2. Consider the equation $R_n = R_p = \frac{np - n_i^2}{\tau_{p0}(n+n') + \tau_{n0}(p+p')} \equiv R$, where $\tau_{p0} = \frac{1}{N_t C_p}$ and $\tau_{n0} = \frac{1}{N_t C_n}$. Let $\tau_{p0} = 10^{-7}s$ and $\tau_{n0} = 5 \times 10^{-7}s$. Also let $n' = p' = n_i = 10^{15} cm^{-3}$. Assume very low injection that $\delta n \ll n_i$. Calculate $R/\delta n$ for a semiconductor which is (a) n-type ($n_0 \gg p_0$), (b) intrinsic ($n_0 = p_0 = n_i$), and (c) p-type ($p_0 \gg n_0$).

Assume $\delta p = \delta n$

$n = n_0 + \delta n$, $p = p_0 + \delta n$

$$\Rightarrow R_n = R_p = \frac{(n_0+\delta n)(p_0+\delta n) - n_i^2}{\tau_{p0}(n_0+\delta n+n_i) + \tau_{n0}(p_0+\delta n+n_i)}$$

Since $\delta n \ll n_i$ and $n_0 p_0 = n_i^2$

$$R = \frac{\delta n(n_0+p_0)}{\tau_{p0}(n_0+n_i) + \tau_{n0}(p_0+n_i)}$$

(a) $n_0 \gg p_0$, $n_0 \gg n_i$

$$\Rightarrow R \approx \frac{\delta n\, n_0}{\tau_{p0}(n_0+n_i)} \approx \frac{\delta n}{\tau_{p0}}$$

$$\Rightarrow \frac{R}{\delta n} = \frac{1}{\tau_{p0}} = 10^7 s^{-1}$$

(b) $n_0 = p_0 = n_i$

$$R = \frac{\delta n \cdot 2n_i}{(\tau_{p0}+\tau_{n0}) \cdot 2n_i}$$

$$\Rightarrow \frac{R}{\delta n} = \frac{1}{\tau_{p0}+\tau_{n0}} = 1.67 \times 10^6 s^{-1}$$

(c) Similar to (a)

$$\frac{R}{\delta n} = \frac{1}{\tau_{n0}} = 2\times 10^6 s^{-1}$$

3. Consider an n-type semiconductor as shown in the figure, doped at $N_d = 10^{16} cm^{-3}$ and with a uniform excess carrier generation rate equal to $g' = 10^{21} cm^{-3}s^{-1}$. Assume that $D_p = 10 cm^2/s$ and $\tau_{p0} = 10^{-7}s$. The electric field is zero. (a) Determine the steady-state excess minority carrier concentration versus x if the surface recombination velocity at x=0 is (i) s=0, (ii) s=2000cm/s, and (iii) s=∞. (b) Calculate the excess minority carrier concentration at x=0 for (i) s=0, (ii) s=2000cm/s, and (iii) s=∞.

For different X_i:

$$D_p \left.\frac{d\Delta p(x)}{dx}\right|_{x=X_i} = s \cdot \Delta p(X_i)$$

(velocity same)

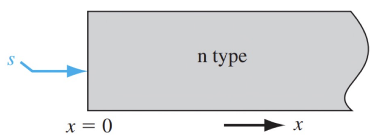


Figure 1. Diagram for problem 3

(a) $$\frac{d\,\delta p}{dt} = D_p \frac{d^2 p}{dx^2} - \mu_p\left(E\frac{dp}{dx} + p\frac{dE}{dx}\right) + g_p' - \frac{\delta p}{\tau_{p0}}$$

$(B=0)$

$$\Rightarrow D_p \frac{d^2 p}{dx^2} = \frac{\delta p}{\tau_{p0}} - g' \Rightarrow \delta p(x) = Ae^{-\lambda x} + Be^{\lambda x} + g'\tau_{p0} \text{ where } \lambda = \frac{1}{\sqrt{D_p \tau_{p0}}} = \frac{1}{10^{-3}}$$

(i) $x=0$, $-D_p \frac{A}{\sqrt{D_p \tau}} = s\cdot(A + g'\tau) \Rightarrow A = -\frac{g'\tau s}{s + \frac{D_p}{L_p}} = 0 \Rightarrow \delta p(x) = 10^{14} cm^{-3}$

