
VE311 Electronic Circuit Homework 4

Due: July 4th

Note:

- 1) Please use A4 size paper or page.*
- 2) Please clearly state out your final result for each question.*
- 3) Please attach the screenshot of Pspice simulation result if necessary.*

Question 1. MOSFET DC Biasing

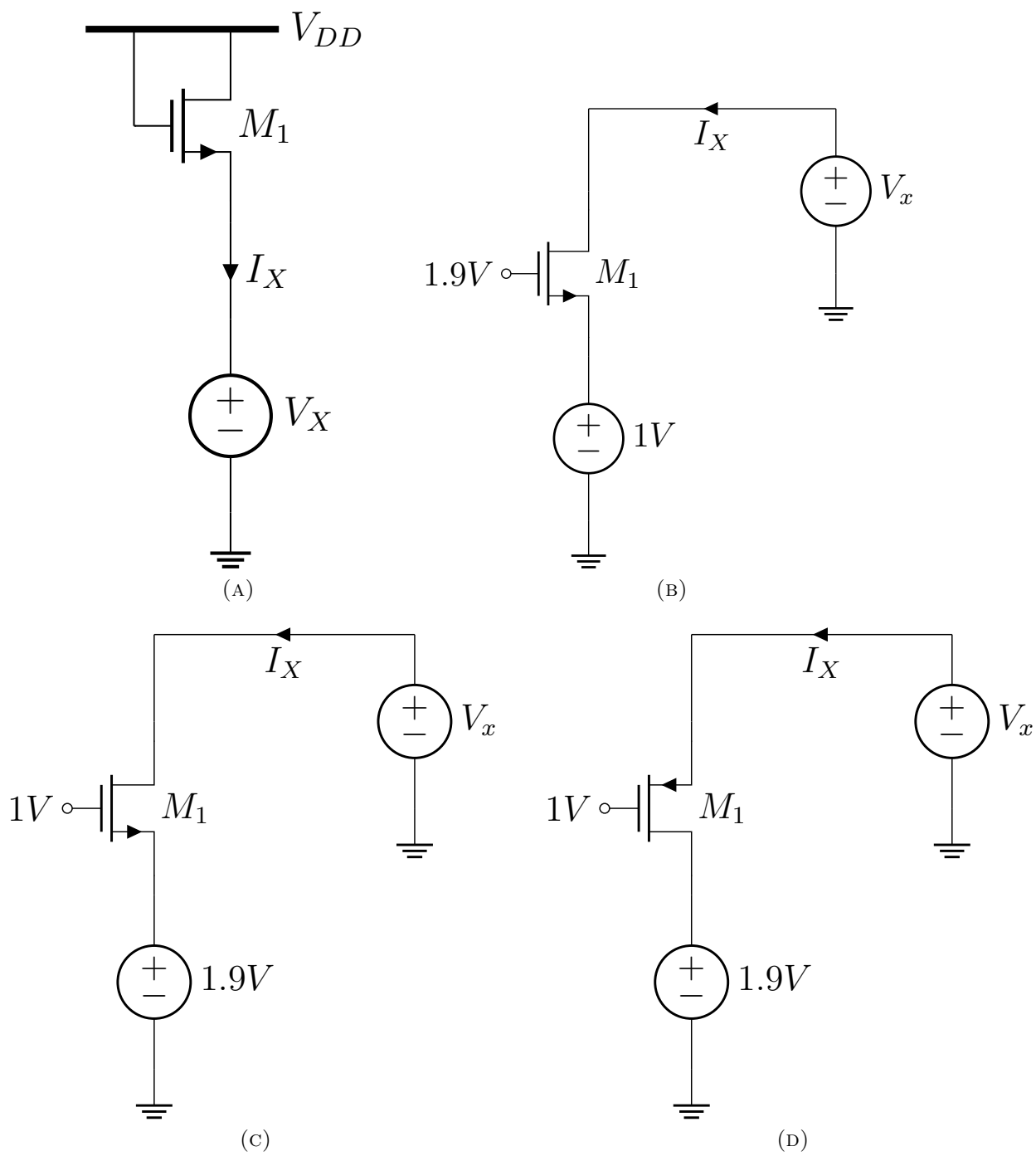
Use the drain current equations below. Don't consider channel-length modulation and body effect. Assuming $\mu_n = 350 \times 10^{-4} \text{m}^2/\text{V}/\text{s}$, $\mu_p = 350 \times 10^{-4} \text{m}^2/\text{V}/\text{s}$, $V_{TH} = 0.7\text{V}$ (NMOS), $V_{TH} = -0.8\text{V}$ (PMOS), $W_{drawn}/L_{drawn} = 20\mu\text{m}/2\mu\text{m}$, $t_{ox} = 9 \times 10^{-9}\text{m}$, $L_D = 0.08\mu\text{m}$, sketch I_X of M_1 as a function of V_X increasing from 0V to $V_{DD} = 5\text{V}$.

