Lecture 22

VE 311 Analog Circuits

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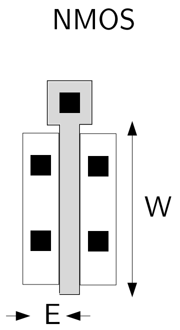
Recap of Last Lecture

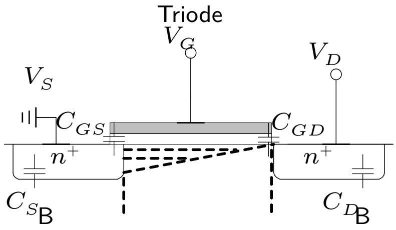
- Review of 216 Frequency Response
- Bode plot Amplitude and Phase Estimation

Topic to be covered

- Source of capacitance in Transistors
- Bandwidth of MOSFET amplifiers

Parasitic Capacitance





Parasitic Capacitance

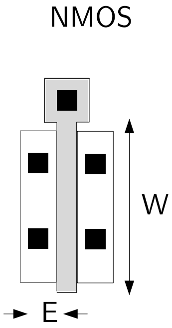
$$C_{GS} = W_{ov} + 1/2\,(WLC_{ox}) \qquad (1)$$

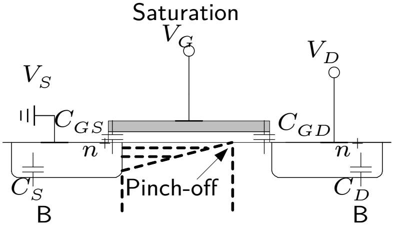
$$C_{GD} = WC_{ov} + 1/2\,(WLC_{ox}) \qquad (2)$$

$$C_{SB} = WEC_j + 2(W+E)C_{jsw} \qquad (3)$$

$$C_{DB} = WEC_j + 2(W+E)C_{jsw} \qquad (4)$$

Parasitic Capacitance





Parasitic Capacitance

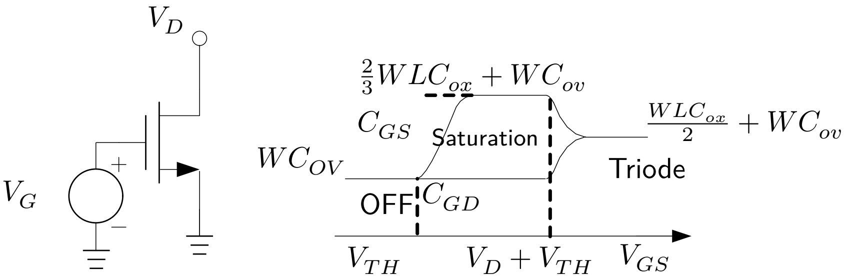
$$C_{GS} = WC_{ov} + 2/3\,(WLC_{ox}) \quad (5)$$

$$C_{GD} = WC_{ov} \quad (6)$$

$$C_{SB} = WC_j + 2(W+E)C_{jsw} \quad (7)$$

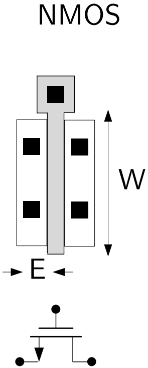
$$C_{DB} = WEC_j + 2(W+E)C_{jsw} \quad (8)$$

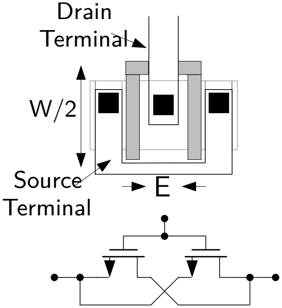
Parasitic Capacitance



Parasitic Capacitance

Calculate C_{SB} and C_{DB} of the two structures below.





