Lecture 23: General Time Constant Approach

VE311 Electronic Circuits

Xuyang Lu
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上海交通大學
SHANGHAI JIAO TONG UNIVERSITY

Recap of Last Lecture

- Bandwidth estimation:
  - Solving for KCL and KVL
  - Miller effect + Pole associated with nodes

Topic to be Covered

- General Time constant approach

Cascode

$$\omega_{p,A} = \frac{1}{R_S\left[C_{GS1} + \left(1 + \frac{gm_1}{gm_2+gmb_2}\right)C_{GD1}\right]} \tag{1}$$

$$\omega_{p,X} = \frac{1}{\frac{1}{gm_2+gmb_2}\left[C_{DB1} + C_{SB2} + C_{GS2} + \left(1 + \frac{gm_2+gmb_2}{gm_1}\right)C_{GD1}\right]} \tag{2}$$

$$\omega_{p,Y} = \frac{1}{R_D\left[C_{GD2} + C_{DB2} + C_L\right]} \tag{3}$$

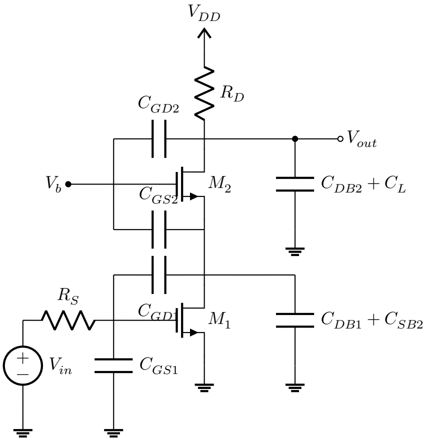
$$\frac{v_{\text{out}}}{v_{\text{in}}} = -gm_1 R_D \frac{1}{\left(1 + \frac{S}{\omega_{p,A}}\right)\left(1 + \frac{S}{\omega_{p,X}}\right)\left(1 + \frac{S}{\omega_{p,Y}}\right)} \tag{4}$$

Cascode

Recall for CS stages:

$$\omega_{\text{in}} = \frac{1}{R_S\left[C_{GS} + (1 + gm_1 R_D)\, C_{GD}\right]} \tag{5}$$

Cascode



V_{DD}
R_D
C_{GD2}
V_{out}
V_b
M_2
C_{GS2}
$C_{DB2} + C_L$
R_S
C_{GD1}
M_1
$C_{DB1} + C_{SB2}$
V_{in}
C_{GS1}

First Order Systems

Read 10.1 and 10.4 for BJTs of Sedra Smith (OCTC & SCTC for UM VE311).
Hajimiri for General Method

- No energy is store in the circle. In general, a one pole one zero system
- $H(S) = \frac{a_0 + a_1 S}{1 + bS}$

First Order Systems

- The low frequency is represented by a_0
- $a_0 = H(s)\,|_{c_1=0} = H^{\circ}$
- The time constant determines b_1 (This is the pole of the system)
- $b_1 = \tau = RC_1$
- The ratio of a_0 and a_1 determines the location of the zero

First Order Systems

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$$H(s) = \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_m s^m}{1 + b_1 s + b_2 s^2 + \cdots + b_n s^n} = a_0 \cdot \frac{\left(1 - \frac{s}{z_1}\right)\left(1 - \frac{s}{z_2}\right)\ldots\left(1 - \frac{s}{z_m}\right)}{\left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right)\ldots\left(1 - \frac{s}{p_n}\right)} \quad (6)$$

