Lecture 15: MOSFET Review

VE311 Electronic Circuits

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Recap of Last Lecture

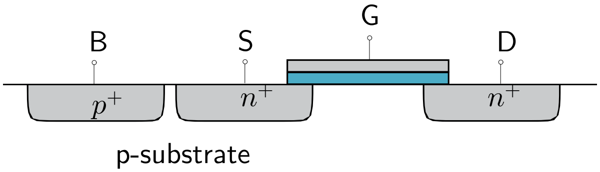
- Review for the midterm

Topics to Be Covered

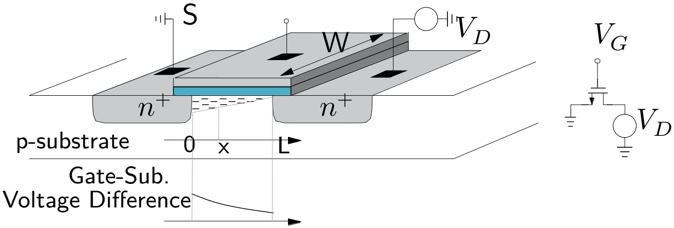
- Review for the midterm (MOSFET part)
- Source Follower

NMOS FET

Reading: Razavi Chapter 2.

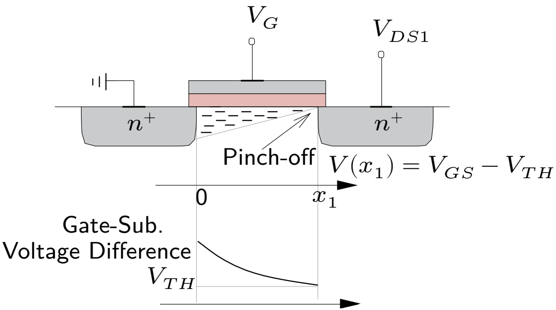


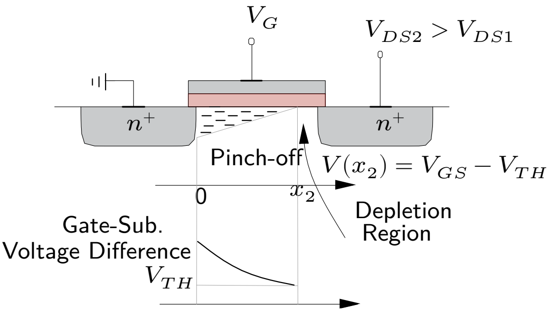
NMOS I-V Characteristics (Triode)



$$I_D = \mu_n C_{ox} \frac{W}{L_{eff}} \left[(V_{GS} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^2 \right] \quad (1)$$

Saturation Region





Saturation Region

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L'}\left(V_{GS} - V_{TH}\right)^2 \quad (2)$$

- I_D: constant along channel
- L': the point at which Q_d drops to zero
- V_{GS}–V_{TH}: the overdrive voltage
- Electron velocity ($v = I_D/Q_d$) becomes tremendously high at the pinch off point ($Q_d \rightarrow 0$), such that electrons shoot through the depletion region and arrive at the drain terminal.

Channel-Length Modulation

$$r_o = \frac{\partial V_{DS}}{\partial I_D} = 1/\frac{\partial I_D}{\partial V_{DS}} \tag{3}$$

$$= \frac{1}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2 \cdot \lambda} \tag{4}$$

$$\approx \frac{1}{I_D \cdot \lambda} \tag{5}$$

Body Effect

$$V_{TH} = V_{TH0} + \gamma(\sqrt{|2\Phi_F + V_{SB}|} - \sqrt{|2\Phi_F|}) \tag{6}$$

$$\Phi_F = \frac{kT}{q} \ln \frac{N_{sub}}{n_i} \tag{7}$$

$$\gamma = \frac{\sqrt{2q\epsilon_{Si} N_{sub}}}{C_{ox}} \tag{8}$$

$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L'} (V_{GS} - V_{TH})^2 \tag{9}$$

I will ask you how the body shall be connected.

NMOS Formula Table

NMOS Transistor Mathematical Model Summary

The following equations represent the complete model for the $i-v$ behavior of the NMOS transistor.

