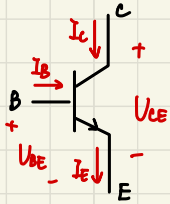
BJT

① 放大电路概述

U_{BE}	发射极与基极电压
I_B	基极电流
I_C	集电极电流
U_{CE}	管压降



* 静态分析：直流通路求静态工作点 Q：U_{BE} . I_B . I_c . U_{CE}

电容当断路

其中：硅管 $|U_{BE}| = 0.7V$，锗管 $|U_{BE}| = 0.3V$

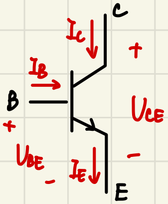
* 动态分析：交流通路求动态性能指标：A_u . R_i . R_o

电容短路，去掉直流独立电源

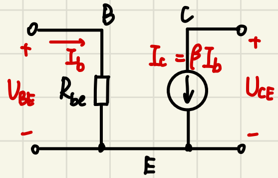
+ 微变等效电路模型

A_u	放大倍数	← $\frac{U_o}{U_i}$
R_i	输入电阻	← $\frac{U_i}{I_i}$
R_0	输出电阻	← 去源，R_L开路，$= \frac{U_o}{I_o}$

② 微变等效电路模型



⇒



BE之间连一个等效电阻 R_{be}

CE之间连一个电流源 $I_c = \beta I_b$

$$R_{be} = R_{bb'} + (1+\beta)\frac{26\,(mV)}{I_E\,(mA)}$$

↓

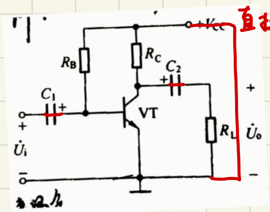
基区体电阻（题目会给 $R_{bb'}$）

③ 当三极管在放大区时

$$\begin{cases} I_c = \beta I_b \\ I_e = (1+\beta) I_b \\ I_e = I_b + I_c \end{cases}$$

④ 画微变等效电路

电容看做短路，直流电源作“去源”处理，擦去三极管并用等效电路替换



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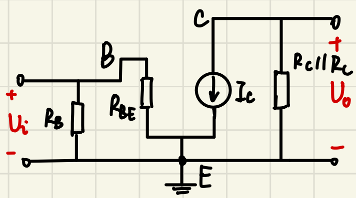
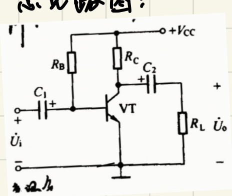


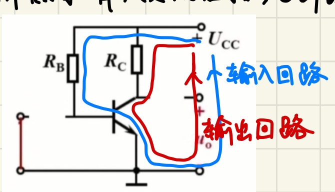
⑤ BJT 双极型三极管的共射放大电路

共射电路：基电极 B 输入，集电极 C 输出

总电路图：



静态工作点求解：（交流短接，C断路）



step 1 求 I_B：$I_B R_B + U_{BE} - U_{CC} = 0$

step 2 求 I_c：$I_c = \beta I_B$

step 3 求 U_{CE}：$I_c R_c + U_{CE} - U_{CC} = 0$

输入电压 V_{GS} 会由直流的 V_{GS} 和交流的 v_{gs} 组成 ($V_{GS}=V_{GS}+v_{gs}$)

$L_{eff}=L_{drawn}-2L_D$

对于直流部分，我们可以用直流电路来求出 I_D：
- 先比 V_{DS} 是否 $>V_{GS}-V_{TH}$ 来判断是否饱和
- 再看是否需要考虑 channel-length modulation

(例子.公式均为NMOS管)

对于 I_D：① 不饱和 (triode region) $\rightarrow V_{GS}-V_{TN}\geq V_{DS}>0$

$$I_D=\underbrace{\underbrace{\mu_n C_{ox}}_{k_n'}\left(\frac{W}{L_{eff}}\right)}_{k_n}\left[(V_{GS}-V_{TH})V_{DS}-\frac{1}{2}V_{DS}^2\right]$$

② 饱和并且考虑 channel-length modulation

$$I_D=\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L_{eff}}\right)(V_{GS}-V_{TH})^2(1+\lambda V_{DS}) \text{ 或 } I_D=\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L_{eff}-\Delta L}\right)(V_{GS}-V_{TH})^2$$

③ 饱和但不考虑 channel-length modulation

$$I_D=\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L_{eff}}\right)(V_{GS}-V_{TH})^2$$

