VE320 – Summer 2024

Introduction to Semiconductor Devices

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Chapter 4 The Semiconductor in Equilibrium

Outline

4.1 Charge carriers in semiconductors

4.2 Dopant atoms and energy levels

4.3 The extrinsic semiconductor

4.4 Statistics of donors and acceptors

4.5 Charge neutrality

4.6 Position of Fermi energy level

Outline

4.1 Charge carriers in semiconductors

4.2 Dopant atoms and energy levels

4.3 The extrinsic semiconductor

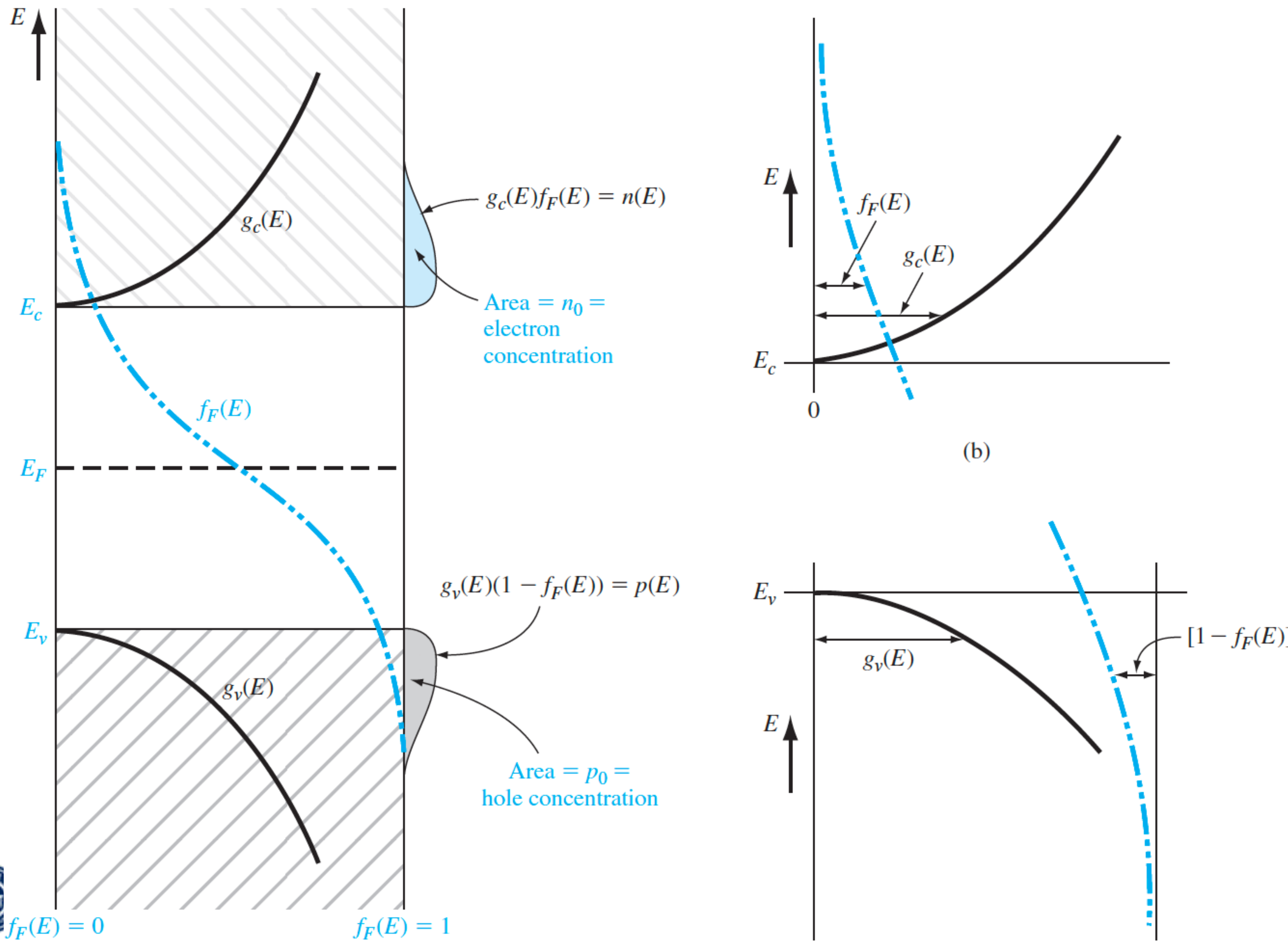
4.4 Statistics of donors and acceptors

4.5 Charge neutrality

4.6 Position of Fermi energy level

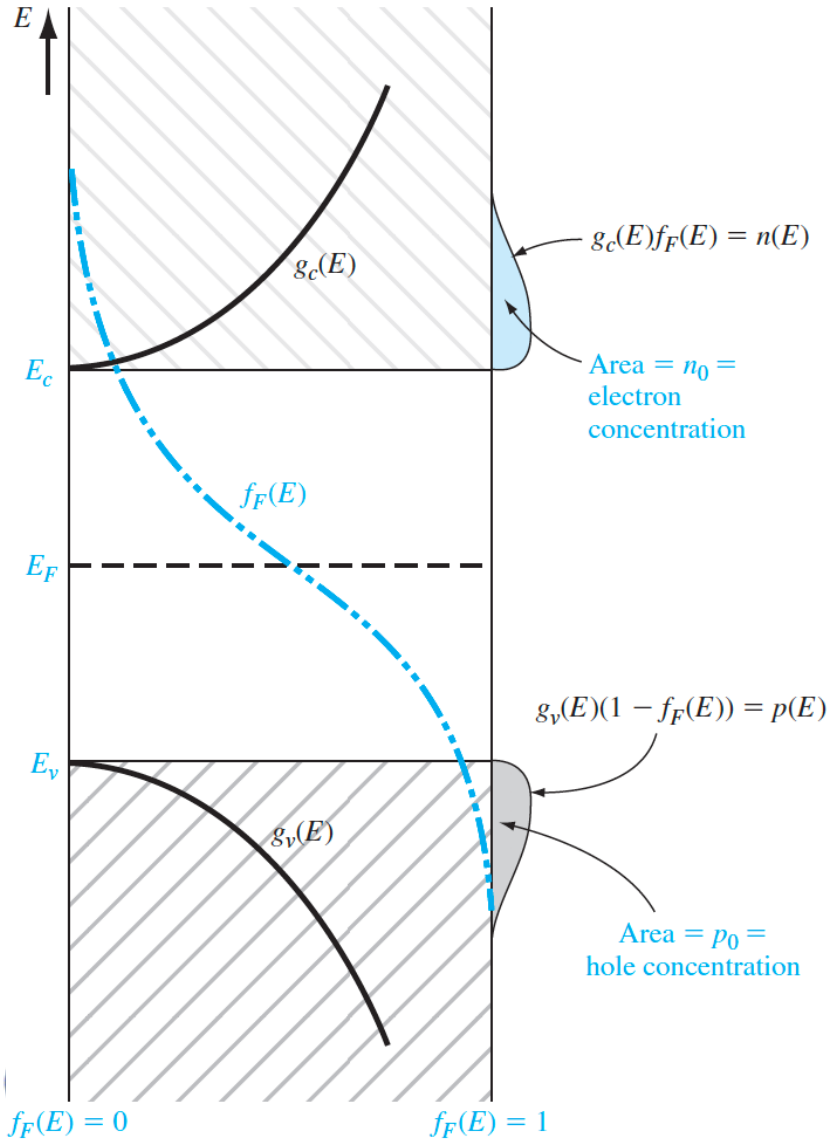
4.1 Charge carriers in semiconductors

Equilibrium distribution of electrons and holes



4.1 Charge carriers in semiconductors

The n_0 and p_0 equations



$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E) dE$$

$$p_0 = \int_{-\infty}^{E_v} g_v(E)[1 - f_F(E)] dE$$

4.1 Charge carriers in semiconductors

The n_0 and p_0 equations

$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E) dE$$

$$= \int_{E_c}^{+\infty} \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} 4\pi \frac{(2m^*)^{\frac{3}{2}}}{h^3} (E - E_c\ )^{\frac{1}{2}} dE$$

4.1 Charge carriers in semiconductors

The n_0 and p_0 equations

$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E) dE$$

$$= \int_{E_c}^{+\infty} \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)} 4\pi \frac{(2m^*)^{\frac{3}{2}}}{h^3} (E-E_c\ )^{\frac{1}{2}} dE$$

$$\eta = \frac{E-E_c}{kT} \qquad = 4\pi \frac{(2m^*)^{\frac{3}{2}}}{h^3} \int_{E_c}^{+\infty} \frac{(E-E_c\ )^{\frac{1}{2}}}{1+\exp\left(\frac{E-E_F}{kT}\right)} dE$$

$$\eta_F = \frac{E_F-E_c}{kT} \qquad = 4\pi \frac{(2m^* kT)^{\frac{3}{2}}}{h^3} \int_{0}^{+\infty} \frac{\eta^{\frac{1}{2}}}{1+\exp(\eta-\eta_F)} d\eta$$

Not analytically integrate-able

4.1 Charge carriers in semiconductors

$$\boxed{if\ \exp(\eta - \eta_F) > 10 \Leftrightarrow \frac{E - E_F}{kT} > 3 \Leftrightarrow E - E_F > 3kT}$$ (2^{nd} time approximation)

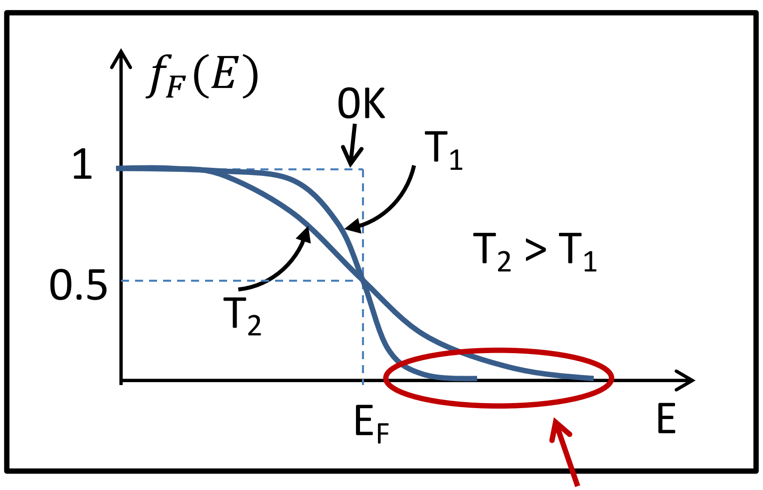
$$\boxed{f_F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}}$$

Fermi-Dirac Distribution

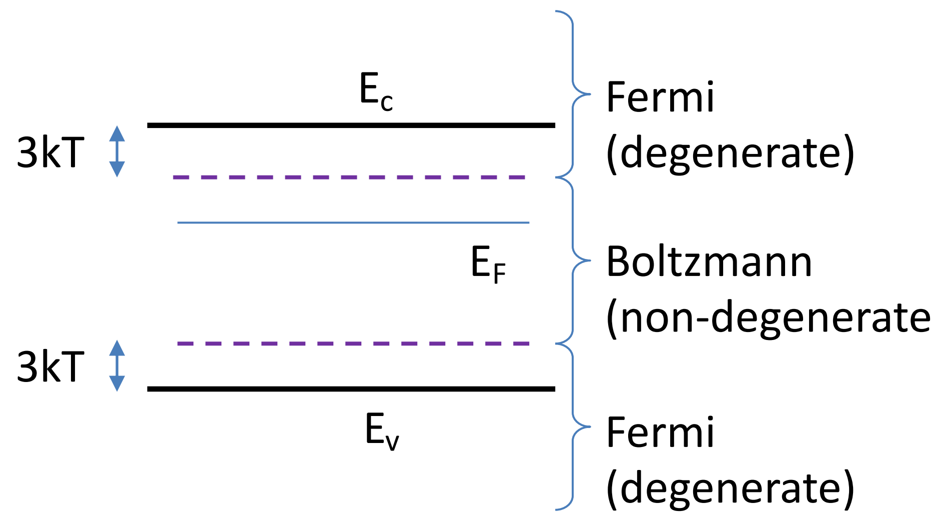
$$\boxed{f_F(E) = \exp(\frac{E_F - E}{kT})}$$

Boltzmann Distribution

Fermi-Dirac



Boltzmann



4.1 Charge carriers in semiconductors

$$\boxed{if\ \exp(\eta-\eta_F)>10 \Leftrightarrow \frac{E-E_F}{kT}>3 \Leftrightarrow E-E_F>3kT}$$

$$\boxed{f_F(E)=\frac{1}{1+\exp(\frac{E-E_F}{kT})}} \Rightarrow \boxed{f_F(E)=\exp(\frac{E_F-E}{kT})}$$

Fermi-Dirac Distribution

Boltzmann Distribution

$$n_0=\int_{E_c}^{\infty} g_c(E)f_F(E)dE \ = 4\pi\frac{(2m^*)^{\frac{3}{2}}}{h^3}\int_{E_c}^{+\infty}(E-E_c\ )^{\frac{1}{2}}\exp\left(\frac{E_F-E}{kT}\right)dE$$

$$=\ 4\pi\frac{(2m^*kT)^{\frac{3}{2}}}{h^3}\int_0^{+\infty}\sqrt{\eta}\exp(\eta_F-\eta)d\eta$$

