Lecture 19

EECS 311 Analog Circuits

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Recap of Last Lecture

- Differential Amplifier (Half circuit method)
  1. Current Equation method
  2. Superposition method
  3. Half circuit method
- More on differential amplifier topologies
- Current mirror

Topics to Be Covered

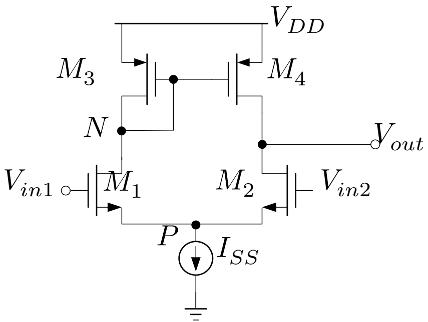
- Differential amplifier topologies
- Frequency Response

Course Outline

- Frequency Response (L19, L20, L21, L22)
  Frequency Response will be another emphasis
  1. Transfer function and bode plot
  2. Estimation of transfer function of simple amplifiers
- Design Examples (Not going to be the focus of the exam, tentative)
  1. Revisit of Biasing (L23)
  2. Amplifier Design Examples (L24)
  3. VLSI Primer (Circuit and Programming) (L24)
- Course Review (L25, L26)

Current Mirror

Asymmetric Differential Pair

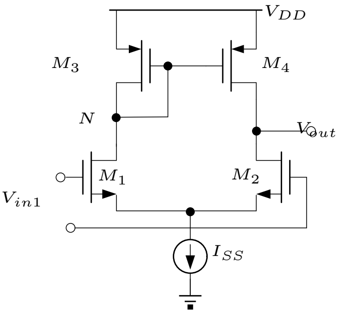


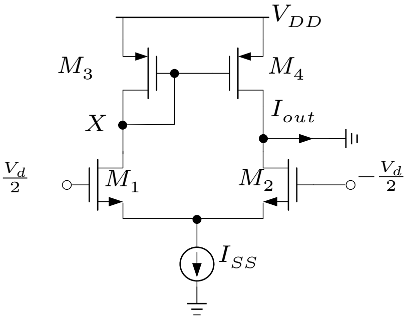
Why N is not on the right branch?

- Caveat: Because of the vastly different resistance magnitude at the drains of M_1 and M_2, the voltage swings at these two nodes are different and therefore node P cannot be viewed as a virtual ground when $V_{in2} = -V_{in1}$.

Differential Pair with Current Mirror

- Small-signal Analysis, Half-Circuit Method, Differential-Mode





- Almost fully symmetric, we can still approximate

Differential Pair with Current Mirror

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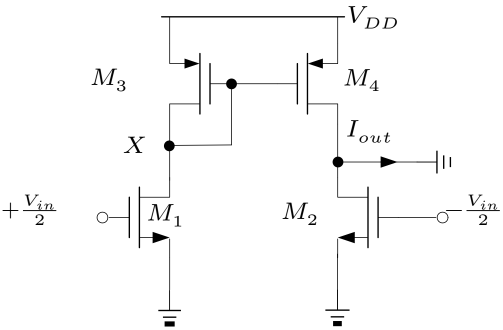


$$R_{in} = \frac{1}{g_{m3}} \parallel r_{o3} \quad \text{(small)} \tag{1}$$

$$i_{out} = g_{m1,2}\left(\frac{u_d}{2}\right) \times 2 \tag{2}$$

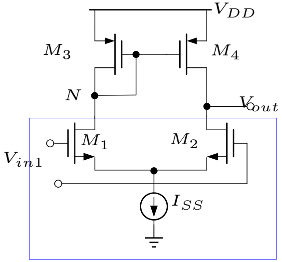
$$G_m = g_{m1,2} \tag{3}$$

Differential Pair with Current Mirror

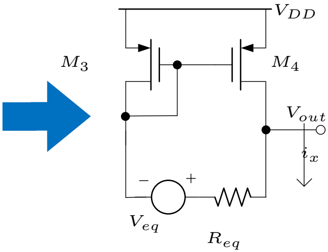
Small-signal Analysis

Half-circuit method

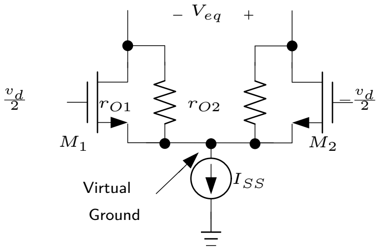
Differential-Mode



Fully Symmetric



Differential Pair with Current Mirror



$$i_X = \frac{V_{out} - V_{eq}}{R_{eq} + \left(\frac{1}{g_{m3,4}} \parallel r_{o3,4}\right)} \tag{4}$$

$$2i_X + \frac{V_{out}}{r_{o3,4}} = 0 \tag{5}$$

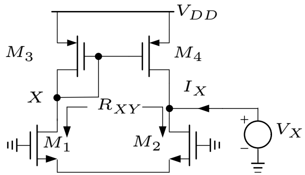
$$A_v = \frac{V_{out}}{V_{in}} \approx g_{m1,2}(r_{o2} \parallel r_{o4}) \tag{6}$$