$$(1) \quad I_D = \mu_n C_{ox} \frac{W}{L_{eff}} \left[(V_{GS} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^2 \right] \text{ (NMOS in triode region)}$$

$$(2) \quad I_D = \mu_n C_{ox} \frac{W}{L_{eff}} (V_{GS} - V_{TH})^2 \text{ (NMOS in saturation region)}$$

$$(3) \quad I_D = \mu_p C_{ox} \frac{W}{L_{eff}} \left[(V_{SG} - |V_{TH}|) V_{SD} - \frac{1}{2} V_{SD}^2 \right] \text{ (PMOS in triode region)}$$

$$(4) \quad I_D = \mu_p C_{ox} \frac{W}{L_{eff}} (V_{SG} - |V_{TH}|)^2 \text{ (PMOS in saturation region)}$$



Q1:

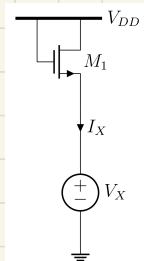
Already know: $\mu_n = 350 \times 10^{-4} \text{ m}^2/\text{V/s}$, $\mu_p = 350 \times 10^{-6} \text{ m}^2/\text{V/s}$, $V_{TH} = 0.7 \text{ V (NMOS)}$

$V_{TH} = -0.8 \text{ V (PMOS)}$, $\frac{W_{drawn}}{L_{drawn}} = \frac{20 \mu\text{m}}{2 \mu\text{m}}$, $t_{ox} = 9 \times 10^{-9} \text{ m}$, $L_D = 0.08 \mu\text{m}$, $V_{DD} = 5 \text{ V}$

$$\Rightarrow L_{eff} = L_{drawn} - 2L_D = 2 - 2 \times 0.08 = 1.84 \mu\text{m} \Rightarrow \frac{W}{L_{eff}} = \frac{20}{1.84} = \frac{250}{23}$$

$$\epsilon_{ox} = 3.9 \epsilon_0 = 3.45 \times 10^{-11} \text{ F/m} \Rightarrow C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = 3.83 \times 10^{-3} \text{ F/m}^2$$

(A)



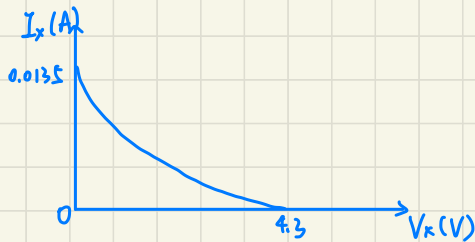
From the graph, we know it's a NMOS. $\Rightarrow V_{TH} = 0.7 \text{ V}$, $V_{GS} = V_{DD} - V_x = V_{GS}$

$\Rightarrow V_{DS} \geq V_{GS} - V_{TH} = V_{DS} - 0.7 \Rightarrow$ it's in saturation region

