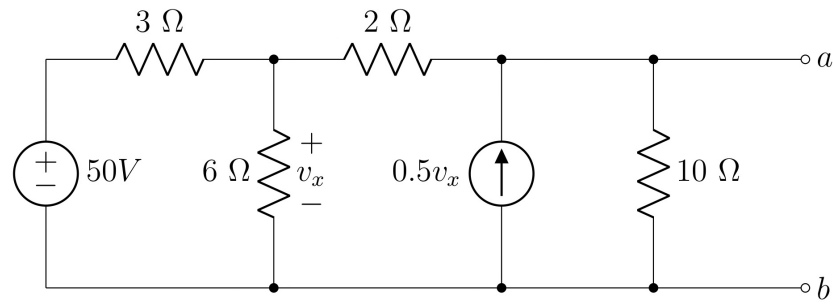


### Question 1. Thevenin Circuit

Obtain the Thevenin equivalent circuit at terminals a-b for the circuit below.

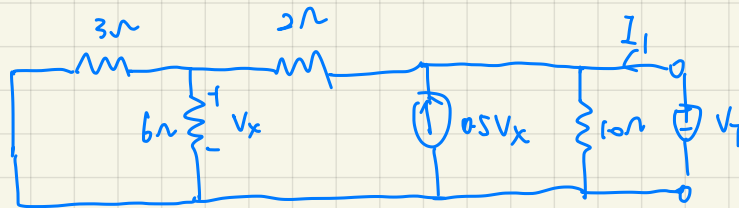


For  $V_{TH}$ :

$$\begin{cases} I_1 = I_2 + I_3 = \frac{V_x}{6} + I_3 \\ V_x = 50 - 3I_1 = 10(I_3 + 0.5V_x) + 2I_3 \end{cases}$$

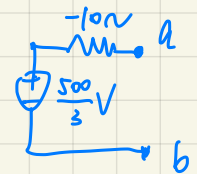
$$\Rightarrow V_{TH} = \frac{500}{3} \text{ V}$$

For  $R_{TH}$ :



$$\begin{cases} V_x = \frac{V_T}{2} \\ 0.5V_x = I_1 + I_2 = \frac{V_T}{4} + I_2 \\ V_T = (I_2 + I_T) \cdot 10 \end{cases} \Rightarrow R_{TH} = -10\Omega$$

Thevenin Circuit:



## Question 2. CMOS Image Sensors (CIS)

You decided to launch a company with innovative light sensors with MOSFET capacitor pixels. Consider the structure shown below where green light is only incident on the gate area and the rest of the area is shielded. Assume that the quantum efficiency is 40 %, which means only 40 out of 100 incident photons excite an electron-hole pair (An imaging-specific process nowadays typically have a quantum efficiency of 75%). The wavelength of green light  $\lambda = 550 \text{ nm}$ . The size of the active area (Yellow area)  $W = L = 5 \mu\text{m}$ . You looked into textbook and found  $h = 6.626 \times 10^{-34}$ .

- (a) The CMOS camera is a PN junction. To understand how it works, you decide to revisit device physics. Consider a p-n diode with an abrupt junction. Assume that the doping densities on the p and n side are uniform and they are  $N_A = 10^{18}/\text{cm}^3$  and  $N_D = 10^{20}/\text{cm}^3$  respectively. Calculate the majority and minority carrier density in each side at thermal equilibrium. Assume that the intrinsic carrier concentration (free electron and hole carrier concentration of pure silicon)  $n_i = 1.5 \times 10^{10}/\text{cm}^3$  at room-temperature of  $25^\circ\text{C}$ .

c.a) For p-type:  $N_A \gg n_i \Rightarrow p = N_A = 10^{18} / \text{cm}^3$   
$$n = \frac{n_i^2}{p} = 2.25 \times 10^2 / \text{cm}^3$$

For n-type:  $N_D \gg N_i \Rightarrow n = N_D = 10^{20} / \text{cm}^3$   
$$p = \frac{n_i^2}{n} = 2.25 / \text{cm}^3$$

- (b) For a CMOS camera to operate properly, you will have to set the bias condition of the photodiode. For a forward bias voltage of  $V_d = 0\text{V}$ ,  $V_d = 0.6\text{V}$  and a reverse bias voltage of  $V_d = -1.6\text{V}$ , calculate the built-in potential ( $\varphi_0$ ) ( $V_d = 0\text{V}$ ) and the potential barrier under the different bias conditions.