Body effect：

$$\gamma=\frac{\sqrt{2q\varepsilon_{si}N_{sub}}}{C_{ox}}$$

不考虑时 $V_{TH}=V_{TH0}$

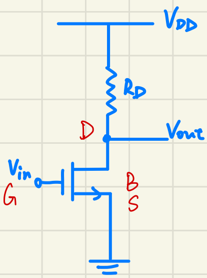
考虑时 $V_{TH}=V_{TH0}+\gamma\left(\sqrt{2\phi_F+V_{SB}}-\sqrt{2\phi_F}\right)$

求完 I_D 之后才能进 small signal 分析：

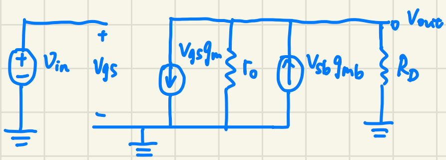
$$g_m=\begin{cases}\dfrac{2I_D}{V_{GS}-V_{TH}}=\sqrt{2\mu_n C_{ox}\dfrac{W}{L_{eff}-\Delta L}I_D}=\mu_n C_{ox}\dfrac{W}{L_{eff}-\Delta L}(V_{GS}-V_{TH}) & \text{不考虑 channel}\\[2ex] \sqrt{2\mu_n C_{ox}\left(\dfrac{W}{L}\right)I_D(1+\lambda V_{DS})}=\mu_n C_{ox}\dfrac{W}{L}(V_{GS}-V_{TH})(1+\lambda V_{DS}) & \text{考虑 channel}\end{cases}$$

$$r_o=\begin{cases}0 & \text{不考虑 channel}\\[2ex] \dfrac{1}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L_{eff}-\Delta L}(V_{GS}-V_{TH})^2\cdot\lambda}=\dfrac{1}{I_D\cdot\lambda} & \text{考虑 channel}\end{cases}$$

$$g_{mb}=\begin{cases}0 & \text{不考虑 body effect}\\[2ex] g_m\cdot\dfrac{\gamma}{2\sqrt{2\phi_F+V_{SB}}}=\eta g_m & \text{考虑 body effect}\end{cases}$$



微变电路

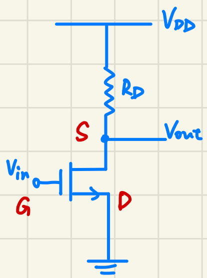


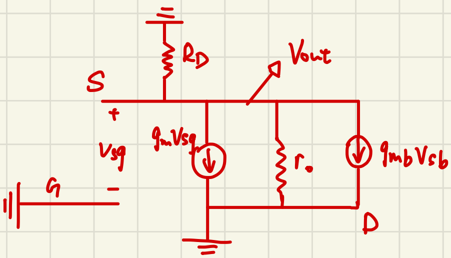
Gate：栅极

Source：源极

Drain：漏极

(ps：PMOS管公式加绝对值就行)





PMOS Formula Table

NMOS Transistor Mathematical Model Summary
The following equations represent the complete model for the $i-v$ behavior of the NMOS transistor.
For all regions,

$$K_p = K_p'\frac{W}{L} \quad K_p' = \mu_p C_{ox}'' \quad i_G = 0 \quad i_B = 0 \tag{15}$$

Cut off region,

$$i_D = 0 \quad \text{for } V_{GS} \geq V_{TP} \tag{16}$$

Triode region,

$$i_D = K_p\left(v_{GS} - V_{TP} - \frac{v_{DS}}{2}\right)v_{DS} \quad \text{for } 0 \leq |v_{DS}| \leq |v_{GS} - V_{TP}| \tag{17}$$

PMOS Formula Table

Saturation region,

$$i_D = \frac{K_p}{2}\left(v_{GS} - V_{TP}\right)^2\left(1 + \lambda|v_{DS}|\right) \quad \text{for } |v_{DS}| \geq |v_{GS} - V_{TP}| \geq 0 \tag{18}$$

Threshold voltage,

$$V_{TP} = V_{TO} + \gamma\left(\sqrt{v_{SB} + 2\phi_F} - \sqrt{2\phi_F}\right) \tag{19}$$

For the enhancement-mode PMOS transistor,$V_{TP} < 0$.Depletion-mode PMOS devices can also be fabricated;$I_{TP} \geq 0$ for these devices.

