RC4

Overview of continuity equation

For an n-type semiconductor,

$$\boxed{\frac{d\Delta p}{dt} = D_p \frac{d^2 p}{dx^2} - \mu_p E \frac{dp}{dx} - p\mu_p \frac{dE}{dx} - R'_p + g'_p}$$

(minority carriers)

$$R'_p = \frac{\Delta p}{\tau_{p0}}$$

$$\boxed{\frac{d\Delta n}{dt} = D_n \frac{d^2 n}{dx^2} + \mu_n E \frac{dn}{dx} + n\mu_n \frac{dE}{dx} - R'_n + g'_n}$$

(majority carriers)

$$R'_n = R'_p = \frac{\Delta p}{\tau_{p0}}$$

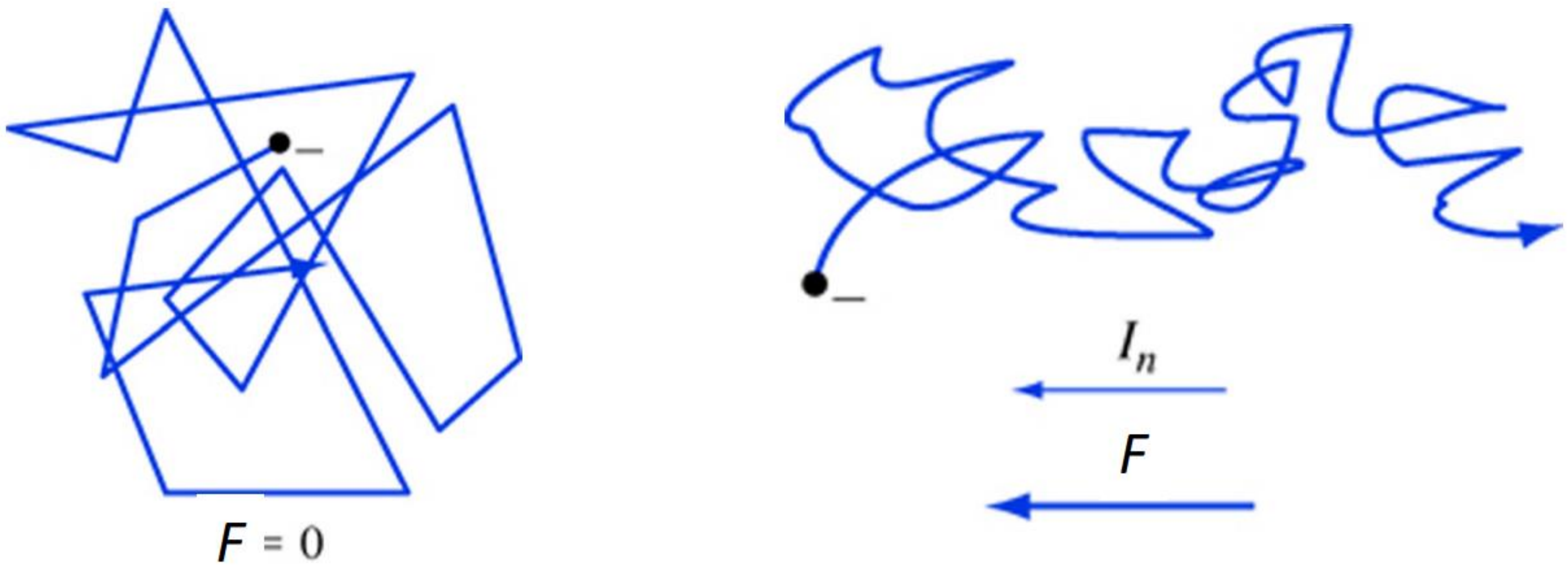
$$g'_n = g'_p$$

Overview of continuity equation

- There are four components contributing to the time and space dynamics of the carrier concentrations.
- Drift current
- Diffusion current
- Net recombination(generation) rate
- External generation effect(light illumination for example)