4.1 Charge carriers in semiconductors

$$\boxed{if\ \ \exp(\eta-\eta_F) > 10 \Leftrightarrow \frac{E-E_F}{kT} > 3 \Leftrightarrow E-E_F > 3kT}$$

$$\boxed{f_F(E) = \frac{1}{1+\exp(\frac{E-E_F}{kT})}}$$

Fermi-Dirac Distribution

$$\boxed{f_F(E) = \exp(\frac{E_F-E}{kT})}$$

Boltzmann Distribution

$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E)\,dE \ = 4\pi\frac{(2m^*)^{\frac{3}{2}}}{h^3}\int_{E_c}^{+\infty}(E-E_c\ )^{\frac{1}{2}}\exp\left(\frac{E_F-E}{kT}\right)dE$$

$$= \ 4\pi\frac{(2m^*kT)^{\frac{3}{2}}}{h^3}\int_0^{+\infty}\sqrt{\eta}\exp(\eta_F-\eta)\,d\eta$$

$$= 4\pi\frac{(2m^*kT)^{\frac{3}{2}}}{h^3}\exp(\frac{E_F-E_c}{kT})\int_0^{+\infty}\sqrt{\eta}\exp(-\eta)\,d\eta$$

$$\eta = \frac{E-E_c}{kT}$$

$$\eta_F = \frac{E_F-E_c}{kT}$$

4.1 Charge carriers in semiconductors

$$\boxed{if\ \ \exp(\eta - \eta_F) > 10 \Leftrightarrow \frac{E - E_F}{kT} > 3 \Leftrightarrow E - E_F > 3kT}$$

$$\boxed{f_F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}} \Rightarrow \boxed{f_F(E) = \exp(\frac{E_F - E}{kT})}$$

Fermi-Dirac Distribution

Boltzmann Distribution

$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E) dE \ = 4\pi \frac{(2m^*)^{\frac{3}{2}}}{h^3} \int_{E_c}^{+\infty} (E - E_c\ )^{\frac{1}{2}} \exp\left(\frac{E_F - E}{kT}\right) dE$$

$$= \ 4\pi \frac{(2m^* kT)^{\frac{3}{2}}}{h^3} \int_0^{+\infty} \sqrt{\eta} \exp(\eta_F - \eta) d\eta$$

$$\eta = \frac{E - E_c}{kT}$$

$$= 4\pi \frac{(2m^* kT)^{\frac{3}{2}}}{h^3} \exp(\frac{E_F - E_c}{kT}) \int_0^{+\infty} \sqrt{\eta} \exp(-\eta) d\eta \qquad = \frac{\sqrt{\pi}}{2}$$

$$\eta_F = \frac{E_F - E_c}{kT}$$

$$\boxed{n_0 = 2 \frac{(2\pi m_n{}^* kT)^{\frac{3}{2}}}{h^3} \exp\left(\frac{E_F - E_c}{kT}\right) = N_c \exp(\frac{E_F - E_c}{kT})}$$

4.1 Charge carriers in semiconductors

$$\boxed{if\ \exp(\eta - \eta_F) > 10 \Leftrightarrow \frac{E - E_F}{kT} > 3 \Leftrightarrow E - E_F > 3kT}$$

$$\boxed{n_0 = \frac{2(2\pi m_n{}^* kT)^{\frac{3}{2}}}{h^3} \exp\left(\frac{E_F - E_c}{kT}\right) = N_c \exp(\frac{E_F - E_c}{kT})}$$

$$\boxed{p_0 = \frac{2\left(2\pi m_p{}^* kT\right)^{\frac{3}{2}}}{h^3} \exp\left(\frac{E_v - E_F}{kT}\right) = N_v \exp(\frac{E_v - E_F}{kT})}$$

4.1 Charge carriers in semiconductors

The intrinsic carrier concentration

$$\boxed{n_0 \times p_0 = {n_i}^2 = N_c N_v \exp\left(\frac{E_v - E_c}{kT}\right) \Rightarrow n_i = \sqrt{N_c N_v}\exp(-\frac{E_g}{2kT})}$$

$$\boxed{n_0 = N_c \exp(\frac{E_F - E_c}{kT}) \qquad p_0 = N_v \exp(\frac{E_v - E_F}{kT})}$$

$$N_c \sim 10^{19} cm^{-3}$$

$$N_v \sim 10^{19} cm^{-3}$$

- The equations are universal for doped and undoped semiconductors

4.1 Charge carriers in semiconductors

Problem Example #1

Determine the thermal-equilibrium concentrations of electrons and holes in silicon at $T = 300K$ if the Fermi energy level E_F is $0.215eV$ above the valence band energy E_V. $N_C = 2.8\times 10^{19}\ cm^{-3}$ and $N_V = 1.04\times10^{19}\ cm^{-3}$. $E_g = 1.12\ eV$ for Si.

$$n_0 = N_c \exp\left(\frac{-(E_c - E_F)}{kT}\right) = N_c \exp\left(\frac{-(E_g - 0.215)}{kT}\right) = 18707\ cm^{-3}$$

$$p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right) = N_v \exp\left(\frac{-0.215}{kT}\right) = 2.58\times10^{15}\ \mathrm{cm}^{-3}$$

4.1 Charge carriers in semiconductors

The intrinsic carrier concentration

Table 4.1 | Effective density of states function and density of states effective mass values

	N_c (cm^{-3})	N_v (cm^{-3})	m_n^*/m_0	m_p^*/m_0
Silicon	2.8×10^{19}	1.04×10^{19}	1.08	0.56
Gallium arsenide	4.7×10^{17}	7.0×10^{18}	0.067	0.48
Germanium	1.04×10^{19}	6.0×10^{18}	0.55	0.37

Table 4.2 | Commonly accepted values of n_i at $T = 300$ K

Silicon	$n_i = 1.5 \times 10^{10}$ cm^{-3}
Gallium arsenide	$n_i = 1.8 \times 10^{6}$ cm^{-3}
Germanium	$n_i = 2.4 \times 10^{13}$ cm^{-3}

4.1 Charge carriers in semiconductors

The intrinsic carrier concentration

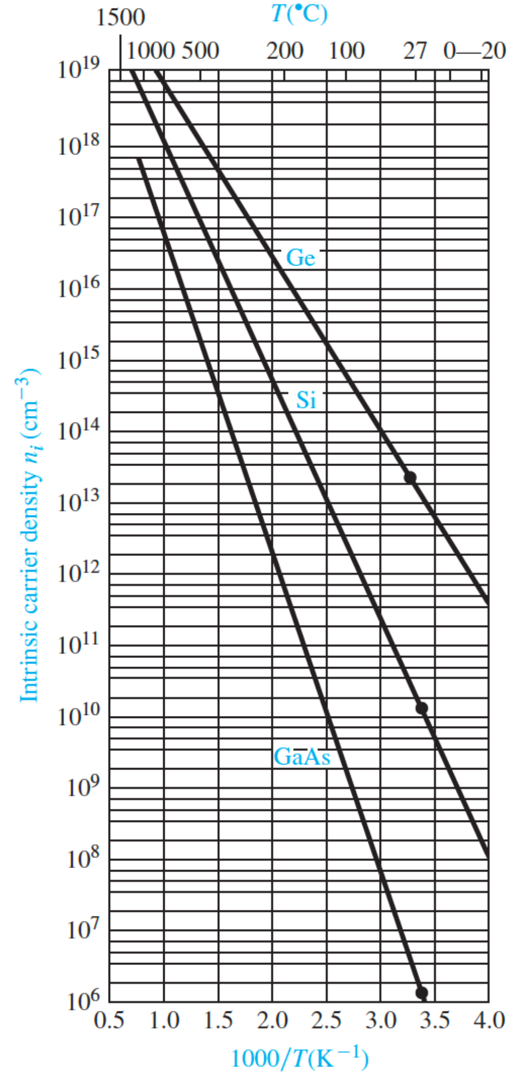


Figure 4.2 | The intrinsic carrier concentration of Ge, Si, and GaAs as a function of temperature.
(From Sze [14].)

Check your understanding

Problem Example #2

Calculate the intrinsic carrier concentration in silicon at T=250K and at 400K.

Objective: Calculate the intrinsic carrier concentration in silicon at $T = 250$ K and at $T = 400$ K.

The values of N_c and N_v for silicon at $T = 300$ K are 2.8×10^{19} cm^{-3} and 1.04×10^{19} cm^{-3}, respectively. Both N_c and N_v vary as $T^{3/2}$. Assume the bandgap energy of silicon is 1.12 eV and does not vary over this temperature range.

■ **Solution**

Using Equation (4.23), we find, at $T = 250$ K

$$n_i^2 = (2.8 \times 10^{19})(1.04 \times 10^{19})\left(\frac{250}{300}\right)^3 \exp\left[\frac{-1.12}{(0.0259)(250/300)}\right]$$

$$= 4.90 \times 10^{15}$$

or

$$n_i = 7.0 \times 10^7 \text{ cm}^{-3}$$

At $T = 400$ K, we find

$$n_i^2 = (2.8 \times 10^{19})(1.04 \times 10^{19})\left(\frac{400}{300}\right)^3 \exp\left[\frac{-1.12}{(0.0259)(400/300)}\right]$$

$$= 5.67 \times 10^{24}$$

or

$$n_i = 2.38 \times 10^{12} \text{ cm}^{-3}$$

4.1 Charge carriers in semiconductors

The intrinsic Fermi-level position

$$n_0 = N_c \exp\left(\frac{E_{Fi} - E_c}{kT}\right) = p_0 = N_v \exp\left(\frac{E_v - E_{Fi}}{kT}\right)$$

$$E_{Fi} = \frac{1}{2}(E_c + E_v) + \frac{1}{2}kTln\left(\frac{N_v}{N_c}\right)$$

$$E_{midgap} = \frac{1}{2}(E_c + E_v)$$

$$E_{Fi} = E_{midgap} + \frac{3}{4}kTln\left(\frac{m_p^*}{m_n^*}\right)$$

Outline

4.1 Charge carriers in semiconductors

4.2 Dopant atoms and energy levels

4.3 The extrinsic semiconductor

4.4 Statistics of donors and acceptors

4.5 Charge neutrality

4.6 Position of Fermi energy level

4.2 Dopant atoms and energy levels

Qualitative description

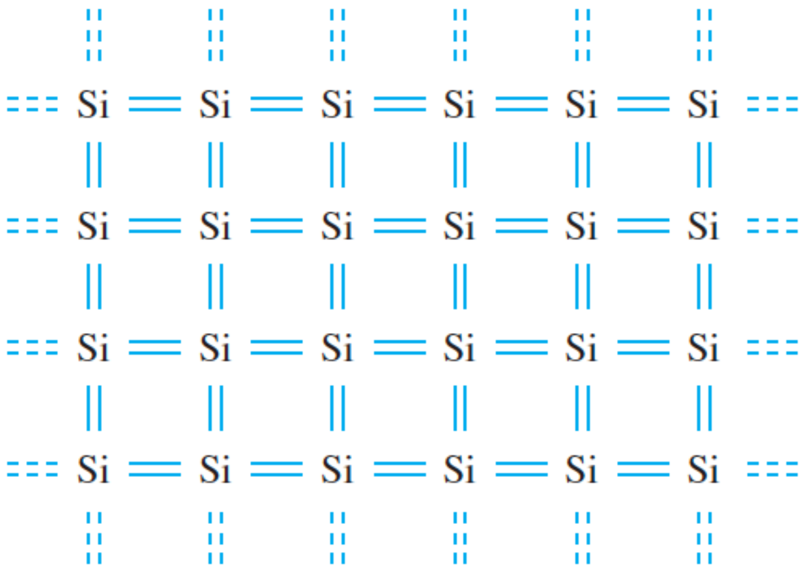


Figure 4.3 | Two-dimensional representation of the intrinsic silicon lattice.

