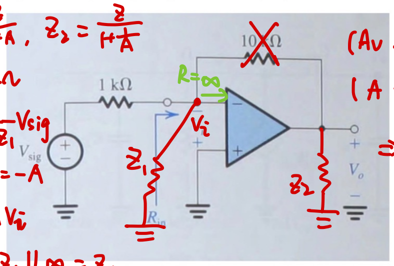
$Z = 10\ k\Omega$

$-V_i = \frac{Z_1}{10^3 + Z_1} V_{sig}$

since $A_v = -A$

$\Rightarrow V_o = A V_i$

$R_{in} = Z_1 \parallel \infty = Z_1$



$\left(A_v = \frac{-V_o}{V_i}\right)$

$\left(A = \frac{V_o}{V_i}\right)$

$\Rightarrow A_v = -A$

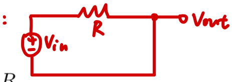
First Order Systems

$$H(s)=\frac{H^0+H^1\tau S}{1+\tau S} \tag{15}$$

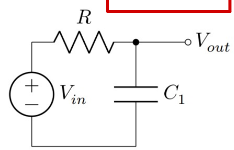
- Time constant: (1) For capacitor: $\tau=RC$; (2) For inductor: $\tau=\frac{L}{R}$
- To find the time constant, remove the cap/ind nulling all the sources and find the resistance.
- To find H^0, use low frequency gain(cap cut off and ind shorted).
- To find H^1, use high frequency gain(cap shorted and ind cut off).

Example 1

算 H_1:

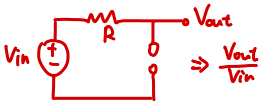


$\Rightarrow V_{out}=0 \Rightarrow \frac{V_{out}}{V_{in}}=0$



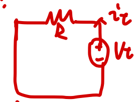
$$H^0=1 \tag{16}$$
$$\tau=RC_1 \tag{17}$$
$$H^1=0 \tag{18}$$
$$H(s)=\frac{1}{1+RCS} \tag{19}$$

算 H_0:



$\Rightarrow \frac{V_{out}}{V_{in}}=1$

算 τ:



$\Rightarrow \tau=\frac{V_t}{i_t}C_1=RC_1$

Exercise 2

For the circuit shown below
(a) Calculate H^0, H^1, τ
(b) Write down the transfer function

① H_0 (C_{GD} 断)

$V_{in}+R_S g_m V_1=V_{out}$

$V_{out}=-R_D g_m V_1$

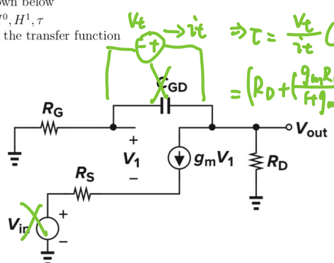
$\Rightarrow H^0=\frac{g_m R_D}{R_S g_m+1}$

② τ

$g_m V_1 R_S=V_t-V_1$

$-i_t R_G+V_t=(i_t-g_m V_1)R_D$

$\Rightarrow \tau=\frac{V_t}{i_t}C_{GD}=\left(R_D+\left(\frac{g_m R_D}{1+g_m R_S}+1\right)R_G\right)\cdot C_{GD}$



③ H^1 (C_{GD} 短)

$V_{in}+R_S g_m V_1=V_{out}-V_1$

$\frac{V_{out}}{R_G}+\frac{V_{out}}{R_D}+g_m V_1=0$

$\Rightarrow H'=\frac{1}{1+(R_S+\frac{1}{g_m})(\frac{1}{R_D}+\frac{1}{R_G})}$

Nth Order Systems

$$H(s)=\frac{a_0+a_1s+a_2s^2}{1+b_1s+b_2s^2} \tag{25}$$

$$b_1=\sum_{i=1}^{N}\tau_i^0 \tag{26}$$

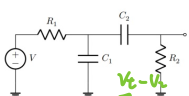
$$b_2=\sum_i^{i<j<N}\sum_j^{0}\tau_j^i \tag{28}$$

$$a_1=\sum_{i=1}^{N}\tau_i^0 H^i \tag{27}$$

$$a_2=\sum^{i<j<N}\sum_j \tau_i^0\tau_j^i H^{ij} \tag{29}$$

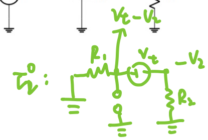
- τ_j^i: the time constant of capacitor j when capacitor i is shorted
- τ_i^0: cut off all other caps

Example



- $\tau_2^1=R_2C_2$
- $\tau_1^2=(R_1\parallel R_2)\,C_1$
- $\tau_1^0\tau_2^1=R_1C_1R_2C_2=\tau_2^0\tau_1^2$
- $\tau_2^0\tau_1^2=(R_1+R_2)\,C_2\frac{R_1R_2}{R_1+R_2}C_1$
- $\tau_2^0=(R_1+R_2)\,C_2$
- $H^1=0$
- $H^2=\frac{R_2}{R_2+R_1}$
- $H^{12}=0$

τ_2^0:



$\frac{V_t-V_2}{R_1}=\frac{V_2}{R_2}=i_t$

$V_t=(1+\frac{R_1}{R_2})V_2 \Rightarrow \frac{V_t}{i_t}=R_1+R_2$

H^1: 断C_2, 短C_1

H^2: 断C_1, 短C_2

H^{12}: 都断

Example

- $H(s)=\frac{a_0+a_1s+a_2s^2}{1+b_1s+b_2s^2}$
- $b_2=\tau_1^0\tau_2^1=\tau_2^0\tau_1^2$
- $a_2=\tau_1^0\tau_2^1H^{12}=\tau_2^0\tau_1^2H^{12}$
- $b_1=\tau_1^0+\tau_2^0$
- $a_1=\tau_1^0H^1+\tau_2^0H^2$

二阶的公式

$$H(s)=\frac{R_2C_2S}{1+(R_1C_1+R_1C_2+R_2C_2)\,S+R_1C_1R_2C_2S^2} \tag{30}$$