Drift current-electrons and holes are accelerated by the electric field

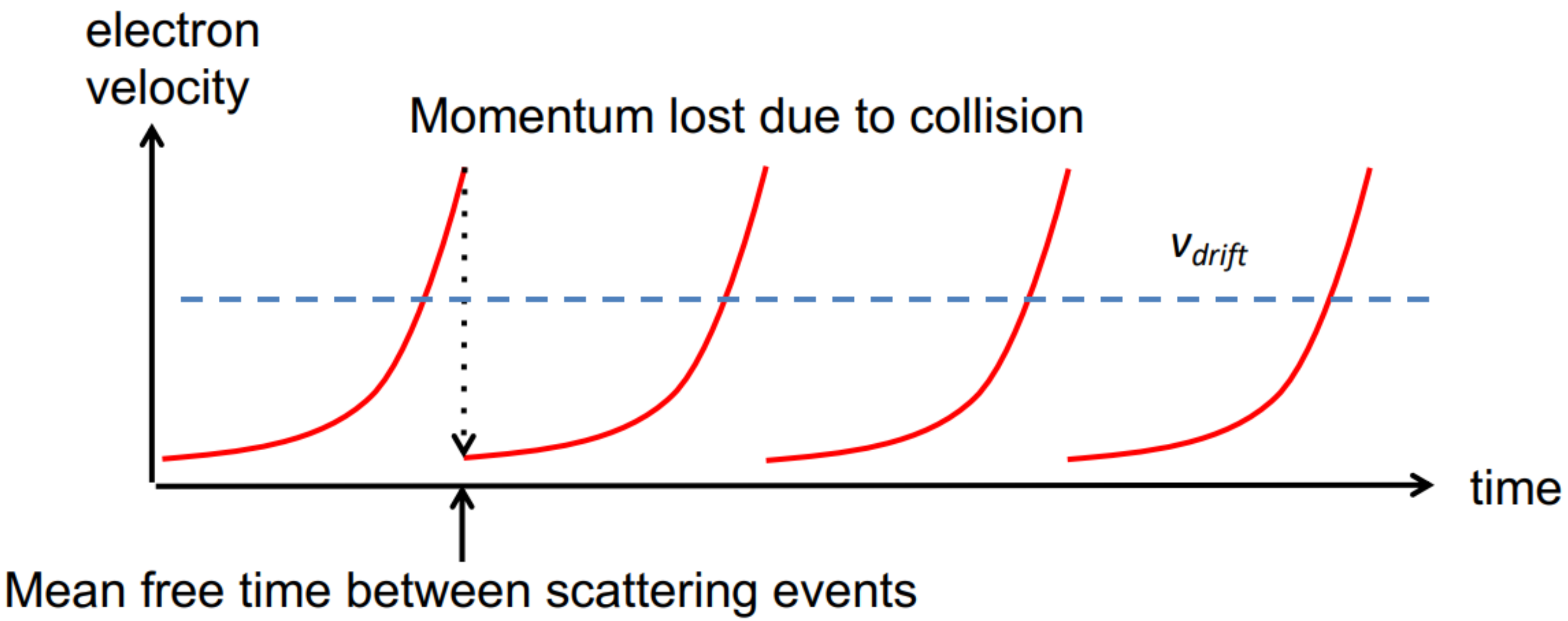
Electrons & holes respond to an electric field



- Carriers are in motion without an electric field (thermal energy)
- Carriers are accelerated by electric field in a particular direction
- Carriers scatter from lattice ions & crystal imperfections
- Average velocity obtained between scattering = v_{drift}

