VE320 Mid1 RC_part2

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Overview

1. Chapter 3
   - Effective Mass
   - Density of States Function
   - Statistical Mechanics
2. Chapter 4
   - Charge Carriers in Semiconductors
   - Intrinsic Semiconductor
   - Extrinsic Semiconductor

Effective Mass

- $F_{ext} = m^* a$
- for electrons in free space:
  $E = \frac{\hbar^2 k^2}{2m}$, $\frac{1}{\hbar}\frac{\mathrm{d}E}{\mathrm{d}k} = v$, $\frac{1}{\hbar^2}\frac{\mathrm{d}^2 E}{\mathrm{d}k^2} = \frac{1}{m}$
- for electrons in crystalline semiconductors:
  use parabola approximation for electrons near the bottom of the conduction band and the top of the valence band
  $\frac{1}{\hbar^2}\frac{\mathrm{d}^2 E}{\mathrm{d}k^2} = \frac{1}{m^*}$
- $E(k) = E_c + \frac{\hbar^2}{2m_n^*}(k - k_1)^2$
- $E(k) = E_v - \frac{\hbar^2}{2m_p^*}(k - k_2)^2$
- k_1, k_2: k axis coordinate of bottom point and top point
  m_n^*, $m_p^* > 0$

Density of States Function

- for electrons in the lattice:

$$g(E) = \frac{4\pi(2m)^{3/2}\sqrt{E}}{h^3}$$

- for electrons at the bottom of conduction band:

$$g_c(E) = \frac{4\pi(2m_n^*)^{3/2}\sqrt{E - E_c}}{h^3}$$

- for electrons at the top of valence band:

$$g_v(E) = \frac{4\pi(2m_p^*)^{3/2}\sqrt{E_v - E}}{h^3}$$

Fermi-Dirac Probability Distribution

- Fermi-Dirac probability distribution function describes the probability density that an quantum state with energy E is occupied.

$$\frac{N(E)}{g(E)} = f_F(E) = \frac{1}{1 + exp(\frac{E - E_F}{kT})}$$

- E_F is the Fermi energy level
- $N(E)$: number of particles per unit volume per unit energy
  $g(E)$: number of quantum states per unit volume per unit energy
  $f_F(E)$: probability that a quantum state at energy level E is occupied by an electron
- probability that a state is empty: $1 - f_F(E)$

Fermi-Dirac Probability Distribution

Temperature T determines the shape of $f_F(E)$:

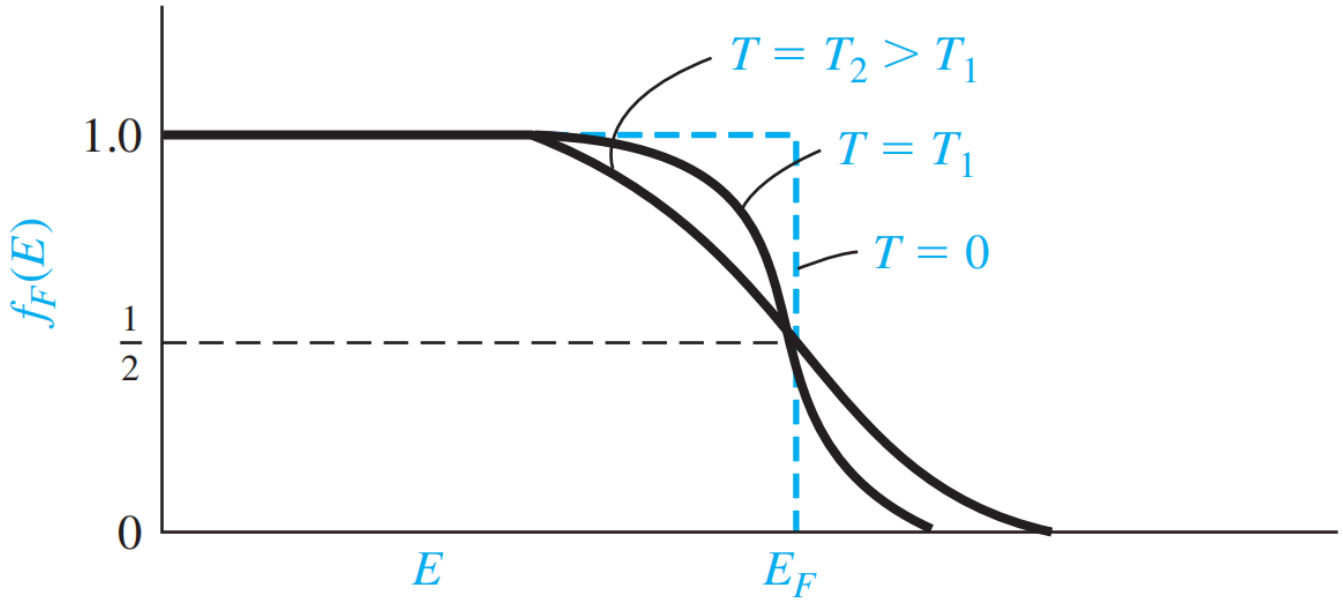


Figure 3.33 | The Fermi probability function versus energy for different temperatures.

Maxwell-Boltzmann Approximation

- Approximation can be applied when

$$E - E_F > 3kT$$

- approximated probability density:

$$f_F(E) = exp(-\frac{E - E_F}{kT})$$

Charge Carriers in Semiconductors

- $n(E) = g_c(E)f_F(E)$
- $p(E) = g_v(E)(1 - f_F(E))$
- concentration for the whole band: integrate the above equations over the entire conduction/valence band
- thermal equilibrium electron concentration:

$$n_0 = \int g_c(E)f_F(E)\mathrm{d}E$$

- thermal equilibrium hole concentration:

$$p_0 = \int g_v(E)(1 - f_F(E))dE$$

Charge Carriers in Semiconductors

- $n_0 = N_c exp(\frac{E_F - E_c}{kT})$
- $p_0 = N_v exp(\frac{E_v - E_F}{kT})$
- $N_c = 2\frac{(2\pi m_n^* kT)^{3/2}}{h^3}$
- $N_v = 2\frac{(2\pi m_p^* kT)^{3/2}}{h^3}$
- N_c: effective density of states function in the conduction band
- N_v: effective density of states function in the valence band

Intrinsic Semiconductor

- $n_0 = p_0 = n_i$
- $n_0 p_0 = n_i^2 = N_c N_v exp(\frac{E_v - E_c}{kT})$
- $n_i = \sqrt{N_c N_v} exp(-\frac{E_g}{2kT})$
- intrinsic Fermi-level position: $E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln(\frac{m_p^*}{m_n^*})$

