

1. (a) Determine the total number ($\#/cm^3$) of energy states in silicon between E_c and $E_c + 2kT$ at (i) $T = 300K$ and (ii) $T = 400K$. (b) Repeat part (a) for GaAs.

$$g_c(E) = \frac{2 \cdot 2\pi (2m^*)^{\frac{3}{2}}}{h^3} \sqrt{E - E_c}$$

$$\begin{aligned} \# \text{ energy states} &= \int_{E_c}^{E_c + 2kT} g_c(E) dE = \frac{2 \cdot 2\pi (2m^*)^{\frac{3}{2}}}{h^3} \int_{E_c}^{E_c + 2kT} \sqrt{E - E_c} dE \\ &= \frac{2 \cdot 2\pi (2m^*)^{\frac{3}{2}}}{h^3} \left. \frac{2}{3} (E - E_c)^{\frac{3}{2}} \right|_{E=E_c}^{E=E_c + 2kT} \end{aligned}$$

$$(a) \text{ For Si: } \# \text{ energy states} = \frac{4\pi (2 \times 1.08 \times 9.11 \times 10^{-31})^{\frac{3}{2}}}{(6.625 \times 10^{-34})^3} \cdot \frac{2}{3} (2 \times 1.38 \times 10^{-23} \times T)^{\frac{3}{2}}$$

$$(i) T = 300 \Rightarrow \# = 5.99 \times 10^{25}$$

$$(ii) T = 400 \Rightarrow \# = 9.22 \times 10^{25}$$

$$(b) \text{ For GaAs: } \# \text{ energy states} = \frac{4\pi (2 \times 0.067 \times 9.11 \times 10^{-31})^{\frac{3}{2}}}{(6.625 \times 10^{-34})^3} \cdot \frac{2}{3} (2 \times 1.38 \times 10^{-23} \times T)^{\frac{3}{2}}$$

$$(i) T = 300 \Rightarrow \# = 9.26 \times 10^{23}$$

$$(ii) T = 400 \Rightarrow \# = 1.42 \times 10^{24}$$

2. (a) For silicon, find the ratio of the density of states in the conduction band at $E = E_c + kT$ to the density of states in the valence band at $E = E_v - kT$. (b) Repeat part (a) for Ge.

$$(a) \text{ Density of states: } g(E)_1 = 2 \frac{2\pi (2m_c^*)^{\frac{3}{2}}}{h^3} \sqrt{E - E_c} \quad (E > E_c), \quad g(E)_2 = 2 \frac{2\pi (2m_v^*)^{\frac{3}{2}}}{h^3} \sqrt{E_v - E} \quad (E < E_v)$$

$$\Rightarrow \frac{g(E)_1}{g(E)_2} = \left(\frac{m_c^*}{m_v^*} \right)^{\frac{3}{2}}$$

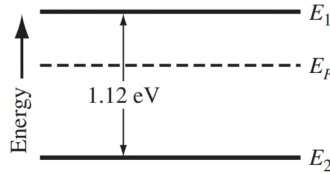
$$\text{For silicon (Si): } m_c^* = 1.08 m_0, \quad m_v^* = 0.56 m_0$$

$$\Rightarrow \text{The ratio is } \left(\frac{1.08}{0.56} \right)^{\frac{3}{2}} = 2.678$$

$$(b) \text{ Similarly, for germanium: } m_c^* = 0.55 m_0, \quad m_v^* = 0.37 m_0$$

$$\Rightarrow \text{The ratio is } \left(\frac{0.55}{0.37} \right)^{\frac{3}{2}} = 1.812$$

3. Consider the energy levels shown in Figure 1. Let $T = 300\text{K}$. (a) If $E_1 - E_F = 0.20\text{eV}$, determine the probability that an energy state at $E = E_1$ is occupied by an electron and the probability that an energy state at $E = E_2$ is empty. (b) Repeat part (a) if $E_F - E_2 = 0.40\text{eV}$.