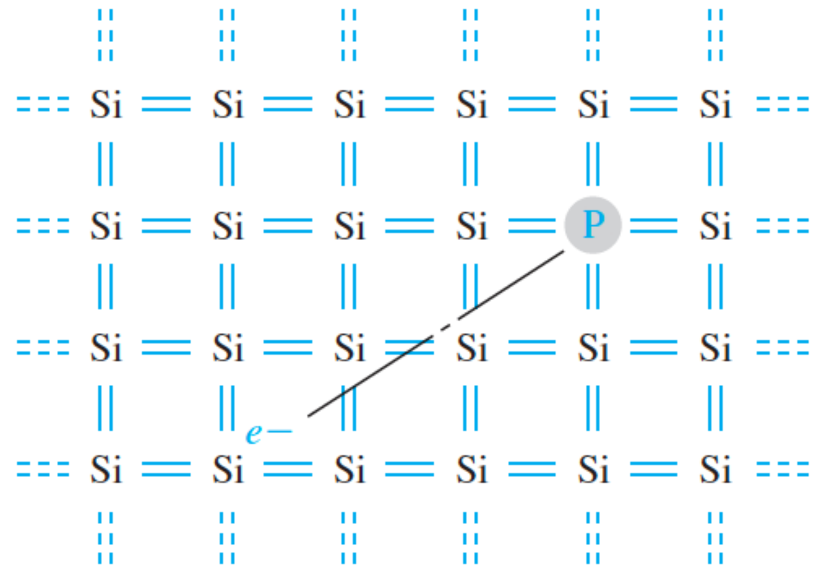
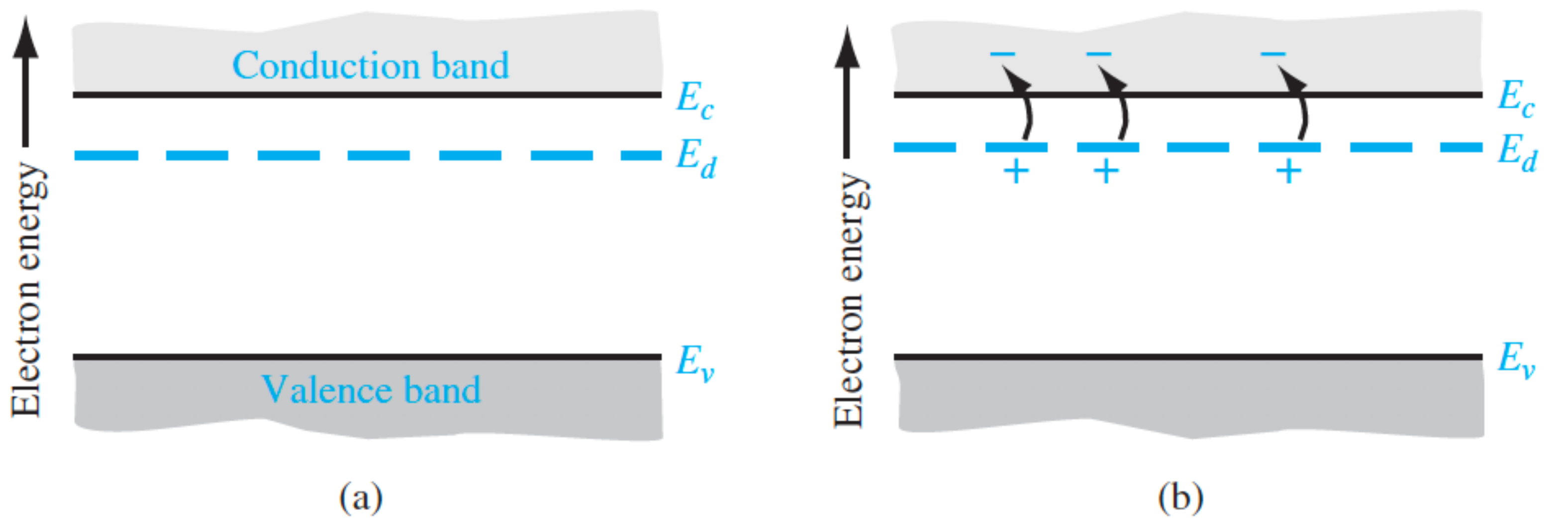


Figure 4.4 | Two-dimensional representation of the silicon lattice doped with a phosphorus atom.

4.2 Dopant atoms and energy levels

Qualitative description



4.2 Dopant atoms and energy levels

Qualitative description

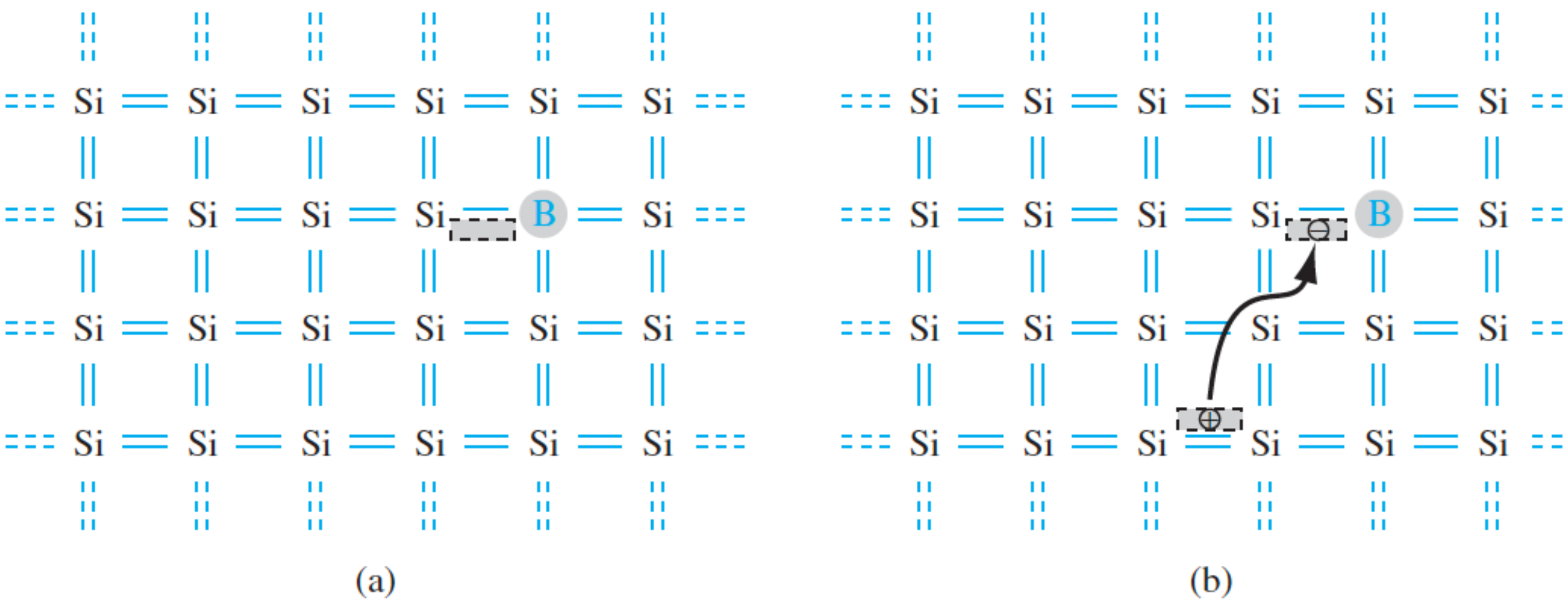


Figure 4.6 | Two-dimensional representation of a silicon lattice (a) doped with a boron atom and (b) showing the ionization of the boron atom resulting in a hole.

4.2 Dopant atoms and energy levels

Qualitative description

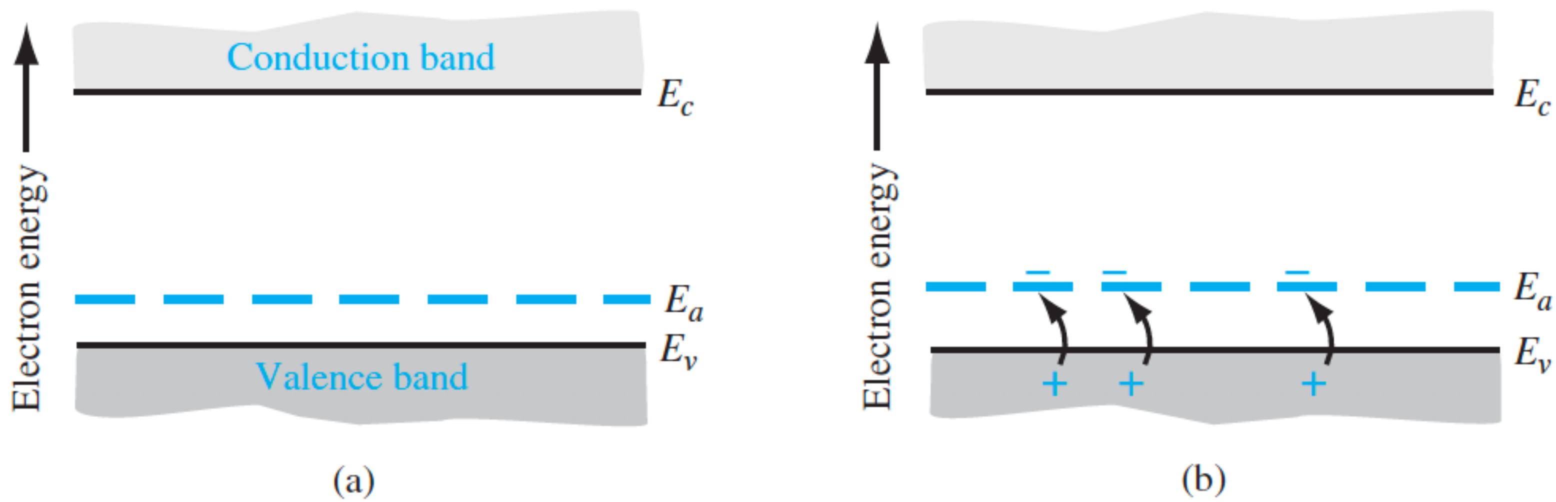
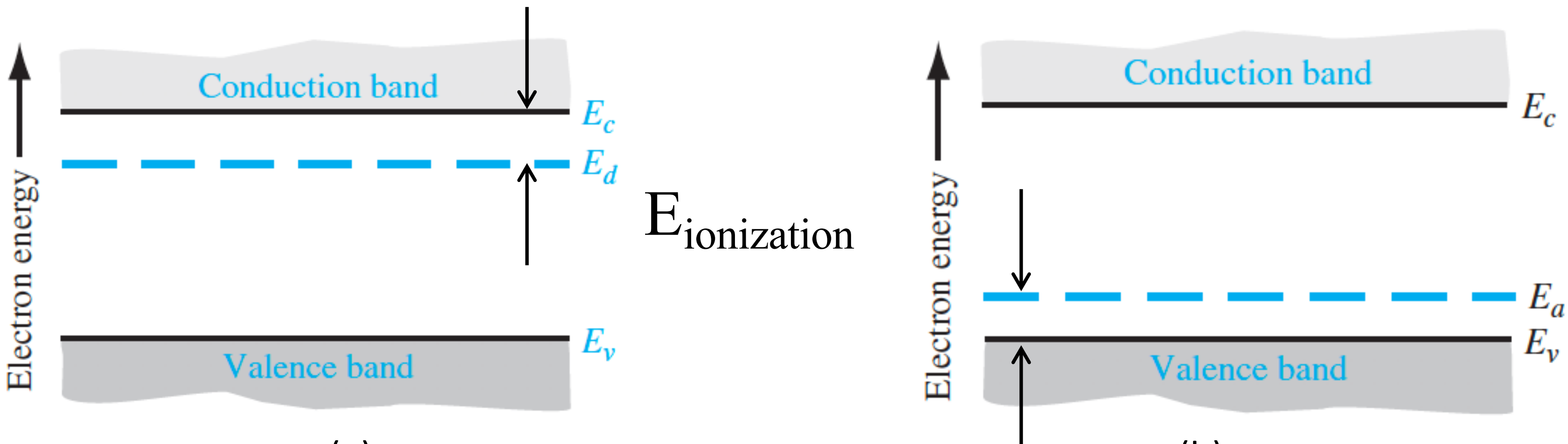


Figure 4.7 | The energy-band diagram showing (a) the discrete acceptor energy state and (b) the effect of an acceptor state being ionized.

4.2 Dopant atoms and energy levels

Ionization energy



(a) (b)

$$E_{ionization}=E_c-E_d$$

$$E_{ionization}=E_a-E_v$$

4.2 Dopant atoms and energy levels

<u>Ionization energy</u>

Table 4.3 | Impurity ionization energies in silicon and germanium

	Ionization energy (eV)	
Impurity	**Si**	**Ge**
Donors		
Phosphorus	0.045	0.012
Arsenic	0.05	0.0127
Acceptors		
Boron	0.045	0.0104
Aluminum	0.06	0.0102

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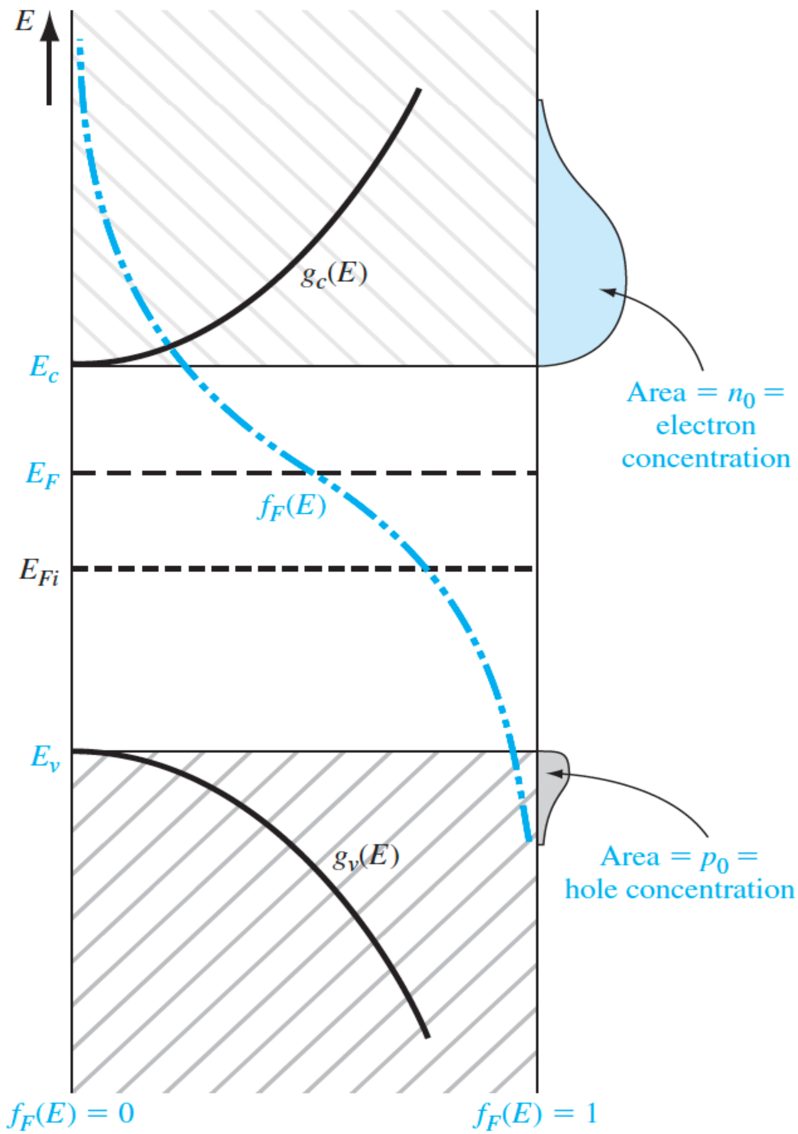
4.3 The extrinsic semiconductor