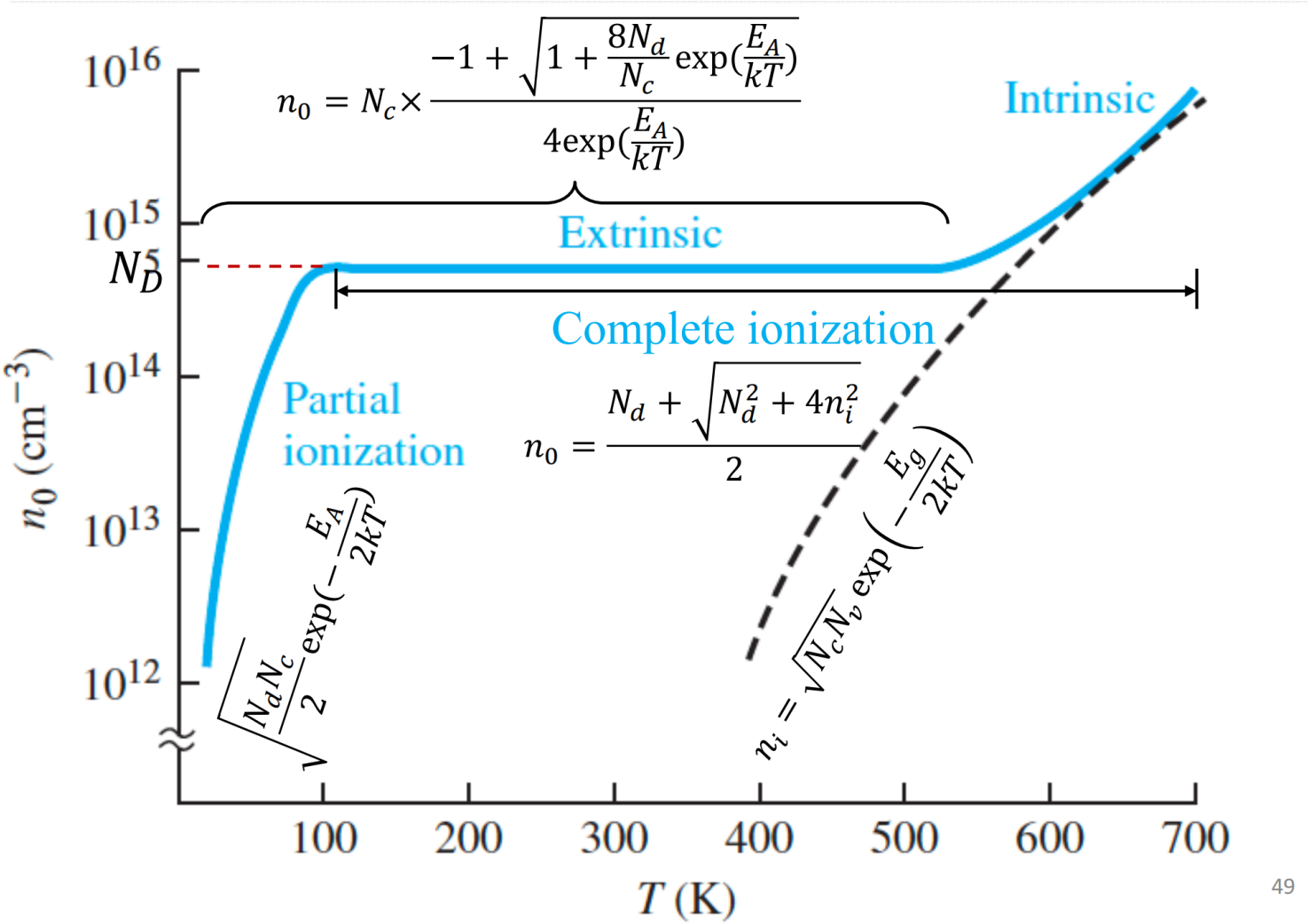
Extrinsic Semiconductor

- n-type: $n_0 > p_0$
- p-type: $p_0 > n_0$
- $n_0 = N_c exp(\frac{E_F - E_c}{kT}) = n_i exp(\frac{E_F - E_{Fi}}{kT})$
- $p_0 = N_v exp(\frac{E_v - E_F}{kT}) = n_i exp(\frac{-(E_F - E_{Fi})}{kT})$
- $n_0 p_0 = n_i^2$

Statistics of Donors and Acceptors

- $n_d = \dfrac{N_d}{1 + \frac{1}{g} exp(\frac{E_d - E_F}{kT})}$, $g = 2$
- $n_a = \dfrac{N_a}{1 + \frac{1}{g} exp(\frac{E_F - E_a}{kT})}$, $g = 4$
- $n_d = N_d - N_d^+$
- $n_a = N_a - N_a^-$
- N_d: concentration of donor atoms
  N_d^+: concentration of ionized donors
- complete ionization: at room temperature, $N_d^+ \approx N_d$
- freeze out: at $0K$, $n_d \approx N_d$

