VE320 – Summer 2024

Introduction to Semiconductor Devices

Instructor: Yaping Dan （但亚平）
yaping.dan@sjtu.edu.cn

Chapter 12 Bipolar Junction Transistor

Outline

12.1 Review and example

12.2 Bipolar Junction transistor

12.3 Early Effect

12.4 Summary

12.5 Quantitative analysis of BJT gain

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12.1 Review and example

12.2 Bipolar Junction transistor

12.3 Early Effect

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12.5 Quantitative analysis of BJT gain

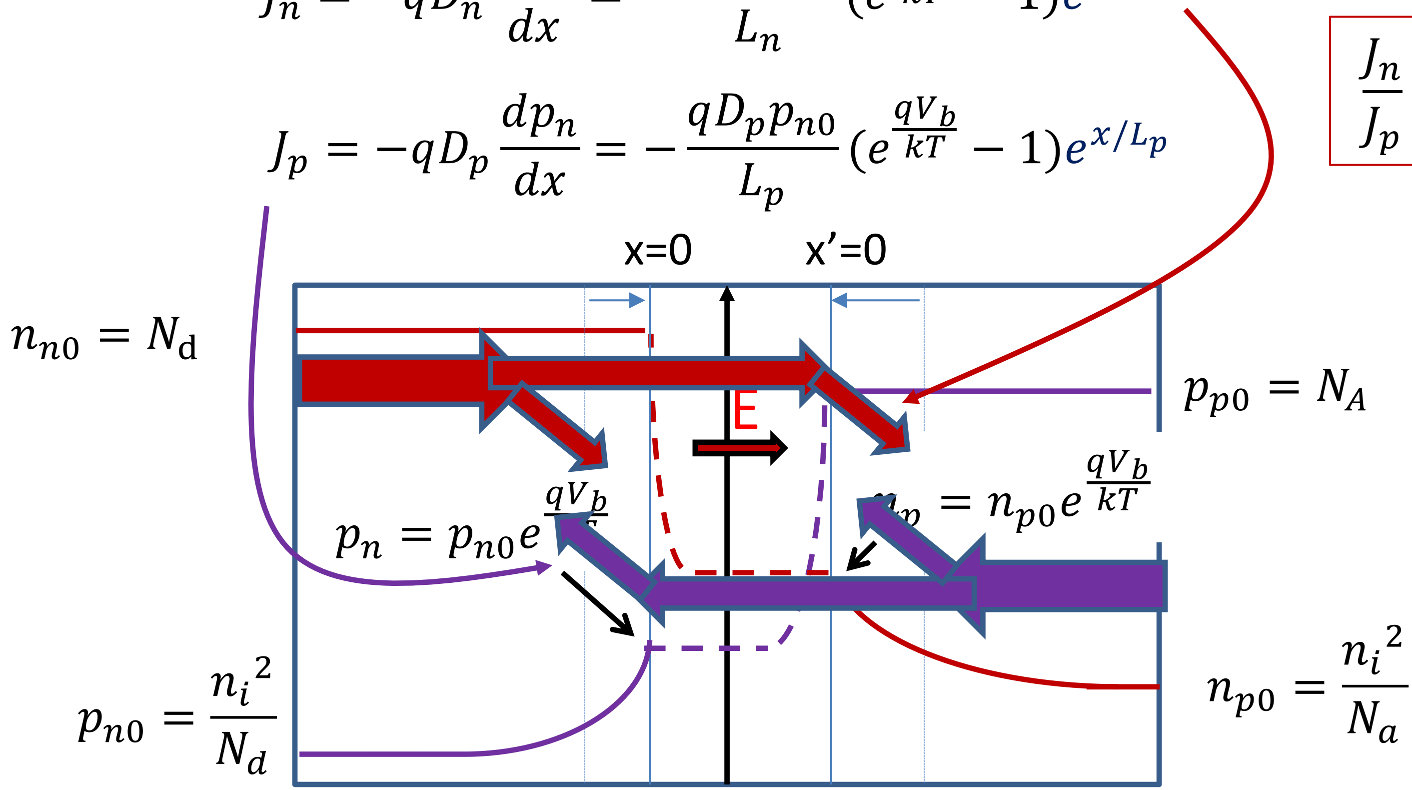
12.1 Previously: pn Junction Current

- charge carrier transport: forward bias: current ratio

$$J_n = -qD_n \frac{dn_p}{dx} = -\frac{qD_n n_{p0}}{L_n}\left(e^{\frac{qV_b}{kT}} - 1\right)e^{-x/L_n}$$

$$J_p = -qD_p \frac{dp_n}{dx} = -\frac{qD_p p_{n0}}{L_p}\left(e^{\frac{qV_b}{kT}} - 1\right)e^{x/L_p}$$

$$\boxed{\frac{J_n}{J_p} = \frac{D_n N_d / L_n}{D_p N_a / L_p}}$$



Assumption: No recombination-generation in depletion region.

12.1 Example: pn Junction Current

Finding L_n, τ_n in **p-type** region because electrons are minority carriers.

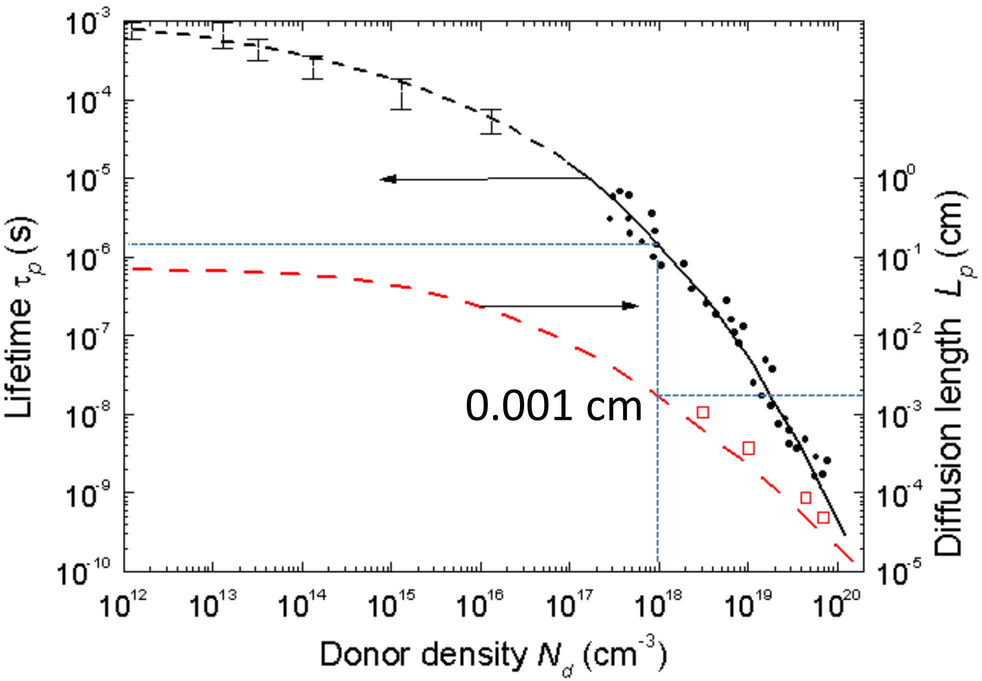
For $N_a=10^{16}\ cm^{-3}$ $\quad L_n = 0.04cm$ $\quad \tau_\mathrm{n} = 5\times10^{-5}s$

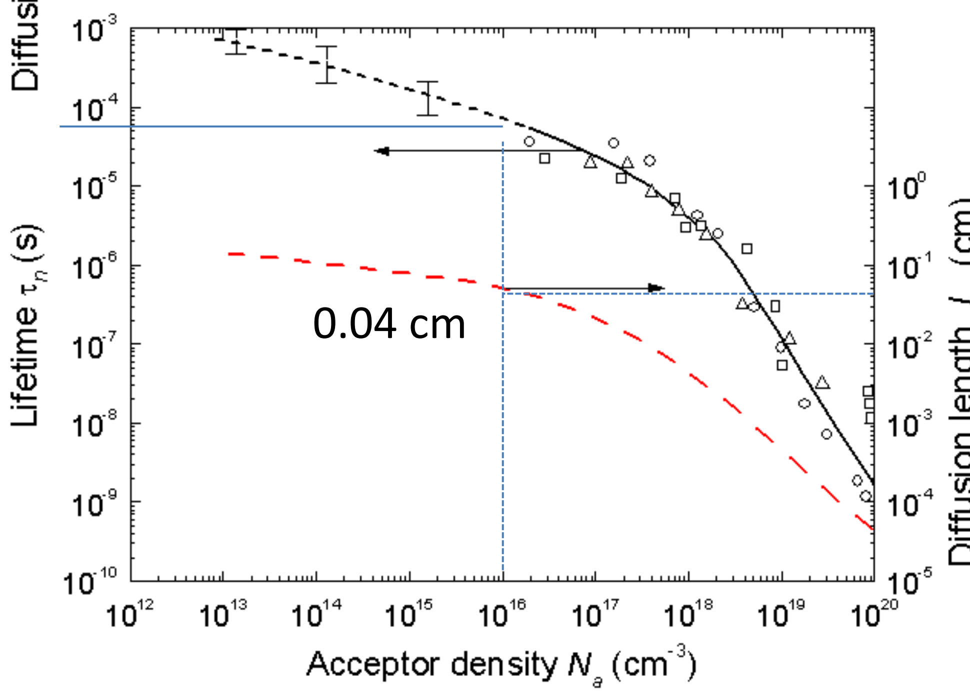
Finding L_p, τ_p in **n-type** region because holes are minority carriers.

For $N_d=10^{18}\ cm^{-3}$ $\quad L_p = 0.0015cm$ $\quad \tau_p = 1.5\times10^{-6}s$

$$\boxed{\frac{J_n}{J_p} = \frac{D_n N_d/L_n}{D_p N_a/L_p} = \frac{\mathrm{L}_n/\tau_n}{L_p/\tau_p}\frac{N_d}{N_a} \approx \frac{\frac{4\times10^{-2}}{5\times10^{-5}}}{\frac{1.5\times10^{-3}}{1.5\times10^{-6}}} \times \frac{10^{18}}{10^{16}} = 80}$$

