# VE320 – Summer 2024

# Introduction to Semiconductor Devices

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## Chapter 7 The pn Junction

# Outline

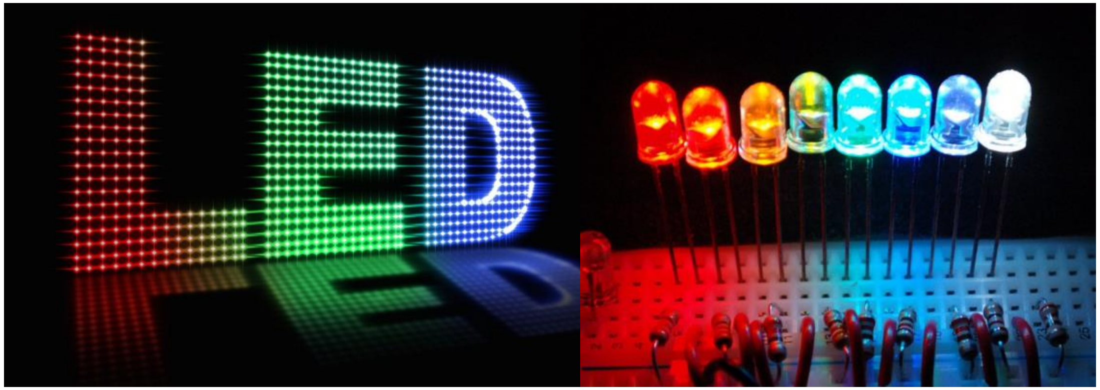
7.0 Introduction to semiconductor devices

7.1 Basic structure of the pn junction

7.2 Zero applied bias

7.3 Reverse applied bias

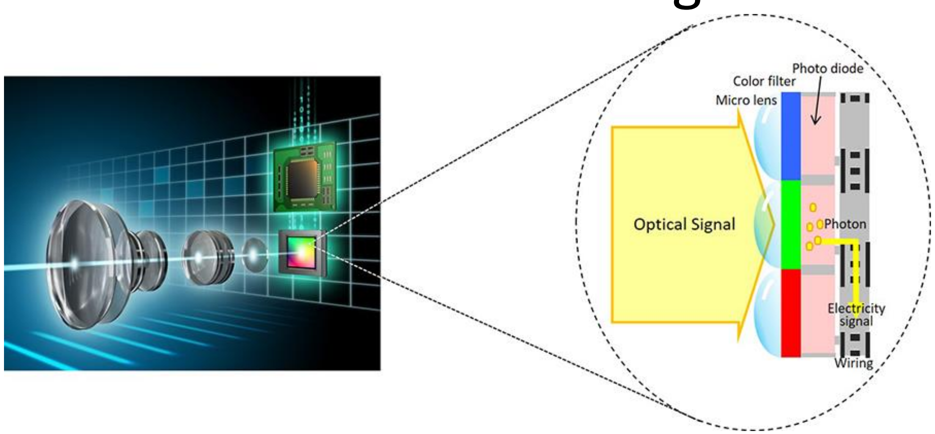
# 7.0 Introduction to semiconductor devices

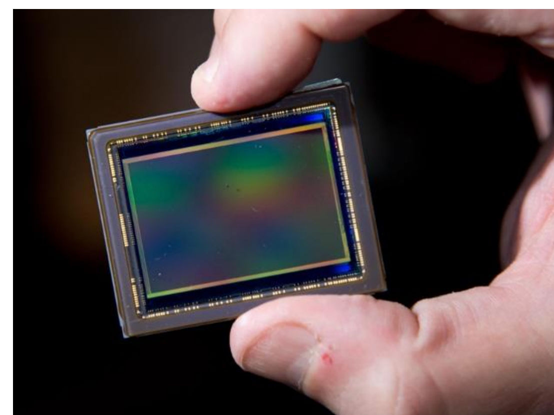



Light emitting diodes

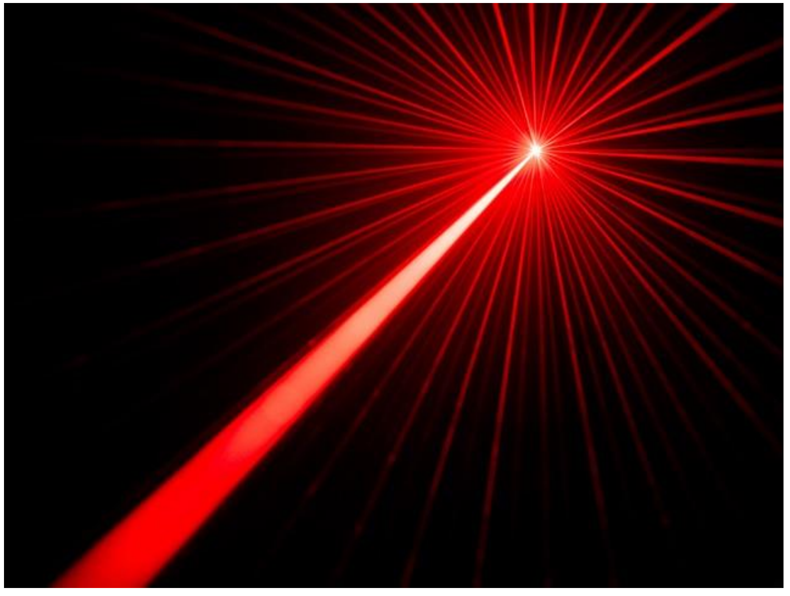
Cold light source

Photodetector: CMOS image sensor



# 7.0 Introduction to semiconductor devices



Semiconductor lasers

Solar cells

# Outline

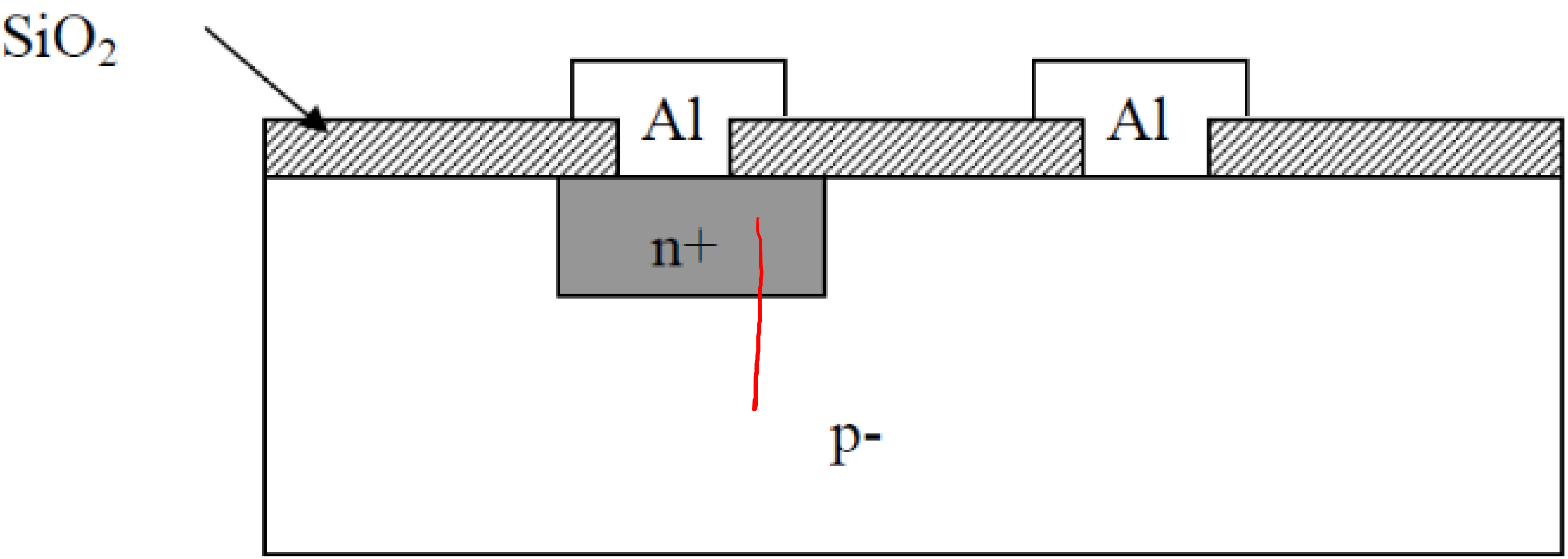
7.0 Introduction to semiconductor devices