Parasitic Capacitance

Solution:

$$\text{Left: } C_{DB} = \frac{W}{2}EC_j + 2\left(\frac{W}{2} + E\right)C_{jsw} \tag{9}$$

$$\text{Right: } C_{DB} = C_{SB} = WEC_j + 2(W + E)C_{jsw} \tag{10}$$

$$\begin{aligned} C_{SB} &= 2\left[\tfrac{W}{2}EC_j + 2\left(\tfrac{W}{2} + E\right)C_{jsw}\right] \\ &= WEC_j + 2(W + 2E)C_{jsw} \end{aligned} \tag{11}$$

- Drain junction capacitance is greatly reduced

Complete Small-Signal Model

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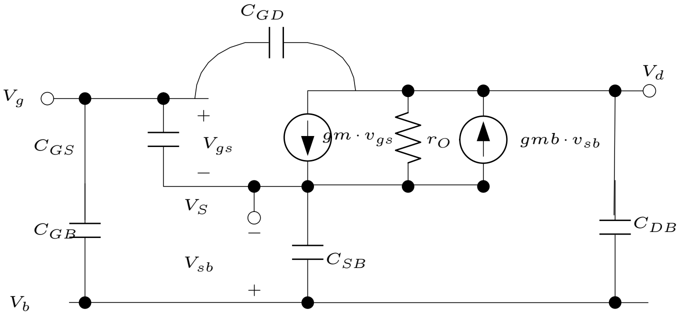


Only when MOSFET is off should we need to consider C_{GB}, which includes the gate oxide capacitance and the depletion region capacitance in series.

Spice Model

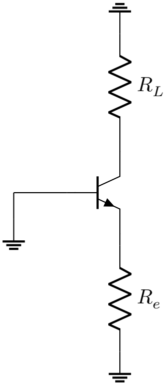
NMOS Model

LEVEL $= 1$	VTO $=0.7$	GAMMA $= 0.45$	PHI $= 0.9$
NSUB $= 9e + 14$	LD $= 0.08e - 6$	UO $= 350$	LAMBDA $= 0.1$
TOX $= 9e - 9$	PB $= 0.9$	CJ $= 0.56e - 3$	CJSW $= 0.35e - 11$
MJ $= 0.45$	MJSW $= 0.2$	CGDO $= 0.4e - 9$	JS $= 1.0e - 8$

PMOS Model

LEVEL $= 1$	VTO $= -0.8$	GAMMA $= 0.4$	PHI $= 0.8$
NSUB $= 5e + 14$	LD $= 0.09e - 6$	UO $= 100$	LAMBDA $= 0.2$
TOX $= 9e - 9$	PB $= 0.9$	CJ $= 0.94e - 3$	CJSW $= 0.32e - 11$
MJ $= 0.5$	MJSW $= 0.3$	CGDO $= 0.3e - 9$	JS $= 0.5e - 8$

Impedances

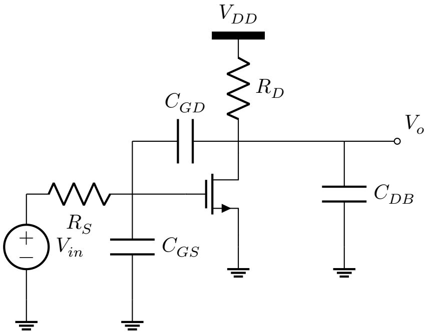


$$R_c = r_o + (R_e \parallel r_\pi) + g_m r_o (R_e \parallel r_\pi) \tag{12}$$

$$= r_e + (g_m r_o)(R_e \parallel r_\pi) \tag{13}$$

$$R_{in} = r_\pi \parallel \left[\frac{r_o + R_L}{g_m r_o + 1} \right] = \frac{1}{g_m} + \frac{R_L}{g_m r_o} \tag{14}$$

Transfer Function Using KCL



$$\frac{V_x - V_{in}}{R_S} + \frac{V_x}{\frac{1}{SC_{GS}}} + \frac{V_x - V_{out}}{\frac{1}{SC_{GD}}} = 0 \tag{15}$$

$$\frac{V_{out} - V_x}{\frac{1}{SC_{GD}}} + \frac{V_{out}}{\frac{1}{SC_{DB}}} + \frac{V_{out}}{R_D} + V_x g_{m1} = 0 \tag{16}$$