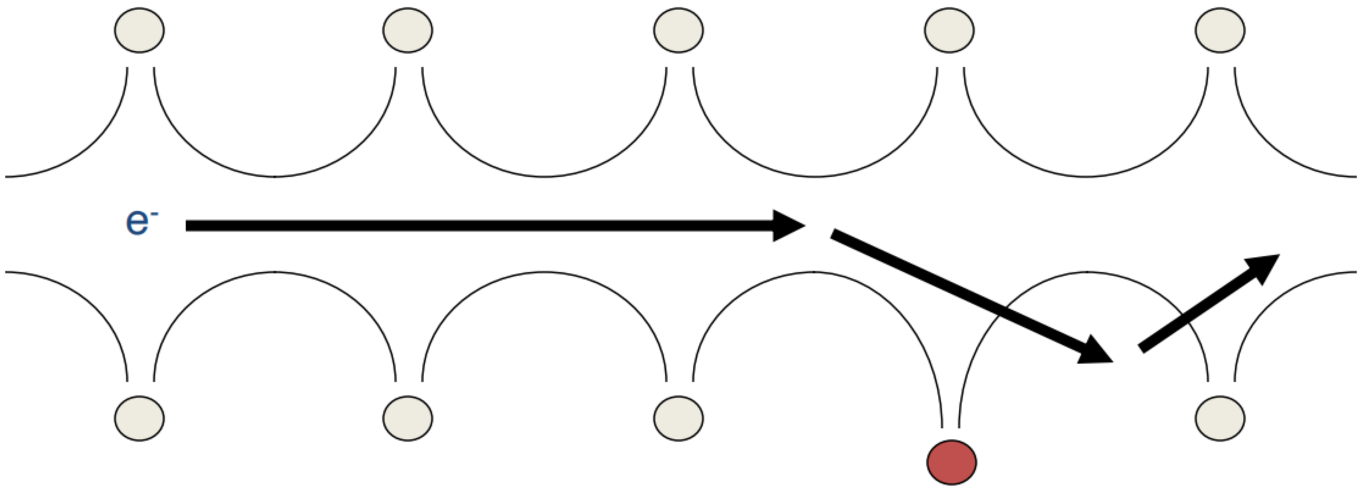
Drift current-drift velocity due to scattering

In semiconductors, a constant Coulomb force moves carriers at a *constant* velocity = **drift velocity**.



Drift current-two scattering mechanisms

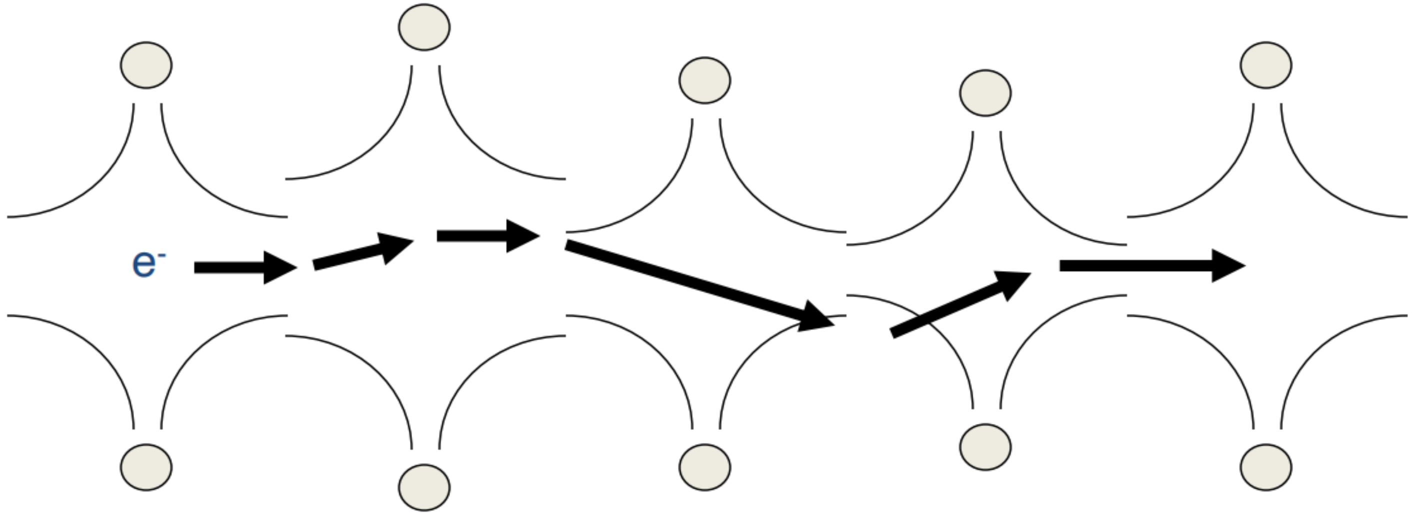
Mobility shows clear dependence on doping