12.1 Example: pn Junction Current





http://www.ioffe.ru/SVA/NSM/Semicond/Si/

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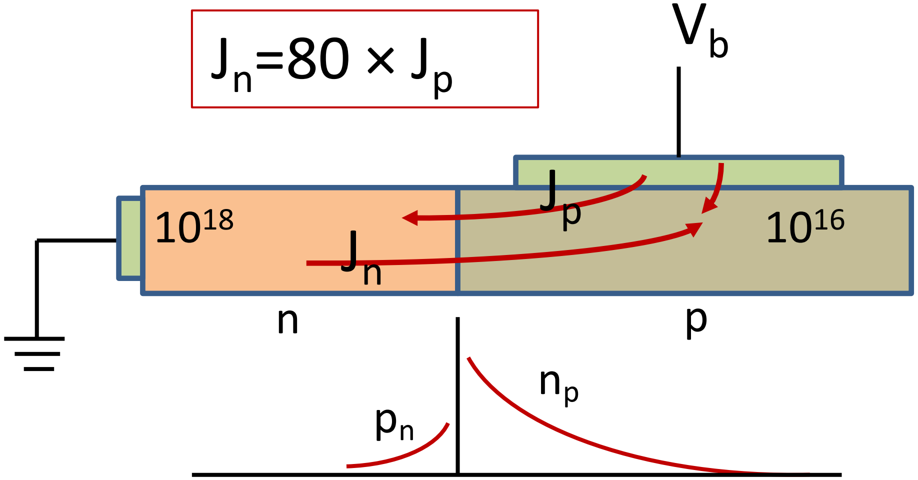
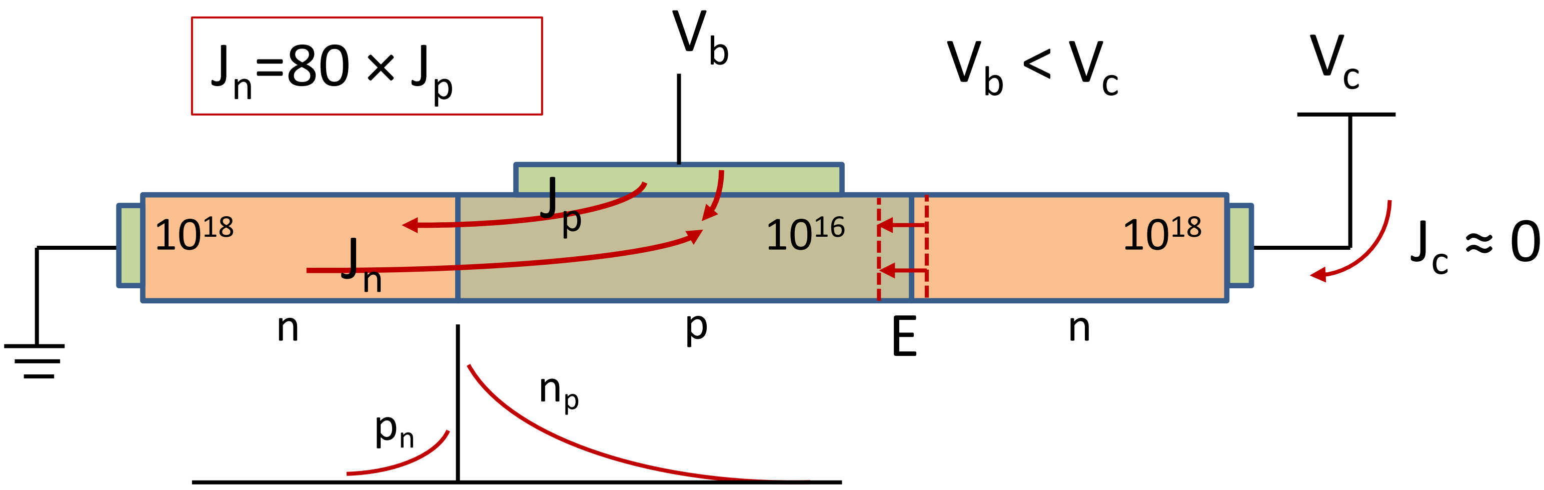
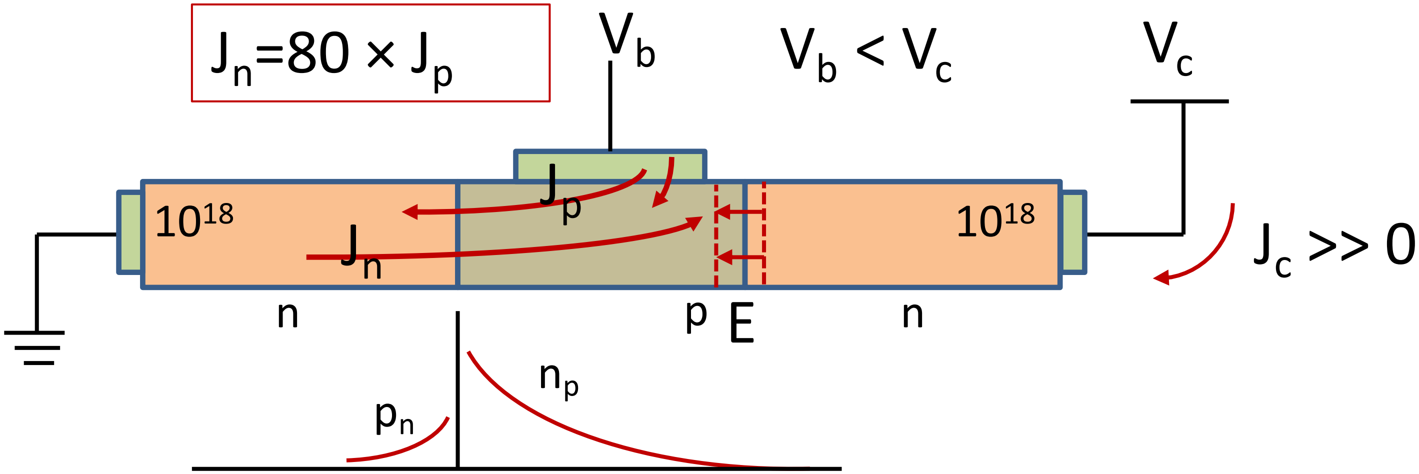
12.2 Bipolar Junction transistor



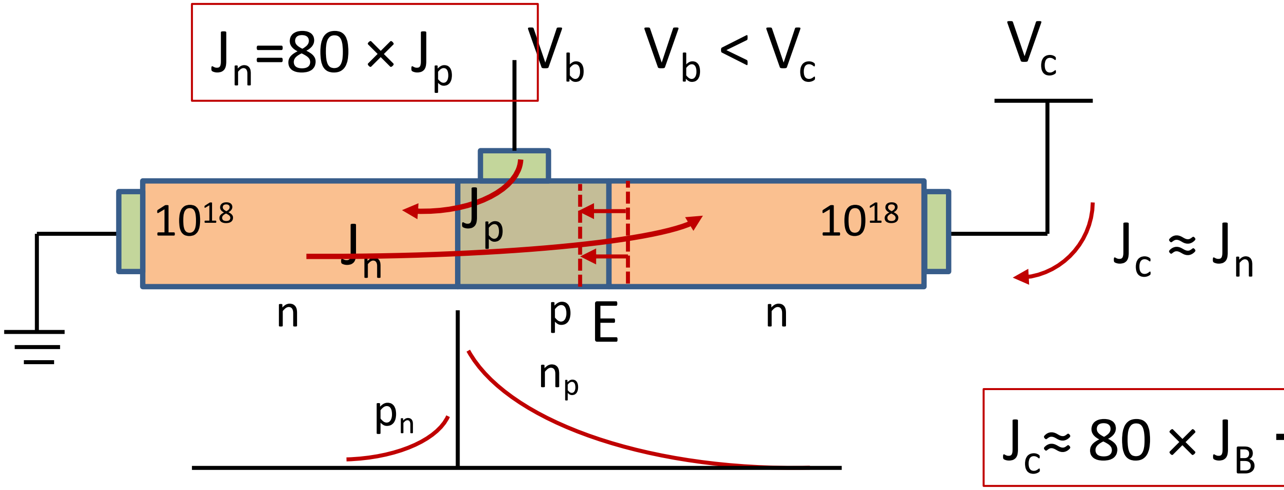
12.2 Bipolar Junction transistor



12.2 Bipolar Junction transistor

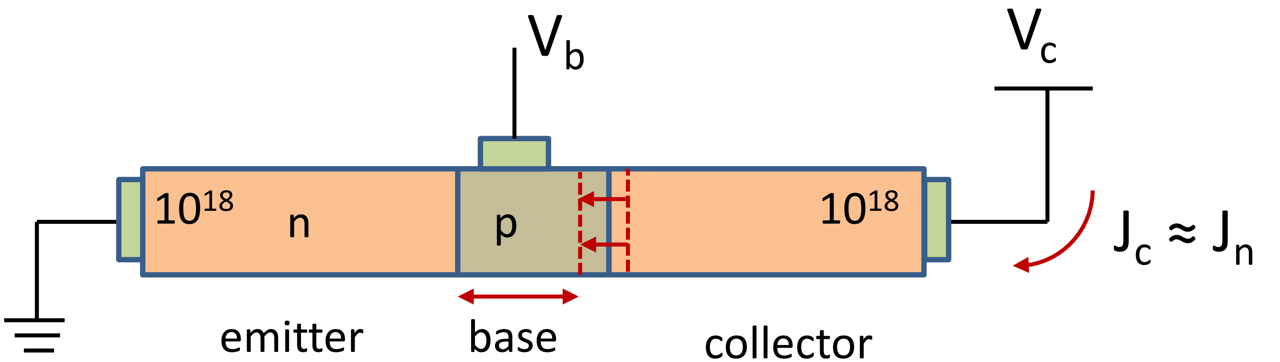


12.2 Bipolar Junction transistor



$J_c \approx 80 \times J_B \rightarrow$ Gain β=80

12.2 Bipolar Junction transistor

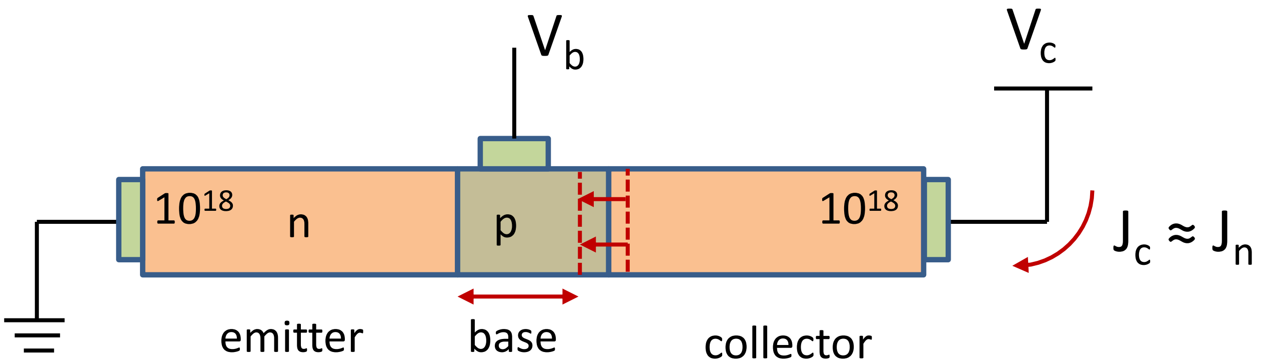


$$\boxed{J_c \approx 80 \times J_B \rightarrow \text{Gain } \beta = 80}$$

BJT Charateristics:

1. Base width smaller → higher gain
2. Larger emitter-base concentration ratio → higher gain

12.2 Bipolar Junction transistor: I-V



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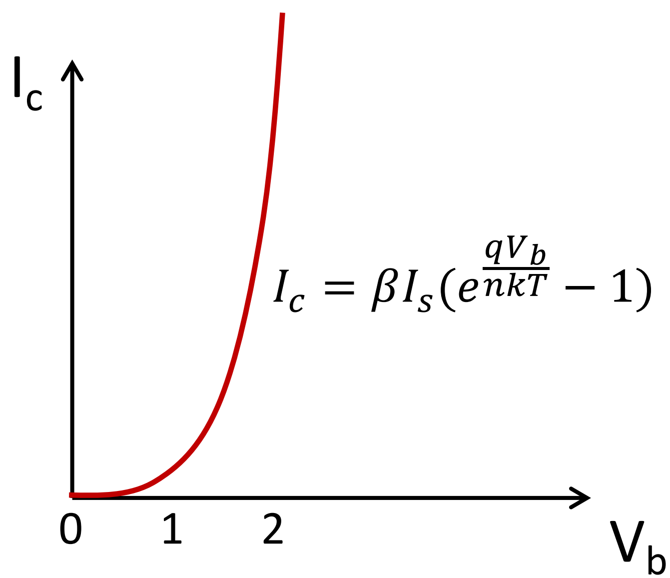
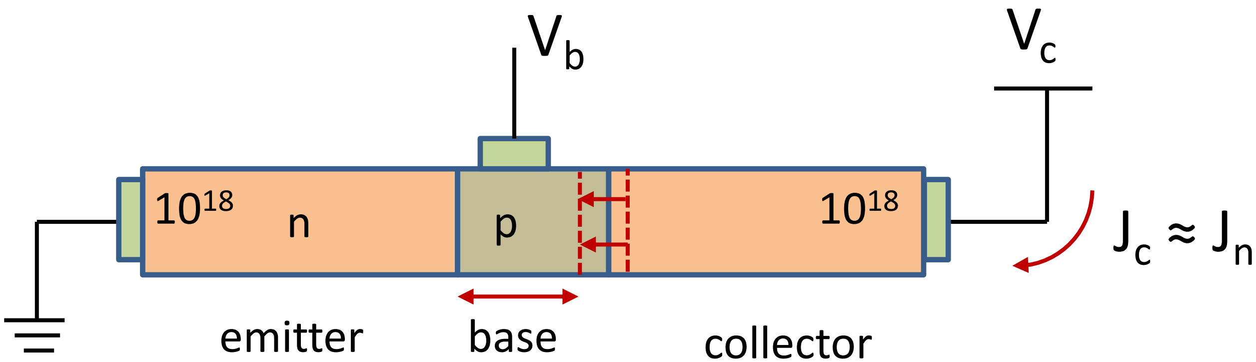


