Lecture 12

VE 311 Analog Circuits

Xuyang Lu

2023 Summer

上海交通大學
SHANGHAI JIAO TONG UNIVERSITY

Recap of Last Lecture

- MOSFET Circuits

Topic to be covered

- MOSFET Circuits

Common-Source

$$I_D = \mu_n C_{ox} \frac{W}{L_{eff}}[(V_{GS} - V_{TH})V_{DS} - \frac{1}{2}{V_{DS}}^2] \quad (1)$$

$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L_{eff}}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS}) \quad (2)$$

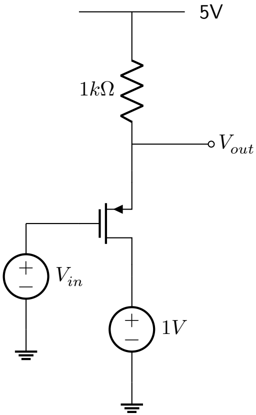
$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \frac{W}{L'}(V_{GS} - V_{TH}) = \sqrt{2\mu_n C_{ox} \frac{W}{L'} I_D} = \frac{2I_D}{V_{GS} - V_{TH}} \quad (3)$$

$$r_o = \frac{\partial V_{DS}}{\partial I_D} = 1 \Big/ \frac{\partial I_D}{\partial V_{DS}} \approx \frac{1}{I_D \cdot \lambda} \quad (4)$$

$$V_{TH} = V_{TH0} + \gamma(|\sqrt{2\Phi_F + V_{SB}|} - \sqrt{|2\Phi_F|}) \quad (5)$$

Common-Source

$\lambda \neq 0$, $\gamma \neq 0$



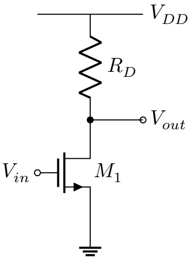
$$V_{in} = 1.8V + 0.001\sin(2\pi \cdot 100t)$$

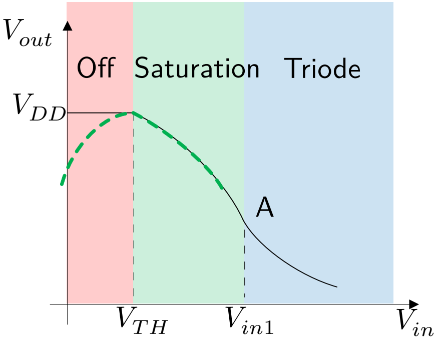
Common-Source with Resistive Load

Reading: Razavi Chapter 3

DC Analysis

$\lambda = 0$
$\gamma = 0$





- $V_{in} < V_{TH} \rightarrow M_1$ off

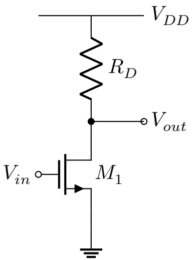
$V_{out} = V_{DD}$

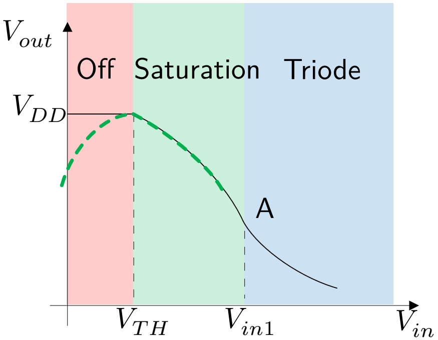
Common-Source with Resistive Load

DC Analysis

$\lambda = 0$

$\gamma = 0$





- $V_{in1} > V_{in} > V_{TH} \rightarrow M_1$ in Saturation

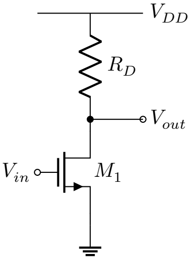
$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2$$

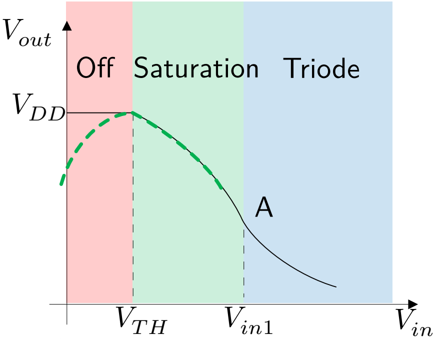
Common-Source with Resistive Load

DC Analysis

$\lambda = 0$

$\gamma = 0$





- $V_{in} > V_{in1} \rightarrow M_1$ in Triode

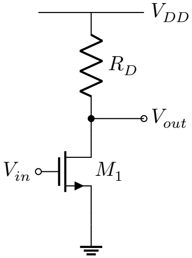
$$V_{out} = V_{DD} - R_D \mu_n C_{ox} \frac{W}{L}((V_{in} - V_{TH})V_{out} - \frac{1}{2}{V_{out}}^2)$$