Dopants are impurities in the crystal that cause local changes to the crystal potential seen by a moving electron

Drift current-two scattering mechanisms

Mobility also depends on temperature



Vibration of crystal atoms due to temperature causes a varying crystal potential as seen by a moving electron

Drift current-drift velocity and mobility

Drift current density

$$\boxed{scatterings}$$

$$\downarrow$$

$$\boxed{v \sim E}$$

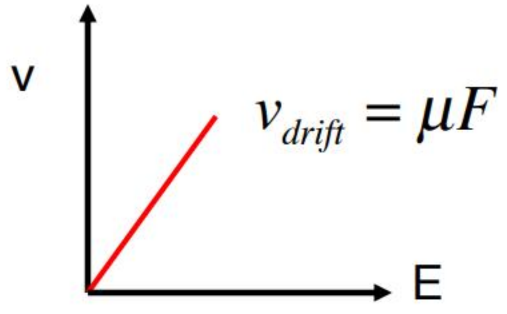
$$\downarrow$$

$$\boxed{v = \mu E}$$

$$\swarrow \qquad \searrow$$

$$\boxed{v_n = \mu_n E} \qquad \boxed{v_p = \mu_p E}$$

Electrons Holes

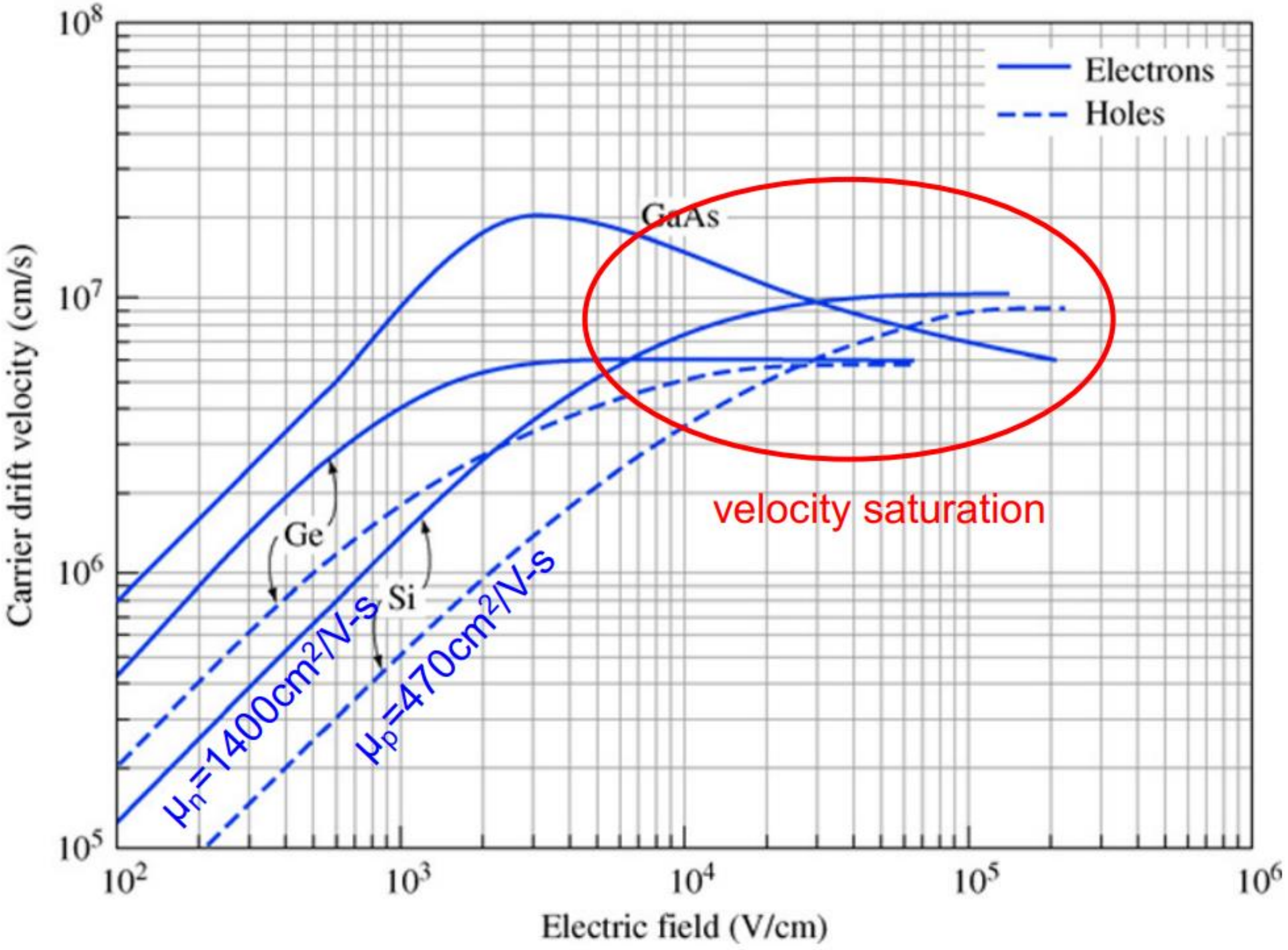