$\gamma \neq 0, \lambda = 0$

Transfer Function Using KCL

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$$\frac{v_{out}}{v_{in}} = \frac{\left(C_{GD}S - g_{m1}\right)R_D}{R_S R_D \xi S^2 + \left[R_S\left(1+g_{m1}R_D\right)C_{GD} + R_S C_{GS} + R_D\left(C_{DB}+C_{GD}\right)\right]S + 1} \quad (17)$$

$$\xi = C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB} \quad (18)$$

Transfer Function Using KCL

$$= \frac{-g_{m1}R_D\left(1-\frac{S}{\omega_z}\right)}{\left(1+\frac{S}{\omega_{p1}}\right)\left(1+\frac{S}{\omega_{p2}}\right)} \tag{19}$$

$$= \frac{-g_{m1}R_D\left(1-\frac{S}{\omega_z}\right)}{\frac{S^2}{\omega_{p1}\omega_{p2}}+\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)S+1} \tag{20}$$

If $\omega_{p1} \ll \omega_{p2}$, $\frac{1}{\omega_{p2}}$ can be ignored.

Transfer Function Using KCL

$$\omega_{p1} = \frac{1}{R_S\left(1+g_{m1}R_D\right)C_{GD} + R_SC_{GS} + R_D\left(C_{DB}+C_{GD}\right)} \tag{21}$$

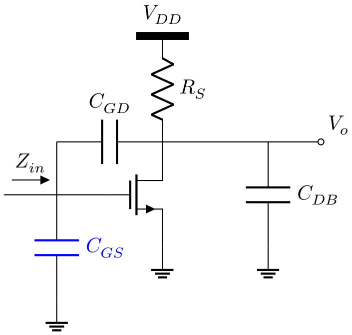
$$\approx \frac{1}{R_S\left(1+gm_1R_D\right)C_{GD} + R_SC_{GS}} \tag{22}$$

$$\omega_{p2} = \frac{R_S\left(1+g_{m1}R_D\right)C_{GD} + R_SC_{GS} + R_D\left(C_{DB}+C_{GD}\right)}{R_SR_D\left(C_{GS}C_{GD}+C_{GS}C_{DB}+C_{GD}C_{DB}\right)} \tag{23}$$

$$\approx \frac{R_SC_{GS}}{R_SR_D\left(C_{GS}C_{GD}+C_{GS}C_{DB}\right)} = \frac{1}{R_D\left(C_{GD}+C_{DB}\right)} \tag{24}$$

If C_{GS} dominates.

Input Impedance

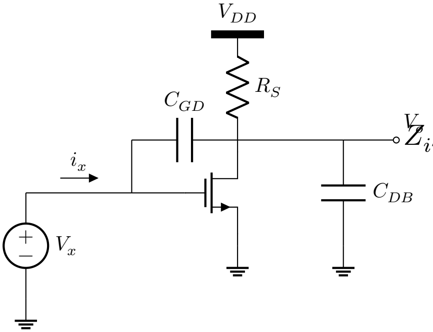


Z_{in1} is the input impedance excluding C_{GS}

$$Z_{in} = Z_{in1} \parallel \frac{1}{SC_{GS}} \tag{25}$$

$\gamma \neq 0, \lambda = 0$

Input Impedance

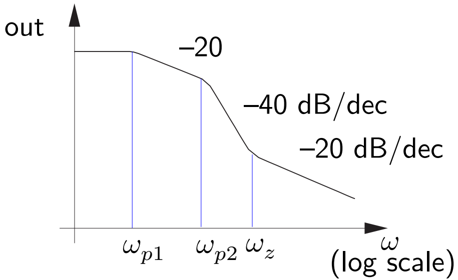


$$i_x = \frac{v_x - v_{out}}{\frac{1}{SC_{GD}}} = \frac{v_{out}}{R_D} + \frac{v_{out}}{\frac{1}{SC_{DB}}} + v_x g_{m1} \tag{26}$$

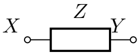
$$Z_{in1} = \frac{v_x}{i_x} = \frac{1 + R_D (C_{GD} + C_{DB}) S}{C_{GD} S (1 + g_{m1} R_D + R_D C_{DB} S)} \tag{27}$$