$$I_c = \beta I_s(e^{\frac{qV_b}{nkT}} - 1)$$

12.2 Bipolar Junction transistor: I-V



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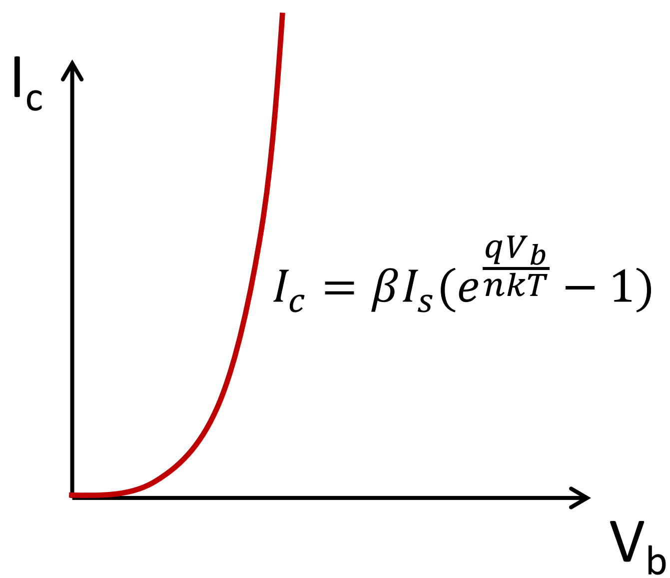

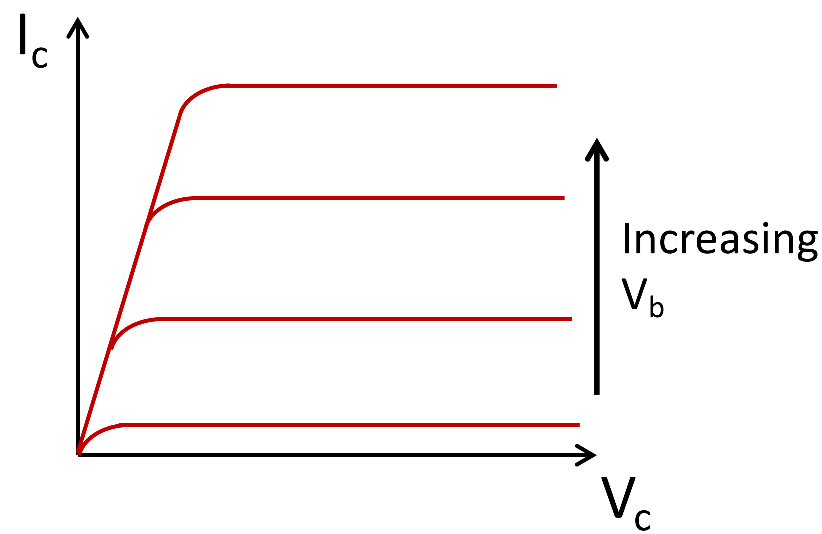

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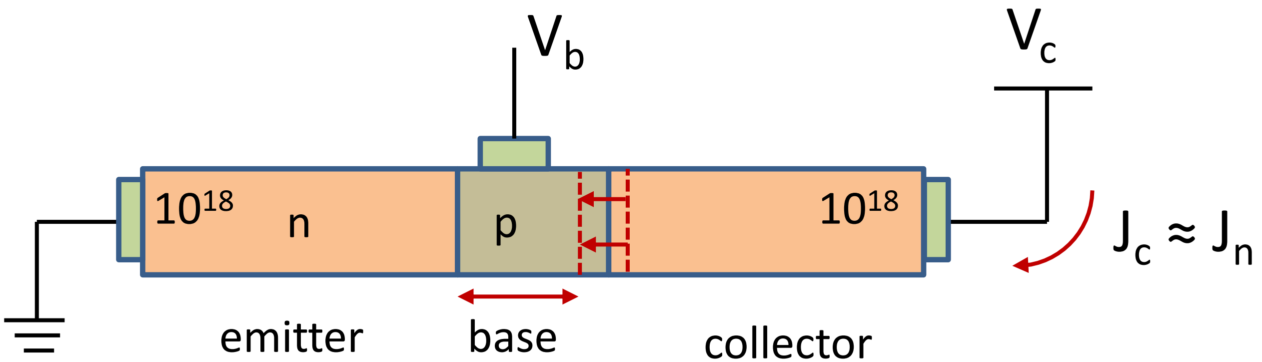
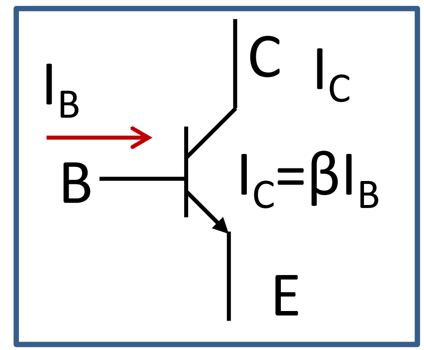


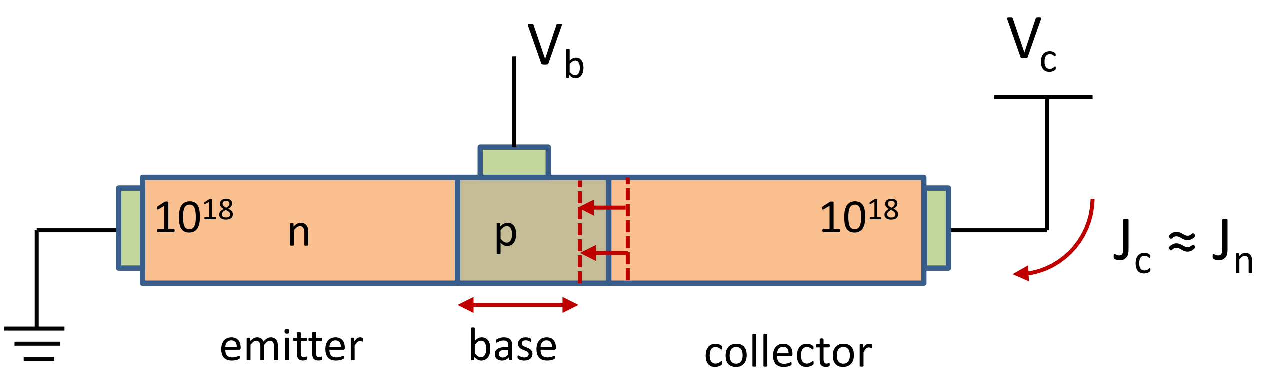
$J_c \approx 80 \times J_B \rightarrow$ Gain $\beta=80$

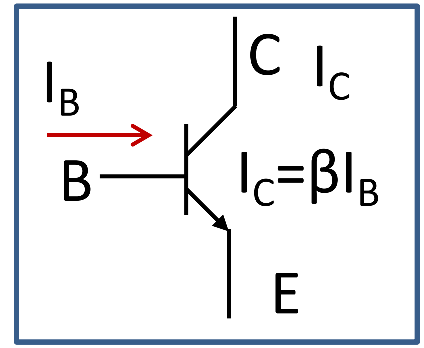


Basic facts:

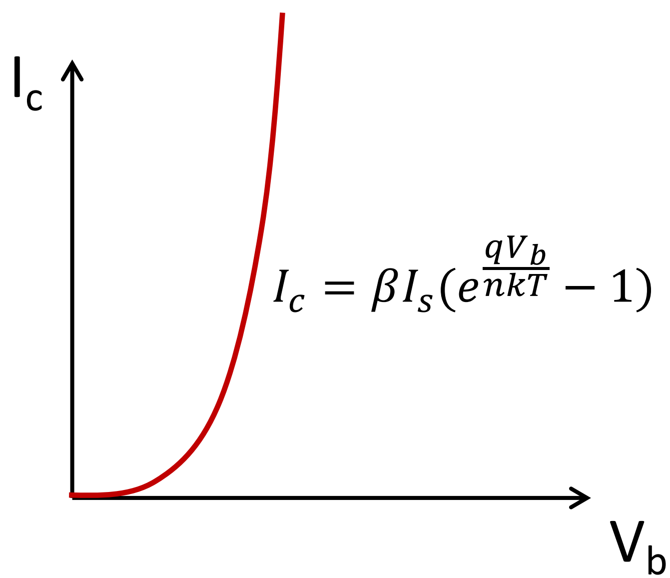
1. Narrower base → larger gain
2. $\beta \approx N_D/N_A$, higher emitter-to-base doping ratio → higher gain