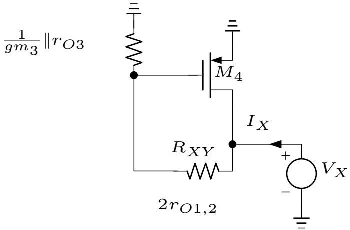
Differential Pair with Current Mirror

Small-signal Analysis

Half-circuit method

Differential-Mode





$$R_{out} \approx r_{o2} \parallel r_{o4} \tag{7}$$

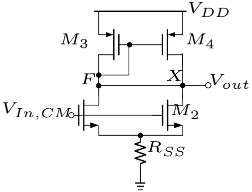
$$I_X = \frac{V_X}{2r_{o1,2} + \frac{1}{g_{m3}} \parallel r_{o3}} \left[1 + \left(\frac{1}{g_{m3}} \parallel r_{O3}\right) g_{m4}\right] + \frac{V_X}{r_{o4}} \tag{8}$$

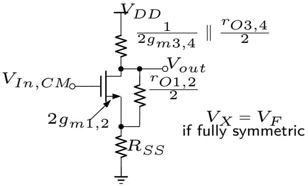
$$A_v = G_m R_{out} \approx g_{m1,2}(r_{o2} \parallel r_{o4}) \tag{9}$$

Differential Pair with Current Mirror

Small-signal Analysis

Common-Mode



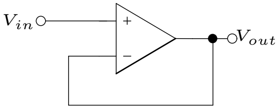


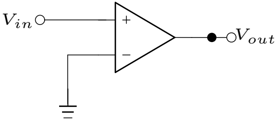
$$A_{CM} = \frac{-\left(2g_{m1,2}\right)\left(\frac{r_{o1,2}}{2}\right)\left(\frac{1}{2g_{m3,4}} \parallel \frac{r_{o3,4}}{2}\right)}{R_{SS} + \left(\frac{r_{o1,2}}{2}\right) + \left(2g_{m1,2} + 2g_{mb1,2}\right)\left(\frac{r_{o1,2}}{2}\right)R_{SS} + \left(\frac{1}{2g_{m3,4}} \parallel \frac{r_{o3,4}}{2}\right)} \approx \frac{-1}{2g_{m3,4}R_{SS}} \quad (10)$$

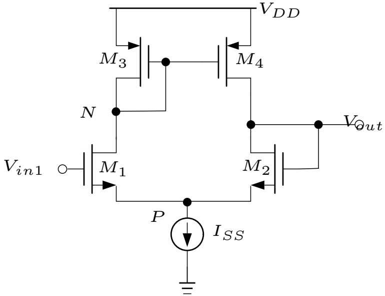
$$CMRR = \left| \frac{A_{DM}}{A_{CM}} \right| = g_{m1,2}\left(r_{o2} \parallel r_{o4}\right) 2g_{m3,4}R_{SS} \quad \text{(large)} \quad (11)$$

Simple Op-amp Topologies

Calculate the input voltage range and the closed-loop output impedance. Each transistor has a threshold voltage 0.7 V and an overdrive 0.3 V.







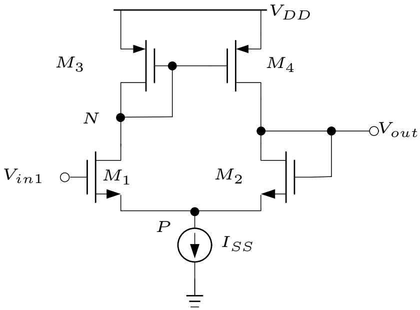
Simple Op-amp Topologies

If each device (including the current source) has a threshold voltage of 0.7 V and an overdrive of 0.3 V

$$\frac{v_{out}}{v_{in,open}} = g_{mN}(r_{oN} \parallel r_{oP}) \quad (12)$$

$$R_{out,open} = (r_{oN} \parallel r_{oP}) \quad (13)$$

$$V_{in,min} = 0.3V + (0.3V + 0.7V) \quad (14)$$

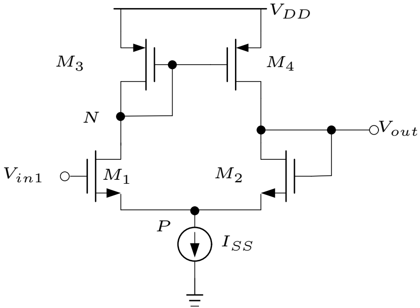


Simple Op-amp Topologies

$$V_{in,max} = V_{DD} - \mid V_{GS3} \mid + V_{TH1} \tag{15}$$

- When M_1 is in triode.

