



Lecture 3

EECS 311 Analog Circuits

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2023 Summer



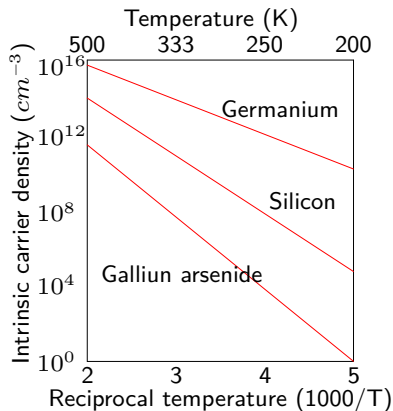
上海交通大學
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Recap of Last Lecture



- Scaling
- Review of Thevenin and Norton Equivalent
- Semiconductor Basics
- Diodes

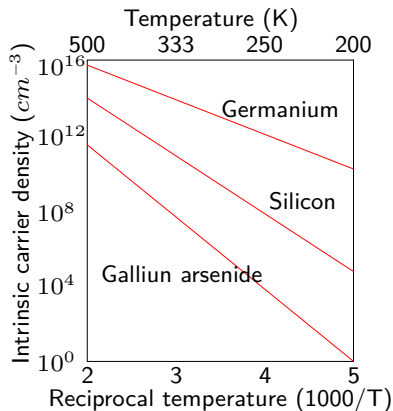
Intrinsic Si



$$np = n_i^2 = BT^3 \exp\left(-\frac{E_G}{kT}\right) \quad (1)$$

- n Free electrons in the conduction band.
- p Free holes in the valence band.
- n_i Intrinsic carrier density.

Intrinsic Si

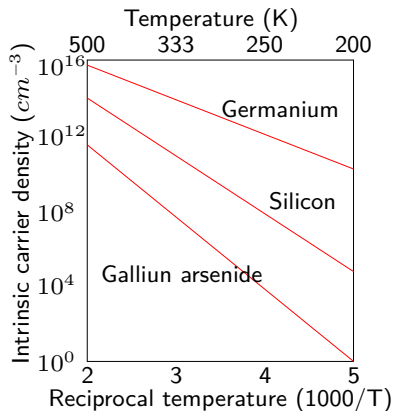


k (Boltzmann's Constant)

$$= 1.38 \times 10^{-23} \text{ J/K} = 8.62 \times 10^{-5} \text{ eV/K} \quad (2)$$

	$B \text{ (K}^{-3} \cdot \text{cm}^{-6})$	$E_G \text{ (eV)}$
Si	1.08×10^{31}	1.12
Ge	2.31×10^{30}	0.66
GaAs	1.27×10^{29}	1.42

Intrinsic Si

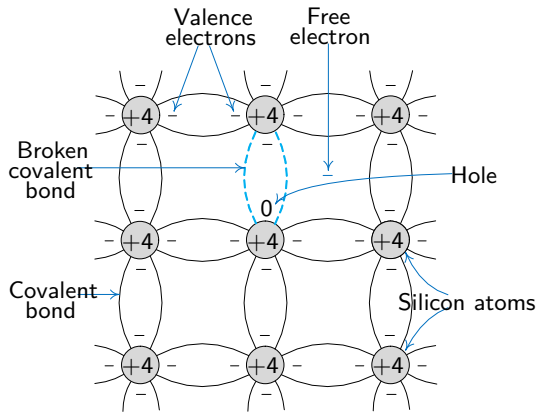
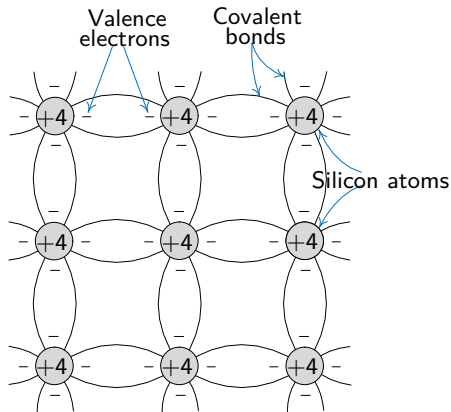


At 300K

$$\begin{aligned} n_i^2 &= (1.08 \cdot 10^{31}) \cdot 300^3 \cdot e^{\frac{-1.12}{(8.62 \cdot 10^{-5}) \cdot 300}} \\ &= 4.52 \times 10^{19} (\text{1/cm}^6) \end{aligned} \quad (3)$$

$$n_i = 6.73 \times 10^9 (\text{1/cm}^3) \approx 10^{10} (\text{1/cm}^3) \quad (4)$$

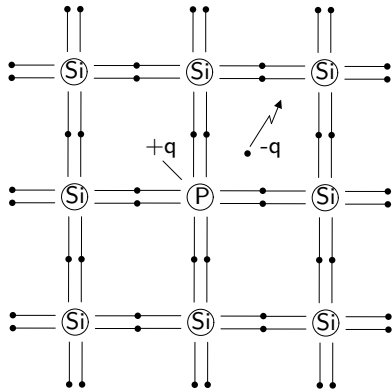
Electrons and Holes



Doped Si (N-type)



- At room temperature, nearly all phosphorus dopants are ionized.
- Each dopant donates one electron away, which creates an electron in the conduction band.
- P: $1s^2 2s^2 2p^6 3s^2 3p^3$
- B: $1s^2 2s^2 2p^1$



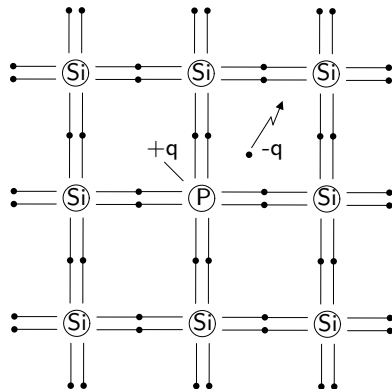
Doped Si (N-type)

- If the n-type dopant concentration $N_d \gg n_i$,
 $n = N_d$ and $p = n_i^2/N_d$
- charge neutrality