Body effect（体效应），也称作substrate effect（底座效应），是指在MOSFET（金属氧化物半导体场效应晶体管）中，由于栅极和底座（substrate）之间的电压差异，导致晶体管特性发生变化的现象。

MOSFET是由P型或N型的底座（衬底）和两个与之电性相反的源极（Source）和漏极（Drain）组成。栅极（Gate）位于底座上，通过控制栅极电压可以控制源极和漏极之间的电流。在MOSFET中，底座与源极之间的电压差会影响晶体管的工作特性，这就是体效应。

具体来说，当在MOSFET的底座与源极之间施加反向偏置（对于N型MOSFET是正向偏置），底座的电势会影响通道区域的形成，从而影响源极和漏极之间的电流。这是因为底座与通道之间的电势差会改变通道中的载流子浓度。底座与源极电压之间的这种关系可以通过MOSFET的参数之一，称为阈值电压（Threshold Voltage），来表示。

在集成电路设计和应用中，需要考虑体效应，尤其是在高度集成的CMOS电路中。通过了解体效应的影响，工程师可以更好地优化MOSFET的特性，以确保电路的性能和可靠性。

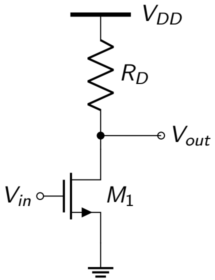
Channel-Length Modulation（沟道长度调制）是指在MOSFET（金属氧化物半导体场效应晶体管）中，当沟道长度发生变化时，晶体管的特性也会相应地改变的现象。

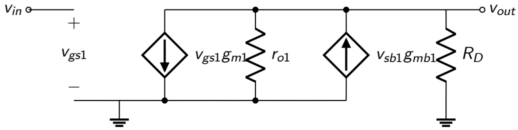
MOSFET的沟道长度指的是源极和漏极之间形成的通道区域的长度。当MOSFET处于饱和区（Saturation Region）时，沟道长度的变化会导致漏极电流（ID）发生调制，即漏极电流与漏极电压（VDS）之间的关系不再是简单的线性关系。

具体来说，当VDS增大时，沟道长度会因为底座与源极之间的电场而增加。由于沟道长度的增加，通道中的电荷载流子数量增多，从而导致漏极电流ID增加。这种现象就是沟道长度调制。

在高电压或高电流的工作条件下，沟道长度调制会成为一个显著影响MOSFET工作的因素。这可以导致MOSFET的输出电流不再是一个固定的值，而是与VDS有关。为了准确建模和设计MOSFET电路，工程师需要考虑沟道长度调制效应，特别是在高性能和高精度的电路应用中。

CS with Resistive Load





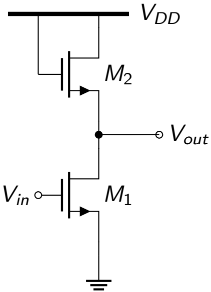
If no channel-length and body effect:

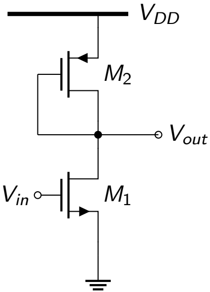
$$A_v = \frac{v_{out}}{v_{in}} = -g_{m1} R_D \tag{1}$$

No body effect:

$$A_v = -g_{m1}(R_D \parallel r_{o1}) \tag{2}$$

CS with Diode-connected Load





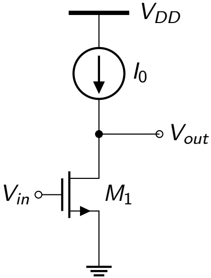
NMOS:

$$A_\mathrm{v} = -\sqrt{\frac{(W/L)_1}{(W/L)_2}}\frac{1}{1+\eta} \qquad \eta = g_\mathrm{mb2}/g_\mathrm{m2} \tag{3}$$

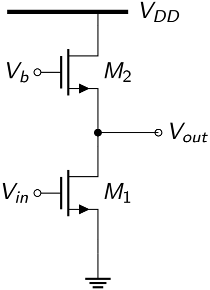
PMOS:

$$A_\mathrm{v} = -\sqrt{\frac{\mu_\mathrm{n}(W/L)_1}{\mu_\mathrm{p}(W/L)_2}} \tag{4}$$