For all regions,

$$K_n = K'_n \frac{W}{L} \quad K'_n = \mu_n C''_{ox} \quad i_G = 0 \quad i_B = 0 \tag{10}$$

Cut off region,

$$i_D = 0 \quad \text{for } v_{GS} \le V_{TN} \tag{11}$$

Triode region,

$$i_D = K_n \left(v_{GS} - V_{TN} - \frac{v_{DS}}{2} \right) v_{DS} \quad \text{for } v_{GS} - V_{TN} \ge v_{DS} \ge 0 \tag{12}$$

NMOS Formula Table

Saturation region,

$$i_D = \frac{K_n}{2}\left(v_{GS} - V_{TN}\right)^2 \left(1 + \lambda v_{DS}\right) \quad \text{for } v_{DS} \geq \left(v_{GS} - V_{TN}\right) \geq 0 \tag{13}$$

Threshold voltage,

$$V_{TN} = V_{TO} + \gamma\left(\sqrt{v_{SB} + 2\phi_F} - \sqrt{2\phi_F}\right) \tag{14}$$

$V_{TN} > 0$ for enhancement-mode NMOS transistors.Depletion-mode NMOS devices can also be fabricated,and $I_{TN} \leq 0$ for these transistors.

PMOS Formula Table

NMOS Transistor Mathematical Model Summary
The following equations represent the complete model for the $i - v$ behavior of the NMOS transistor.
For all regions,

$$K_p = K_p' \frac{W}{L} \quad K_p' = \mu_p C_{ox}'' \quad i_G = 0 \quad i_B = 0 \tag{15}$$

Cut off region,

$$i_D = 0 \quad \text{for } V_{GS} \geq V_{TP} \tag{16}$$

Triode region,

$$i_D = K_p \left(v_{GS} - V_{TP} - \frac{v_{DS}}{2} \right) v_{DS} \quad \text{for } 0 \leq |v_{DS}| \leq |v_{GS} - V_{TP}| \tag{17}$$

PMOS Formula Table

Saturation region,

$$i_D = \frac{K_p}{2}(v_{GS} - V_{TP})^2 (1 + \lambda |v_{DS}|) \quad \text{for } |v_{DS}| \geq |v_{GS} - V_{TP}| \geq 0 \tag{18}$$

Threshold voltage,

$$V_{TN} = V_{TO} + \gamma \left(\sqrt{v_{SB} + 2\phi_F} - \sqrt{2\phi_F} \right) \tag{19}$$

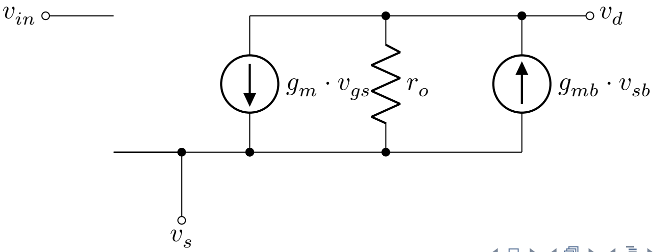
For the enhancement-mode PMOS transistor,$V_{TP} < 0$.Depletion-mode PMOS devices can also be fabricated;$I_{TP} \geq 0$ for these devices.

Small Signal Model

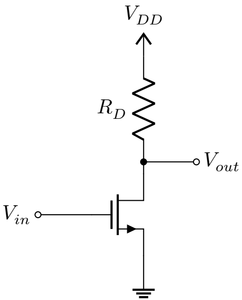
$$g_m = \mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_{TH}\right) \left(1 + \lambda V_{DS}\right) \quad (20)$$

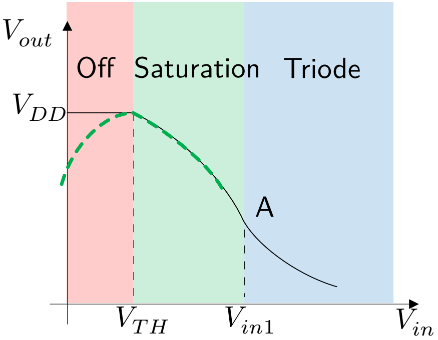
$$r_o = \frac{\partial V_{DS}}{\partial I_D} = 1/\frac{\partial I_D}{\partial V_{DS}} \approx \frac{1}{I_D \cdot \lambda} \quad (21)$$

SS of NMOS = SS of PMOS



Common-Source

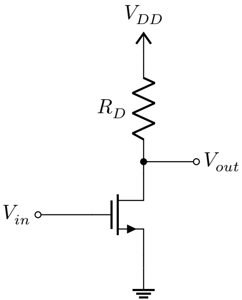




I will ask you how to determine the DC operation region.