Equilibrium distribution of electrons and holes

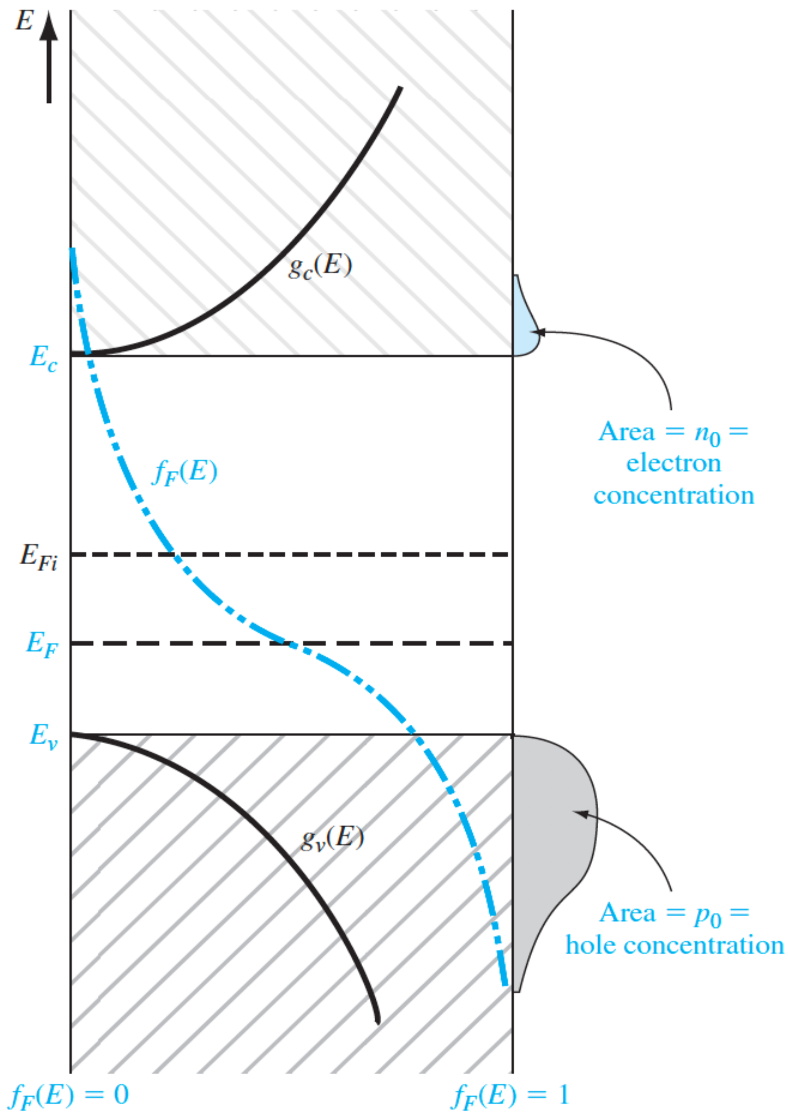


$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

$$p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

4.3 The extrinsic semiconductor

Equilibrium distribution of electrons and holes



$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

$$p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

4.3 The extrinsic semiconductor

The n_0p_0 product

$$n_0 = N_c \exp(\frac{E_F - E_c}{kT}) \qquad p_0 = N_v \exp(\frac{E_v - E_F}{kT})$$

$$n_0p_0 = N_cN_v \exp(\frac{E_F - E_c}{kT})\exp(\frac{E_v - E_F}{kT})$$

$$= N_cN_v \exp\left(\frac{E_v - E_c}{kT}\right) = N_cN_v \exp\left(-\frac{E_g}{kT}\right)$$

$$= constant$$

If $n_0 = p_0 = n_i$, this constant is equal to $n_i^2 = n_0p_0$

4.3 The extrinsic semiconductor

The n_0p_0 product

$$E_{Fi} = \frac{1}{2}(E_c + E_v) + \frac{1}{2}kTln(\frac{N_v}{N_c})$$

$$n_0 = N_c \exp(\frac{E_F - E_c}{kT}) \qquad p_0 = N_v \exp(\frac{E_v - E_F}{kT})$$

$$n_0 = N_c \exp(\frac{E_F - \mathrm{E}_i + \mathrm{E}_i - E_c}{kT}) = N_c \exp(\frac{E_F - \mathrm{E}_i}{kT})\exp(\frac{\mathrm{E}_i - E_c}{kT})$$

$$= N_c \exp(\frac{E_F - \mathrm{E}_i}{kT})\exp(\frac{\frac{1}{2}(E_v - E_c) + \frac{1}{2}kTln(\frac{N_v}{N_c})}{kT})$$

$$= N_c \exp(\frac{E_F - \mathrm{E}_i}{kT})\exp(\frac{E_v - E_c}{2kT})\sqrt{\frac{N_v}{N_c}}$$

$$= \exp\left(\frac{E_F - \mathrm{E}_i}{kT}\right)\exp\left(\frac{E_v - E_c}{2kT}\right)\sqrt{N_c N_v} = n_i \exp\left(\frac{E_F - \mathrm{E}_i}{kT}\right)$$

4.3 The extrinsic semiconductor

<u>The n_0p_0 product</u>

$$n_0 = N_c \exp(\frac{E_F - E_c}{kT})$$

$$p_0 = N_v \exp(\frac{E_v - E_F}{kT})$$

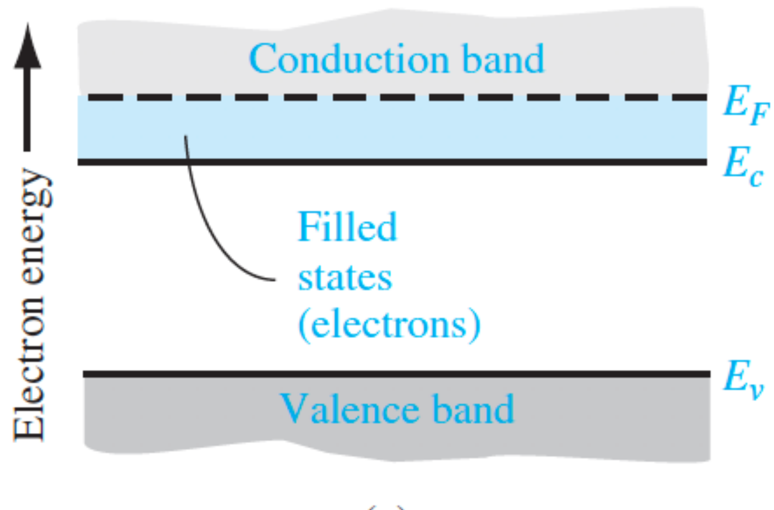
$$n_0 = n_i \exp(\frac{E_F - E_i}{kT})$$

$$p_0 = n_i \exp(\frac{E_i - E_F}{kT})$$

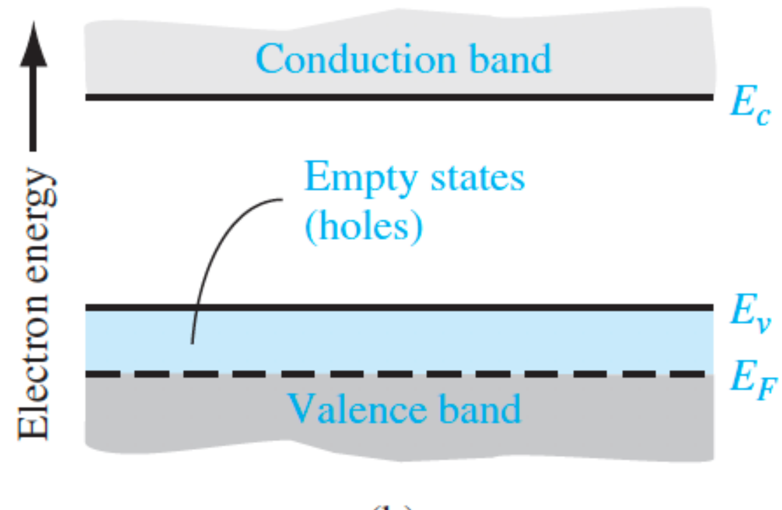
$$\boxed{n_i^2 = n_0 p_0}$$

4.3 The extrinsic semiconductor

Degenerate and nondegenerate semiconductors



(a)



(b)

Degenerate semiconductors:

- Extremely high doping concentration
- Fermi level in the band
- Electron cloud in dopants overlap,
- dopant energy level splitting

4.3 The extrinsic semiconductor

Problem Example #3

Determine the thermal-equilibrium concentrations of electrons and holes in silicon at T = 300K if the Fermi energy level E_F is 0.215eV above the valence band energy E_V. $N_V = 1.04 \times 10^{19}$ cm^{-3}, $n_i = 1.5 \times 10^{10}$ cm^{-3}.

$$
\begin{aligned}
p_0 &= N_v \exp\left(\frac{E_v - E_F}{kT}\right) \\
&= N_v \exp\left(\frac{-0.215}{kT}\right) \\
&= 2.58\times10^{15}\ \mathrm{cm}^{-3}
\end{aligned}
$$

$$
\begin{aligned}
n_0 &= \frac{n_i^2}{p_0} \\
&= 87155
\end{aligned}
$$

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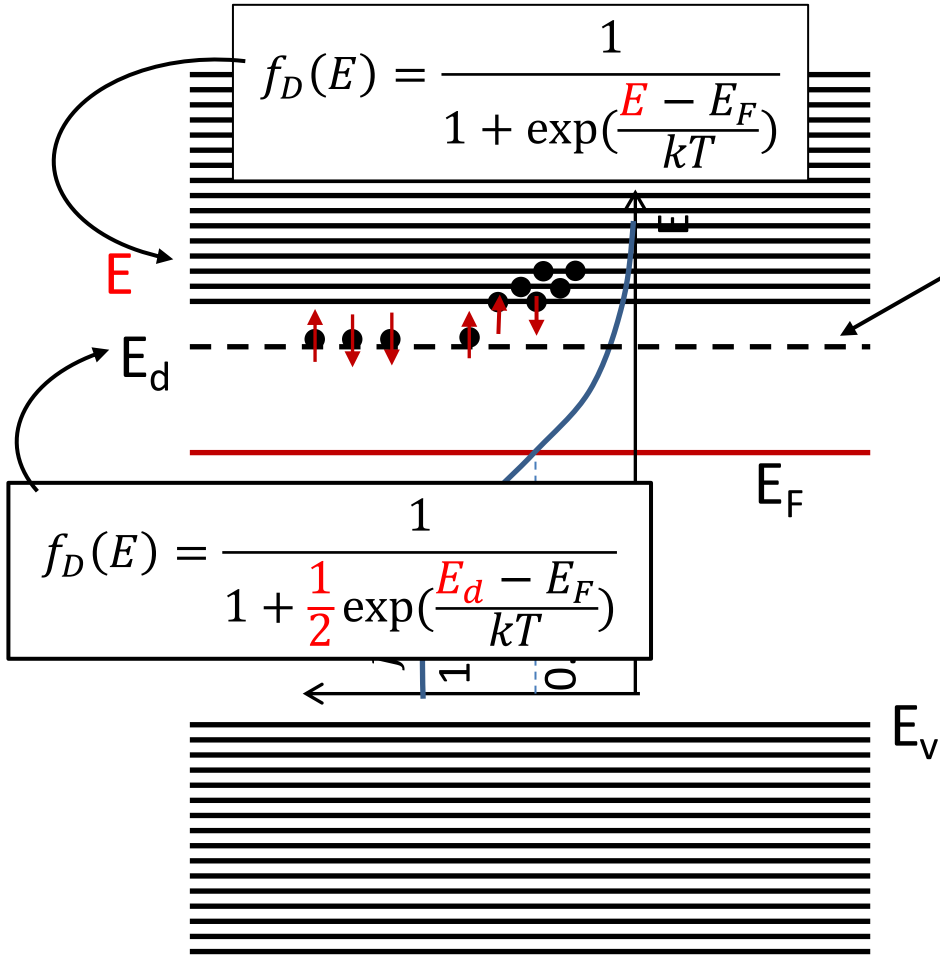
4.4 Statistics of donors and acceptors