$$V_{in.max} = V_{DD} - (0.3V + 0.7V) + 0.7V \tag{16}$$



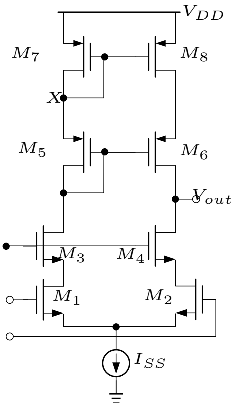
Simple Op-amp Topologies

$$\left.\frac{v_{out}}{v_{in}}\right|_{closed} = \frac{g_{mN}\left(r_{oN} \parallel r_{oP}\right)}{1 + g_{mN}\left(r_{oN} \parallel r_{oP}\right)} \tag{17}$$

$$R_{\text{out,close}} = \frac{\left(r_{ON} \parallel r_{OP}\right)}{1 + g_{mN}\left(r_{ON} \parallel r_{OP}\right)} \approx \frac{1}{g_{mN}} \tag{18}$$

- The closed-loop output impedance is relatively independent of the open-loop output impedance. Allowing the design high-gain op amps by increasing the open-loop output impedance while still achieving a relatively low closed-loop output impedance

Telescopic Cascode



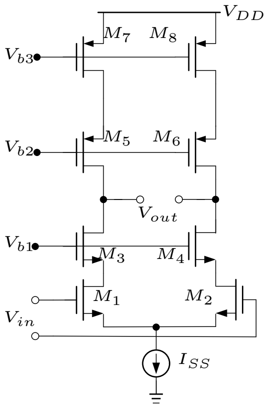
$$A_{\text{DM,open}} = \frac{v_{\text{out}}}{v_{\text{in1}} - v_{\text{in2}}} \approx g_{mN}\left(g_{mN}r_{oN} \parallel g_{mP}r_{oP}\right) \tag{19}$$

- Higher gain at the cost of voltage headroom

$$V_b - V_{TH4} \leq V_{\text{out}} \leq V_{DD} - V_{SG7} - V_{SG5} + V_{TH6} \tag{20}$$

$$V_{b,min} = V_{SS} + V_{ov2} + V_{GS4} \tag{21}$$

Telescopic Cascode



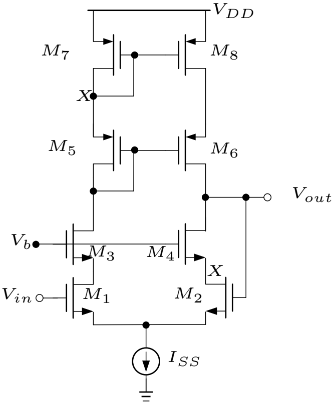
$$A_{\text{DM,open}} = \frac{v_{\text{out}} v_{\text{out1}} - v_{\text{out2}}}{v_{\text{in1}} - v_{\text{in2}}} \tag{22}$$

$$\approx g_{mN} \left(g_{mN} r_{oN} r_{oN} \parallel g_{mP} r_{oP} r_{oP} \right) \tag{23}$$

- Higher gain at the cost of voltage headroom
- Double output swing compared to single-ended

$$V_{b1} - V_{TH3,4} \leq V_{\text{out }1,2} \leq V_{b3} + V_{TH5,6} \tag{24}$$

Example



Due to feedback, $V_{out} \approx V_{in}$.

- Calculate the input voltage range.
- For M_4 in saturation, $V_{\text{out}} \geq V_b - V_{TH4}$
- For M_2 in saturation, $V_X = V_b - V_{GS4} \geq V_{\text{out}} - V_{TH2}$
- Input voltage range: $V_b - (V_{GS4} -$
- $V_{TH2}) \geq V_{\text{out}} \geq V_b - V_{TH4}$
- $(V_{GS4} - V_{TH2}) \leq V_{TH4}$
- $(V_{GS4} - V_{TH4}) \leq V_{TH2}$
- Make M_4 overdrive as small as possible to maximize the available range.

