RC3

Mid-term exam

- Next Thursday's class time
- Chapter 0-4
- Open-book
- Calculating and Conceptual questions

Clarification of HW2

- You can refer to appendix B in the RC files on Canvas
- Some "constants" may not be self consistent with each other. Don't worry. Start with any constant you like.

Clarification of Appendix B

Use density of state effective mass will be fine

Effective mass $\left(\dfrac{m^*}{m_0}\right)$

Electrons	$m_l^* = 0.98$	0.067	1.64
	$m_t^* = 0.19$		0.082
Holes	$m_{lh}^* = 0.16$	0.082	0.044
	$m_{hh}^* = 0.49$	0.45	0.28
Density of states effective mass			
Electrons $\left(\dfrac{m_{dn}^*}{m_o}\right)$	1.08	0.067	0.55
Holes $\left(\dfrac{m_{dp}^*}{m_o}\right)$	0.56	0.48	0.37
Conductivity effective mass			
Electrons $\left(\dfrac{m_{cn}^*}{m_o}\right)$	0.26	0.067	0.12
Holes $\left(\dfrac{m_{cp}^*}{m_o}\right)$	0.37	0.34	0.21

If you are really curious about why there's ml and mt for silicon…

the result we would predict from Fig. 3–15b.

In Eq. (3–16a), m_n^* is the density-of-states effective mass for electrons. To illustrate how it is obtained from the band curvature effective masses mentioned in Section 3.2.2, let us consider the 6 equivalent conduction band minima along the X-directions for Si. Looking at the cigar-shaped equi-energy surfaces in Fig. 3–10b, we find that we have more than one band curvature to deal with in calculating effective masses. There is a longitudinal effective mass m_l along the major axis of the ellipsoid, and the transverse effective mass m_t along the two minor axes. Since we have $(m_n^*)^{3/2}$ appearing in the density-of-states expression Eq. (3–16a), by using dimensional equivalence and adding contributions from all 6 valleys, we get

$$(m_n^*)^{3/2} = 6(m_l m_t^2)^{1/2} \qquad (3\text{–}16b)$$

It can be seen that this is the geometric mean of the effective masses.

Fig. 1 shows the six constant energy surfaces for the energies just above the conduction band edge E_C in silicon. The surface is ellipsoidal and is described by two effective masses; m_t^* for the transverse mass and m_l^* for the longitudinal mass. For the ellipsoid along the z-axis with the center at (k_{x0}, k_{y0}, k_{z0}), for example, the surface of constant energy is

$$E(\mathbf{k}) = E_C + \frac{\hbar^2(k_x - k_{x0})^2}{2m_t^*} + \frac{\hbar^2(k_y - k_{y0})^2}{2m_t^*} + \frac{\hbar^2(k_z - k_{z0})^2}{2m_l^*}$$

Show that

$$\frac{m_l^*}{m_t^*} = \left(\frac{\text{Length of the ellipsoid along the axis of revolution}}{\text{Maximum width of the ellipsoid perpendicular to the axis of revolution}}\right)^2$$

Hint: The standard equation of an ellipsoid centered at (x_0, y_0, z_0) of a Cartesian coordinate system and aligned with the axes is

$$\frac{(x - x_0)^2}{\alpha^2} + \frac{(y - y_0)^2}{\beta^2} + \frac{(z - z_0)^2}{\gamma^2} = 1$$

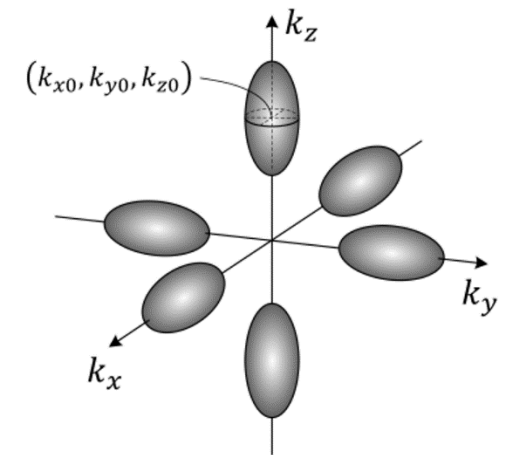where the semi-axes are of lengths α, β, and γ.

Fig. 1 Ellipsoidal constant energy surfaces in a silicon crystal.

HW1-P1-look at the solution

1) Consider a particle of mass m confined by the following potential:

$$V(x) = \begin{cases} \infty & x \leq -\frac{L}{2} \\ 0 & -\frac{L}{2} < x < \frac{L}{2} \\ \infty & x \geq \frac{L}{2} \end{cases}$$

a) What is the wave function for $|x| > L/2$?

b) Consider the time independent Schrodinger equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$$

What quantity from classical mechanics do the first and second quantity in the left side of the equation most closely represent?

c) First, independent of the boundary conditions, what is the most general solution that solves the Schrodinger equation above in the region where the potential vanishes.

d) State the boundary conditions that the wave function must satisfy.

e) Derive and normalize expressions for the allowed energies and energy eigenfunctions.

f) Sketch the first three eigenfunctions.