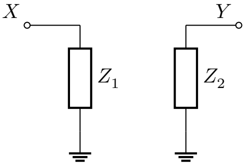
$$\approx \frac{1}{C_{GD} S (1 + g_{m1} R_D)} \tag{28}$$

Effect of C_{GD}



Miller Effect





$$Z_1 = \frac{Z}{1 - A_v} \tag{29}$$

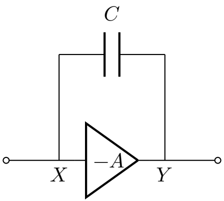
$$Z_2 = \frac{Z}{1 - \frac{1}{A_v}} \tag{30}$$

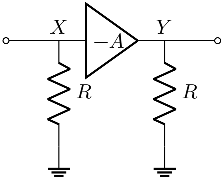
Proof:

$$\frac{X - Y}{Z} = \frac{X}{Z_1} \to Z_1 = \frac{Z}{1 - \frac{Y}{X}} \tag{31}$$

$$\frac{X - Y}{Z} = \frac{-Y}{Z_2} \to Z_2 = \frac{Z}{1 - \frac{X}{Y}} \tag{32}$$

Calculate the Input Capacitance





$$Z_1 = \frac{\frac{1}{SC_F}}{1+A} = \frac{1}{S(1+A)C_F} \tag{33}$$

$$Z_2 = \frac{\frac{1}{SC_F}}{1+\frac{1}{A}} = \frac{1}{S\left(1+\frac{1}{A}\right)C_F} \tag{34}$$

Miller Effect Example

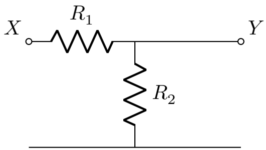
The amount of charge flowing into C_F from X is $C_F(1+A)\Delta V$, as if the input capacitance increased to $(1+A)C_F$.

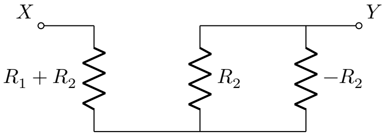
$$R_{in} = \frac{1}{S(1+A)C_F} \tag{35}$$

$$R_{out} = 0 \tag{36}$$

Limitation on Miller Effect

If the impedance Z forms the ONLY signal path between X and Y, then Miller conversion is often invalid.





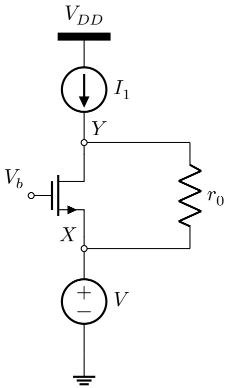
$$A_v = \frac{Y}{X} = \frac{R_2}{R_1 + R_2} \tag{37}$$

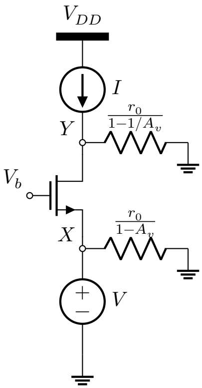
Limitation on Miller Effect

$$Z_1 = \frac{R_1}{1 - \frac{R_2}{R_1+R_2}} = R_1 + R_2 \tag{38}$$

$$Z_2 = \frac{R_1}{1 - \frac{R_1+R_2}{R_2}} = -R_2 \tag{39}$$

Miller Effect Example





Miller Effect Example

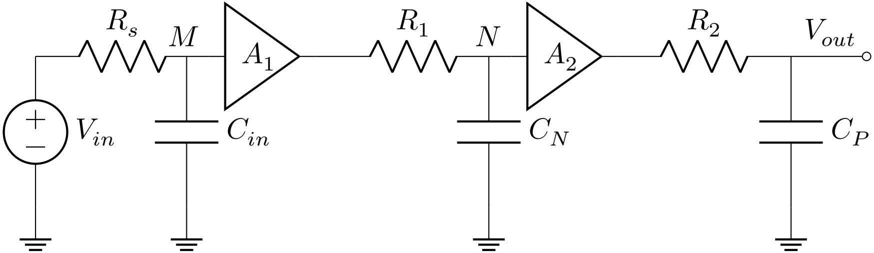
$$A_v = \frac{Y}{X} = 1 + (g_{m1} + g_{mb1})\, r_{o1} \tag{40}$$

$$Z_1 = \frac{r_{01}}{1 - [1 + (g_{m1} + g_{mb1})\, r_{01}]} = \frac{-1}{g_{m1} + g_{mb1}} \tag{41}$$

