VE311 Final Review - Frequency Response (L19-L22)

Zeyu Zhang

University of Michigan-Shanghai Jiao Tong University Joint Institute

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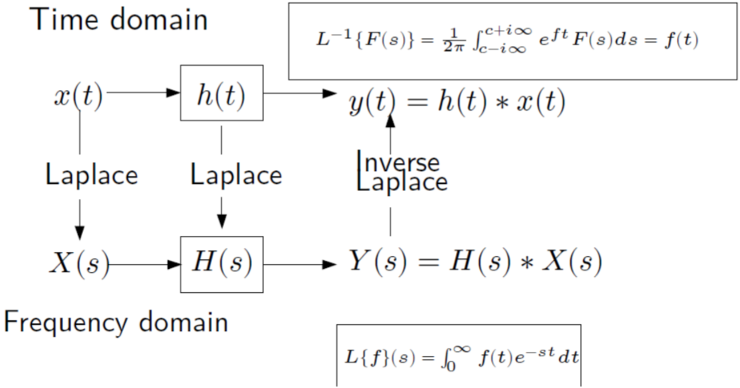
Overview

- Review of ve216
- Finite op amp bandwidth
- Miller effect

Circuit as system

- Input signal $\Longrightarrow$ $\boxed{\textit{Circuit}}$ $\Longrightarrow$ Output signal
- Time domain $\Longleftrightarrow$ Frequency domain (s-domain)



Calculation

Transfer function:

$$H(s) = \frac{Y(s)}{X(s)} \tag{1}$$

Useful Laplace transform:

$$\frac{K}{(s+\alpha)(s+\beta)} \Leftrightarrow \frac{K}{(\beta-\alpha)}\left(e^{-\alpha t} - e^{-\beta t}\right) u(t) \tag{2}$$

Zero and pole

$$H(s) = H_0 \frac{\prod_z (s - s_z)}{\prod_p (s - s_p)} = H_0' \frac{\prod_z (\tau_z s - 1)}{\prod_p (\tau_p s - 1)} \tag{3}$$

- Zeros: s_z
- Poles: s_p
- Magnitude:

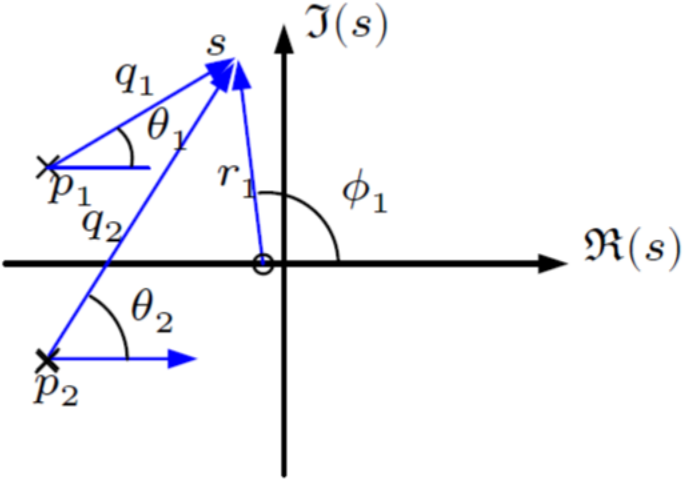
$$|H(s)| = K \frac{\prod_{i=1}^{m} |(s - z_i)|}{\prod_{i=1}^{n} |(s - p_i)|} \tag{4}$$

- Phase:

$$\angle H(s) = \sum_{i=1}^{m} \angle (s - z_i) - \sum_{i=1}^{n} \angle (s - p_i) \tag{5}$$

Pole-zero representation

- Complex plane



Pole-zero representation

- Bode plot (magnitude)
  - Zero contributes 20dB/decade slope if $s > s_z$
  - Pole contributes -20dB/decade slope if $s > s_p$
  - Complex conjugate zeros contributes 40dB/decade slope if $s >\mid s_z \mid$
  - Complex conjugate poles contributes -40dB/decade slope if $s >\mid s_p \mid$
- Bode plot (phase)
  - 90° if $s_z = 0$
  - -90° if $s_p = 0$
  - Increase by 90° and passes though the midpoint of 45° at the break point $s = s_z \neq 0$
  - Decrease by 90° and passes though the midpoint of -45° at the break point $s = s_p \neq 0$