μ depends on mean time between scattering, τ, and m^*

$$\mu = \frac{q\tau}{m^*}$$

Drift current-drift velocity saturation

At high fields, mobility not constant, velocity "saturates"



Drift current-current density and conductivity

Drift current density

Hole drift current

$$J_{p|drf} = qp_0\mu_pE$$

Electron drift current

$$J_{n|drf} = qn_0\mu_nE$$

Both electrons and holes contribute to current:

$$J_{drf} = q(p_0\mu_p + n_0\mu_n)E$$

Table 5.1 | Typical mobility values at $T = 300$ K and low doping concentrations

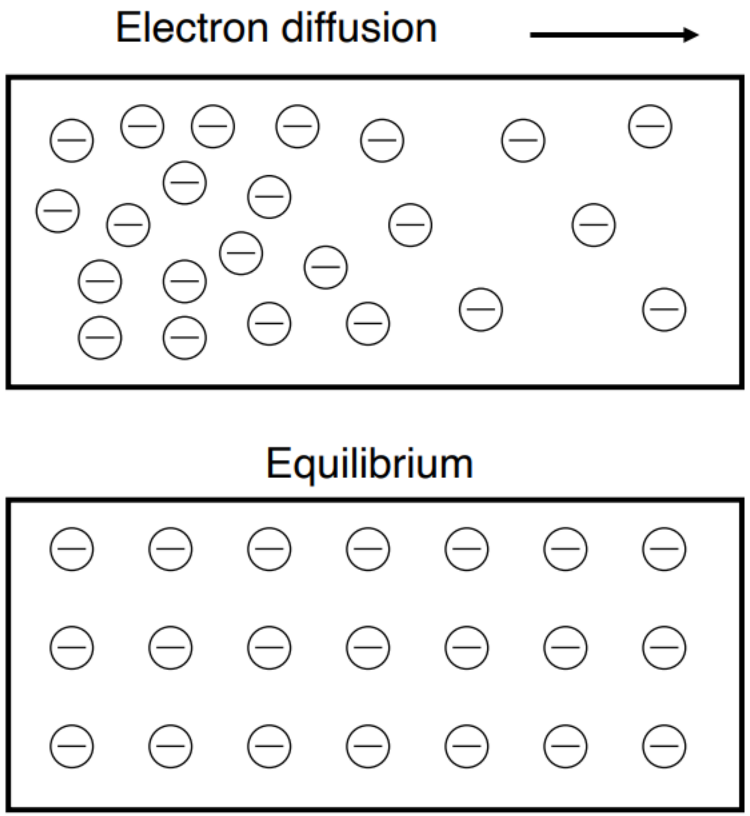
	μ_n (cm^2/V-s)	μ_p (cm^2/V-s)
Silicon	1350	480
Gallium arsenide	8500	400
Germanium	3900	1900