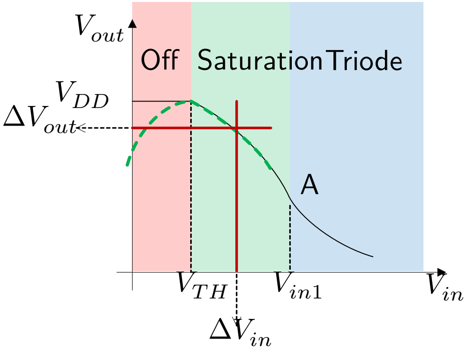
Common-Source with Resistive Load

DC Analysis

$\lambda = 0$

$\gamma = 0$





- $A_V = \frac{\partial V_{out}}{\partial V_{in}} = -R_D \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH}) = -g_m \cdot R_D$

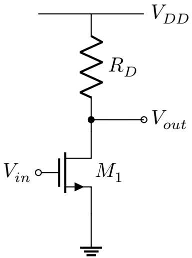
V_{gs} increases by $\partial V_{in} \rightarrow I_d$ increases by $\partial V_{in} \cdot g_m \rightarrow V_{out}$ decreases by $\partial V_{in} \cdot (g_m \cdot R_D)$

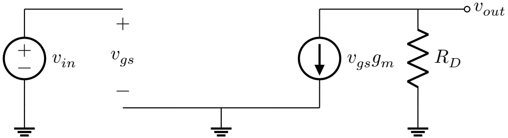
Common-Source with Resistive Load

Small-signal Analysis

$\lambda = 0$

$\gamma = 0$





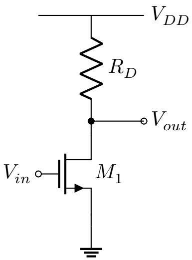
$$A_v = \frac{v_{out}}{v_{in}} = -g_m \cdot R_D \tag{6}$$

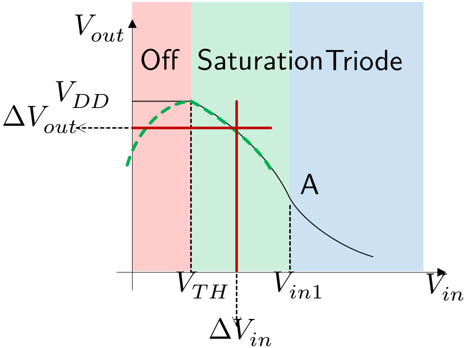
- Small-signal analysis leads to the same result as DC analysis.

Common-Source with Resistive Load

DC Analysis

$\lambda \neq 0$
$\gamma \neq 0$





$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = \frac{\partial \left[V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2(1 + \lambda V_{out})\right]}{\partial V_{in}} \tag{7}$$

Common-Source with Resistive Load

$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = \frac{\partial \left[V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2(1+\lambda V_{out})\right]}{\partial V_{in}}$$
$$= -R_D \mu_n C_{\mathsf{ox}} \frac{W}{L}(V_{in} - V_{TH})(1+\lambda V_{out}) - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2 \lambda \frac{\partial V_{out}}{\partial V_{in}} \tag{8}$$

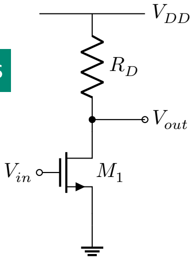
$$A_v = \frac{-g_m R_D}{1 + R_D I_D \lambda} = -g_m \frac{1}{\frac{1}{R_D} + \frac{1}{r_O}} = -g_m (R_D \parallel r_O) \tag{9}$$

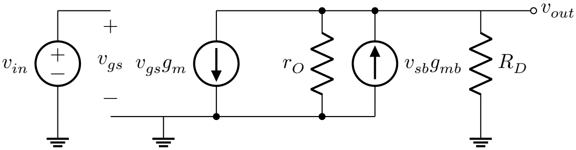
Common-Source with Resistive Load

Small-signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$



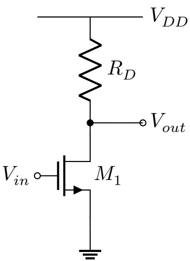


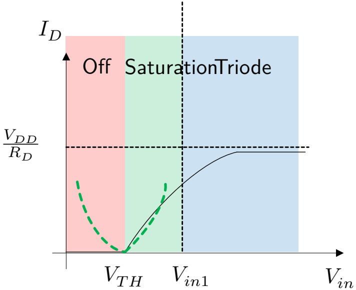
$$A_v = \frac{v_{out}}{v_{in}} = -g_m \cdot (R_D \parallel r_O) \tag{10}$$