$$b_1 = -\sum_i \frac{1}{p_i} \quad (7)$$

$$b_2 = \sum_i \sum_j^{i<j} \frac{1}{p_i p_j} \quad (8)$$

$$b_n = \frac{(-1)^n}{p_1 p_2 \cdots p_n} \quad (9)$$

$$b_1 = -\sum_i \frac{1}{p_i} \quad (10)$$

$$b_2 = \sum_i \sum_j^{i<j} \frac{1}{p_i p_j} \quad (11)$$

$$b_n = \frac{(-1)^n}{p_1 p_2 \cdots p_n} \quad (12)$$

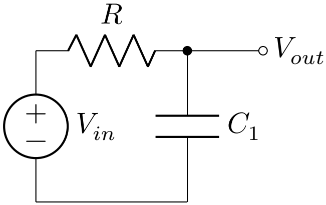
First Order Systems

- $H(S) = \frac{a_0 + a_1 S}{1 + bS}$
- $H_i(s) = \frac{a_0 + \alpha_1^i C_i s}{1 + \beta_1^i C_i s}$
- (It is easy to convince ourselves that a_1 is also related to the capacitor. For a capacitor, $1/j\omega C$ means C and S always come together.)
- The transfer function shall be valid for all capacitor values including zero and infinity.
- the first denominator coefficient b_1, is simply given by the sum of these zero-value time constants (ZVT)
- $b_1 = \sum_{i=1}^{N} \tau_i^0$

First Order Systems

- $H_i(s) = \frac{a_0 + \alpha_1^i C_i s}{1 + \beta_1^i C_i s}$ If C_i goes to ∞, $\mathrm{H(s)} = \frac{\alpha_1}{\beta_1}$
- $H(s) = \frac{H^0 + H^1 \tau S}{1 + \tau S}$
- If it is an inductor, $\tau = \frac{L_1}{R_0}$
- To find the time constant, remove the cap/ind nulling all the sources, find the resistance.
- To find transfer constant H^0, it is just the low frequency gain.
- To find the transfer constant H^1, we look into high frequency response, so the cap shall be shorted. For inductor it is the opposite.

First Order Systems



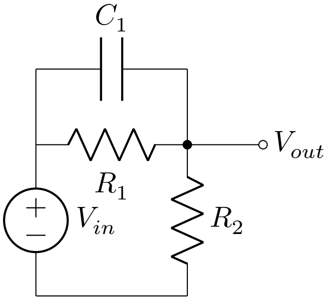
$$H^0 = 1 \tag{13}$$

$$\tau = RC_1 \tag{14}$$

$$H^1 = 0 \tag{15}$$

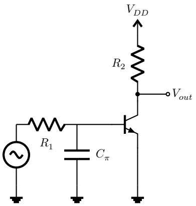
$$H(s) = \frac{1}{1 + RCS} \tag{16}$$

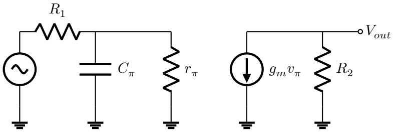
First Order Systems



- We do zero frequency response to find out H^0
- $H^0 = \frac{R_2}{R_1+R_2}$
- We null sources to find τ_1
- $\tau_1 = (R_1 \parallel R_2)\, C_1$
- We short circuit to find H^1
- $H^1 = 1$
- $\mathrm{H(S)} = \frac{R_1}{R_1+R_2}\frac{1+\mathrm{R_2C_1S}}{1+\mathrm{(R_1\|R_2)C_1S}}$

First Order Systems





$$H^0 = -\frac{r_\pi}{R_1 + r_\pi} g_m R_2 \qquad (17)$$

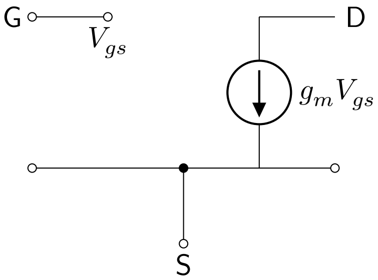
$$\tau = (R_1 \parallel r_\pi) C_\pi \qquad (18)$$

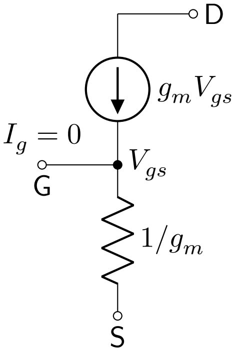
$$H^\pi = 0 \qquad (19)$$

$$H(S) = \frac{H^0}{1 + \tau S} \qquad (20)$$

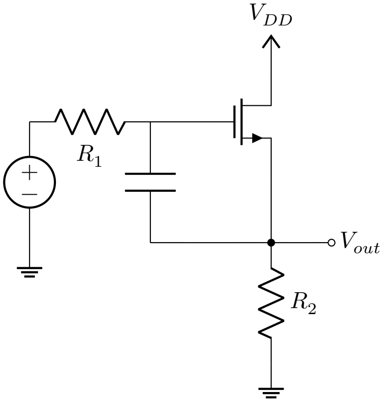
If there is a zero in the system, then we can test it with a shorted cap / open ind and see if the output still have some value.