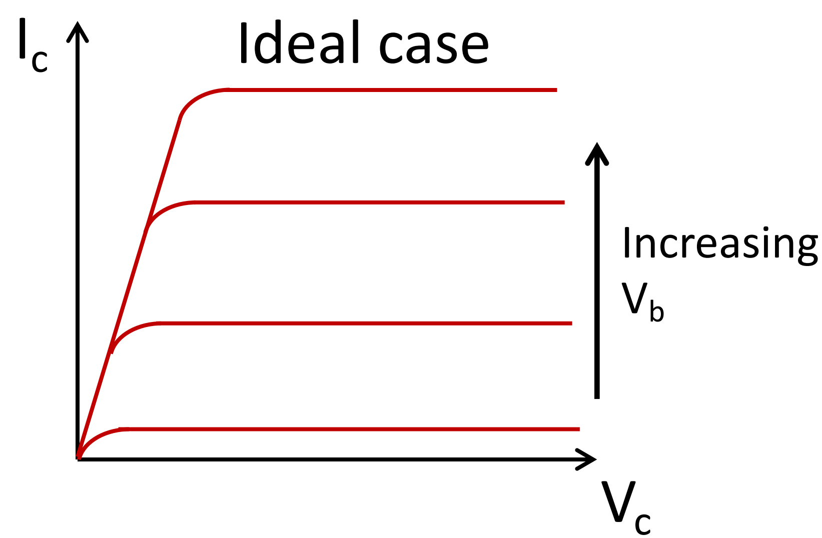
12.2 Bipolar Junction transistor: I-V





$J_c \approx 80 \times J_B \rightarrow$ Gain β=80





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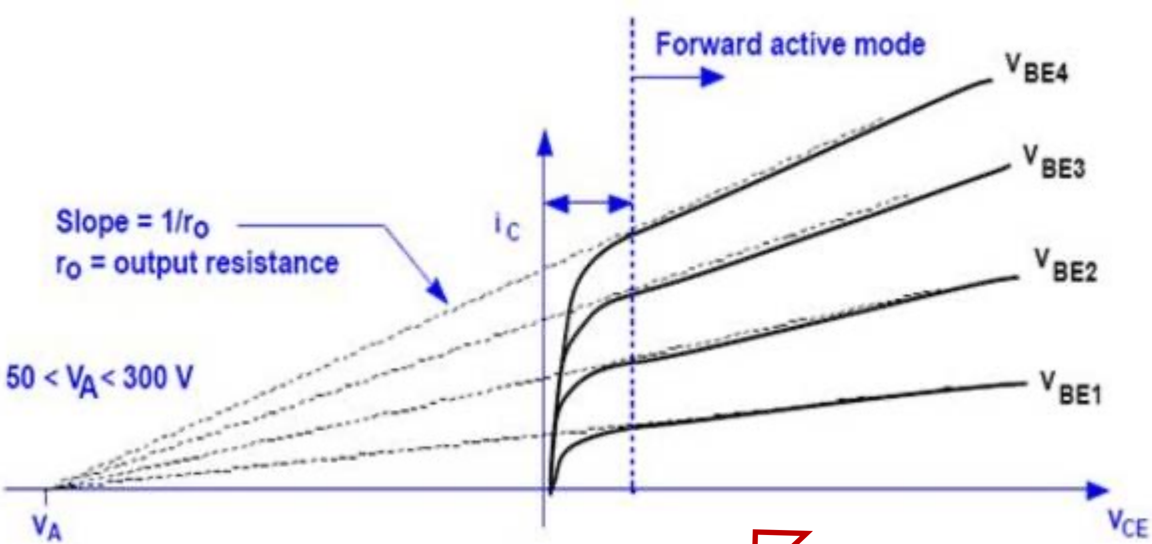
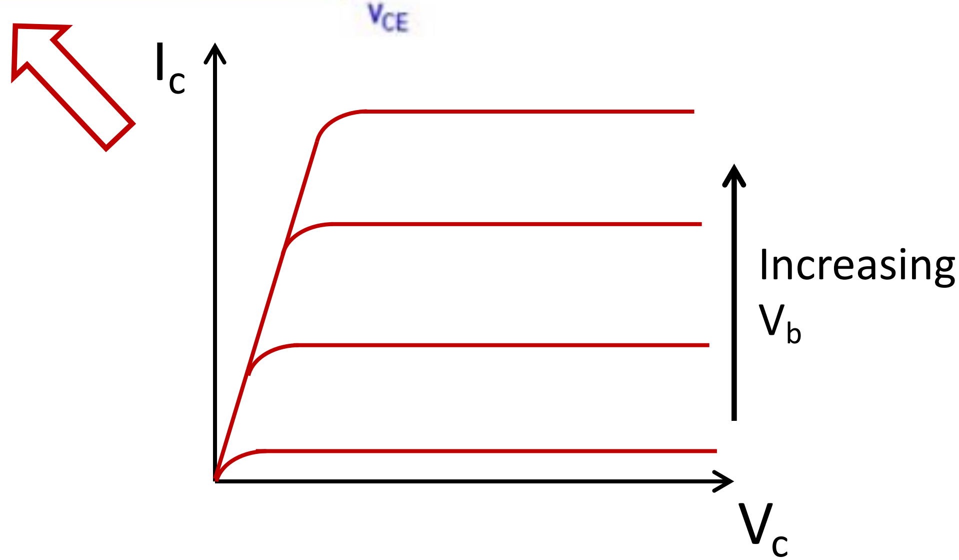
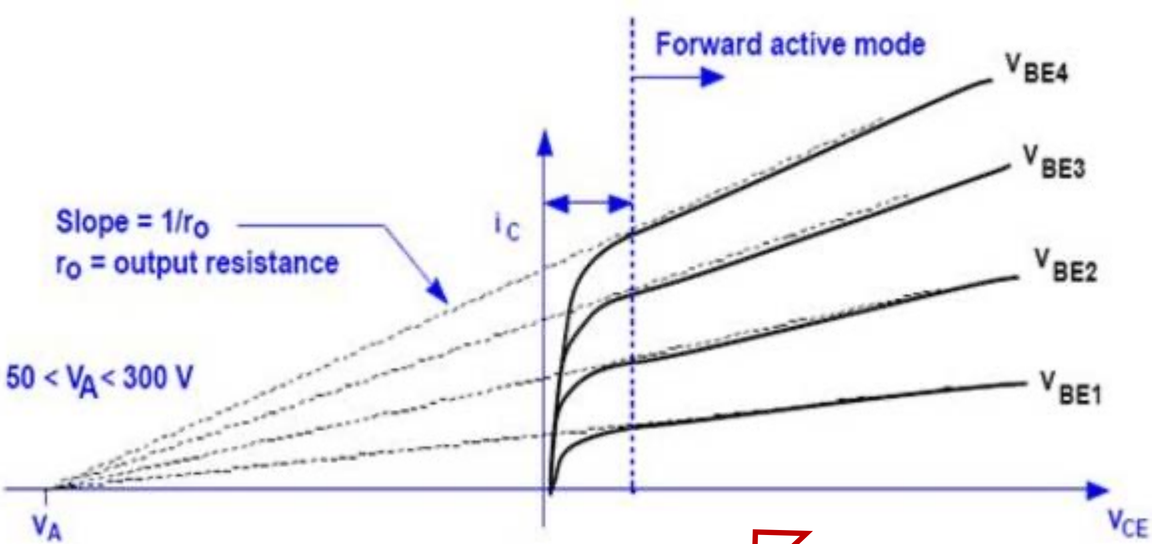
12.3 Early Effect

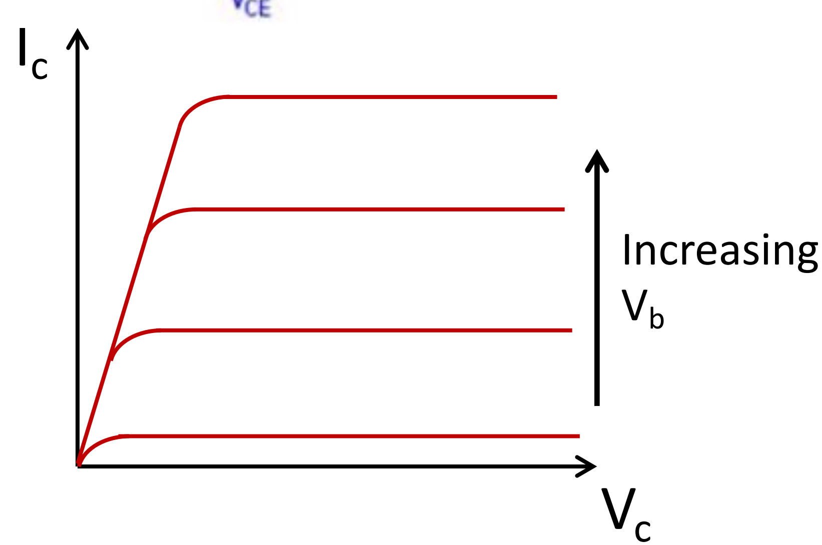




12.3 Early Effect



James M. Early



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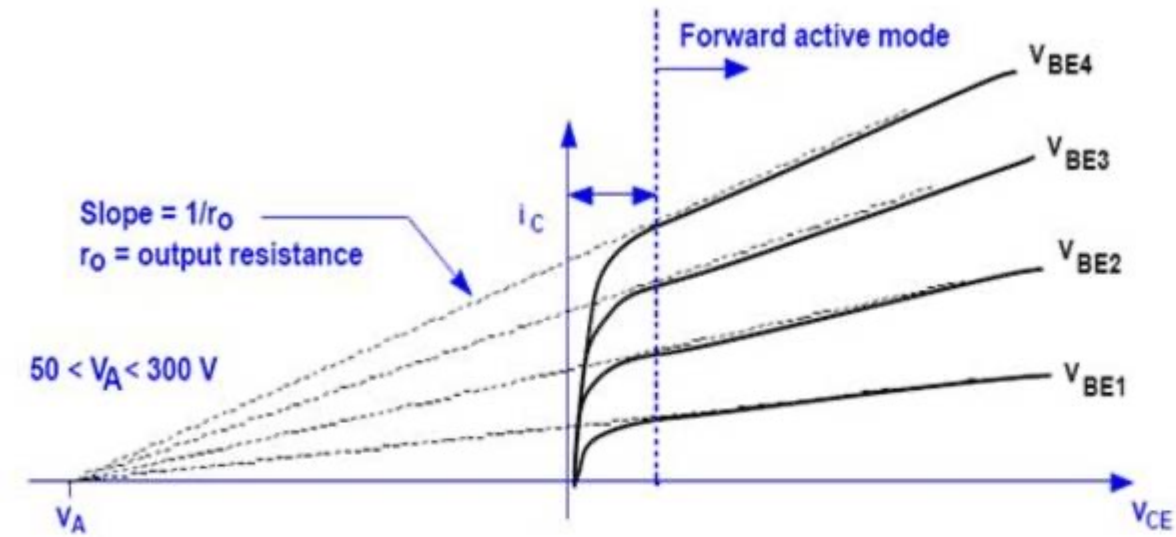

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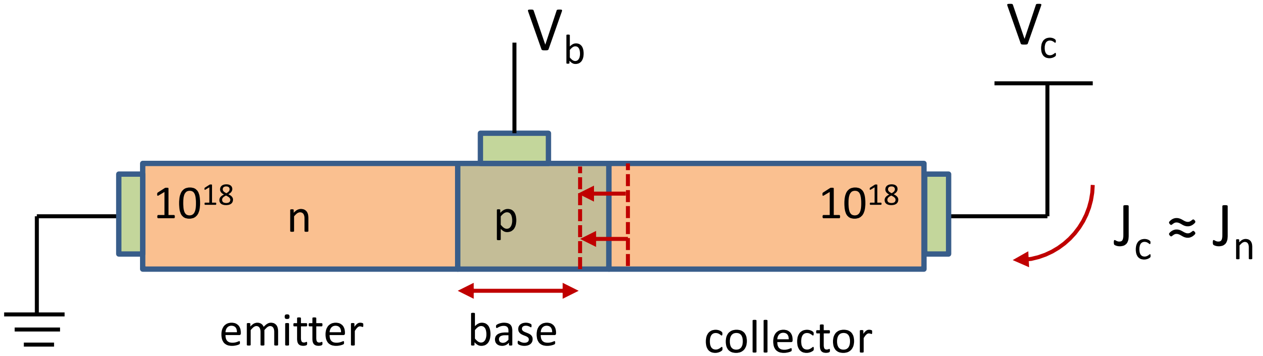

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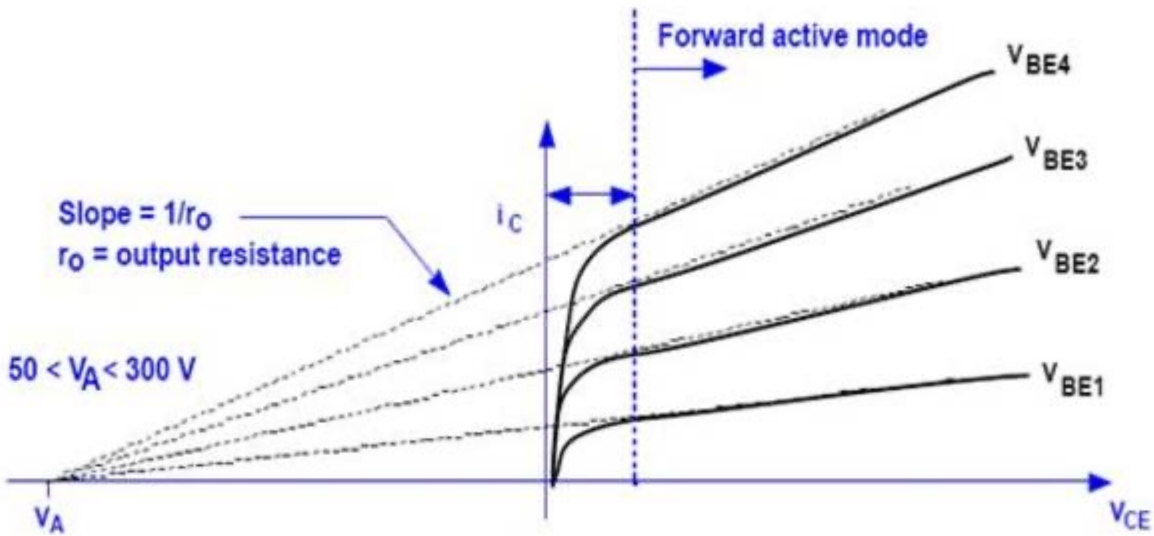

