Problem Set 2 Solution

1. (20 points) 5'+5'+5'+5'

(a)(i) At $T=300K$, $m_n^* = 1.08 m_0$

$$N=\int_{E_c}^{E_c+2kT} \frac{4\pi(2m_n^*)^{3/2}}{h^3}\cdot\sqrt{E-E_c}\,dE \quad \cdots (2')$$

$$= \frac{4\pi(2m_n^*)^{3/2}}{h^3}\cdot\frac{2}{3}\cdot\left[(E-E_c)^{\frac{3}{2}}\right]_{E_c}^{E_c+2kT}$$

$$= \frac{4\pi(2m_n^*)^{3/2}}{h^3}\cdot\frac{2}{3}\cdot(2kT)^{3/2} \quad \text{—— (2': details of calculation)}$$

$$= \frac{4\pi\times(2\times1.08\times9.11\times10^{-31})^{3/2}}{(6.625\times10^{-34})^3}\times\frac{2}{3}\times(2\times1.38\times10^{-23}\times300)^{\frac{3}{2}}$$

$$\simeq 5.992\times10^{25}/m^3 = \boxed{5.992\times10^{19}/cm^3} \quad \cdots (1')$$

(ii) At $T=400K$, similarly,

$$N=\int_{E_c}^{E_c+2kT} \frac{4\pi(2m_n^*)^{3/2}}{h^2}\cdot\sqrt{E-E_c}\,dE \quad \cdots (2')$$

$$= \frac{4\pi\times(2\times1.08\times9.11\times10^{-31})^{3/2}}{(6.625\times10^{-34})^3}\times\frac{2}{3}\times(2\times1.38\times10^{-23}\times400)^{3/2} \quad \cdots (2')$$

$$\simeq 9.225\times10^{25}/m^3 = \boxed{9.225\times10^{19}/cm^3} \quad \cdots (1')$$

(b)(i) At $T=300K$, $m_n^* = 0.067 m_0$

$$N=\int_{E_c}^{E_c+2kT} \frac{4\pi(2m_n^*)^{3/2}}{h^3}\cdot\sqrt{E-E_c}\,dE \quad \cdots (2')$$

$$= \frac{4\pi\times(2\times0.067\times9.11\times10^{-31})^{3/2}}{(6.625\times10^{-34})^3}\times\frac{2}{3}\times(2\times1.38\times10^{-23}\times300)^{3/2} \quad \cdots (2')$$

$$\simeq 9.259\times10^{23}/m^3 = \boxed{9.259\times10^{17}/cm^3} \quad \cdots (1')$$

(ii) At $T=400K$, similarly

$$N=\int_{E_c}^{E_c+2kT} \frac{4\pi(2m_n^*)^{3/2}}{h^3}\cdot\sqrt{E-E_c}\,dE \quad \cdots (2')$$

$$= \frac{4\pi\times(2\times0.067\times9.11\times10^{-31})^{3/2}}{(6.625\times10^{-34})^3}\times\frac{2}{3}\times(2\times1.38\times10^{-23}\times400)^{3/2} \quad \cdots (2')$$

$$\simeq 1.425\times10^{24}/m^3 = \boxed{1.425\times10^{18}/cm^3} \quad \cdots (1')$$

2. (10 points) 5'+5'

(a) $$g_c(E) = \frac{4\pi(2m_n^*)^{3/2}\sqrt{E-E_c}}{h^3} \qquad g_v(E) = \frac{4\pi(2m_p^*)^{3/2}\sqrt{E_v-E}}{h^3} \quad \cdots (3')$$

Thus, $$\frac{g_c(E_c+kT)}{g_v(E_v-kT)} = \frac{(m_n^*)^{\frac{3}{2}}}{(m_p^*)^{\frac{3}{2}}} = \left(\frac{1.08}{0.56}\right)^{3/2} \simeq \boxed{2.678} \quad \cdots (2')$$

(b) Similar to (a). $$\frac{g_c(E_c+kT)}{g_v(E_v-kT)} = \frac{(m_n^*)^{\frac{3}{2}}}{(m_p^*)^{\frac{3}{2}}} \quad \cdots (3')$$

$$= \left(\frac{0.55}{0.37}\right)^{1.5} \simeq \boxed{1.812} \quad \cdots (2')$$

3. (20 points) 5' + 5' + 5' + 5'

(a)(i) $E = E_1$ is occupied by an electron:

$f(E_1) \simeq e^{-(E_1-E_F)/kT}$ --- (3')

$= e^{-0.2/0.025}$ --- (1': detail of calculation)

$\simeq \boxed{4.430\times10^{-4}}$ --- (1')