Folded Cascode

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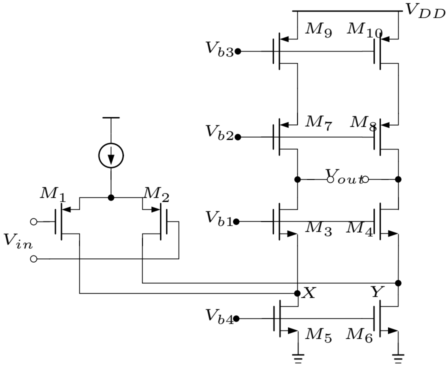
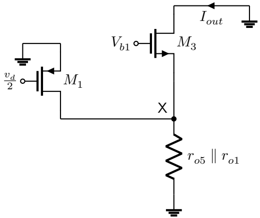
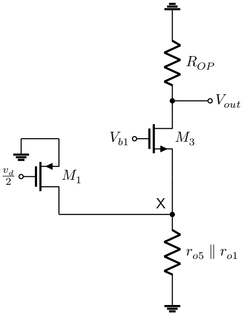


Fully symmetric

Fully differential

Half-circuit Method





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$$V_{b1} - V_{TH3,4} \leq V_{out1,2} \leq V_{b2} + V_{TH7,8}$$

- $G_m = \frac{i_{\text{out}}}{v_d/2} \approx -gm_1$
- $V_{b1} - V_{TH3,4} \leq V_{out1,2} \leq V_{b2} + V_{TH7,8}$
- $V_{b2,\max} = V_{DD} - V_{ov9,10} - V_{SG7,8}$
- $V_{b1,\min} = V_{ov5,6} + V_{GS3,4}$

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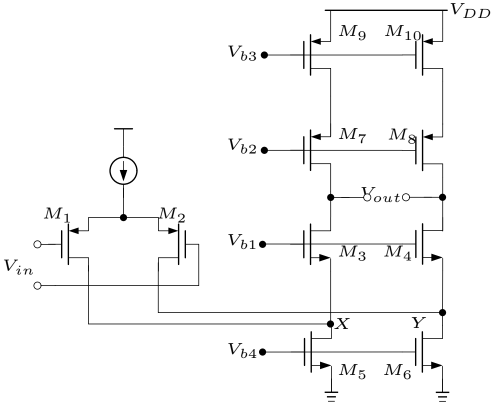

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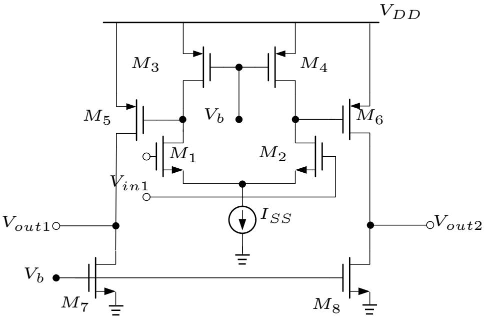
$$G_m R_{out} = \frac{v_{out1}}{v_d/2} \tag{25}$$

$$\approx -g_{m1}\left[\left(g_{m3}+g_{mb3}\right) r_{o3}\left(r_{o5} \parallel r_{o1}\right)\right] \parallel \left[\left(g_{m7}+g_{mb7}\right) r_{o7} r_{o9}\right] \tag{26}$$

$$A_{DM} = \frac{v_{out1}-v_{out2}}{v_d} \tag{27}$$

$$\approx -g_{m1,2}\left[\left(g_{m3,4}+g_{m3,4}\right) r_{o3,4}\left(r_{o,6,6} \parallel r_{o1,2}\right)\right] \parallel \left[\left(g_{m7,8}+g_{mb7,8}\right) r_{o7,8} r_{o,10}\right] \tag{28}$$

Implementation #1



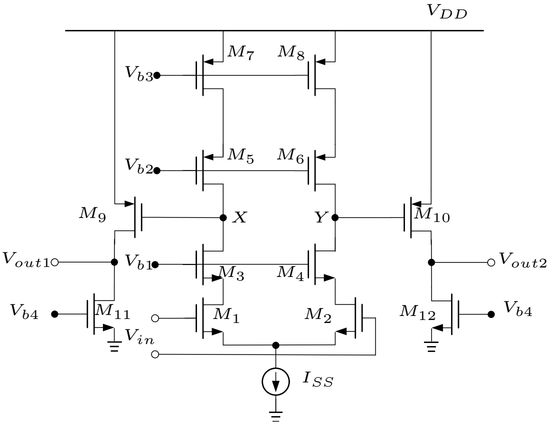
Implementation #1

$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_{in1} - v_{in2}} = \frac{v_{out1} - v_{out2}}{v_d} \tag{29}$$

$$= g_{m1,2}\left(r_{o1,2} \parallel r_{o3,4}\right) \times g_{m5,6}\left(r_{o5,6} \parallel r_{o7,8}\right) \tag{30}$$