$$\varphi_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} = 1.04 \text{ V}$$

For forward bias  $0.6\text{V}$ :  $V_F = \varphi_0 - V_d = 0.44 \text{ V}$

For reverse bias  $-1.6\text{V}$ :  $V_R = \varphi_0 + V_d = 2.64 \text{ V}$

(c) Potential across the Junction

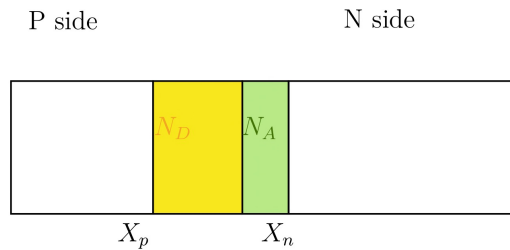


FIGURE 3. PN junction

By continuity at the junction, we have

$$N_A X_p = N_D X_n$$

What is the potential across the junction?

$$V_{\text{junction}} = \varphi_0 - V_d$$

In thermal equilibrium  $V_d = 0 \text{ V}$ , the potential barrier is equal to the built-in potential

$$\Rightarrow V_{\text{junction}} = \varphi_0$$

Since  $\varphi_0 = V_T \cdot \ln \frac{N_A N_D}{n_i^2}$ , Given that  $N_A \cdot X_p = N_D \cdot X_n$

$$\Rightarrow \varphi_0 = V_T \cdot \ln \frac{N_D \cdot X_n \cdot N_A}{n_i^2}$$

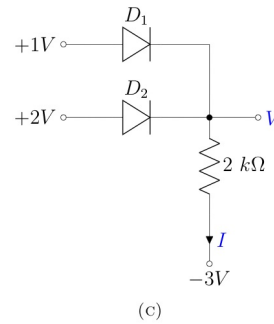
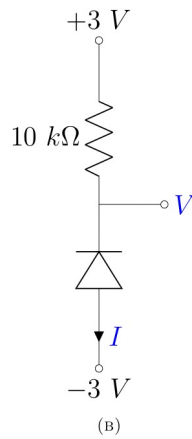
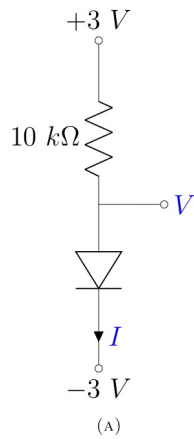
(d) We've explored the previous calculations that, by applying a reverse bias to the PN junction, we can sweep photo excited electron-hole pairs to the contact, inducing a current. if the reverse bias voltage is too large, for example  $E_{\text{max}} = 1 \times 10^6 \text{ V/cm}$ , the PN junction breaks down. Whats the reverse bias voltage at this moment?

$$\parallel 10^8 \text{ V/m}$$

$$V_d = E_{\text{max}} \cdot W = 10^8 \cdot 5 \times 10^{-6} = 500 \text{ V}$$

### Question 3. Diode Circuit Exercise 1

For each circuit below, assuming **ideal diode model**, calculate the labeled voltages and currents (V and I):



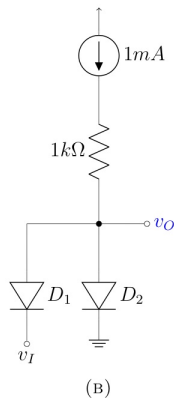
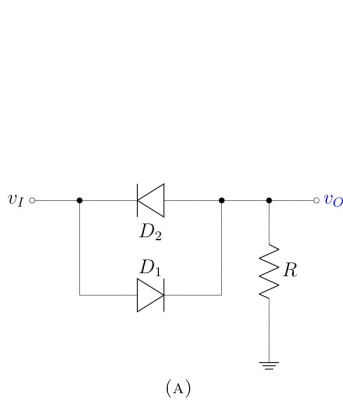
(A)  $V = -3V$ ,  $I = 6 \times 10^{-4} A$