CS with Current Source Load

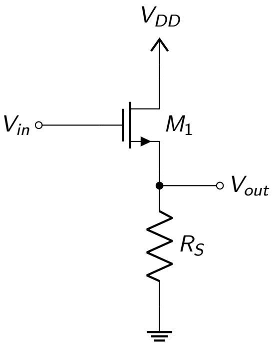


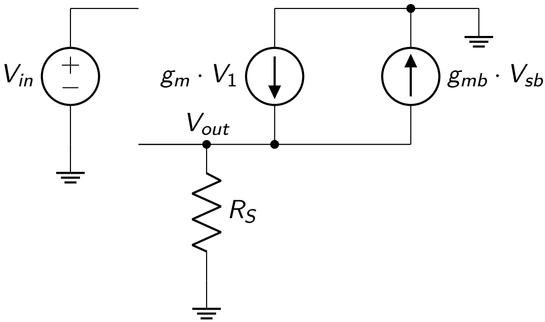
or



$$A_v = -g_{m1}(r_{o2} \parallel r_{o1}) \tag{5}$$

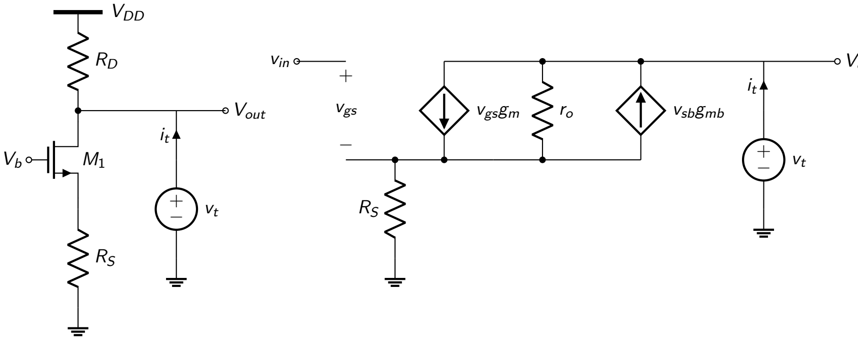
Source Follower





$$A_v = \frac{g_m R_S}{1 + g_m R_S(1+\eta)} = \frac{g_m R_S}{1 + (g_m + g_{mb}) R_S} \approx \frac{1}{1+\eta} \quad (6)$$

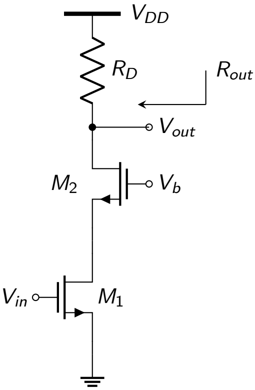
Common Gate



$$R_{in} = \frac{R_D + r_o}{1 + (g_m + g_{mb})r_o} \begin{cases} \text{If } R_D = 0 & R_{in} = r_o \parallel \frac{1}{g_m} \parallel \frac{1}{g_{mb}} \\ \text{If } R_D = \infty & R_{in} = \infty \end{cases} \tag{7}$$

$$R_{out} = [R_S + r_{o1} + (g_{m1} + g_{mb1})r_{o1}R_S] \parallel R_D \tag{8}$$

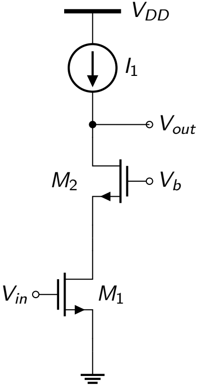
Cascode



$$G_m = -g_{m1} \frac{r_{o1}}{r_{o1} + \left(r_{o2} \parallel \frac{1}{g_{m2}+g_{mb2}} \right)} \quad (9)$$

$$R_{out} = [r_{o1}+r_{o2}+(g_{m2}+g_{mb2})r_{o2}r_{o1}] \parallel R_D \quad (10)$$

Cascode

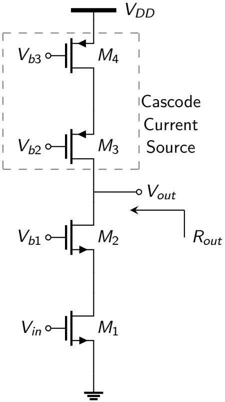


$$G_m = -g_{m1}\frac{r_{o1}}{r_{o1} + \left(r_{o2} \parallel \frac{1}{g_{m2}+g_{mb2}}\right)} \tag{11}$$

$$R_{out} = r_{o1} + r_{o2} + (g_{m2} + g_{mb2})r_{o2}r_{o1} \tag{12}$$

$$A_v = G_m R_{out} \tag{13}$$

Cascode

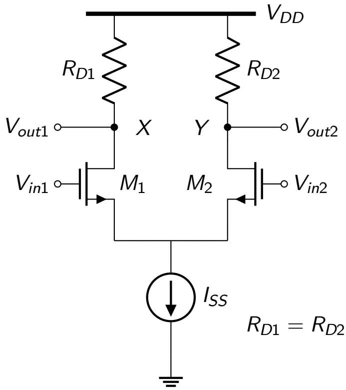


$$G_m = -g_{m1}\frac{r_{o1}}{r_{o1} + (r_{o2} \parallel \frac{1}{g_{m2}g_{mb2}})} \tag{14}$$

$$R_{out} = [r_{o1} + r_{o2} + (g_{m2} + g_{mb2})r_{o2}r_{o1}] \parallel [r_{o3} + r_{o4} + (g_{m3} + g_{mb3})r_{o3}r_{o4}] \tag{15}$$