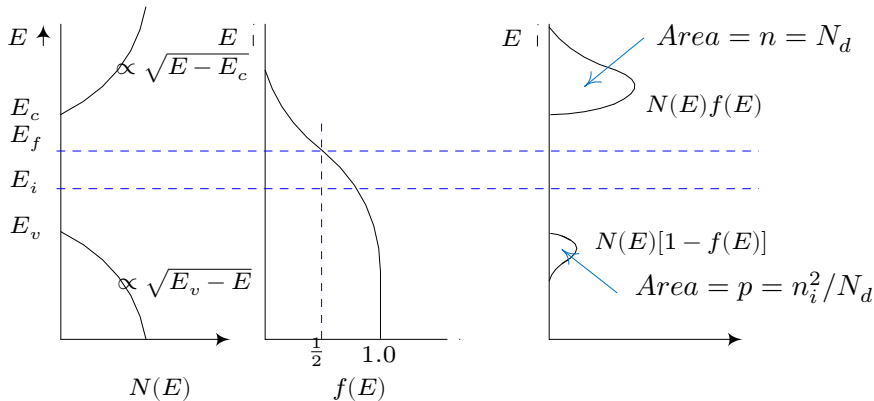
$$q(N_D + p - N_A - n) = 0 \quad (5)$$

$$n^2 - (N_D - N_A)n - n_i^2 = 0 \quad (6)$$

$$n = \frac{(N_D - N_A) + \sqrt{(N_D - N_A)^2 + 4n_i^2}}{2} \quad (7)$$

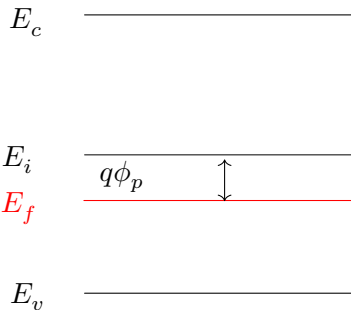


Density of States (N-type)



- Notice that the fermi level is shifted.

P-Type



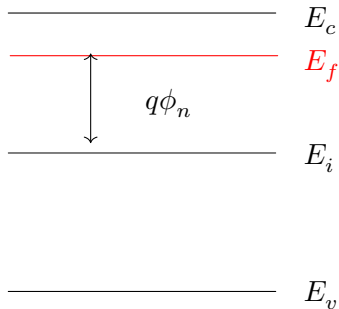
$$np = n_i^2 \quad (8)$$

$$p = N_a = n_i e^{\frac{E_i - E_f}{kT}} = n_i e^{\frac{q\phi_p}{kT}} \quad (9)$$

$$n = \frac{n_i^2}{N_a} = n_i e^{\frac{E_f - E_i}{kT}} = n_i e^{\frac{-q\phi_p}{kT}} \quad (10)$$

- Hu, Chenming. Modern semiconductor devices for integrated circuits.

N-Type



$$np = n_i^2 \quad (11)$$

$$n = N_d = n_i e^{\frac{E_f - E_i}{kT}} = n_i e^{\frac{q\phi_n}{kT}} \quad (12)$$

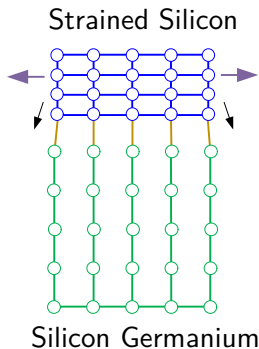
$$p = \frac{n_i^2}{N_d} = n_i e^{\frac{E_i - E_f}{kT}} = n_i e^{\frac{-q\phi_n}{kT}} \quad (13)$$

Carrier Concentration Practice



- By ion implantation on crystalline Si with phosphorous concentration equal to 10^{17} cm^{-3}
- What is the value of n , p , and E_f ?

Strained Silicon for Higher Mobility



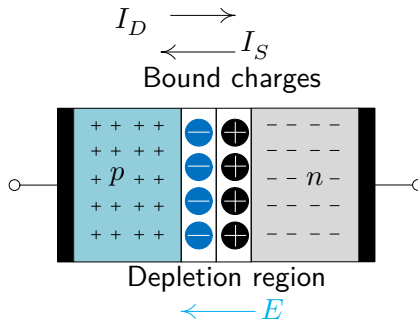
Johnson's figure of merit

Material	Saturation velocity $\times 10^{-5}$ m/s	$V_{breakdown}$ MV/cm	JFM
Silicon	1.0	0.3	1.0
GaAs	1.5	0.4	2.7
SiC	2.0	3.5	20
InP	0.67	0.5	0.33
GaN	2.5	3.3	27.5

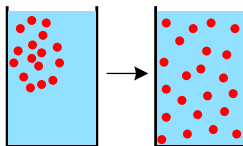
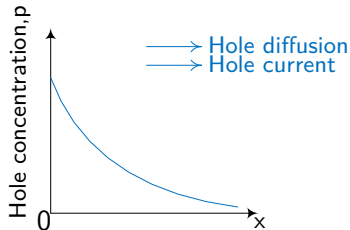
PN Junction



- Diffusion current
- space-charge (Depletion) region
- Drift current



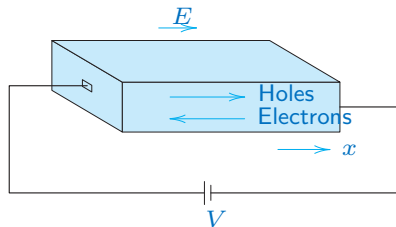
Diffusion



- Magnitude of the current at any point is proportional to the slope of the concentration profile, or the **concentration gradient** , at that point

$$J_{D,n} = qD_n \frac{dn(x)}{dx} \quad (14)$$

Drift Current



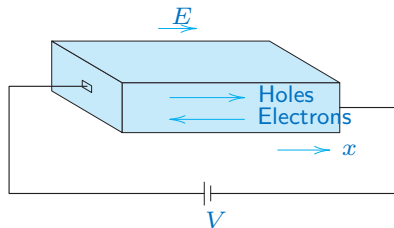
$$I_p = Aqp v_{p\text{-drift}} \quad (15)$$