Electron Concentration versus Temperature



49

Charge Neutrality

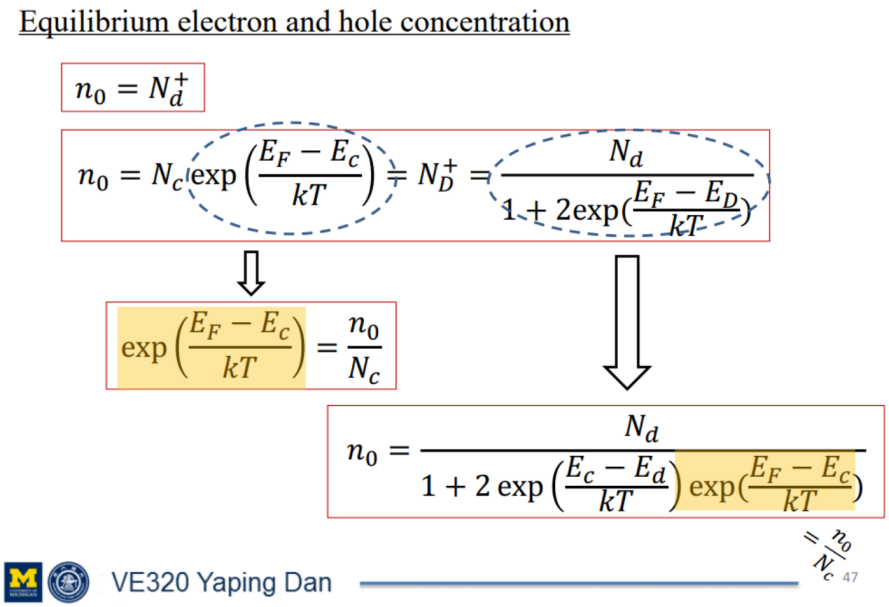
- $n + N_a^- = N_d^+ + p$
- $n_0 p_0 = n_i^2$
- Assume complete ionization:

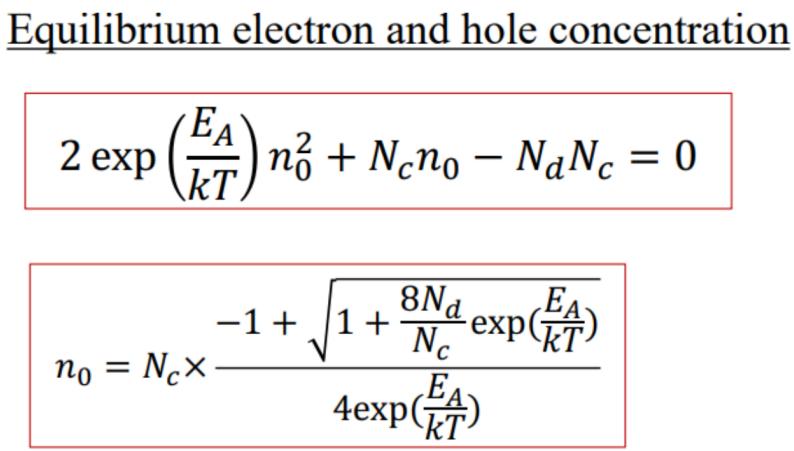
$$n_0 = \frac{N_d - N_a}{2} + \sqrt{(\frac{N_d - N_a}{2})^2 + n_i^2}$$

$$p_0 = \frac{N_a - N_d}{2} + \sqrt{(\frac{N_a - N_d}{2})^2 + n_i^2}$$

Derivation

For more details, please refer to Chapter 4 slides:





Position of Fermi Energy Level

- E_F is not fixed for a material, depends on doping
- $E_c - E_F = kT\ln(\frac{N_c}{n_0})$
- $E_F - E_v = kT\ln(\frac{N_v}{p_0})$
- $E_F - E_{Fi} = kT\ln(\frac{n_0}{n_i})$
- $E_{Fi} - E_F = kT\ln(\frac{p_0}{n_i})$

Reference

1. Semiconductor Physics and Devices: Basic Principles 4th ed. Donald A. Neamen.
2. 2023Summer Ve320_RC_2, Shuo Deng
3. 2023Summer ve320_mid_rc_part2, Shuo Deng
4. 2023Summer VE320 RC3, Jiajun Sun
5. 2023Summer Mid1 RC - part3, Xinyi Zhou