Lecture 19

EECS 311 Analog Circuits

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Representation of Poles and Zeros

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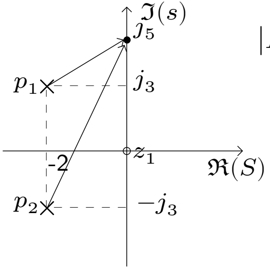


$$|H(s)| = K\frac{\sqrt{(0-5)^2}}{\sqrt{(0-(-2))^2+(5-3)^2}\sqrt{(0+2)^2+(5+3)^2}} = K\frac{5}{4\sqrt{34}} \quad (1)$$

$$\angle H(s) = \tan^{-1}(5/0) - \tan^{-1}(2/2) - \tan^{-1}(8/2) = -31^\circ \quad (2)$$

Frequency Response

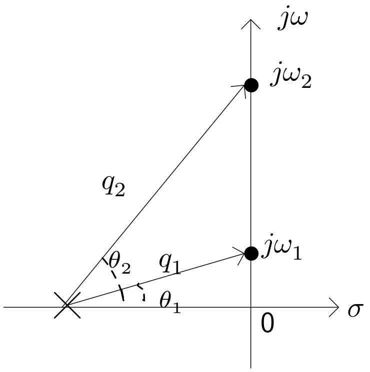
y substituting $j\omega$ for s

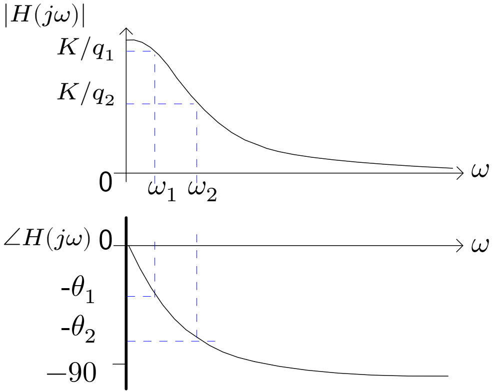
$$H(j\omega) = K\frac{(j\omega - z_1)(j\omega - z_2)\ldots(j\omega - z_{m-1})(j\omega - z_m)}{(j\omega - p_1)(j\omega - p_2)\ldots(j\omega - p_{n-1})(j\omega - p_n)} \tag{3}$$

$$\mid j\omega - p_i \mid = \sqrt{\sigma_i^2 + (\omega - \omega_i)^2} \tag{4}$$

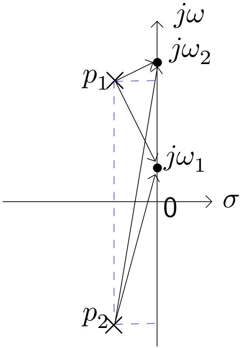
$$\angle(s - p_i) = \tan^{-1}\left(\frac{\omega - \omega_i}{-\sigma_i}\right) \tag{5}$$

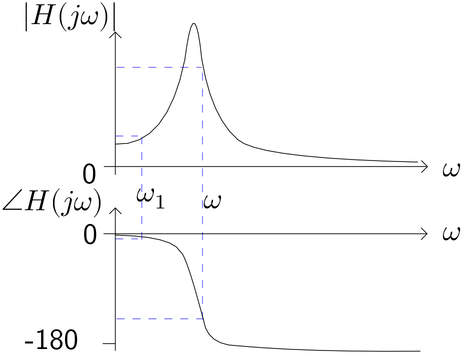
Frequency Response Example





Frequency Response Example





Frequency Response Example

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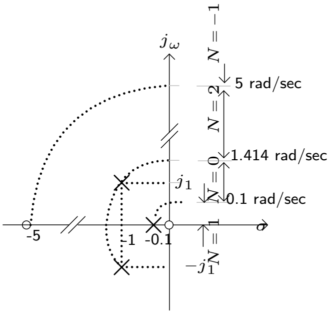
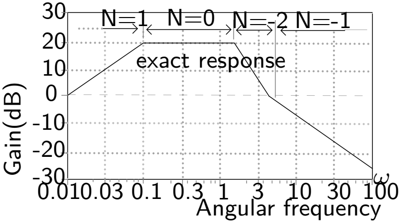




- Each pole or zero contributes a change in the slope of the asymptotic plot of $\pm$ 20 dB/decade above its break frequency. A complex conjugate pole or zero pair gives a total change in the slope of $\pm$ 40 dB/decade.