$$A_v = G_m R_{out} \tag{16}$$

Differential Pair



$R_{D1} = R_{D2} = R_D$

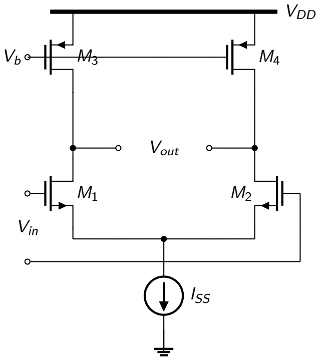
$$A_{DM} = \frac{V_{out1} - V_{out2}}{v_d} = -g_m(R_D \parallel r_o) \quad (17)$$

If the circuit is fully symmetric,

$$A_{CM-DM} = \frac{V_{out1} - V_{out2}}{v_{in,CM}} = 0 \quad (18)$$

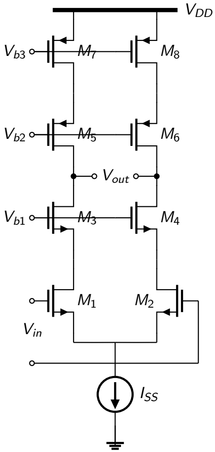
$$CMRR = \left| \frac{A_{DM}}{A_{CM-DM}} \right| = \infty \quad (19)$$

Differential Pair with MOS Loads



$$A_{DM} = -g_{m1,2}(r_{o1,2} \parallel r_{o3,4}) \tag{20}$$

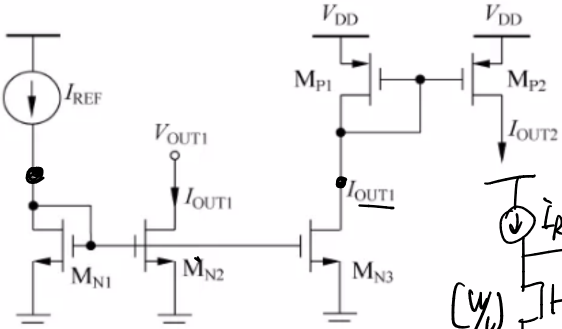
Differential Pair with Cascode Loads

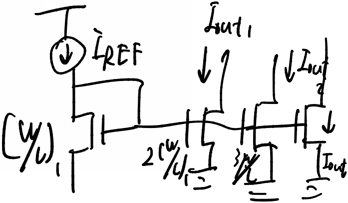


$$A_{DM} \cong -g_{m1,2}[(g_{m3,4} + gmb_{3,4})r_{o3,4}r_{o1,2} \parallel (g_{m5,6} + g_{mb5,6})r_{o5,6}r_{o7,8}] \tag{21}$$

Current Mirror

at most 4 MOSFET





$$I_{OUT1} = \frac{(W/L)_{N2}}{(W/L)_{N1}} I_{REF}$$

$$I_{OUT2} = \frac{(W/L)_{N3}}{(W/L)_{N1}} \frac{(W/L)_{P2}}{(W/L)_{P1}} I_{REF}$$