$$V_b - V_{TH7,8} \leq V_{out1,2} \leq V_{DD} - V_{ov5,6} \tag{31}$$

Implementation #2

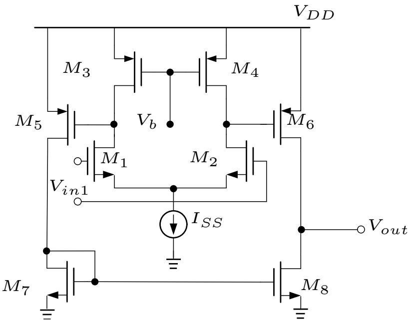


Implementation #2

$$
\begin{aligned}
A_{DM} &= \frac{v_{out1} - v_{out2}}{v_{in1} - v_{in2}} = \frac{v_{out1} - v_{out2}}{v_d} \\
&\cong \left\{ g_{m1,2} \left[\left(g_{m3,4} + g_{mb3,4} \right) r_{o3,4} r_{o1,2} \right] \parallel \left[\left(g_{m5,6} + g_{mb5,6} \right) r_{o5,6} r_{o7,8} \right] \right\} \\
&\times g_{m9,10} \left(r_{o9,10} \parallel r_{o11,12} \right)
\end{aligned} \tag{32}
$$

$$
V_{ov11,12} \leq V_{out1,2} \leq V_{DD} - V_{ov9,10} \tag{33}
$$

Implementation #3

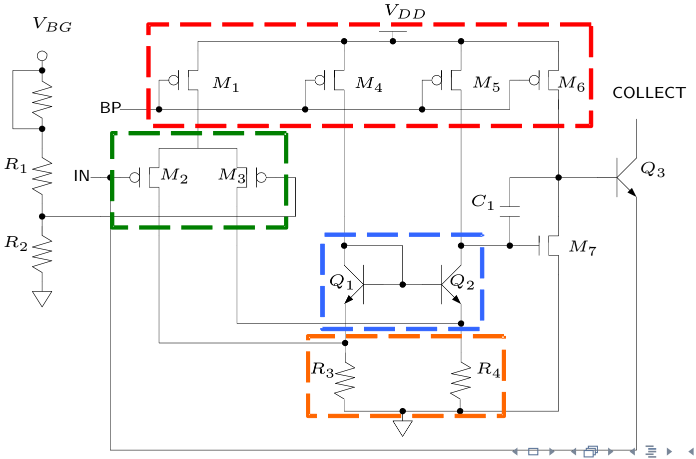


Implementation #3

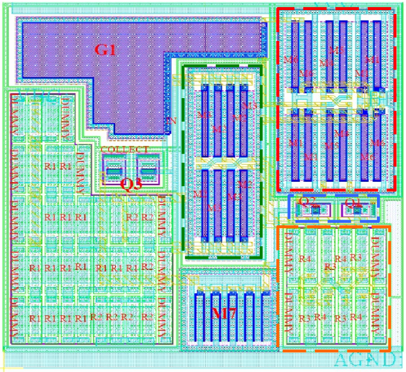
$$A_{\text{DM}} = \frac{v_{\text{out}}}{v_{\text{in1}} - v_{\text{in2}}} = -g_{m1,2}\left(r_{o1,2} \parallel r_{o3,4}\right) \times g_{m5,6}\left(r_{o5,6} \parallel r_{o7,8}\right) \tag{34}$$

$$V_{\text{ov8}} \le V_{\text{out}} \le V_{\text{DD}} - V_{\text{ov6}} \tag{35}$$

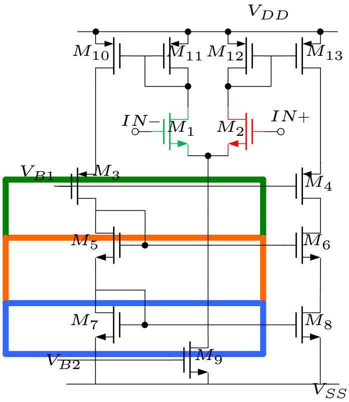
OPA Layout



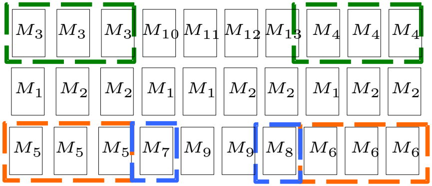
OPA Layout



OPA Layout

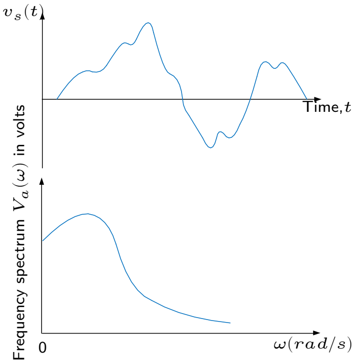


Schematic



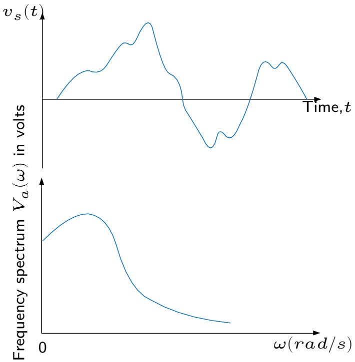
Floor Plan

Time and Frequency Domain



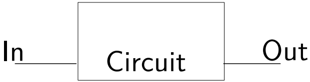
- Most signals of interest are time- varying (AC) signals, meaning their values change over time, such as an acoustic signals.
- For example, a voltage waveform as a function of time is shown to the right.