For example, What is the value of V_{in1}

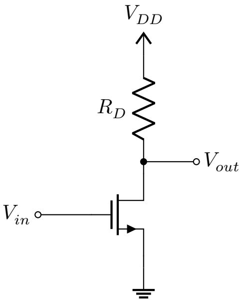
Common-Source and its Derivations



- Gain, bias, and small-signal model
- Input and output impedance
- Advantages:
  - Large input impedance
  - Large gain
  - Widely used
  - No current through Gate, easily biased.

Of course, there is going to be a question related to a common source amplifier and its derivations.

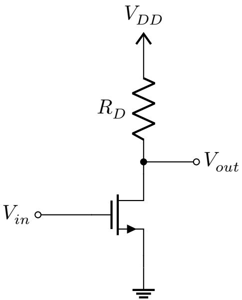
Common-Source and its Derivations

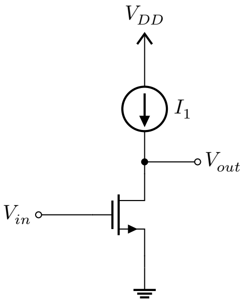


- Cons:
  - R_D and M_1 process variation
  - (Operating condition?) Swing limited by R_D
  - Output DC voltage cannot be chosen arbitrarily
  - Output impedance and gain tradeoff

Of course, there is going to be a question related to a common source amplifier and its derivations.

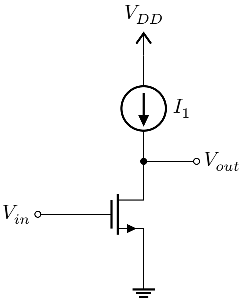
Common-Source and its Derivations

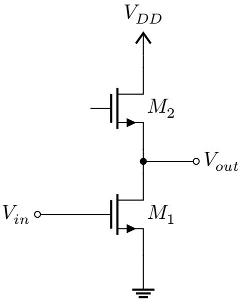




- Current Source, provide high impedance
- Problem: there is no ideal current source in the world.

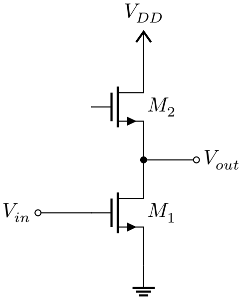
Common-Source and its Derivations

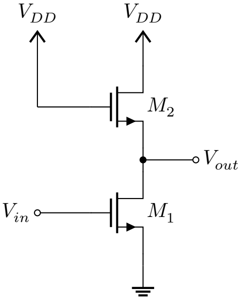




- DC biased NMOS Load: Good approximation of current source.
- Pros: Less process variation.
- Cons: Additional Bias to be generated, Body effect and therefore Linearity problem.

Common-Source and its Derivations



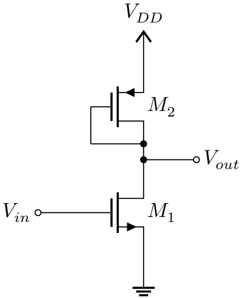


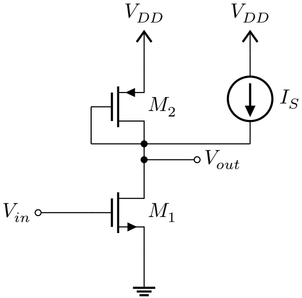
- No need to generate an additional voltage. M_2 is always in saturation.
- Still, output impedance and linearity

Common-Source and its Derivations

- PMOS to improve linearity
- No more body effect.
- Cons: Now how is the DC defined? Limited output swing.
- Large input capacitance.