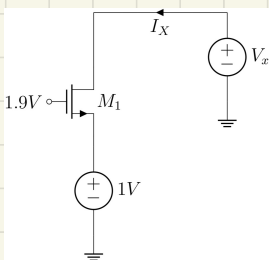
$V_{GS} \leq V_{TH} \Leftrightarrow 5 - V_x \leq 0.7 \Leftrightarrow V_x \geq 4.3 \text{ V} \Rightarrow$ it's in cut off region $I_x = 0 \text{ A}$

$$V_x \leq 4.3 \text{ V} \Rightarrow \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V_{GS} - V_{TH})^2 = \frac{1}{2} \cdot 350 \times 10^{-4} \cdot 3.83 \times 10^{-3} \cdot \frac{250}{23} (4.3 - V_x)^2$$

$$\Rightarrow I_x = 7.285 \times 10^{-4} \times (V_x - 4.3)^2$$



(B)



From the graph, we know it's a NMOS. $\Rightarrow V_{TH} = 0.7 \text{ V}$

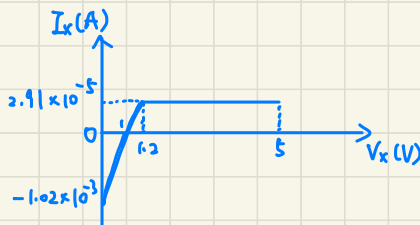
$V_{GS} = 0.9 \text{ V} > V_{TH}$, $V_{DS} = V_x - 1 \Rightarrow V_{GS} - V_{TH} = 0.2 \text{ V}$

① $V_{DS} < 0.2 \text{ V} \Leftrightarrow V_x < 1.2 \text{ V} \Rightarrow$ Triode Region

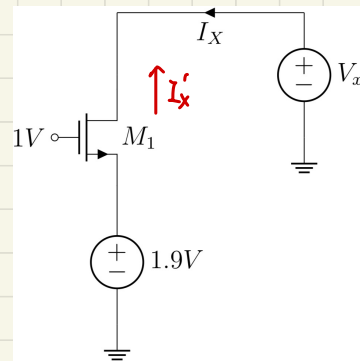
$$I_D = \mu_n C_{ox} \frac{W}{L_{eff}} \left[(V_{GS} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^2 \right] = 7.285 \times 10^{-4} \cdot \left[(V_x - 1.2)^2 - \frac{1}{2} \right]$$

② $V_{DS} \geq 0.2 \text{ V} \Leftrightarrow V_x \geq 1.2 \text{ V} \Rightarrow$ Saturation Region

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V_{GS} - V_{TH})^2 = 2.91 \times 10^{-5} \text{ A}$$



(C)



From the graph, we know it's a NMOS. $\Rightarrow V_{TH} = 0.7V$

$$V_{GS} = -0.9V < V_{TH}. \quad V_{DS} = V_X - 1.9$$

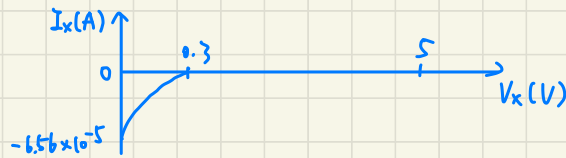
$$V_X > 1.9V \Rightarrow \text{cut off region } I_X = 0A$$

$$V_X < 1.9V \Rightarrow \text{change drain \& source terminal}$$

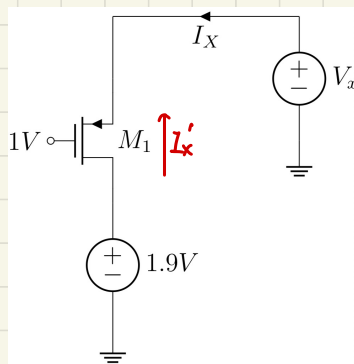
$$\Rightarrow V'_{GS} = 1 - V_X \quad V_X \geq 0.3V \Rightarrow V'_{GS} \leq V_{TH} \cdot \text{cutoff region}$$

$$V_X < 0.3V \Rightarrow V_{DS} = 1.9 - V_X > V'_{GS} - V_{TH} = 0.3 - V_X \Rightarrow \text{saturation}$$

$$I'_X = \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V'_{GS} - V_{TH})^2 \Rightarrow I_X = -7.285 \times 10^{-4} \cdot (V_X - 0.3)^2$$



