

Lecture 4: Analog Circuits

VE 311 Electronic Circuits

Xuyang Lu 2023 Fall



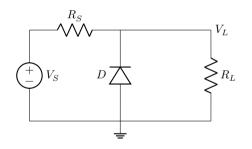
Recap of Last Lecture



- Diodes
- Diode Circuits

Line Regulation



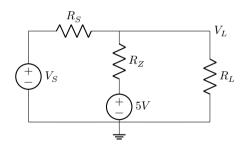


• Line Regulation: how sensitive the output voltage (V_L) is to input voltage (V_S) changes, when $R_L=\infty$.

Line Regulation =
$$\frac{\partial V_L}{\partial V_S} = \frac{R_Z}{R_S + R_Z}$$
 (1)

Load Regulation

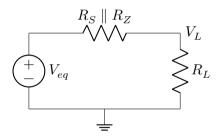




 Load Regulation: output impedance of the voltage regulator.

Thevenin Equivalent of a Voltage Regulator





$$V_{eq} = 5 + \frac{V_S - 5}{R_S + R_Z} R_Z \qquad (3)$$

$$V_L = V_{eq} \frac{R_L}{(R_S \| R_Z) + R_L} \qquad \text{(4)}$$

Thevenin Equivalent of a Voltage Regulator



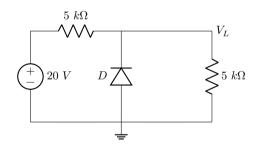
Numerical Test: $V_S=20~V, R_S=5~k\Omega, R_Z=0.1~k\Omega, V_Z=~5~V, R_L=5~k\Omega$

$$V_{eq} = 5 + \frac{20 - 5}{5 \ k + 0.1 \ k} 0.1 \ k = 5.2941 \ V \tag{5}$$

$$V_L = 5.2941 \times \frac{5 \ k}{\frac{5 \ k \times 0.1 \ k}{5 \ k + 0.1 \ k} + 5 \ k} = 5.1923 \ V \tag{6}$$

Line Regulation



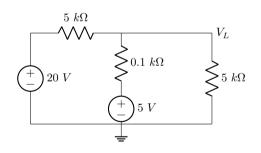


• Line Regulation

$$\frac{R_Z}{R_Z + R_S} = \frac{0.1k}{5k + 0.1k} = 19.6 \ mV/V \quad (7)$$

Load Regulation



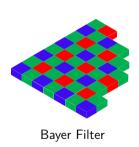


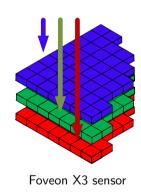
Load Regulation

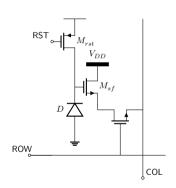
$$R_Z \parallel R_S = 5 \ k \parallel 0.1 \ k = 98 \ \Omega$$
 (8)

CMOS Image Sensors



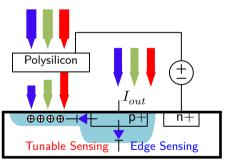




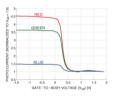


CIS Applications





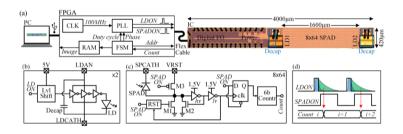




D. Ho et al., "CMOS Tunable-Color Image Sensor With Dual-ADC Shot-Noise-Aware Dynamic Range Extension," in IEEE Transactions on Circuits and Systems I: Regular Papers.

CIS Applications





Jaebin Choi et al. "Fully Integrated Time-Gated 3D Fluorescence Imager for Deep Neural Imaging", in IEEE Transactions on Biomedical Circuits and Systems.

Sensitivity of Diode current to its Voltage



• Calculate V_a for a silicon diode with $I_S=0.1~fA$ and I_D increasing from 300 μA to 10 mA at 300 K.

$$300 \times 10^{-6} = (0.1 \times 10^{-15}) \left(e^{0.025875} - 1 \right) \tag{9}$$

$$V_a = 0.743 \ V \tag{10}$$

$$10 \times 10^{-3} = (0.1 \times 10^{-15}) \left(e^{\frac{V_a}{0.025875}} - 1 \right)$$
 (11)

$$V_a = 0.834 \ V \tag{12}$$

Example



• Calculate I_S for a silicon diode with $I_D=2.5~mA$ and $V_a=0.736~V$ at 50 $^{\circ}C$.

$$2.5 \times 10^{-3} = I_S \left(\frac{1.6 \times 10^{-19} \cdot 0.736}{e^{1.38 \times 10^{-23} \cdot 323}} - 1 \right)$$
 (13)

$$I_S = 8.4 fA \tag{14}$$

Example



• Calculate the required V_a for I_D of a silicon diode to increase by a factor 10 at 300 K. Assume $I_D\gg I_S$.