**7.1 Basic structure of the pn junction**

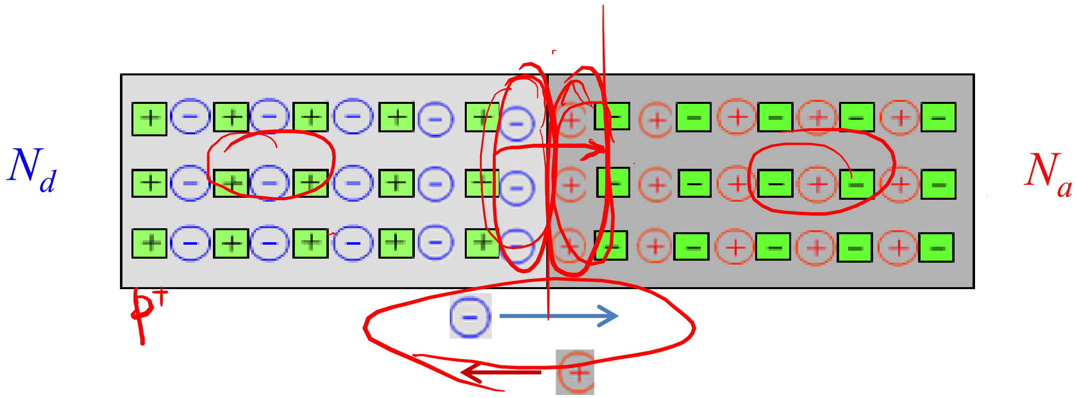
7.2 Zero applied bias

7.3 Reverse applied bias

# 7.1 Basic structure of pn junction

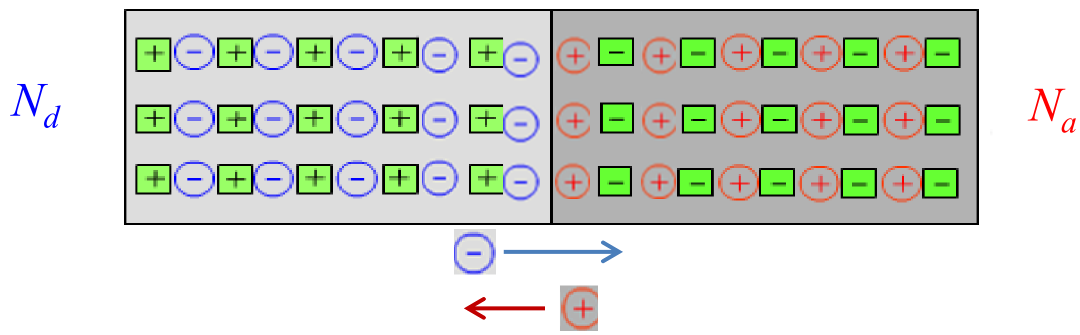


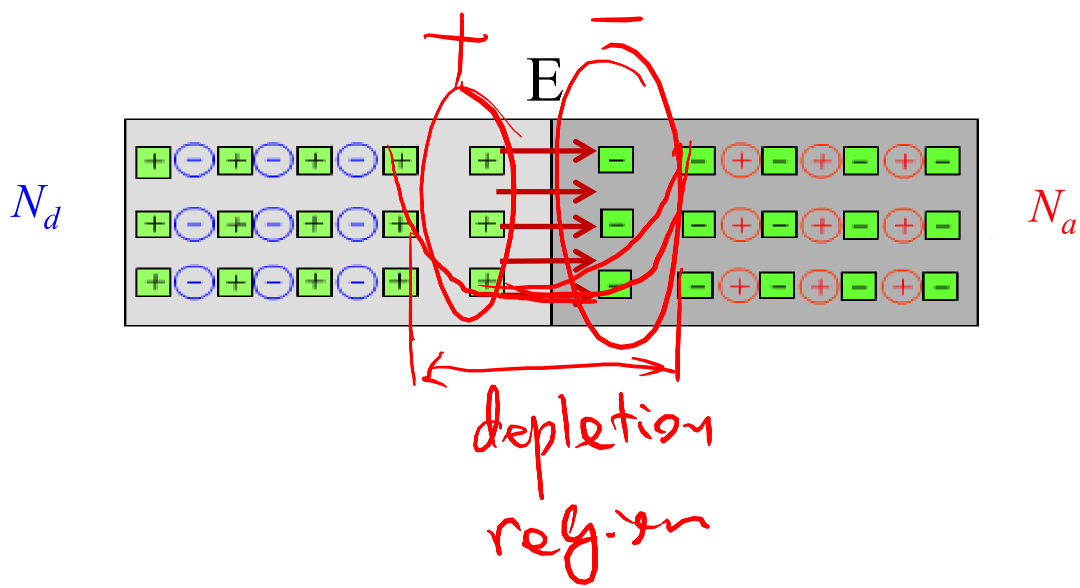
# 7.1 Basic structure of pn junction




$$\boxed{n_0 \cdot p_0 = n_i^2}$$

# 7.1 Basic structure of pn junction



# Outline

7.1 Basic structure of the pn junction

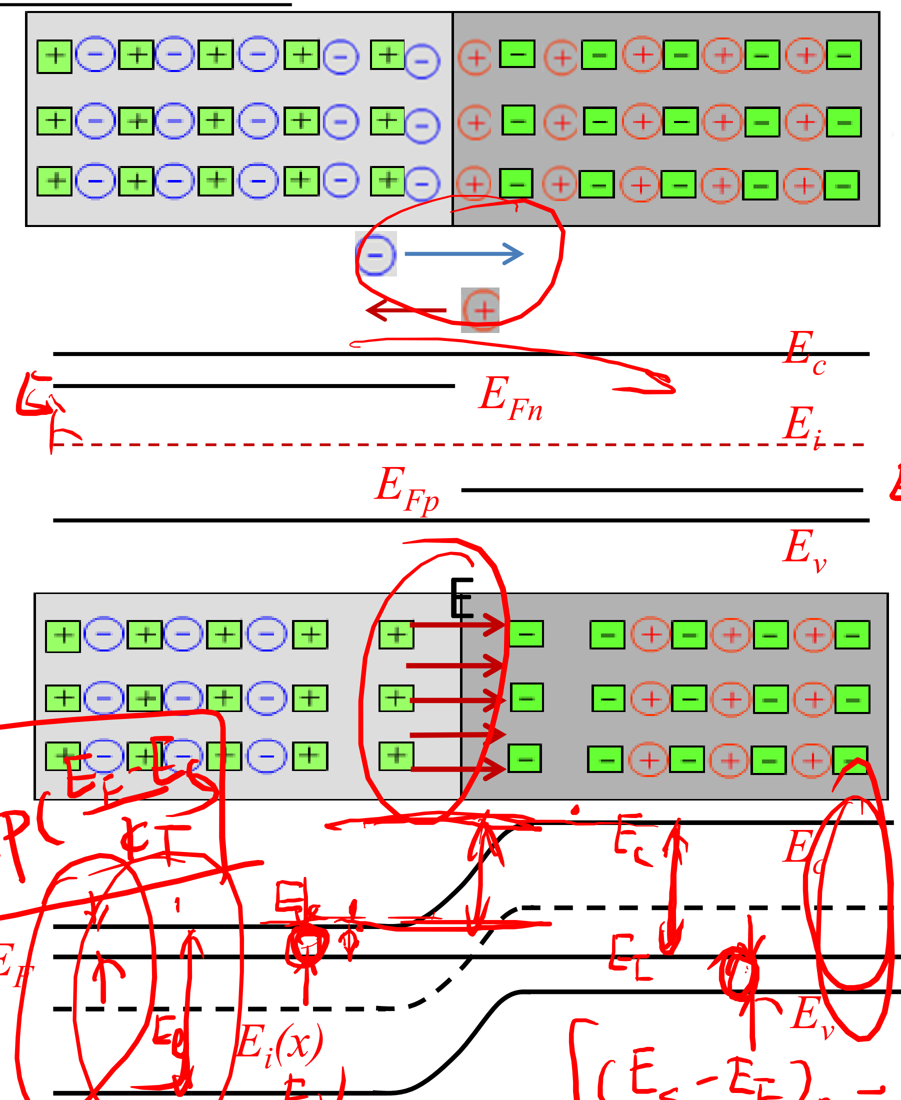
**7.2 Zero applied bias**

7.3 Reverse applied bias

# 7.2 Zero applied bias

$$n(x) = N_c \exp\left(\frac{E_F - E_c(x)}{kT}\right)$$

## Built-in potential barrier




$$\frac{dE_F(x)}{dx} = 0$$

for all $x$'s

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

$$p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

$$\left[(E_c - E_F)_p - (E_c - E_F)_n\right] / q$$

# 7.2 Zero applied bias

Built-in potential barrier

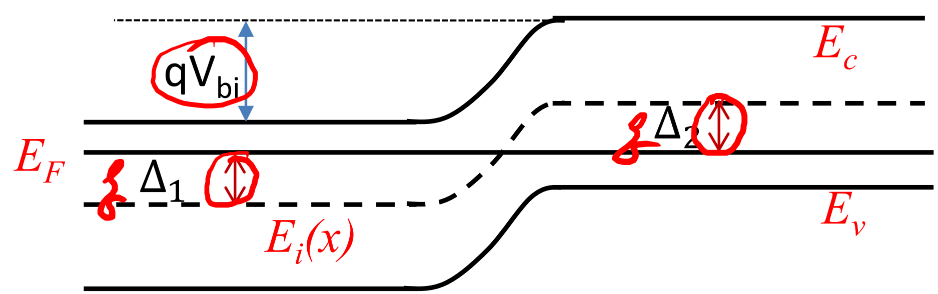
n-side: $n_{n0} = n_i \exp\left(\frac{E_F - E_i}{kT}\right) = n_i \exp\left(\frac{q\Delta_1}{kT}\right)$

p-side: $p_{p0} = n_i \exp\left(\frac{E_i - E_F}{kT}\right) = n_i \exp\left(\frac{q\Delta_2}{kT}\right)$

$$\underline{V_{bi}} = \Delta_1 + \Delta_2 = \frac{kT}{q}\ln\frac{n_{n0}}{n_i} + \frac{kT}{q}\ln\frac{p_{p0}}{n_i} = \frac{kT}{q}\ln\frac{n_{n0}\cdot p_{p0}}{n_i^2}$$