Conductivity depends on both carrier density and mobility

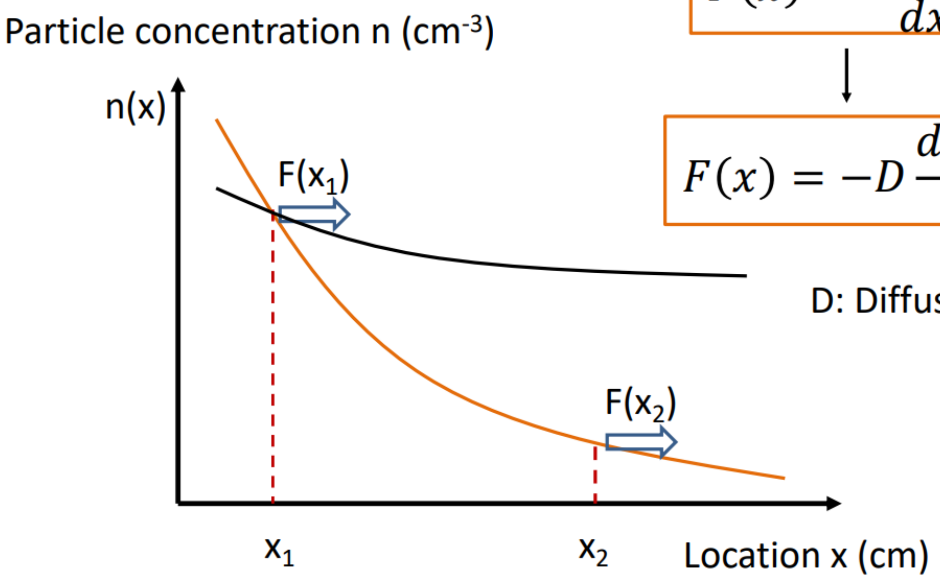
$$\sigma = qn\mu_n + qp\mu_p$$

Diffusion current

Carriers flow from high to low concentration



Diffusion current



$$F(x) \sim \frac{dn(x)}{dx}$$

$$F(x) = -D\frac{dn(x)}{dx}$$

D: Diffusivity

<u>Diffusion current density</u>

Electron diffusion current density: $J_{nx|dif} = -qF_n = qD_n\frac{dn}{dx}$

D_n is called the electron diffusion coefficient

Hole diffusion current density: $J_{px|dif} = qF_p = -qD_p\frac{dp}{dx}$

D_p is called the hole diffusion coefficient

Total current - drift + diffusion current

$$J = J_{drf} + J_{dif} = J_{n|drf} + J_{p|drf} + J_{n|dif} + J_{p|dif}$$

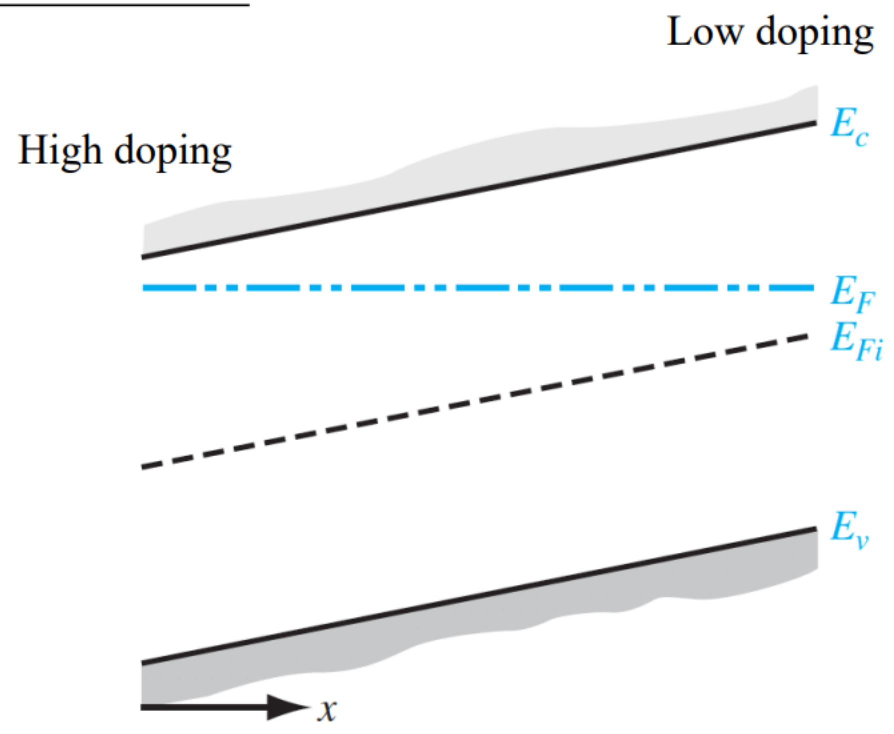
$$= qn\mu_n E_x + qp\mu_p E_x + qD_n \frac{dn}{dx} - qD_p \frac{dp}{dx}$$

$$\mathrm{J} = qn\mu_n E_x + qp\mu_p E_x + qD_n \nabla \mathrm{n} - qD_p \nabla p$$

How to link diffusion and drift

- Note in this case there's no net combination(generation) rate since $np = n_i^2$
- If there's no external generation rate and it's in steady state($\Delta n = \Delta p = 0$), then diffusion current should be equal to the drift current

<u>Induced electric field</u>



How to link diffusion and drift

- Refer to the slide for detailed derivation