Bode plot

- Example: Find the Bode log magnitude and phase angle plot for the transfer function,

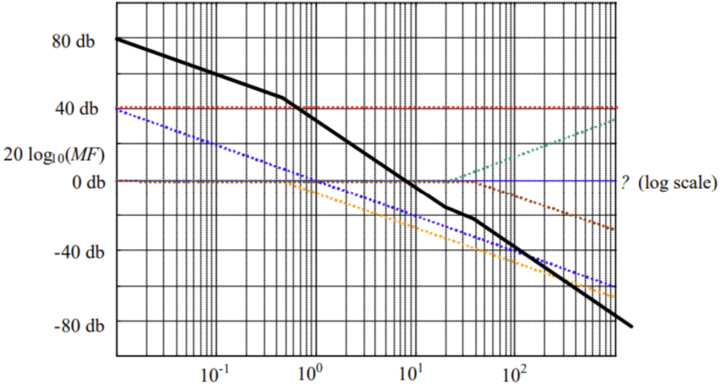
$$H(s) = \frac{200(s+20)}{s(2s+1)(s+40)} \tag{6}$$

- Solution: Firstly, change the form to

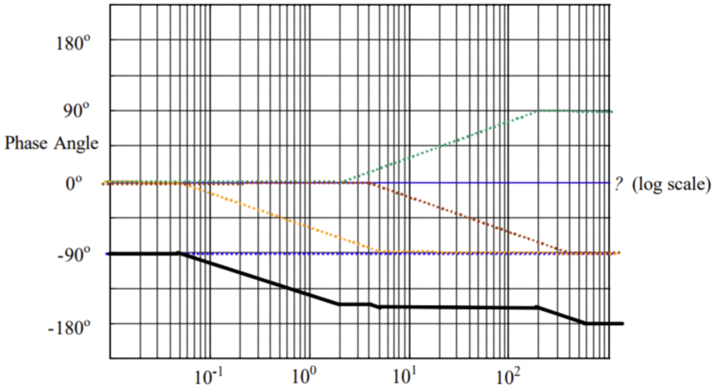
$$H(s) = \frac{100(s/20+1)}{s(s/0.5+1)(s/40+1)} \tag{7}$$

Bode plot

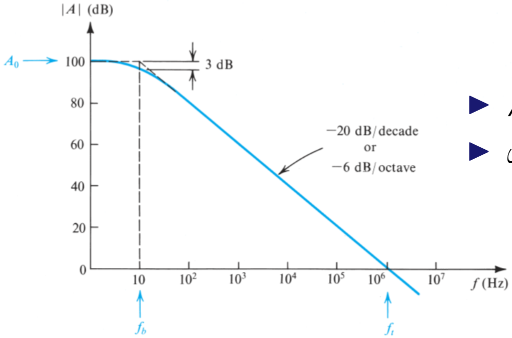
- Magnitude



- Phase



Finite op amp bandwidth



$$A(j\omega) = \frac{A_0}{1 + j\omega/\omega_b} \qquad (8)$$

- A_0: dc gain
- ω_b: 3-dB frequency

Finite op amp bandwidth

- Magnitude:

$$| A(j\omega) |= \frac{A_0\omega_b}{\omega} \tag{9}$$

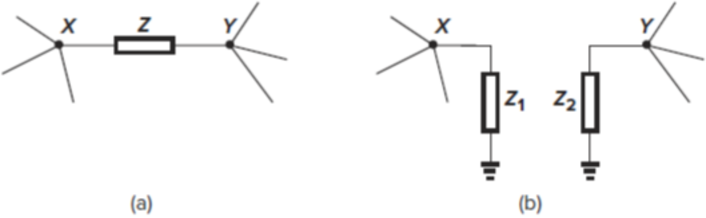
- Unity gain:

$$A(j\omega_t) = 1 \tag{10}$$

- unity-gain bandwidth (gain–bandwidth product)

$$f_t = \frac{\omega_t}{2\pi} = \frac{A_0\omega_b}{2\pi} \tag{11}$$

Miller effect

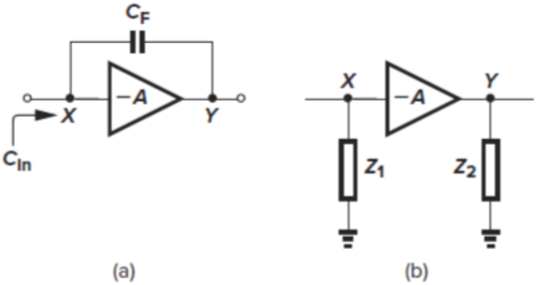


$$A_v = \frac{V_Y}{V_X} \tag{12}$$

$$Z_1 = \frac{Z}{1 - A_v} \tag{13}$$

$$Z_2 = \frac{Z}{1 - A_v^{-1}} \tag{14}$$

Miller effect



$$Z_1 = \frac{1}{1+A}\frac{1}{sC_F} \tag{15}$$

Input Capacitance:

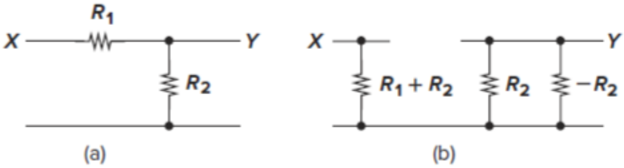
$$C_{in} = C_F(1+A) \tag{16}$$

Reason:

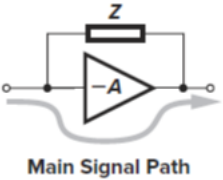
$$\Delta Q = C_F(1+A)\Delta V = C_{in}\Delta V \tag{17}$$

Miller effect

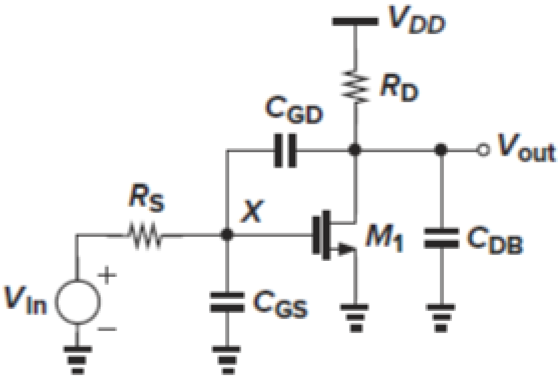
Invalid:



valid:



Example:



Miller effect

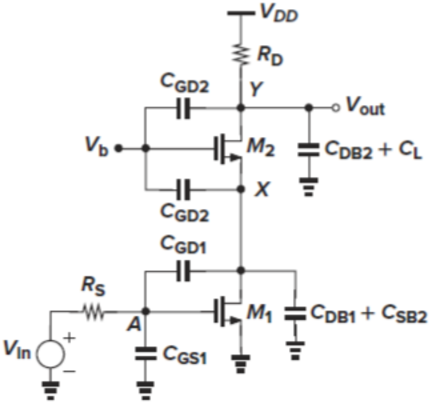
$$A_v = -g_m R_D \tag{18}$$

$$\omega_{in} = \frac{1}{R_S \left[C_{GS} + \left(1 + g_m R_D \right) C_{GD} \right]} \tag{19}$$

$$\omega_{out} = \frac{1}{R_D \left[C_{DB} + \left(1 + 1/(g_m R_D) \right) C_{GD} \right]} \tag{20}$$

$$\frac{V_{\text{out}}}{V_{\text{in}}}(s) = \frac{-g_m R_D}{\left(1 + \frac{s}{\omega_{\text{in}}}\right)\left(1 + \frac{s}{\omega_{\text{out}}}\right)} \tag{21}$$

Cascode



V_{DD}
R_D
C_{GD2}
Y
V_{out}
V_b
M_2
$C_{DB2} + C_L$
X
C_{GD2}
C_{GD1}
R_S
M_1
$C_{DB1} + C_{SB2}$
A
V_{in}
+
−
C_{GS1}

Cascode

$$A_v = -\frac{g_{m1}}{g_{m2} + g_{mb2}}, \quad \text{negligible channel-length modulation} \tag{22}$$

$$\omega_{p,A} = \frac{1}{R_S \left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}+g_{mb2}} \right) C_{GD1} \right]} \tag{23}$$

$$\omega_{p,X} = \frac{g_{m2} + g_{mb2}}{C_{DB1} + C_{SB2} + C_{GS2} + \left(1 + \frac{g_{m2}+g_{mb2}}{g_{m1}} \right) C_{GD1}} \tag{24}$$

$$\omega_{p,Y} = \frac{1}{R_D \left(C_{DB2} + C_L + C_{GD2} \right)} \tag{25}$$

Good Luck