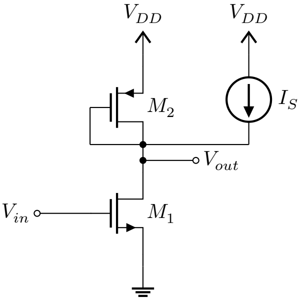
Common-Source and its Derivations

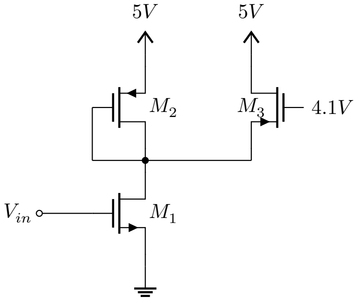




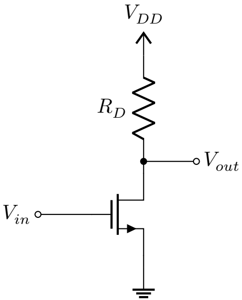
- Additional Current path
- Pros: Improve voltage swing, reduce input cap size.

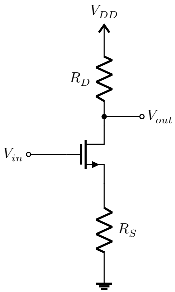
Common-Source and its Derivations





Common-Source and its Derivations

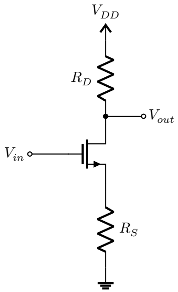


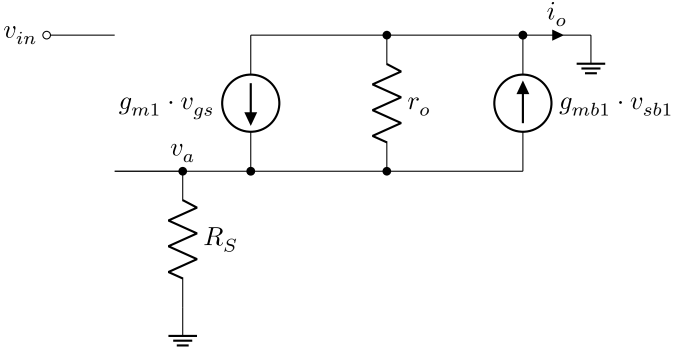


- Pros: Linearity
- Cons: Gain

$$Gain = G_m \cdot R_{out} \tag{22}$$

CS with Source Degradation





CS with Source Degradation

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$$i_o = \frac{-v_a}{R_S} \tag{23}$$

$$\left(v_{in} - v_a\right) gm_1 + i_o = \frac{v_a}{r_{o1}} + v_a gmb_1 \tag{24}$$

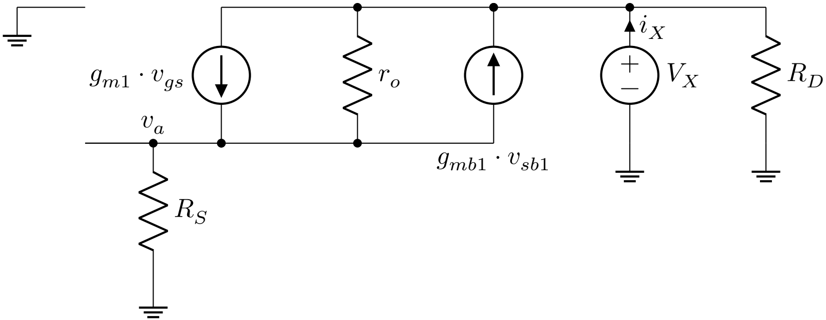
$$G_m = \frac{i_o}{v_{in}} = \frac{-gm_1 r_{o1}}{R_S + r_{o1} + \left(g_{m1} + g_{mb1}\right) r_{o1} R_S} \approx -\frac{1}{R_S} \tag{25}$$

