VE320 – Summer 2023

Introduction to Semiconductor Devices

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Chapter 11 Metal-Oxide-Semiconductor Field Effect Transistors: More Concepts

Outline

Nonideal Effects:

11.1 Channel length modulation

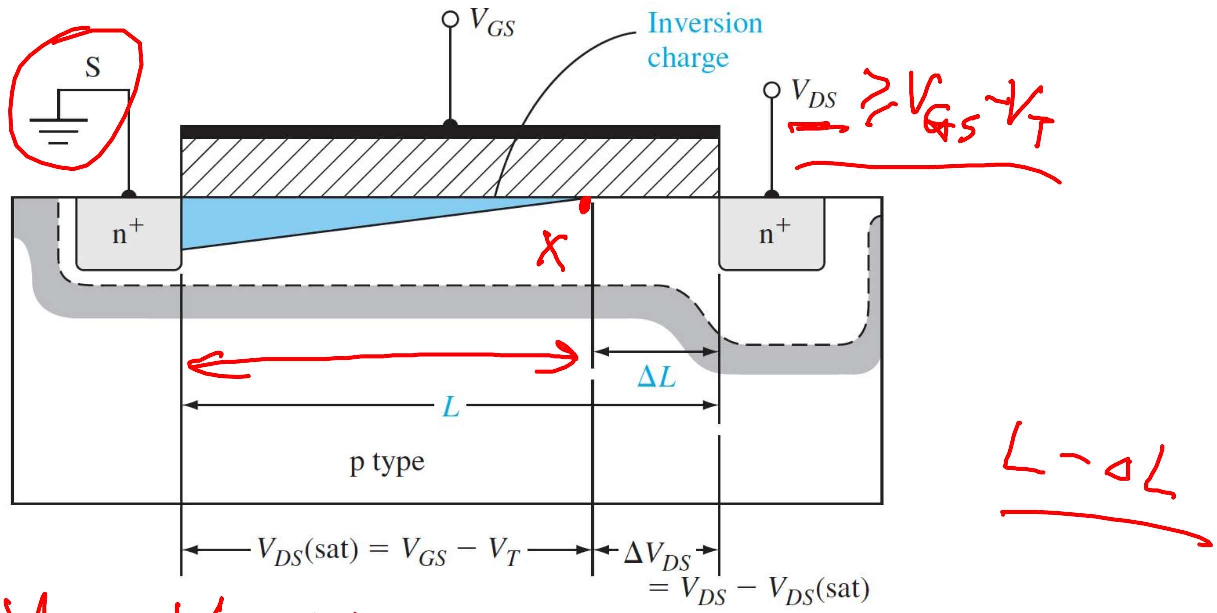
11.2 Conductance and transconductance

11.3 Subthreshold conduction

11.4 Velocity Saturation

11.5 Short Channel Effect

11.1 Channel length modulation



$V_x = V_{GS} - V_T$

$\lambda = 0.01/V$

$$I_D' = \frac{k_n'}{2} \cdot \frac{W}{L} \cdot \left[(V_{GS} - V_T)^2 \, (1 + \lambda V_{DS}) \right]$$

11.1 Channel length modulation

ideal

$$
I_D = \begin{cases} 0 & V_{GS} - V_T < 0 \\ \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_T)^2 & 0 \le V_{GS} - V_T < V_{DS} \\ \mu_n C_{ox}\frac{W}{L}\left[(V_{GS} - V_T)V_{DS} - \frac{1}{2}V_{DS}^2\right] & V_{GS} - V_T \ge V_{DS} \end{cases}
$$

Saturat ~~linear~~

~~Saturatio~~ linear

channel-length modulation

$$
I_D = \begin{cases} 0 & V_{GS} - V_T < 0 \\ \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_T)^2(1 + \lambda V_{DS}) & 0 \le V_{GS} - V_T < V_{DS} \\ \mu_n C_{ox}\frac{W}{L}\left[(V_{GS} - V_T)V_{DS} - \frac{1}{2}V_{DS}^2\right] & V_{GS} - V_T \ge V_{DS} \end{cases}
$$

Saturation

linear

Check your understanding

Problem Example 1

$\mu_n C_{ox}$

Consider an n-channel silicon MOSFET. The parameters are $k_n' = 75\ \mu A/V^2$, $W/L = 10$, and $V_T = 0.35$ V. The applied drain-to-source voltage is $V_{DS} = 1.5$ V. (*a*) For $V_{GS} = 0.8$ V, find (*i*) the ideal drain current, (*ii*) the drain current if $\lambda = 0.02\ V^{-1}$, and (*iii*) the output resistance for $\lambda = 0.02\ V^{-1}$. (*b*) Repeat part (*a*) for $V_{GS} = 1.25$ V.

(i) $V_{GS} = 0.8V$ $\quad V_{GS} - V_T = 0.45V$ $\quad V_{DS} = 1.5V$ $\quad V_{GS} - V_T < V_{DS}$

pinch-off saturation

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_T)^2$$

$$= \frac{1}{2} \times 75\times10^{-6} \times 10 \times 0.45 \times 0.45$$

$$= 7.59\times10^{-5} A = 75.9\,\mu A$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_T)^2(1+\lambda V_{DS})$$

$$= 75.9 \times (1 + 0.02\times1.5) = 78.2\,\mu A$$

Outline

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11.5 Short Channel Effect

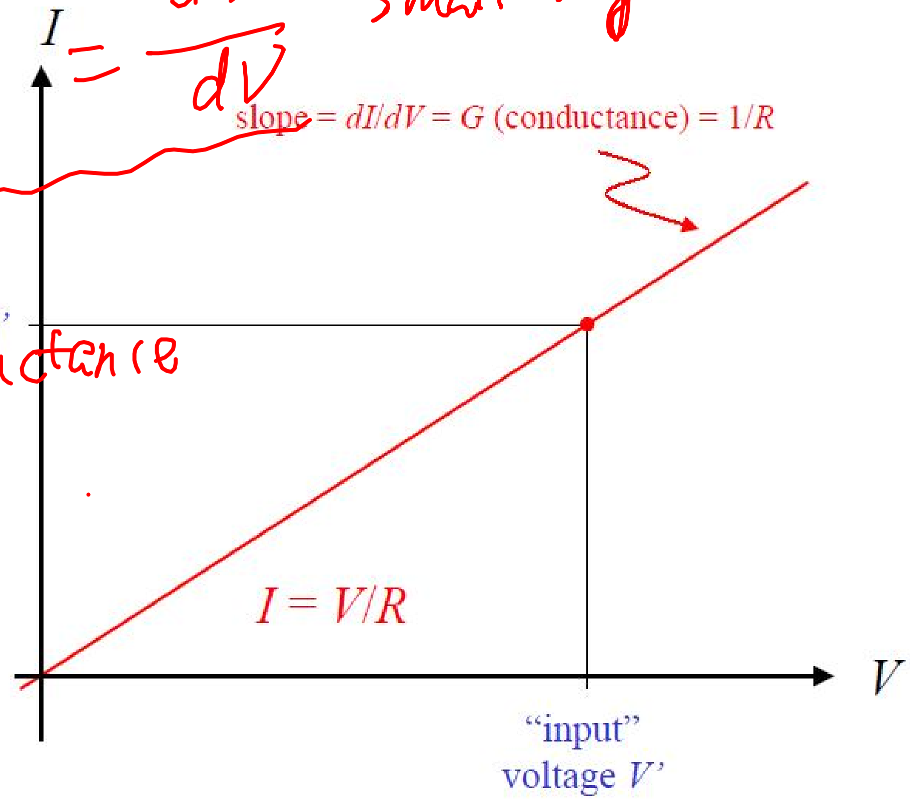
11.2 conductance and transconductance

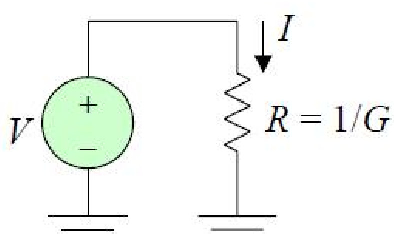
C'