- Note the Einstein relationship has some restrictions. The Boltzmann approximation must can be applied to the semiconductor.

$$\boxed{D_n = \frac{\mu_n kT}{q}}$$

$$J_{n,drift} = qn(x)\mu_n|E| = qD_n\frac{dn(x)}{dx} = J_{n,diff}$$

$$qn(x)\mu_n\left(\frac{1}{q}\frac{kT}{n(x)}\frac{dn(x)}{dx}\right) = qD_n\frac{dn(x)}{dx}$$

$$\cancel{q}\cancel{n(x)}\mu_n\left(\frac{1}{q}\frac{kT}{\cancel{n(x)}}\frac{\cancel{dn(x)}}{\cancel{dx}}\right) = \cancel{q}D_n\frac{\cancel{dn(x)}}{\cancel{dx}}$$

Net combination(generation) rate

- In one sentence, $np - n_i^2 = (n_0 + \Delta n)(p_0 + \Delta p) - n_i^2$ is the key parameter.
- If $np - n_i^2 = 0$, no net combination(generation) rate. But generation and recombination still exist. The rate is 0 because they are balanced.

Excess carrier generation and recombination

Net recombination rate

$$\frac{d\Delta p}{dt} = -(R_n - G_0) = -[\alpha_r(n_0 + \Delta n)(p_0 + \Delta p) - \alpha_r n_i^2]$$

$$= -\alpha_r \cdot \Delta p \cdot (p_0 + n_0) - \alpha_r \cdot (\Delta p)^2$$

$$if\ p_0 + n_0 \gg \Delta p \qquad \approx -\alpha_r \cdot \Delta p \cdot (p_0 + n_0)$$

(Small injection condition)

$$\Delta p(t) = \Delta p(0)\exp(-\frac{t}{\tau_{p0}}) \qquad \tau_{p0} = \frac{1}{\alpha_r(p_0 + n_0)}$$

Net combination(generation) rate

- Note the small injection condition
- Note that $\Delta n, \Delta p$ can depend on both x and t