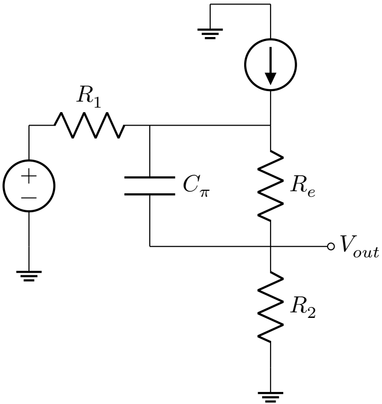
MOSFET T Model





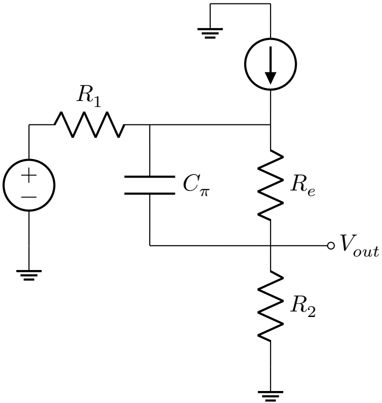
First Order Source Follower





First Order Source Follower

- Low frequency, $V_x = V_{in}$
- V_{out} is voltage divider,
- $H^0 = \frac{R_2}{R_2+r_e}$ (assume it' s a BJT)
- $H^1 = \frac{R_2}{R_2+r_1}$
- $R_2\left(g_m v_x - i_x\right) + v_x = R_1 i_x$
- $R = \frac{R_1+R_2}{1+g_m R_2}$
- You can imagine the existence of a zero and a pole.



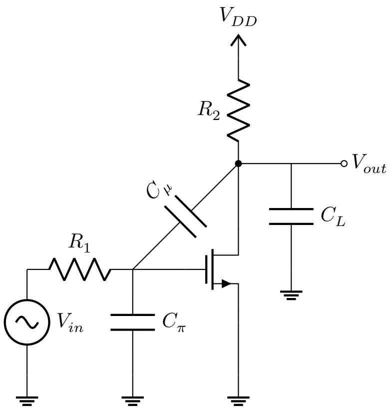
First Order Source Follower

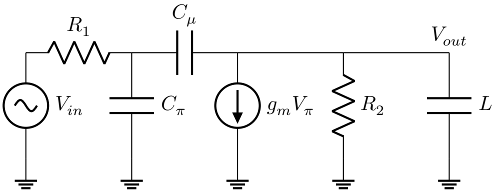
$$H(s) = \frac{R_2}{R_2 + R_e} \cdot \frac{1 + \frac{R_e+R_2}{R_1+R_2} \cdot \frac{R_1+R_2}{1+g_e R_2} C_\pi S}{1 + R_\pi^0 C_\pi S} \tag{21}$$

$$= \frac{R_2}{R_2 + r_e} \cdot \frac{1 + r_e C_\pi S}{1 + R_\pi^0 C_\pi S} \tag{22}$$

- $Z = -\frac{g_m}{C_\pi}$ @ high frequency
- $P = -\frac{1}{R_\pi^0 C_\pi}$ @ high frequency
- Therefore, source follower is quite wide band

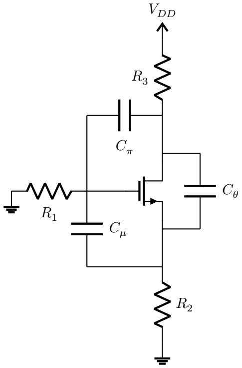
First Order Common Source





First Order Common Source

- $R_\pi^0 = r_\pi \parallel \frac{R_1+R_2}{1+g_m R_2}$
- $R_\mu^0 =$
  $R_{\text{left}} + R_{\text{right}} + G_m R_{\text{left}} R_{\text{right}}$
- For BJT we have
  $R_{\text{left}} = R_B \parallel [r_\pi + (1+\beta) R_E]$
- $R_{\text{right}} = R_3$
- $G_m = \frac{g_m}{1+g_m R_2}$
- $R_\theta^0 \approx \frac{R_2+R_3}{1+g_m R_2}$
- $b_1 = \sum_{i=1}^{N} \tau_i^0$