(D)



From the graph, we know it's a NMOS. $\Rightarrow V_{TH} = -0.8V$

$$V_{GS} = 1 - V_X, \quad V_{DS} = 1.9 - V_X$$

$$\textcircled{1} V_X \leq 1.9V \Rightarrow \text{change drain \& source}$$

$$V_{GS} = -0.9V, \quad V_{DS} = V_X - 1.9$$

$$\text{i) } V_{GS} - V_{TH} \leq V_{DS} \text{ triode region}$$

$$\Rightarrow -0.9 + 0.8 \leq V_X - 1.9 \Rightarrow V_X \geq 1.8V$$

$$I'_X = \mu_p C_{ox} \frac{W}{L_{eff}} [(V_{GS} - |V_{TH}|) V_{SD} - \frac{1}{2} V_{SD}^2]$$

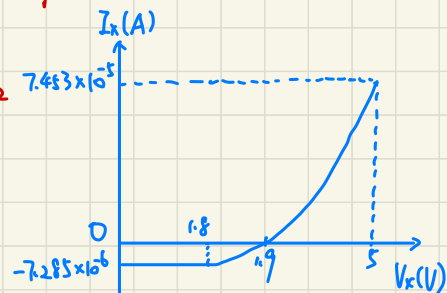
$$= -7.285 \times 10^{-4} (V_X - 1.7) (V_X - 1.9)$$

$$\text{ii) } V_{GS} - V_{TH} > V_{DS} \text{ saturation region}$$

$$\Rightarrow V_X < 1.8V$$

$$I'_X = \frac{1}{2} \mu_p C_{ox} \frac{W}{L_{eff}} (V_{GS} - |V_{TH}|)^2$$

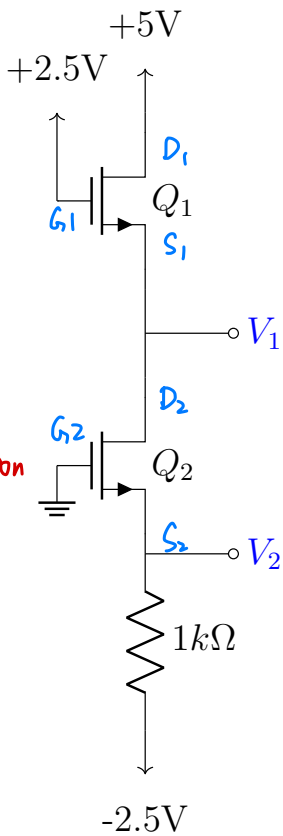
$$= 7.285 \times 10^{-6} A$$



Question 2. Combination of MOSFET

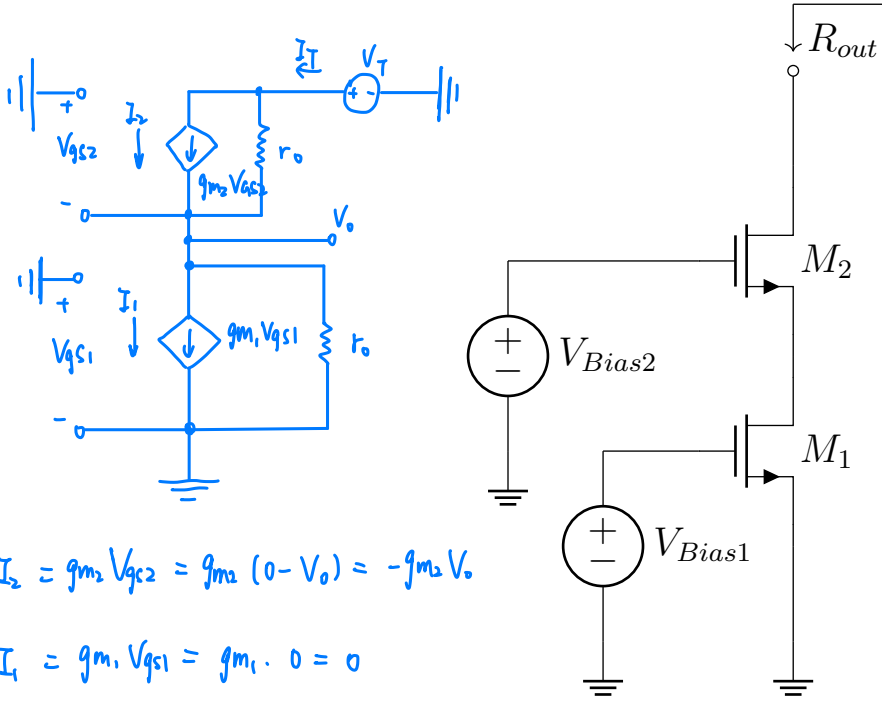
For the circuit below, find the labeled node voltages. The NMOS transistor has $V_{TH} = 0.9V$, $k_n = \mu_n C_{ox}(W/L) = 1.5mA/V^2$.