If,

$$\left(gm_1 + gmb_1\right) r_{01} R_S \gg r_{01} \text{ and } R_S \tag{26}$$

CS with Source Degradation

$$\lambda \neq 0, \gamma \neq 0 \tag{27}$$



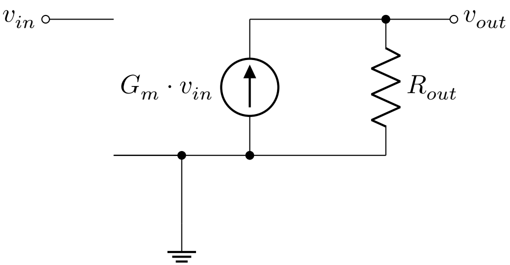
CS with Source Degradation

$$i_x = \frac{v_a}{R_S} \tag{28}$$

$$v_a gm_1 + v_a gmb_1 + \frac{v_a - v_x}{r_o} + i_x = 0 \tag{29}$$

$$R_{\text{out}} = R_{\text{out1 }1} \parallel R_D = \left[R_S + r_{o1} + \left(gm_1 + gmb_1\right) r_{o1} R_S\right] \parallel R_D \approx R_D \tag{30}$$

CS with Source Degradation



CS with Source Degradation

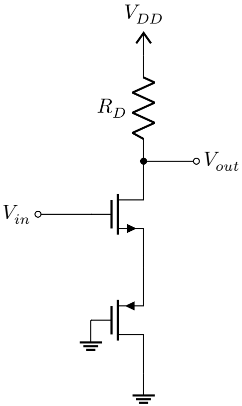
$$A_v = \frac{v_{\text{out}}}{v_{\text{in}}} = G_m R_{\text{out}} \tag{31}$$

$$= \frac{-gm_1 r_{o1}}{R_S + r_{o1} + (gm_1 + gmb_1) r_{o1} R_S} \cdot \frac{[R_S + r_{o1} + (gm_1 + gmb_1) r_{o1} R_S] R_D}{[R_S + r_{o1} + (gm_1 + gmb_1) r_{o1} R_S] + R_D} \tag{32}$$

$$\approx -\frac{R_D}{R_S} \quad \text{If } (gm_1 + gmb_1) r_{01}\text{, the intrinsic gain, is large.} \tag{33}$$

Example

Assuming $\lambda = \gamma = 0$, calculate the small signal voltage gain of the circuit below.



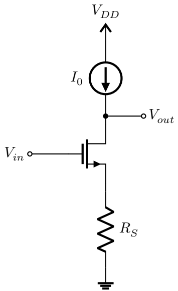
$$G_m = -\frac{1}{\frac{1}{gm_1} + \frac{1}{gm_2}} \tag{34}$$

$$R_{\text{out}} = R_D \tag{35}$$

$$A_v = G_m R_{out} \tag{36}$$

Example

Calculate the small signal voltage gain of the circuit below.



$$G_m = \frac{-gm_1 r_{01}}{r_{01} + R_S + (gm_1 + gmb_1)\, r_{01} R_S} \quad (37)$$

$$R_{out} = r_{o1} + R_S + (gm_1 + gmb_1)\, r_{o1} R_S \quad (38)$$

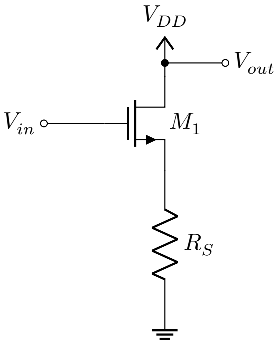
$$A_v = G_m R_{out} = -gm_1 r_{o1} \quad (39)$$

- I_0 is ideal current source $\rightarrow$ Voltage across R_S is constant $\rightarrow$ M_1 source shorted to ground
- R_D replaced by current source $\rightarrow$ Nonlinearity issue arises again

Ideal Amplifier

- For driving a low impedance load, source follower, as a buffer, provides no gain but large input impedance and low output impedance.

Source Follower



$$V_{DD} - V_{\text{out}} = V_{\text{in1}} - V_{\text{out}} - V_{TH} \quad (40)$$

$$\rightarrow V_{in1} = V_{DD} + V_{TH} \quad (41)$$

Source Follower

$$\mathbf{V}_{\text{in}} < \mathbf{V}_{TH} \rightarrow M_1 \text{ Off} \tag{42}$$

$$V_{out} = 0 \tag{43}$$

$$V_{\text{in }1} > V_{\text{in}} > V_{TH} \rightarrow M_1 \text{ in Saturation} \tag{44}$$

$$R_S \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{out} - V_{TH})^2 = V_{out} \tag{45}$$

$$V_{\text{in}} > V_{\text{in }1} \rightarrow M_1 \text{ in Triode} \tag{46}$$

$$R_S \mu_n C_{\text{ox}} \frac{W}{L} \left[(V_{\text{in}} - V_{\text{out}} - V_{TH}) (V_{DD} - V_{\text{out}}) - \frac{1}{2} (V_{DD} - V_{\text{out}})^2 \right] = V_{\text{out}} \tag{47}$$