Example: $N_a = 10^{17}\,cm^{-3}$ $N_d = 10^{17}\,cm^{-3}$ $= \frac{kT}{q}\ln\frac{N_a N_d}{n_i^2}$

$$V_{bi} = 0.0259 \times \ln\frac{10^{17}\times 10^{17}}{(1.5\times 10^{10})^2} = 0.84V$$



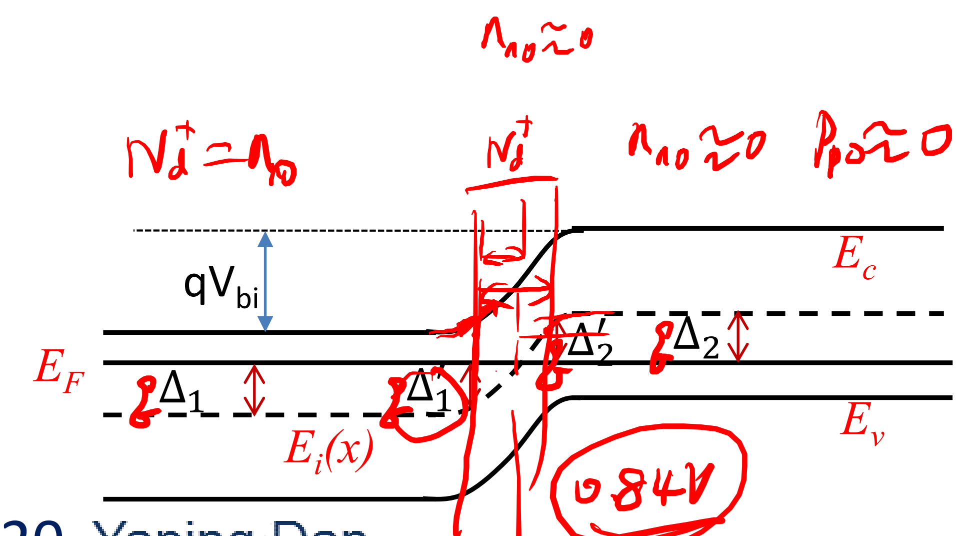
# 7.2 Zero applied bias

## Charge carrier distribution

$$n(x) = n_i \exp\left[\frac{E_F - E_i(x)}{kT}\right] = n_i \exp\left(\frac{\Delta_1(x)}{kT}\right) = N_d \exp\left(-\frac{0.1}{0.0259}\right) \approx \frac{N_d}{50}$$

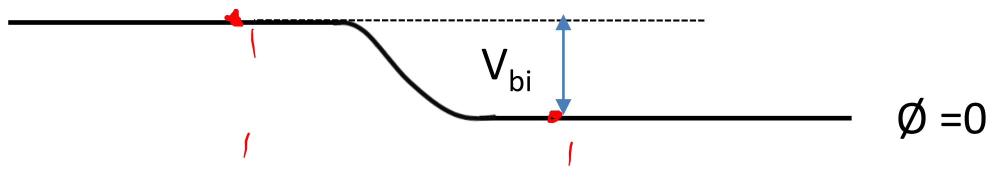
0.078eV

$$p(x) = n_i \exp\left[\frac{E_i(x) - E_F}{kT}\right] = N_a \exp\left(-\frac{0.1eV}{0.0259eV}\right) = \frac{N_a}{50}$$

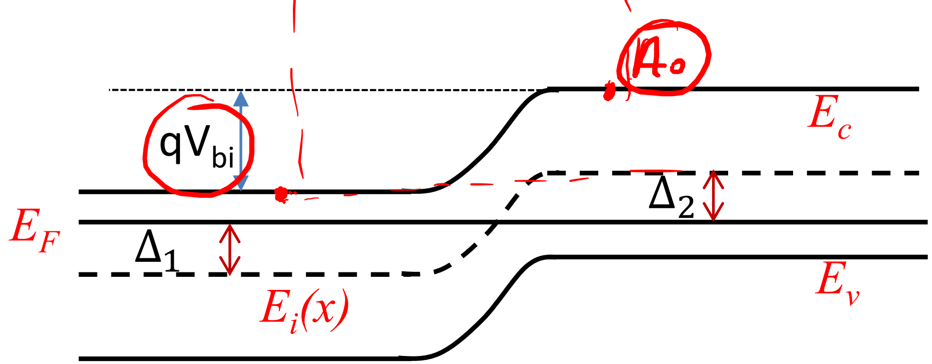


# 7.2 Zero applied bias

## Potential profile




Potential profile

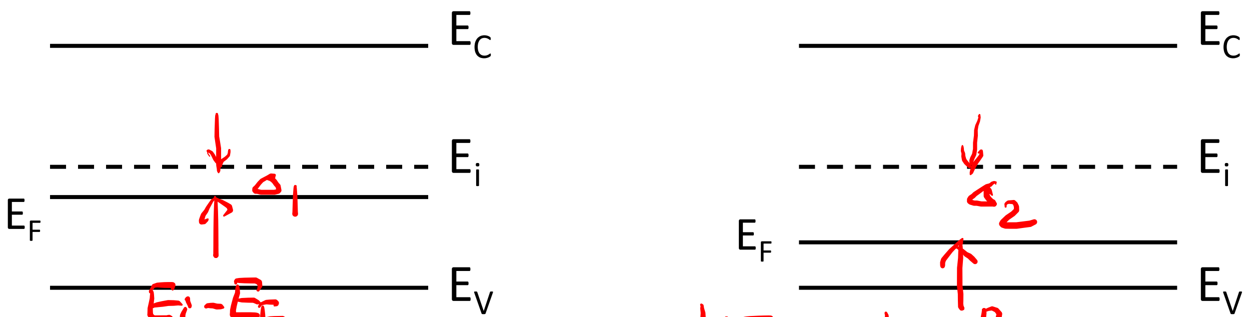



Energy band diagram

# Check your understanding

## Problem Example #1

Two pieces of p-type silicon are in contact. The doping concentrations are $10^{16}$ cm$^{-3}$ and $10^{18}$ cm$^{-3}$. Calculate the built-in potential between these two pieces of silicon and plot the energy band bending diagram.




$$p_0 = n_i \exp\left(\frac{E_i - E_F}{kT}\right) \Rightarrow E_i - E_F = kT \ln\frac{p_0}{n_i} = 0.0259 \times \ln\frac{10^{16}}{1.5\times10^{10}}$$

$$= 0.0259 \times 13.4 = 0.347eV$$

$$\Delta_1 = 0.347eV$$

$E_F$

$$\Delta_2 = kT \ln\frac{10^{18}}{1.5\times10^{10}} = 0.259 \times 18 = 0.466eV$$