$$I_p = Aqp \mu_p E \quad (16)$$

- For intrinsic silicon $\mu_p = 480 \text{ cm}^2/\text{V} \cdot \text{s}$.
- For the electron mobility, $\mu_n = 1350 \text{ cm}^2/\text{V}$

$$J_S = J_{S,p} + J_{S,n} = q(p\mu_p + n\mu_n) E \quad (17)$$

Drift Current

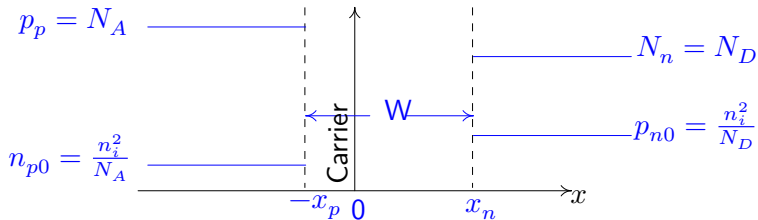
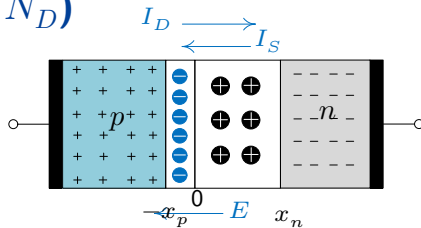


- The Junction Built-in Voltage

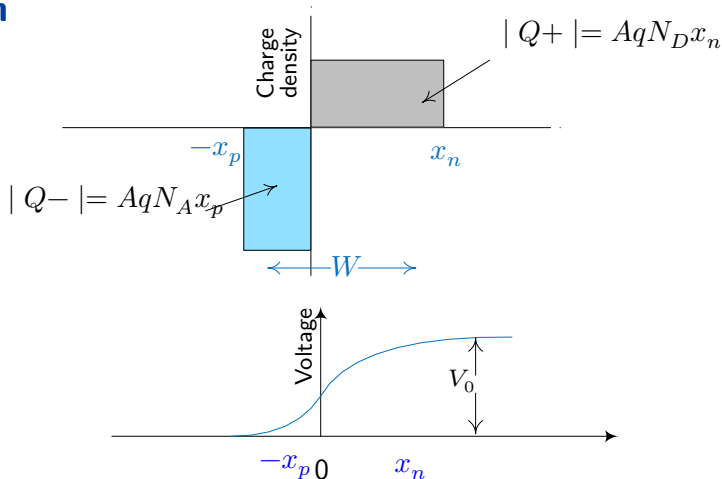
$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) \quad (18)$$

- Derivation: see Streetman and Bannerjee

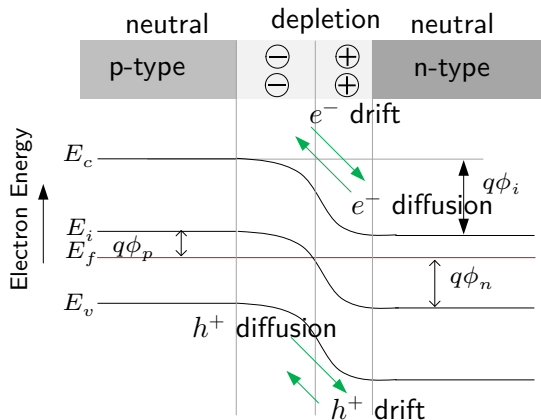
PN Junction ($N_A > N_D$)



PN Junction

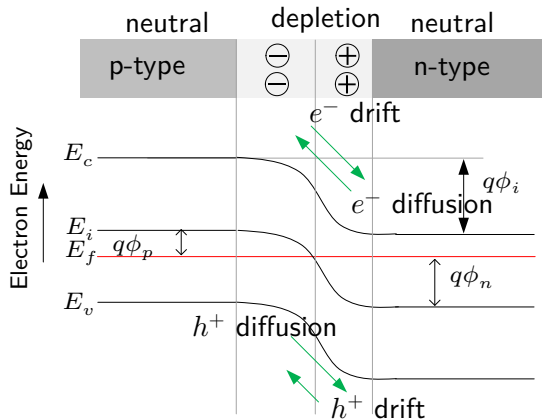


PN Junction



- 1 Electrons/holes near the junction diffuse to the opposite sides.
- 2 Ionized dopants, fixed in the lattice, are left behind.
- 3 Formation of built-in electric field and energy barrier ($q\phi_i$) for diffusion.

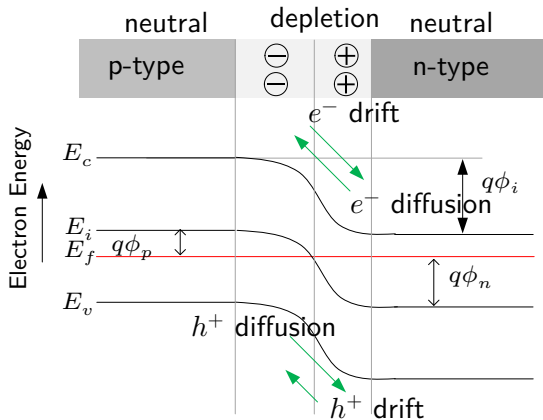
PN Junction



Note:

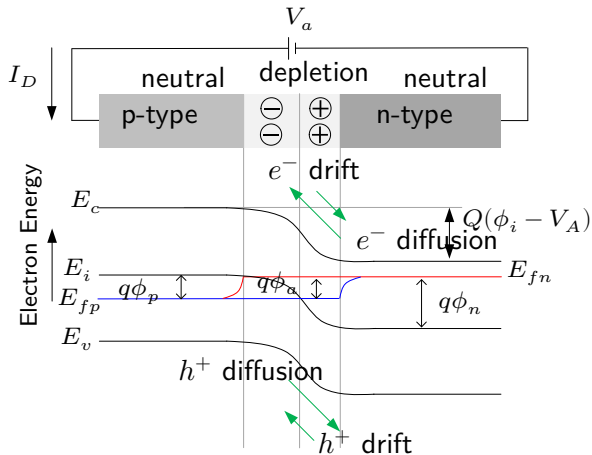
- E_C , E_i and E_V are parallel to each other.
- E_C , E_i and E_V bending means there is electric field.
- E_f bending means there is current.