$$\frac{I}{V} = \frac{\Delta I}{\Delta V} = \frac{dI}{dV}$$

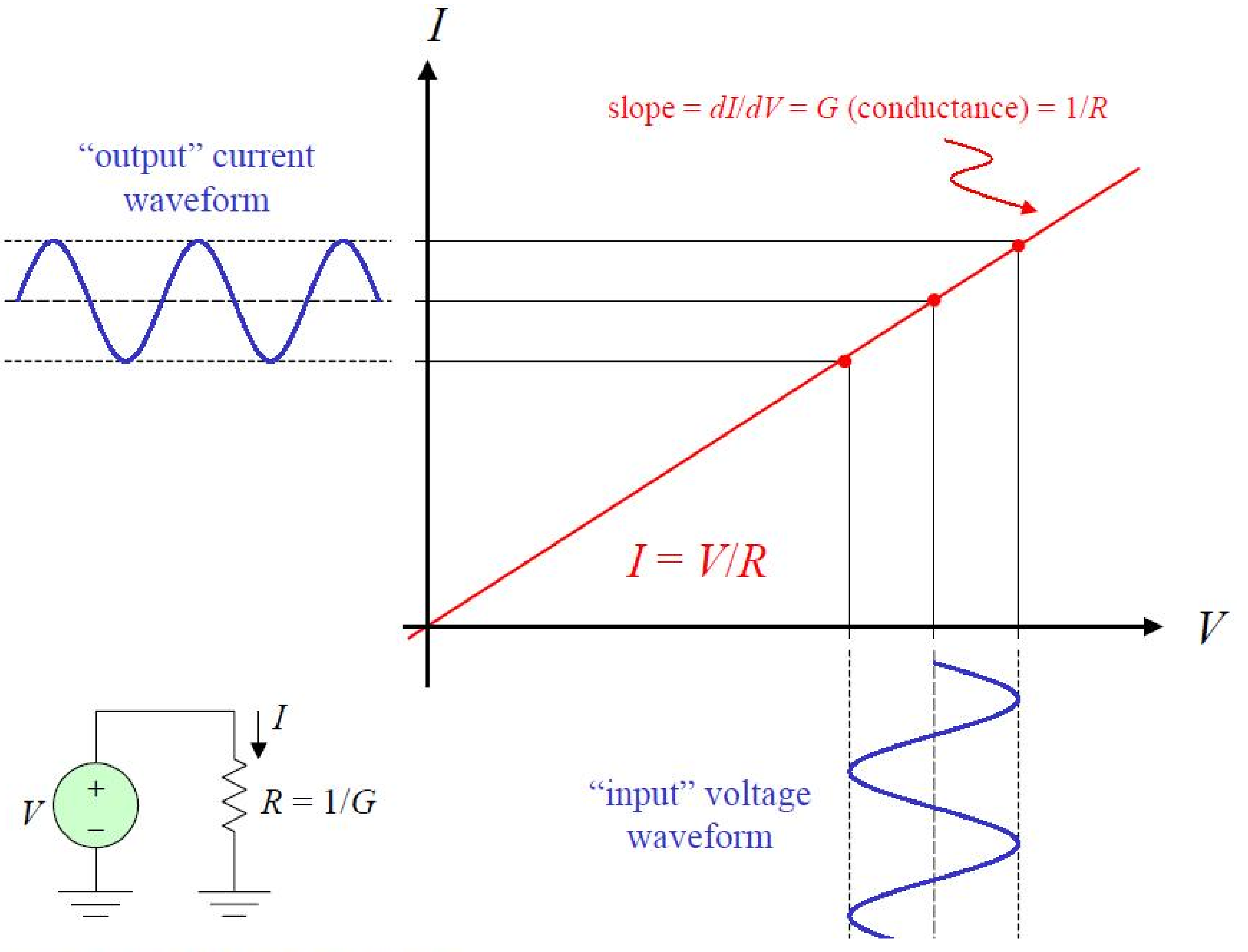
small signal

large signal conductance





11.2 conductance and transconductance



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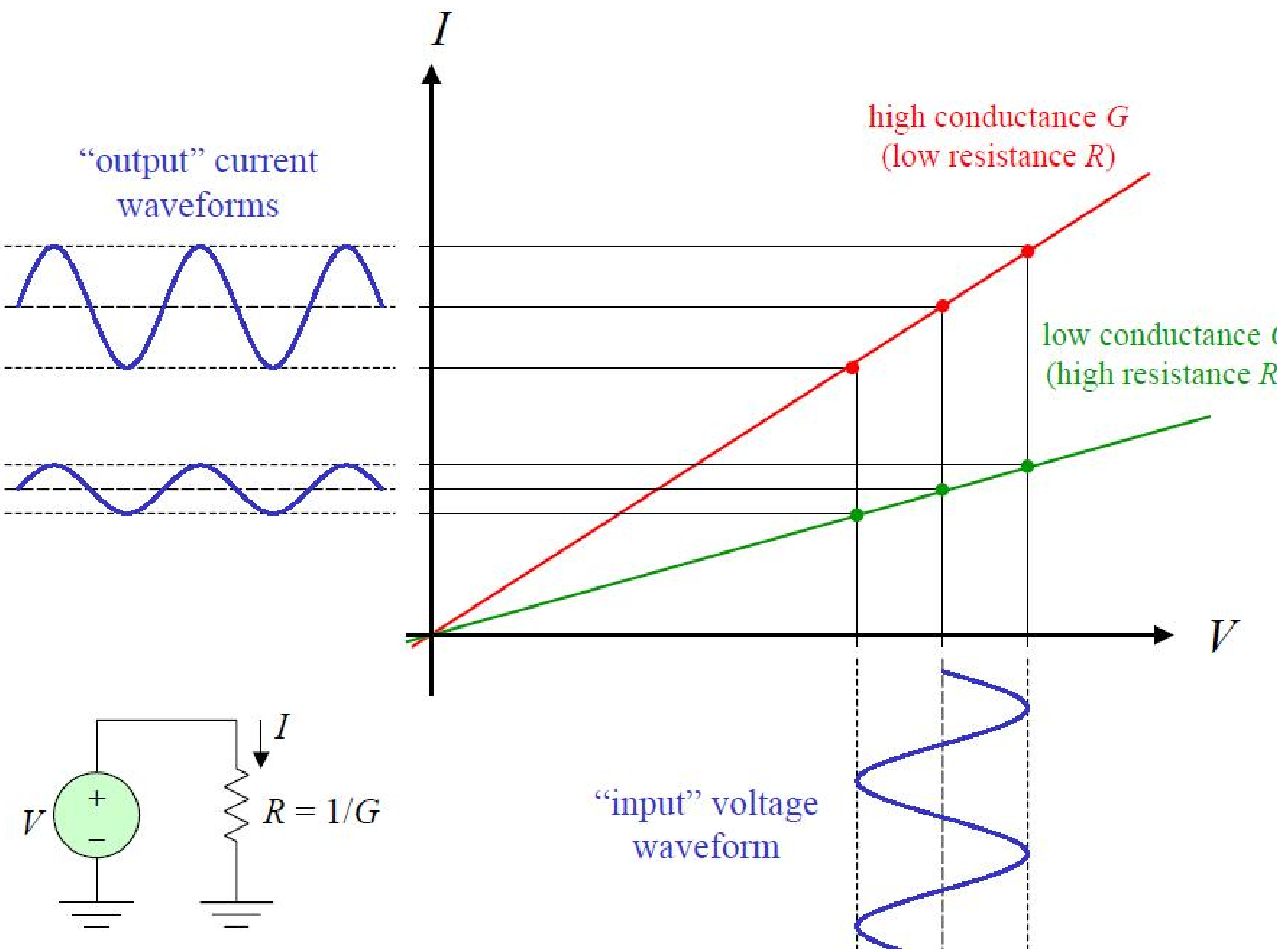
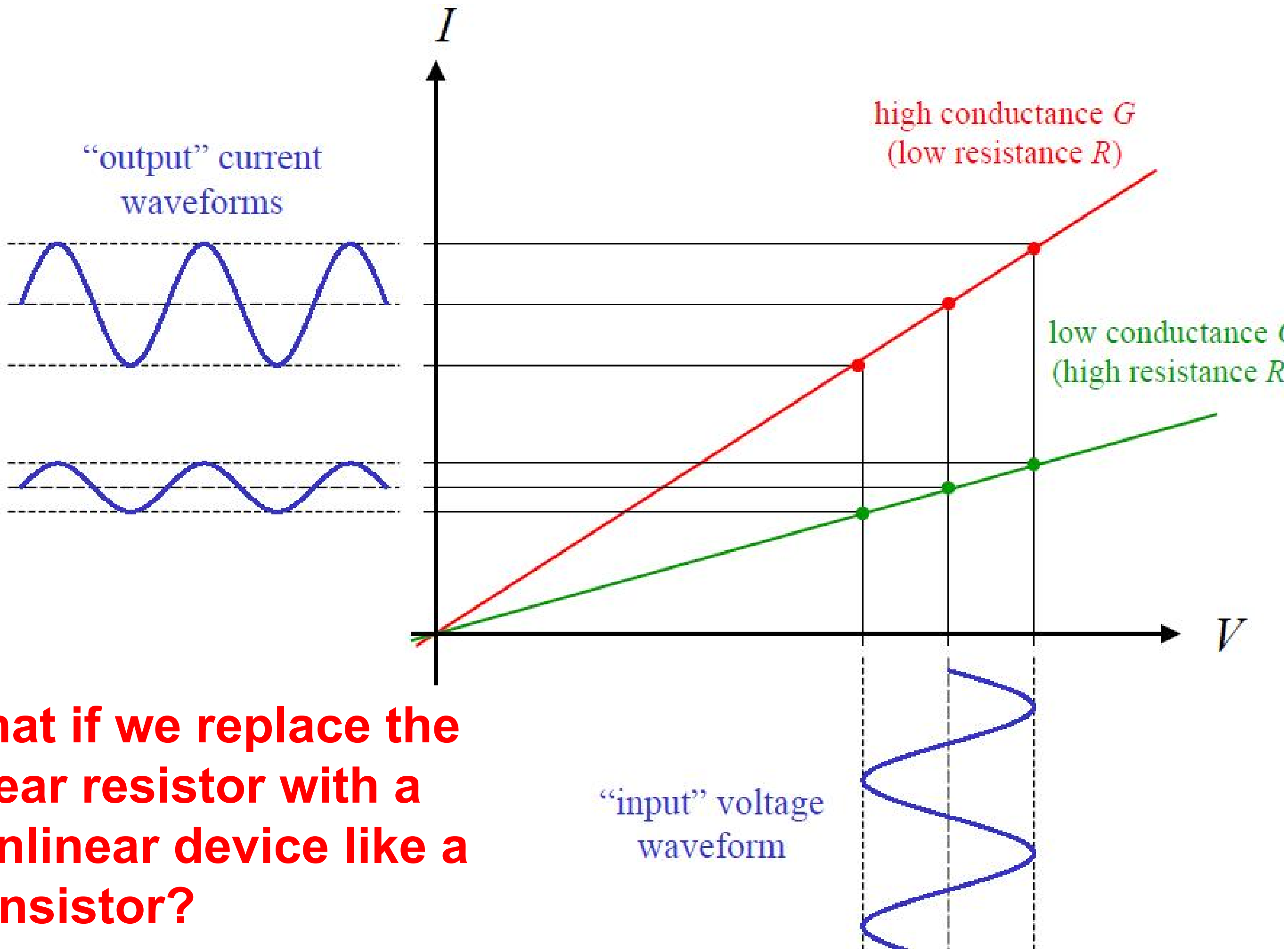
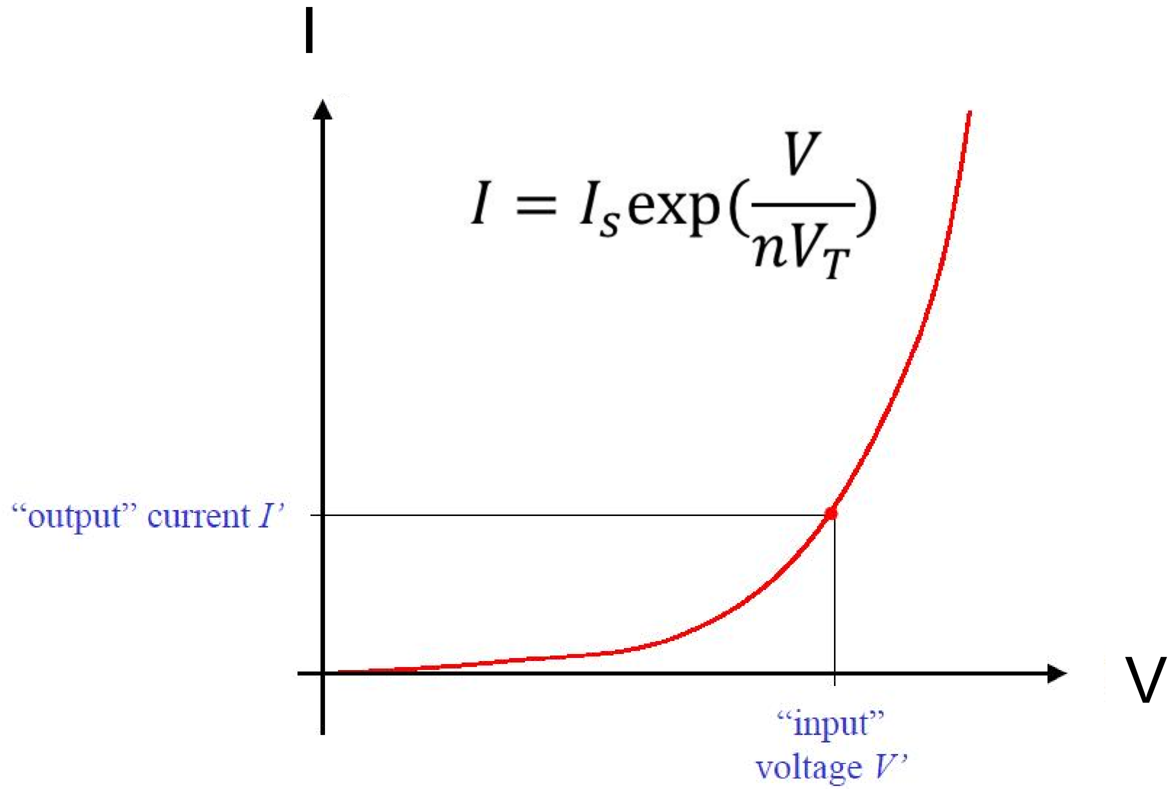


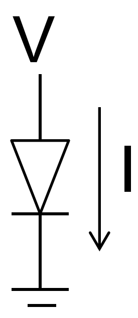
11.2 conductance and transconductance



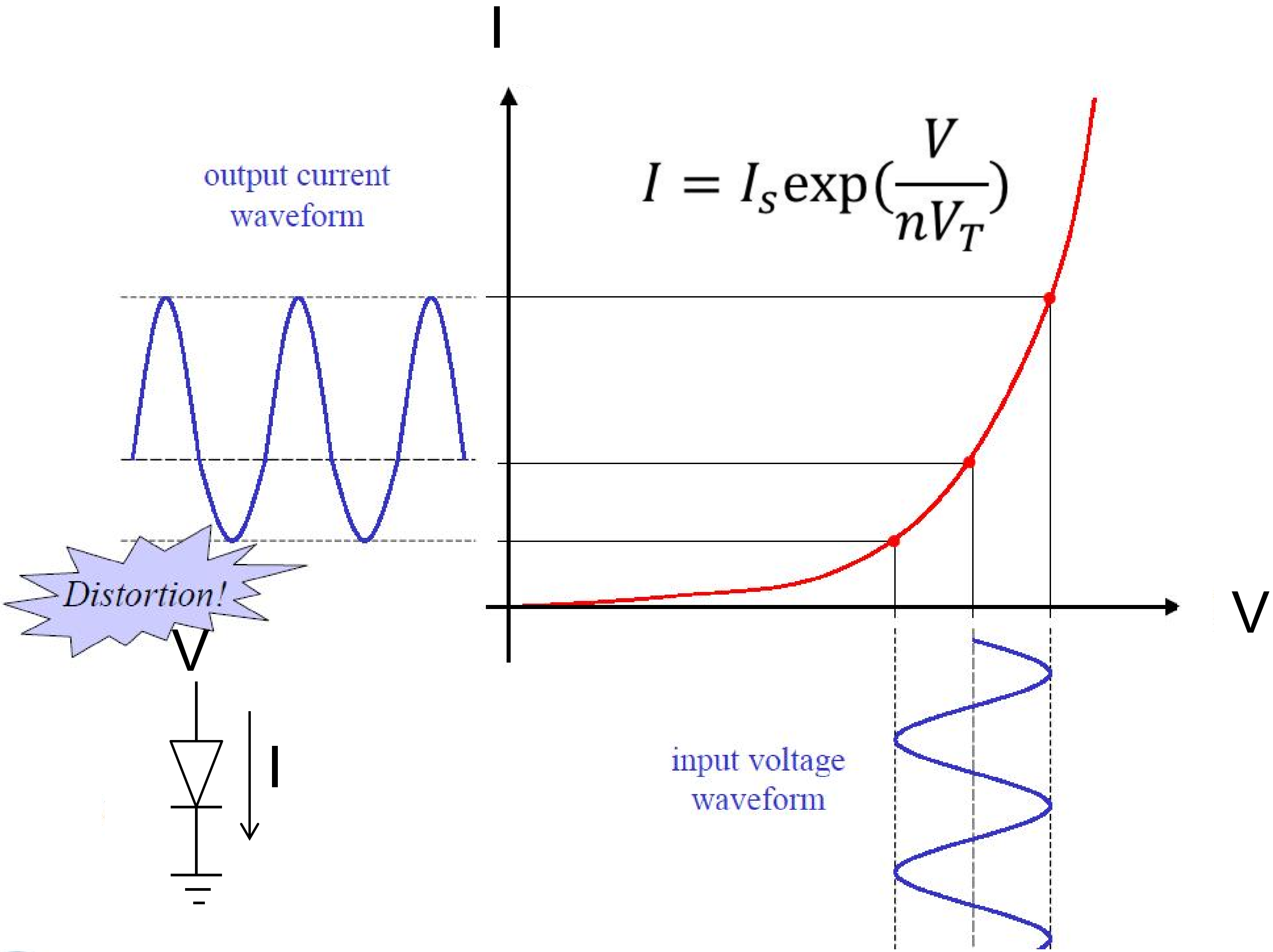
What if we replace the linear resistor with a nonlinear device like a transistor?