$$\begin{aligned}
 &V_{GS1} = 2.5 - V_1, V_{DS1} = 5 - V_1 \\
 &\Rightarrow V_{GS1} - V_{TH} = 1.6 - V_1 < V_{DS1} \\
 &\Rightarrow M_1 \text{ in saturation } (V_1 \leq 1.6V) \\
 &V_{GS2} = -V_2, V_{DS2} = V_1 - V_2 \\
 &\Rightarrow V_{GS2} - V_{TH} = -V_2 - 0.9 \\
 &\text{Assume } M_2 \text{ also in saturation region} \\
 &V_{GS2} \geq 0.9V \Rightarrow V_2 \leq -0.9V \\
 &\Rightarrow I_1 = I_2 \Leftrightarrow \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V_{GS1} - V_{TH})^2 \\
 &\quad = \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V_{GS2} - V_{TH})^2 \\
 &\Rightarrow |V_{GS1} - 0.9| = |V_{GS2} - 0.9| \\
 &\Leftrightarrow |1.6 - V_1| = |V_2 + 0.9| \Rightarrow V_1 - V_2 = 2.5V \\
 &\text{Meanwhile } V_2 = -2.5 + 10^3 I_2 \Rightarrow I_2 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (-V_2 - V_{TH})^2 = \frac{1}{2} \times 1.5 \times 10^{-3} (2.5 - 10^3 I_2 - 0.9)^2 \\
 &\Rightarrow I_2 = 6.857 \times 10^{-4} A \\
 &\Rightarrow \begin{cases} V_1 = 0.66V \\ V_2 = -1.84V \end{cases} \quad \left(\Rightarrow V_{GS2} - V_{TH} = 0.94V < V_{DS2} = 2.5V \Rightarrow \text{assumption is correct} \right)
 \end{aligned}$$



Question 3. Small Signal of of MOSFET

The circuit shown below is a MOSFET cascode amplifier. Draw the small signal model and derive R_{out} for the amplifier. Assume transistors M_1 and M_2 are in saturation and include r_o in your calculation.



$$I_2 = g_{m2} V_{gs2} = g_{m2} (0 - V_o) = -g_{m2} V_o$$

$$I_1 = g_{m1} V_{gs1} = g_{m1} \cdot 0 = 0$$

$$\frac{V_T - V_o}{r_o} + I_2 = \frac{V_o}{r_o}$$

$$\Rightarrow V_o = \frac{V_T}{r_o} \cdot \frac{1}{\frac{2}{r_o} + g_{m2}}$$

$$\Rightarrow I_T = \frac{V_o}{r_o} = \frac{V_T}{r_o} \cdot \frac{1}{2 + g_{m2} r_o} \Rightarrow R_{out} = \frac{V_T}{I_T} = r_o (2 + g_{m2} r_o)$$