$I_{out1} = 2 I_{REF}$

$I_{out2} = 3 I_{REF}$

Bode 图绘制

① 化成标准形式：

如 $H(s) = \dfrac{25(0.1s+1)}{s^2(0.2s+1)}$

此时有比例环节（1个）：25 $\boxed{k}$ ⇒ 低频部分会过 $(1, 20\lg k)$ ⟶ 此处 $(1, 20\lg 25)$

积分环节（2个）：$\dfrac{1}{s^2}$

导前环节（1个）：$0.1s+1$

惯性环节（1个）：$0.2s+1$

比例环节：$G(s)=k$

积分环节：$G(s)=\dfrac{1}{s}$

微分环节：$G(s)=s$

惯性环节：$G(s)=\dfrac{1}{Ts+1}$

导前环节：$G(s)=Ts+1$

确定了第一条线

Ⅱ型系统（有n个积分环节就是n型系统）

⇓

$V=2$ ⇒ 低频部分斜率为 $-20\ dB/dec \cdot V = -40\ dB/dec$ 此处

② 求频率特性（把 s 变为 jw）

$$H(jw) = \frac{25(j0.1w+1)}{(jw)^2(j0.2w+1)}$$

对导前环节 $j0.1w+1$，$w_2 = \dfrac{1}{0.1} = 10\ s^{-1}$

对惯性环节 $j0.2w+1$，$w_1 = \dfrac{1}{0.2} = 5\ s^{-1}$

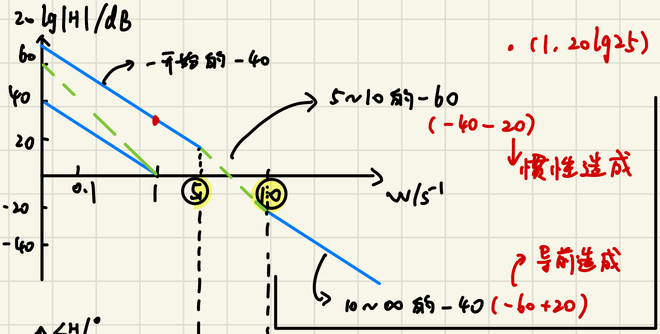
（小的标1）

对导前环节 w_2 开始斜率上升 20 dB/dec

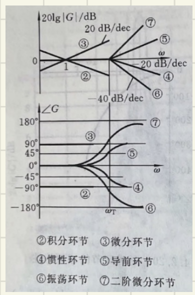
对惯性环节 w_1 开始斜率下降 20 dB/dec

③ 画图

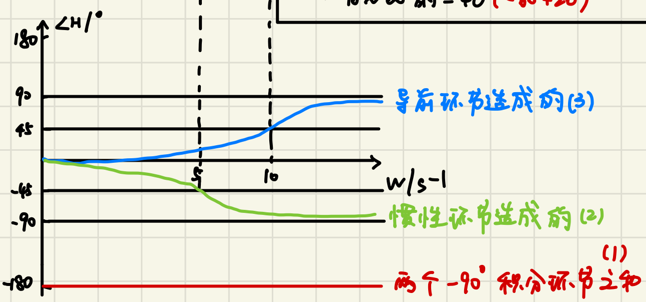
幅频图 ⟶



不用管比例环节



相频图 ⟶



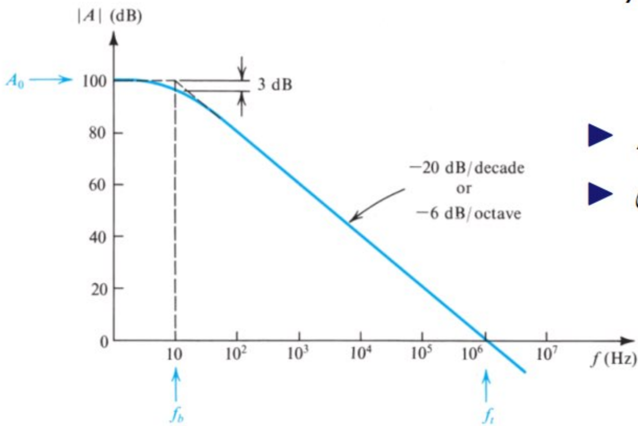
⇓

把(1)(2)(3)加起来即可得到结果

Pole-zero representation

- Bode plot (magnitude)
  - Zero contributes 20dB/decade slope if $s > s_z$
  - Pole contributes -20dB/decade slope if $s > s_p$
  - Complex conjugate zeros contributes 40dB/decade slope if $s >\mid s_z \mid$
  - Complex conjugate poles contributes -40dB/decade slope if $s >\mid s_p \mid$
- Bode plot (phase)
  - 90° if $s_z = 0$
  - -90° if $s_p = 0$
  - Increase by 90° and passes though the midpoint of 45° at the break point $s = s_z \neq 0$
  - Decrease by 90° and passes though the midpoint of -45° at the break point $s = s_p \neq 0$

Finite op amp bandwidth



$$A(j\omega) = \frac{A_0}{1 + j\omega/\omega_b} \tag{8}$$

- A_0: dc gain
- ω_b: 3-dB frequency

Finite op amp bandwidth

- Magnitude:

$$| A(j\omega) | = \frac{A_0 \omega_b}{\omega} \tag{9}$$

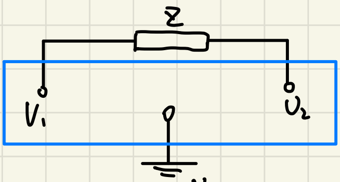
- Unity gain:

$$A(j\omega_t) = 1 \tag{10}$$

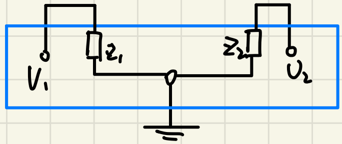
- unity-gain bandwidth (gain–bandwidth product)

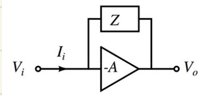
$$f_t = \frac{\omega_t}{2\pi} = \frac{A_0 \omega_b}{2\pi} \tag{11}$$

Miller Effect :