11.2 conductance and transconductance



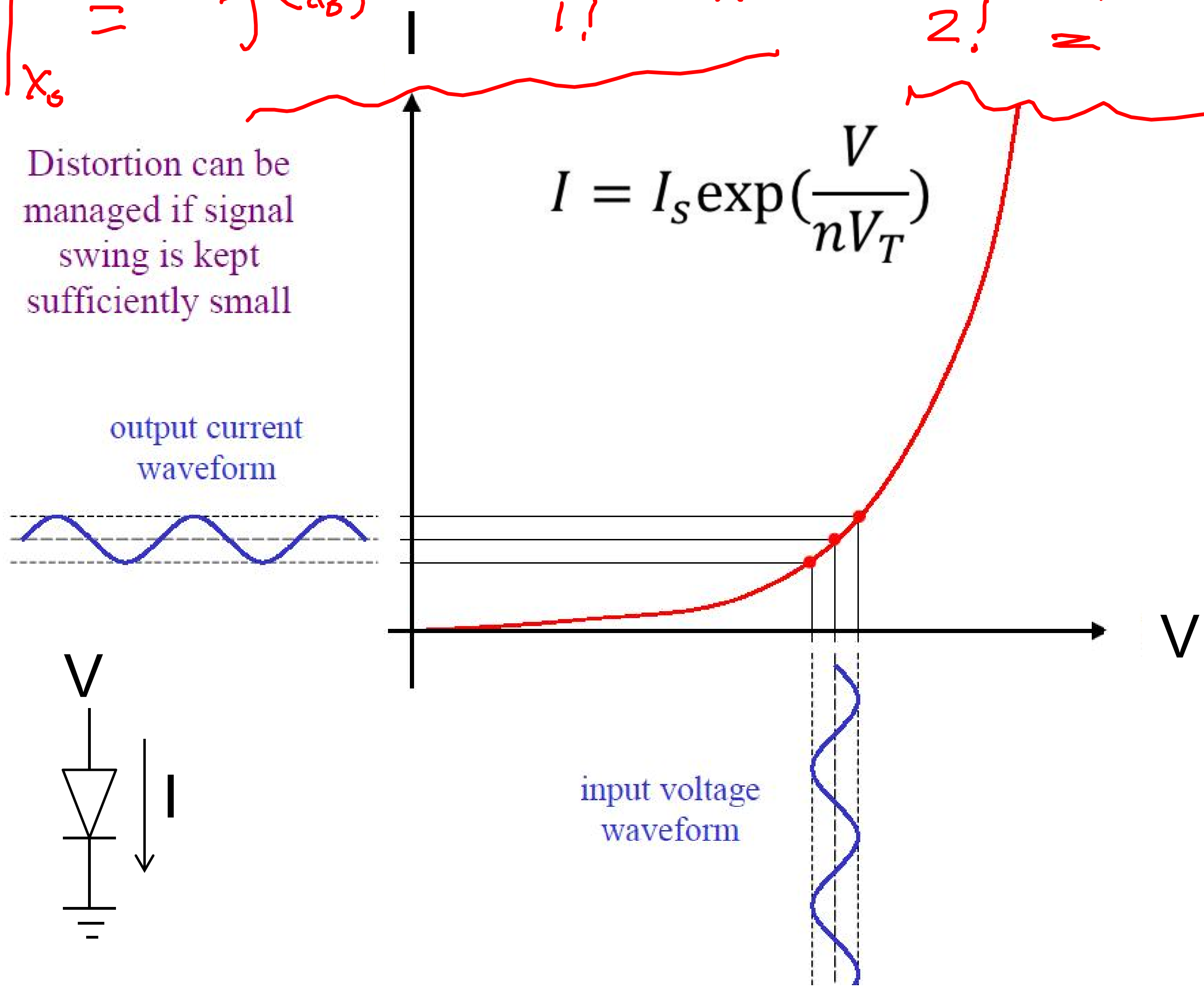


11.2 conductance and transconductance



11.2 conductance and transconductance

$$f(x)\Big|_{x_0} = \underbrace{f(x_0) + \frac{f'(x_0)}{1!}\cdot\Delta x}_{} + \underbrace{\frac{f''(x_0)}{2!}\Delta x^2}_{\to 0} =$$



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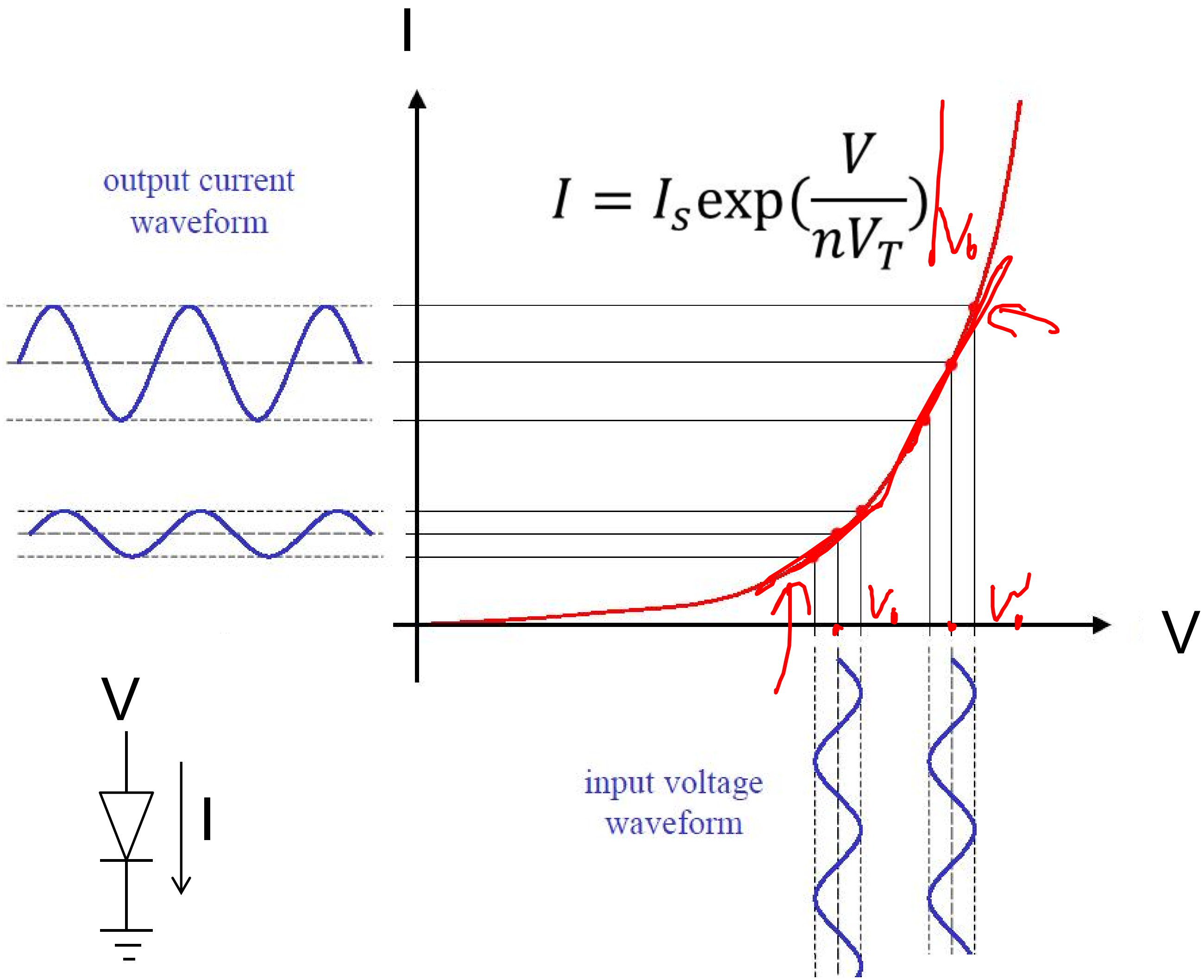

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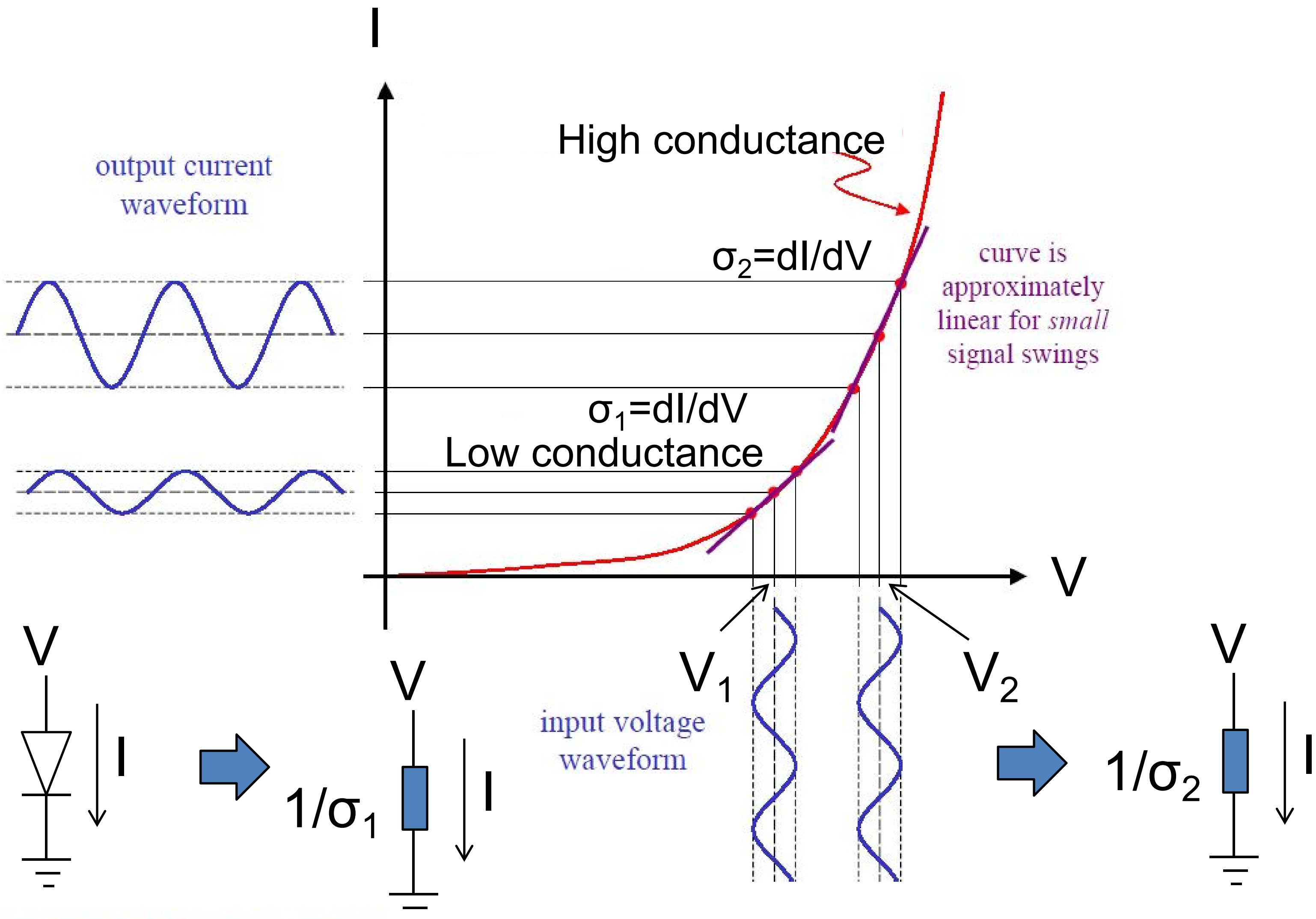