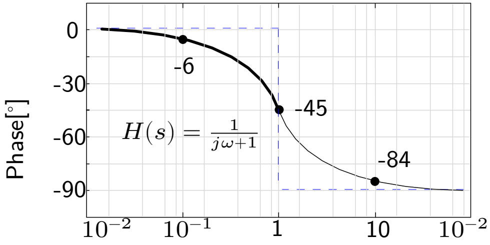
Constructing Magnitude Bode Plot

- For lefthand plane
  - Slope changes by –20 dB/decade
  - Phase decreases by 90°
- Zero
  - Slope changes by 20 dB/decade
  - Phase increase by 90°

Constructing Magnitude Bode Plot

- $H(s) = \frac{1}{s+1} \Rightarrow H(j\omega) = \frac{1}{j\omega+1}$
- $\angle H(j\omega) = -\tan^{-1}(\omega)$
- $\angle H(j\omega) = \begin{cases} 0^\circ & \omega \ll 1 \\ -90^\circ & \omega \gg 1 \end{cases}$

The behavior for a zero is similar. The phase increases by 90° and passes though the midpoint of 45° at the break point



Constructing Magnitude Bode Plot

We can extend the results for simple repeated poles and zeroes as before using the more general function

$$H(s) = (s+a)^{\pm r} \Rightarrow H(j\omega) = (j\omega + a)^{\pm r} \tag{6}$$

$$\angle H(j\omega) = \begin{cases} 0^\circ & \omega \ll a \\ \pm r 90^\circ & \omega \gg a \end{cases} \tag{7}$$

$$\angle H(ja) = \pm r 45^\circ \tag{8}$$

Unstable (right half plane) poles and zeros have opposite behavior

Zero @ Origin

- $H(s) = s$
- $\mid H(j\omega) \mid = \omega \Rightarrow \omega = 1 \Rightarrow \log \mid H(j\omega) \mid = 0dB$
- Slope 20 dB/decade

Pole @ Origin

- $H(s) = \frac{1}{s}$ and $s = j\omega$
- $\angle H(j\omega) = -90^{\circ}$ Phase always at -90°
- $\mid H(j\omega) \mid = \frac{1}{\omega} \Rightarrow \omega = 1 \Rightarrow \log(1) = 0 \text{ dB}$
- Slope $-20 \text{ dB}/$ decade

Bode Plot Example

- $H(s) = \frac{s}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{10^4}\right)}$

Zero @ 0
Pole @ -10
Pole @ -10^4

Bode Plot Example

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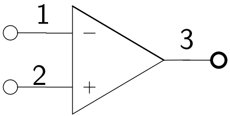
$$H(s) = \frac{1000}{s+100} = \frac{10}{1+\frac{s}{100}} \tag{9}$$

$$G_{DC} = 10 = 20\log 10 = 20 \text{ dB} \tag{10}$$

Pole at -100.

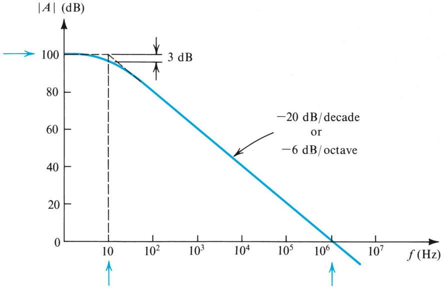
Finite Op-amp Bandwidth



- Remember for a RC circuit
- $V_{\text{out}} = \frac{1}{1+j\omega CR} V_{in}$
- $\omega_p = -\frac{1}{RC} \Rightarrow$
- $Av(\omega) = \frac{1}{1+j\omega/\omega_p}$

- Ideal : Gain is Infinite
- Gain is finite, bandwidth infinite
- Gain is finite, bandwidth finite
- Assume single pole amplifier
- $A0$ is the DC gain,
- $A(j\omega) = \frac{Ao}{1+j\omega/\omega p}$

Finite Op-amp Bandwidth



$$A(j\omega) = \frac{A_0}{1 + j\omega/\omega_p} \qquad (11)$$

Typical op-amps have every low 3dB bandwidth

Finite Op-amp Bandwidth

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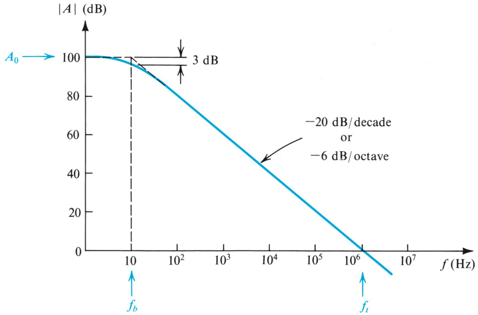

- ω_t is ω Where $|A(j\omega)| = 1$
- $A(j\omega) = \frac{A_0}{1+j\omega/\omega_b}$
- $1 = \frac{A_0}{\omega_{T/\omega_b}} \Rightarrow \omega_T = A_0\omega_b$
- $\omega_T = A_0\omega_b$
- Gain bandwidth product (GBW) important parameter

Finite Op-amp Bandwidth

- ω_t is ω Where $\mid A(j\omega) \mid = 1$
- $A(j\omega) = \frac{A_0}{1+j\omega/\omega_b}$
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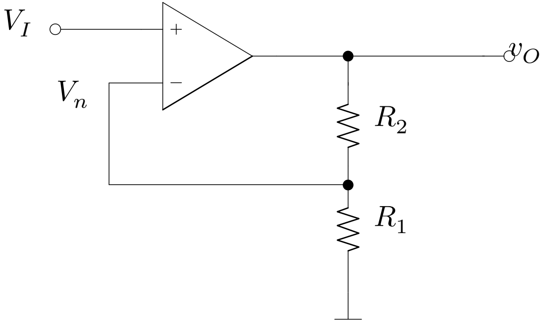
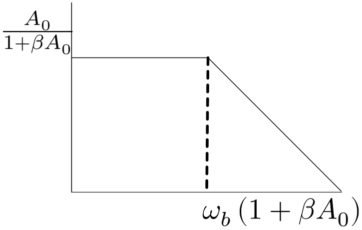