(ii) $$A = -\frac{g'\tau s}{s+\frac{D_p}{L_p}} = -\frac{10^{21}\cdot 10^{-7}\cdot 2000}{2000 + \frac{10}{10^{-3}}} = -1.67\times 10^{13}$$

$$\Rightarrow \delta p(x) = 10^{14} - 1.67\times 10^{13} e^{\frac{-x}{10^{-3}}}$$

(iii) $$A = -\frac{g'\tau}{1 + D_p/L_p s} \approx -g'\tau = -10^{21}\cdot 10^{-7} = -10^{14}$$

(b) From (a), we can calculate the result easily:

(i) $\delta p(0) = 10^{14}$ (ii) $\delta p(0) = 8.33\times 10^{13}$ (iii) $\delta_p(0) = 0$

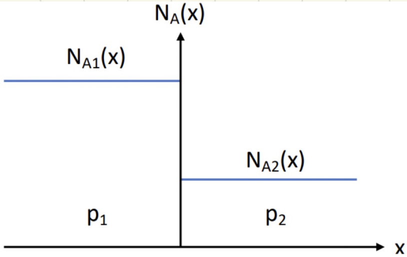
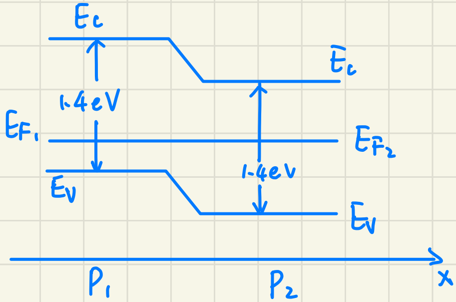


Figure 2. Diagram for problem 4

(a) Draw the energy band diagram of the junction assuming that the doping is non-degenerate throughout. Assume an energy gap of 1.4eV.

(b) Derive an expression for Vbi that exists across the junction in equilibrium.

(a)



(b)

$$V_{bi} \cdot e = (E_{C_1} - E_{F_1}) - (E_{C_2} - E_{F_2})$$

$$E_{C_1} - E_{F_1} = -kT \ln \frac{N_{A_1}}{N_c}$$

$$E_{C_2} - E_{F_2} = -kT \ln \frac{N_{A_2}}{N_c}$$

$$\Rightarrow V_{bi} = \frac{1}{e} kT \left(\ln \frac{N_{A_2}}{N_c} - \ln \frac{N_{A_1}}{N_c} \right) = \frac{1}{e} kT \ln \frac{N_{A_2}}{N_{A_1}}$$

5. A Ge diode has a p-side doping of $N_a = 5\times10^{16} cm^{-3}$ and an n-side doping of half that value.

(a) Calculate depletion widths on both sides of the junction and draw the equilibrium energy level diagram as a function of position. Carefully label all energy levels (Ec, Ev and Ef) and boundaries of the depletion region.

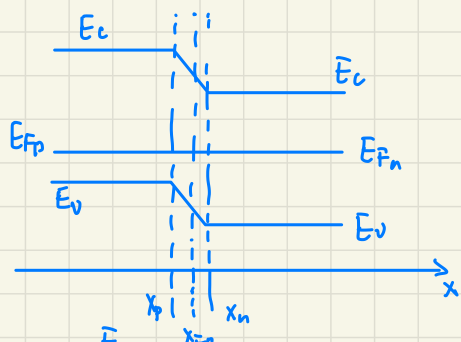
(b) Now apply a reverse voltage 0.1V and repeat part a. Include a sketch of the approximate positions of the quasi-Fermi energies.

(a)

$$V_{bi} = \frac{kT}{e} \ln \frac{N_A N_D}{n_i^2} = 0.378 V$$

$$x_n = \sqrt{\frac{2 \varepsilon_s V_{bi}}{e} \cdot \frac{N_A}{N_D} \cdot \frac{1}{N_A + N_D}} = 1.336 \times 10^{-5} \text{ cm}$$

$$x_p = \sqrt{\frac{2 \varepsilon_s V_{bi}}{e} \cdot \frac{N_D}{N_A} \cdot \frac{1}{N_A + N_D}} = 6.68 \times 10^{-6} \text{ cm}$$