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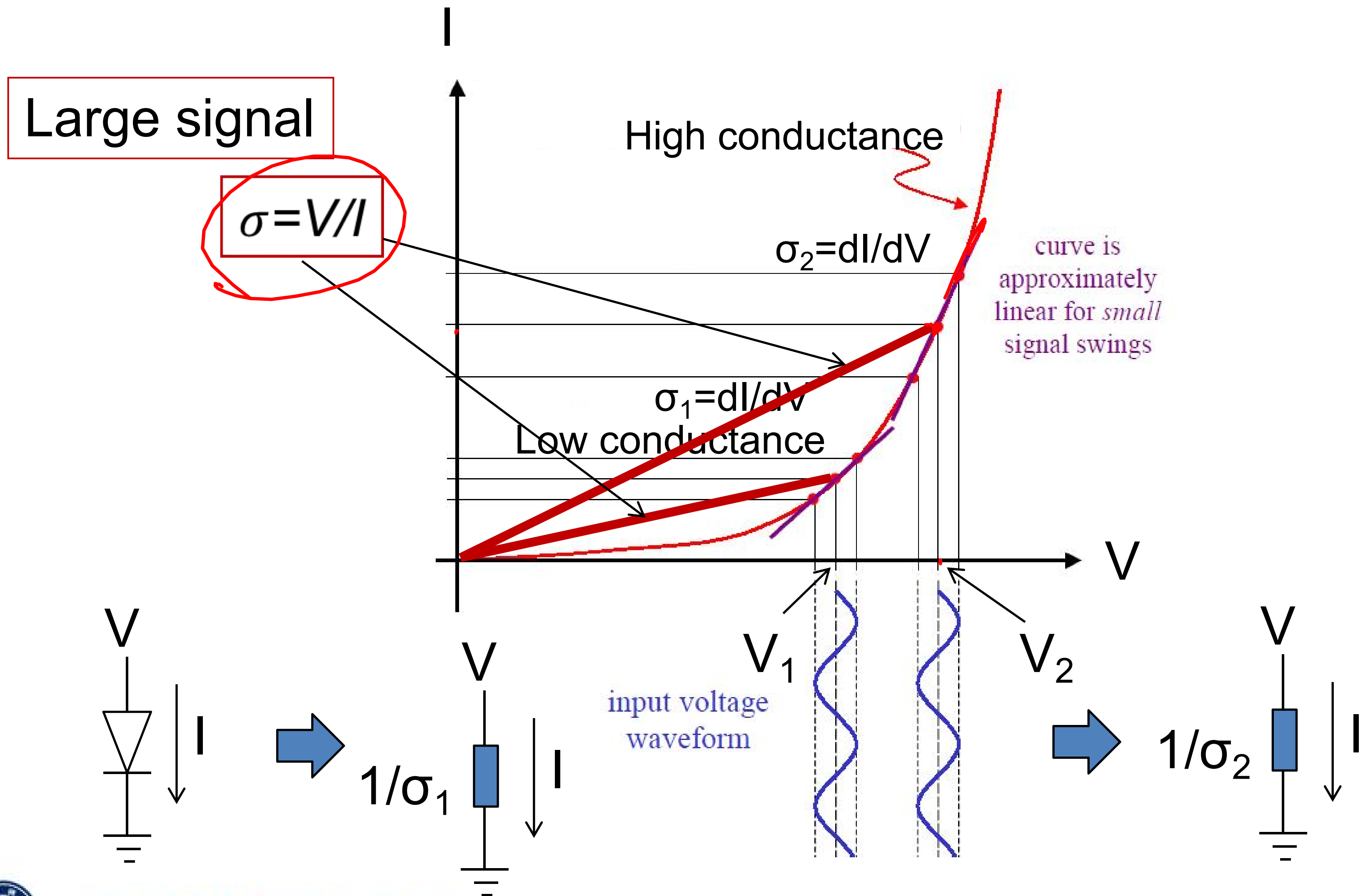
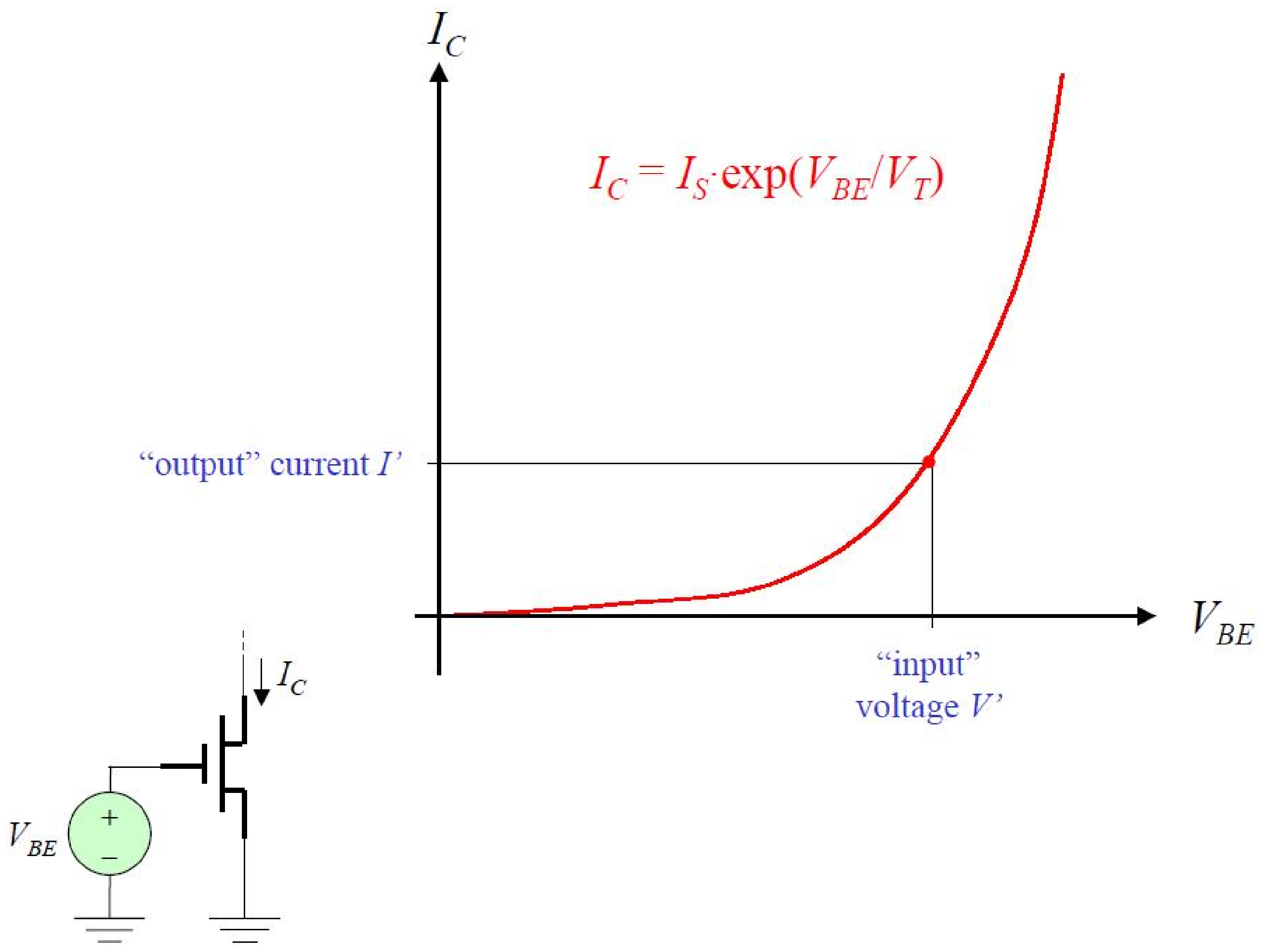


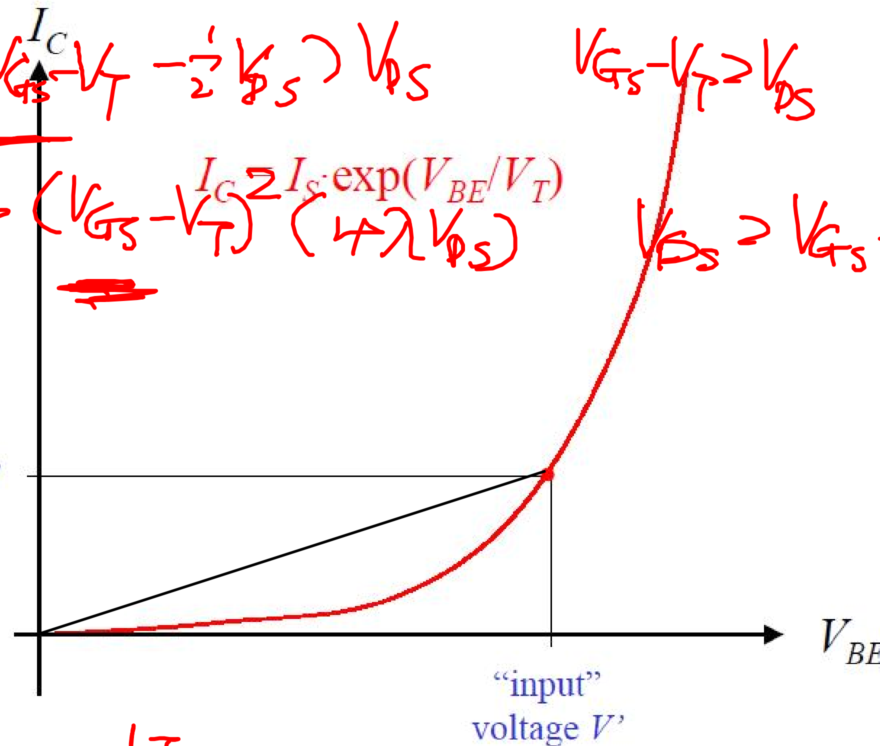
11.2 conductance and transconductance



11.2 conductance and transconductance

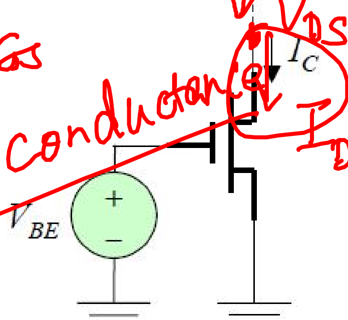
$$I_D = \begin{cases} 0 & V_{GS} < V_T \\ \mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_T - \frac{1}{2} V_{DS}\right) V_{DS} & V_{GS} - V_T > V_{DS} \\ \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) & V_{DS} > V_{GS} - V_T \end{cases}$$

$$I_D = f(V_{DS}, V_{GS})$$

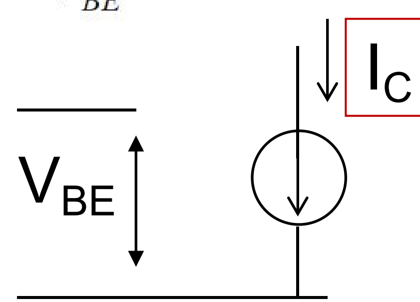


$$\frac{dI_D}{dV_{GS}}$$

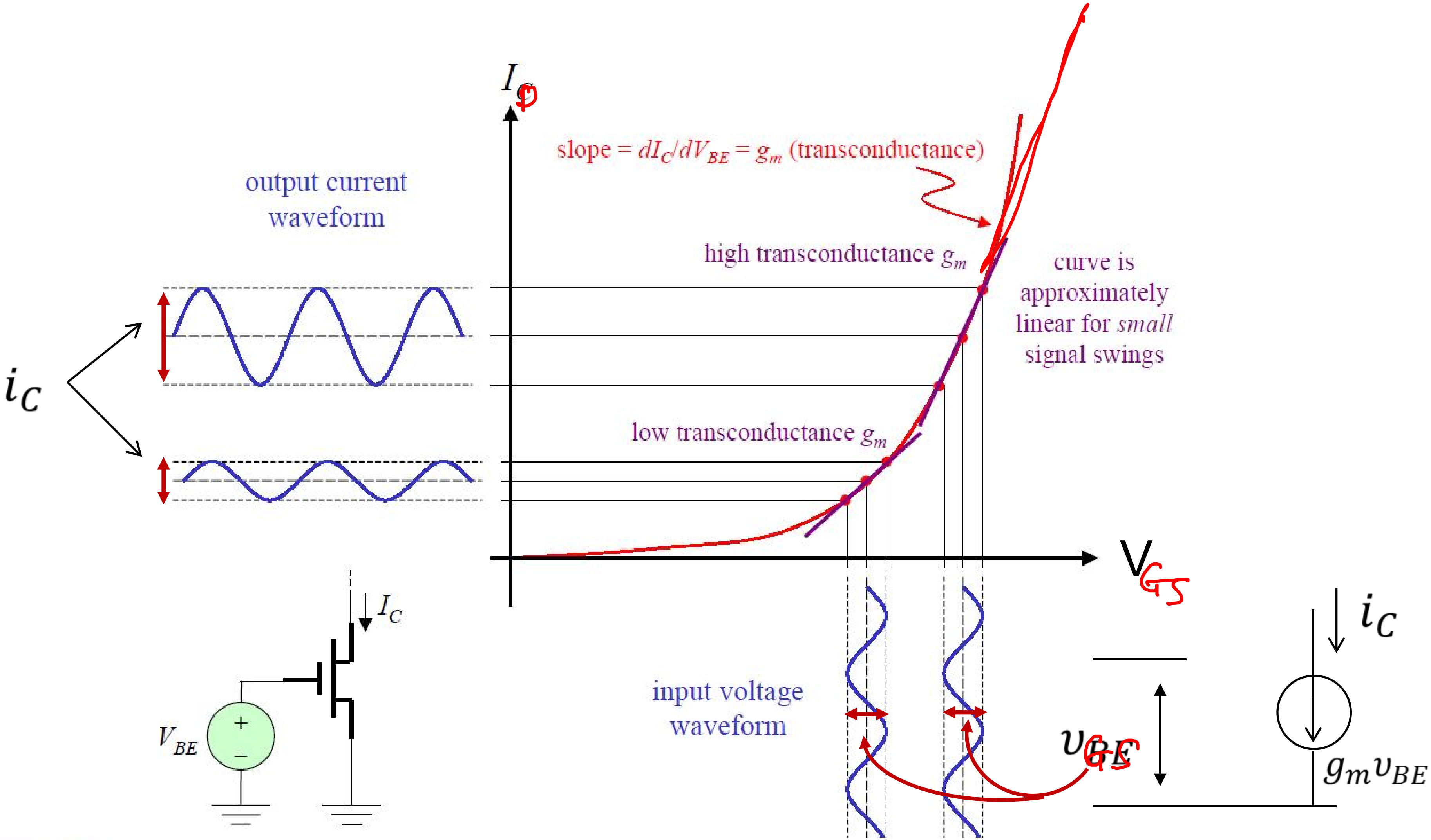
transconductance



$$\sigma = \frac{dI_{DS}}{dV_{DS}}$$



11.2 conductance and transconductance



11.2 conductance and transconductance

$$I_d = \begin{cases} 0 & V_{gs} - V_T < 0 \\ \frac{1}{2}k_n\left(V_{gs} - V_T\right)^2(1 + \lambda V_{ds}) & 0 \le V_{gs} - V_T < V_{ds} \\ k_n[\left(V_{gs} - V_T\right)V_{ds} - \frac{1}{2}V_{ds}^2] & V_{gs} - V_T \ge V_{ds} \end{cases}$$

11.2 conductance and transconductance

Transconductance

NMOS

$$I_d = \begin{cases} 0 & V_{gs} - V_T < 0 \\ \frac{1}{2}k_n(V_{gs} - V_T)^2(1 + \lambda V_{ds}) & 0 \le V_{gs} - V_T < V_{ds} \\ k_n[(V_{gs} - V_T)V_{ds} - \frac{1}{2}V_{ds}^2] & V_{gs} - V_T \ge V_{ds} \end{cases}$$

(Saturation)

(linear/triode

$g_n = \frac{dI_d}{dV_{gs}} = k_n(V_{ds}) = k_n V_{ds}$

g_m transconductance

NMOS PMOS

g_n g_p

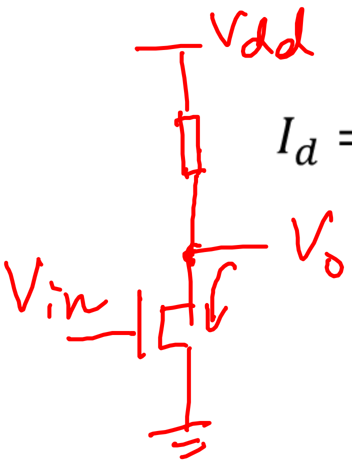
$g_n = \frac{dI_d}{dV_{gs}} = \frac{1}{2} \times 2 k_n (V_{gs} - V_T)(1+\lambda V_{ds})$