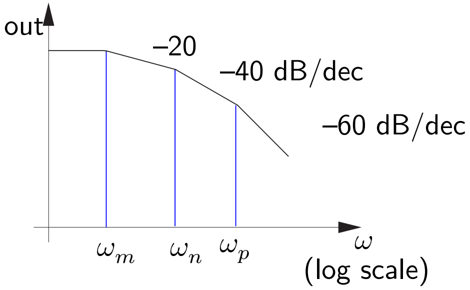
$$R_{in} = \frac{-1}{g_{m1} + g_{mb1}} \parallel \frac{1}{g_{m1} + g_{mb1}} = \infty \tag{42}$$

- r_o in parallel with the main signal path. Miller conversion is valid.
- The result is the same as direct small-signal analysis.

Association of Poles with Nodes



Association of Poles with Nodes



Association of Poles with Nodes

$$v_{out} = v_{in} \frac{\frac{1}{sC_{in}}}{R_S + \frac{1}{SC_{in}}} A_1 \frac{\frac{1}{SC_N}}{R_1 + \frac{1}{sC_N}} \ A_2 \frac{\frac{1}{SC_P}}{R_2 + \frac{1}{sC_P}} \tag{43}$$

$$\frac{v_{out}}{v_{in}}(\omega) = \frac{A_1}{1 + SR_SC_{in}} \frac{A_2}{1 + SR_1C_N} \frac{1}{1 + SR_2C_P} \tag{44}$$

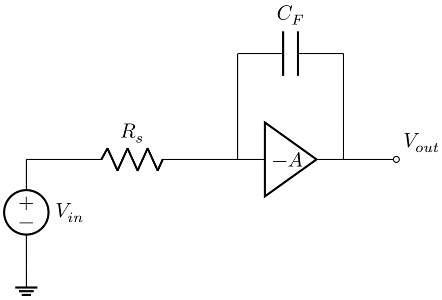
$$= A_1 \ A_2 \frac{1}{\left(1 + \frac{s}{\omega_M}\right)\left(1 + \frac{s}{\omega_N}\right)\left(1 + \frac{s}{\omega_P}\right)} \tag{45}$$

$$\omega_M = \frac{1}{R_SC_{in}} \quad (46) \qquad \omega_N = \frac{1}{R_1C_N} \quad (47) \qquad \omega_P = \frac{1}{R_2C_P} \ (rad/s) \quad (48)$$

Miller conversion often discards the zeros in the transfer function.

Method 1 (Association Poles With Nodes)

Simplification through Miller effect and then association of poles with nodes.



$$\frac{v_{out}}{v_{in}}(\omega \text{ close to } DC) = -A \tag{49}$$

$$\frac{v_{out}}{v_{in}} = \frac{-A}{1 + SR_S C_F(1+A)} \tag{50}$$

Method 2 (Derivation of transfer function using KCL)

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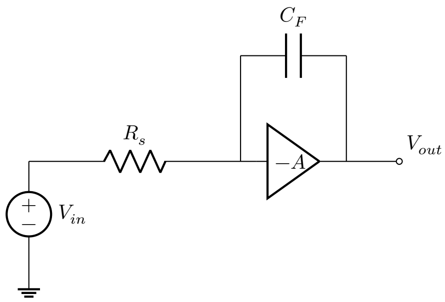

$$v_x - \frac{v_{in} - v_x}{R_S}\frac{1}{SC_F} = v_{out} \tag{51}$$

$$v_{out} = v_x(-A) \tag{52}$$

$$\frac{v_{out}}{v_{in}} = \frac{-A}{1 + SR_SC_F(1+A)} \tag{53}$$

Method 1 (Association Poles With Nodes)

