Problem Set 5 Solution

1. (10 points) 5'+5'

Built-in potential: the potential difference across the depletion region of p-n junction in thermal equilibrium without external voltage. (5')
How it maintains thermal equilibrium: by the balance between diffusion current due to concentration gradient and drift current due to electric field in depletion region.) (5')

2. (15 points) 5'+5'+5'

(a) Plug in $n=n_0+\delta n$, $p=p_0+\delta n$.

$$R=\frac{(n_0+\delta n)(p_0+\delta n)-n_i^2}{\tau_{p0}(n_0+\delta n+n')+\tau_{n0}(p_0+\delta n+p')}=\frac{n_0p_0+(n_0+p_0)\delta n+(\delta n)^2-n_i^2}{\tau_{p0}(n_0+\delta n+n')+\tau_{n0}(p_0+\delta n+p')}$$

$$\simeq\frac{\delta n(n_0+p_0)}{\tau_{p0}(n_0+n_i)+\tau_{n0}(p_0+n_i)}$$ (Since $\delta n \ll n_i$ and $n_i^2=n_0p_0$) (3')

When n-type ($n_0 \gg p_0$) $\frac{R}{\delta n}=\frac{1}{\tau_{p0}}=\boxed{10^7\,s^{-1}}$ (2')

(b) When intrinsic ($n_0=p_0=n_i$), $\frac{R}{\delta n}=\frac{\delta n\cdot 2n_i}{(\tau_{p0}\cdot 2n_i+\tau_{n0}\cdot 2n_i)\delta n}=\frac{1}{\tau_{p0}+\tau_{n0}}$ (3')

$=\frac{1}{10^{-7}+5\times10^{-7}}\simeq\boxed{1.667\times10^6\,s^{-1}}$ (2')

(c) When p-type ($p_0 \gg n_0$), $\frac{R}{\delta n}=\frac{p_0}{\tau_{n0}\cdot p_0}=\frac{1}{\tau_{n0}}$ (3') $=\frac{1}{5\times10^{-7}}=\boxed{2\times10^6\,s^{-1}}$ (2')

3. (30 points) 5'+5'+5'+5'+5'+5'

(a) $0=D_p\frac{d^2\Delta p}{dx^2}-\frac{\Delta p}{\tau_{p0}}+g'$

The solution may look like this: $\Delta p=C_1e^{-x/L_p}+C_2e^{x/L_p}+g'\tau_{p0}$

Then use boundary conditions to determine coefficients (C_1, C_2)

When $x\to\infty$, $\Delta p=g'\tau_{p0}$, then $C_2=0$

$\Rightarrow \Delta p=C_1e^{-x/L_p}+g'\tau_{p0}$

And $-D_p\frac{d(\Delta p)}{dx}\Big|_{x=0}=s(\Delta p)\Big|_{x=0}$, plug $\Delta p=C_1e^{-x/L_p}+g'\tau_{p0}$ in

$$-D_p\left[\frac{d}{dx}\left(C_1e^{-x/L_p}+g'\tau_{p0}\right)\right]_{x=0}=s(C_1+g'\tau_{p0})\Rightarrow C_1=\frac{-sg'\tau_{p0}}{\frac{D_p}{L_p}+s}$$

Thus, $$\Delta p=-\frac{sg'\tau_{p0}}{\frac{D_p}{L_p}+s}\cdot e^{-x/L_p}+g'\tau_{p0}=g'\tau_{p0}\left[1-\frac{s}{\frac{D_p}{L_p}+s}\cdot e^{-x/L_p}\right]$$

Here, $L_p = \sqrt{D_p \tau_{p0}} = \sqrt{10 \cdot 10^{-7}} = 10^{-3}\,cm$.

