VV311
Electronic Circuits
RC Mid

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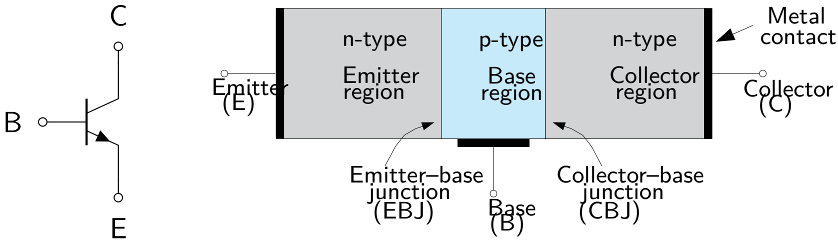
1. BJT

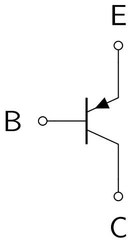
Bipolar Junction Transistors (BJTs)

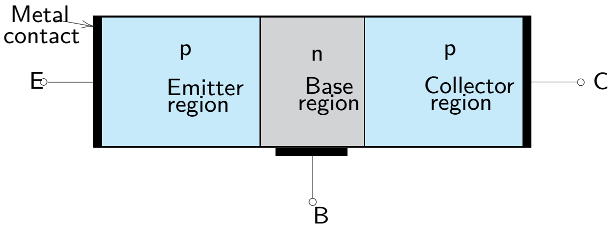
- These three–terminal devices are very useful for amplification, since an input signal can be applied across one pair of terminals and the output signal can be taken from another pair of terminals.
- bipolar junction transistor (BJTs) are still widely used in many analog circuits, including amplifiers, filters, mixers, etc.
- These are very fast devices, but also consume more power. This is the main reason CMOS has become the mainstream technology for digital circuits.

npn BJT Transistors



pnp BJT Transistors

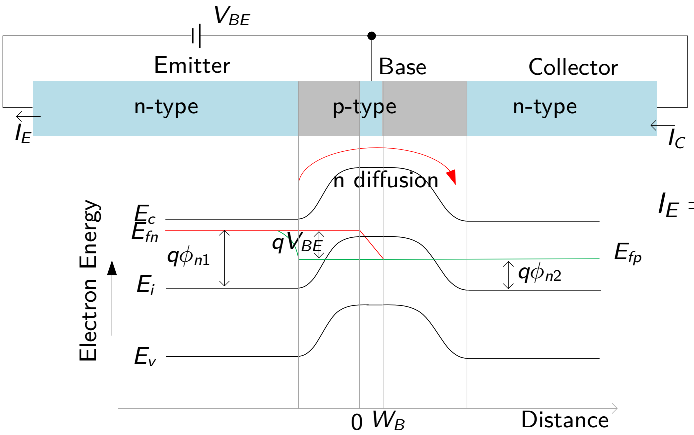




npn & pnp BJT Transistors

- There are three terminals: emitter, collector, and base
- There are two pn junctions, the base-emitter junction, and the base-collector junction, like two diodes that are connected back to back.
- However, the operation of the transistor is very different than just two back to back connected diodes.

$V_{BE} > 0$ and $V_{CB} = 0$ ($N_{d1} \gg N_a$, W_B very short)



$$I_B \cong 0 \qquad (1)$$

$$I_C \cong I_E \qquad (2)$$

$$I_E = I_C + I_B \qquad (3)$$

$$\frac{I_C}{I_E} = \alpha \qquad (4)$$

$$\frac{I_C}{I_B} = \beta \qquad (5)$$

BJT Operation Summary

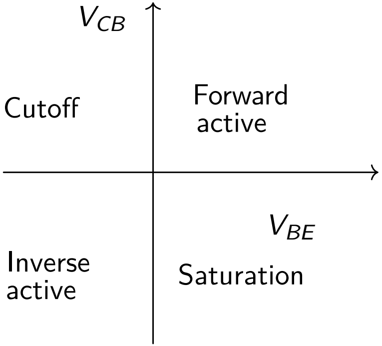
$$i_E = i_C + i_B \tag{6}$$

$$i_E = i_C + \frac{i_C}{\beta} = \frac{i_C(\beta + 1)}{\beta} \tag{7}$$

$$i_C = \frac{\beta}{\beta + 1} i_E = \alpha i_E \tag{8}$$

$$\beta \approx 50 - 200\ (100) \tag{9}$$

α is less than 1, but ≈ 1
What is the idel case?