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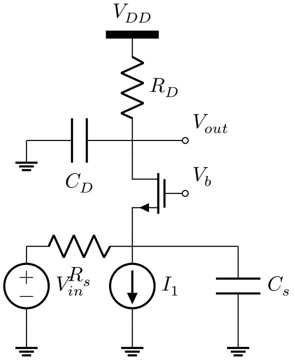

$$\frac{v_{out}}{v_{\text{in}}} = \frac{(g_{m1}+g_{mb1})R_D}{1+(g_{m1}+g_{mb1})R_s}\frac{1}{\left(1+\frac{s}{\omega_X}\right)\left(1+\frac{s}{\omega_Y}\right)} \tag{54}$$

$$\omega_X = \frac{1}{\left(\frac{1}{g_{m1}+g_{mb1}} \parallel R_S\right)C_s}(rad/s) \tag{55}$$

$$\omega_Y = \frac{1}{R_D C_D} \quad (rad/s) \tag{56}$$

Method 2 (Derivation of transfer function using KCL)

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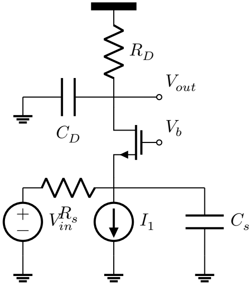
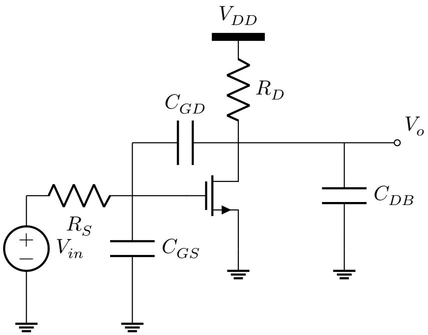


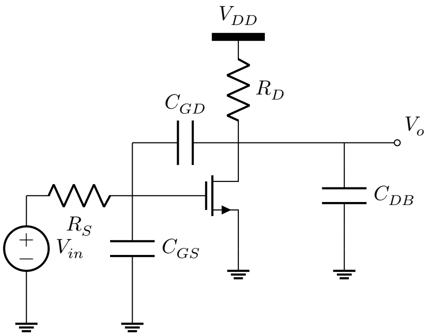
$$\frac{v_{in}}{R_S + \left(\frac{1}{g_{m1}+g_{mb1}} \parallel \frac{1}{SC_S}\right)} \frac{\frac{1}{SC_S}}{\frac{1}{SC_S} + \frac{1}{g_{m1}+g_{mb1}}} \left(R_D \parallel \frac{1}{SC_D}\right) = v_{out} \tag{57}$$

Method 1 (Association Poles With Nodes)



- C_{GD} in parallel with the main signal path. So, Miller conversion is valid.
- Zeros are neglected in the transfer function.
- $A_v = -g_{m1}R_D$ is the low-frequency gain. In reality, it changes with frequency.

Method 1 (Association Poles With Nodes)

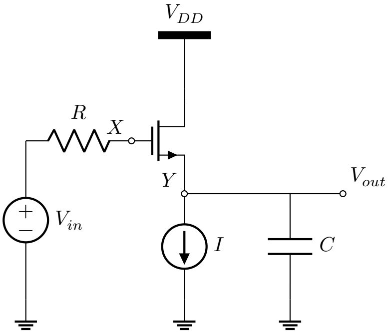


$$\omega_{in} = \frac{1}{R_S\left[C_{GS} + (1 + g_{m1}R_D)\, C_{GD}\right]} \tag{58}$$

$$\omega_{out} \approx \frac{1}{R_D\left(C_{DB} + C_{GD}\right)} \tag{59}$$

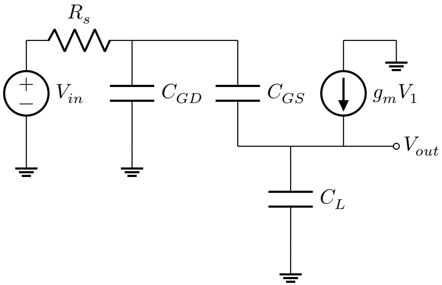
$$\frac{v_{out}}{v_{in}} = \frac{-g_{m1}R_D}{\left(1 + \frac{S}{\omega_{in}}\right)\left(1 + \frac{S}{\omega_{out}}\right)} \tag{60}$$

Method 1 (Association Poles With Nodes)



- C_{GS} forms the only signal path between V_{in} and V_{out}. Miller conversion may not be applied here.
- Strong interaction between V_{in} and V_{out} through C_{GS}. Association of poles with nodes method may not be applied here.