So, $\Delta p = 10^{21} \times 10^{-7} \times \left[1 - \dfrac{S}{\frac{10}{10^{-3}} + S} \cdot e^{-x/10^{-3}}\right]$

$= 10^{14} \times \left[1 - \dfrac{S}{10^4 + S} \cdot e^{-10^3 x}\right]$

(i) when $S = 0$, $\Delta p = \boxed{10^{14}\,cm^{-3}}$ --- (5')

(ii) when $S = 2000\,cm/s$, $\Delta p = 10^{14}\left(1 - \dfrac{2000}{2000+10^4} \times e^{-10^3 x}\right) = \boxed{10^{14}\left(1 - \frac{1}{6} e^{-10^3 x}\right)}$ --- (5')

(iii) when $S = \infty$. $\Delta p = \boxed{10^{14}(1 - e^{-10^3 x})\,cm^{-3}}$ --- (5')

(b) $\Delta p(0) = 10^{14}\left(1 - \dfrac{S}{10^4 + S}\right)$

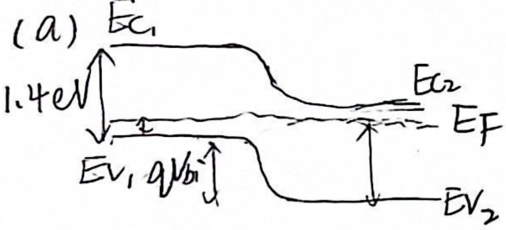
(i) when $S = 0$, $\Delta p = 10^{14} \times \left(1 - \dfrac{1}{10^4}\right) \simeq \boxed{9.999 \times 10^{13}\,cm^{-3}}$ --- (5'),

(ii) when $S = 2000\,cm/s$, $\Delta p = 10^{14}\left(1 - \dfrac{2000}{2000+10^4}\right) \simeq \boxed{8.333 \times 10^{13}\,cm^{-3}}$ --- (5')

(iii) when $S = \infty$, $\Delta p \simeq 10^{14}(1-1) = \boxed{0\,cm^{-3}}$ --- (5')

4. (10 points) 5'+5'

(a)



--- (5')

(b) $qV_{bi} = (E_F - E_{V2}) - (E_F - E_{V1})$ ------ (2')

$= -kT \ln \dfrac{N_{A2}}{N_V} - \left(-kT \ln \dfrac{N_{A1}}{N_V}\right)$ ----- (2')

$= kT \ln \dfrac{N_{A1}}{N_{A2}}$

$\Rightarrow V_{bi} = \boxed{\dfrac{kT}{q} \ln \dfrac{N_{A1}}{N_{A2}}}$ ---- (1')

5. (10 points) 5'+5'

(a) $V_{bi} = \dfrac{kT}{q} \ln \dfrac{N_A N_D}{n_i^2} = 0.0259 \times \ln \dfrac{5\times10^{16} \times 2.5\times10^{16}}{(2.4\times10^{13})^2} \simeq 0.378\,V$

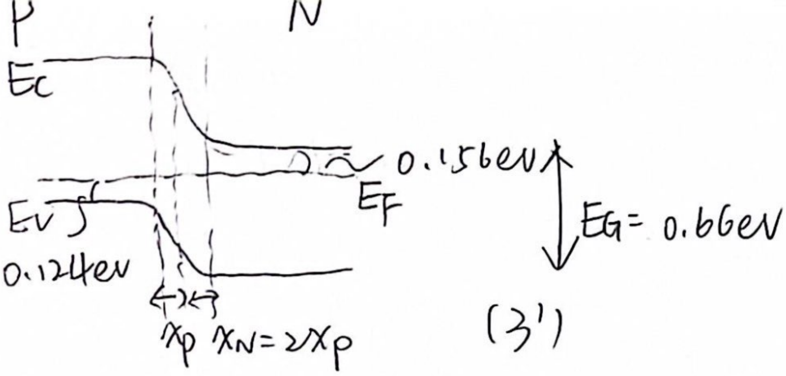
$x_p = \sqrt{\dfrac{2\varepsilon V_{bi}}{q} \dfrac{N_D}{N_A} \dfrac{1}{N_D + N_A}} = \sqrt{\dfrac{2\times16\times8.85\times10^{-14}\times0.378}{1.602\times10^{-19}} \times \dfrac{2.5\times10^{16}}{5\times10^{16}} \times \dfrac{1}{2.5\times10^{16} + 5\times10^{16}}}$

$\simeq \boxed{6.674\times10^{-6}\,cm}$ (1')

$x_n = \dfrac{N_A}{N_D} x_p = \boxed{1.335\times10^{-5}\,cm}$ (1')

$E_F - E_V = -kT \ln \dfrac{N_A}{N_V} = -0.0259 \times \ln \dfrac{5\times10^{16}}{6\times10^{18}} \simeq 0.124\,eV$