Probability function



$$f_D(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}$$

$$f_D(E) = \frac{1}{1 + \frac{1}{2}\exp(\frac{E_d - E_F}{kT})}$$

Given the concentration of donors is N_d

The concentration of electrons on these donors is n_d

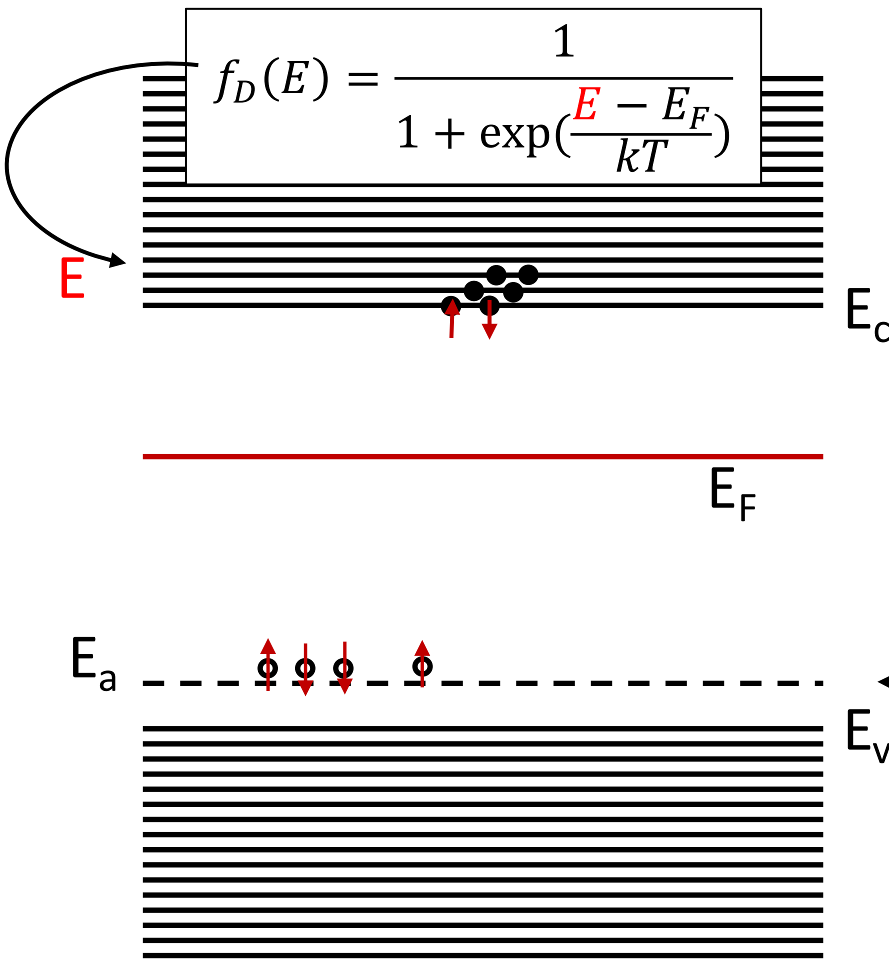
$$n_d = N_d - N_d^+$$

$$= \frac{N_d}{1 + \frac{1}{2}\exp(\frac{E_d - E_F}{kT})}$$

4.4 Statistics of donors and acceptors

Probability function

$$f_D(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}$$



The concentration of holes on these acceptors is n_d

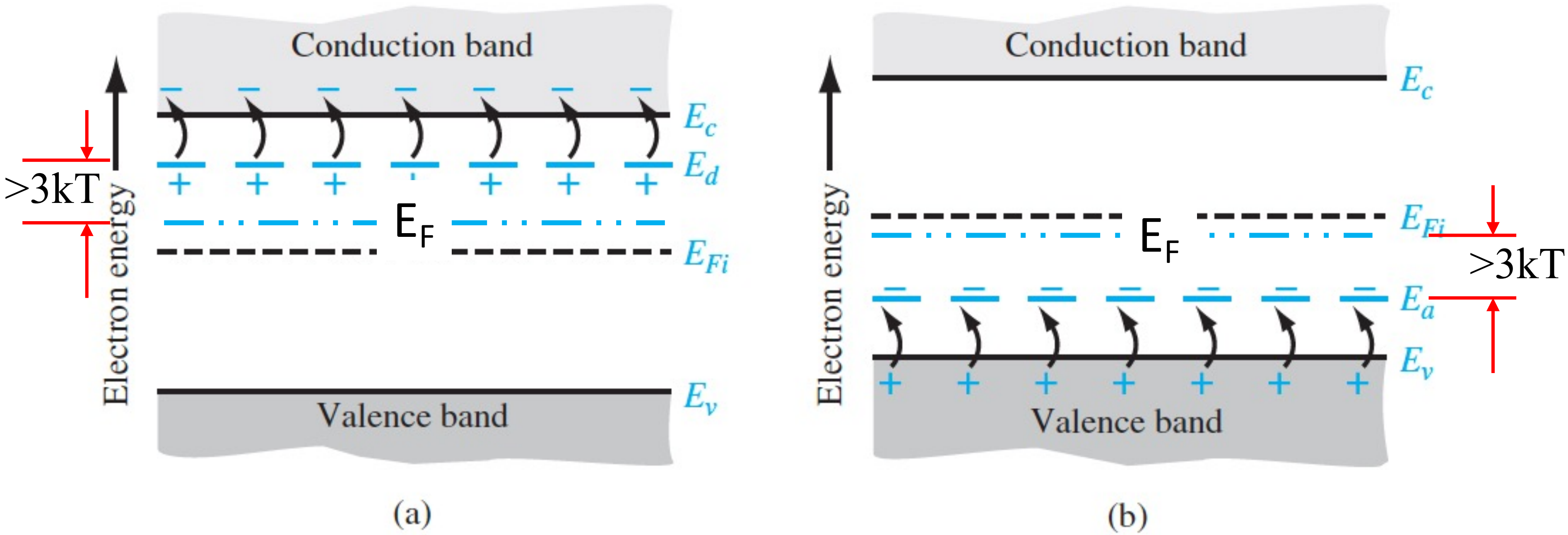
$$p_a = N_a - N_a^- = \frac{N_a}{1+\frac{1}{g}\exp\left(\frac{E_d-E_F}{kT}\right)}$$

(g=4 for Si, GaAs …)

Given the concentration of acceptors is N_a

4.4 Statistics of donors and acceptors

<u>Complete ionization</u>

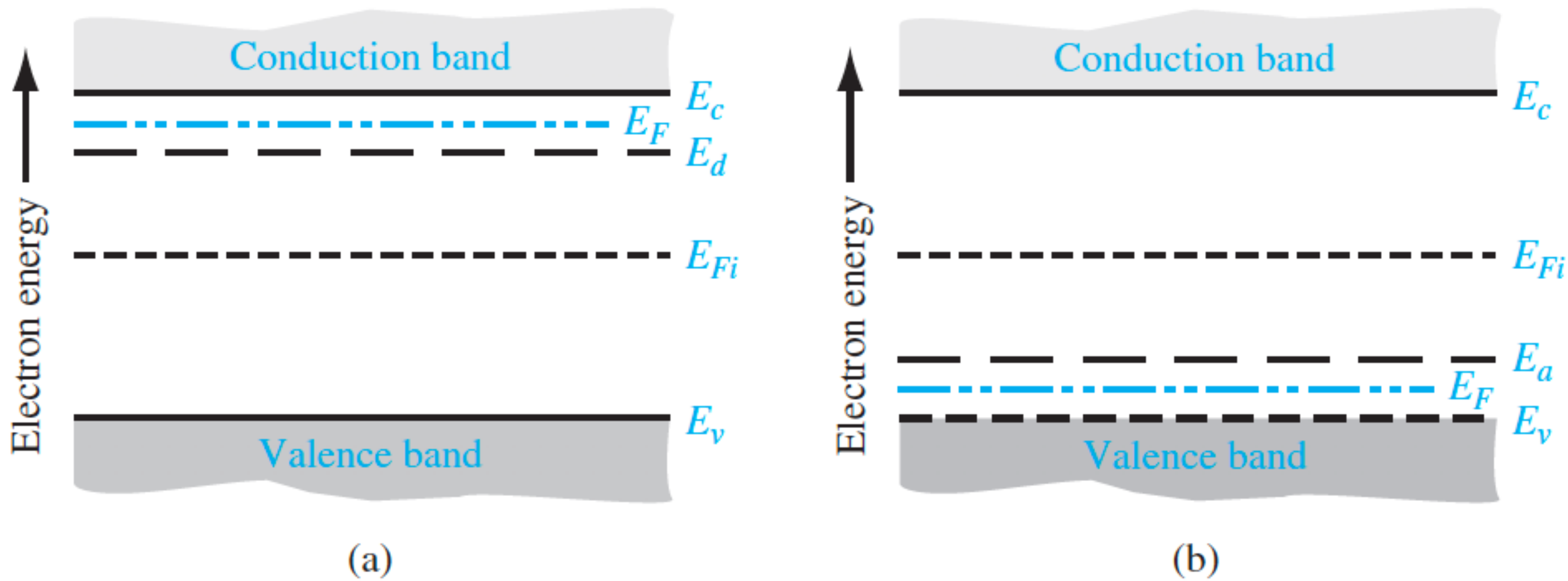


$$n_d = N_d - N_d^+ = \frac{N_d}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)} = 2N_d \exp\left(-\frac{E_d - E_F}{kT}\right)$$

>10

4.4 Statistics of donors and acceptors

<u>Complete freeze-out</u>



$$n_d = N_d - N_d^+ = \frac{N_d}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)} \approx N_d$$

$\rightarrow 0$

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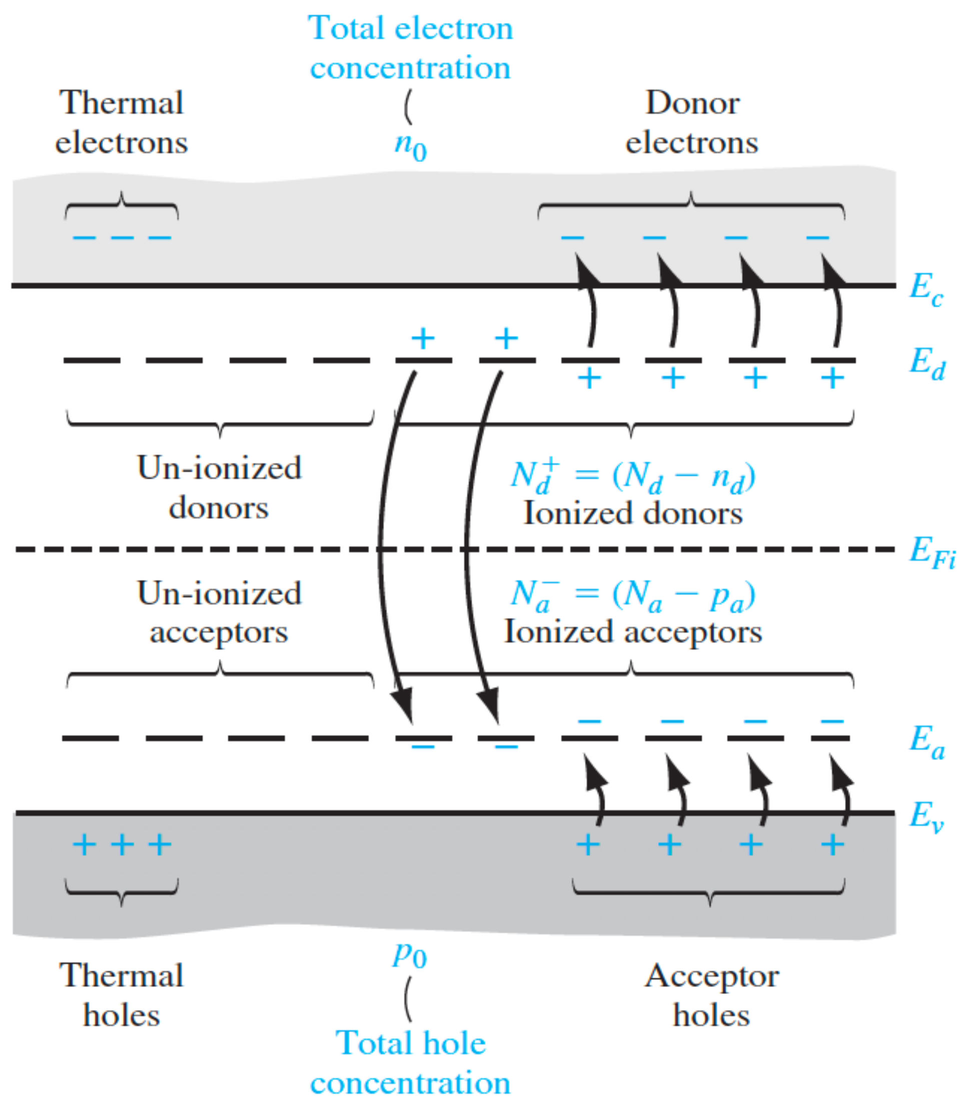
4.5 Charge neutrality

<u>Compensated semiconductor</u>

- $N_d > N_a$:
  n-type compensated (N_d-N_a)
- $N_a > N_d$:
  p-type compensated (N_a-N_d)
- $N_d = N_a$:
  completely compensated, like intrinsic semiconductors