⇐⇒





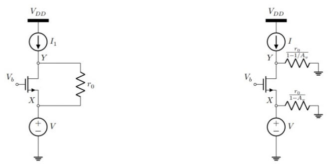
$$I_i = \frac{V_i - V_o}{Z} = V_i\left(\frac{1+A}{Z}\right)$$

$$Z_{in} = \frac{V_i}{I_i} = \frac{Z}{1+A}$$

设已经 $A = \frac{V_2}{V_1}$

$Z_1 = \frac{Z}{1-A}$, $Z_2 = \frac{Z}{1-\bar{A}}$

Example 1

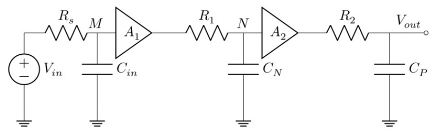


$$A_v = \frac{Y}{X} = 1 + (g_{m1} + g_{mb1})\, r_{o1} \qquad (4)$$

$$Z_1 = \frac{r_{01}}{1 - [1 + (g_{m1} + g_{mb1})\, r_{01}]} = \frac{-1}{g_{m1} + g_{mb1}} \qquad (5)$$

$$R_{in} = \frac{-1}{g_{m1} + g_{mb1}} \parallel \frac{1}{g_{m1} + g_{mb1}} = \infty \qquad (6)$$

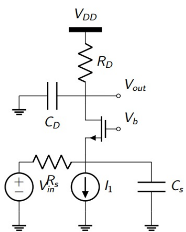
Association of Poles with Nodes



$$\frac{V_{out}}{V_{in}}(\omega) = A_1\, A_2 \frac{1}{\left(1+\frac{s}{\omega_M}\right)\left(1+\frac{s}{\omega_N}\right)\left(1+\frac{s}{\omega_P}\right)} \qquad (8)$$

$$\omega_M = \frac{1}{R_S C_{in}} \quad (9) \qquad \omega_N = \frac{1}{R_1 C_N} \quad (10) \qquad \omega_P = \frac{1}{R_2 C_P}\,(rad/s) \quad (11)$$

Example 2



$$\frac{V_{out}}{V_{in}} = \frac{(g_{m1} + g_{mb1})\, R_D}{1 + (g_{m1} + g_{mb1})\, R_s} \frac{1}{\left(1+\frac{s}{\omega_X}\right)\left(1+\frac{s}{\omega_Y}\right)} \qquad (12)$$

$$\omega_X = \frac{1}{\left(\frac{1}{g_{m1}+g_{mb1}} \parallel R_S\right) C_s}\,(rad/s) \qquad (13)$$

$$\omega_Y = \frac{1}{R_D C_D} \quad (rad/s) \qquad (14)$$

Exercise 1

Assume the op amp to be ideal except for having a finite differential gain A and $V_{sig} = 1V$. Use Miller's theorem to find R_{in}, V_i, V_o for each of the following values of A : 10, 100, 1000 (**without using knowledge of op-amp circuit analysis**)

$Z_1 = \frac{Z}{1+A}$, $Z_2 = \frac{Z}{1+\bar{A}}$

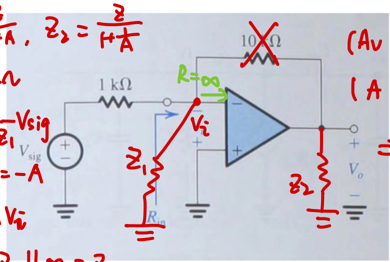
$Z = 10\ k\Omega$

$-V_i = \frac{Z_1}{10^3 + Z_1} V_{sig}$

since $A_v = -A$

$\Rightarrow V_o = A V_i$

$R_{in} = Z_1 \parallel \infty = Z_1$



$(A_v = \frac{-V_o}{V_i})$

$(A = \frac{V_o}{V_i})$

$\Rightarrow A_v = -A$

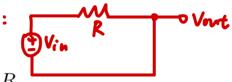
First Order Systems

$$H(s)=\frac{H^0+H^1\tau S}{1+\tau S} \qquad (15)$$

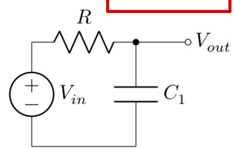
- Time constant: (1) For capacitor: $\tau = RC$; (2) For inductor: $\tau=\frac{L}{R}$
- To find the time constant, remove the cap/ind nulling all the sources and find the resistance.
- To find H^0, use low frequency gain(cap cut off and ind shorted).
- To find H^1, use high frequency gain(cap shorted and ind cut off).