PN Junction



- ④ Some electrons/holes in the neutral regions with sufficient energy continuously diffuse to the opposite sides. → Formation of diffusion current.
- ⑤ Some electrons/holes wandering into the in the depletion region get swept by the built-in electric field. → Formation of drift current.
- ⑥ Diffusion current cancels drift current. No net current flowing.

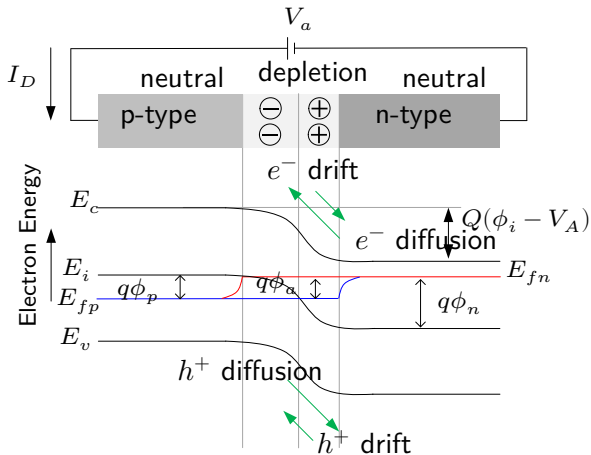
Forward Bias (When $V_a > 0$)



$$I_D = I_S \left(e^{\frac{qV_a}{kT}} - 1 \right) > 0 \quad (19)$$

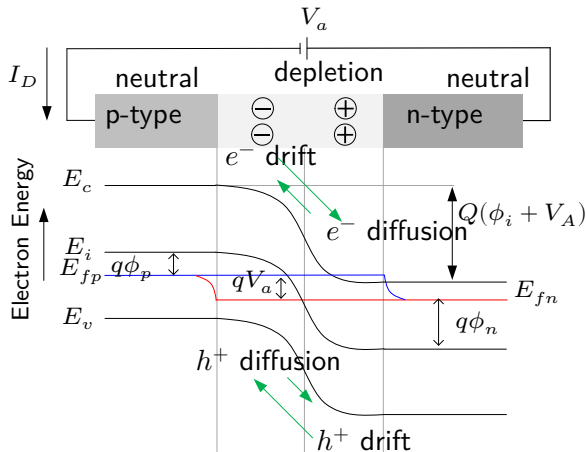
- ① $e^{\frac{qV_a}{kT}}$ is the diffusion current.
- ② -1 stands for the drift current.
- ③ The energy barrier formed by the built-in electric field becomes smaller, $q(\phi_i - V_a)$.
- ④ Neamen, Donald A. Semiconductor Physics and Devices Basic Principles, chapter 8

Forward Bias (When $V_a > 0$)



- 1 More electrons/holes diffuse to the opposite sides. → Diffusion current increases, while drift current remains the same.
- 2 There is (+) net current flowing.
- 3 The depletion width becomes narrower.

Reverse Bias ($V_a < 0$)

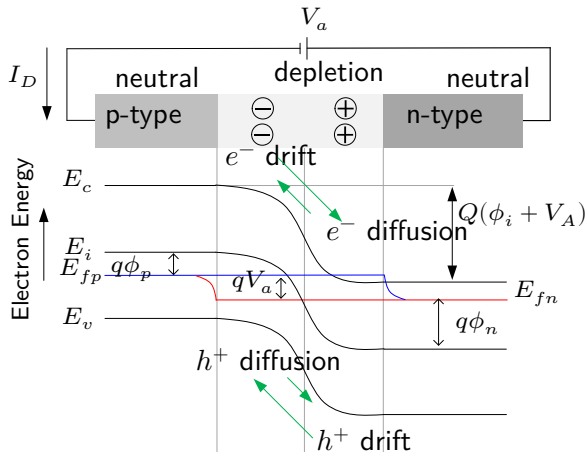


$$I_D = I_S \left(e^{\frac{qV_a}{kT}} - 1 \right) < 0 \quad (20)$$

- 1 The energy barrier formed by the built-in electric field becomes larger, $q(\phi_i + V_a)$.



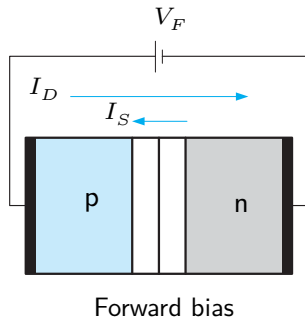
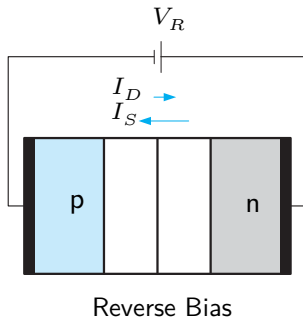
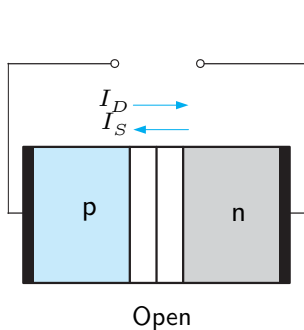
Reverse Bias ($V_a < 0$)



$$I_D = I_S \left(e^{\frac{qV_a}{kT}} - 1 \right) < 0 \quad (21)$$

- ① Less electrons/holes diffuse to the opposite sides. → Diffusion current decreases, while drift current remains the same.
- ② There is (-) net current flowing.
- ③ The depletion width becomes wider.