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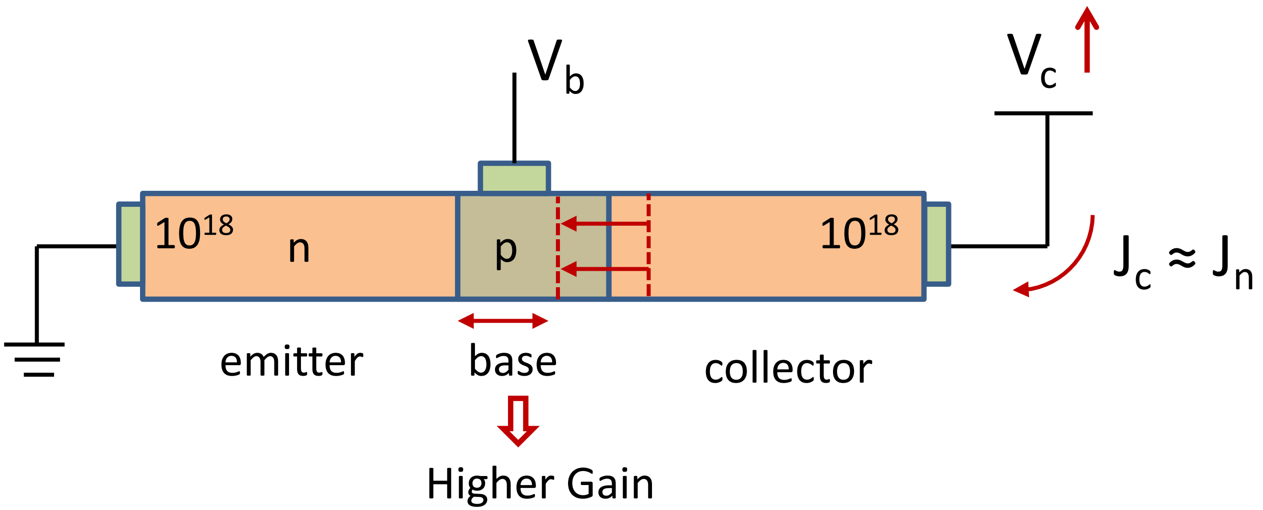
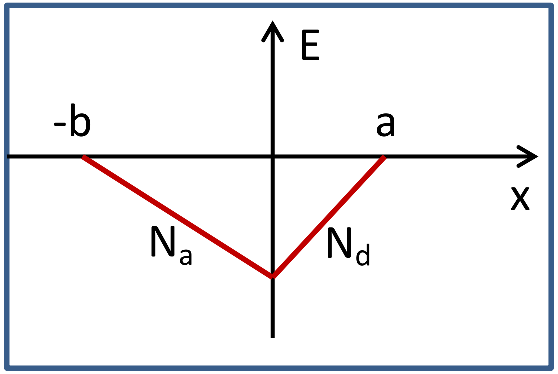
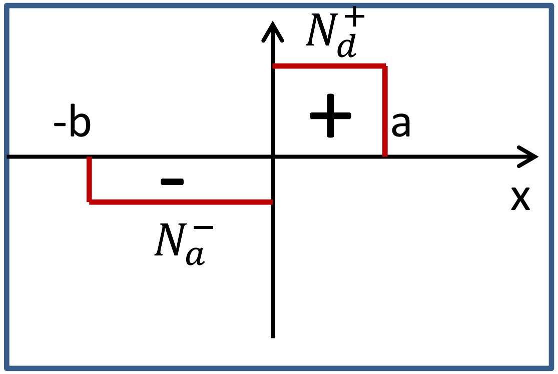


Previously…

$$a = \sqrt{\frac{2\varepsilon(V_{bi}-V_R)}{q}\frac{N_a}{N_d}\frac{1}{N_a+N_d}}$$

$$N_A^- b = N_D^+ a \Rightarrow b = \sqrt{\frac{2\varepsilon(V_{bi}-V_R)}{q}\frac{N_d}{N_a}\frac{1}{N_a+N_d}}$$





Previously…

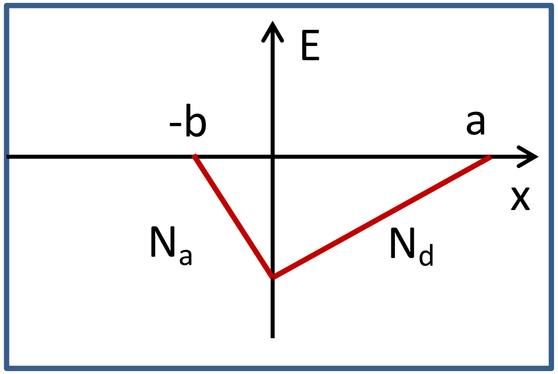
$$a = \sqrt{\frac{2\varepsilon(V_{bi} - V_R)}{q}\frac{N_a}{N_d}\frac{1}{N_a + N_d}}$$

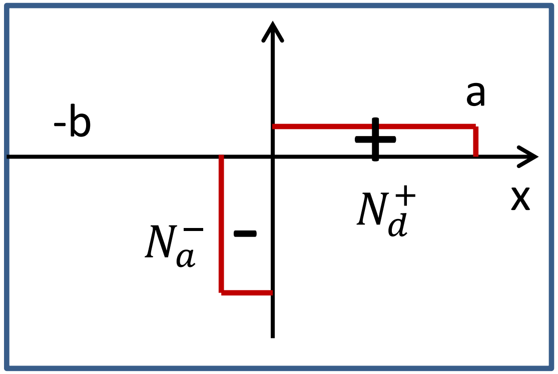
$$N_A^- b = N_D^+ a \Rightarrow b = \sqrt{\frac{2\varepsilon(V_{bi} - V_R)}{q}\frac{N_d}{N_a}\frac{1}{N_a + N_d}}$$

$N_a = 100\ N_d$

$$a = \sqrt{\frac{2\varepsilon(V_{bi} - V_R)}{q}\frac{\cancel{100N_d}}{N_d}\frac{1}{\cancel{100N_d}}}$$

$$b = \sqrt{\frac{2\varepsilon(V_{bi} - V_R)}{q}\frac{\cancel{N_d}}{100\cancel{N_d}}\frac{1}{100N_d}}$$



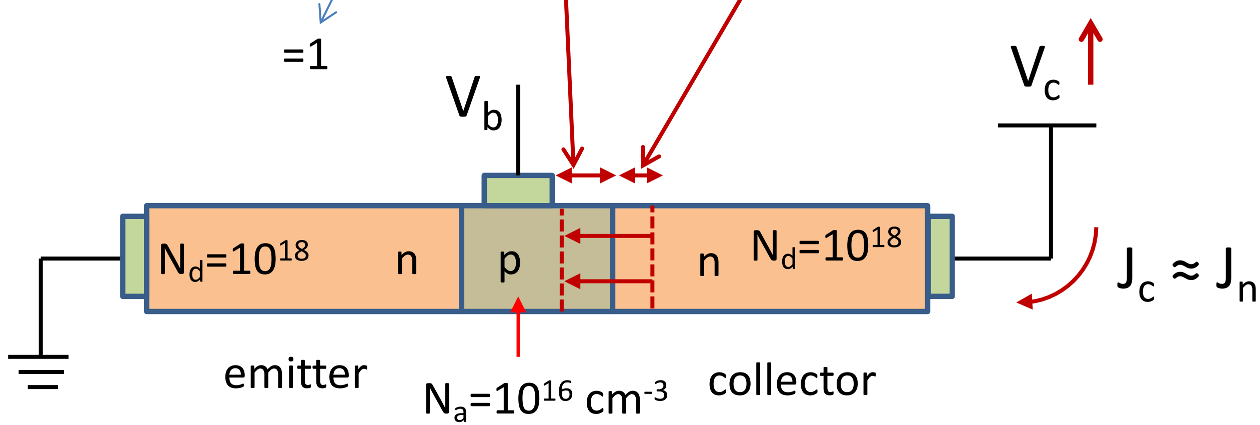


12.3 Early Effect

$$a = \sqrt{\frac{2\varepsilon(V_{bi}-V_R)}{q}\underbrace{\frac{10^{16}}{10^{18}}\frac{1}{10^{16}+10^{18}}}_{=1/10000}}$$

$$b = \sqrt{\frac{2\varepsilon(V_{bi}-V_R)}{q}\underbrace{\frac{10^{18}}{10^{16}}\frac{1}{10^{16}+10^{18}}}_{=1}}$$

James M. Early

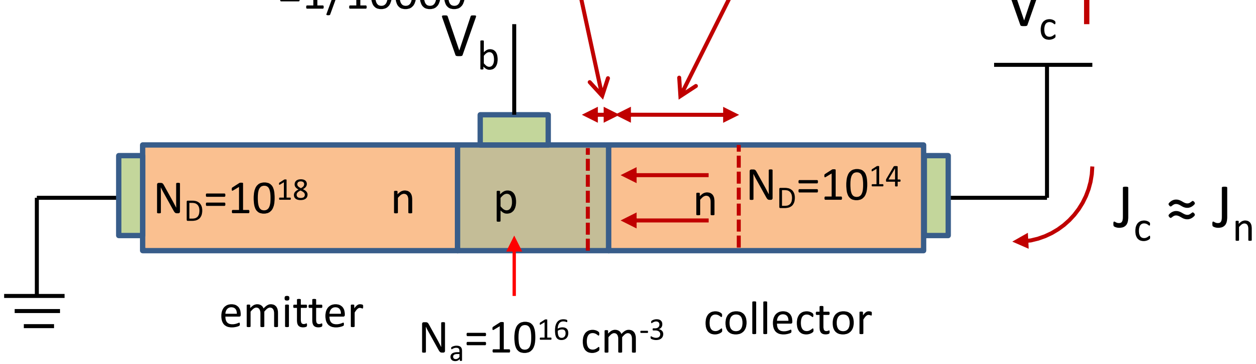


12.3 Early Effect

$$a = \sqrt{\frac{2\varepsilon(V_{bi}-V_R)}{q}\underbrace{\frac{10^{16}}{10^{14}}\frac{1}{10^{16}+10^{14}}}_{=1}}$$

$$b = \sqrt{\frac{2\varepsilon(V_{bi}-V_R)}{q}\underbrace{\frac{10^{14}}{10^{16}}\frac{1}{10^{16}+10^{14}}}_{=1/10000}}$$

James M. Early



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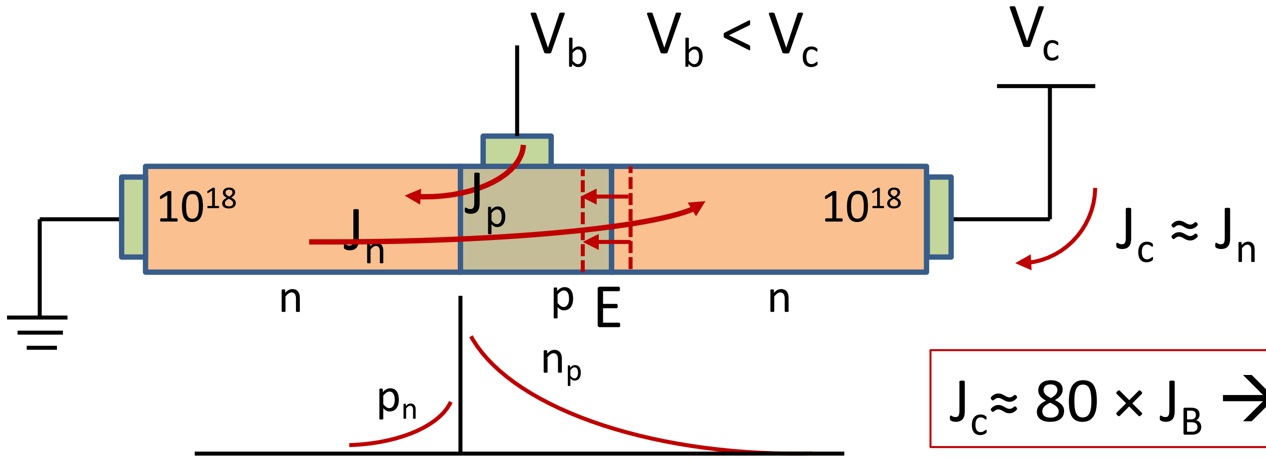
12.4 Summary



$J_c \approx 80 \times J_B \rightarrow$ Gain β=80

1. highest doping concentration is limited by solubility ($<10^{20}$)
2. Lowest doping concentration is limited by n_i and fabrication process

Basic facts:

1. Narrower base → larger gain
2. $\beta \approx N_D/N_A$, higher emitter-to-base doping ratio → higher gain
3. Trade-off for base doping concentration (gain and Early effect)

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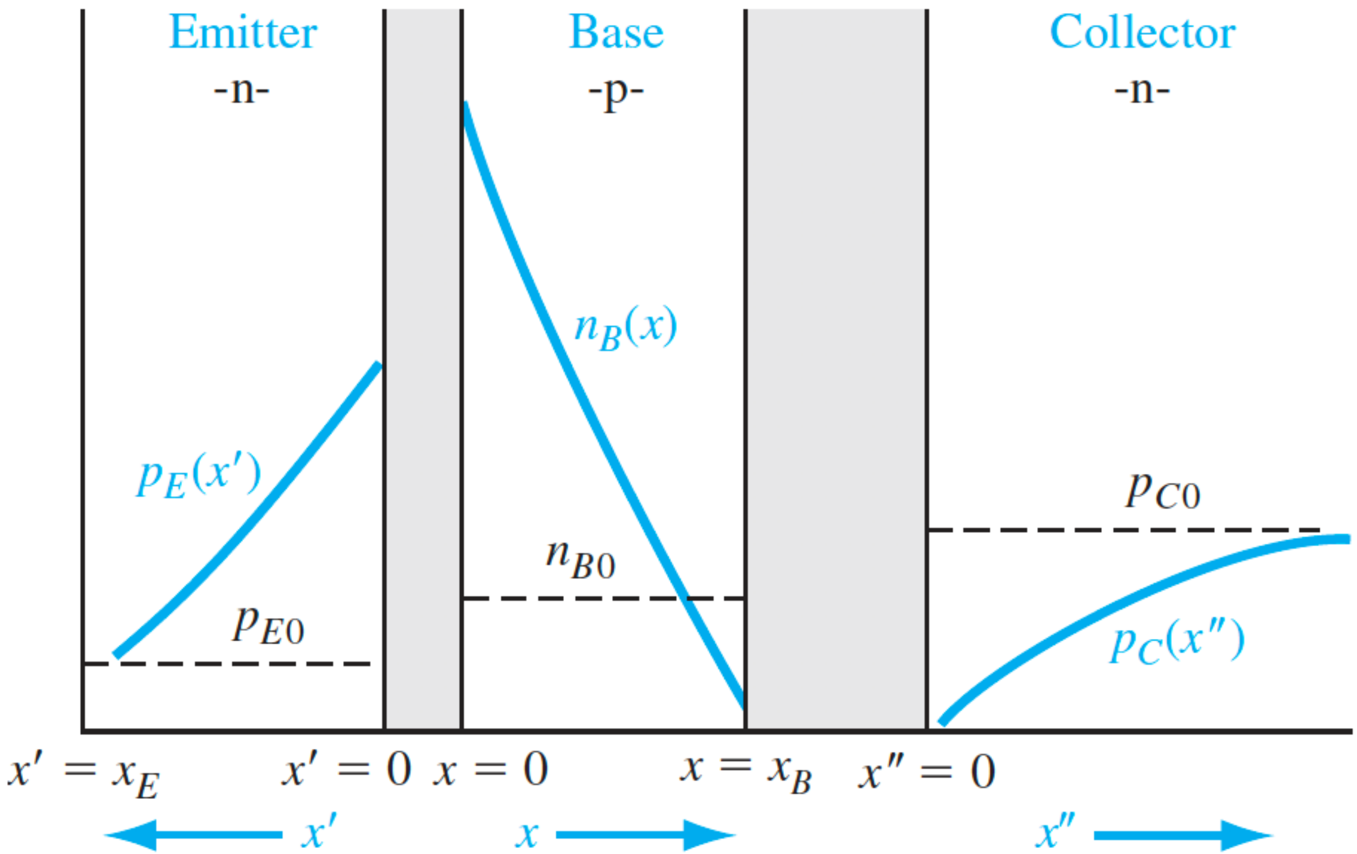
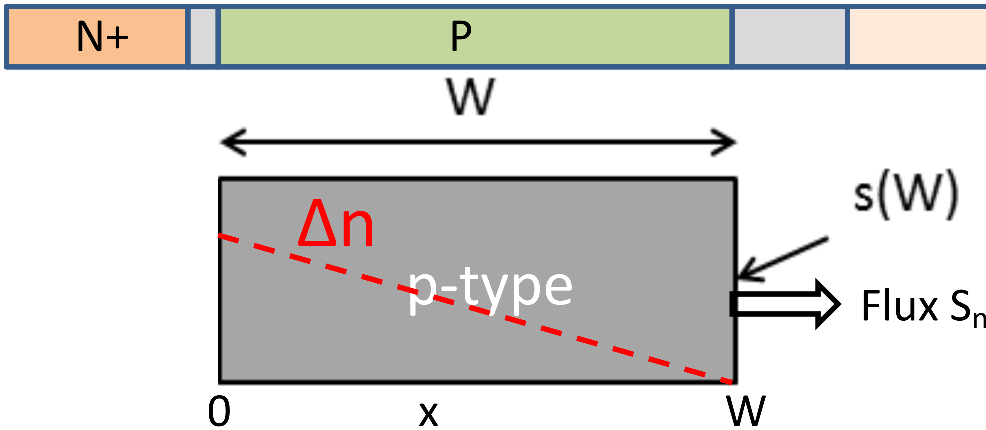


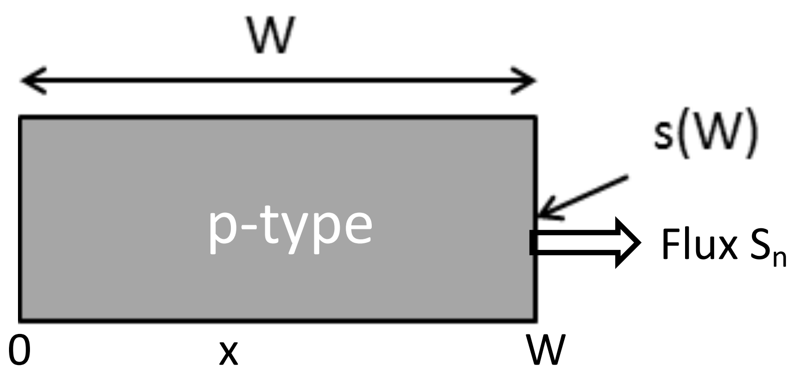
12.5 Quantitative analysis of BJT gain



$N_a = 10^{17}$ cm^{-3}, $D_n = 10$ cm^2/s, $\tau_n = 10^{-7}$ s, SRV $s(x=W) = \infty$
$\Delta n\ (x=0) = 10^{14}$ cm^{-3}

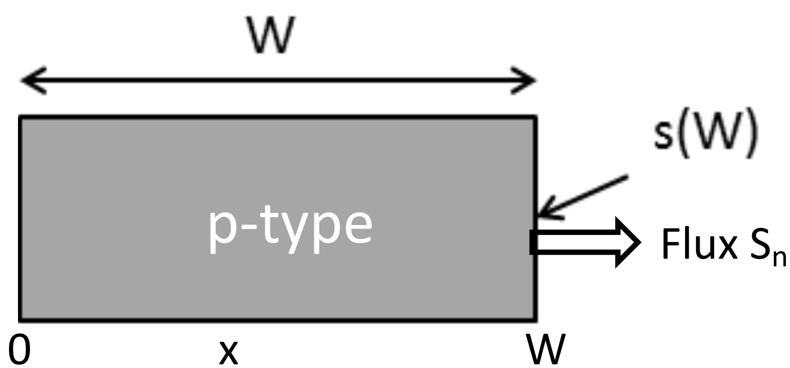
Find the electron flux Sn at x=0 and W, if
1) W=20um
2) W=2um

12.5 Quantitative analysis of BJT gain



$$\boxed{0 = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau}} \quad \Rightarrow \Delta n = Aexp\left(-\frac{x}{L_n}\right) + Bexp\left(\frac{x}{L_n}\right)$$

12.5 Quantitative analysis of BJT gain

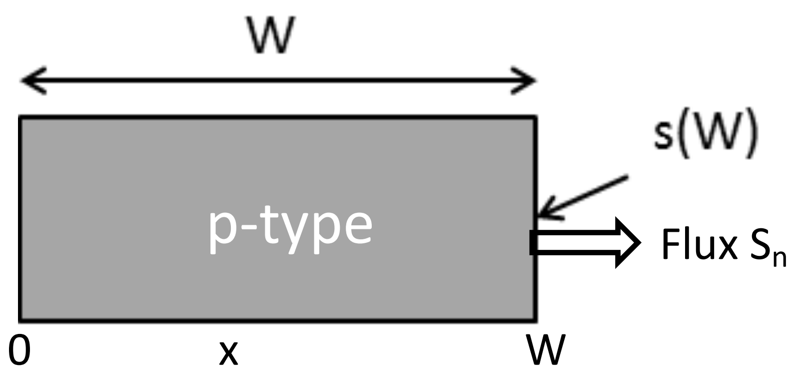


$$\boxed{0 = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau}}$$

$$\Rightarrow \Delta n = Aexp\left(-\frac{x}{L_n}\right) + Bexp\left(\frac{x}{L_n}\right)$$

$$\begin{cases} x = 0 \Rightarrow \Delta n_0 = \Delta n(x = 0) = A + B \\ x = W \Rightarrow \Delta n = Aexp\left(-\frac{W}{L_n}\right) + Bexp\left(\frac{W}{L_n}\right) = 0 \end{cases}$$

12.5 Quantitative analysis of BJT gain

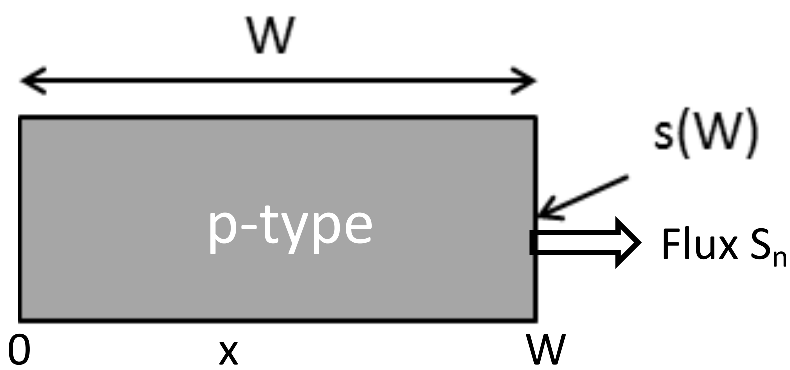


$$\boxed{0 = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau}}$$

$$\Rightarrow \Delta n = A exp\left(-\frac{x}{L_n}\right) + B exp\left(\frac{x}{L_n}\right)$$

$$A = (\Delta n)_0 \frac{\exp\left(\frac{W}{L_n}\right)}{\exp\left(\frac{W}{L_n}\right) - \exp\left(-\frac{W}{L_n}\right)} \qquad B = (\Delta n)_0 \frac{\exp\left(-\frac{W}{L_n}\right)}{\exp\left(\frac{W}{L_n}\right) - \exp\left(-\frac{W}{L_n}\right)}$$

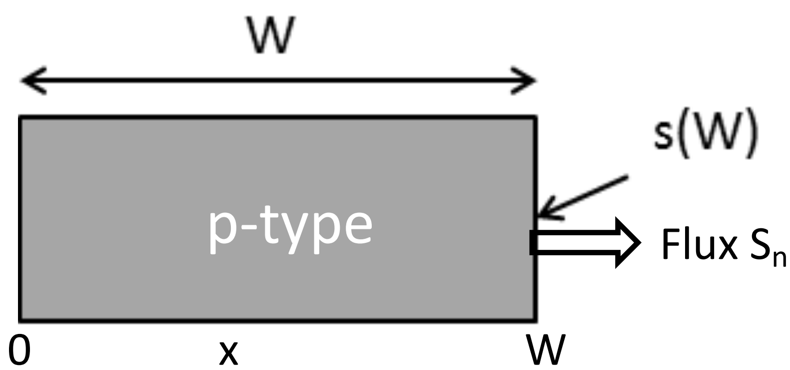
12.5 Quantitative analysis of BJT gain



$$\boxed{0 = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau}} \quad \Rightarrow \Delta n = Aexp\left(-\frac{x}{L_n}\right) + Bexp\left(\frac{x}{L_n}\right)$$

$$\Delta n(x) = (\Delta n)_0 \frac{\mathrm{sh}\left(\frac{W-x}{L_n}\right)}{\mathrm{sh}\left(\frac{W}{L_n}\right)}$$