$E_C - E_F = -kT \ln \dfrac{N_D}{N_C} = -0.0259 \times \ln \dfrac{2.5\times10^{16}}{1.04\times10^{19}} \simeq 0.156\,eV$

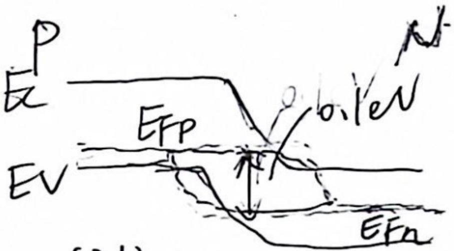


(3')

(b) $x_p = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q}\frac{N_D}{N_A}\cdot\frac{1}{N_A+N_D}} = \sqrt{\frac{2\times16\times8.85\times10^{-14}\times(0.378+0.1)}{1.602\times10^{-19}}\times\frac{1}{2}\times\frac{1}{7.5\times10^{16}}}$

$\simeq \boxed{7.506\times10^{-6}\,cm}$ (1')

$x_N = \frac{N_A}{N_D}x_p = \boxed{1.501\times10^{-5}\,cm}$ (1')



(3')

6. (10 points) 5'+5'

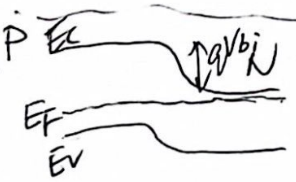
(a) $C = \sqrt{\frac{q\varepsilon N_A N_D}{2(V_{bi}+V_R)(N_D+N_A)}} = \sqrt{\frac{q\varepsilon}{2(V_{bi}+V_R)}\cdot\frac{1}{\frac{1}{N_A}+\frac{1}{N_D}}} \simeq \sqrt{\frac{q\varepsilon N_A}{2(V_{bi}+V_R)}}$ $(N_D >> N_A)$

$qV_{bi} \simeq E_G - (E_F - E_V)_p = E_G + kT\ln\frac{N_A}{N_V} = 1.12 + 0.0259\times\ln\frac{10^{15}}{1.04\times10^{19}} \simeq 0.880eV$

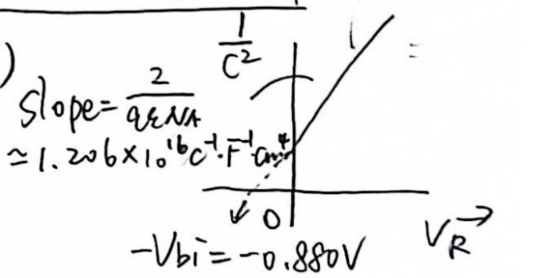
$\Rightarrow V_{bi} = 0.880V$

$C = \sqrt{\frac{1.602\times10^{-19}\times11.7\times8.85\times10^{-14}\times10^{15}}{2\times(0.880+1)}} \simeq \boxed{6.642\times10^{-9}\,F/cm^2}$

(3')



Since $N_D >> N_A$, on n-side, E_F almost coincides with E_c, Hence, $qV_{bi} \simeq E_G - (E_F - E_V)_p$



(2')

(b) $V_{bi} = 0.880V$

$C = \sqrt{\frac{1.602\times10^{-19}\times11.7\times8.85\times10^{-14}\times10^{15}}{2\times(0.880+5)}}$ (2') $\simeq \boxed{3.756\times10^{-9}\,F/cm^2}$ (3')

7. (15 points) 5'+5'+5'

(a) $\boxed{\text{Forward-biased.}}$ (3') Bias voltage = $\boxed{0.5V}$ (2')

(b) $qV_{bi} - qV_F = 0.28eV \overset{(3')}{\Rightarrow} qV_{bi} - 0.5eV = 0.28eV \Rightarrow V_{bi} = \boxed{0.78V}$ (2')

(c) $N_A = N_V e^{-(E_{Fp} - E_V)/kT} = 1.04 \times 10^{19} \times e^{-0.36/0.0259} \simeq \boxed{9.561 \times 10^{12}\,cm^{-3}}$ --(2')

$N_D = \frac{n_i^2}{N_A} e^{qV_{bi}/kT} = \frac{(1.5 \times 10^{10})^2}{9.561 \times 10^{12}} \times e^{0.78/0.0259} \simeq \boxed{2.824 \times 10^{20}\,cm^{-3}}$ --(3')