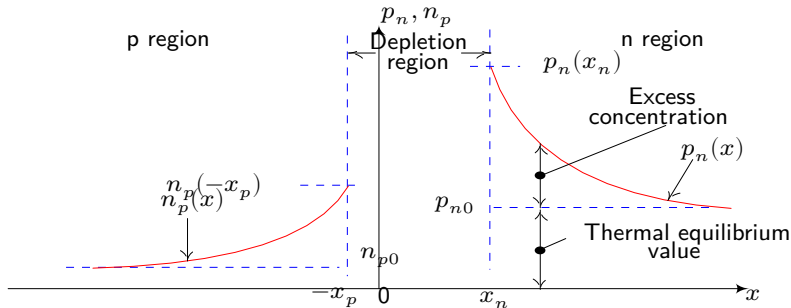
Operating Conditions of Diodes



Diode Current (Forward Bias, $N_A \gg N_D$)

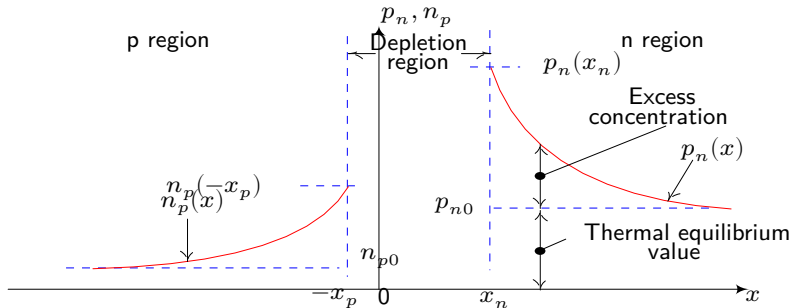


Sedra: 3.5.2



- a greater number of holes (electrons) to overcome the barrier and diffuse into the n (p) region

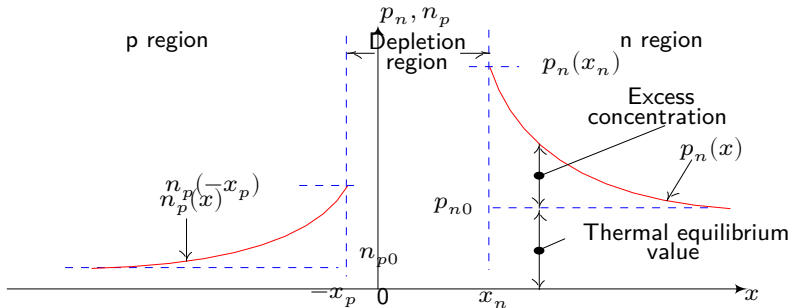
Diode Current



the steady-state concentration at the edge of the depletion region will be

$$p_n(x_n) = p_{n0} e^{V/V_T} \quad (22)$$

Location-Dependent Recombination



$$p_n(x) = Pn_0 + pn_0(e^{V/N_T} - 1)e^{-(x-x_n)/L_p} \quad (23)$$

L_p is the diffusion length

Diode Current



the value of the hole-diffusion current density, D is diffusivity.

$$J_p(x) = -qD_p \frac{dp_n(x)}{dx} \quad (24)$$

$$p_n(x) = p_{n0} + p_{n0} (e^{V/V_T} - 1) e^{-(x-x_n)/L_p} \quad (25)$$

$$J_p(x) = q \left(\frac{D_p}{L_p} \right) p_{n0} (e^{V/V_T} - 1) e^{-(x-x_n)/L_p} \quad (26)$$

$J_p(x)$ is highest at $x = x_n$,

$$J_p(x_n) = q \left(\frac{D_p}{L_p} \right) p_{n0} (e^{V/V_T} - 1) \quad (27)$$

Derivation of PN Junction Current Equation

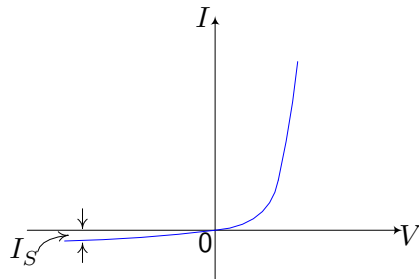


$$I = A(J_p + J_n) \quad (28)$$

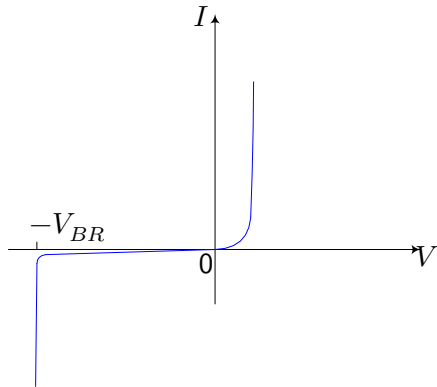
$$I = Aq \left(\frac{D_p}{L_p} p_{n0} + \frac{D_n}{L_n} n_{p0} \right) (e^{V/V_T} - 1) \quad (29)$$

Remember the following equation

$$I = I_S (e^{V/V_T} - 1) \quad (30)$$

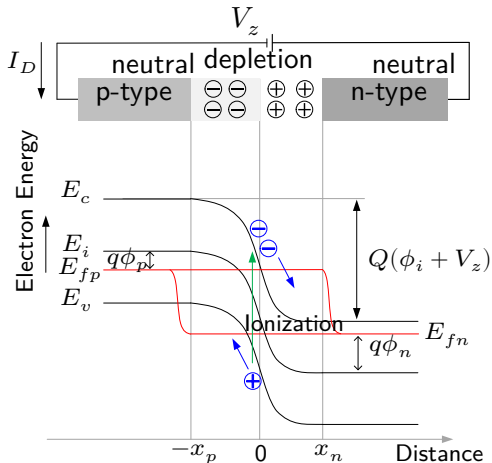


Reverse Breakdown



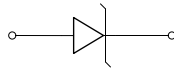
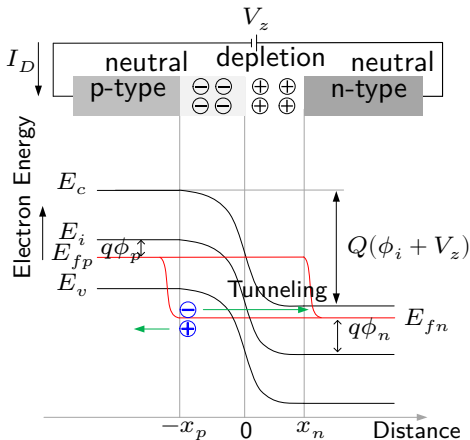
- the reverse-breakdown current is limited by the external circuit to a “safe” value.
- Two possible mechanisms for pn junction breakdown: the zener effect and the avalanche effect.