$= \frac{2 \times \frac{1}{2} k_n (V_{gs}-V_T)^2 (1+\lambda V_{ds})}{V_{gs} - V_T}$

$= \frac{2I_d}{V_{gs}-V_T}$

11.2 conductance and transconductance

Output Impedance



$$I_d = \begin{cases} 0 & V_{gs} - V_T < 0 \\ \frac{1}{2} k_n (V_{gs} - V_T)^2 (1 + \lambda V_{ds}) & 0 \le V_{gs} - V_T < V_{ds} \\ k_n [(V_{gs} - V_T) V_{ds} - \frac{1}{2} V_{ds}^2] & V_{gs} - V_T \ge V_{ds} \end{cases}$$

Saturation

linear

$$r_o = \frac{1}{\sigma_o} = \frac{1}{\frac{dI_d}{dV_{ds}}} = \frac{1+\lambda V_{ds}}{\lambda I_d}$$

$\mu_n C_{ox}$

Saturation $\sigma_o = \frac{dI_d}{dV_{ds}} = \frac{1}{2} k_n (V_{gs} - V_T)^2 \cdot \lambda$

$$= \lambda \frac{\frac{1}{2} k_n (V_{gs} - V_T)^2 (1 + \lambda V_{ds})}{1 + \lambda V_{ds}}$$

$$= \frac{\lambda I_d}{1 + \lambda V_{ds}}$$

How the knowledge used by engineers and scientists

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Diameter-Dependent Electron Mobility of InAs Nanowires

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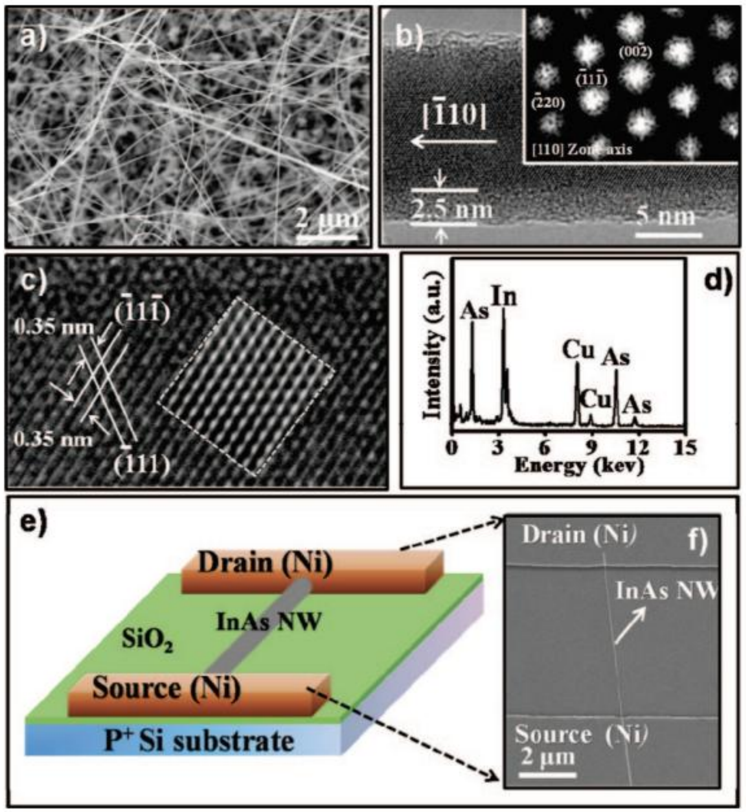
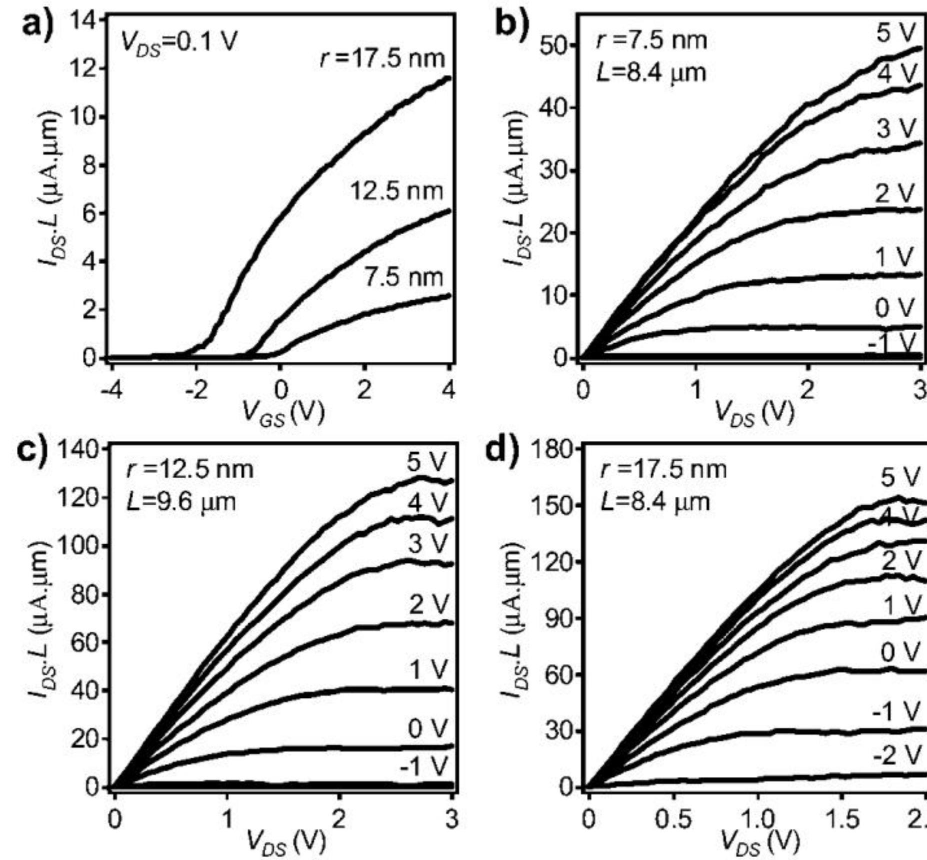
How the knowledge used by engineers and scientists





How the knowledge used by engineers and scientists

We next assess the field-effect electron mobility of InAs NW FETs by using the low-bias ($V_{DS} = 0.1$ V) transconductance

$$g_m = \left.\frac{dI_{DS}}{dV_{GS}}\right|_{V_{DS}}$$

and the analytical expression

$$\mu_n = g_m \frac{L^2}{C_{ox}} \frac{1}{V_{DS}}$$

MOSFETs. The field-effect and effective mobilities are, however, deduced from the $I-V$ characteristics by using different analytical models. Specifically, the effective mobility is deduced from the drain conductance

$$g_D = \left.\frac{dI_{DS}}{dV_{DS}}\right|_{V_{GS}}$$

with

$$\mu_{n,eff} = g_D \frac{L^2}{C_{ox}} \frac{1}{(V_{GS} - V_t)}$$

Check your understanding

Problem example 2

$V_{gs} = 1V$

The transconductance of an n-channel MOSFET is found to be $g_m = \partial I_D / \partial V_{GS} = 1.25$ mA/V when measured at $V_{DS} = 50$ mV. The threshold voltage is $V_T = 0.3$ V. (*a*) Determine the conductance parameter K_n. (*b*) What is the current at $V_{GS} = 0.8$ V and $V_{DS} = 50$ mV? (*c*) Determine the current at $V_{GS} = 0.8$ V and $V_{DS} = 1.5$ V.

$\boxed{V_{GS} - V_T < V_{ds}}$

$$g_m = \frac{2 I_d}{V_{gs} - V_T} = 1.25 \text{ mA/V}$$

0.05

(b) $V_{GS} - V_T = 0.5V > V_{ds}$

linear

$$1.25 \text{ mA/V} = K_n (V_{gs} - V_T)(1 + \lambda V_{ds})$$

$$= K_n (1 - 0.3)(1 + 0)$$

$$\Rightarrow K_n = \frac{1.25 \text{ mA/V}}{0.7 V} \Rightarrow K_n = 1.8 \text{ mA/V}^2$$

$$I_d = \mu_n C_{ox} \frac{W}{L} \left((V_{gs} - V_T) - \frac{1}{2} V_{ds} \right) V_{ds}$$

$$= 1.8 \times (0.5) \times 0.05$$

$$= 0.045 \text{ mA}$$

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11.3 Subthreshold conduction

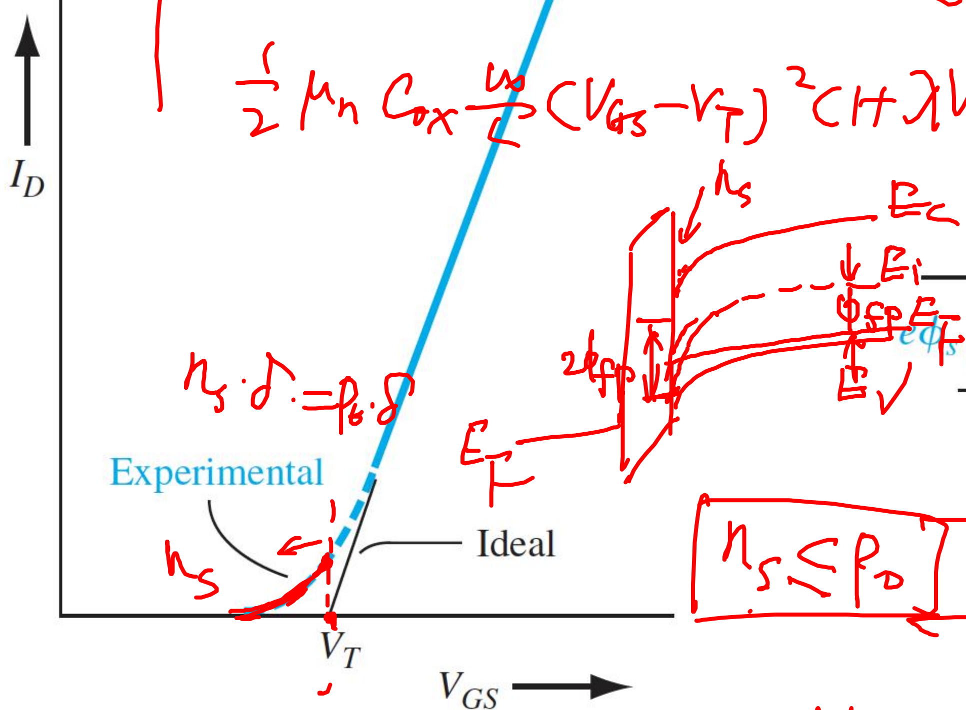
11.4 Velocity Saturation

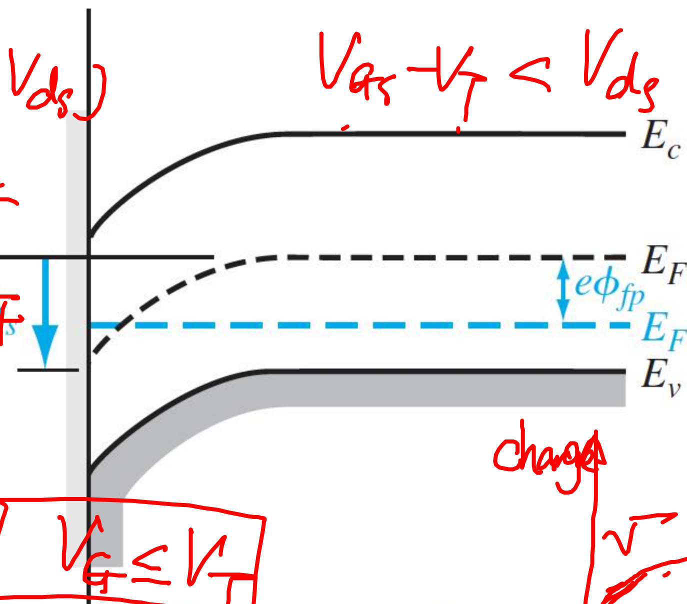
11.5 Short Channel Effect

11.3 Subthreshold conduction

$V_{GS} \le V_T$ $V_{GS}-V_T<0$

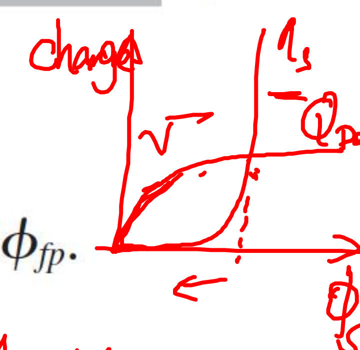
$$I_d = \begin{cases} 0 & V_{GS}-V_T<0 \\ \mu_n C_{ox}\frac{W}{L}\left(V_{GS}-V_T-\frac{1}{2}V_{ds}\right)V_{ds} & V_{ds}<V_{GS}-V_T \\ \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_T)^2(1+\lambda V_{ds}) & V_{GS}-V_T<V_{ds} \end{cases}$$