Non-Inverting Amplifier

$$v_O = \frac{A_0}{1 - \frac{s}{\omega_p}} \left(v_I - \beta v_O\right) \quad (12)$$

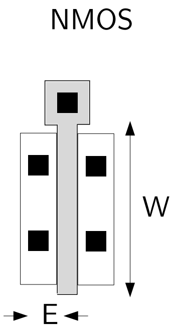
$$v_O \left(1 - \frac{s}{\omega_p}\right) + A_0 \beta v_O = A_0 v_I \quad (13)$$

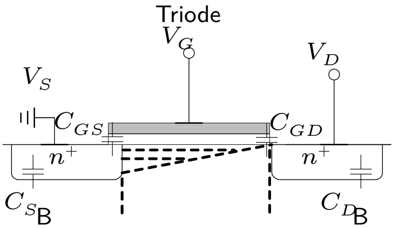
$$\frac{v_O}{v_I} = \frac{A_0}{1 + \beta A_0 - s/\omega_p} = \frac{A_0}{1 + \beta A_0} \frac{1}{1 - \frac{s}{\omega_p (1+\beta A_0)}} \quad (14)$$



- This tells me the bandwidth of a noninverting amplifier is $\omega_p \left(1 + \beta A_0\right)$

Parasitic Capacitance





Parasitic Capacitance

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$$C_{GS} = W_{ov} + 1/2\,(WLC_{ox}) \quad (15)$$

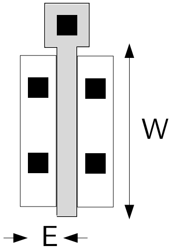
$$C_{GD} = WC_{ov} + 1/2\,(WLC_{ox}) \quad (16)$$

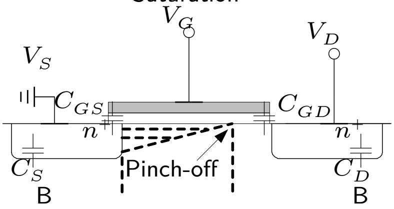
$$C_{SB} = WEC_j + 2(W+E)C_{jsw} \quad (17)$$

$$C_{DB} = WEC_j + 2(W+E)C_{jsw} \quad (18)$$

Parasitic Capacitance

NMOS





Parasitic Capacitance

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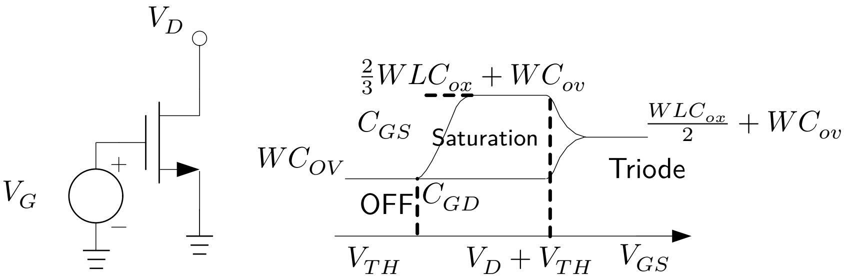
$$C_{GS} = WC_{ov} + 2/3\,(WLC_{ox}) \quad (19)$$

$$C_{GD} = WC_{ov} \quad (20)$$

$$C_{SB} = WC_j + 2(W+E)C_{jsw} \quad (21)$$

$$C_{DB} = WEC_j + 2(W+E)C_{jsw} \quad (22)$$

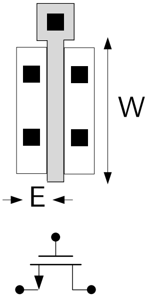
Parasitic Capacitance

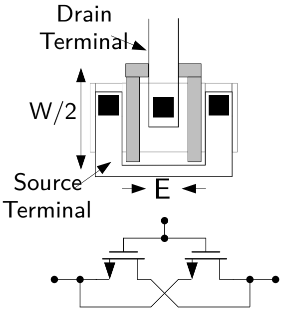


Parasitic Capacitance

Calculate C_{SB} and C_{DB} of the two structures below.

NMOS





Parasitic Capacitance

Solution:

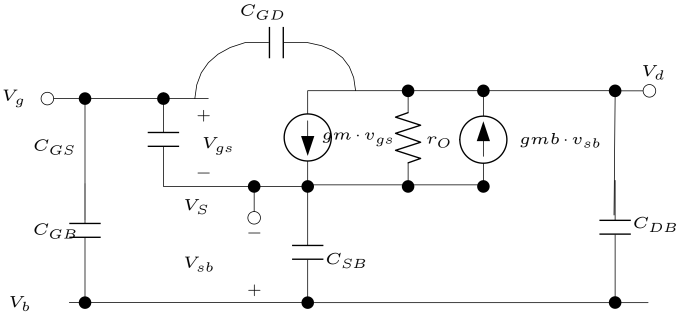
$$\text{Left: } C_{DB} = \frac{W}{2} E C_j + 2\left(\frac{W}{2} + E\right) C_{jsw} \tag{23}$$

$$\text{Right: } C_{DB} = C_{SB} = WEC_j + 2(W+E)C_{jsw} \tag{24}$$

$$\begin{aligned} C_{SB} &= 2\left[\tfrac{W}{2} E C_j + 2\left(\tfrac{W}{2} + E\right) C_{jsw}\right] \\ &= WEC_j + 2(W+2E)C_{jsw} \end{aligned} \tag{25}$$

- Drain junction capacitance is greatly reduced

Complete Small-Signal Model



Only when MOSFET is off should we need to consider C_{GB}, which includes the gate oxide capacitance and the depletion region capacitance in series.