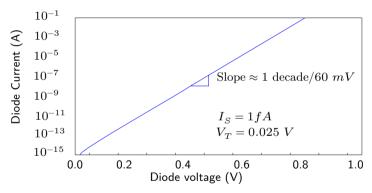
$$\begin{cases}
I_{D1} = I_S \left(e^{\frac{qV_{a1}}{kT}} - 1 \right) \approx I_S e^{\frac{qV_{a1}}{kT}} \\
I_{D2} = I_S \left(e^{\frac{qV_{a2}}{kT}} - 1 \right) \approx I_S e^{\frac{qV_{a2}}{kT}}
\end{cases}$$
(15)

$$\frac{I_{D2}}{I_{D1}} = 10 = \frac{I_S e^{\frac{qV_{a2}}{kT}}}{I_S e^{\frac{qV_{a1}}{kT}}} = e^{\frac{V_{a2} - V_{a1}}{0.025875}}$$
(16)

$$V_{a2} - V_{a1} = 0.025875 \times \ln 10 = 0.05958 \ V \approx 60 \ mV$$
 (17)

Sensitivity of Diode Current



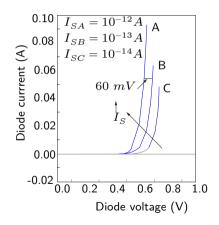


Diode i-v characteristic on semilog scale

• The diode voltage changes by about 60 mV per decade change in diode current.

I_D and I_S versus V_a



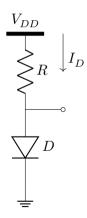


$$I_D = I_S(e^{\frac{qV_a}{kT}} - 1)$$
 (18)

- \bullet At the same $I_D,$ when I_S increases by 10, V_a decreases by 60 mV.
- \bullet At the same $I_S,$ when I_D increases by 10, V_a increases by 60 mV.

Large Signal Model

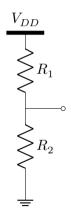


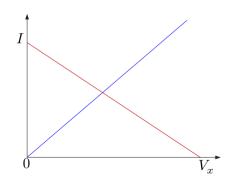


$$I_D = \frac{V_{DD} - V_D}{R} \tag{19}$$

Graphical "Load Line" Method

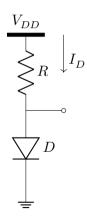


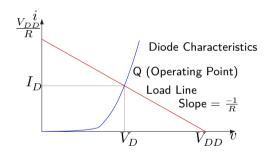




Graphical "Load Line" Method



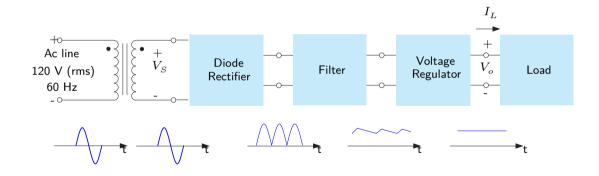




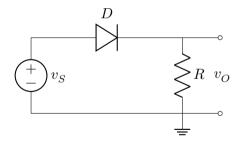
- Can be used for non-linear devices as well.
- It is very useful in future when we discuss transistors.

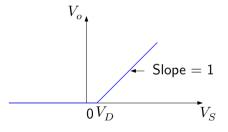
Block Diagram of a DC Power Supply



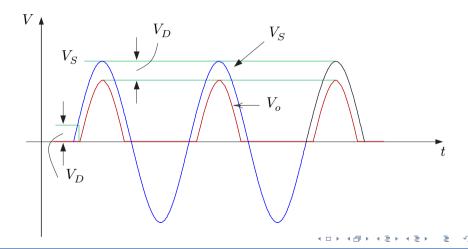








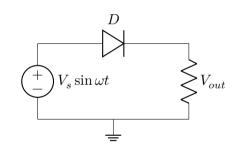






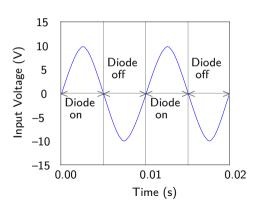
$$V_s = 10V \tag{20}$$

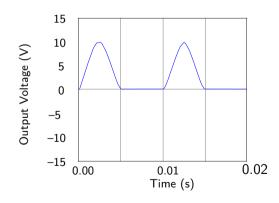
$$\omega = 2\pi f = 2\pi \frac{1}{0.01} = 200\pi (\text{rad/sec})$$
 (21)



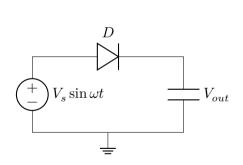
• V_{out} is not DC.