# 7.2 Zero applied bias

$N_d^+ + p_0 = n_0 + N_a^-$

3rd time approximation

Poisson's equation

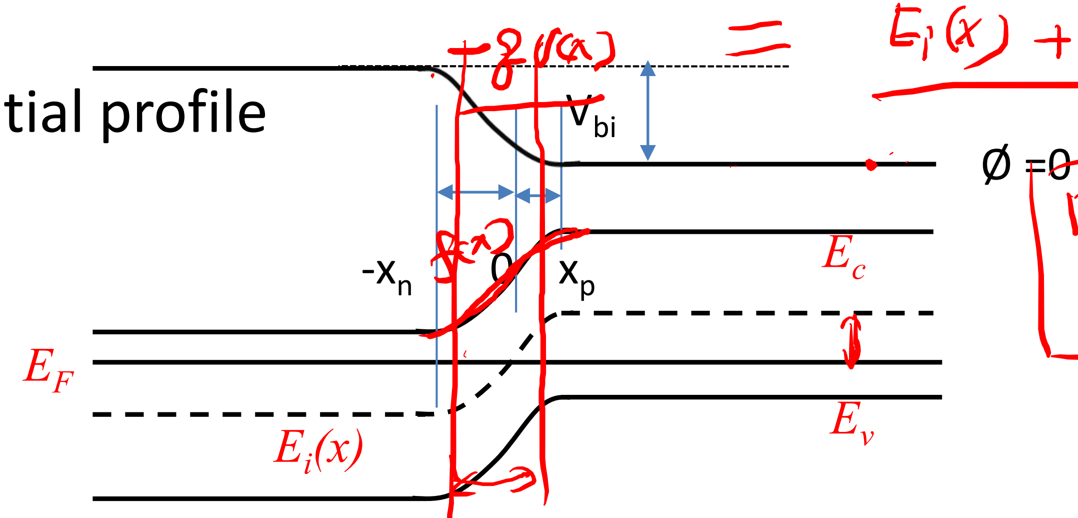
$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon} = -\frac{q}{\varepsilon}\left[N_d^+(x) + p(x) - N_a^-(x) - n(x)\right]$$

$$= -\frac{q}{\varepsilon}\left[N_d(x) - N_a(x) + n_i\exp\left(\frac{E_i(x)-E_F}{kT}\right) - n_i\exp\left(\frac{E_F-E_i(x)}{kT}\right)\right]$$

$$= -\frac{q}{\varepsilon}\left[N_d(x) - N_a + n_i\exp\left(\frac{-qV(x) - E_F + \Delta}{kT}\right) - n_i\exp\left(\frac{E_F + qV(x)}{kT}\right)\right]$$

$-qV(x) = E_i(x) + \Delta$

Potential profile




$\boxed{\text{No analytical solution}}$

# 7.2 Zero applied bias

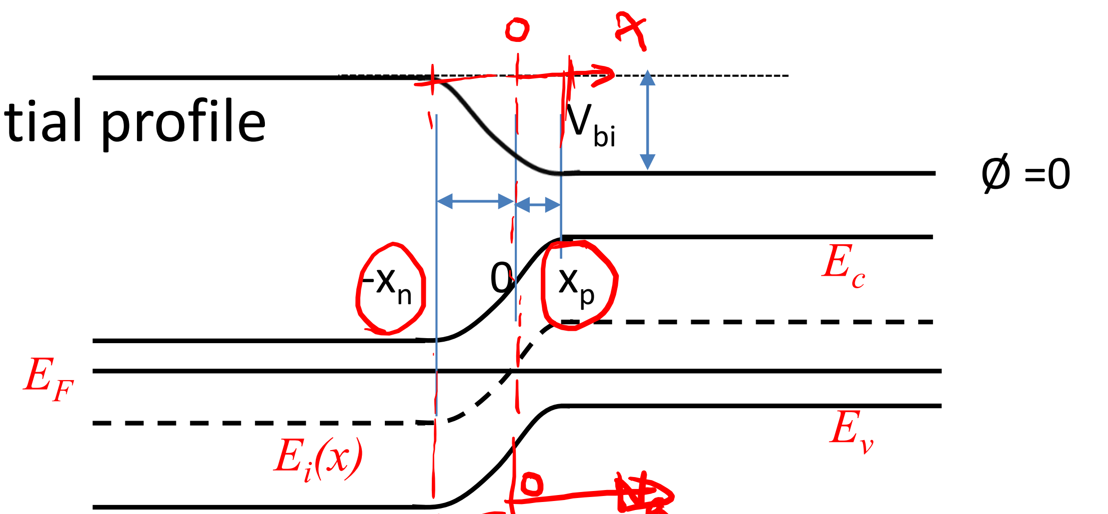
$$\mathcal{E} = -\frac{dV(x)}{dx}$$

Poisson's equation

$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon} = -\frac{q}{\varepsilon}\left[N_d(x) - N_a(x)\right] = \begin{cases} \frac{q}{\varepsilon} N_a & 0 \le x \le x_p \\ -\frac{q}{\varepsilon} N_d & -x_n \le x < 0 \end{cases}$$

$$\int \frac{d^2V(x)}{dx^2}\,dx = \frac{dV(x)}{dx} = \begin{cases} +\frac{q}{\varepsilon} N_a x + C_1 & 0 \le x \le x_p \\ -\frac{q}{\varepsilon} N_d \cdot x + C_2 & -x_n \le x < 0 \end{cases}$$

Potential profile



# 7.2 Zero applied bias

$-\frac{q}{\varepsilon}N_a x_p + A_1 = 0 \Rightarrow A_1 = \frac{q}{\varepsilon}N_a x_p$

Third time approximation

$-\frac{q}{\varepsilon}N_d x_n + A_2 = 0 \Rightarrow A_2 = \frac{q}{\varepsilon}N_d x_n$

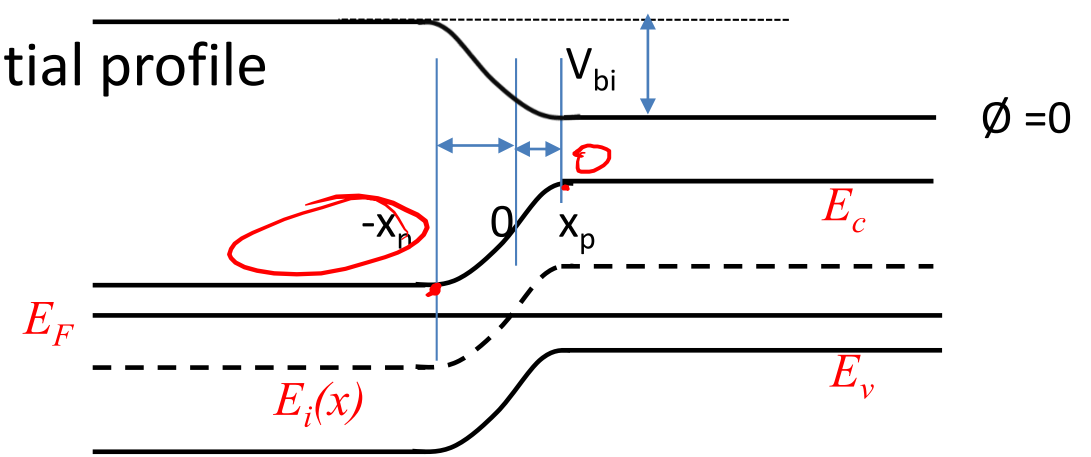
Poisson's equation

$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon}$$

$$E = -\frac{dV(x)}{dx} = \begin{cases} -\frac{q}{\varepsilon}N_a x + A_1 & 0 \le x \le x_p \\ \frac{q}{\varepsilon}N_d x + A_2 & -x_n \le x < 0 \end{cases}$$