BJT Operation Summary

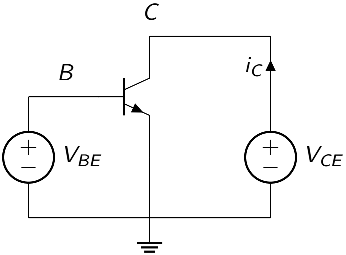
What is the idel case?

$$\mathbf{I}_{\mathrm{C}} = \mathbf{I_s}\left(\mathbf{e}^{\frac{\mathbf{q}V_{BE}}{\mathrm{kT}}} - \mathbf{1}\right)$$

$$\boldsymbol{\alpha} = \frac{\mathbf{I_C}}{\mathbf{I}_{\mathrm{E}}} = \mathbf{1}$$

$$\boldsymbol{\beta} = \frac{\mathbf{I_C}}{\mathbf{I}_{\mathrm{B}}} = \infty \text{ If } I_\beta = 0$$

Summary in Forward - Active Mode



$$V_{CE} \geq V_{BE} \tag{10}$$

$$I_C = I_S \left(e^{\frac{qV}{BT}} - 1 \right) \tag{11}$$

$$\alpha = \frac{I_C}{I_E} \cong 1 \tag{12}$$

$$\beta = \frac{I_C}{I_B} = \frac{\alpha}{1 - \alpha} \tag{13}$$

I_s is a constant in the spice model

Early Effect

- As the collector voltage changes the width of the depletion region at the BC junction changes, and this causes the effective distance between the edge of the BE junction and BC junction to reduce. Thus smaller base width $\rightarrow$ a higher collector current as there is less recombination.

$$i_C \approx I_s e^{v_{EB}/V_T}(1 + \frac{v_{CE}}{V_A}) \qquad (14)$$

- V_A is called the Early voltage. Typically is 50-100 V.

I_C vs V_{CE} (with Early Effect)

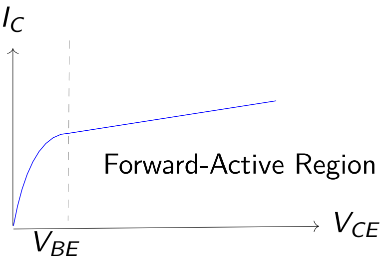


Figure: *

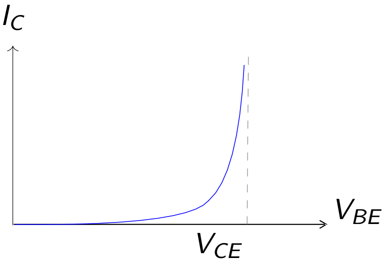


Figure: *

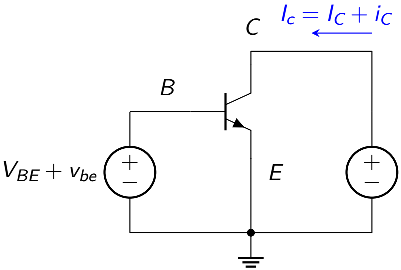
At given V_{BE}, DC sweep V_{CE}

At given V_{CE}, DC sweep V_{BE}

- For $V_{CE} \geq V_{BE}$,

$$I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right) \tag{15}$$

Hybrid-π Model (g_m and r_π)

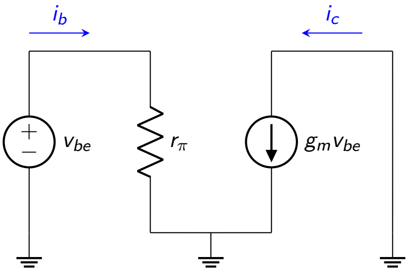


$V_{CE} \geq V_{BE} \Rightarrow$ Forward—Active

$$I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right) \tag{16}$$

Hybrid-π Model (Derivation of g_m and r_π)

Small-signal circuit:



$$r_\pi = \frac{dV_{BE}}{dI_B} = \frac{1}{\frac{dI_C}{\beta dV_{BE}}} \tag{17}$$

$$= \frac{1}{\frac{g_m}{\beta}} = \frac{\beta}{g_m} \tag{18}$$

$$g_m = \frac{dI_C}{dV_{BE}} \cong \frac{I_C}{kT/q} \tag{19}$$

Models with the Early Effect Included

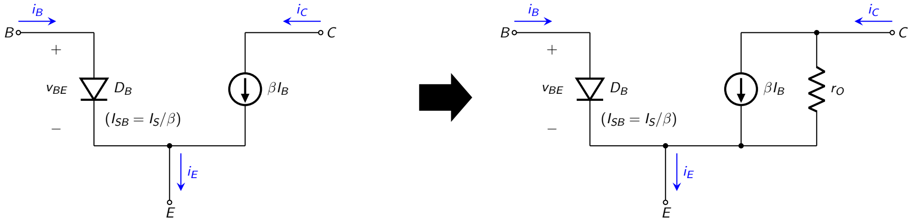
- As we see from this relationship, the collector current should change when the collector-emitter voltage changes:

$$i_C \approx I_S e^{v_{EB}/V_T}\left(1 + \frac{v_{CE}}{V_A}\right) \tag{20}$$

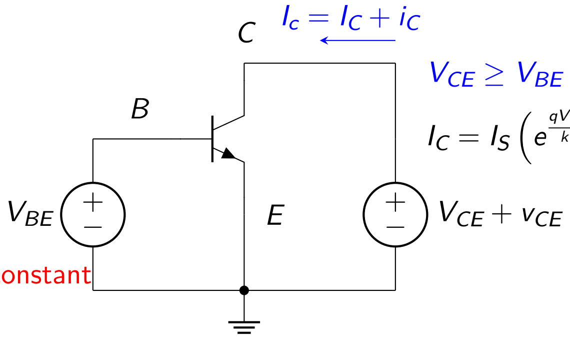
- So, this effect is included in the model shown below where:

$$r_o = \frac{V_A}{I_C} = \frac{\Delta v_{CE}}{\Delta i_C} \tag{21}$$

Models with the Early Effect Included



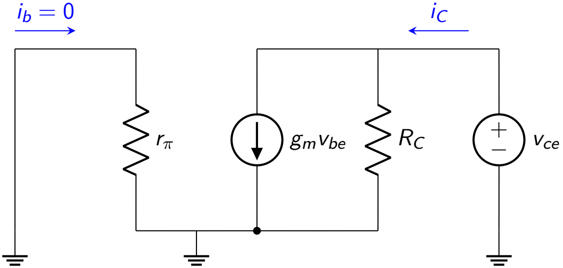
Hybrid-π Model (how to get r_o)



$V_{CE} \geq V_{BE} \Rightarrow$ Forward—Active

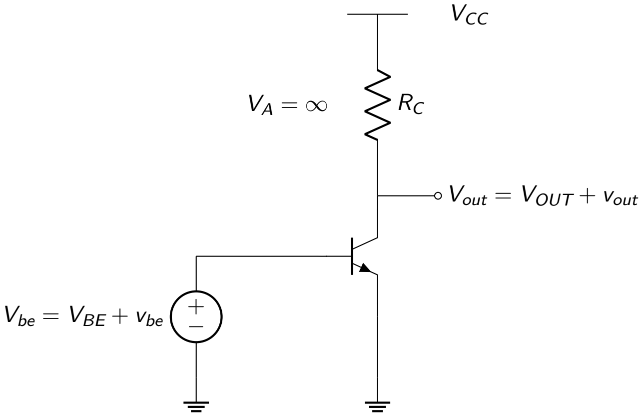
$$I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right) \tag{22}$$

Hybrid-π Model (Derivation of r_o)

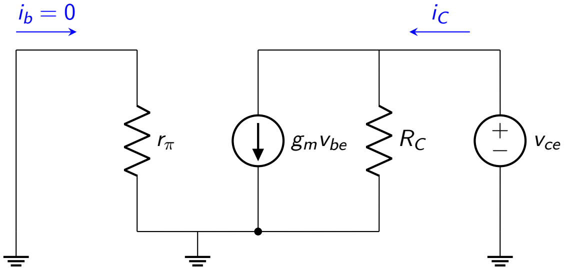


$$r_o = \frac{1}{\frac{dI_C}{dV_{CE}}} \cong \frac{V_A}{I_C} \tag{23}$$

Common-Emitter Amplifier ($V_A = \infty$)



Common-Emitter Amplifier ($V_A = \infty$)



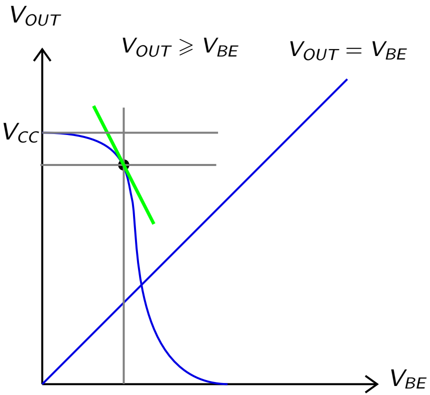
$$A_v = \frac{v_{out}}{v_{be}} = -g_m\left(R_C \parallel r_O\right) \tag{24}$$

$$= -g_m R_C \quad (\text{since } r_o = \infty) \tag{25}$$

Common-Emitter Amplifier (V_A = finite)