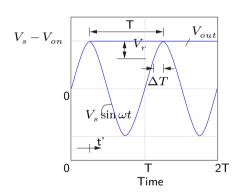






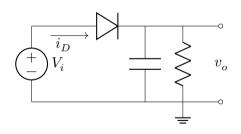
Half-Wave Rectifier with Capacitive Load (Peak Rectifier Via Capacitive Via Capacitive Load (Peak Rectifier Via Capacitive Via Capa

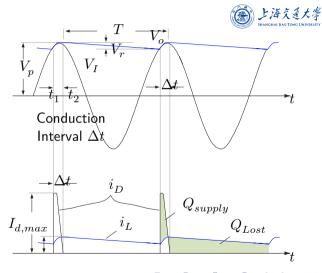




 V_{out} is DC

Peak Rectifier with Load





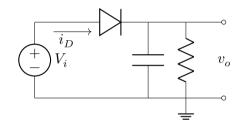
Peak Rectifier with Load



$$i_L = V_o/R \tag{22}$$

$$i_D=i_C+i_L=C\frac{dv_I}{dt}+i_L \hspace{0.5cm} \mbox{(23)}$$

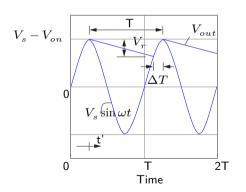
$$V_p - V_r \simeq V_p e^{-T/CR} \qquad (24)$$



A very large capacitor is required to reduce the ripple

Half-Wave Rectifier with RC Load (I)





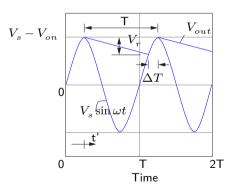
Here "DC" means we approximate the discharging period with a constant voltage.

$$V_{dc} = V_s - V_{on} \tag{25}$$

$$I_{dc} = \frac{V_{dc}}{R} \tag{26}$$

Calculating Ripple Voltage





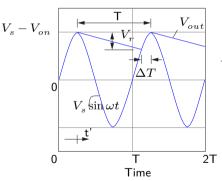
$$V_r = (V_s - V_{on}) (1 - e^{-\frac{T - \Delta T}{RC}}) \tag{27}$$

$$\cong (V_s - V_{on}) \left(\frac{T - \Delta T}{RC} \right) \text{ if } (T - \Delta T) \ll RC \quad \text{(28)}$$

$$\cong (V_s - V_{\text{on}}) \left(\frac{T}{RC}\right) \text{ if } \Delta T \ll T$$
 (29)

Calculating Conduction Angle and Interval





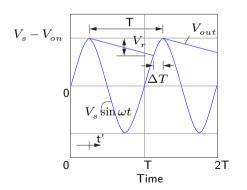
$$V_s \sin \left[\omega \left(\frac{5T}{4} - \Delta T\right)\right] - V_{on} = (V_s - V_{on}) - V_r$$
 (30)

$$V_s \sin\left(\frac{5\pi}{2} - \theta_c\right) - V_{on} = (V_s - V_{on}) - V_r$$
 (31)

$$V_s \cos \theta_c = V_s - V_r \tag{32}$$

Half-Wave Rectifier with RC Load





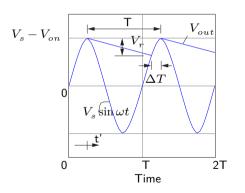
$$\cos \theta_c = rac{V_s - V_r}{V_c} \cong 1 - rac{\theta_c^2}{2}$$
 if θ_c very small (33)

$$\theta_c = \sqrt{\frac{2V_r}{V_s}} \tag{34}$$

$$\Delta T = \frac{\theta_c}{\omega} = \frac{1}{\omega} \sqrt{\frac{2V_r}{V_s}} \tag{35}$$