Time and Frequency Domain



- These AC signals consist of different frequency components. We often need to analyze and manipulate signals to control what frequency components should be processed (amplified, filtered, digitized, etc.).
- Therefore, we often need to switch between time and frequency domains, and need to convert signals from one domain to the other.

Circuit Input-Output Relationships



- We are interested in understanding how a circuit behaves in response to a specific type of signal. In so doing, we can also design circuits to deliver a specific behavior.
- There are several different kinds of responses we are interested in, and different ways that we can analyze these responses.

Time Domain $\Longleftrightarrow$ Frequency Domain

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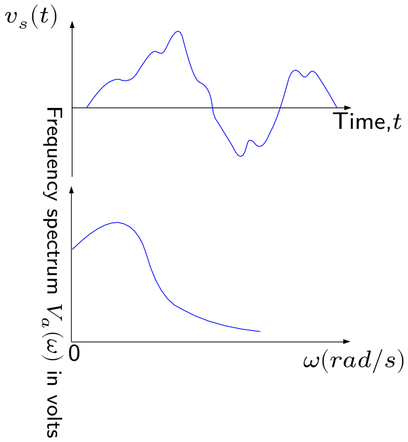
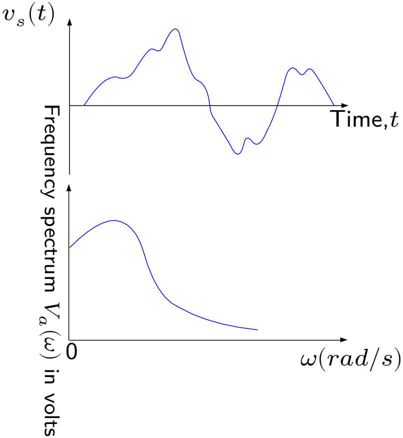


- Use Fourier and Laplace transform to convert signals between the two domains.
- Remember that any signal v(t) can be considered as a superposition (sum) of a number of sinusoids at given frequency.
- So we can analyze the response of a circuit to a sinusoid of a given frequency, and then obtain the complete response of a circuit to a signal v(t) by adding up all of the individual sinusoidal responses at frequency components that are contained within v(t).

Time Domain $\Longleftrightarrow$ Frequency Domain

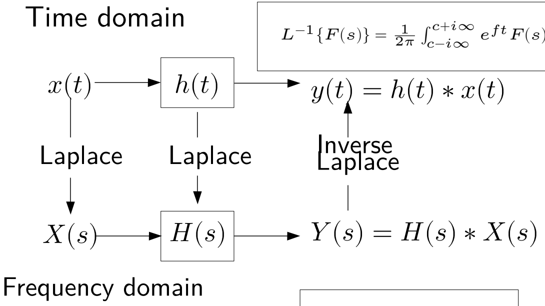


- So we need to be able to switch between time and frequency domains.
- In addition, it is much easier to analyze circuits containing energy soring components, such as L/C, in the frequency domain since we do not heed to solve complicated differential equations.

Use s-domain to Get Time Response

- Figure stuff out in the s-domain (frequency domain) $f(t) \Longleftrightarrow F(s)$
  ....Then go back to time domain

$$L^{-1}\{F(s)\} = \frac{1}{2\pi}\int_{c-i\infty}^{c+i\infty} e^{ft}F(s)ds = f(t)$$



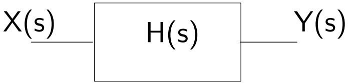
$$L\{f\}(s) = \int_0^{\infty} f(t)e^{-st}dt$$

Example: Step Unit Function

Time- and Frequency Responses

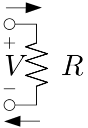
- Analyze the circuit, which includes reactive components (L/C), in the frequency (s-domain).

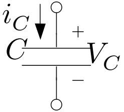
$$H(s)=Y(s)/X(s)$$

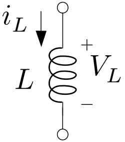


- Then revert back to the time domain, or continue to analyze in the s-domain to find the transfer function in the s-domain in response to a specific input X(s), and then use inverse transform to find the time domain response.
- Note that finding the time domain response to inputs like a step function is analytically difficult when circuits get complicated.
- But things get a lot easier in ther frequency s-domain as we will see.