Spice Model

NMOS Model

LEVEL $= 1$	VTO $=0.7$	GAMMA $= 0.45$	PHI $= 0.9$
NSUB $= 9e+14$	LD $= 0.08e-6$	UO $= 350$	LAMBDA $= 0.1$
TOX $= 9e-9$	PB $= 0.9$	CJ $= 0.56e-3$	CJSW $= 0.35e-11$
MJ $= 0.45$	MJSW $= 0.2$	CGDO $= 0.4e-9$	JS $= 1.0e-8$

PMOS Model

LEVEL $= 1$	VTO $= -0.8$	GAMMA $= 0.4$	PHI $= 0.8$
NSUB $= 5e+14$	LD $= 0.09e-6$	UO $= 100$	LAMBDA $= 0.2$
TOX $= 9e-9$	PB $= 0.9$	CJ $= 0.94e-3$	CJSW $= 0.32e-11$
MJ $= 0.5$	MJSW $= 0.3$	CGDO $= 0.3e-9$	JS $= 0.5e-8$

First Order Systems

- No energy is store in the circle. In general, a one pole one zero system
- $H(S) = \frac{a_0 + a_1 S}{1 + bS}$

First Order Systems

- The low frequency is represented by a_0
- $a_0 = H(s)\mid_{c_1=0} = H^\circ$
- The time constant determines b1 (This is the pole of the system)
- $b_1 = \tau = RC_1$
- The ratio of a_0 and a_1 determines the location of the zero

First Order Systems

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- $H(S) = \frac{a_0 + a_1 S}{1 + bs}$
- $H_i(s) = \frac{a_0 + \alpha_1^i C_i s}{1 + \beta_1^i C_i s}$
- (It is easy to convince ourselves that a1 is also related to the capacitor.)
- The transfer function shall be valid for all capacitor values including zero and infinity.
- the first denominator coefficient $b1$, is simply given by the sum of these zero-value time constants (ZVT)
- $b_1 = \sum_{i=1}^{N} \tau_i^0$

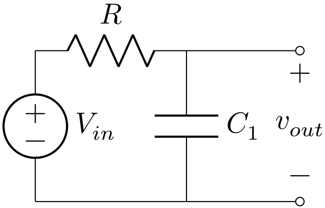
First Order Systems

- $H_i(s) = \frac{a_0+\alpha_1^i C_i s}{1+\beta_1^i C_i s}$C1 goes to inf H(s) $= \frac{\alpha_1}{\beta_1}$

$$H(s) = \frac{H^0 + H^1 \tau s}{1+\tau s} \tag{26}$$

- If it is an inductor, $\tau = \frac{L_1}{R_0}$
- To find the time constant, remove the cap/ind nulling all the sources, find the resistance.
- To find transfer constant H_0, it is just the low frequency gain.
- To find the transfer constant H_1, we look into high frequency response, so the cap shall be shorted. For inductor it is the opposite.

First Order Systems



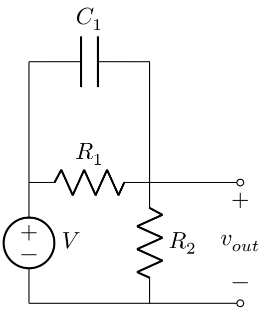
$$H^0 = 1 \tag{27}$$

$$\tau = RC_1 \tag{28}$$

$$H^1 = 0 \tag{29}$$

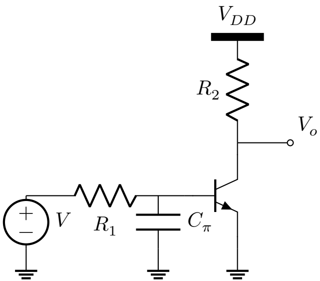
$$H(s) = \frac{1}{1 + RCS} \tag{30}$$

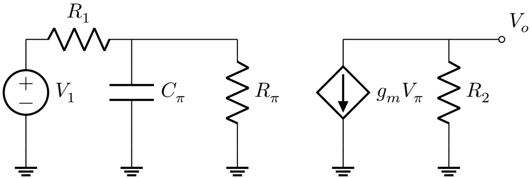
First Order Systems



- We do zero frequency response to find out H_0
- $H^0 = \frac{R_1}{R_1+R_2}$
- We null sources to find τ_1
- $\tau_1 = (R_1 \parallel R_2)\, C_1$
- We short circuit to find H_1
- $H^1 = 1$
- $H(s) = \frac{R_1}{R_1+R_2} \frac{1+R_1 C_1 S}{1+(R_1 \| R_2) C_1 S}$