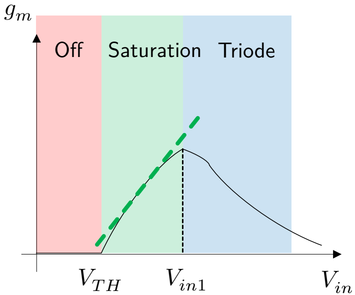
- Small-signal analysis leads to the same result as DC analysis.
- g_m is a function of V_{GS} and V_{DS}, while r_O is a function of I_D. $\rightarrow$ **Nonlinearity**

Example

Sketch the drain current and transconductance of M_1 as a function of input voltage. Assume $\lambda = \gamma = 0$.





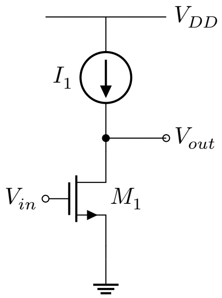


$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{in} - V_{TH}\right)^2$$

$$g_m = \mu_n C_{ox}\frac{W}{L}\left(V_{in} - V_{TH}\right)$$

Example

Assuming M_1 in saturation, calculate its small-signal gain.



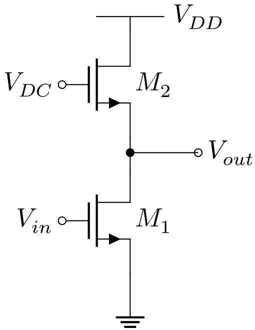
- Small-signal Analysis:

$$A_V = \frac{V_{out}}{V_{in}} = -gm_1 r_{o1} \tag{11}$$

- DC Analysis:

$$I_1 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2(1 + \lambda V_{DS}) \tag{12}$$

Example



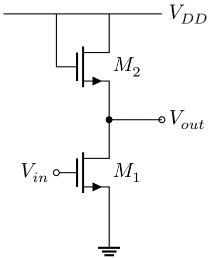
How to choose $V_{in,DC}$?

Example

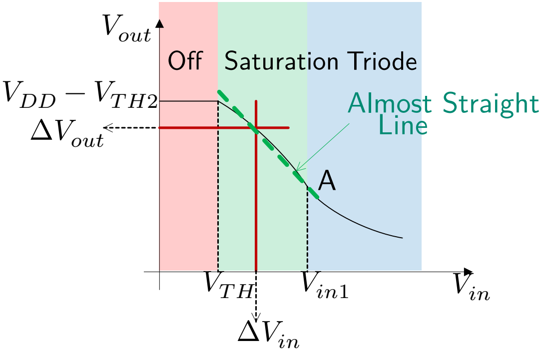
Diode-Connected Load

DC Analysis

$\lambda = 0$
$\gamma \neq 0$



$$V_{out} = V_{in1} - V_{TH1}$$



$$\frac{1}{2}\mu_n C_{ox}(\frac{W}{L})_1(V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}(\frac{W}{L})_2[V_{DD} - (V_{in1} - V_{TH1}) - V_{TH2}]^2 \quad (13)$$

Diode-Connected Load

- $V_{in1} > V_{in} > V_{TH} \to M_1$ in Saturation

$$\frac{1}{2}\mu_n C_{ox}(\frac{W}{L})_1(V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}(\frac{W}{L})_2[V_{DD} - V_{out} - V_{TH2}]^2 \tag{14}$$

$$\sqrt{(\frac{W}{L})_1}(V_{in} - V_{TH1}) = \sqrt{(\frac{W}{L})_2}(V_{DD} - V_{out} - V_{TH2}) \tag{15}$$

$$\sqrt{(\frac{W}{L})_1} = \sqrt{(\frac{W}{L})_2}(-\frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH2}}{\partial V_{in}}) \tag{16}$$

Diode-Connected Load

DC Analysis $\lambda = 0$, $\gamma \neq 0$

$$\sqrt{(\frac{W}{L})_1} = \sqrt{(\frac{W}{L})_2}(-\frac{\partial V_{out}}{\partial V_{in}} - \boxed{\frac{\partial V_{TH2}}{\partial V_{out}}}\frac{\partial V_{out}}{\partial V_{in}})$$

$$= \eta = \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}$$

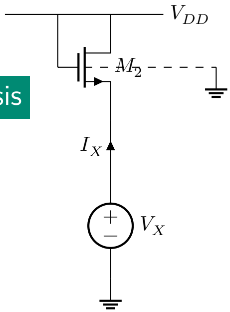
$$A_V = \frac{\partial V_{out}}{\partial V_{in}} = -\sqrt{\frac{(W/L)_1}{(W/L)_2}}\frac{1}{1+\eta} \tag{17}$$