(a)

$$f_F(E_1) = \frac{1}{1 + \exp\left(\frac{E_1 - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{0.2}{8.62 \times 10^{-5} \cdot 300}\right)} \approx \exp\left(\frac{-0.2}{0.02586}\right) = 4.38 \times 10^{-4}$$

$$f_F(E_2) = \frac{1}{1 + \exp\left(\frac{E_2 - E_F}{kT}\right)} \approx 3.54 \times 10^{-16}$$

(b)

Similarly: $f_F(E_1) = 8.45 \times 10^{-13}$ $f_F(E_2) = 1.96 \times 10^{-7}$

4. (a) The carrier effective masses in a semiconductor are $m_n^* = 1.21m_0$ and $m_p^* = 0.70m_0$. Determine the position of the intrinsic Fermi level with respect to the center of the bandgap at $T = 300\text{K}$. (b) Repeat part (a) if $m_n^* = 0.80m_0$ and $m_p^* = 0.75m_0$.

$$E_i = \frac{E_C + E_V}{2} + \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$\Rightarrow \text{(a)} E_i - \frac{E_C + E_V}{2} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right) = \frac{3}{4} \times 1.38 \times 10^{-23} \cdot 300 \cdot \ln \frac{0.7}{1.21} = -0.01062\text{eV}$$

$\Rightarrow E_i$ is 0.01062eV below the center of the bandgap

$$\text{(b)} E_i - \frac{E_C + E_V}{2} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right) = \frac{3}{4} \times 1.38 \times 10^{-23} \cdot 300 \cdot \ln \frac{0.75}{0.80} = -0.00126\text{eV}$$

$\Rightarrow E_i$ is 0.00126eV below the center of the bandgap

5. Semiconductor A has a band gap of 1eV , while semiconductor B has a band gap of 2eV . What is the ratio of the intrinsic carrier concentrations in the two materials (n_{iA}/n_{iB}) at 300K . Assume any differences in the carrier effective masses may be neglected.

$$\text{From } n_i = \sqrt{N_C N_V} e^{-\frac{E_g}{2kT}}$$

$$\text{we can know that: } \frac{n_{iA}}{n_{iB}} = \frac{\sqrt{N_{CA} N_{VA}} e^{-\frac{E_{gA}}{2kT}}}{\sqrt{N_{CB} N_{VB}} e^{-\frac{E_{gB}}{2kT}}} = e^{\frac{E_{gB} - E_{gA}}{2kT}} = e^{\frac{2-1}{2.8617 \cdot 10^{-5} \cdot 300}} = e^{1.935} = 2.3 \times 10^8$$

6. The value p_0 in Silicon at $T = 300\text{K}$ is $2 \times 10^{16} \text{cm}^{-3}$. (a) Determine $E_F - E_V$. (b) Calculate the value of $E_C - E_F$. (c) What is the value of n_0 ? (d) Determine $E_{Fi} - E_F$.

$$(a) \quad p_0 = N_V \exp\left(\frac{E_V - E_F}{kT}\right) \Rightarrow E_F - E_V = -\ln\left(\frac{p_0}{N_V}\right) \cdot kT = 0.162 \text{ eV}$$

$$(b) \quad E_C - E_F = E_C - E_V + E_V - E_F = E_G - (E_F - E_V) = 1.12 - 0.162 = 0.958 \text{ eV}$$

$$(c) \quad n_0 = \frac{n_i^2}{p_0} = \frac{(1.5 \times 10^{-10})^2}{2 \times 10^{16}} = 1.125 \times 10^4 \text{ cm}^{-3}$$

$$(d) \quad E_{Fi} - E_F = \frac{E_C + E_V}{2} - E_F = \frac{1}{2}(E_C - E_F) + \frac{1}{2}(E_V - E_F) = \frac{0.958 - 0.162}{2} = 0.398 \text{ eV}$$

7. The electron concentration in silicon at $T = 300\text{K}$ is $n_0 = 2 \times 10^5 \text{cm}^{-3}$. (a) Determine the position of the Fermi level with respect to the valence band energy level. (b) Determine p_0 . (c) Is it n- or p-type material?

$$(a) \quad n_0 = N_C \exp\left(\frac{E_F - E_C}{kT}\right) \Rightarrow E_F - E_C = kT \cdot \ln\left(\frac{n_0}{N_C}\right) \Rightarrow E_F - E_V = E_G + E_F - E_C =$$

$$= 1.12 + 300 \cdot 8.62 \times 10^{-5} \ln\left(\frac{2 \times 10^5}{2.8 \times 10^{19}}\right)$$

$$= 1.12 - 0.842 = 0.278 \text{ eV}$$

$$(b) \quad p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{-10})^2}{2 \times 10^5} = 1.125 \times 10^{15}$$

(c)

Since $p_0 \gg n_0$, we can know that it's an n-type material