Boundary condition $E(x = x_p) = 0$ $E(x = -x_n) = 0$

Potential profile



# 7.2 Zero applied bias

Poisson's equation

$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon}$$

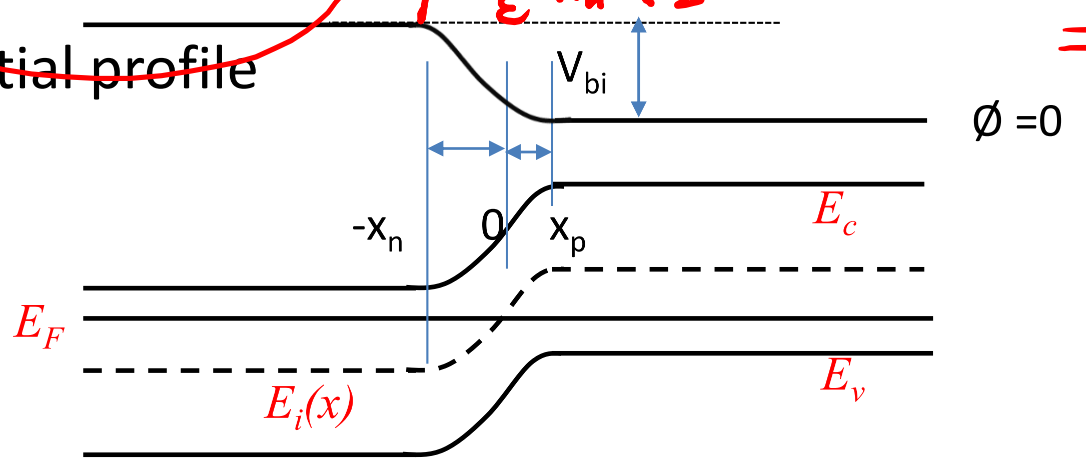
$$E = -\frac{dV(x)}{dx} = \begin{cases} -\frac{q}{\varepsilon} N_a x + \frac{q}{\varepsilon} N_a \cdot x_p & 0 \le x \le x_p \\ \frac{q}{\varepsilon} N_d x + \frac{q}{\varepsilon} N_d x_n & -x_n \le x < 0 \end{cases}$$

$$-V(x) = + \int_{-x_n}^{x_p} E\, d(x) = \begin{cases} -\frac{q}{\varepsilon} N_a \left(\frac{1}{2}x^2 - x_p \cdot x + C_1\right) \\ \frac{q}{\varepsilon} N_d \left(\frac{1}{2}x^2 + x_n \cdot x + C_2\right) \end{cases}$$

$$x = 0 \Rightarrow \frac{q}{\varepsilon} N_a x_p = \frac{q}{\varepsilon} N_d x_p$$

$$\Rightarrow N_a \cdot x_p = N_d x_n$$

Potential profile



# 7.2 Zero applied bias

$$\frac{x_p^2}{2}$$

$$V(x) = \begin{cases} \frac{q}{\varepsilon} N_a \left(\frac{1}{2}x^2 - x_p \cdot x + C_1\right) & 0 \le x \le x_p \\ -\frac{q}{\varepsilon} N_d \left(\frac{1}{2}x^2 + x_n \cdot x + C_2\right) & -x_n \le x < 0 \end{cases}$$

$$\Rightarrow C_1 = \frac{x_p^2}{2}$$

boundary condition: $V(x = x_p) = 0 \rightarrow \frac{1}{2}x_p^2 - x_p^2 + C_1 = 0$

$$V(x = -x_n) = V_{bi} = \frac{kT}{q} \ln \frac{N_a \cdot N_d}{n_i^2}$$

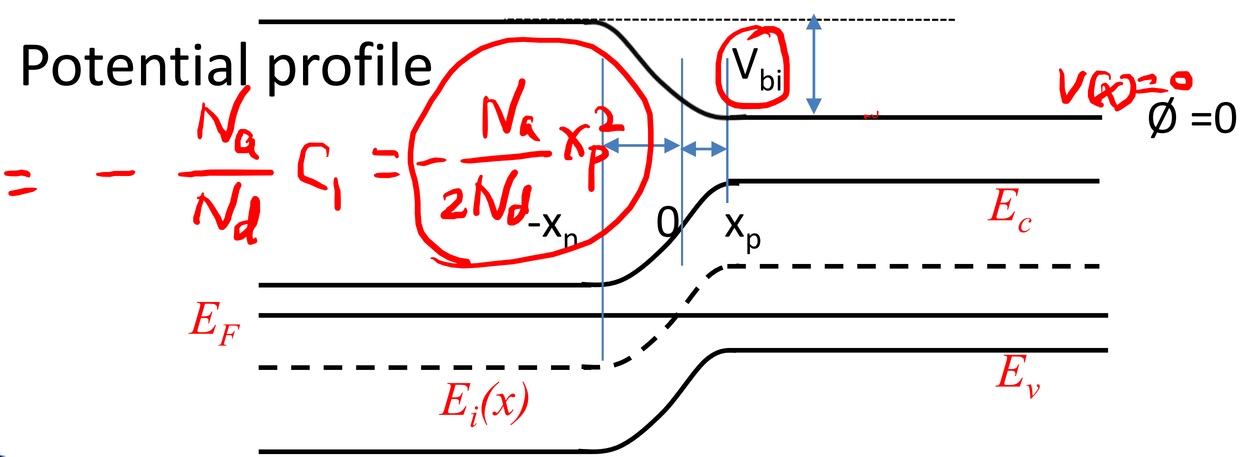
$$\Downarrow$$

$$-\frac{q}{\varepsilon} N_d \left(\frac{1}{2}x_n^2 - x_n^2 + C_2\right) = V_{bi}$$

continuous $x = 0$

$$\frac{q}{\varepsilon} N_a \cdot C_1 = -\frac{q}{\varepsilon} N_d C_2$$

$$C_2 = -\frac{N_a}{N_d} C_1 = -\frac{N_a}{2N_d} x_p^2$$



# 7.2 Zero applied bias

$$V(x) = \begin{cases} \frac{q}{\varepsilon} N_a \left(\frac{1}{2}x^2 - x_p \cdot x + \frac{x_p^2}{2}\right) & 0 \le x \le x_p \\ -\frac{q}{\varepsilon} N_d \left(\frac{1}{2}x^2 + x_n \cdot x - \frac{N_a}{N_d} \cdot \frac{x_p^2}{2}\right) & -x_n \le x < 0 \end{cases}$$

$$E_c(x) = -qV(x)$$

① $\boxed{V(-x_n) = V_{bi}}$

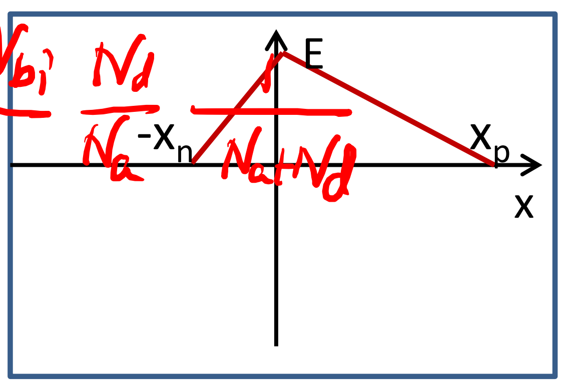
② $N_d x_n = N_a \cdot x_p$

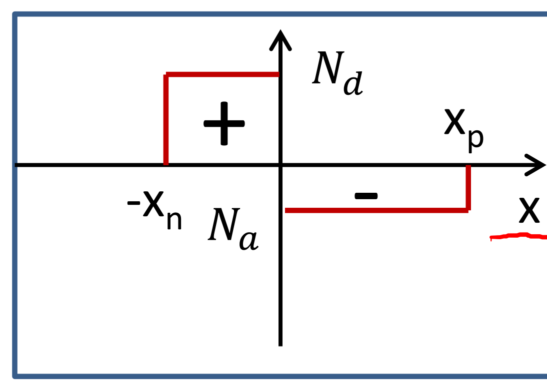
$$-\frac{q}{\varepsilon} N_d \left(\frac{1}{2}x_n^2 - x_n^2 - \frac{N_a}{N_d}\frac{x_p^2}{2}\right) = V_{bi}$$

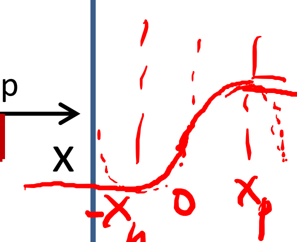
$$+\frac{q}{\varepsilon} N_d \left(+\frac{1}{2}\frac{N_a^2}{N_d^2}x_p^2 + \frac{N_a}{N_d}\frac{x_p^2}{2}\right) = V_{bi}$$

$$x_n = \frac{N_a}{N_d} x_p$$

$$\Rightarrow x_p = \frac{2\varepsilon V_{bi}}{q} \, \frac{N_d}{N_a} \, \frac{\,}{N_a + N_d}$$