Avalanche Breakdown



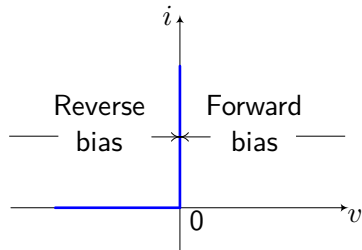
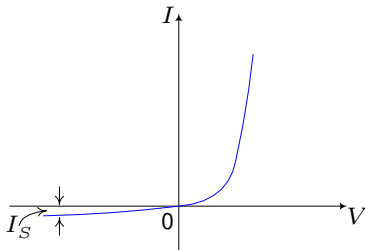
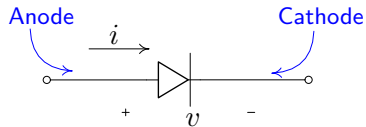
- Collision of electrons with atoms
- Si diode with breakdown voltages greater than about 5.6 V enter breakdown through an avalanche mechanism.
- Carriers accelerated by electric field gain sufficient energy to break covalent bonds upon impact, thereby creating electron-hole pairs.

Zener Breakdown

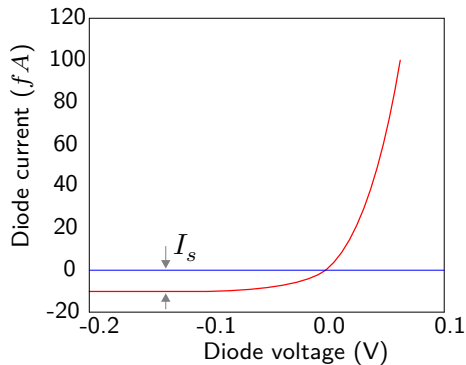
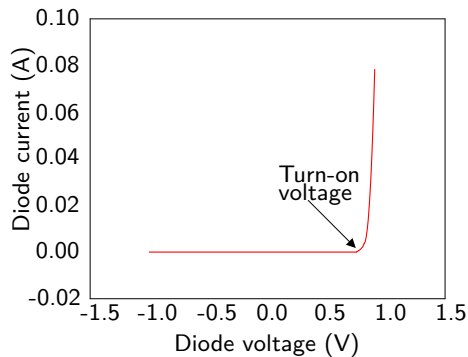


- Si diode with very heavy doping (i.e. very narrow depletion region) easily enter into Zener breakdown under reverse bias.
- Electrons tunnel directly between valence and conduction bands.

Diode



Diode



Diode



$$I_D = I_S \left(e^{\frac{qV_a}{kT}} - 1 \right) \quad (31)$$

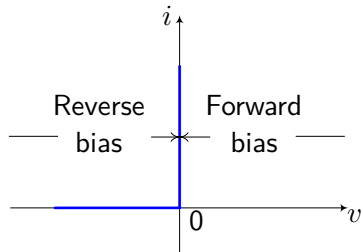
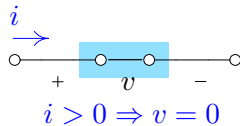
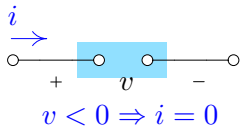
- Turn-on voltage typically 0.5 to 0.7 V
- Saturation current (I_S) typically 10^{-18} to 10^{-9} A
- $kT/q = 0.025875$ V at 300 K

Diode Models

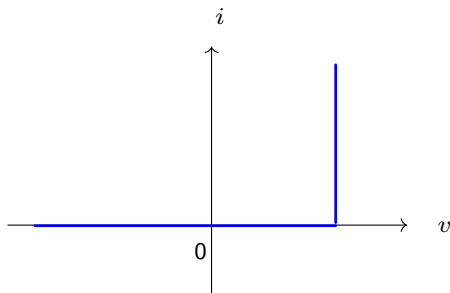


- 1. Ideal
- 2. Constant voltage drop
- 3. Large signal model

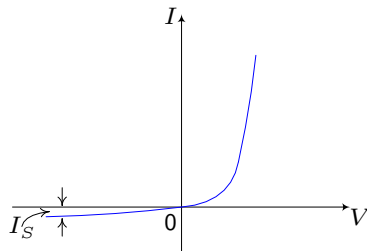
Ideal Model



Different Diode Models



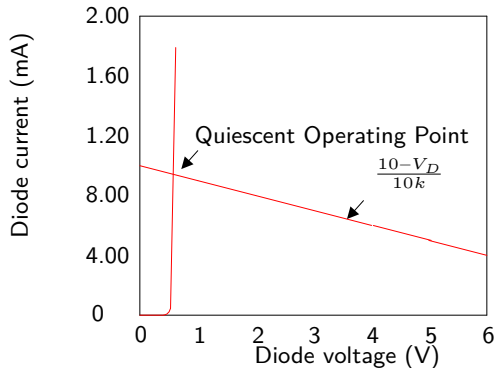
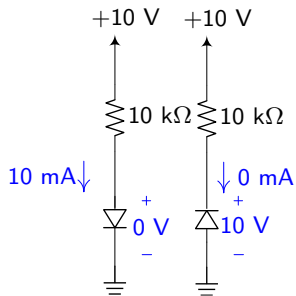
Constant Voltage Drop Model



Large signal model

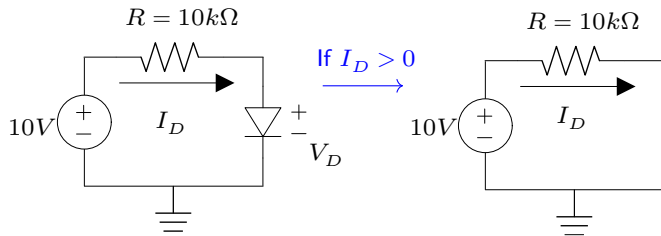
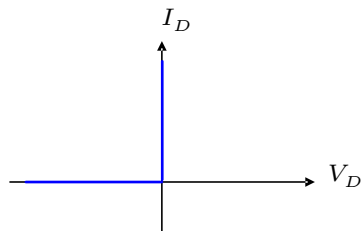
$$I_D = I_S \left(e^{\frac{qV_a}{kT}} - 1 \right) \quad (32)$$