(B)  $V = 3V$ ,  $I = 0 A$

(C)  $V = 2V$ ,  $I = 2.5 \times 10^{-3} A$

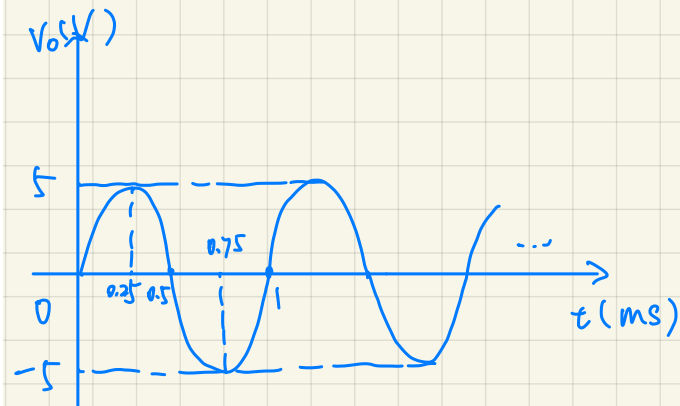
### Question 4. Diode Circuit Exercise 2

In each of the ideal-diode circuits shown below,  $v_I$  is a 1-kHz, 5-V peak sine wave. Sketch the output waveform  $v_O$  as a function of time, showing the important values, such as the peak voltage values (positive and negative).

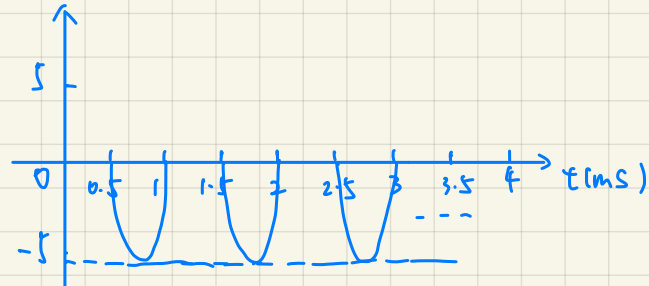


$V_I = 5 \sin(2000\pi t)$

(A)

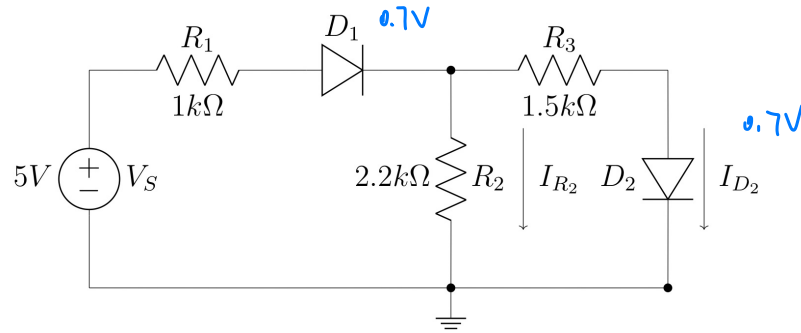


(B)  $V_O (V)$



### Question 5. Diode Circuit Exercise 3

In the circuit below, assume the constant voltage drop model for the diodes and assume the turn-on voltage is 0.7V. Calculate the values for current  $I_{R_2}$  and  $I_{D_2}$ .

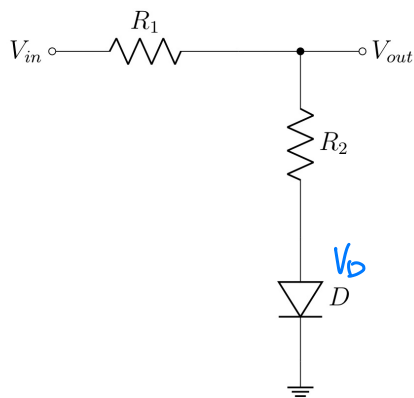


Assume all diodes are activated :

$$\begin{cases} 5 - 0.7 - R_1 \cdot (I_{R_2} + I_{D_2}) = V_0 \\ V_0 = 2.2 \times 10^3 \cdot I_{R_2} \\ V_0 - 0.7 = 1.5 \times 10^3 \cdot I_{D_2} \end{cases} \Rightarrow \begin{cases} I_{R_2} = 1.02 \times 10^{-3} \text{ A} \\ I_{D_2} = 1.03 \times 10^{-3} \text{ A} \end{cases}$$

### Question 6. Diode Circuit Exercise 4

Plot the input/output characteristics of the circuit below using the CVD model.



① When  $V_{in} > V_D$  ,  $V_{out} = \frac{R_1 V_D + R_2 V_{in}}{R_1 + R_2}$

② When  $V_{in} < V_D$  ,  $V_{out} = V_{in}$