# 7.2 Zero applied bias

Space charge width

$$x_p = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_d}{N_a}\frac{1}{N_a+N_d}}$$

$$N_d \cdot x_n = N_a \cdot x_p$$

$$x_n = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_a}{N_d}\frac{1}{N_a+N_d}}$$

$$W_{dep} = x_n + x_p = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{1}{N_a+N_d}}\left(\sqrt{\frac{N_d}{N_a}}+\sqrt{\frac{N_a}{N_d}}\right)$$

$$= \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_a+N_d}{N_a \cdot N_d}}$$

$$V_{bi} = \frac{kT}{q}\ln\frac{N_a \cdot N_d}{n_i^2}$$

$$\frac{\sqrt{N_d}}{\sqrt{N_a}} + \frac{N_a}{\sqrt{N_d}} = \frac{N_d+N_d}{\sqrt{N_a N_d}}$$

# 7.2 Zero applied bias

Space charge width

$$V(x) = \begin{cases} \dfrac{q}{\varepsilon} N_a \left( \dfrac{1}{2}x^2 - x_p x + \dfrac{x_p^2}{2} \right) & 0 \le x \le x_p \\ -\dfrac{q}{\varepsilon} N_d \left( \dfrac{1}{2}x^2 + x_n \cdot x - \dfrac{N_a}{N_d}\dfrac{x_p^2}{2} \right) & -x_n \le x < 0 \end{cases}$$

$$x_n = \sqrt{\frac{2\varepsilon V_{bi}}{q} \frac{N_d}{N_a} \frac{1}{N_d + N_a}}$$

$$x_p = \sqrt{\frac{2\varepsilon V_{bi}}{q} \frac{N_a}{N_d} \frac{1}{N_d + N_a}}$$

# 7.2 Zero applied bias

## Space charge width

# 7.2 Zero applied bias

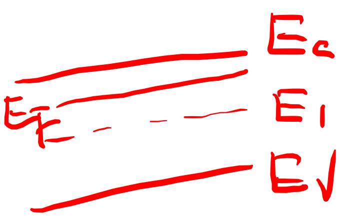
Space charge width

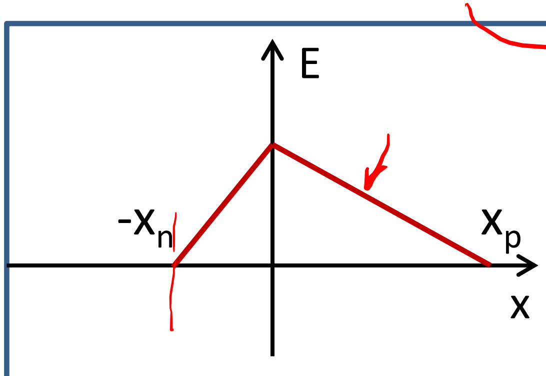
electric field intensity

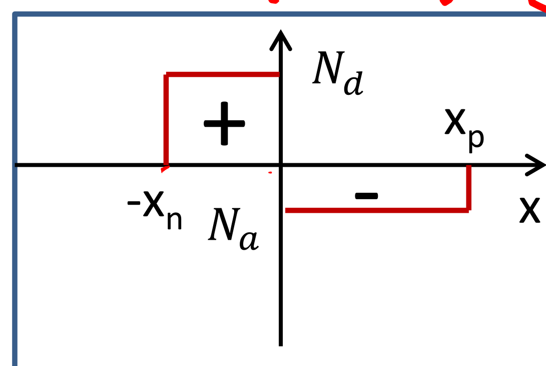
$$E = -\frac{dV(x)}{dx} = \begin{cases} -\frac{q}{\varepsilon} N_a x + \frac{q}{\varepsilon} N_a x_p & 0 \le x \le x_p \\ \frac{q}{\varepsilon} N_d x + \frac{q}{\varepsilon} N_d x_n & -x_n \le x \le 0 \end{cases}$$

$$E_{max}(x=0) = \frac{q}{\varepsilon} N_a x_p \qquad \frac{q}{\varepsilon} N_a x_p = N_d x_n$$

$$= \frac{q}{\varepsilon} N_a \sqrt{\frac{2\varepsilon V_{bi}}{q} \frac{N_d}{N_a} \frac{1}{N_a+N_d}} = \sqrt{\frac{2q V_{bi}}{\varepsilon} \frac{N_a N_d}{N_a+N_d}}$$



# Check your understanding

$$|E_{max}| = \frac{2V_{bi}}{W_{dep}} = \frac{2\times 0.718\,V}{452.1\,nm}$$

$$= 3.18\times10^4\ V/cm$$

## Problem Example #2

A silicon pn junction at T=300K with zero applied bias has doping concentration of $N_d = 5 \times 10^{16}\ cm^{-3}$ and $N_a = 5\times10^{15}\ cm^{-3}$. Determine $x_n$, $x_p$, W and $|E_{max}|$.

$$V_{bi} = \frac{kT}{q}\ln\frac{N_a \cdot N_d}{n_i^2} = 0.0259\times\ln\frac{5\times10^{16}\times5\times10^{15}}{(1.5\times10^{10})^2} = 0.0259\times27.3$$

$$= 0.718\,V$$

$$x_n = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_a}{N_d}\frac{1}{N_a+N_d}} = \sqrt{\frac{2\times11.7\times8.85\times10^{-14}\times0.718}{1.6\times10^{-19}}\times\frac{5\times10^{15}}{5\times10^{16}}\times\frac{1}{5\times10^{15}+5\times10^{16}}}$$