$\boxed{n_s \le p_0}$ $\boxed{V_G \le V_T}$

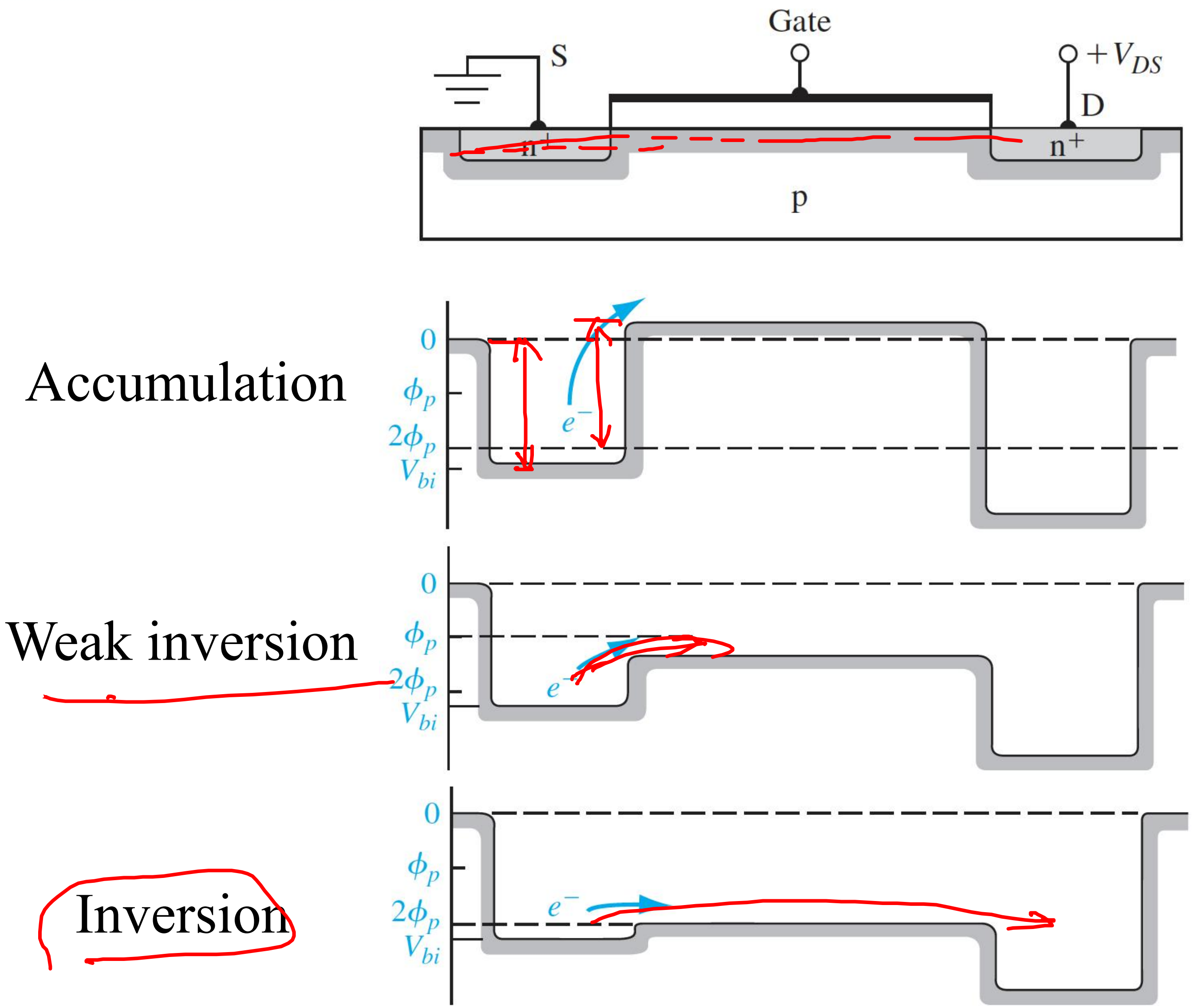
$\phi_{fp} < \phi_s < 2\phi_{fp}.$



$\underline{V_{GS}-V_T} \sim n_s$
>0

$V_{GS}-V_T$
<0

11.3 Subthreshold conduction

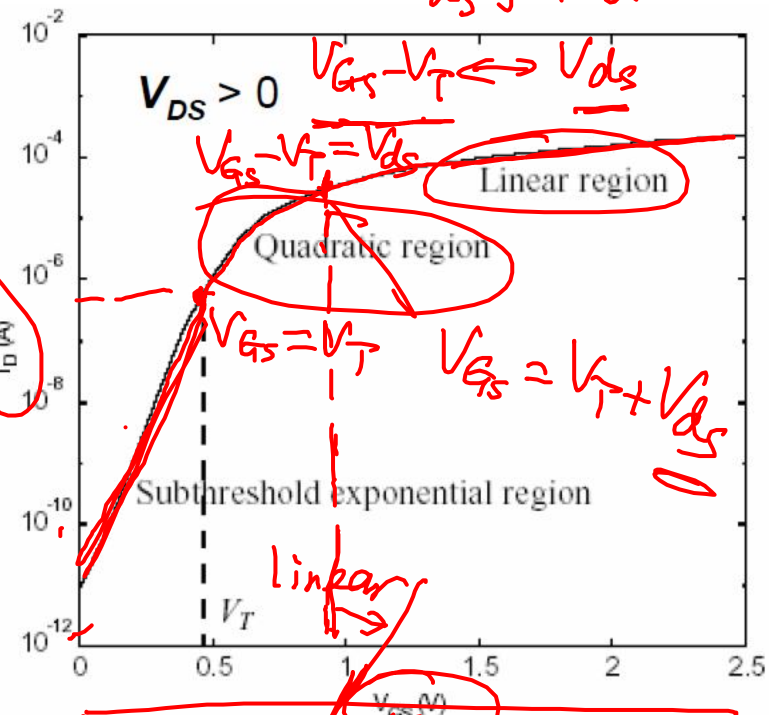


11.3 Subthreshold conduction

Subthreshold Conduction (Leakage Current)

- The transition from the ON state to the OFF state is gradual. This can be seen more clearly when I_D is plotted on a logarithmic scale:

Vds fixed



- In the subthreshold ($V_{GS} < V_T$) region,

$$I_D \propto \exp\left(\frac{qV_{GS}}{nkT}\right)$$

log

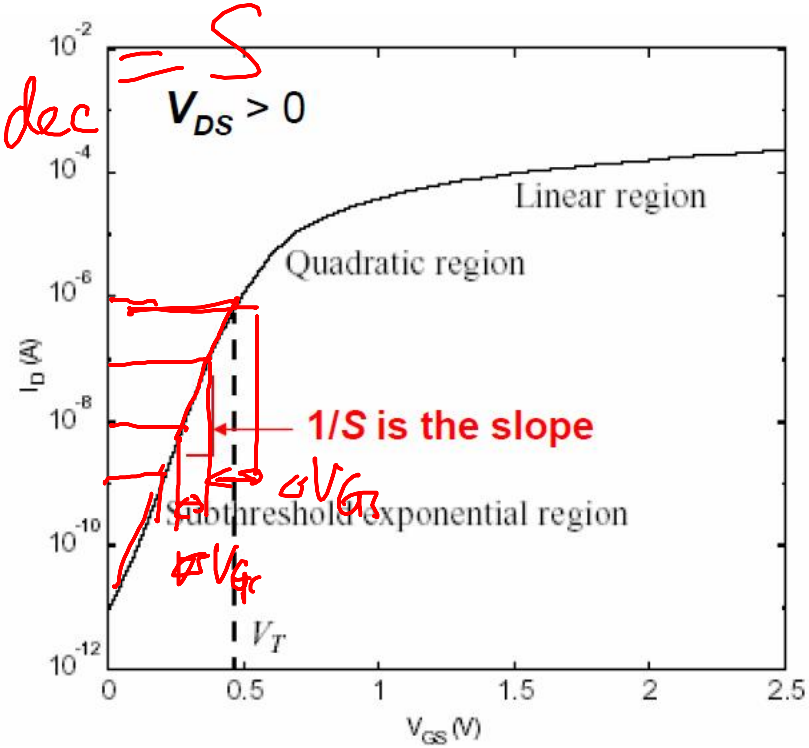
This is essentially the channel-source pn junction current.
(Some electrons diffuse from the source into the channel, if this pn junction is forward biased.)

11.3 Subthreshold conduction

Slope Factor (or Subthreshold Swing) *S*

- ***S*** is defined to be the inverse slope of the log (I_D) *vs.* V_{GS} characteristic in the subthreshold region:

60mV/dec = S



$$S \equiv n\left(\frac{kT}{q}\right)\ln(10)$$

Units: Volts per decade

Note that ***S*** ≥ 60 mV/dec at room temperature:

$$\left(\frac{kT}{q}\right)\ln(10) = 60\,\mathrm{mV}$$

75mV/dec

11.3 Subthreshold conduction

V_T Design Trade-Off

(Important consideration for digital-circuit applications)

- Low V_T is desirable for high ON current

$$I_{DSAT} \propto (V_{DD} - V_T)^{\eta} \qquad 1 < \eta < 2$$

where V_{DD} is the power-supply voltage

Vdd

…but high V_T is needed for low OFF current

10^{-12} A = 1pA $\rightarrow 10^{-2}$

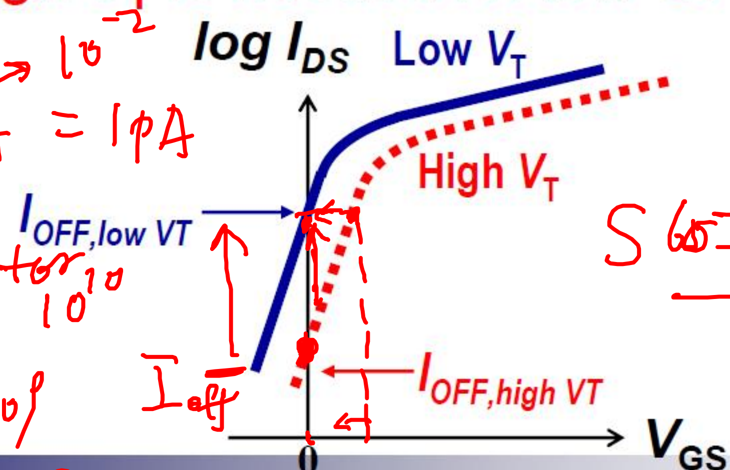
2.5V

10^9 transistor 10^{10}

$2.5 \times 10^{-12} \times 10^9$

$= 2.5 \times 10^{-3} = 2.5$mW

0.6 ~ 0.7V



$V_T \rightarrow 0$

S ~~65~~ 90mV/dec

$2.5 \times 10^{-3} \times 10^{10}$ W

$= 2.5 \times 10^7$ W $= 25$mW

Check your understanding

Problem example 3

Assume that the subthreshold current of a MOSFET is given by

$$I_D = 10^{-15} \exp\left(\frac{V_{GS}}{(2.1)V_t}\right)$$

$V_t = kT/q$

over the range $0 \le V_{GS} \le 1$ volt and where the factor 2.1 takes into account the effect of interface states. Assume that 10^6 identical transistors on a chip are all biased at the same V_{GS} and at $V_{DD} = 5$ V. (*a*) Calculate the total current that must be supplied to the chip at $V_{GS} = 0.5$, 0.7, and 0.9 V. (*b*) Calculate the total power dissipated in the chip for the same V_{GS} values.

$$I_{D,total} = 10^6 \times 10^{-15} \exp\left(\frac{0.5}{2.1 \times 0.0259}\right)$$

$$W_{total} = I_{D,total} \cdot V_{dd}$$

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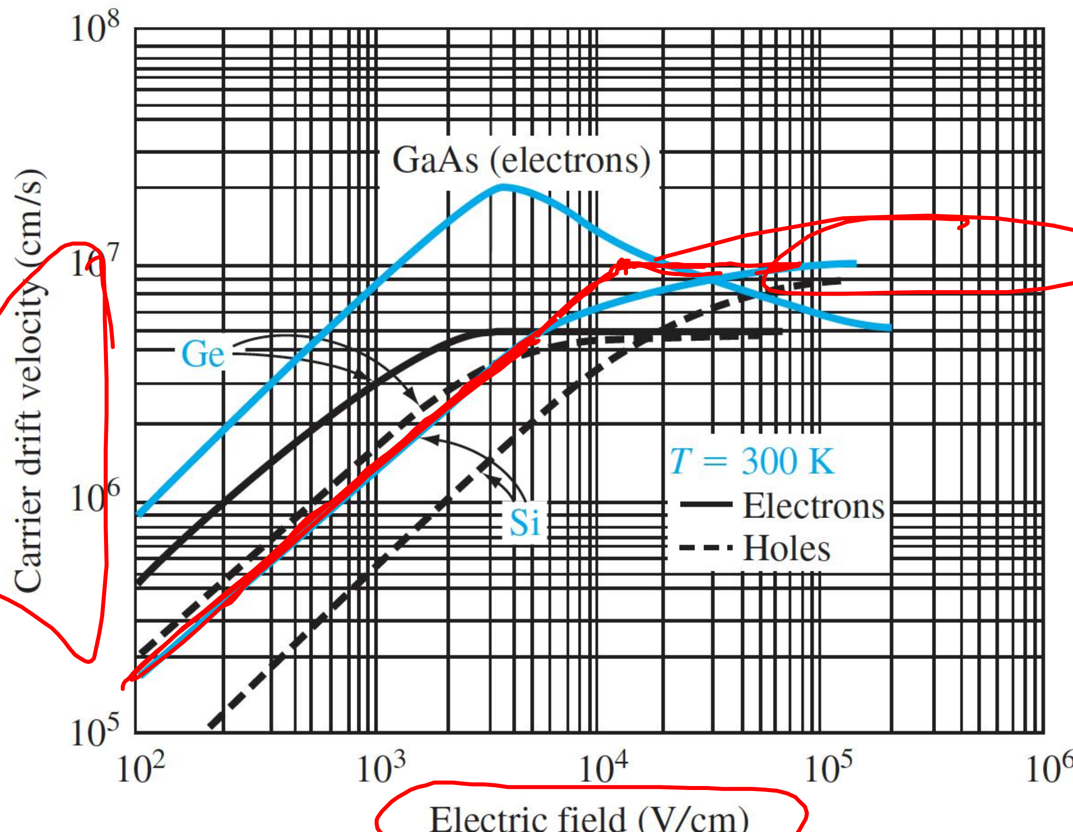
11.4 Velocity Saturation