(ii) $E = E_2$ is empty: $E_F - E_2 = E_G - (E_1 - E_F) = 1.12 - 0.2 = 0.92\,eV$

$1 - f(E_2) \simeq e^{-(E_F-E_2)/kT}$ --- (3')

$= e^{-0.92/0.0259}$ --- (1')

$\simeq \boxed{3.744\times10^{-16}}$ --- (1')

(b)(i) $E = E_1$ is occupied by an electron: $E_1 - E_F = E_G - (E_F - E_2) = 1.12 - 0.4 = 0.72\,eV$

$f(E_1) \simeq e^{-(E_1-E_F)/kT}$ --- (3')

$= e^{-0.72/0.0259}$ --- (1')

$\simeq \boxed{8.452\times10^{-13}}$ --- (1')

(ii) $E = E_2$ is empty:

$1 - f(E_2) \simeq e^{-(E_F-E_2)/kT}$ --- (3')

$= e^{-0.4/0.0259}$ --- (1')

$\simeq \boxed{1.962\times10^{-7}}$ --- (1')

4. (10 points) 5' + 5'

(a) $E_{Fi} - E_{midgap} = \underbrace{\frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)}_{(3')} = \underbrace{\frac{3}{4}\times 0.0259\times\ln\left(\frac{0.7}{1.21}\right)}_{(1')} = \underbrace{\boxed{-10.631\,meV}}_{(1')}$

(b) $E_{Fi} - E_{midgap} = \underbrace{\frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)}_{(3')} = \underbrace{\frac{3}{4}\times 0.0259\times\ln\left(\frac{0.75}{0.8}\right)}_{(1')} = \underbrace{\boxed{-1.254\,meV}}_{(1')}$

5. (10 points)

$n_i = \sqrt{N_c N_v}\, e^{-E_G/(2kT)}$ ---- (3')

Since differences in carrier effective masses can be neglected.

$$\underbrace{\frac{n_{iA}}{n_{iB}} = \frac{e^{-E_{GA}/(2kT)}}{e^{-E_{GB}/(2kT)}}}_{(3')} = e^{(E_{GB}-E_{GA})/(2kT)} = \underbrace{e^{1/(2\times0.0259)}}_{(2')} \simeq \underbrace{\boxed{2.421\times10^{8}}}_{(2')}$$

6. (20 points) 5'+5'+5'+5'

(a) $E_F - E_v = kT \ln\left(\frac{N_v}{p_0}\right) = 0.0259 \times \ln \frac{1.04\times10^{19}}{2\times10^{16}} \simeq \boxed{0.162\,eV}$

(3') (1') (1')

(b) $E_c - E_F = E_G - (E_F - E_v) = 1.12 - 0.162 = \boxed{0.958\,eV}$

(3') (1') (1')

(c) $n_0 = N_c e^{-(E_c - E_F)/kT} = 2.8\times10^{19} \times e^{-0.958/0.0259} \simeq \boxed{2.417\times10^{3}/cm^3}$

(3') (1') (1')

(d) $E_{Fi} - E_F = kT \ln\left(\frac{p_0}{n_i}\right) = 0.0259 \times \ln \frac{2\times10^{16}}{1.5\times10^{10}} \simeq \boxed{0.365\,eV}$

(3') (1') (1')

Another approach: (c)

$n_0 = \frac{n_i^2}{p_0} = \frac{(1.5\times10^{10})^2}{2\times10^{16}} = \boxed{1.125\times10^{4}/cm^3}$

7. (10 points) 4'+4'+2'

(a) $E_c - E_F = kT \ln\left(\frac{N_c}{n_0}\right) = 0.0259 \times \ln \frac{2.8\times10^{19}}{2\times10^{5}} \simeq 0.844\,eV$

(2') (1')

$E_F - E_v = E_G - (E_c - E_F) = 1.12 - 0.844 = \boxed{0.276\,eV}$

(1') (1')

(b) $p_0 = N_v e^{-(E_F - E_v)/kT} = 1.04\times10^{19} \times e^{-0.276/0.0259} \simeq \boxed{2.449\times10^{14}/cm^3}$

(2') (1') (1')

(c) $\boxed{\text{P-type}}$ (2')

Another approach for 7 (a) and (b):

(a) $E_F - E_v = kT \ln\left(\frac{N_v}{p_0}\right) = kT \ln\left(\frac{N_v n_0}{n_i^2}\right) = 0.0259 \times \ln\left(\frac{1.04\times10^{19}\times2\times10^{5}}{(1.5\times10^{10})^2}\right) \simeq \boxed{0.237\,e}$

(b) $p_0 = \frac{n_i^2}{p_0} = \frac{(1.5\times10^{19})^2}{2\times10^{5}} = \boxed{1.125\times10^{15}\,cm^{-3}}$