Example 1

算 H_1:

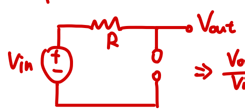


$\Rightarrow V_{out}=0 \Rightarrow \frac{V_{out}}{V_{in}}=0$



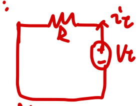
$$H^0=1 \qquad (16)$$
$$\tau = RC_1 \qquad (17)$$
$$H^1=0 \qquad (18)$$
$$H(s)=\frac{1}{1+RCS} \qquad (19)$$

算 H_0:



$\Rightarrow \frac{V_{out}}{V_{in}}=1$

算 τ:



$\Rightarrow \tau=\frac{V_t}{i_t}C_1=RC_1$

Exercise 2

For the circuit shown below
(a) Calculate H^0, H^1, τ
(b) Write down the transfer function

① H_0 (C_{GD} 断)

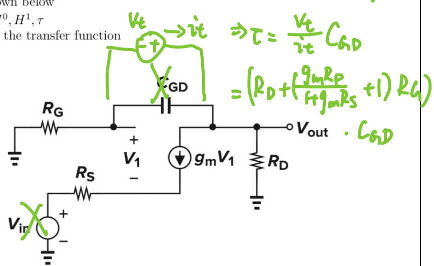
$\begin{cases} V_{in}+R_S g_m V_1 = V_{out} \\ V_{out} = -R_D g_m V_1 \end{cases}$

$\Rightarrow H^0=\frac{g_m R_D}{R_S g_m+1}$

② τ

$\begin{cases} g_m V_1 R_S = V_t - V_1 \\ -i_t R_G + V_t = (i_t - g_m V_1) R_D \end{cases}$

$\Rightarrow \tau = \frac{V_t}{i_t} C_{GD} = \left(R_D+\left(\frac{g_m R_D}{1+g_m R_S}+1\right)R_G\right) \cdot C_{GD}$



③ H^1 (C_{GD} 短)

$\begin{cases} V_{in}+R_S g_m V_1 = V_{out}-V_1 \\ \frac{V_{out}}{R_G}+\frac{V_{out}}{R_D}+g_m V_1=0 \end{cases}$

$\Rightarrow H^1=\frac{1}{1+(R_S+\frac{1}{g_m})(\frac{1}{R_D}+\frac{1}{R_G})}$

Nth Order Systems

$$H(s)=\frac{a_0+a_1 s+a_2 s^2}{1+b_1 s+b_2 s^2} \qquad (25)$$

$$b_1=\sum_{i=1}^{N}\tau_i^0 \qquad (26)$$

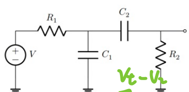
$$b_2=\sum_{i}^{i<j<N}\sum_{j}^{0}\tau_j^i \qquad (28)$$

$$a_1=\sum_{i=1}^{N}\tau_i^0 H^i \qquad (27)$$

$$a_2=\sum\sum_{j}^{i<j<N}\tau_i^0\tau_j^i H^{ij} \qquad (29)$$

- τ_j^i: the time constant of capacitor j when capacitor i is shorted
- τ_i^0: cut off all other caps

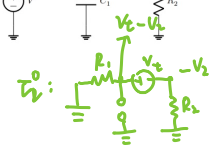
Example



- $\tau_2^1 = R_2 C_2$

- $\tau_1^2 = (R_1 \parallel R_2)\, C_1$
- $\tau_1^0\tau_2^1 = R_1C_1R_2C_2 = \tau_2^0\tau_1^2$
- $\tau_2^0\tau_1^2 = (R_1+R_2)\,C_2\frac{R_1R_2}{R_1+R_2}C_1$
- $\tau_2^0 = (R_1+R_2)\,C_2$
- $H^1=0$
- $H^2=\frac{R_2}{R_2+R_1}$
- $H^{12}=0$

τ_2^0:



$\frac{V_t-V_2}{R_1}=\frac{V_2}{R_2}=i_t$

$V_t=(1+\frac{R_1}{R_2})V_2 \Rightarrow \frac{V_t}{i_t}=R_1+R_2$

H^1: 断C_2, 短C_1

H^2: 断C_1, 短C_2

H^{12}: 都断

Example

- $H(s)=\frac{a_0+a_1s+a_2s^2}{1+b_1s+b_2s^2}$
- $b_2=\tau_1^0\tau_2^1=\tau_2^0\tau_1^2$
- $a_2=\tau_1^0\tau_2^1H^{12}=\tau_2^0\tau_1^2H^{12}$
- $b_1=\tau_1^0+\tau_2^0$
- $a_1=\tau_1^0H^1+\tau_2^0H^2$

二阶的公式

$$H(s)=\frac{R_2C_2S}{1+(R_1C_1+R_1C_2+R_2C_2)\,S+R_1C_1R_2C_2S^2} \qquad (30)$$