$I = q \cdot n \cdot \mu \cdot E = q \cdot n \cdot v$

$$\boxed{v_d \rightarrow v_{th}}$$

$v = \mu \cdot E$

- Electric field is heating up electrons
- Electrons transfer energy to lattice to reach thermal equilibrium



$$v_n = \frac{v_s}{\left[1 + \left(\frac{E_{on}}{E}\right)^2\right]^{1/2}}$$

$$v_p = \frac{v_s}{\left[1 + \left(\frac{E_{op}}{E}\right)\right]}$$

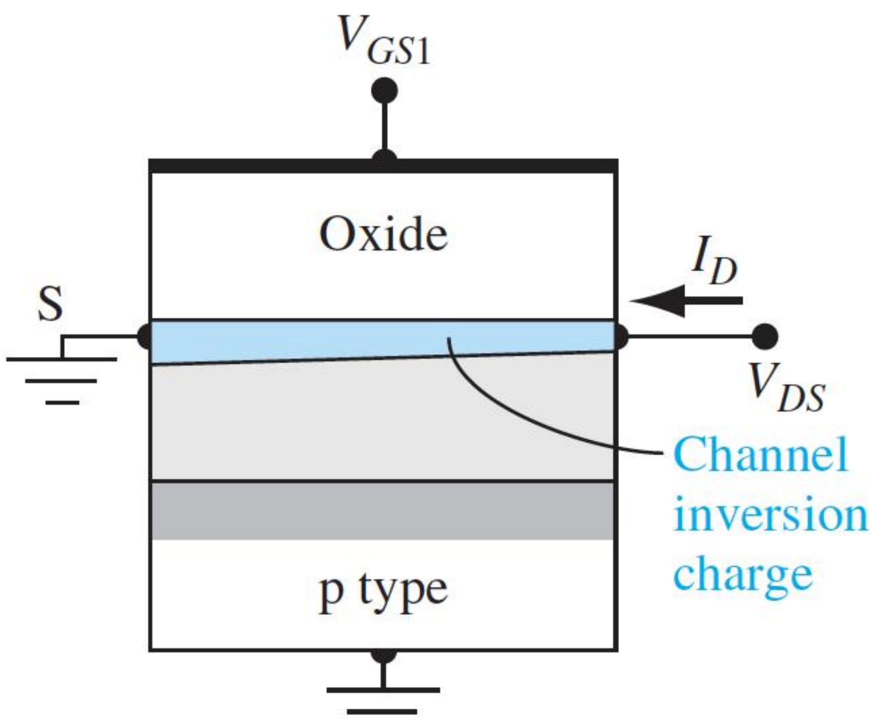
2 1/2

Probably a typo in textbook

11.4 Velocity Saturation

$$\boxed{v_d \to v_{th}}$$

- Electric field is heating up electrons
- Electrons transfer energy to lattice to reach thermal equilibrium



$$E_{DS} = \frac{V_{DS}}{L}$$

As the transistor size scales down, the electric field intensity E increases.

Vdd

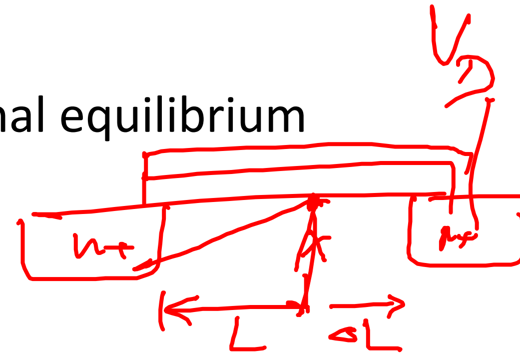
$$E_{on} = \frac{V_{DSAT}}{L}$$

11.4 Velocity Saturation

$$\boxed{v_d \to v_{th}}$$

As the transistor size scales down, the electric field intensity E increases.

- Electric field is heating up electrons
- Electrons transfer energy to lattice to reach thermal equilibrium

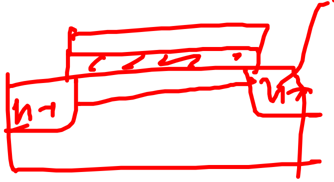


$$I_{DSAT} = \mu_n C_{ox} \frac{W}{L}\left(V_{GS} - VT - \frac{1}{2}V_{DS}\right)V_{DS}$$

V_D

$$= C_{ox} W \left(V_{GS} - VT - \frac{1}{2}V_{DSAT}\right)\frac{V_{DSAT}}{L}\mu_n$$

E_{ox}



$V_X = V_{GS} - V_T$

$$I_{DSAT} = WC_{ox}\left[V_{GS} - V_T - \frac{V_{DSAT}}{2}\right]v_{sat}$$

where $V_{DSAT} = \dfrac{L}{\mu_n}v_{sat}$

E_{ox}

$\dfrac{V_{Dsat}}{L} \cdot \mu_n = V_{sat}$

$\dfrac{V_X}{L} < E_{sat}$

$I_D = \frac{1}{2}\mu_n C_{ox}(V_{GS} - V_T)^2(1+\lambda V_{DS})$ ✓

11.4 Velocity Saturation

$k' \frac{W}{L} \left[(V_{GS}-V_T)(V_{GS}-V_T) - \frac{1}{2}(V_{GS}-V_T)^2 \right]$

$= k' \frac{W}{L} (V_{GS}-V_T)^2$

Unified model

$$I_D = 0 \quad \text{for} \quad V_{GT} \leq 0$$

$$I_D = k' \frac{W}{L}\left(V_{GT}V_{min} - \frac{V_{min}^2}{2}\right) \qquad \text{for } V_{GT} \geq 0$$

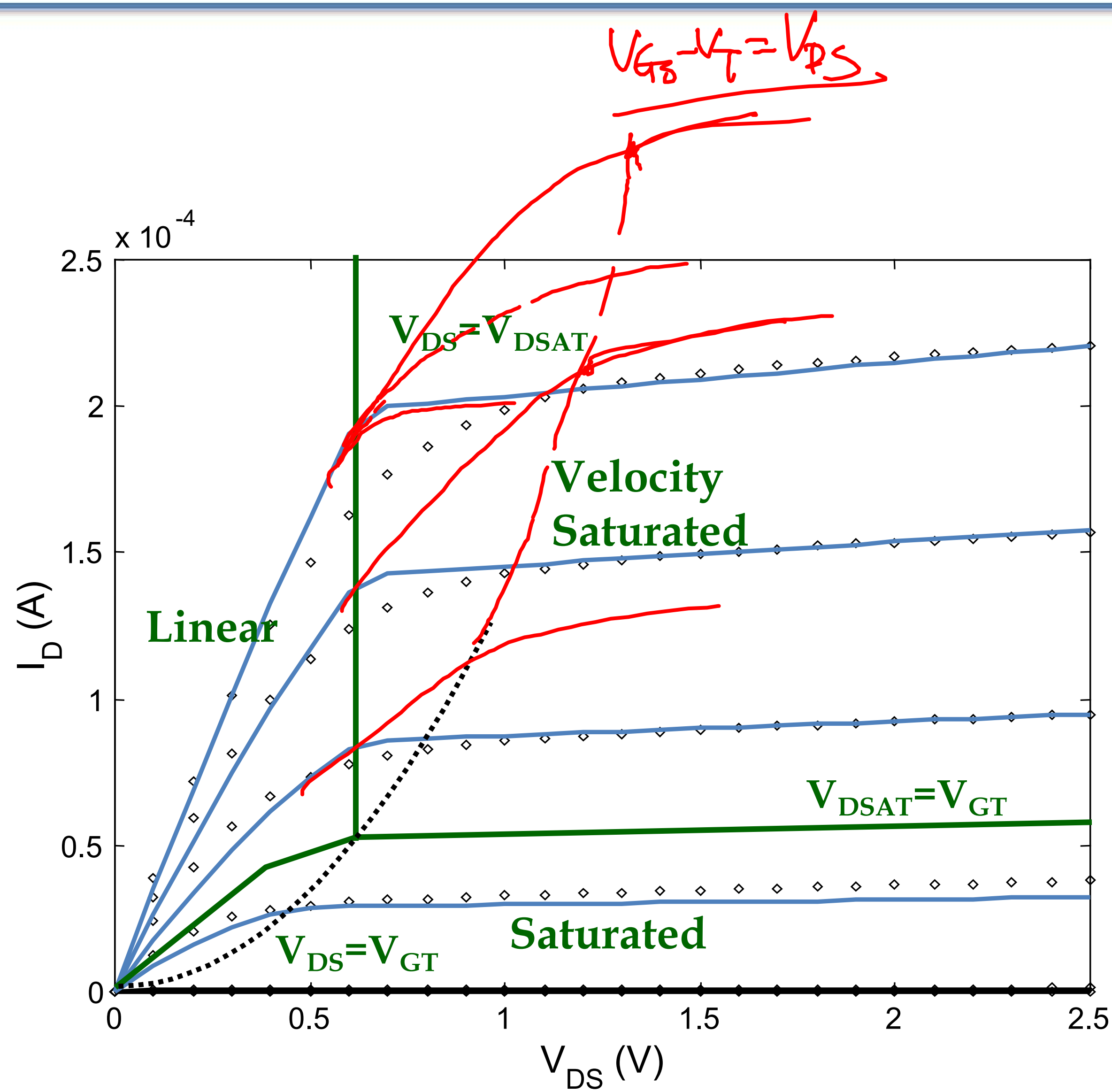
with $V_{min} = \min(V_{GT}, V_{DS}, V_{DSAT})$,

$V_{GT} = V_{GS} - V_T$,

λ

and $V_T = V_{T0} + \gamma(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F})$