Derive Peak Current





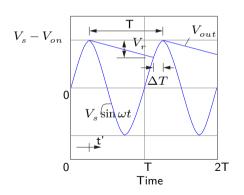
The charge filled on C during ΔT is discharged during $T-\Delta T$

$$Q \cong \frac{I_{\mathsf{peak}} \Delta T}{2} = I_{dc}(T - \Delta T) \cong I_{dc}T \qquad \textbf{(36)}$$

$$I_{\mathsf{peak}} = \frac{2I_{\mathsf{dc}} T}{\Delta T} \tag{37}$$

Derive Surge Current





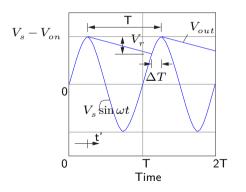
During charging period (ΔT), almost all diode current goes to C.

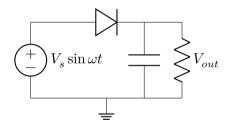
$$\left|\frac{1}{SC}\right| = \frac{1}{2\pi \frac{C}{T}} = \frac{T}{2\pi C} \ll R \quad \text{if } RC \gg T \quad \text{(38)}$$

$$I_{\rm surge} = C \frac{d \left(V_s \sin \omega t - V_{\rm on} \right)}{dt} = \omega C V_s \text{ if } t = 0 \tag{39} \label{eq:surge}$$

Half-Wave Rectifier with RC Load (IV)







Peak-inverse-voltage $\cong 2V_s - V_{on}$ If too large, the diode breaks down.

Example



Find the value of the dc output voltage, dc output current, ripple voltage, conduction interval, conduction angle and diode peak current for a half-wave rectifier driven from a transformer having a secondary voltage of 12.6 V_{rms} (60 Hz) with R=15 Ω and C=25,000 μ F. Assume V_{on} =1 V.

$$V_{dc} = 12.6\sqrt{2} - 1 = 16.8 \ V \tag{40}$$

$$I_{dc} = \frac{16.8}{15} = 1.12 \ A \tag{41}$$

$$V_r \cong V_{dc} \frac{T}{RC} = 16.8 \frac{\frac{1}{60}}{15 \times 25000 \times 10^{-6}} = 0.747 \ V$$
 (42)

Example



$$\theta_c \cong \sqrt{\frac{2 V_r}{V_s}} = \sqrt{\frac{2 \times 0.747}{12.6 \times \sqrt{2}}} = 0.29 \ (rad) \ \text{or} \ 16.6^{\circ}$$
 (43)

$$\Delta T \cong \frac{\theta_c}{\omega} = \frac{0.29}{2\pi \times 60} = 7.69 \times 10^{-4} \ (sec)$$
 (44)

$$I_{\text{peak}} = \frac{2 \times 1.12 \times \frac{1}{60}}{7.69 \times 10^{-4}} = 48.6 A$$
 (45)

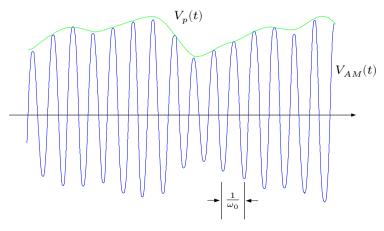
Example



- Make sure all assumptions are valid.
- Since R is small (15 Ω), C needs to be large (25,000 μ F) to maintain a low V_r .
- The diode must be able to handle these repetitively high peak currents.

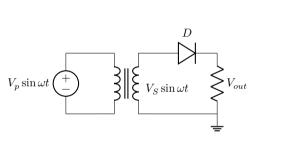
Peak Rectifier in Communication

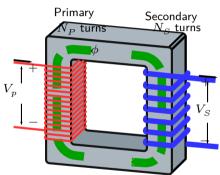




Transformer







• The output of an ideal transformer can be represented as an ideal voltage source.