Source Follower

$$V_{\text{in1}} > V_{\text{in}} > V_{TH} \rightarrow M_1 \text{ in Saturation} \tag{48}$$

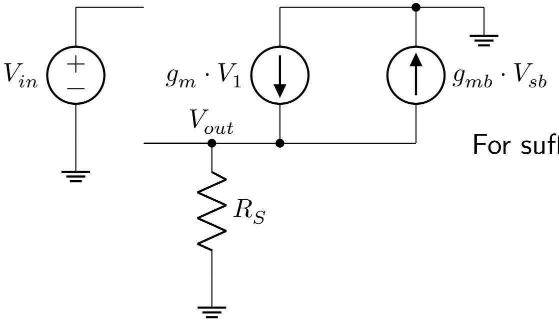
$$R_S \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{\text{in}} - V_{\text{out}} - V_{TH}\right)^2 = V_{\text{out}} \tag{49}$$

$$R_S \frac{1}{2}\mu_n C_{ox}\frac{W}{L}2\left(V_{in} - V_{out} - V_{TH}\right)\left(1 - \frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH}}{\partial V_{in}}\right) = \frac{\partial V_{out}}{\partial V_{in}} \tag{50}$$

$$R_S \mu_n C_{ox}\frac{W}{L}\left(V_{\text{in}} - V_{\text{out}} - V_{TH}\right)\left(1 - \frac{\partial V_{\text{out}}}{\partial V_{\text{in}}} - \frac{\partial V_{TH}}{\partial V_{\text{out}}}\frac{\partial V_{\text{out}}}{\partial V_{\text{in}}}\right) = \frac{\partial V_{\text{out}}}{\partial V_{\text{in}}} \tag{51}$$

$$A_v = \frac{g_m R_S}{1 + g_m R_S(1+\eta)} = \frac{g_m R_S}{1 + (g_m + g_{mb})R_S} \approx \frac{1}{1+\eta} \tag{52}$$

Source Follower



For sufficiently large V_{in}, I_D and thus g_m.

Source Follower

$$G_m = g_m \tag{53}$$

$$R_{out} = R_S \parallel \left(\frac{1}{gm + g_{mb}} \right) \tag{54}$$

$$A_v = \frac{gmR_S}{1 + (gm + g_{mb})R_S} \approx \frac{1}{1+\eta} \tag{55}$$

$$\text{If } (g_m + g_{mb})R_S \gg 1$$

Source Follower

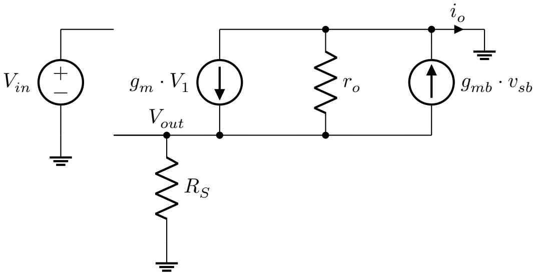
$$\lambda \neq 0, \gamma \neq 0 \tag{56}$$

$$G_m = g_m \tag{57}$$

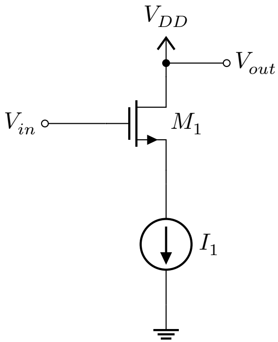
$$R_{out} = r_o \parallel R_S \parallel \left(\frac{1}{gm + g_{mb}} \right) \tag{58}$$

$$A_V = \frac{g_m r_o R_S}{r_o + R_S + (g_m + g_{mb}) r_o R_S} \approx \frac{1}{1+\eta} \tag{59}$$