First Order Systems

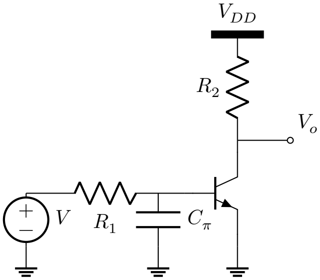


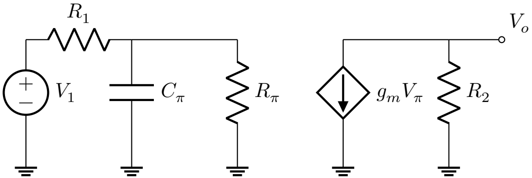


$$H^0 = -\frac{r_\pi}{R_1 + r_\pi} g_m R_2 \tag{31}$$

$$H^\pi = 0 \tag{32}$$

First Order Systems



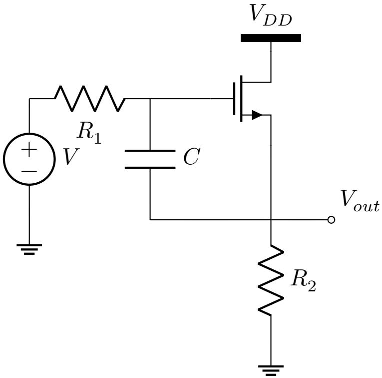


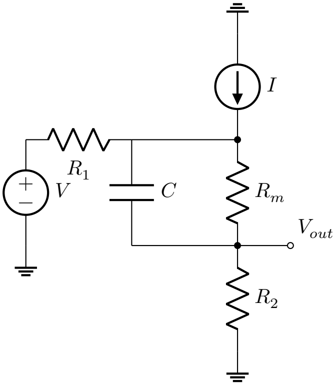
$$\tau = (R_1 \parallel r_\pi) C_\pi \qquad (33)$$

$$H(S) = \frac{H^0}{1 + \tau S} \qquad (34)$$

If there is a zero in the system, then we can test it with a shorted cap/ open ind and see if the output still have some value.

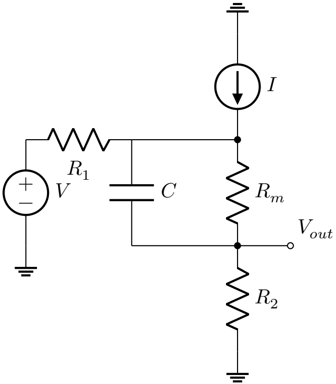
First Order Systems





First Order Systems

- Low frequency, $Vx = V_{in}$
- Vout is voltage divider,
- $H^0 = \frac{R_2}{R_2+r_m}$ (assume it's a bjt)
- $H^1 = \frac{R_2}{R_2+r_1}$
- $R_2\left(g_m v_x - i_x\right) + v_x = R_1 i_x$
- $R = \frac{R_1+R_2}{1+g_m R_2}$
- You can imagine the existence of a zero and a pole.



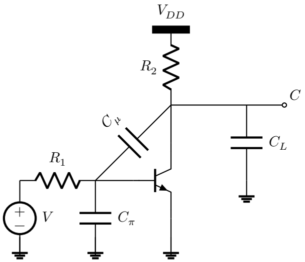
First Order Systems

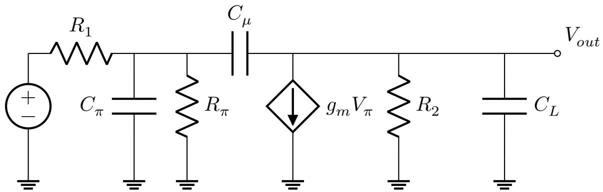
transistor model statement for the 2N696

.model Q2N696 NPN (Is=14.34f Xti=3 Eg=1.11 Vaf=74.03 Bf=65.62 Ne=1.208 Ise=19.48f Ikf=.2385 Xtb=1.5 Br=9.715 Nc=2 Isc=0 Ikr=0 Rc=1 Cjc=9.393p Mjc=.3416 Vjc=.75 Fc=.5 Cje=22.01p Mje=.377 Vje=.75 Tr=58.98n Tf=408.8p Itf=.6 Vtf=1.7 Xtf=3 Rb=10)

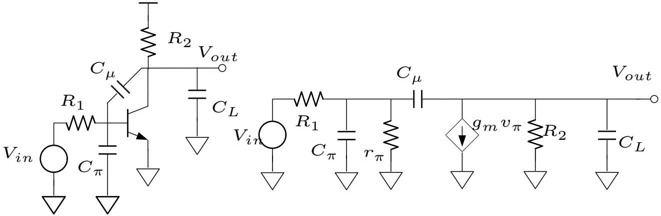
First Order Systems

.model NPN ($C_{je} = 20$ fF, $C_{jc} = 20$ fF, $\beta_0 = 100$, $C_{js} = 50$ fF, $\tau_F = 2$ ps, $C_L = 150$ fF) c_{je} Zero bias B-E depletion capacitance