4.5 Charge neutrality

Equilibrium electron and hole concentration

Charge neutrality:

$$\boxed{n_0 + N_a^- = N_d^+ + p_0}$$

Or

$$\boxed{n_0 = N_d^+ - N_a^- + p_0}$$

Complete ionization

$$\Longrightarrow \boxed{n_0 = N_d - N_a + p_0}$$

$$\Big\Downarrow \; n_0 p_0 = n_i^2$$

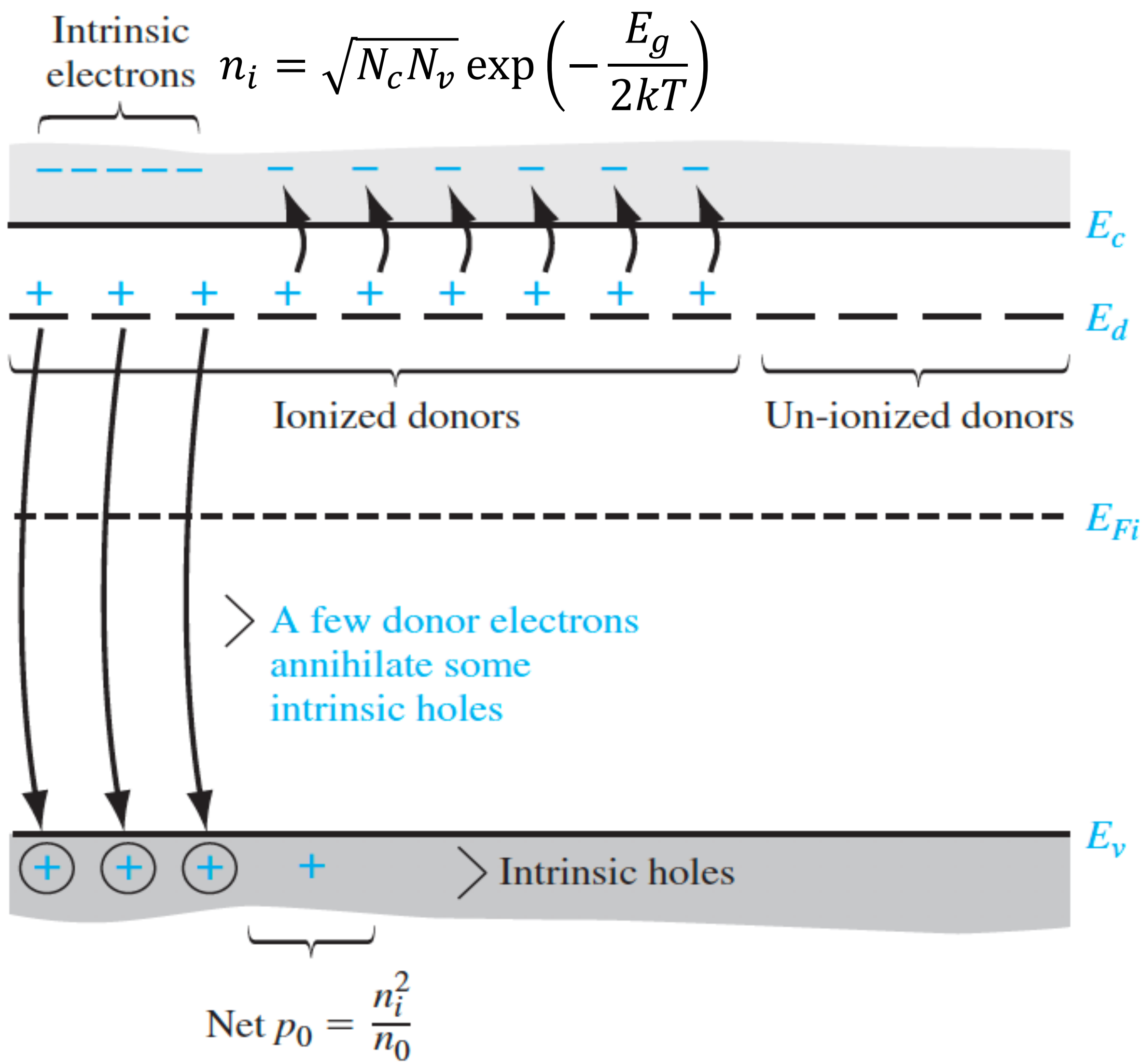
$$n_0 = \frac{N_d - N_a + \sqrt{(N_d - N_a)^2 + 4n_i^2}}{2}$$

Problem Example #4

Determine the thermal-equilibrium electron and hole concentrations in silicon at T = 300K for given doping concentrations. Complete ionization is assumed. (a) Let $N_d = 10^{11} cm^{-3}$ and $N_a=0$. (b) Let $N_d = 10^{12} cm^{-3}$ and $N_a=0$. (c) Let $N_d = 5 \times 10^{15}$ cm^{-3} and $N_a = 2 \times 10^{15}$ cm^{-3}. (b) Let Nd = 10^{11} cm^{-3}.

4.5 Charge neutrality

Equilibrium electron and hole concentration



4.5 Charge neutrality

Equilibrium electron and hole concentration

$$n_0 = \frac{N_d^+ + \sqrt{(N_d^+)^2 + 4n_i^2}}{2} \text{ } (but\ N_d^+ \text{ unknown})$$

① $n_i >> N_d^+ \Rightarrow T\ very\ high$

$$\boxed{n_0 = p_0 = n_i = \sqrt{N_c N_v} \exp(-\frac{E_g}{2kT})}$$

4.5 Charge neutrality

<u>Equilibrium electron and hole concentration</u>

$$n_0 = \frac{N_d^+ + \sqrt{(N_d^+)^2 + 4n_i^2}}{2} \ (but\ N_d^+ \text{ unknown})$$

① $n_i >> N_d^+ \Rightarrow T\ very\ high$

$$\boxed{n_0 = p_0 = n_i = \sqrt{N_c N_v} \exp(-\frac{E_g}{2kT})}$$

② $n_i << N_d^+ \Rightarrow T\ not\ very\ high$ (meaning charge carriers mostly come from dopants, which is often true for a doped semiconductor)

$$\boxed{n_0 = N_d^+}$$

4.5 Charge neutrality

Equilibrium electron and hole concentration

$$\boxed{n_0 = N_d^+}$$

$$\boxed{n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) = N_D^+ = \frac{N_d}{1 + 2\exp(\frac{E_F - E_d}{kT})}}$$

$$\Downarrow$$

$$\boxed{\exp\left(\frac{E_F - E_c}{kT}\right) = \frac{n_0}{N_c}}$$

4.5 Charge neutrality

Equilibrium electron and hole concentration

$$\boxed{n_0 = N_d^+}$$

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) = N_D^+ = \frac{N_d}{1 + 2\exp(\frac{E_F - E_D}{kT})}$$

$$\Downarrow$$

$$\exp\left(\frac{E_F - E_c}{kT}\right) = \frac{n_0}{N_c}$$

$$\Downarrow$$

$$n_0 = \frac{N_d}{1 + 2\exp\left(\frac{E_c - E_d}{kT}\right)\exp(\frac{E_F - E_c}{kT})}$$

$$= \frac{n_0}{N_c}$$

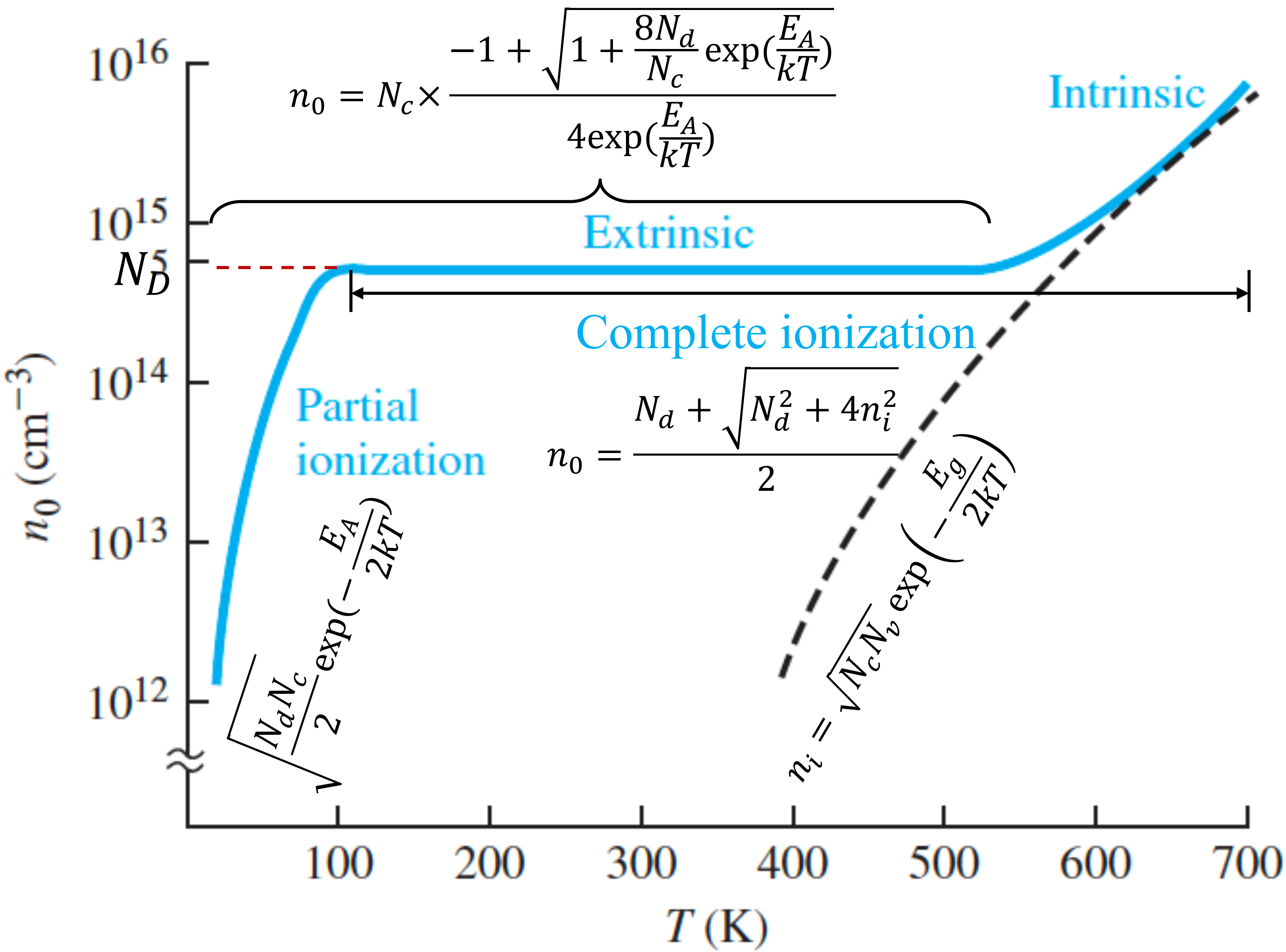
4.5 Charge neutrality

Equilibrium electron and hole concentration

$$\boxed{2\exp\left(\frac{E_A}{kT}\right)n_0^2 + N_c n_0 - N_d N_c = 0}$$

$$\boxed{n_0 = N_c \times \frac{-1 + \sqrt{1 + \frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})}}$$

Ionization of dopants



Outline

4.1 Charge carriers in semiconductors

4.2 Dopant atoms and energy levels

4.3 The extrinsic semiconductor

4.4 Statistics of donors and acceptors

4.5 Charge neutrality

4.6 Position of Fermi energy level

4.6 Position of Fermi energy level

Mathematical Derivation

$$\boxed{n_0 = N_c \times \frac{-1 + \sqrt{1 + \frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})}}$$

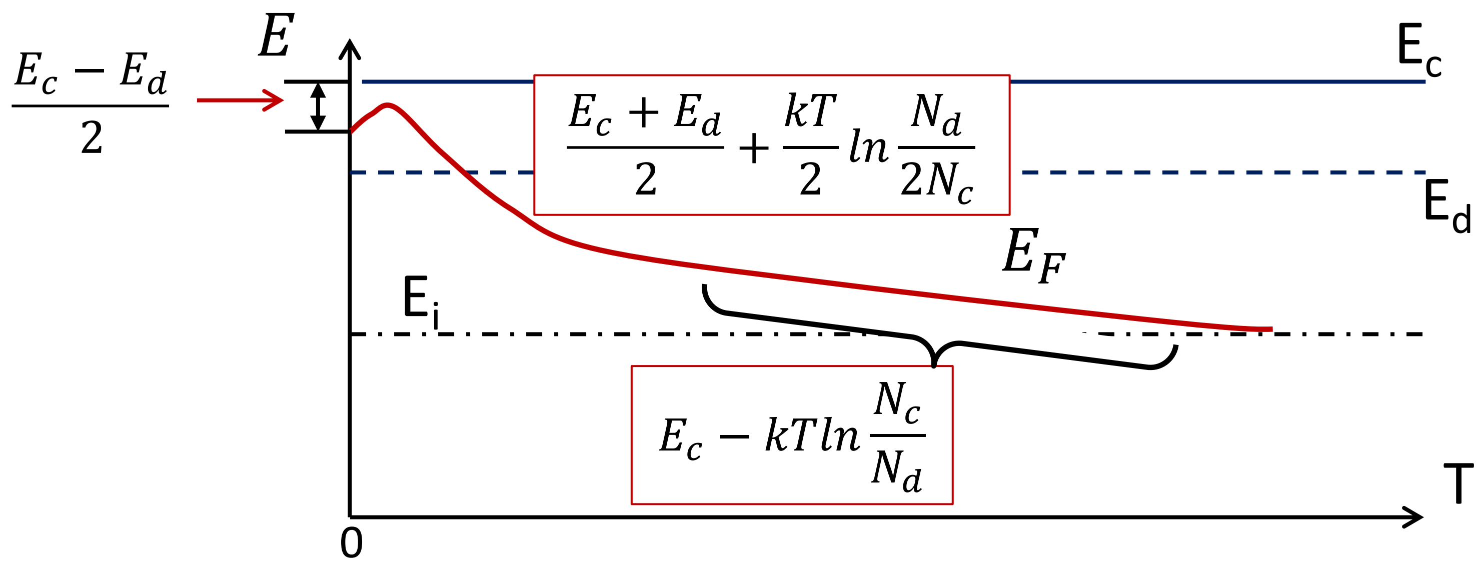
$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) \Rightarrow \ \exp\left(\frac{E_F - E_c}{kT}\right) = \frac{-1 + \sqrt{1 + \frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})}$$

$$E_F = E_c + kT ln(\frac{\sqrt{1 + \frac{8N_d}{N_c}\exp(\frac{E_A}{kT})} - 1}{4\exp(\frac{E_A}{kT})})$$