For n-type semiconductor, net recombination rate

$$R_n - G_0 = R_p - G_0 = \frac{\Delta p(t)}{\tau_{p0}}$$

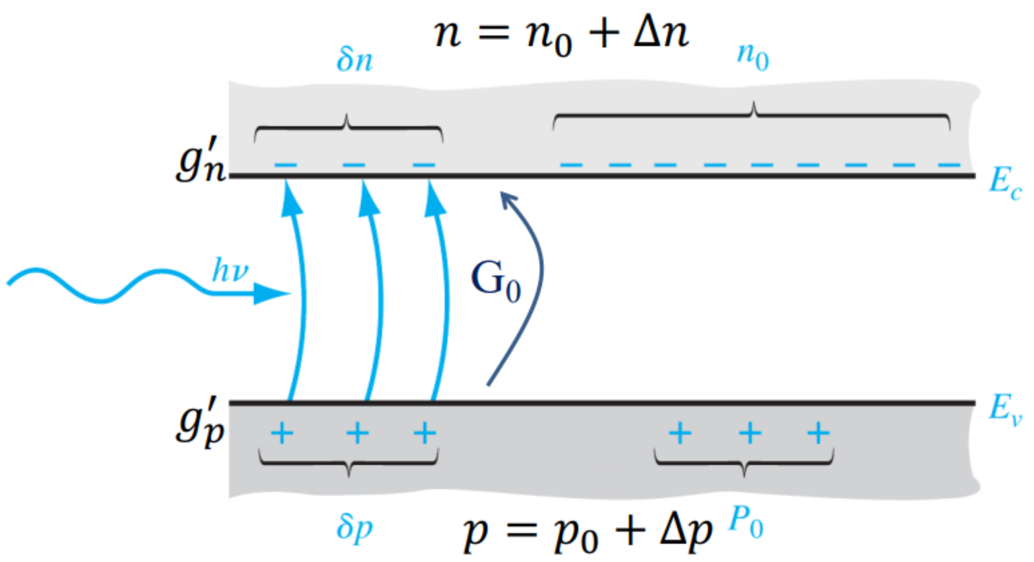
For p-type semiconductor, net recombination rate

$$R_n - G_0 = R_p - G_0 = \frac{\Delta n(t)}{\tau_{n0}}$$

External generation effect

- Note G_0 here is the intrinsic generation rate
- g' is the external generation rate

Excess carrier generation and recombination



g' is not a function of n and p

$g'_n = g'_p = g', \qquad \Delta n = \Delta p$

Combination of those four effects give us the continuity equation

- Details will be left to the next RC

For an n-type semiconductor,

$$\boxed{\frac{d\Delta p}{dt} = D_p \frac{d^2 p}{dx^2} - \mu_p E \frac{dp}{dx} - p\mu_p \frac{dE}{dx} - R_p' + g_p'}$$

(minority carriers)

$$R_p' = \frac{\Delta p}{\tau_{p0}}$$

$$\boxed{\frac{d\Delta n}{dt} = D_n \frac{d^2 n}{dx^2} + \mu_n E \frac{dn}{dx} + n\mu_n \frac{dE}{dx} - R_n' + g_n'}$$

(majority carriers)

$$R_n' = R_p' = \frac{\Delta p}{\tau_{p0}}$$

$$g_n' = g_p'$$

At most cases, you can simplify the continuity equations by some assumptions

Table 6.2 |

Specification	**Effect**
Steady state	$\frac{\partial(\delta n)}{\partial t} = 0, \quad \frac{\partial(\delta p)}{\partial t} = 0$
Uniform distribution of excess carriers (uniform generation rate) + no boundary confinement	$D_n \frac{\partial^2(\delta n)}{\partial x^2} = 0, \quad D_p \frac{\partial^2(\delta n)}{\partial x^2} = 0$
Zero electric field	$\mathrm{E}\frac{\partial(\delta n)}{\partial x} = 0, \quad \mathrm{E}\frac{\partial(\delta p)}{\partial x} = 0$
No excess carrier generation	$g' = 0$
No excess carrier recombination (infinite lifetime)	$\frac{\delta n}{\tau_{n0}} = 0, \quad \frac{\delta p}{\tau_{p0}} = 0$