Simple Diode Circuits with Load-Line Method





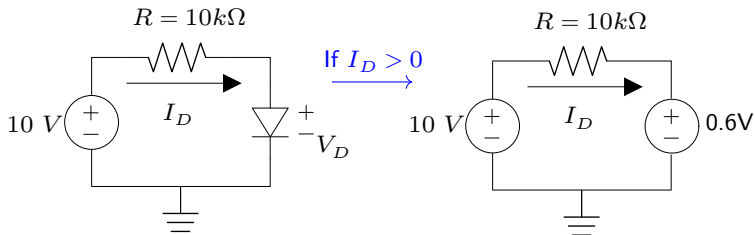
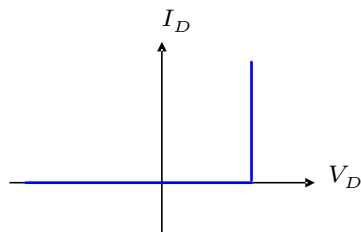
Simplified Analysis (Ideal Diode)



$$V_D = 0 \quad (33)$$

$$I_D = \frac{(10 - 0)V}{10k\Omega} = 1 \text{ mA} \quad (34)$$

Constant Voltage Drop



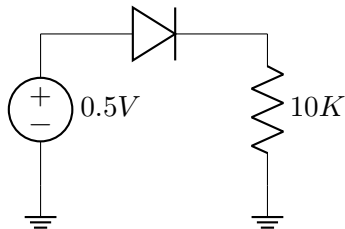
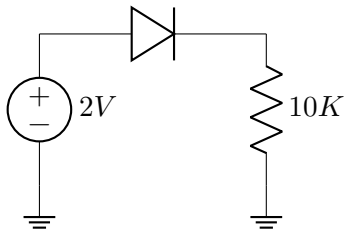
$$I_D = \frac{(10 - 0.6) V}{10 k\Omega} = 0.94 \text{ mA} \quad (35)$$

Comparison of Different Diode Models

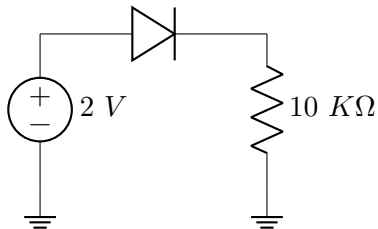


	V_D	I_D
Graphical Analysis	0.6 V	0.95 mA
Mathematical Analysis	0.5742 V	0.944 mA
Ideal Diode Model	0 V	1 mA
Constant Voltage Drop Model	0.6 V	0.94 mA

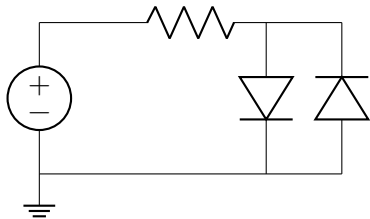
More Diode Circuit Examples (Constant V-drop)



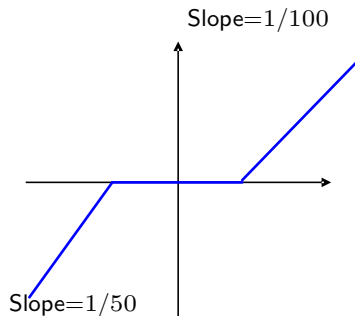
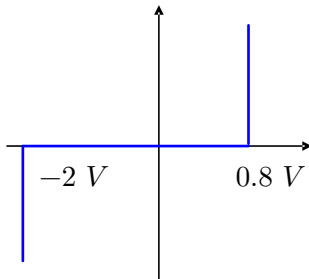
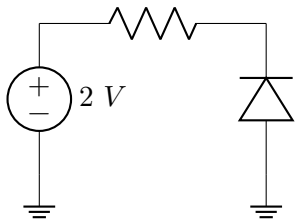
Examples With Diode Resistance of $100\ \Omega$



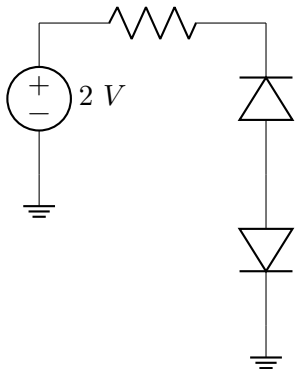
Diodes as a Clipper Circuit



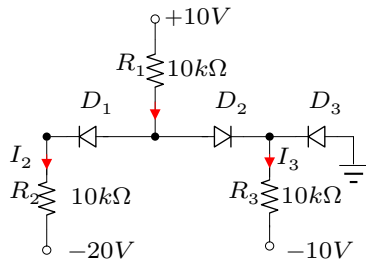
Zener Diode Examples



Examples



Examples with Constant V-Drop Model ($V_{on} = 0.6V$)



Assume no current flowing through D_3

$$\frac{10 - V_B}{10k} = \frac{V_B - 0.6 + 20}{10k} + \frac{V_B - 0.6 + 10}{10k} \quad (36)$$

$$V_B = -6.27V \quad (37)$$

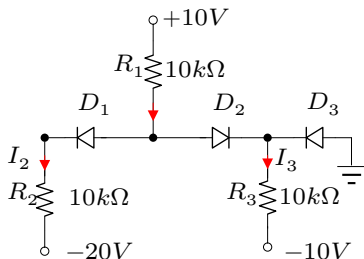
$$V_C = -6.87V \Rightarrow D_3 \text{ in forward bias} \quad (38)$$

Assumption NOT valid

Examples with Constant V-Drop Model ($V_{on} = 0.6V$)



Calculate V_D and I_D of each diode.