Method 1 (Association Poles With Nodes)



$$\frac{v_{\text{out}}}{\frac{1}{SC_L}} + \frac{v_{\text{out}} - v_a}{\frac{1}{SC_{GS}}} + (v_{\text{out}} - v_a)\, g_{m1} = 0 \tag{61}$$

$$\frac{v_a - v_{\text{out}}}{\frac{1}{SC_{GS}}} + \frac{v_a}{\frac{1}{SC_{GD}}} + \frac{v_a - v_{\text{in}}}{R_S} = 0 \tag{62}$$

$\lambda = \gamma = 0$

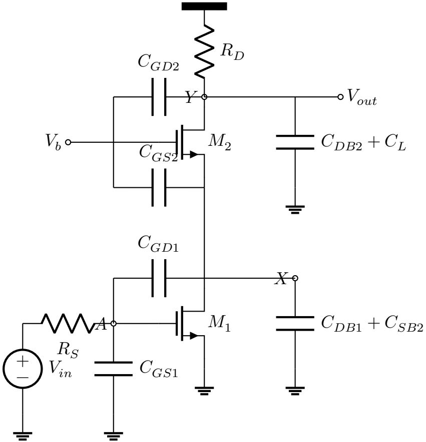
Method 1 (Association Poles With Nodes)

$$\frac{v_{out}}{v_{in}} = \frac{1+\frac{C_{GS}S}{g_{m1}}}{\frac{R_S(C_{GS}C_L+C_{GS}C_{GD}+C_{GD}C_L)}{g_{m1}}S^2+\frac{g_{m1}R_SC_{GD}+C_L+C_{GS}}{g_{m1}}+1} \tag{63}$$

$$= \frac{1+\frac{C_{GS}S}{g_{m1}}}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{S}{\omega_{p2}}\right)} = \frac{1+\frac{C_{GS}S}{g_{m1}}}{\frac{S^2}{\omega_{p1}\omega_{p2}}+\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)S+1} \tag{64}$$

$$\omega_{p1} = \frac{g_{m1}}{g_{m1}R_SC_{GD}+C_L+C_{GS}} = \frac{1}{R_SC_{GD}+\frac{C_L+C_{GS}}{g_{m1}}} \quad \text{If } \omega_{p1} \ll \omega_{p2} \tag{65}$$

Cascode



- No Miller multiplication of capacitance at the CG stage.
- Miller multiplication of capacitance suppressed at the CS stage.

$$\frac{X}{A} = -\frac{g_{m1}}{g_{m2} + g_{mb2}} \tag{66}$$

Cascode

$$\omega_{p,A} = \frac{1}{R_S\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}+g_{mb2}}\right)C_{GD1}\right]} \tag{67}$$

$$\omega_{p,X} = \frac{1}{\frac{1}{g_{m2}+g_{mb2}}\left[C_{DB1} + C_{SB2} + C_{GS2} + \left(1 + \frac{g_{m2}+g_{mb2}}{g_{m1}}\right)C_{GD1}\right]} \tag{68}$$

$$\omega_{p,Y} = \frac{1}{R_D\left[C_{GD2} + C_{DB2} + C_L\right]} \tag{69}$$

Cascode

$$\frac{v_{out}}{v_{in}} = -g_{m1}R_D \frac{1}{\left(1+\frac{s}{\omega_{p,A}}\right)\left(1+\frac{s}{\omega_{p,X}}\right)\left(1+\frac{s}{\omega_{p,Y}}\right)} \tag{70}$$

Recall for CS stages

$$\omega_{in} = \frac{1}{R_S\left[C_{GS} + \left(1+g_{m1}R_D\right)C_{GD}\right]} \tag{71}$$