4.6 Position of Fermi energy level

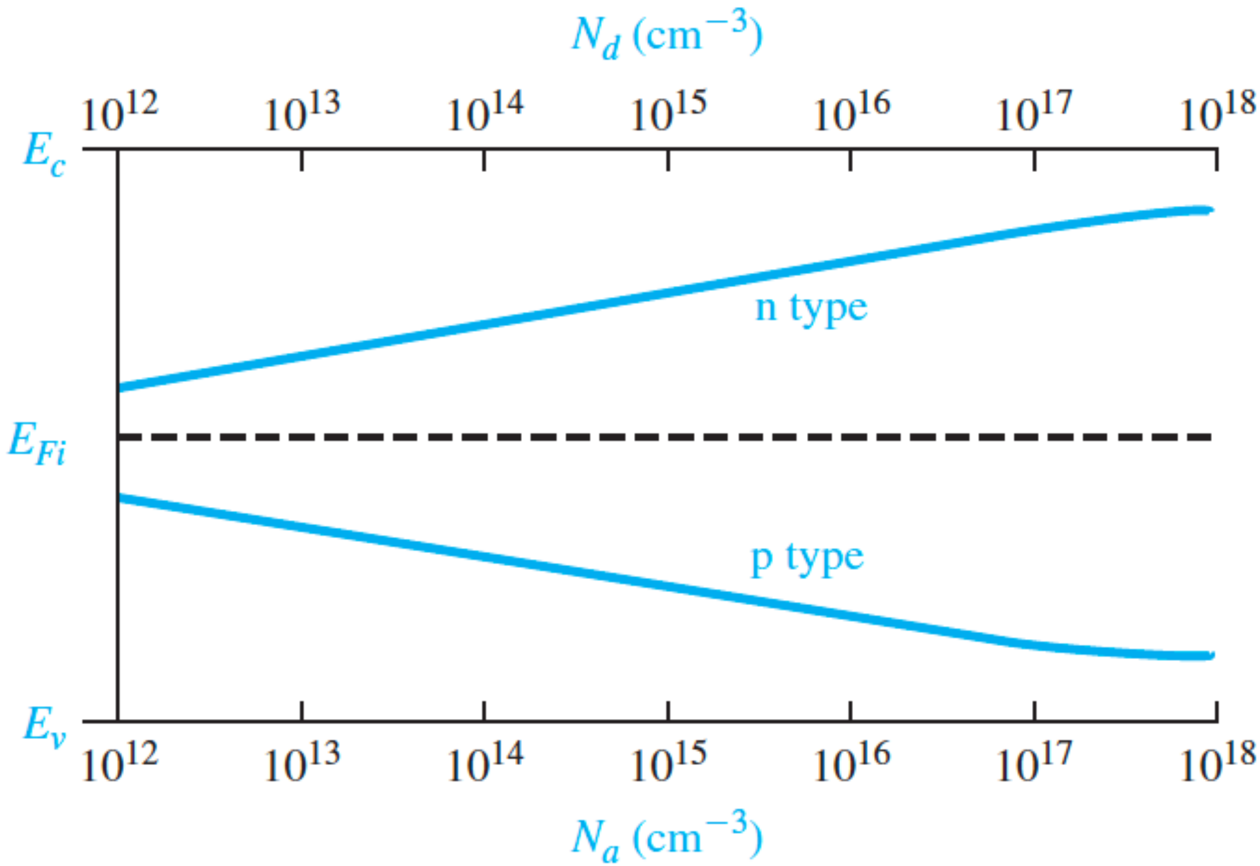
Mathematical Derivation

$$E_F = E_c + kT ln(\frac{\sqrt{1+\frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}-1}{4\exp(\frac{E_A}{kT})}) = \begin{cases} \frac{E_c+E_D}{2}+\frac{kT}{2}ln\frac{N_d}{2N_c} & T\ small \\ E_c - kT ln\frac{N_c}{N_d} & T\ big \end{cases}$$



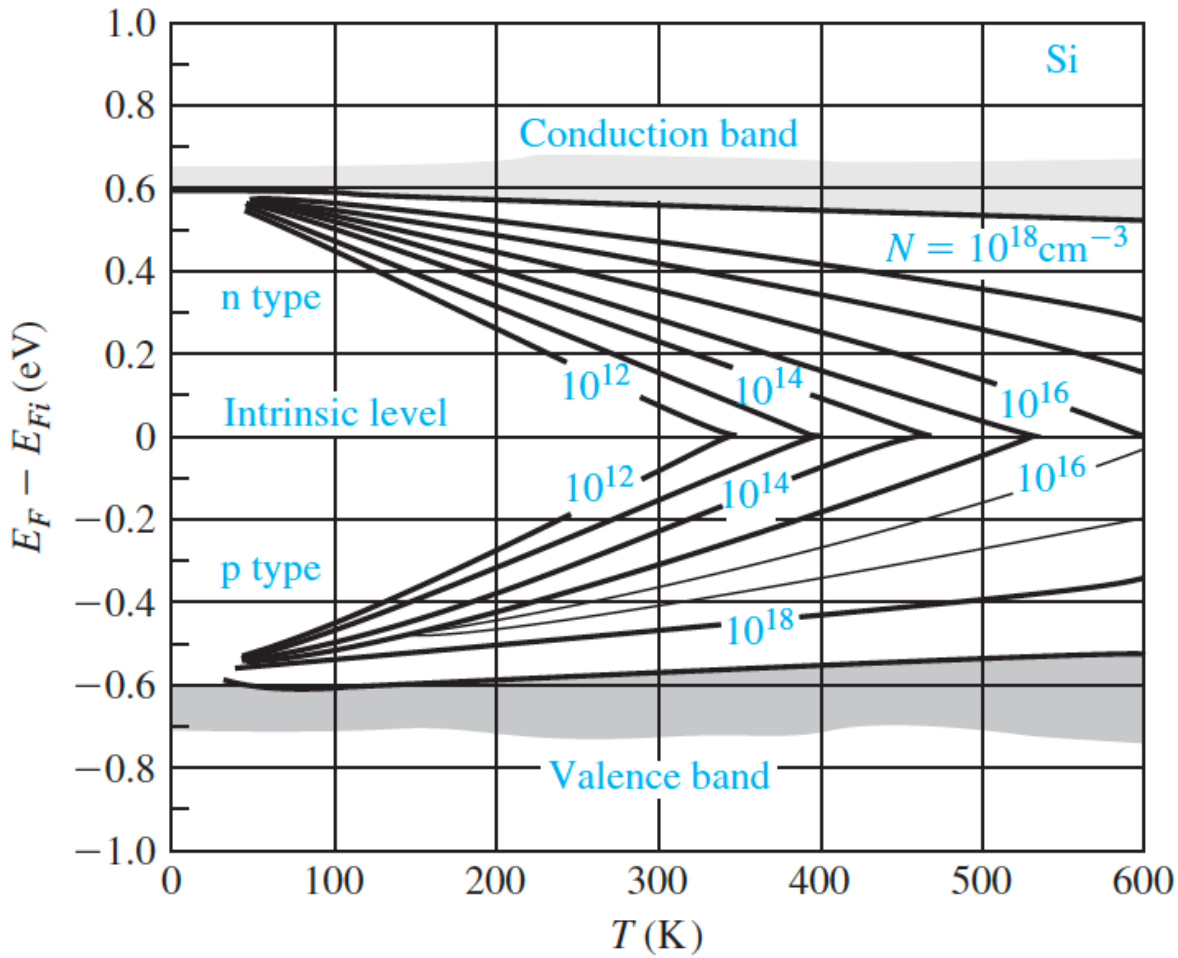
4.6 Position of Fermi energy level

Variation of E_F with doping concentration and temperature



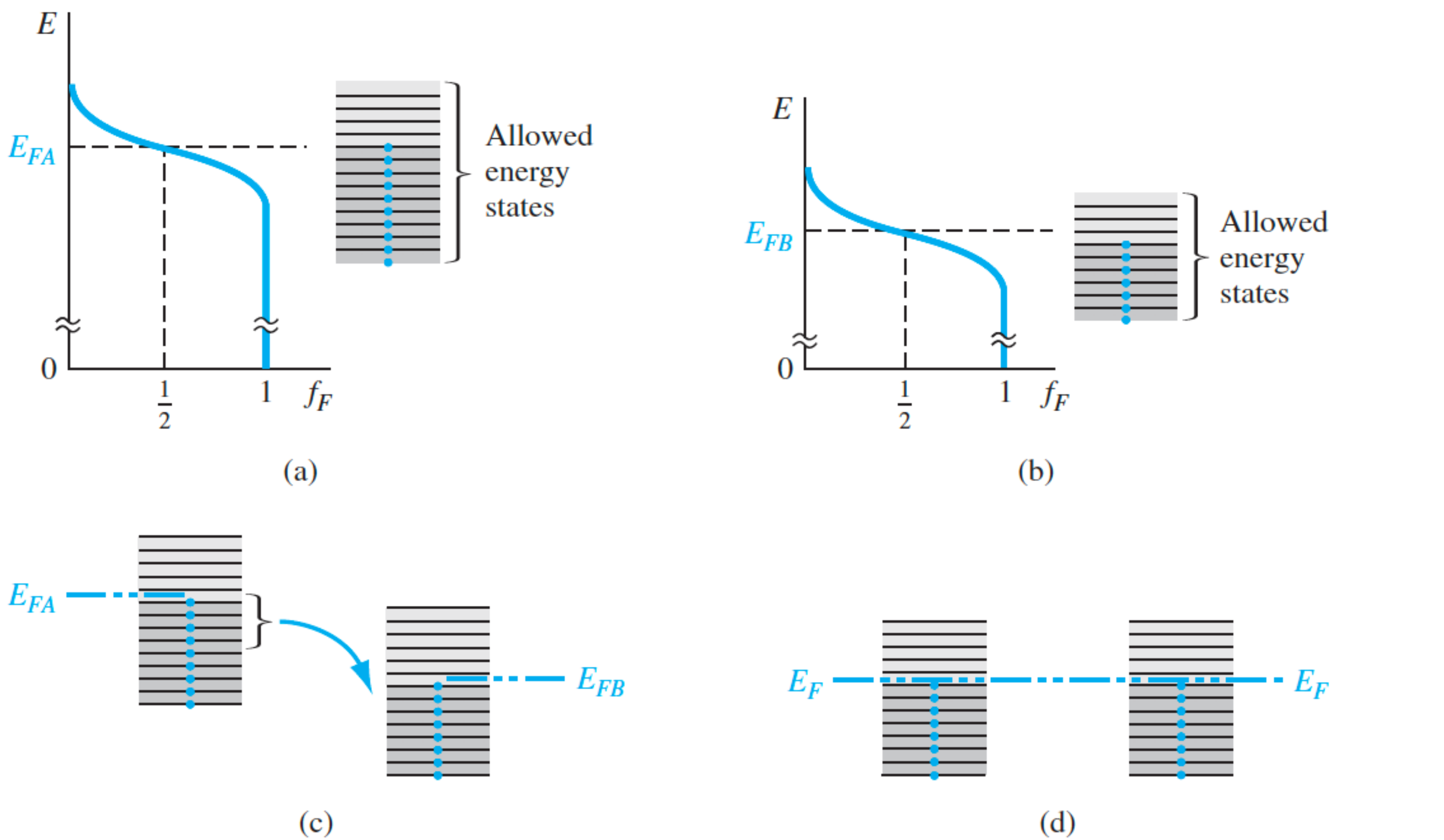
4.6 Position of Fermi energy level

Variation of E_F with doping concentration and temperature



4.6 Position of Fermi energy level

Relevance of Fermi energy



Summary

$$n_0 = N_c \times \frac{-1+\sqrt{1+\frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})} = \begin{cases} \sqrt{\frac{N_d N_c}{2}}\exp(-\frac{E_A}{2kT}) & \textit{partial ionization}, T\ \textit{low} \\ N_d & \textit{complete ionization}, T\ \textit{high} \end{cases}$$