First Order Common Source

$$R_\theta^0 \approx \frac{R_2 + R_3}{1 + g_m R_2} \tag{23}$$

First Order Common Source

$$R_{\text{left}} = R_B \parallel [r_\pi + (1+\beta)R_E] \tag{24}$$

First Order Common Source

.model Q2N696 NPN (Is=14.34f Xti=3 Eg=1.11 Vaf=74.03 Bf=65.62 Ne=1.208 Ise=19.48f Ikf=.2385 Xtb=1.5 Br=9.715 Nc=2 Isc=0 Ikr=0 Rc=1 Cjc=9.393p Mjc=.3416 Vjc=.75 Fc=.5 Cje=22.01p Mje=.377 Vje=.75 Tr=58.98n Tf=408.8p Itf=.6 Vtf=1.7 Xtf=3 Rb=10)

.model NPN NPN

.model PNP PNP

cje Zero bias B-E depletion capacitance

First Order Common Source

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We can use those parameters to determine small signal parameters. a collector current of 1mA which give you a $Gm = 40mS \quad \beta_0 = 100$

$$C_\pi = C_{je} + C_b = 100\text{fF} \tag{25}$$

$$C_L = C_{out} + C_{js} = 200\text{fF} \tag{26}$$

$$C_u = g_m \tau_F = 16fF \tag{27}$$

$$R_1 = 1K\Omega \tag{28}$$

$$R_2 = 1K\Omega \tag{29}$$

$$r_\pi = \frac{\beta + 1}{gm} = \frac{101}{40} \approx 2500\Omega \tag{30}$$

$$R_{\text{left}} = R_B \parallel [r_\pi + (1+\beta)R_E] = \tag{31}$$

$$1\text{K} \parallel (2.5\text{K} + (101)^*0) = 0.7\text{K} \tag{32}$$

$$R_\mu^0 = R_{\text{left}} + R_{\text{right}} + G_m R_{\text{left}} R_{\text{right}} \tag{33}$$

$$= 0.7\text{K} + 2\text{K} + 40\text{e} - 3^*0.7\text{K} * 2\text{K} \tag{34}$$

First Order Common Source

$$b_1 = \sum_i \tau_i^0 = \tau_\pi^0 + \tau_\mu^0 + \tau_L^0 \tag{35}$$

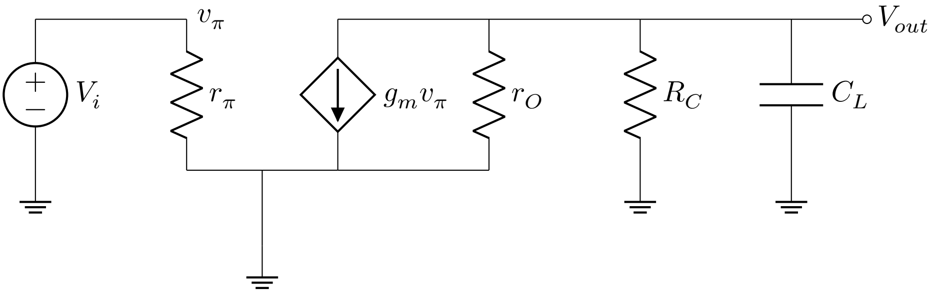
$$H^0 = -57 \tag{36}$$

$$\tau_\pi^0 \approx 70\text{ps} \quad \tau_\mu^0 \approx 1000\text{ps} \quad \tau_L^0 = 400\text{ps} \tag{37}$$

$$\omega_h \approx 1/b_1 \approx 2\pi \cdot 95\text{MHz} \tag{38}$$

- This allows you to determine the lowest operating frequency, and also the contribution of each nodes in the circuit.

Practice on BJT Case



Practice on BJT Case

$$g_m v_i + \frac{v_0}{r_0 \parallel R_c} + v_0 S C_L = 0 \quad (39)$$

$$g_m v_i + v_0 \frac{1 + r_0 \parallel R_c S C_L}{r_0 \parallel R_c} = 0 \quad (40)$$

$$v_0 = -g_m \frac{r_0 \parallel R_c}{1 + r_0 \parallel R_c S C_L} v_i \quad (41)$$

- Single pole at :
- Often $r_O >> RC$