Capacitors and Inductors





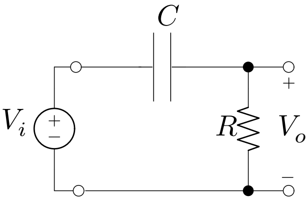


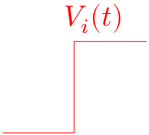
Time-Domain

Frequency-Domain

Capacitors and Inductors

$$V_o = H(s)V_i(s) \tag{36}$$





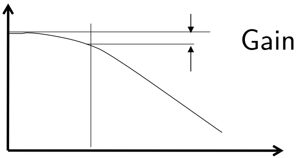
$$V(s) = \frac{sRC}{1 + sRC} \tag{37}$$

What would be the output?

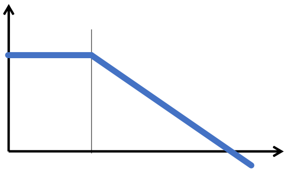
Pole

- $A_v = \frac{1}{1+sRC}$
- $\omega_p = -\frac{1}{RC}$
  What happens at the poles?
- $A_v\left(w_p\right) = \frac{1}{1+j\omega_p CR}$
- $A_v\left(w_p\right) = \frac{1}{1+j\frac{1}{CR}CR} = \frac{1}{1+j}$
- $\mid A_v\left(w_p\right)\mid = \mid \frac{1}{1+j} \mid = \frac{1}{\sqrt{2}}$
- Gains drops by $\sqrt{2}$

Pole



Gains drops by 3dB at ωp



Useful Relationship

- Another important Laplace transform

$$\frac{K}{(s+\alpha)(s+\beta)} \Leftrightarrow \frac{K}{(\beta-\alpha)}\left(e^{-\alpha t}-e^{-\beta t}\right)u(t) \tag{38}$$

- For example

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{s}{\left(1+\frac{s}{10^3}\right)\left(1+\frac{s}{10^4}\right)} \tag{39}$$

- $v_{\text{out}}(t) = \frac{10^7}{10^4-10^3}\left(e^{-1000t}-e^{-10000t}\right)u(t)$

Frequency Response of Circuits

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Clearly capacitors and inductors influence the time response of circuits, and they clearly influence the frequency response as well. As we just saw time-domain and frequency-domain responses are related. The transfer function of a linear dynamic circuit can be described using a generalized system function

$$H(s) = H_0 \frac{\prod_z (s - s_z)}{\prod_p (s - s_p)} = H_0' \frac{\prod_z (\tau_z s - 1)}{\prod_p (\tau_p s - 1)} \tag{40}$$

$$s_z = \frac{1}{\tau_z} \rightarrow \text{ zero at } s_z = \sigma_z + j\omega_z; \sigma_z = \mathrm{Re}\left(s_z\right), \omega_z = \mathrm{Im}\left(s_z\right) \tag{41}$$

$$s_p = \frac{1}{\tau_p} \rightarrow \text{ pole at } s_p = \sigma_p + j\omega_p; \sigma_p = \mathrm{Re}\left(s_p\right), \omega_p = \mathrm{Im}\left(s_p\right) \tag{42}$$

Frequency Response of Circuits

Because the zeros are the roots of the numerator polynomial,

$$\left|H\left(s_z\right)\right| = 0 \tag{43}$$

the poles are the roots of the denominator polynomial

$$\left|H\left(s_p\right)\right| = \infty \tag{44}$$

In physical systems singularities with

$$\mathrm{Im}(s) \neq 0 \tag{45}$$

always occur in complex conjugate pairs:

$$\mathrm{Re}\left(s_1\right) = \mathrm{Re}\left(s_2\right) \tag{46}$$

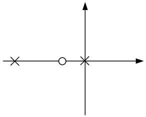
$$\mathrm{Im}\left(s_1\right) = -\mathrm{Im}\left(s_2\right). \tag{47}$$

Frequency Response of Circuits

It is of course possible to mathematically find the magnitude and angle of transfer function, but this could be complicated when there are multiple zeros and poles. Instead, we use complex plane and bode plot.

All Singularities (i.e., poles and zeros) are represented by positions in the complex plane, by vectors drawn from the point of interest in frequency domain to these poles and zeros.

- $G_{(s)} = \frac{s+2}{s^2+5s}$
- Zeros at $s = -2$, poles at $s = 0$ and $s = -5$.