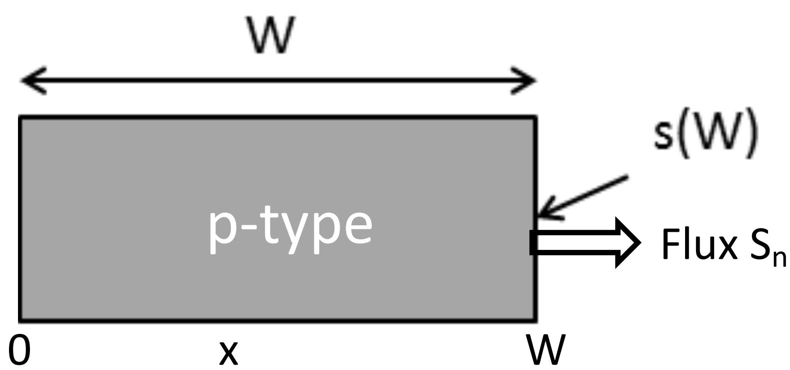
12.5 Quantitative analysis of BJT gain



$$\boxed{0 = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau}} \qquad \Rightarrow \Delta n = Aexp\left(-\frac{x}{L_n}\right) + Bexp\left(\frac{x}{L_n}\right)$$

$$\Delta n(x) = (\Delta n)_0 \frac{\mathrm{sh}\left(\frac{W-x}{L_n}\right)}{\mathrm{sh}\left(\frac{W}{L_n}\right)} \qquad S_n = -D_n \frac{d\Delta n(x)}{dx} = \frac{D_n (\Delta n)_0}{L_n} \frac{ch(\frac{W-x}{L_p})}{sh(\frac{W}{L_p})}$$

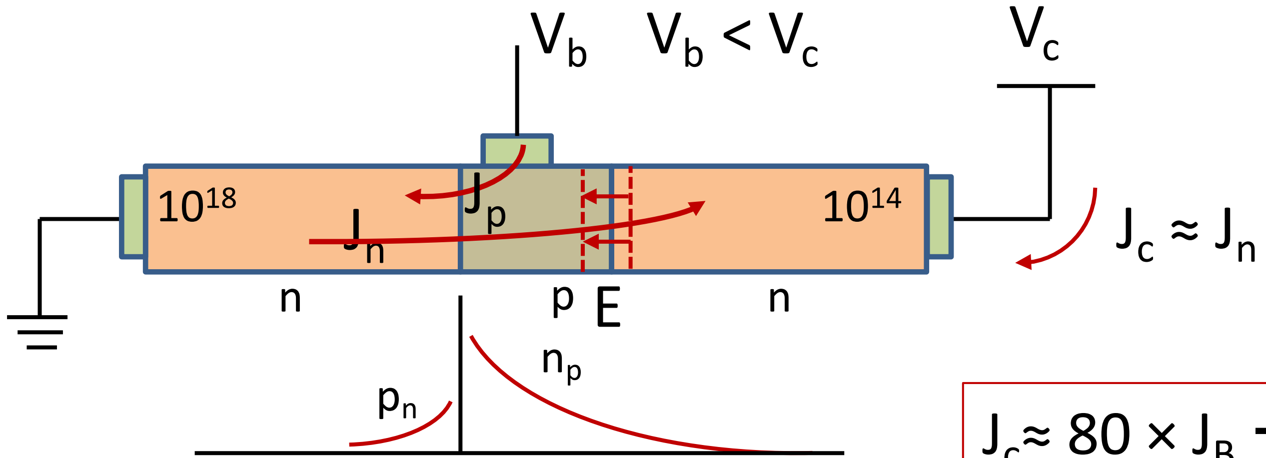
12.5 Quantitative analysis of BJT gain



$$S_n = -D_n \frac{d\Delta n(x)}{dx} = \frac{D_n(\Delta n)_{B0}}{L_n} \frac{ch(\frac{W-x}{L_p})}{sh(\frac{W}{L_p})}$$

$$S_n(0) = \frac{D_n(\Delta n)_0}{L_n} \frac{ch(\frac{W}{L_p})}{sh(\frac{W}{L_p})} \qquad S_n(W) = \frac{D_n(\Delta n)_0}{L_n} \frac{1}{sh(\frac{W}{L_p})}$$

12.5 Quantitative analysis of BJT gain



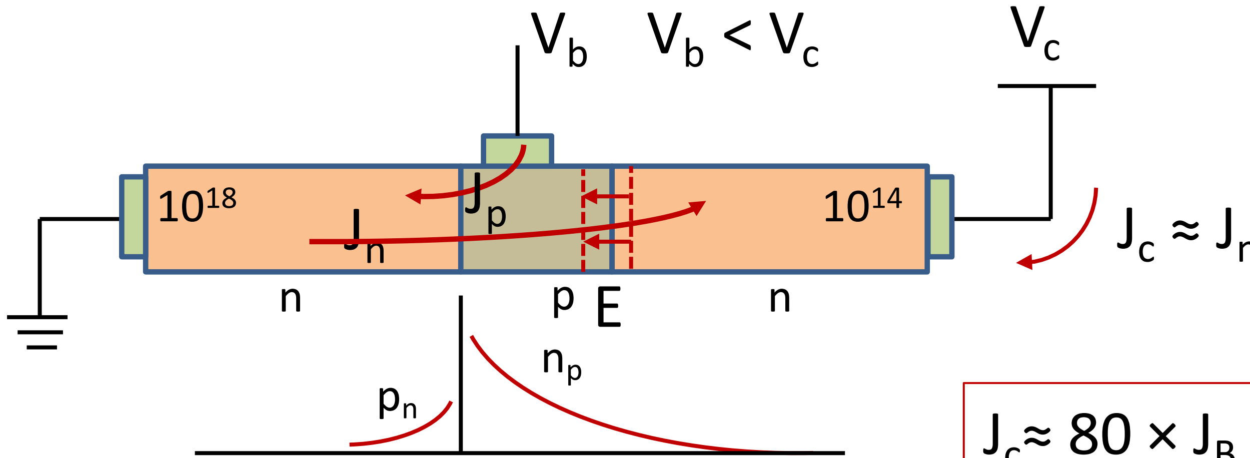
$J_c \approx 80 \times J_B \rightarrow$ Gain $\beta=80$

$$S_n(0) = \frac{D_B \cdot (\Delta n)_{B0}}{L_B} \frac{ch(\frac{W}{L_B})}{sh(\frac{W}{L_B})} \qquad S_n(W_b) = \frac{D_B \cdot (\Delta n)_{B0}}{L_B} \frac{1}{sh(\frac{W}{L_B})}$$

Electron flux from emitter to base: $S_n(0)$

Electron flux from base to collector: $S_n(W_b)$

12.5 Quantitative analysis of BJT gain



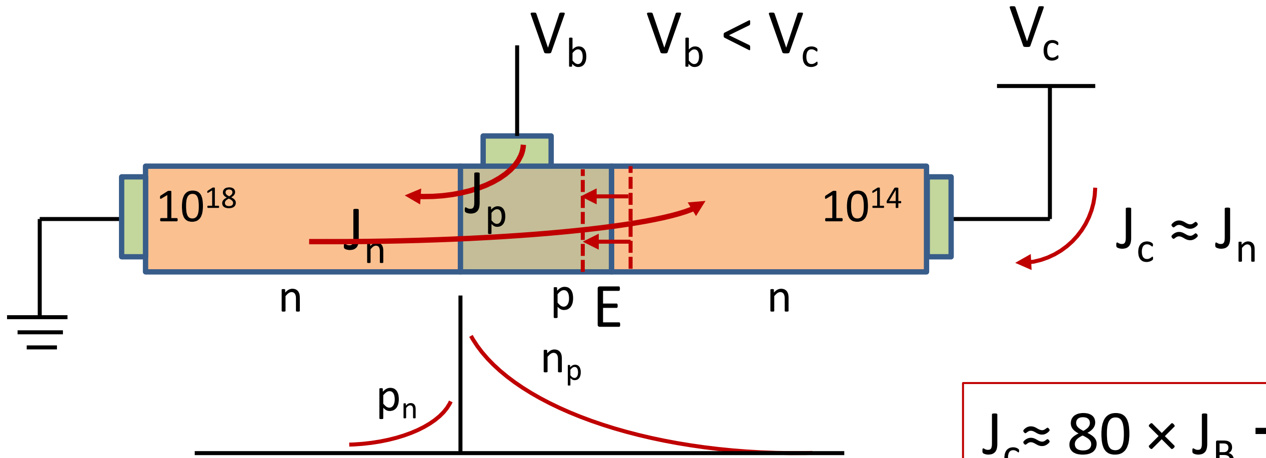
$J_c \approx 80 \times J_B \rightarrow$ Gain β=80

$$S_n(0) = \frac{D_B(\Delta n)_{B0}}{L_B}\frac{ch(\frac{W}{L_B})}{sh(\frac{W}{L_B})} \qquad S_n(W_b) = \frac{D_B(\Delta n)_{B0}}{L_B}\frac{1}{sh(\frac{W}{L_B})}$$

Electron flux from emitter to base: $S_n(0)$
Electron flux from base to collector: $S_n(W_b)$
Hole flux from base to emitter: S_p

$$S_p = D_E\frac{dp_E}{dx} = \frac{D_E p_{E0}}{L_E}$$

12.5 Quantitative analysis of BJT gain



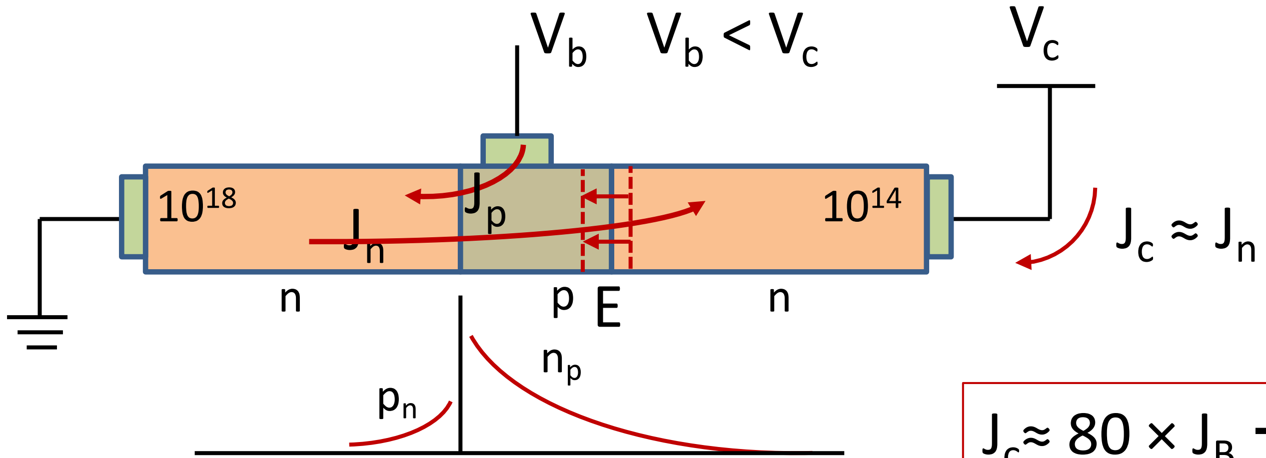
$J_c \approx 80 \times J_B \rightarrow$ Gain β=80

$$S_n(0) = \frac{D_B(\Delta n)_{B0}}{L_B}\frac{ch(\frac{W}{L_B})}{sh(\frac{W}{L_B})} \qquad S_n(W_b) = \frac{D_B(\Delta n)_{B0}}{L_B}\frac{1}{sh(\frac{W}{L_B})}$$