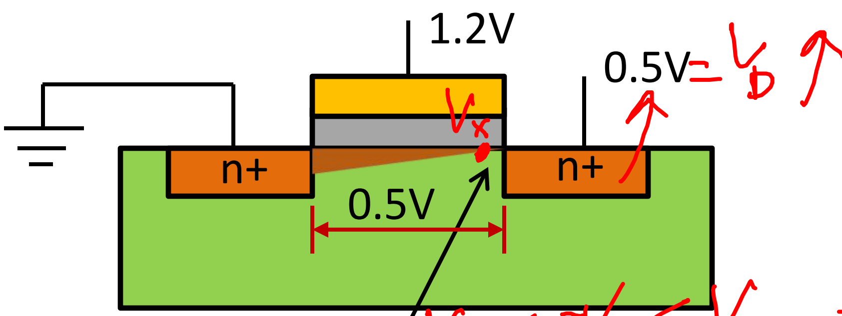
11.4 Velocity Saturation



Example

V_{ds} V_x

V_T=0.7 , V_{gs} = 1.2 V, $\boxed{V_{sat} = 1V}$

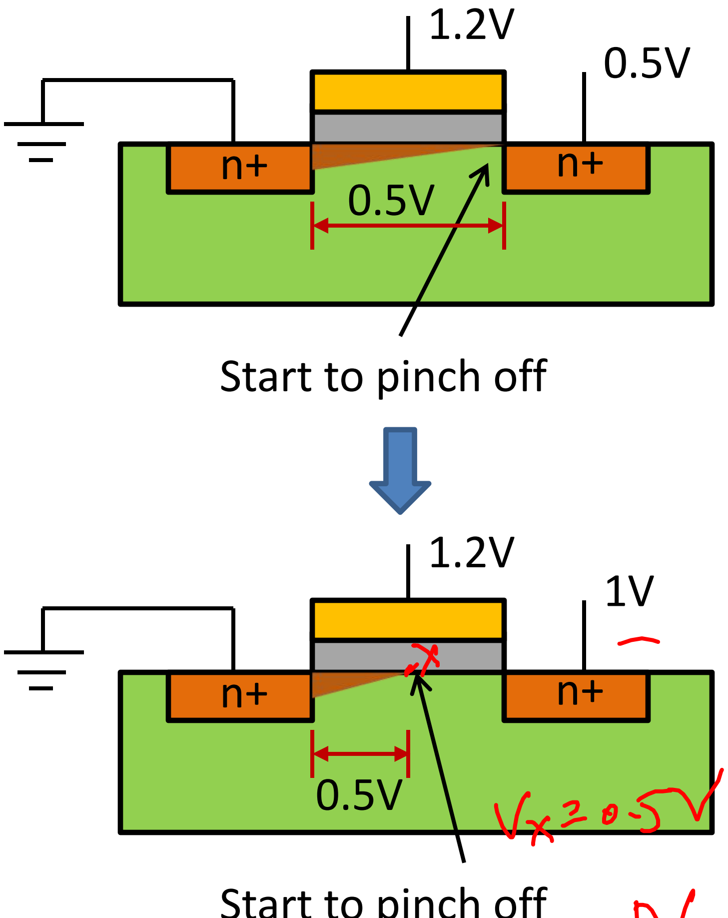


$V_x = 0.5V < V_{sat} = 1V$

Start to pinch off

Example

$V_T = 0.7$, $V_{gs} = 1.2$ V, $V_{sat} = 1$V

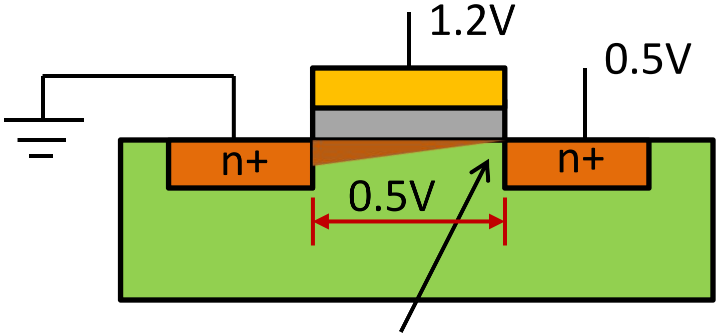


$V_x = 0.5V < V_{sat}$

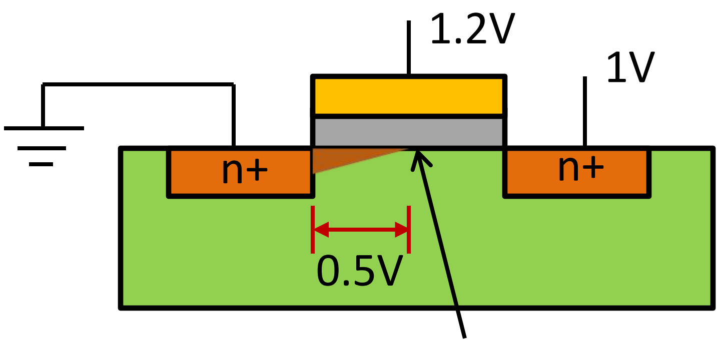
No velocity sat

Example

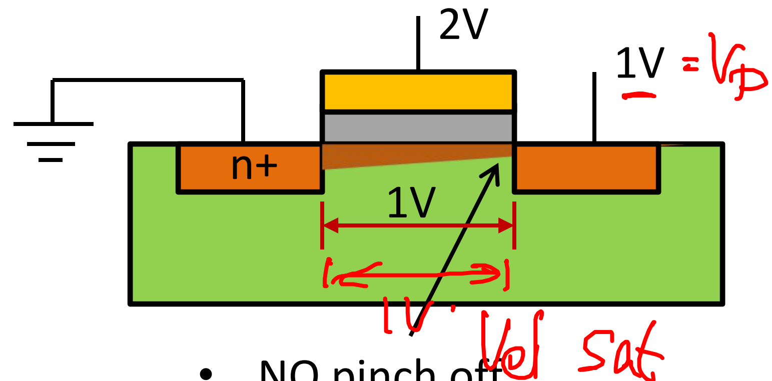
$V_T=0.7$, V_{gs} = 1.2 V, V_{sat} = 1V



Start to pinch off



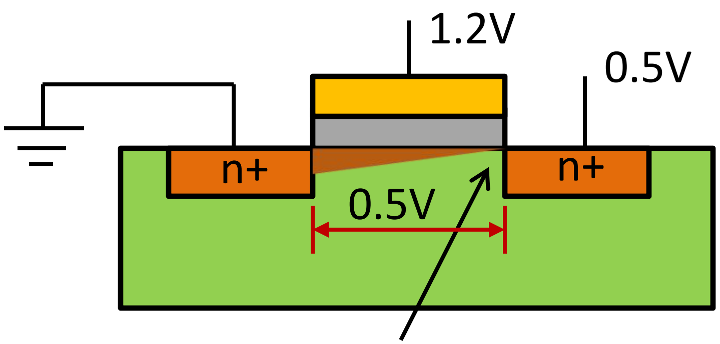
Start to pinch off



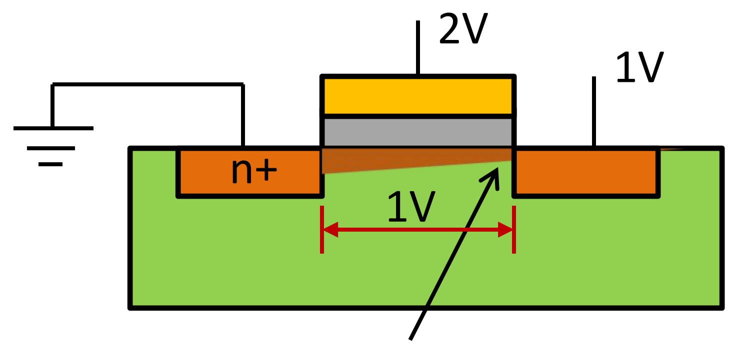
- NO pinch off
- Velocity saturation starts

Example

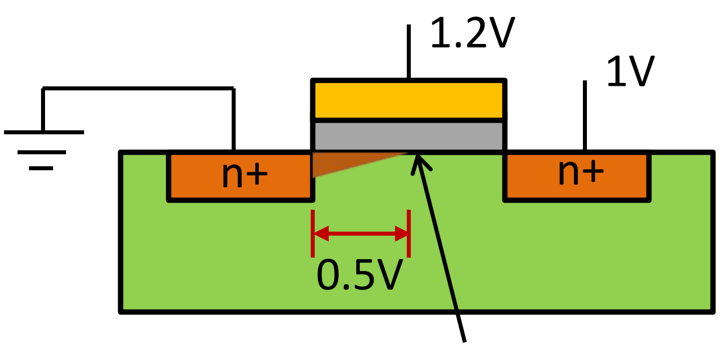
V_T=0.7 , V_{gs} = 1.2 V, V_{sat} = 1V



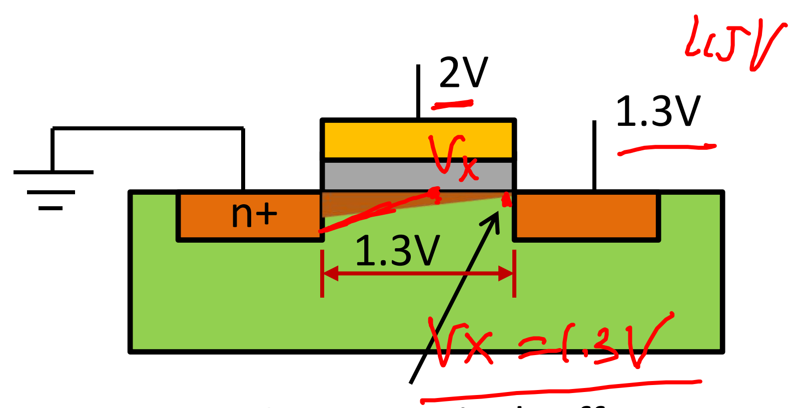
Start to pinch off



- NO pinch off
- Velocity saturation starts



Start to pinch off



- Starts to pinch off
- Velocity saturation

Outline

Nonideal Effects:

11.1 Channel length modulation

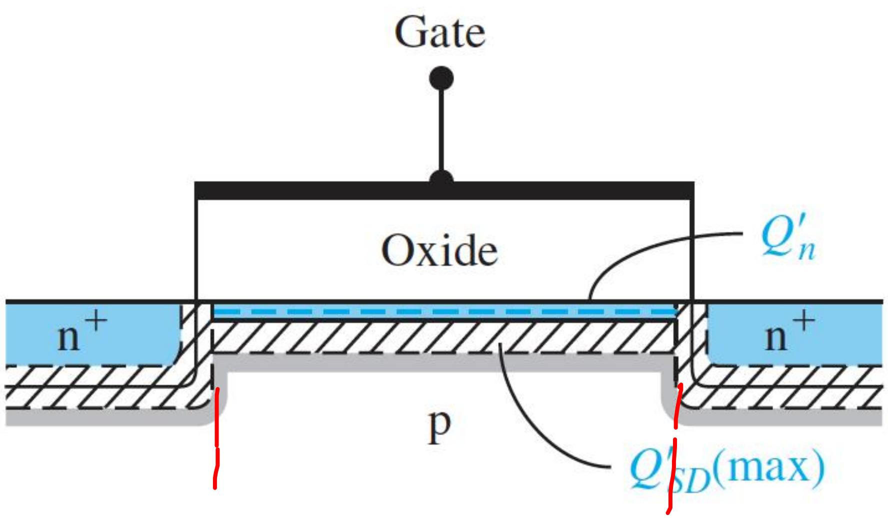
11.2 Conductance and transconductance

11.3 Subthreshold conduction

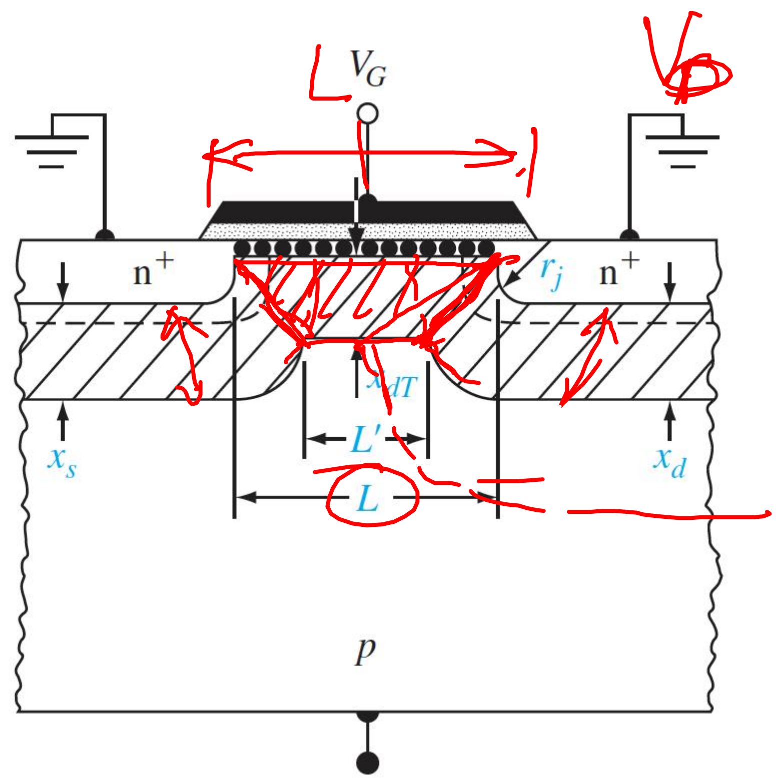
11.4 Velocity Saturation

11.5 Short Channel Effect

11.5 Short Channel Effect



A long channel device



A short channel device

11.5 Short Channel Effect

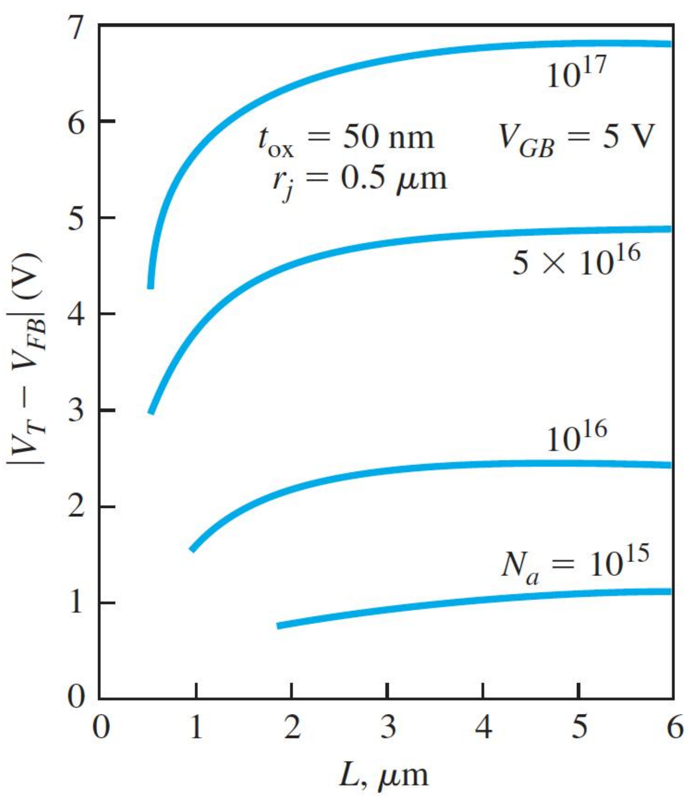


Figure 11.16 | Threshold voltage versus channel length for various substrate dopings. *(From Yau [26].)*

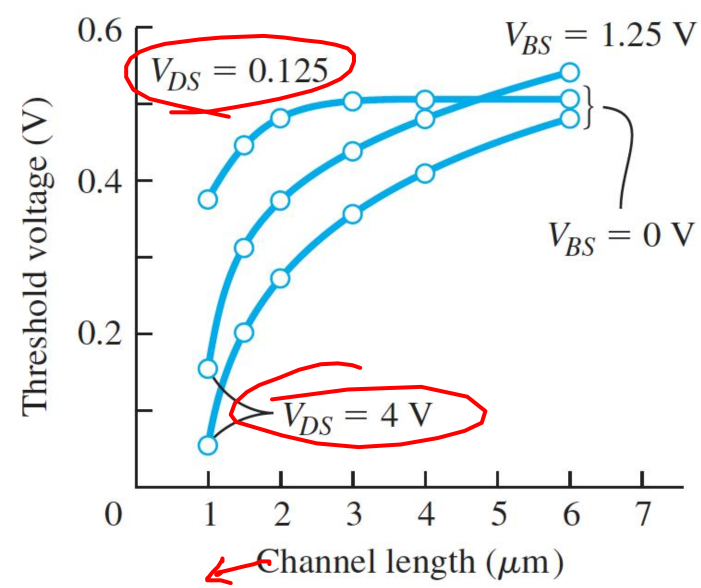
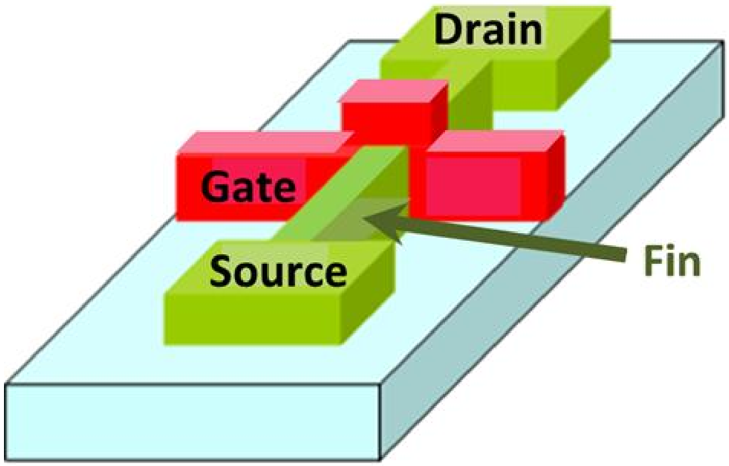


Figure 11.17 | Threshold voltage versus channel length for two values of drain-to-source and body-to-source voltage. *(From Yang [25].)*

11.5 Short Channel Effect



Fin Gate to suppress short channel effect