HW1-P7-Note the unit
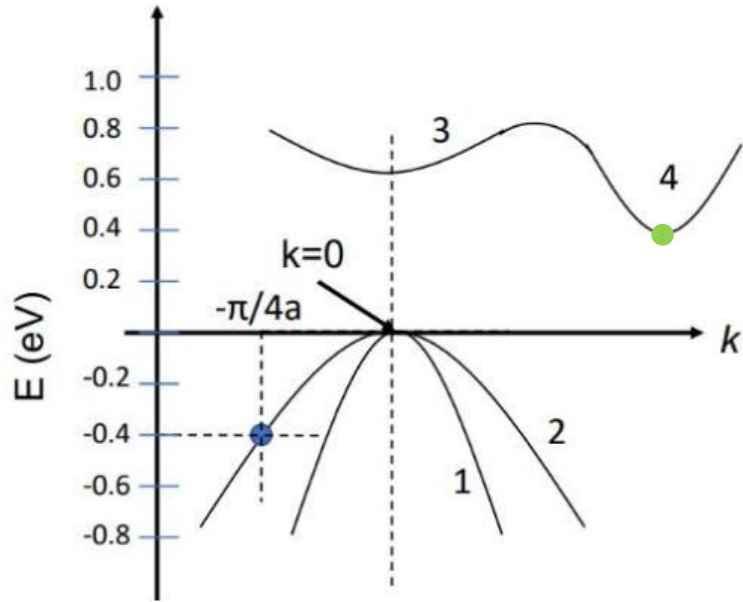
What if I ask you the momentum of electron state at the green point ?
It will be 0

Class problem-4 states not 8 if not consider spin

Problem Example #1

1. In an infinite quantum well, the wavenumber k and energy E is quantized as shown in Figure 1. Please answer the following questions:

a) Write the static wavefunction of the dot ($k = \frac{3}{4}k_0$) that the red arrow is pointing to. Find the wavelength of this wavefunction.

b) If all the states are filled with electrons, how many electrons can be filled in the figure below? The electron spin is not considered.

c) If the effective mass of this quantum system is m_n^*, can this system allow an electron to have a quantized energy of $\frac{\hbar^2 k_0^2}{9m_n^*}$? If yes, mark the state of this electron in the figure below. How about a quantized energy of $\frac{\hbar^2 k_0^2}{8m_n^*}$? If yes, mark the state of this electron in the figure below.
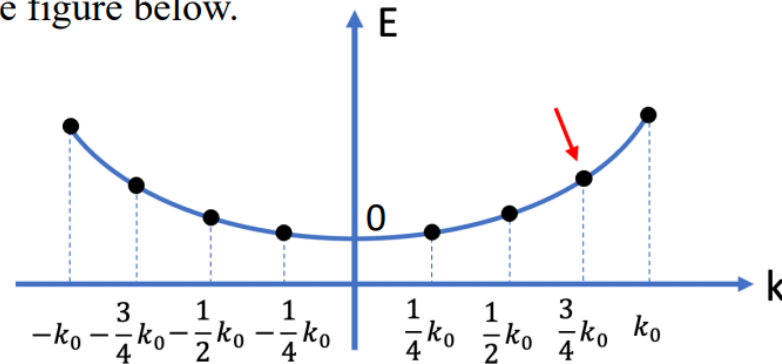
Statistics—applicable to both intrinsic and extrinsic semiconductor
You can view fermi energy as a flag indicating the state of the semiconductor

Density of States

$$g_C(E) = \frac{\sqrt{2}m_n^{*\frac{3}{2}}(E - E_C)^{1/2}}{\pi^2 \hbar^3}$$

$$g_V(E) = \frac{\sqrt{2}m_p^{*\frac{3}{2}}(E_V - E)^{1/2}}{\pi^2 \hbar^3}$$

$$\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k^2}$$

Fermi Distribution Function

$$f(E) = \frac{1}{1 + \exp\left(\dfrac{E - E_f}{kT}\right)}$$

Boltzmann approximation:

$$f(E) = \exp\left(-\frac{E - E_f}{kT}\right)$$

Carrier Statistics

$$n = \int g_C(E)f(E)dE$$

$$p = \int g_V(E)(1 - f(E))dE$$

Carrier densities relative to E_C, E_V
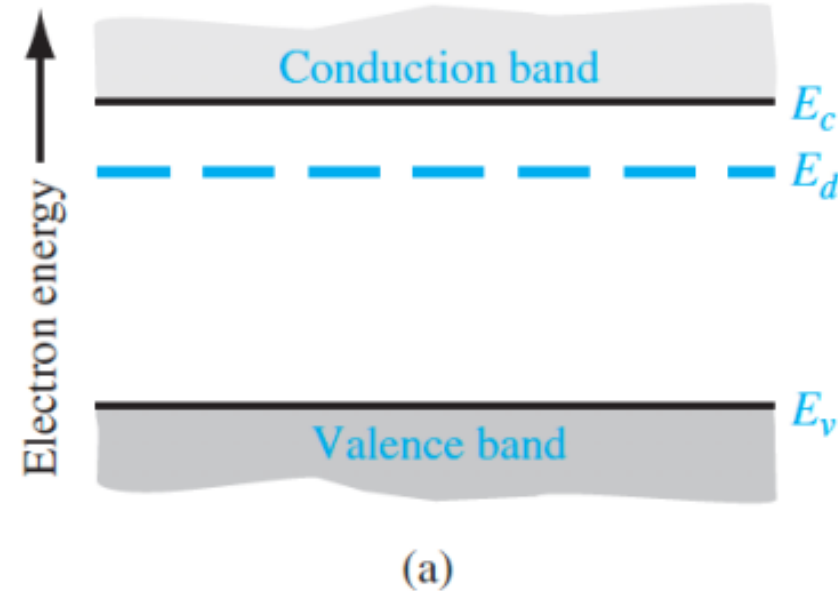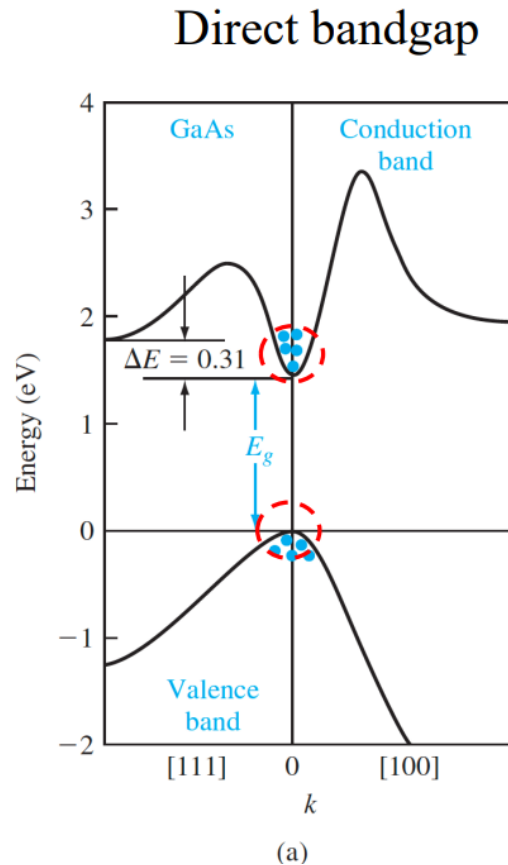
$$n = N_C \exp\left(\frac{E_f - E_C}{kT}\right)$$

$$p = N_V \exp\left(\frac{E_V - E_f}{kT}\right)$$

$$N_C = 2\left(\frac{m_n^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$N_V = 2\left(\frac{m_p^* kT}{2\pi\hbar^2}\right)^{3/2}$$

Note the difference between band structure and band diagram

- E-K relationship is band structure
- E-X relationship is band diagram

Direct bandgap

GaAs

Conduction band

$\Delta E = 0.31$

E_g

Valence band

Energy (eV)

[111]   0   [100]

k

(a)

Electron energy

Conduction band

E_c

E_d

E_v

Valence band

(a)