If $(g_m + g_{mb}) r_0 R_S \gg r_0$ and R_S



SF with Current Source



$$A_v = \frac{1}{1+\eta} \tag{60}$$

If $\gamma = 0, A_v = 1$

SF with Current Source

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$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{\text{in}} - V_{\text{out}} - V_{TH}\right)^2 = I_1 \tag{61}$$

$$\frac{1}{2}\mu_n C_{\text{ox}}\frac{W}{L}2\left(V_{\text{in}} - V_{\text{out}} - V_{TH}\right)\left(1 - \frac{\partial V_{\text{out}}}{\partial V_{\text{in}}} - \frac{\partial V_{TH}}{\partial V_{\text{in}}}\right) = 0 \tag{62}$$

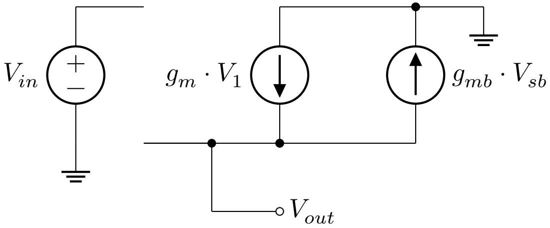
$$\mu_n C_{\text{ox}}\frac{W}{L}\left(V_{\text{in}} - V_{\text{out}} - V_{TH}\right)\left(1 - \frac{\partial V_{\text{out}}}{\partial V_{\text{in}}} - \frac{\partial V_{TH}}{\partial V_{\text{out}}}\frac{\partial V_{\text{out}}}{\partial V_{\text{in}}}\right) = 0 \tag{63}$$

SF with Current Source

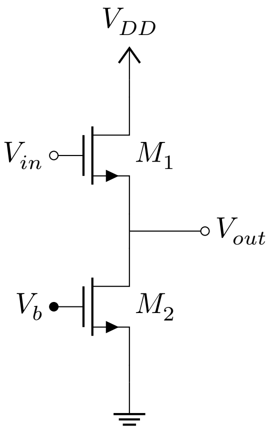
$$G_m = g_m \tag{64}$$

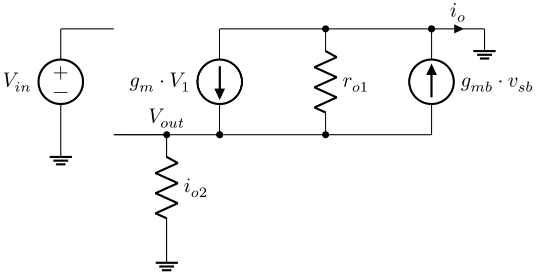
$$R_{out} = \frac{1}{g_m + g_{mb}} \tag{65}$$

$$A_v = \frac{1}{1+\eta} \quad \text{If } \gamma = 0, A_v = 1 \tag{66}$$



SF with Current Source





SF with Current Source

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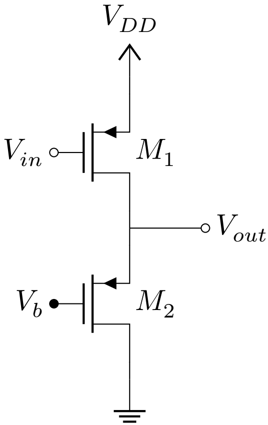
$$G_m = g_{m1} \tag{67}$$

$$R_{out} = r_{o1} \parallel r_{o2} \parallel \left(\frac{1}{g_{m_1} + g_{mb_1}} \right) \tag{68}$$

$$A_v = \frac{g_m r_{o1} r_{o2}}{r_{o1} + r_{o2} + (g_m + g_{mb}) r_{o1} r_{o2}} \tag{69}$$

If r_{o1} and r_{o2} large, A_v is linear.

SF with Current Source ($V_{SB} = 0$)



SF with Current Source ($V_{SB} = 0$)

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$$G_m = g_{m1} \tag{70}$$

$$R_{out} = r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m1}} \tag{71}$$

$$A_V = \frac{g_{m1} r_{o1} r_{o2}}{r_{o1} + r_{o2} + g_{m1} r_{o1} r_{o2}} \tag{72}$$

- The sacrifice here is the higher output impedance due to smaller mobility of holes relative to electrons.