Representation of Poles and Zeros

- A system has a pair of complex conjugate poles $p_1, p_2 = -1 \pm j2$, a single real zero $z_1 = -4$, and a gain factor $K = 3$. Find the differential equation representing the system. The transfer function is
- The transfer function is

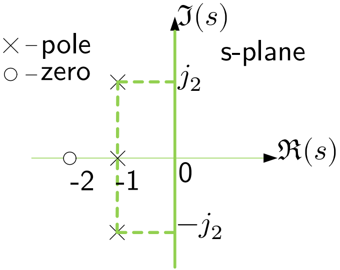
$$\begin{aligned} H(s) &= K \frac{s - z}{(s - p_1)(s - p_2)} \\ &= 3 \frac{s - (-4)}{(s - (-1 + j2))(s - (-1 - j2))} \\ &= 3 \frac{(s + 4)}{s^2 + 2s + 5} \end{aligned} \tag{48}$$

Representation of Poles and Zeros

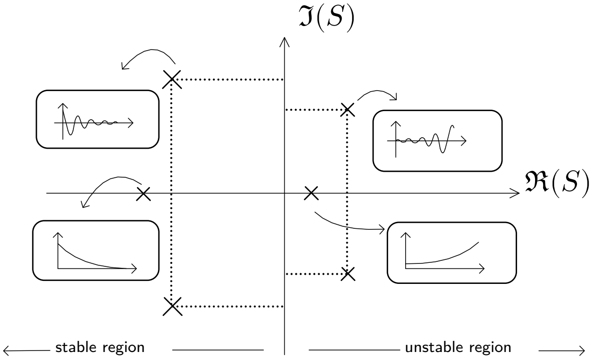
$$H(s) = 3\frac{(s+4)}{s^2+2s+5} \qquad (49)$$

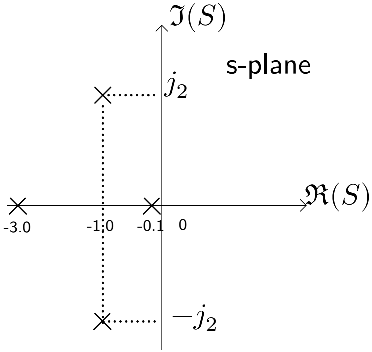
and the differential equation is

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 3\frac{du}{dt} + 12u \qquad (50)$$



Time Domain vs. Poles and Zeros





Time Domain vs. Poles and Zeros

$$y_h(t) = C_1 e^{-3t} + C_2 e^{-0.1t} + Ae^{-t}\sin(2t + \phi) \tag{51}$$

decays rapidly decays in approximately 4 seconds. (4τ)

Calculate τ and decays pretty much for 4-5 τ

Representation of Poles and Zeros

$$|H(s)| = K\frac{\prod_{i=1}^{m} |(s-z_i)|}{\prod_{i=1}^{n} |(s-p_i)|} \tag{52}$$

$$\angle H(s) = \sum_{i=1}^{m} \angle(s-z_i) - \sum_{i=1}^{n} \angle(s-p_i) \tag{53}$$

The magnitude of each of the component is the distance of the point s from the pole or zero on the s-plane.

Representation of Poles and Zeros

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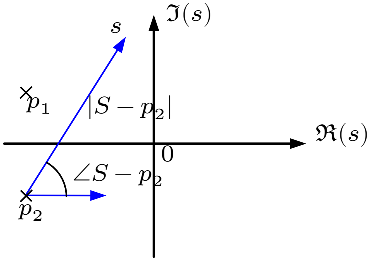
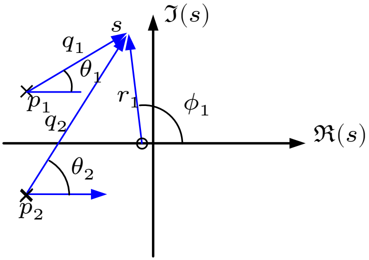
- if the vector from the pole pi to the point s on a pole-zero plot has a length qi and an angle i from the horizontal, and the vector from the zero zi to the point s has a length ri and an angle ϕ_1
- $|H(s)| = K\frac{r_1 \dots r_m}{q_1 \dots q_n}$
- $\angle H(s) = (\phi_1 + \dots + \phi_m) - (\theta_1 + \dots + \theta_n)$ The magnitude of the transfer function is proportional to the product of the geometric distances on the s-plane from each zero to the point s divided by the product of the distances from each pole to the point.

Representation of Poles and Zeros

The angle of the transfer function is the sum of the angles of the vectors associated with the zeros minus the sum of the angles of the vectors associated with the poles



$$|H(s)| = K\frac{r}{q_1 q_2} \tag{54}$$



$$\angle H(s) = \phi_1 - \theta_1 - \theta_2 \tag{55}$$

Representation of Poles and Zeros

- A second-order system has a pair of complex conjugate poles at $s = -2 \pm j3$ and a single zero at the origin of the s-plane. Find the transfer function and use the pole-zero plot to evaluate the transfer function at $s = 0 + j5$.

$$H(s) = K \frac{s}{(s - (-2 + j3))(s - (-2 - j3))} = K \frac{s}{s^2 + 4s + 13} \tag{56}$$