Intrinsic semiconductor—no dopants

$$n = p = ni$$
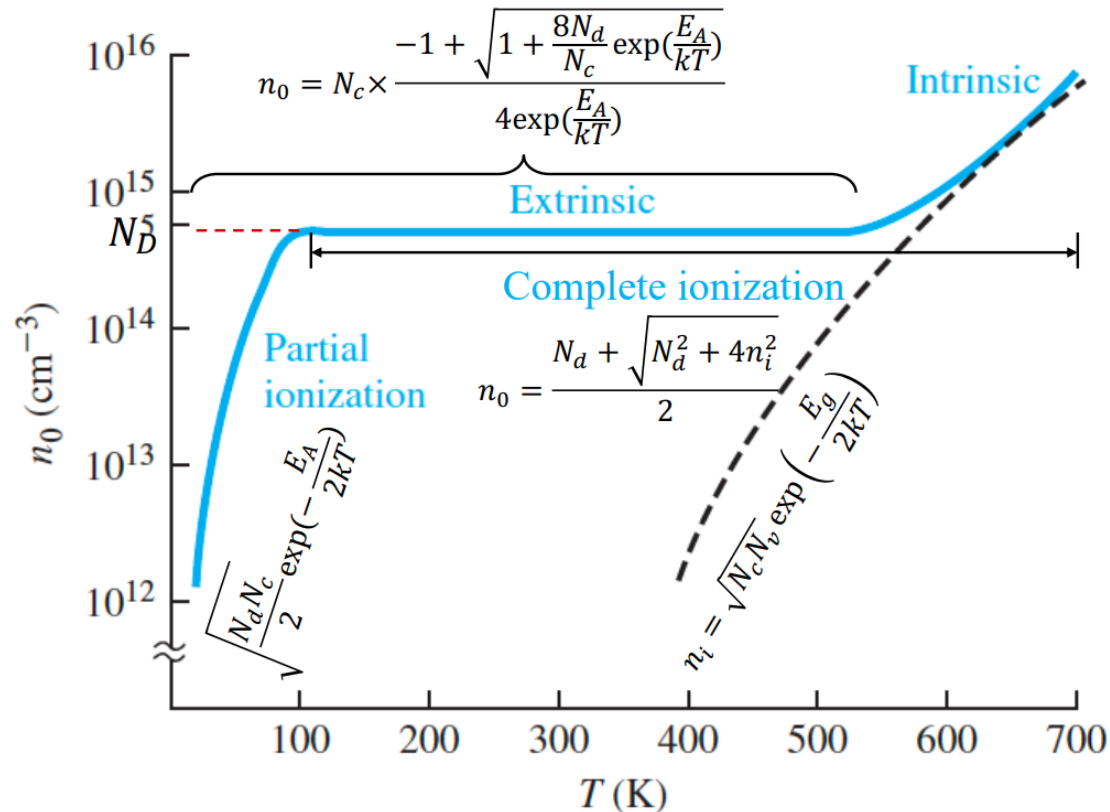
Intrinsic Carrier Concentration

$$n_i = 2\left(\frac{kT}{2\pi\hbar^2}\right)^{3/2} \left(m_n^* m_p^*\right)^{3/4} \exp\left(-\frac{E_G}{2kT}\right)$$

$$n_i = \sqrt{N_C N_V}\, \exp\left(-\frac{E_G}{2kT}\right)$$

$$E_i = \frac{E_C + E_V}{2} + \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$$

Extrinsic semiconductor—with dopants

- N_d^+: the ionized dopant (contribute electrons)
- N_d: both ionized and not ionized dopant
- There are three cases depending on T

Extrinsic semiconductor—statistics applicable to all cases

- N_d^+: concentration of the ionized dopant (contribute electrons)
- N_d: concentration of both ionized and not ionized dopant

$$np = n_i^2$$

$$n - N_D^+ - p + N_A^- = 0$$

T: 300K (usual case)-complete ionization

- Nearly all the dopants are ionized
- $N_d^+ = N_d$
- Solving this equation $np = n_i^2$
$$n - N_D^+ - p + N_A^- = 0$$ will give you

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

$$p = \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2}$$

T: High temperature

- Still complete ionization, but now n_i will be much greater than $N_d^+=N_d$. Note n_i is dependent on temperature. So the semiconductor behaves more like an intrinsic semiconductor.

- So you can refer to the previous intrinsic semiconductor statistics

Intrinsic Carrier Concentration

$$n_i = 2\left(\frac{kT}{2\pi\hbar^2}\right)^{3/2}\left(m_n^* m_p^*\right)^{3/4}\exp\left(-\frac{E_G}{2kT}\right)$$

$$n_i = \sqrt{N_C N_V}\exp\left(-\frac{E_G}{2kT}\right)$$

T: low temperature- not complete ionization

- N_d^+ is not equal to N_d in this case
- Refer to the slide for detailed calculation

Equilibrium electron and hole concentration

$$n_0 = N_d^+$$

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) = N_D^+ = \frac{N_d}{1 + 2\exp\left(\frac{E_F - E_D}{kT}\right)}$$

⇩

$$\exp\left(\frac{E_F - E_c}{kT}\right) = \frac{n_0}{N_c}$$

$$n_0 = \frac{N_d}{1 + 2\exp\left(\frac{E_c - E_d}{kT}\right)\exp(\frac{E_F - E_c}{kT})}$$

$$= \frac{n_0}{N_c}$$

Equilibrium electron and hole concentration

$$2\exp\left(\frac{E_A}{kT}\right)n_0^2 + N_c n_0 - N_d N_c = 0$$

$$n_0 = N_c \times \frac{-1 + \sqrt{1 + \frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})}$$