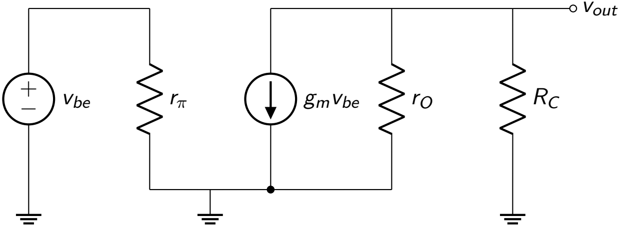
$$A_V = \frac{dV_{OUT}}{dV_{BE}} \quad (26)$$

$$\cong g_m(R_C \parallel r_O) \quad (27)$$



Common-Emitter Amplifier (V_A = finite)

- Small-Signal Analysis



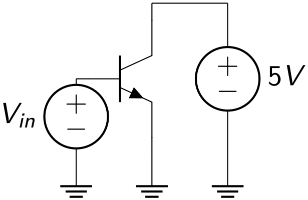
$$A_v = \frac{v_{out}}{v_{be}} = -g_m(R_C \parallel r_O) \quad (28)$$

Examples

See Blackboard

Example 1: $I_S = 1e{-}15$ $\beta = 100$ $V_{AF} = 50$

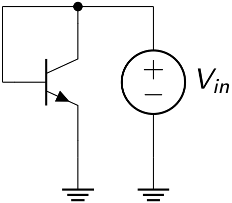
- What is current?
- What is small signal gain?
- What is the output if input $= 2 + 0.01\sin(2 \cdot \pi \cdot 100t)$



$$I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right) \tag{29}$$

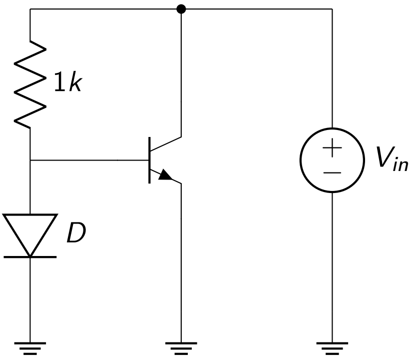
Example 2: $I_S = 10^{-15}$, $\beta = 100$, $V_{AF} = 50$

Find the output if input $= 2 + 0.01\sin(2\pi 100 t)$



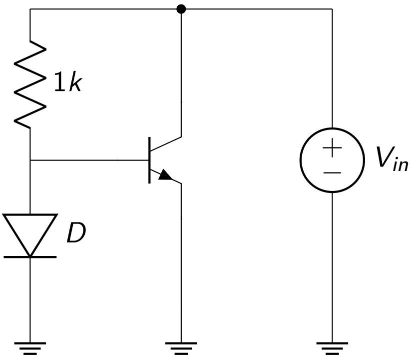
Example 3: $I_S = 1e{-}15, \beta = \infty, V_{AF} = \infty, V_{ON} = 1V$

- Find the output if input $= 2 + 0.01\sin(2\pi 100t)$



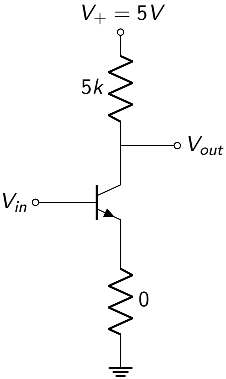
Example 4: $I_S = 1e{-}15, \beta = \infty, V_{AF} = 100$

- Find the output if input $= 2 + 0.01\sin(2\pi 100 t)$



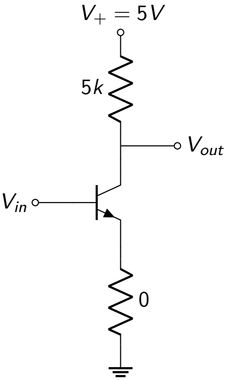
Example 5: $I_s = 5 \times 10^{-16}, \beta = 200, V_{AF} = \infty$

- Find the output if input $= 0.75 + 0.01 \sin(2\pi 100 t)$



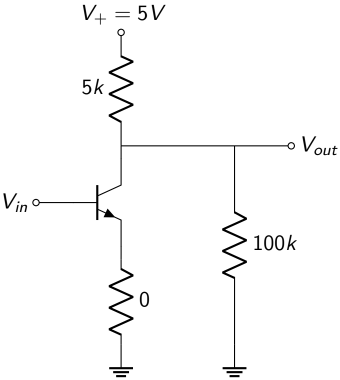
Example 6: $I_s = 5 \times 10^{-16}, \beta = 200, V_{AF} = 100$

- Find the output if input $= 0.75 + 0.01\sin(2\pi 100t)$



Example 7: $I_s = 5 \times 10^{-16}, \beta = 200, V_{AF} = 100$

- Find the output if input $= 0.75 + 0.01\sin(2\pi 100t)$



END

Thanks