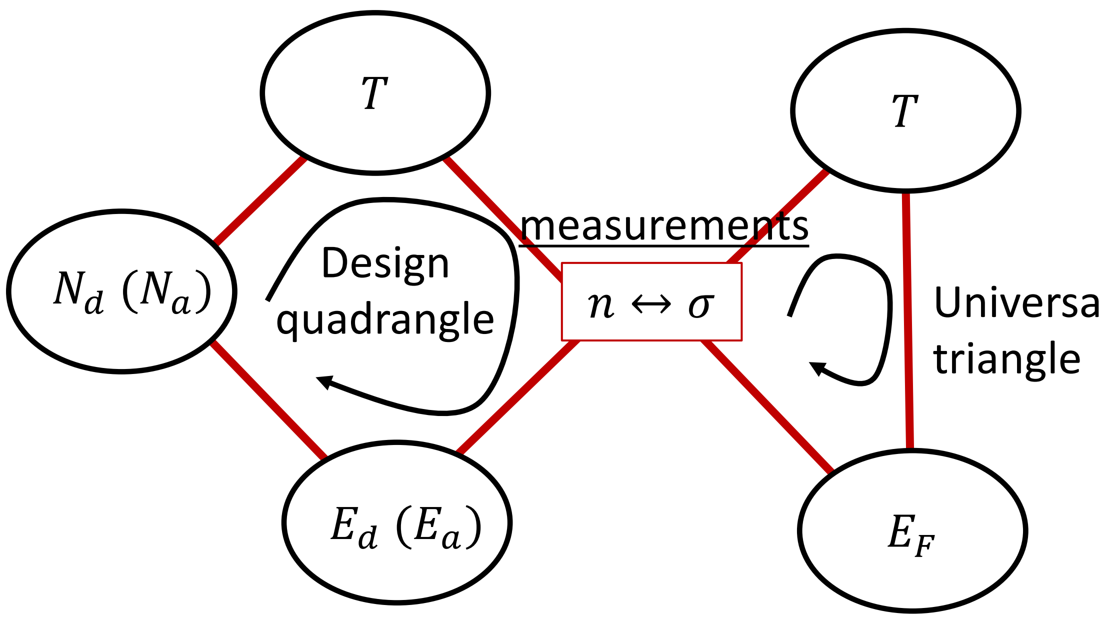
$$n_0 = \frac{N_d + \sqrt{N_d^2 + 4n_i^2}}{2}$$

Complete ionization at high T to intrinsic ionization at very high T

n_0 → p_0 and E_F → ionization rate

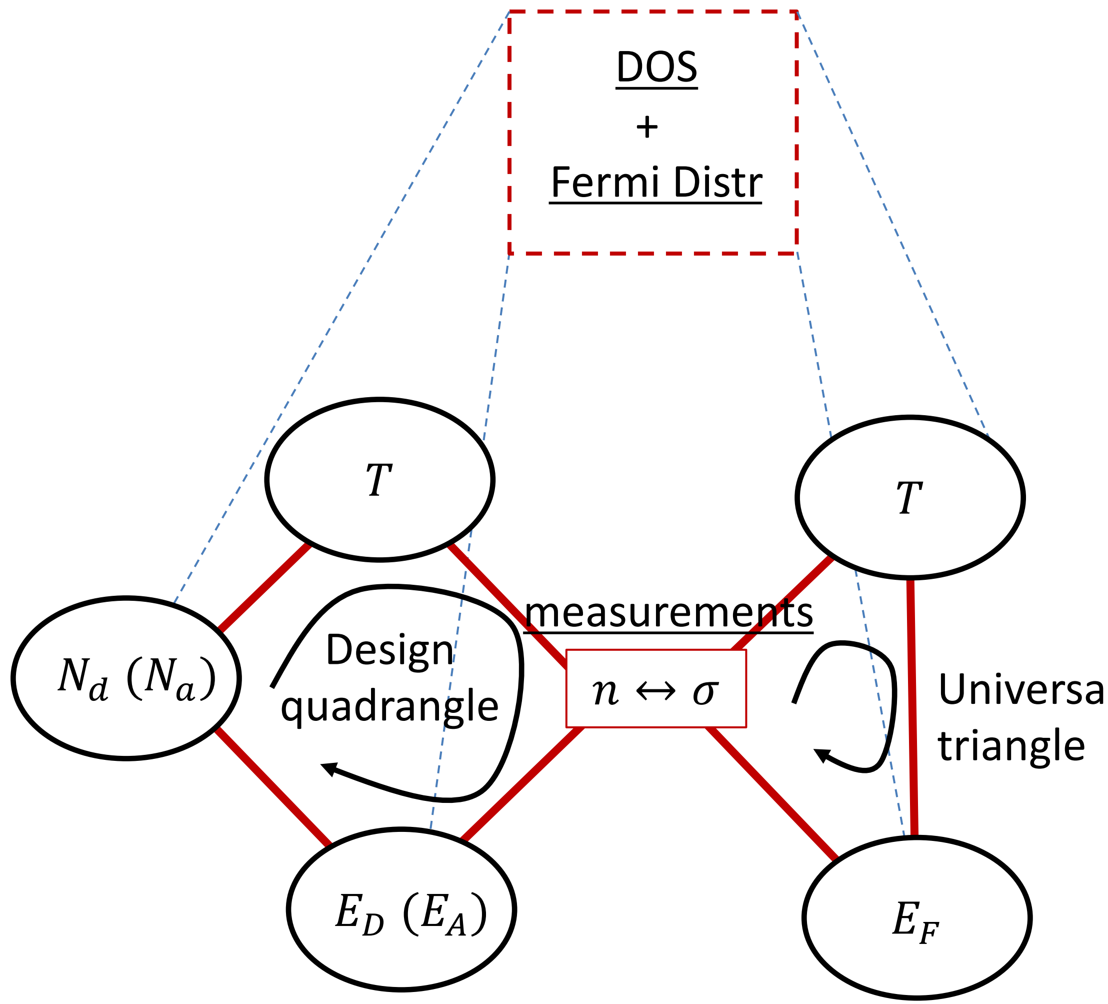
$$n_0 = N_c \times \frac{-1+\sqrt{1+\frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})} = \begin{cases} \sqrt{\frac{N_d N_c}{2}}\exp(-\frac{E_A}{2kT}) & \textit{partial ionization}, T\ \textit{low} \\ N_d & \textit{complete ionization}, T\ \textit{high} \end{cases}$$

Summary



T
N_d (N_a)
Design
quadrangle
E_d (E_a)
measurements
$n \leftrightarrow \sigma$
T
Universal
triangle
E_F

Summary


DOS
+
Fermi Distr
T
T
N_d (N_a)
Design
quadrangle
measurements
$n \leftrightarrow \sigma$
Universal
triangle
E_D (E_A)
E_F

Problem Example #5

Given a piece of silicon that is uniformly doped with impurities. The concentration of the impurities is 10^{17} cm^{-3} and the energy level of the impurities is 0.1eV below the conduction band. Calculate the electron concentration and Fermi energy level in silicon at 100K. N_c = 5.4x10^{18} cm^{-3} at 100K.

$$p_0 = \frac{n_i^2}{n_0}(T = 300K) = \frac{(1.5\times 10^{10}\times(1/3)^{3/2})^2}{1.578\times 10^{15}}$$

$$= \frac{(0.289\times 10^{10})^2}{1.578\times 10^{15}} = 5.28\times 10^3 cm^{-3}$$

$$\mathrm{n}_0 = \sqrt{\frac{N_d N_c}{2}}\exp\left(-\frac{E_A}{2kT}\right)$$

$$= \sqrt{\frac{10^{17}\times 5.4\times 10^{18}}{2}}\exp\left(-\frac{0.1\times 1.6\times 10^{-19}}{2\times 1.38\times 10^{-23}\times 100}\right)$$

$$= 1.578\times 10^{15}\ cm^{-3}$$

$$\mathrm{n}_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

$$E_F = \mathrm{E_c} + \mathrm{kT}\ln\frac{n_0}{N_c} = \mathrm{E_c} + \frac{0.0259}{3}\ \ln\frac{1.578\mathrm{e}15}{5.4\mathrm{e}18} = \mathrm{E_c} + \frac{0.0259}{3}\times(-8.13)$$

$$= E_c\ - 0.0702\ eV$$

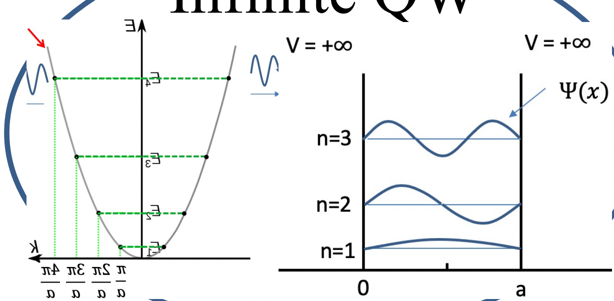
Overview from Chapter 1-4

2nd order differential equation → Wave function $Ae^{ikx} + Be^{-ikx}$

Schrodinger Equation ← Particle-wave duality

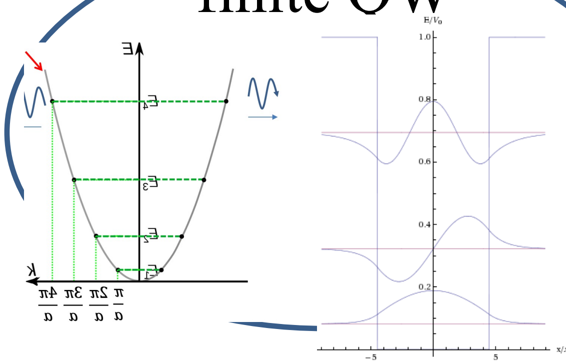
↓

Infinite QW



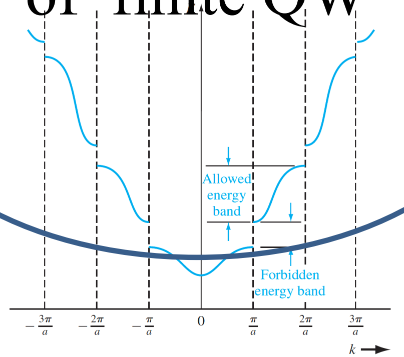
→

finite QW

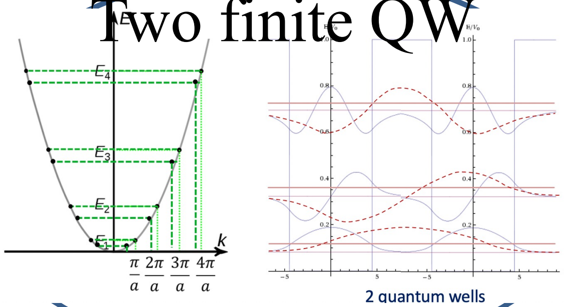


Overview from Chapter 1-4

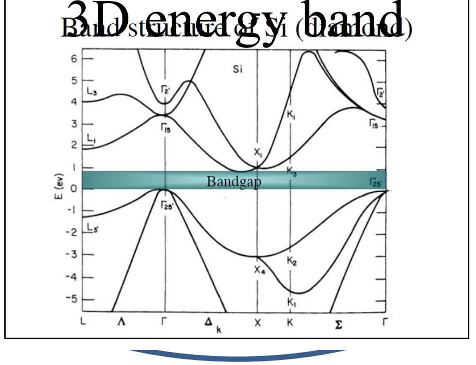
Infinite number of finite QW



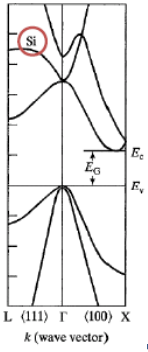
Two finite QW



3D energy band



Effective mass and E ~ k



Conduction Band:

$$E = E(k) = E_c + \frac{\hbar^2}{2m_n^*}(k-k_0)^2$$

Valence Band:

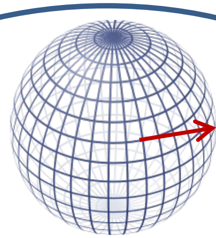
$$E = E(k) = E_c - \frac{\hbar^2}{2m_p^*}(k-k_0')^2$$

Overview from Chapter 1-4

To find electron concentration in conduction band

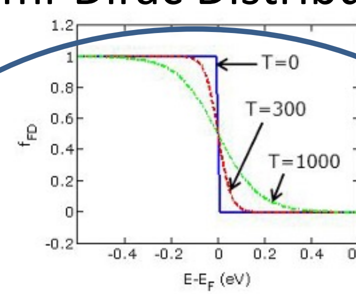
$$n_0 = \int_{E_C}^{+\propto} g(E) \cdot f_F(E) dE$$

Density of states

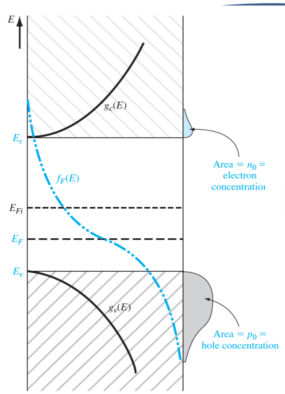


$$g(E) = 2\frac{2\pi(2m^*)^{3/2}}{h^3}\sqrt{E - E_c}$$

Fermi-Dirac Distribution



$$f_{FD}(E) = \frac{1}{\exp\left(\frac{E - E_F}{k_B T}\right) + 1}$$

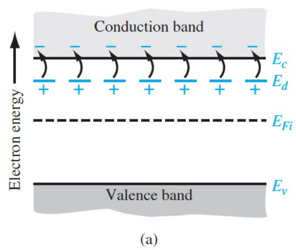


$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

$$p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

Overview from Chapter 1-4

Modified Fermi
Distribution for dopants



(a)

$$f_D(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}$$

$$n_0 = f(N_D, E_A, T)$$

$$n_0 = N_c \times \frac{-1 + \sqrt{1 + \frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})}$$