Transformer



(48)

• By Faraday's law of induction

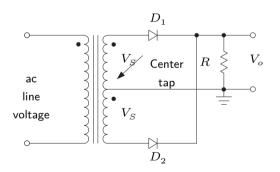
$$V_s = -N_s \frac{d\phi}{dt} \tag{46}$$

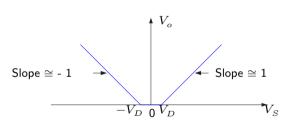
$$V_p = -N_p \frac{d\phi}{dt}$$
 (47)

$$rac{N_s}{N_p} = rac{N_s}{N_p}$$

Full Wave Rectifier

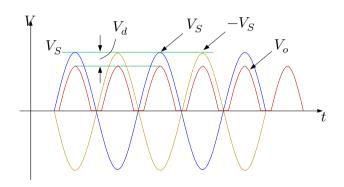






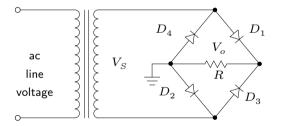
Full Wave Rectifier

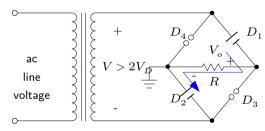




Full Wave Rectifier ($V > 2V_D$)

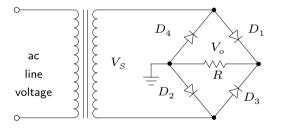


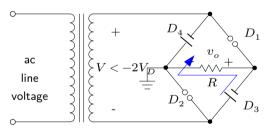




Full Wave Rectifier ($V < -2V_D$)

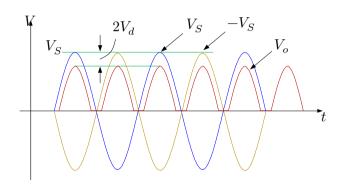






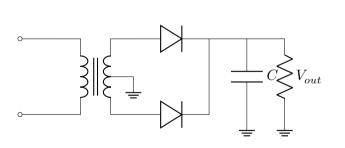
Full Wave Rectifier

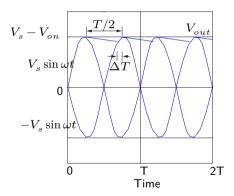




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Full Wave Rectifier(I)





Full Wave Rectifier(I)



$$V_{dc} = V_s - V_{on} \tag{49}$$

$$I_{dc} = \frac{V_{dc}}{R} \tag{50}$$

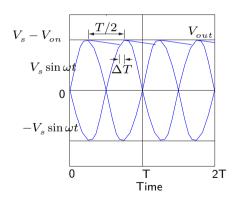
$$V_r = (V_s - V_{on}) \left(1 - e^{-\frac{T/2 - \Delta T}{RC}} \right)$$
 (51)

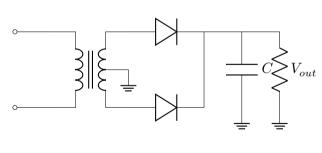
$$\cong (V_s - V_{on}) \left(\frac{T/2 - \Delta T}{RC}\right) \text{ if } \left(\frac{T}{2} - \Delta T\right) \ll RC$$
 (52)

$$\cong (V_s - V_{on}) \left(\frac{T}{2RC}\right) \text{ if } \Delta T \ll \frac{T}{2}$$
 (53)

Full Wave Rectifier(II)







Full Wave Rectifier(II)



$$-V_s \sin\left[\omega\left(\frac{3T}{4} - \Delta T\right)\right] - V_{on} = (V_s - V_{on}) - V_r \tag{54}$$

$$-V_s \sin\left(\frac{3\pi}{2} - \theta_c\right) - V_{on} = (V_s - V_{on}) - V_r \tag{55}$$

$$V_s \cos \theta_c = V_s - V_r \tag{56}$$

$$\theta_c = \sqrt{\frac{2V_r}{V_s}} \tag{57}$$

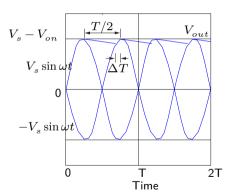
Full Wave Rectifier(II)

$$\cos \theta_c = \frac{V_s - V_r}{V_s} \cong 1 - \frac{\theta_c^2}{2} \text{ if } \theta_c \text{ very small}$$
 (58)

$$\Delta T = \frac{\theta_c}{\omega} = \frac{1}{\omega} \sqrt{\frac{2V_r}{V_s}} \tag{59}$$

Full Wave Rectifier(III)





$$Q \cong rac{I_{
m peak} \; \Delta T}{2} = I_{dc} \left(rac{T}{2} - \Delta T
ight) \cong I_{dc} rac{T}{2} \end{60}$$

$$I_{\mathsf{peak}} = \frac{I_{dc}T}{\Delta T} \tag{61}$$

$$I_{\mathsf{surge}} = \omega C V_s$$
 (62)

$$PIV = 2V_s - V_{on} \tag{63}$$