Electron flux from emitter to base: $S_n(0)$
Electron flux from base to collector: $S_n(W_b)$
Hole flux from base to emitter: S_p
Base electrode flux: $S_p + S_n(0) - S_n(W_b)$

$$S_p = D_E\frac{dp_E}{dx} = \frac{D_E p_{E0}}{L_E}$$

12.5 Quantitative analysis of BJT gain



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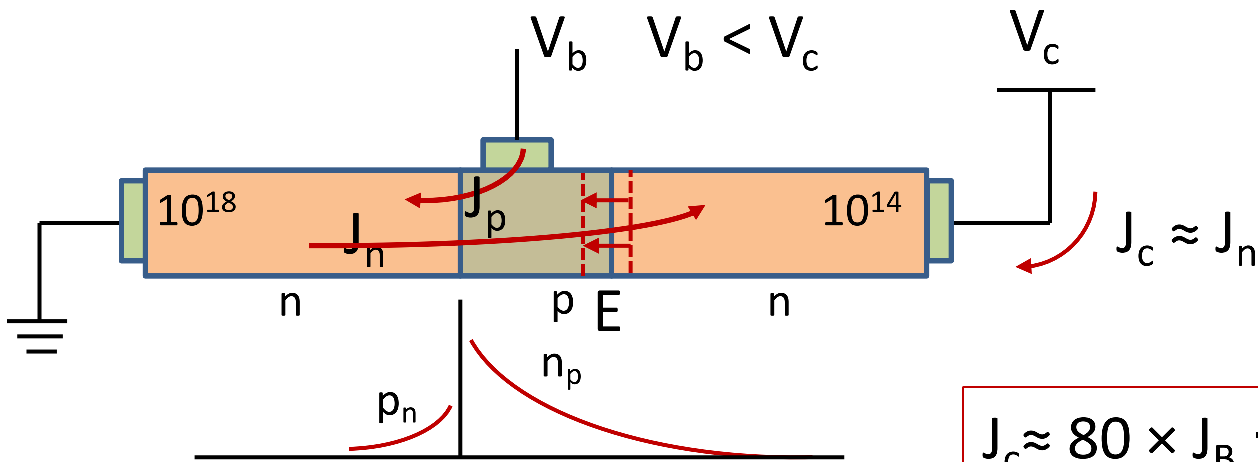
Base electrode flux: $S_p + S_n(0) - S_n(W_b)$
Gain β = collector flux /base electrode flux = $S_n(W_b)/(S_p+S_n(0)-S_n(W_b))$

12.5 Quantitative analysis of BJT gain

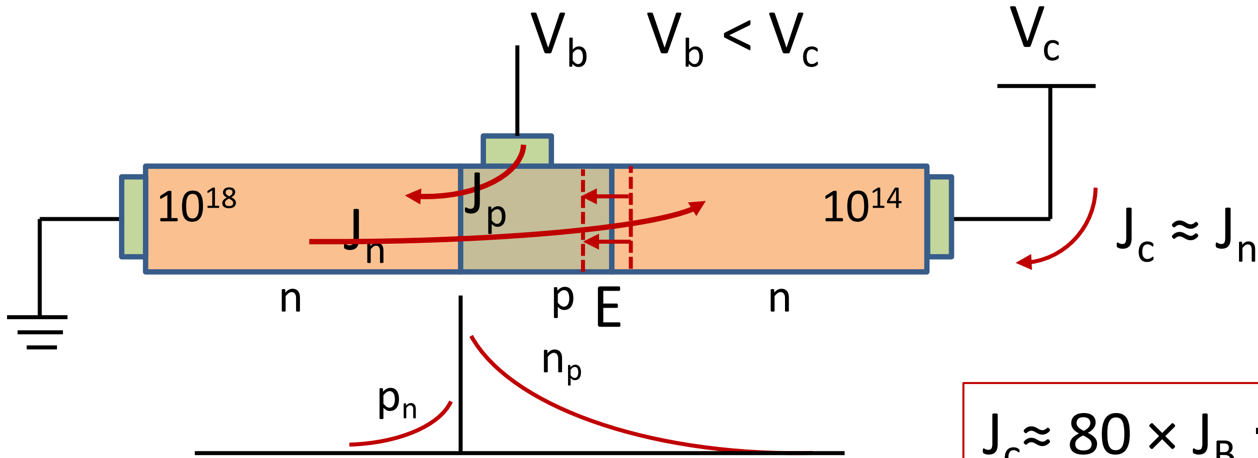


$J_c \approx 80 \times J_B \rightarrow$ Gain β=80

$$S_n(0) = \frac{D_B(\Delta n)_{B0}}{L_B}\frac{ch(\frac{W}{L_B})}{sh(\frac{W}{L_B})} \qquad S_n(W_b) = \frac{D_B(\Delta n)_{B0}}{L_B}\frac{1}{sh(\frac{W}{L_B})}$$

$$\beta = \frac{\frac{D_B(\Delta n)_{B0}}{L_B}\frac{1}{sh(\frac{W}{L_B})}}{\frac{D_E(\Delta p)_{E0}}{L_E} + \frac{D_B(\Delta n)_{B0}}{L_B}\frac{ch(\frac{W}{L_B})}{sh(\frac{W}{L_B})} - \frac{D_B(\Delta n)_{B0}}{L_B}\frac{1}{sh(\frac{W}{L_B})}}$$

12.5 Quantitative analysis of BJT gain



$J_c \approx 80 \times J_B \rightarrow$ Gain β=80

$$S_n(0) = \frac{D_B(\Delta n)_{B0}}{L_B}\frac{ch(\frac{W}{L_p})}{sh(\frac{W}{L_p})} \qquad S_n(W_b) = \frac{D_B(\Delta n)_{B0}}{L_B}\frac{1}{sh(\frac{W}{L_p})}$$

$$\beta = \frac{\frac{D_B(\Delta n)_{B0}}{L_B}}{\frac{D_E(\Delta p)_{E0}}{L_E}sh(\frac{W}{L_B}) + \frac{D_B(\Delta n)_{B0}}{L_B}[ch\left(\frac{W}{L_p}\right) - 1]}$$

12.5 Quantitative analysis of BJT gain

$$sh\left(\frac{W}{L_B}\right) = \frac{1}{2}\left[\exp\left(\frac{W}{L_B}\right) - exp\left(-\frac{W}{L_B}\right)\right]$$

$$\exp\left(\frac{W}{L_B}\right) = 1 + \frac{W}{L_B} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2 + \cdots$$

$$\exp\left(-\frac{W}{L_B}\right) = 1 - \frac{W}{L_B} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2 - \cdots$$

$$\beta = \frac{\frac{D_B(\Delta n)_{B0}}{L_B}}{\frac{D_E(\Delta p)_{E0}}{L_E} sh\left(\frac{W}{L_B}\right) + \frac{D_B(\Delta n)_{B0}}{L_B}\left[ch\left(\frac{W}{L_p}\right) - 1\right]}$$

12.5 Quantitative analysis of BJT gain

$$sh\left(\frac{W}{L_B}\right) = \frac{1}{2}\left[\exp\left(\frac{W}{L_B}\right) - exp\left(-\frac{W}{L_B}\right)\right] = \frac{W}{L_B}$$

$$\exp\left(\frac{W}{L_B}\right) = 1 + \frac{W}{L_B} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2 + \cdots$$

$$if\ \frac{W}{L_p} < 1$$

$$\exp\left(-\frac{W}{L_B}\right) = 1 - \frac{W}{L_B} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2 - \cdots$$

$$\beta = \frac{\frac{D_B(\Delta n)_{B0}}{L_B}}{\frac{D_E(\Delta p)_{E0}}{L_E} sh\left(\frac{W}{L_B}\right) + \frac{D_B(\Delta n)_{B0}}{L_B}\left[ch\left(\frac{W}{L_p}\right) - 1\right]}$$

12.5 Quantitative analysis of BJT gain

$$ch\left(\frac{W}{L_B}\right) - 1 = \frac{1}{2}\left[\exp\left(\frac{W}{L_B}\right) + \exp\left(-\frac{W}{L_B}\right)\right] - 1$$

$$\exp\left(\frac{W}{L_B}\right) = 1 + \frac{W}{L_B} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2 + \cdots$$

$$\exp\left(-\frac{W}{L_B}\right) = 1 - \frac{W}{L_B} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2 - \cdots$$

$$\beta = \frac{\frac{D_B(\Delta n)_{B0}}{L_B}}{\frac{D_E(\Delta p)_{E0}}{L_E}\frac{W}{L_B} + \frac{D_B(\Delta n)_{B0}}{L_B}\left[ch\left(\frac{W}{L_p}\right) - 1\right]}$$

12.5 Quantitative analysis of BJT gain

$$ch\left(\frac{W}{L_B}\right)-1=\frac{1}{2}\left[\exp\left(\frac{W}{L_B}\right)+\exp\left(-\frac{W}{L_B}\right)\right]-1=\frac{1}{2}\left(\frac{W}{L_B}\right)^2$$

$$\exp\left(\frac{W}{L_B}\right)=1+\frac{W}{L_B}+\frac{1}{2}\left(\frac{W}{L_B}\right)^2+\cdots$$

$$if\ \frac{W}{L_B}<1$$

$$\exp\left(-\frac{W}{L_B}\right)=1-\frac{W}{L_B}+\frac{1}{2}\left(\frac{W}{L_B}\right)^2-\cdots$$

$$\beta=\frac{\frac{D_B(\Delta n)_{B0}}{L_B}}{\frac{D_E(\Delta p)_{E0}}{L_E}\frac{W}{L_B}+\frac{D_B(\Delta n)_{B0}}{L_B}\left[\frac{1}{2}\left(\frac{W}{L_B}\right)^2\right]}$$

12.5 Quantitative analysis of BJT gain

$$ch\left(\frac{W}{L_B}\right) - 1 = \frac{1}{2}\left[\exp\left(\frac{W}{L_B}\right) + \exp\left(-\frac{W}{L_B}\right)\right] - 1 = \frac{1}{2}\left(\frac{W}{L_B}\right)^2$$

$$\exp\left(\frac{W}{L_B}\right) = 1 + \frac{W}{L_B} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2 + \cdots$$

$$if\ \frac{W}{L_B} < 1$$

$$\exp\left(-\frac{W}{L_B}\right) = 1 - \frac{W}{L_B} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2 - \cdots$$

$$\beta = \frac{\frac{D_B(\Delta n)_{B0}}{L_B}}{\frac{D_E(\Delta p)_{E0}}{L_E}\frac{W}{L_B} + \frac{D_B(\Delta n)_{B0}}{L_B}\left[\frac{1}{2}\left(\frac{W}{L_B}\right)^2\right]} = \frac{1}{\frac{D_E(\Delta p)_{E0}W}{D_B(\Delta n)_{B0}L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

12.5 Quantitative analysis of BJT gain

$$ch\left(\frac{W}{L_B}\right) - 1 = \frac{1}{2}\left[\exp\left(\frac{W}{L_B}\right) + \exp\left(-\frac{W}{L_B}\right)\right] - 1 = \frac{1}{2}\left(\frac{W}{L_B}\right)^2$$

$$\exp\left(\frac{W}{L_B}\right) = 1 + \frac{W}{L_B} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2 + \cdots$$

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$$\boxed{\beta = \frac{1}{\frac{N_B D_E W}{N_E D_B L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}}$$

12.5 Quantitative analysis of BJT gain

$$ch\left(\frac{W}{L_B}\right) - 1 = \frac{1}{2}\left[\exp\left(\frac{W}{L_B}\right) + \exp\left(-\frac{W}{L_B}\right)\right] - 1 = \frac{1}{2}\left(\frac{W}{L_B}\right)^2$$

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$$\beta = \frac{1}{\frac{N_B D_E W}{N_E D_B L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

$\beta = S_n(W_b)/(S_p + \underbrace{S_n(0) - S_n(W_b)}_{})$