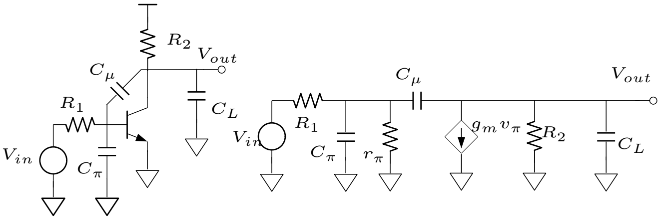
First Order Systems



We can use those parameters to determine small signal parameters. a collector current of 1mA which give you a $Gm = 40mS \quad \beta_0 = 100$

- $C_\pi = C_{je} + C_b = 100\text{fF}$
- $C_L = C_{\text{out}} + C_{js} = 200\text{fF}$
- $C_b = g_m \tau_F = 80fF$
- $R_1 = 1K\Omega$
- $R_2 = 2K\Omega$

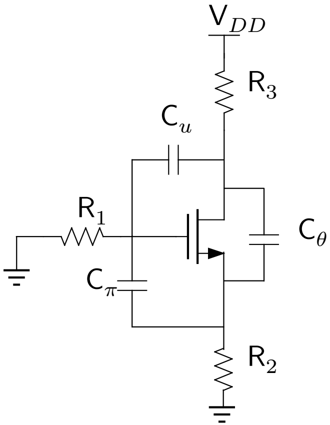
First Order Systems



- $b_1 = \sum_i \tau_i^0 = \tau_\pi^0 + \tau_\mu^0 + \tau_L^0$
- $H^0 = -57$
- $\tau_\pi^0 \approx 70\text{ps} \quad \tau_\mu^0 \approx 1200\text{ps} \quad \tau_L^0 = 400\text{ps}$
- $\omega_h \approx 1/b_1 \approx 2\pi \cdot 95\text{MHz}$
- This allows you to determine the lowest operating frequency, and also the contribution of each nodes in the circuit.

First Order Systems

- $R_\pi^0 = r_\pi \parallel \frac{R_1+R_2}{1+g_m R_2}$
- $R_\mu^0 =$
  $R_{\text{left}} + R_{\text{right}} + G_m R_{\text{left}} R_{\text{right}}$
- $R_{\text{left}} \equiv R_B \parallel [r_\pi + (1+\beta) R_E]$
- $R_{\text{right}} \equiv R_C$
- $G_m = \frac{g_m}{1+g_m R_2}$
- $R_\theta^0 \approx \frac{R_2+R_3}{1+g_m R_2}$
- $b_1 = \sum_{i=1}^{N} \tau_i^0$



Further discussion on P and Z

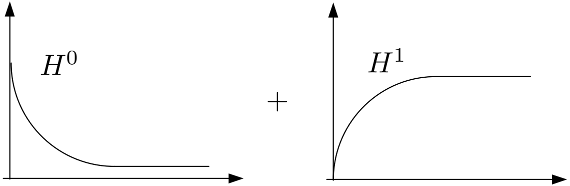
- $H(s) = \frac{H^0 + H^1 \tau s}{1+\tau s} = H^\circ \frac{1+\frac{H^1}{H^0}\tau s}{1+\tau s}$
- $P = -\frac{1}{\tau}$
- $Z = -\frac{1}{H^1} = \frac{H^0}{H^0}\tau\left(-\frac{1}{\tau}\right)$
- $Z = \frac{H^0}{H^1}P$

Further discussion on P and Z

$$H(s) = \frac{H^0 + H^1\tau S}{1+\tau S} = H^0\frac{1}{1+\tau S} + H^1\frac{\tau S}{1+\tau S} \tag{35}$$

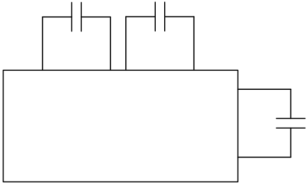
Low path filter High path filter

$$s(t) = H^0\left(1 - e^{-t/\tau}\right)u(t) + H^1 e^{-t/\tau}u(t) \tag{36}$$



The final waveform also tells you the existence of poles and zeros.

Nth order system



$$H(s) = \frac{a_0 + a_1 S + a_2 S^2 + \dots}{1 + b_1 S + b_2 S^2 + \dots} \tag{37}$$

- Only the caps and inductors produces S.
- To get a_1 we have to have a cap (or inductor)

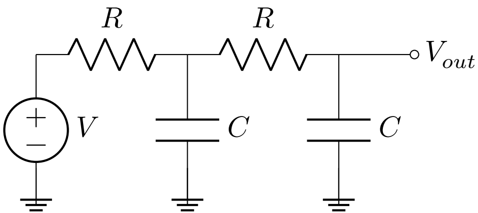
Nth order system

- We can also infer that the s^2 term comes from two capacitors.
- $H(s) = \frac{a_0+\left(\sum_{i=1}^{N}\alpha_1^i C_i\right)s+\left(\sum_i^{1\leqslant i<j\leqslant N}\alpha_j^{ij} C_i C_j\right)s^2+\ldots}{1+\left(\sum_{i=1}^{N}\beta_1^i C_i\right)s+\left(\sum_i^{1\leqslant i<j\leqslant N}\sum_j^{ij}\beta_2^{ij} C_i C_j\right)s^2+\ldots}$
- If we set all c's except ci as zeros
- $H_i(s) = \frac{a_0+\alpha_1^i C_i s}{1+\beta_1^i C_i s}$
- $\tau_i^0 = R_i^0 C_i$
- $\beta_1^i = R_i^0$
- $b_1 = \sum_{i=1}^{N}\tau_i^0$