Question 4. Common-Source with Resistive Load

Assume $\lambda = 0$ and $\gamma = 0$. For $V_{DD} = 5V$, $V_{in} = 0.9V + \text{small signal}$, $R_D = 15k\Omega$ and $L_{drawn} = 2\mu m$, find out the value W_{drawn} to obtain a voltage gain $|A_v| > 10$ and V_{OUT} (the DC biasing voltage at the output) close to 2.5 V as much as possible.

$$|A_v| = g_m R_D = k_n' \frac{W}{L_{eff}} (V_{GS} - V_{TH}) R_D > 10$$

$$\Rightarrow k_n' \frac{W}{L_{eff}} > \frac{1}{300}$$

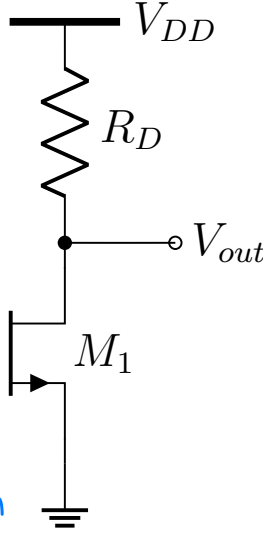
$$V_{out} \approx 2.5V \Rightarrow I_D = \frac{V_{DD} - V_{out}}{R_D} = \frac{1}{6000} A$$

$$\Rightarrow V_{DS} = 2.5V > V_{GS} - V_{TH} = 0.2V \Rightarrow \text{saturation}$$

$$\Rightarrow I_D = \frac{1}{2} k_n' \frac{W}{L_{eff}} (V_{GS} - V_{TH})^2 = \frac{1}{6000} A$$

$$\Rightarrow k_n' \frac{W}{L_{eff}} = \frac{1}{6000} \times 2 \times \frac{1}{0.2^2} > \frac{1}{300} \quad \text{valid}$$

$$\Rightarrow W = \frac{1}{120} \cdot \frac{L_{eff}}{k_n'} = \frac{1}{120} \cdot \frac{L_{drawn} - 2L_D}{\mu_n C_{ox}} = 114.2 \text{ nm}$$



NMOS Model

LEVEL=1	VTO=0.7	GAMMA=0.45	PHI=0.9
NSUB=9e+14	LD=0.08e-6	UO=350	LAMBDA=0.1
TOX=9e-9	PB=0.9	CJ=0.56e-3	CJSW=0.35e-11
MJ=0.45	MJSW=0.2	CGDO=0.4e-9	JS=1.0e-8

PMOS Model

LEVEL=1	VTO=-0.8	GAMMA=0.4	PHI=0.8
NSUB=5e+14	LD=0.09e-6	UO=100	LAMBDA=0.2
TOX=9e-9	PB=0.9	CJ=0.94e-3	CJSW=0.32e-11
MJ=0.5	MJSW=0.3	CGDO=0.3e-9	JS=0.5e-8

VTO : threshold voltage with zero V_{SB} (unit : V)

GAMMA : body effect coefficient (unit : $V^{1/2}$)

PHI : $2\Phi_F$ (unit : V)

TOX : gate oxide thickness (unit : m)

NSUB : substrate doping (unit : cm^{-3})

LD : source/drain side diffusion (unit : m)

UO : channel mobility (unit : cm^2/Vs)

LAMBDA : channel-length modulation coefficient (unit : V^{-1})

CJ : source/drain bottom-plate junction capacitance per unit area (unit : F/m^2)

CJSW : source/drain sidewall junction capacitance per unit length (unit : F/m)

PB : source / drain junction built-in potential (unit : V)

MJ : exponent in CJ equation (unitless)

MJSW : exponent in CJSW equation (unitless)

CGDO: gate-drain overlap capacitance per unit width (unit : F/m)

CGSO : gate-source overlap capacitance per unit width (unit : F/m)

JS : source/drain leakage current per unit area (unit : A/m^2)

Vacuum permittivity (ϵ_o) = 8.85×10^{-12} (F/m)

Silicon oxide dielectric constant (ϵ_r) = 3.9