$$= 4.11\times10^{-6}\ cm$$

$$= 41.1\,nm$$

$$x_n \cdot N_d = x_p \cdot N_a$$

$$x_p = \frac{N_d}{N_a}x_n = 10\times41.1\,nm$$

$$= 411\,nm$$

$$W_{dep} = x_n + x_p = 452.1\,nm \qquad |E_{max}| = 3.18\times10^4\ V/cm$$

# Outline

7.1 Basic structure of the pn junction

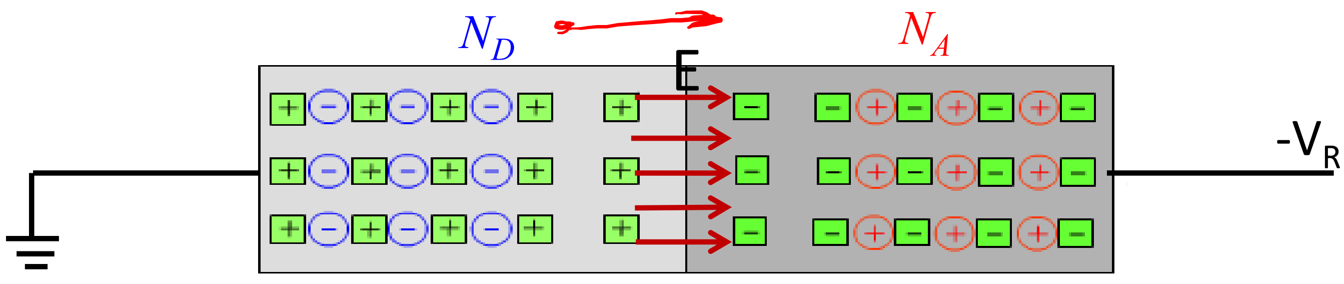
7.2 Zero applied bias

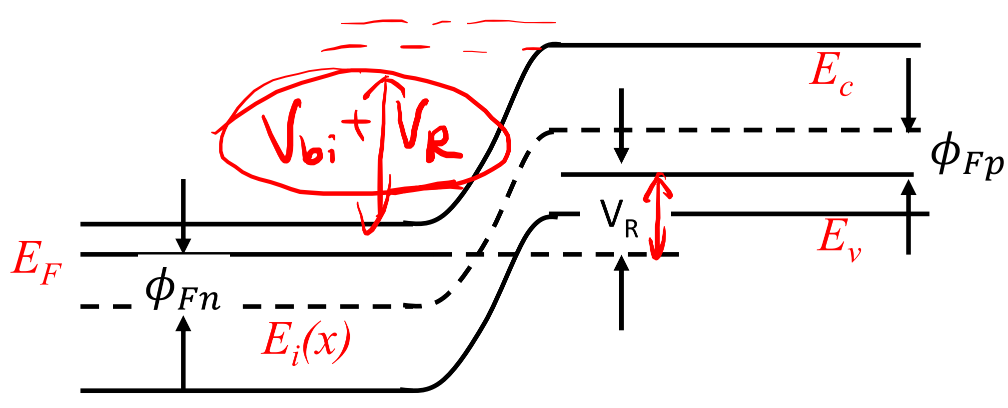
**7.3 Reverse applied bias**

# 7.3 Reverse applied bias

Space charge width and electric field

$$V_{\text{total}} = |\phi_{Fn}| + |\phi_{Fp}| + V_R$$



# 7.3 Reverse applied bias

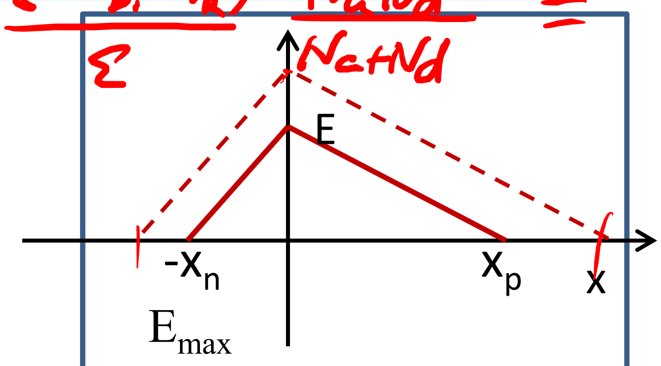
Space charge width and electric field

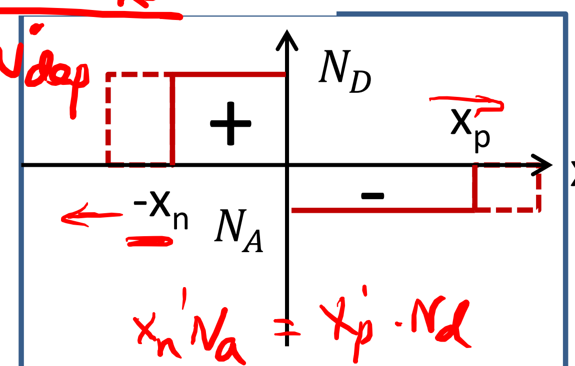
$$x_p = \sqrt{\frac{2\varepsilon V_{bi}}{e}\frac{N_d}{N_a}\frac{1}{N_a+N_d}} \qquad x_p' = \sqrt{\frac{2\varepsilon (V_{bi}+V_R)}{e}\frac{N_d}{N_a}\frac{1}{N_a+N_d}}$$

$$W_{dep} = \sqrt{\frac{2\varepsilon V_{bi}}{e}\frac{N_a+N_d}{N_a N_d}} \qquad W_{dep}' = \sqrt{\frac{2\varepsilon (V_{bi}+V_R)}{e}\frac{N_a+N_d}{N_a \cdot N_d}}$$

$$E_{max} = \sqrt{\frac{2e V_{bi}}{\varepsilon}\frac{N_a N_d}{N_a+N_d}} = \frac{2V_{bi}}{W_{dep}}$$

$$E_{max}' = \sqrt{\frac{2e (V_{bi}+V_R)}{\varepsilon}\frac{N_a N_d}{N_a+N_d}} = \frac{2(V_{bi}+V_R)}{W_{dep}'}$$




$$x_n' N_a = x_p' \cdot N_d$$

# 7.3 Reverse applied bias

Junction capacitance

$$Q^+ = q x_p \cdot N_d$$

$$= q N_d \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q} \frac{N_d}{N_a(N_a+N_d)}}$$

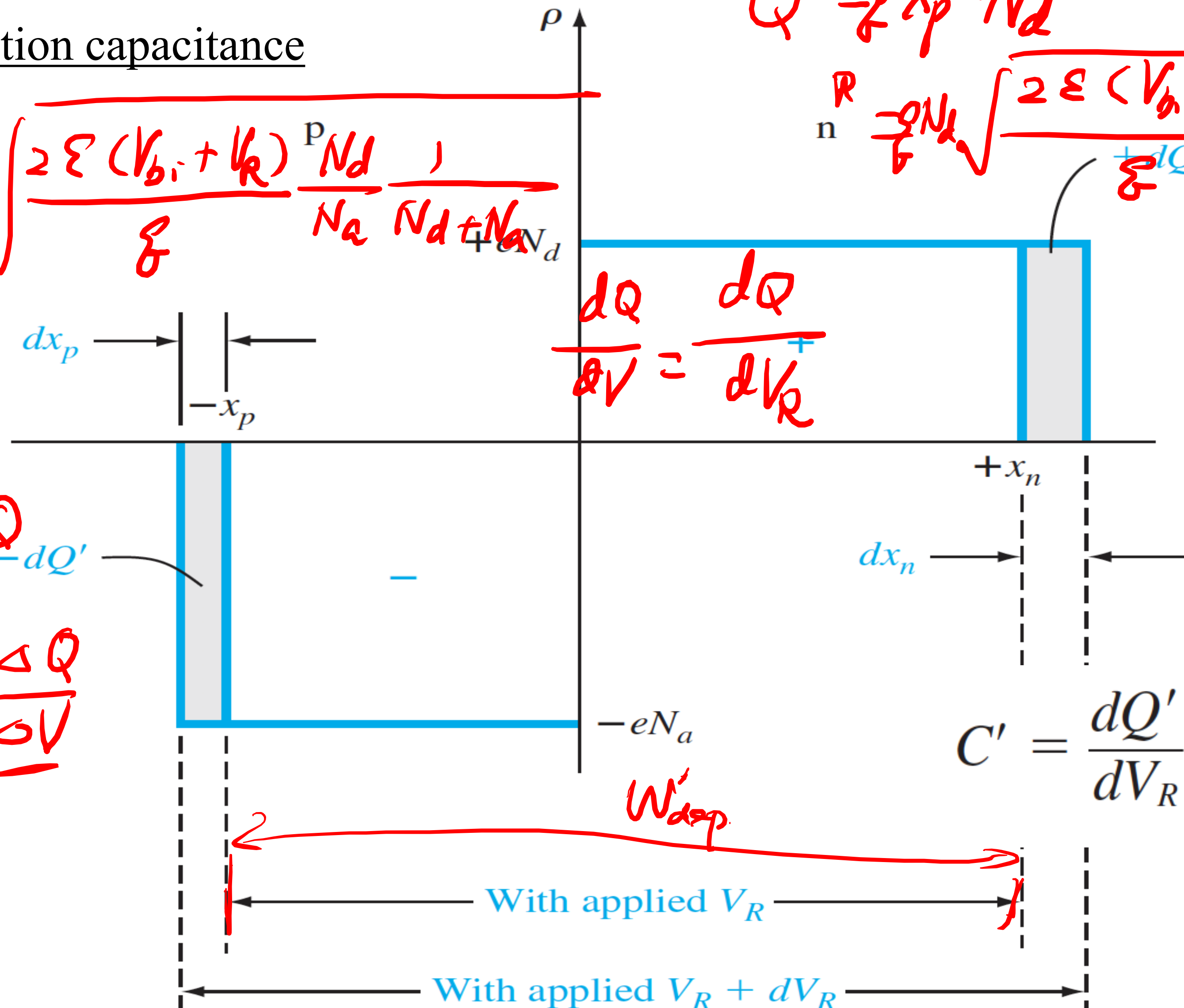
$$x_p = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q} \frac{N_d}{N_a} \frac{1}{N_d+N_a}}$$

$$\frac{dQ}{dV} = \frac{dQ^+}{dV_R}$$

$$CV = Q$$

$$C = \frac{\Delta Q}{\Delta V}$$

ac




$$C' = \frac{dQ'}{dV_R}$$

# 7.3 Reverse applied bias

Junction capacitance

$$C' = \frac{dQ'}{dV_R}.$$

$$dQ' = qN_d \cdot dx_n = qN_a \cdot dx_p$$

$$W'_{dep} = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q} \cdot \frac{N_a+N_d}{N_a \cdot N_d}}$$

$$x_n = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q} \frac{N_a}{N_d} \frac{1}{N_a+N_d}}$$

$$= \sqrt{V_{bi}+V_R} \cdot \sqrt{\frac{2\varepsilon}{q} \frac{N_a}{N_d} \frac{1}{N_a+N_d}}$$

$$dx_n = \frac{1}{2} \frac{dV_R}{\sqrt{V_{bi}+V_R}} \cdot \sqrt{\frac{2\varepsilon}{q} \frac{N_a}{N_d} \frac{1}{N_a+N_d}}$$

$$C' = \frac{dQ'}{dV_R} = \frac{qN_d \cdot dx_n}{dV_R} = qN_d \cdot \frac{1}{2} \frac{1}{\sqrt{V_{bi}+V_R}} \sqrt{\frac{2\varepsilon}{q} \frac{N_a}{N_d} \frac{1}{N_a+N_d}}$$

$$= \frac{1}{2} \frac{1}{\sqrt{V_{bi}+V_R}} \sqrt{2q\varepsilon N_a \cdot N_d/(N_a+N_d)}$$

$$= \frac{\varepsilon}{W'_{dep}} =$$

$$= \frac{\varepsilon}{W'_{dep}} \Big|_{V_R = V_{R0}}$$

# Check your understanding

## Problem Example #3

$n_i = 12.1 \times 10^6$

Consider a GaAs pn junction at T = 300K doped to $N_a = 5 \times 10^{15}$ cm$^{-3}$ and $N_d = 2 \times 10^{16}$ cm$^{-3}$. (a) Calculate $V_{bi}$. (b) Determine the junction capacitance C’ for $V_R$ =4V.