N th order system

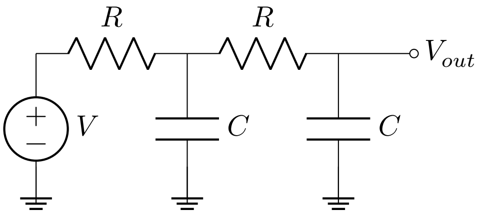
- $b_1 = \sum_{i=1}^{N} \tau_i^0$
- coefficient b_1 is the sum of all zero valued time constant.
- $a_1 = \sum_{i=1}^{N} \tau_i^0 H^i$
- This allows us to find out the dominant time constant.

N th order system



It is a two cap system, therefore the number of poles are two.
It is a system with an infinite response of zero if we short both capacitors, so there is no zeros.

Nth order system



- $H(S) = \frac{1}{1+3RCS+(RC)^2S^2}$
- $b_1 = \sum_{i=1}^{N} \tau_i^0 = \tau_1^0 + \tau_2^0$
- $\tau_1^0 = RC$
- $\tau_2^0 = 2RC$
- $b_1 = 3R_c$

Nth order system

$$H(S) = \frac{1}{1 + 3RCS + (RC)^2 S^2} \tag{38}$$

- If the impedance seen by one cap does not change as we open or short the other cap, we say that the two-time constant are uncoupled to each other, and the expression can be written as

$$H(s) = \frac{H^0}{(1 + \tau_1 S)(1 + \tau_2 S)} \tag{39}$$

- We now have the $3RC$ term. The question is whether it is true in this case to determine $(RC)^2 S^2$

$$H(s) = 1 \qquad 1 \qquad = \tag{40}$$

N th order system

We now know how to calculate a_1, b_1, and a_0.

$$H(s) = \frac{a_0 + \left(\sum_{i=1}^{N} \alpha_1^i C_i\right) s + \left(\sum_i^{1\leqslant i<j\leqslant N} \alpha_j^{ij} C_i C_j\right) s^2 + \ldots}{1 + \left(\sum_{i=1}^{N} \beta_1^i C_i\right) s + \left(\sum_i^{1\leqslant i<j\leqslant N} \sum_j \beta_2^{ij} C_i C_j\right) s^2 + \ldots} \tag{42}$$

A_O is the zero frequency response. B_1 is the summation of time constant A_1 can be obtained from infinite time response.

Nth order system

$$b_2 = \sum_{i}^{i<j<N} \sum_{j}^{0} \tau_j^i \tag{43}$$

- That means you don't repeat $\tau 12$ and $\tau 21$
- τ_j^i means the time constant of element j when element I is infinite frequency

$$a_2 = \sum \sum_{j}^{i<j<N} \tau_i^0 \tau_j^i H^{ij} \tag{44}$$

- We can expect b_n is a multiple summation of the product of many time constants

Nth order system

$$b_n = \sum_{i}^{1\leqslant i<j<k} \sum_{j} \sum_{k\ldots}^{\ldots} \ldots \tau_i^0 \tau_j^i \tau_k^{ij} \ldots \tag{45}$$

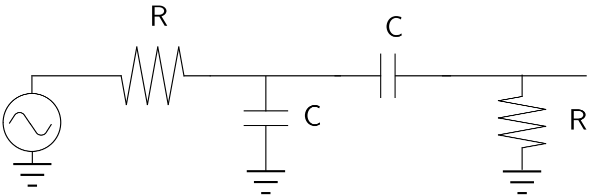
$$a_n = \sum_{i}^{1\leqslant i<j<k} \sum_{j}^{\ldots\ldots N} \ldots \tau_i^0 \tau_j^i \tau_k^{ij} \ldots H^{ijk\ldots} \tag{46}$$

I will skip the derivations and show you how to use it.

Nth order system

- $H(s) = \frac{a_0 + a_1 s + a_2 s^2}{1 + b_1 s + b_2 s^2}$
- $b_2 = \tau_1^0 \tau_2^1 = \tau_2^0 \tau_1^2$
- $a_2 = \tau_1^0 \tau_2^1 H^{12} = \tau_2^0 \tau_1^2 H^{12}$
- $b_1 = \tau_1^0 + \tau_2^0$
- $a_1 = \tau_1^0 H^1 + \tau_2^0 H^2$