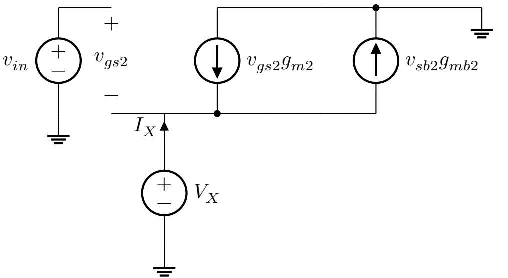
- η is a function of V_{SB}
- A_V is almost linear for M_1 in saturation

Diode-Connected Load

Small-signal Analysis

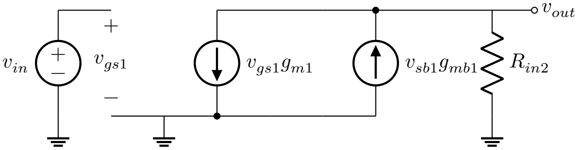
$\lambda = 0$
$\gamma \neq 0$





$$R_{in2} = \frac{V_x}{i_x} = \frac{1}{g_{m2} + g_{mb2}} \quad (18)$$

Diode-Connected Load



$$A_V = \frac{V_{out}}{V_{in}} = \frac{-g_{m1}}{g_{m2} + g_{mb2}} \tag{19}$$

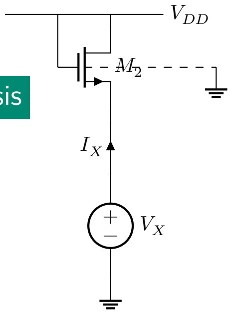
$$= -\sqrt{\frac{(W/L)_1}{(W/L)_2}} \frac{1}{1+\eta} \tag{20}$$

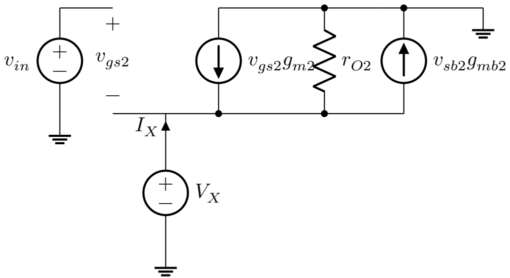
- Small-signal analysis leads to the same result as DC analysis.

Diode-Connected Load

Small-signal Analysis

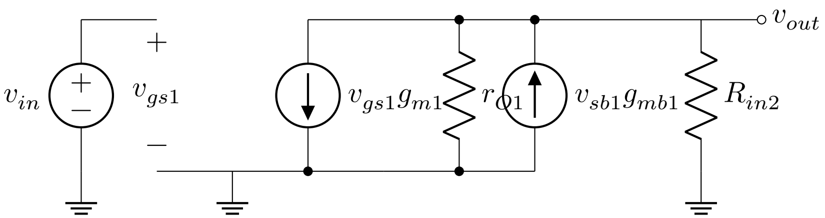
$\lambda \neq 0$
$\gamma \neq 0$





$$R_{in2} = \frac{V_x}{i_x} = \frac{1}{g_{m2}} \parallel \frac{1}{g_{mb2}} \parallel r_{o2} \tag{21}$$

Diode-Connected Load



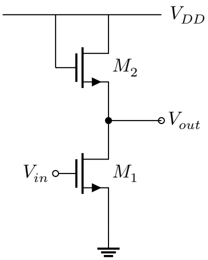
$$A_V = \frac{V_{out}}{V_{in}} = -g_{m1}\left(\frac{1}{g_{m2}} \parallel \frac{1}{g_{mb2}} \parallel r_{o2} \parallel r_{o1}\right) \tag{22}$$

Diode-Connected Load

M_2 has no body effect

Small-signal Analysis

$\lambda \neq 0$
$\gamma \neq 0$



$$
\begin{aligned}
A_V &= \frac{V_{out}}{V_{in}} \\
&= -g_{m1}\left(\frac{1}{g_{m2}} \parallel \underset{\uparrow}{\overset{r_o >> 1/gm}{}}\boxed{r_{o2} \parallel r_{o1}}\right) \approx -\frac{g_{m1}}{g_{m2}} \\
&= -\sqrt{\frac{\mu_n (W/L)_1}{\mu_p (W/L)_2}} \\
&= -\frac{V_{SG2} - V_{TH2}}{V_{GS1} - V_{TH1}}
\end{aligned}
\qquad (23)
$$

- For $A_V = 10$, $(\frac{W}{L})_1 >> (\frac{W}{L})_2 \rightarrow$ Disproportionally large transistor
- For $A_V = 10$, $(V_{SG2} - V_{TH2}) = 10(V_{GS1} - V_{TH1}) \rightarrow$ Limited output swing