$$V_{bi} = \frac{kT}{q} \ln \frac{N_a \cdot N_d}{n_i^2} = 0.0259 \times \ln \frac{5\times10^{15} \times 2\times10^{16}}{(12.1\times10^6)^2}$$

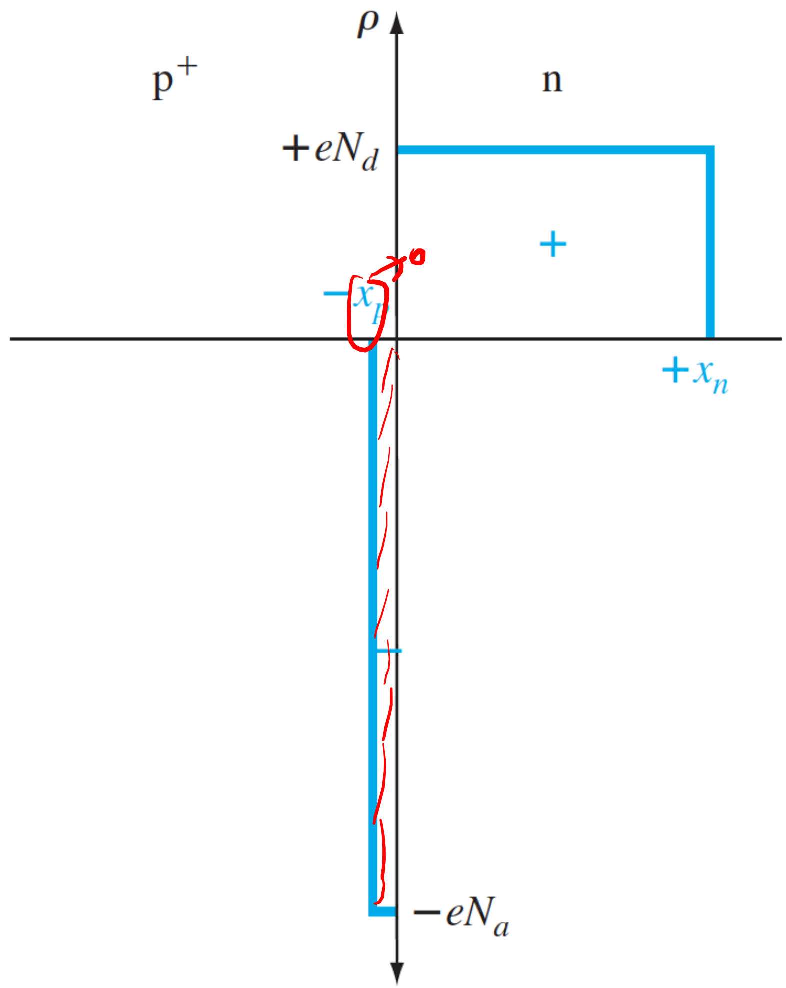
$$= 0.0259 \times 44.57 = \underline{1.154\,V}$$

$$C' = \frac{\varepsilon}{W_{dep}'}$$

$$W_{dep}(@ V_R = 4V) = \sqrt{(V_{bi} + 4V)}$$

# 7.3 Reverse applied bias

One-sided junction




$$W_{dep} = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{e}\,\frac{N_d+N_a}{N_d \cdot N_a}}$$

$N_a \to +\infty$

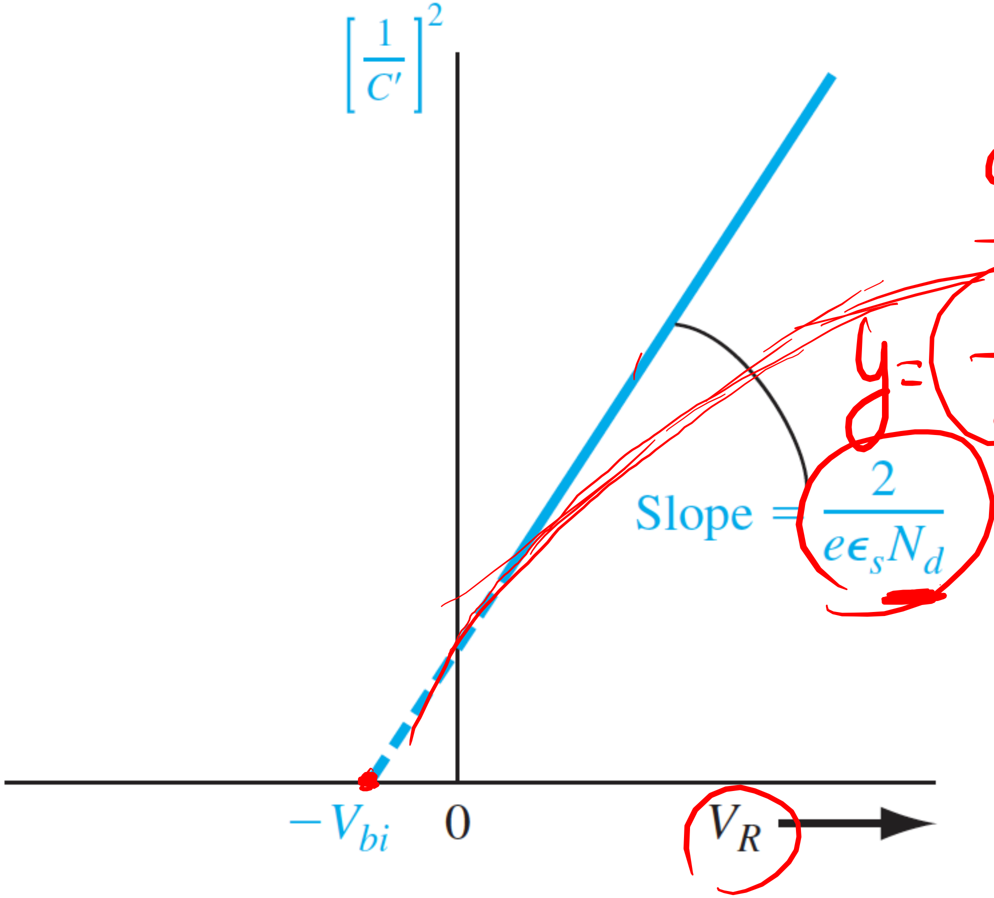
$N_a \gg N_d$

$$W_{dep} = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{e}\,\frac{1}{N_d}} \approx x_n$$

$$C' = \frac{\varepsilon}{W_{dep}} = \sqrt{\frac{e\varepsilon N_d}{2(V_{bi}+V_R)}}$$

# 7.3 Reverse applied bias

<u>One-sided junction</u>




$$C' = \frac{\varepsilon}{W_{dep}} = \sqrt{\frac{e\varepsilon N_d}{2(V_{bi}+V_R)}}$$

$$C'^2 = \frac{e\varepsilon N_d}{2(V_{bi}+V_R)}$$

$$y = \frac{1}{C'^2} = \frac{2(V_{bi}+V_R)}{e\varepsilon N_d}$$

$$= \frac{2}{e\varepsilon N_d}(V_{bi}+V_R)$$

$$y = \frac{1}{C'^2} = \frac{2}{e\varepsilon N_d}(V_{bi}+x)$$

# Check your understanding

## Problem Example #4

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Control sample： Au is in contact with a uniform doped n-type Si substrate forming a device similar to a pn junction.
SAMM-doped sample: Au is in contact with Si that is doped with SAMM
Take Au as $p^{++}$ doping in this case.

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Deep level transient spectroscopic investigation of phosphorus-doped silicon by self-assembled molecular monolayers

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