N^{th} order system

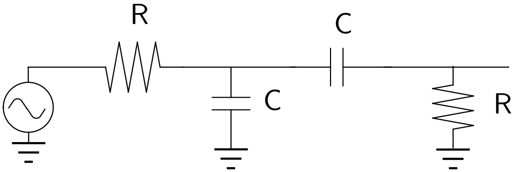


- Two pole and one zero
- How do we test this?
- $H^0 = 0$

$$\tau_1^0 = R_1 C_1 \quad (47)$$

$$\tau_2^0 = (R_1 + R_2) C_2 \quad (48)$$

Nth order system



$$\tau_2^1 = R_2 C_L \qquad (49)$$

$$\tau_1^2 = (R_1 \parallel R_2)\, C_1 \qquad (50)$$

$$\tau_1^0 \tau_2^1 = R_1 C_1 R_2 C_2 = \tau_2^0 \tau_1^2 \qquad (51)$$

$$H^1 = 0 \qquad (53)$$

$$H^2 = \frac{R_2}{R_2 + R_1} \qquad (54)$$

Nth order system

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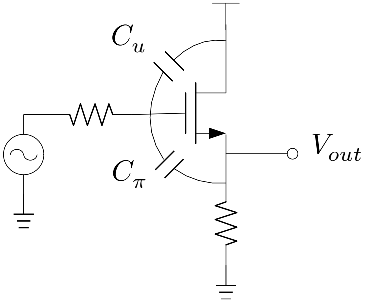
- Two poles and one zero
- $H^0 = \frac{R_2}{R_2+r_m}$
- $\tau_\pi^0 = C_\pi \frac{R_1+R_2}{1+g_m R_2}$
- $\tau_\mu^0 = C_\mu R_1$
- $\tau_\mu^\pi = C_\mu \left(R_1 \parallel R_2\right)$
- $H^\mu = 0$
- $H^\pi = \frac{R_2}{R_1+R_2}$
- $H^{\mu\pi} = 0$



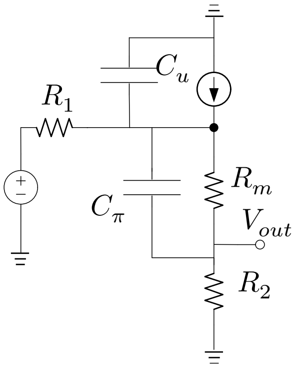
Nth order system

- Now we can write out the transfer function. If we assume that $R_2 >> R_1$
- And $R_2 >> R_m$

$$H^0 = \frac{R_2}{R_2 + r_m} \quad (57)$$

$$\tau_\pi^0 = C_\pi \frac{R_1 + R_2}{1 + g_m R_2} = r_m C_\pi \quad (58)$$

$$\tau_\mu^0 = C_\mu R_1 \quad (59)$$



Nth order system

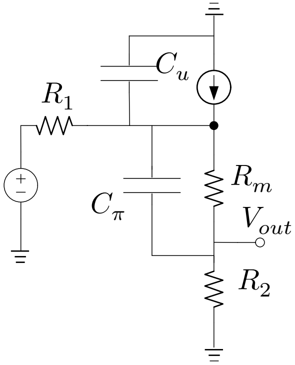
$$\tau_\mu^\pi = C_\mu (R_1 \parallel R_2) = C_\mu R_1 \qquad (60)$$

$$H^\mu = 0 \qquad (61)$$

$$H^\pi = \frac{R_2}{R_1 + R_2} \qquad (62)$$

$$\tau_\pi^0 \tau_\mu^\pi = r_m R_1 C_\pi C_\mu \qquad (63)$$

$$H^\pi \tau_\pi^0 = r_m C_\pi \frac{R_2}{r_m + R_2} = H^0 r_m C_\pi \qquad (64)$$



Nth order system

- $H(S) = H^0 \frac{1+r_m C_\pi S}{1+(r_m C_\pi + R_1 C_\mu)S + r_m C_\pi R_1 C_\mu S^2} = \frac{H^0}{1+R_1 C_\mu S}$
- This essentially tells us the dominate pole is the C_μ, because C_π shares current between the capacitor and the resistor, so that it tells us to improve the bandwidth of operation, we need to use a inductor or some topology to cancel the effect of C_μ

Bandwidth Estimation

- The whole system can be expressed as the product of a high pass and a low pass transfer function.
- If I'm designing an analog circuit, I can use transfer function to estimate bandwidth, assuming I care about the lowpass one.
- For low frequency system, in many cases we can assume a zeroless system.
- For example, in common-source stage and the source-follower stage, the zero's frequencies are comparable to the cut-off frequency of the transistor itself

Bandwidth Estimation

- $H(s) \approx \frac{a_0}{1+b_1 s+b_2 s^2+...+b_n s^n}$
- At dc $(\omega = 0)$, the only term in the denominator that matters is the leading 1.
- As the frequency goes up and starts approaching the wh, the first term that becomes non-negligible would be $b1$, so in the vicinity of the ωh
- The system is pretty much
- $H(s) \approx \frac{a_0}{1+b_1 s}$
- This tells us, in order to find out the cutoff frequency,
- We calculate b1

Bandwidth Estimation

- $\omega_h \approx \frac{1}{b_1} = \frac{1}{\sum_{i=1}^{N} \tau_i^0}$
- $\frac{a_1}{1+b_1 S+b_2 S^2} \Rightarrow H(j\omega) = \frac{a_0}{(1-b_2\omega^2)+j\omega b_1}$
- It is a conservative estimation of bandwidth