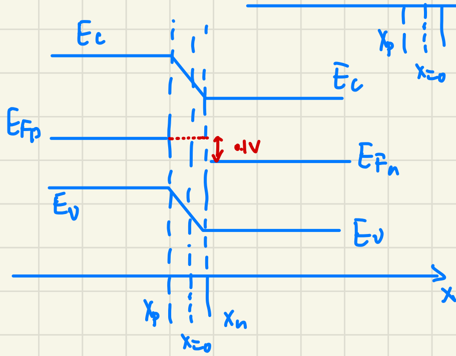


(b)

$$V' = V_{bi} + V_r = 0.378 + 0.1 = 0.478 V$$

$$x_n = 1.502 \times 10^{-5} \text{ cm}$$

$$x_p = \frac{1}{2} x_n = 7.51 \times 10^{-6} \text{ cm}$$



7. In the diagram below (the material is Si) :

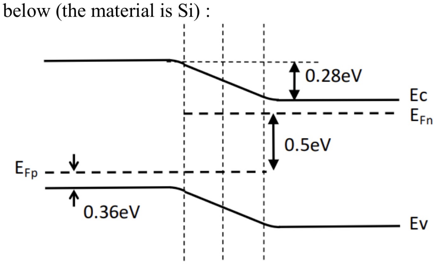


Figure 3. Diagram for problem 7

(a) Is the diode under equilibrium or forward biased or reverse biased? If biased, what is the bias voltage?

(b) Determine the built-in potential of the diode under equilibrium.

(c) Determine N_a and N_d.

(a) Forward Biased, $V_{biased} = 0.5V$ (b) $V_{bi} - V_{biased} = 0.28\,V$, $V_{bi} = 0.78V$

(c) $E_{Fp} - E_v = KT \ln \frac{N_v}{N_A} = 0.0259 \ln \frac{1.04 \times 10^{19}}{N_A} = 0.36 \Rightarrow N_A = 9.56 \times 10^{12}\ cm^{-3}$

$$V_{bi} = \frac{KT}{e} \ln \frac{N_A N_d}{n_i^2} = 0.0259 \ln \frac{9.56 \times 10^{12} N_d}{(1.5 \times 10^{10})^2} = 0.78 \Rightarrow N_d = 2.82 \times 10^{20}\ cm^{-3}$$

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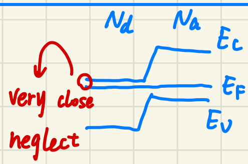
6. Calculate the capacitance and plot $\frac{1}{C^2}$ vs V_R for the following Si n⁺p junctions:

$Na = 10^{15} cm^{-3}$

(a) Reverse bias voltage = 1V; (b) reverse bias voltage = 5V.

(For n⁺p junctions, $N_d \gg N_a$. Use a suitable approximation in your calculation.)



$$\frac{1}{C^2} = \frac{2(V_{bi} + V_R)}{q\varepsilon} \cdot \frac{N_a + N_d}{N_a N_d} = \frac{2}{q\varepsilon} \cdot \left[\frac{KT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right) + V_R\right] \cdot \frac{N_a + N_d}{N_a \cdot N_d}$$

$$= \frac{2}{q\varepsilon N_A} \cdot \left[\frac{KT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right) + V_R\right] = \frac{2}{q\varepsilon N_A} \cdot (V_{bi} + V_R) = 1.4 \times 10^{17} (V_{bi} + V_R)$$

since $N_a >> N_a$, at N_d side, E_c will be very close to E_c. therefore, $qV_{bi} = E_G - (E_c - E_F)_n - (E_F - E_v)_p \approx E_G - (E_F - E_v)_p = E_G + KT \ln \frac{N_A}{N_v} = 1.12 + 8.62 \times 10^{-5} \times 300 \ln \frac{10^{15}}{1.04 \times 10^{19}} = 0.88eV$

$\Rightarrow V_{bi} = 0.88V$

Plot: $\frac{1}{C^2} = k \cdot V_R + b$

where: $b = 1.23 \times 10^{17}$

$k = 1.4 \times 10^{17}$

(a) $\frac{1}{C^2} = 1.4 \times 10^{17} \times (V_{bi} + 1) = 2.63 \times 10^{17}$

(b) $\frac{1}{C^2} = 1.4 \times 10^{17} \times (V_{bi} + 5) = 8.23 \times 10^{17}$

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1. Define the built-in potential voltage and describe how it maintains thermal equilibrium.

The built-in potential voltage is the electric potential difference across a PN junction due to the diffusion of electrons and holes when the junction forms, resulting in a depletion region. It maintains thermal equilibrium by balancing the diffusion current of carriers with an equal and opposite drift current caused by the electric field in the depletion region. This equilibrium ensures that no net current flows across the junction in the absence of an external bias.