Assume no current flowing through D_2

$$\frac{10 - V_B}{10k} = \frac{V_B - 0.6 + 20}{10k} \quad (39)$$

$$V_B = -4.7 V$$

$V_C = -0.6 V \Rightarrow D_2$ indeed in reverse bias

Assumption valid

$$V_{D1} = 0.6 V \quad V_{D2} = -4.1 V \quad V_{D3} = 0.6 V$$

$$I_{D1} = 1.47 mA \quad I_{D2} = 0 mA \quad I_{D3} = 0.94 mA$$

Examples with Ideal Model ($V_{on} = 0 \text{ V}$)

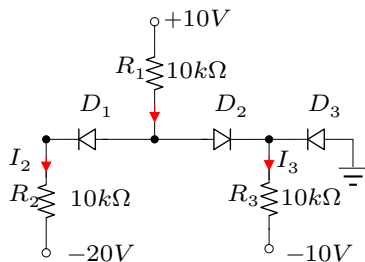


Assume D_3 in reverse bias

$$\frac{10 - V_B}{10k} = \frac{V_B + 20}{10k} + \frac{V_B + 10}{10k} \quad (40)$$

$$V_B = -6.67 \text{ V} \quad (41)$$

$$V_C = -6.67 \text{ V} \Rightarrow D_3 \text{ in forward bias} \quad (42)$$

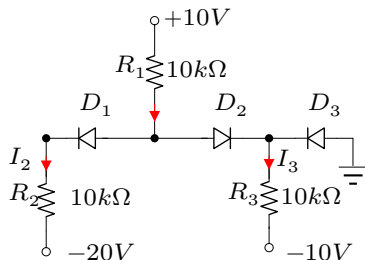


Assumption NOT valid

Examples with Ideal Model ($V_{on} = 0 \text{ V}$)



Assume D_2 in reverse bias



$$\frac{10 - V_B}{10k} = \frac{V_B + 20}{10k} \quad (43)$$

$$V_B = -5 \text{ V}$$

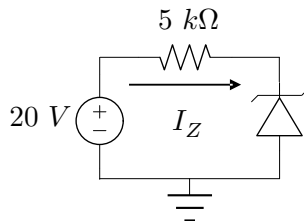
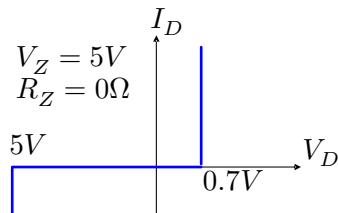
$V_C = 0 \text{ V} \Rightarrow D_2$ indeed in reverse bias

Assumption valid

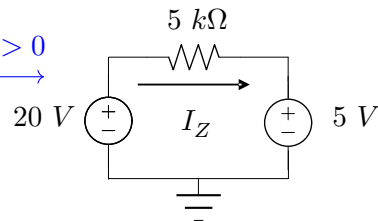
$$V_{D1} = 0 \text{ V} \quad V_{D2} = -5 \text{ V} \quad V_{D3} = 0 \text{ V}$$

$$I_{D1} = 1.5 \text{ mA} \quad I_{D2} = 0 \text{ mA} \quad I_{D3} = 1 \text{ mA}$$

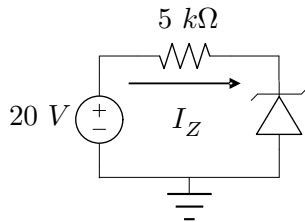
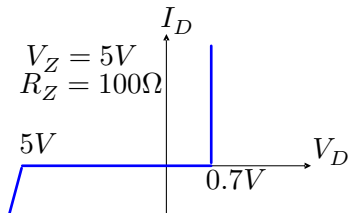
Zener Diode ($R_Z = 0$)



If $I_Z > 0$
→

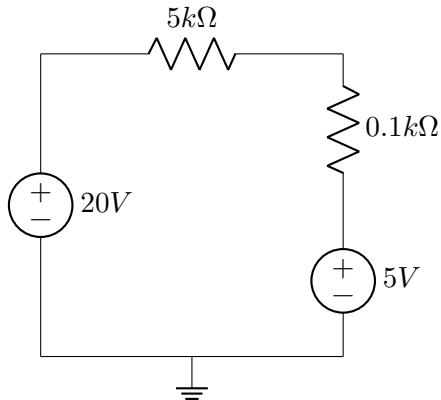


Zener Diode ($R_Z \neq 0$)

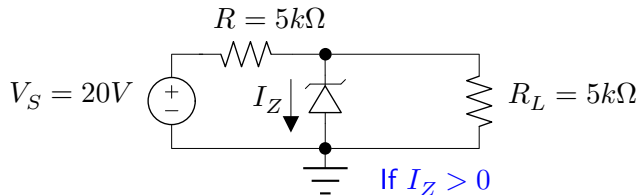


If $I_Z > 0$

Zener Diode ($R_Z \neq 0$, If $I_Z > 0$)



Regulator Using Zener ($R_Z=0$)



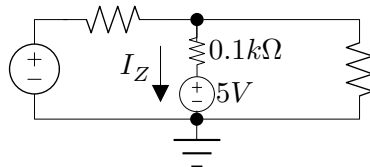
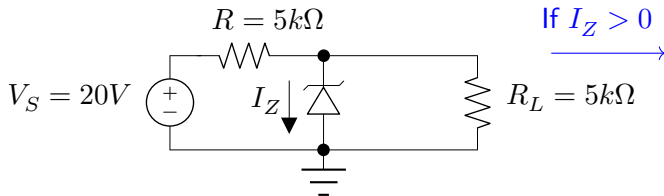
What is the smallest R_L for reverse breakdown to happen?

$$R_{L,min} = 1.67 \text{ k}\Omega \quad (44)$$

As long as the zener diode operates in reverse breakdown ($I_Z > 0$), a constant voltage (5V) appears across R_L .



Zener Diode ($R_Z \neq 0$)



$$\frac{20 - V_L}{5k} = \frac{V_L - 5}{0.1k} + \frac{V_L}{5k} \quad (45)$$

$$I_Z = \frac{5.19 - 5}{0.1k} = 1.9 \text{ mA} > 0 \quad (46)$$

$$V_L = 5.1923 \text{ V} \quad (47)$$