Mid2 RC_part2

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Overview

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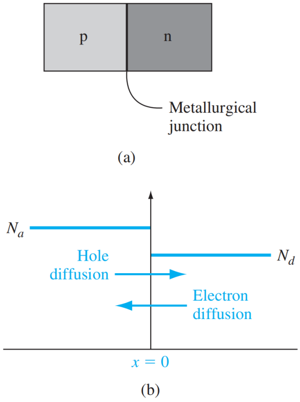
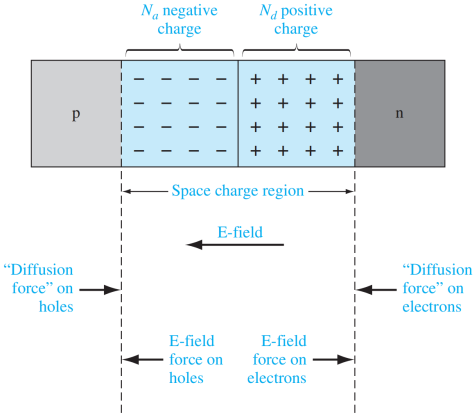
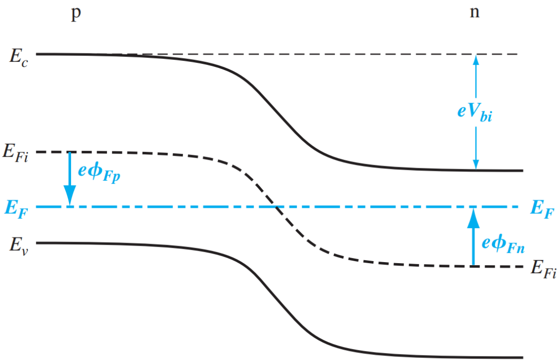
   - Zero Applied Bias
   - Reverse Applied Bias
2. Chapter 8
   - pn Junction Current
   - Generation-Recombination Currents

Basic Structure





Zero Applied Bias



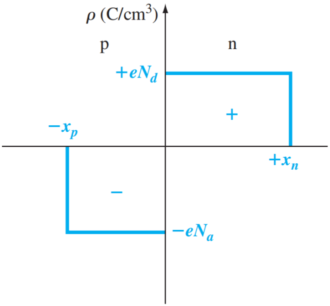
Built-in Potential Barrier

$$
\begin{aligned}
\phi_{Fn} &= -\frac{kT}{e}\ln(\frac{N_d}{n_i}) \\
\phi_{Fp} &= \frac{kT}{e}\ln(\frac{N_a}{n_i}) \\
V_{bi} &= |\phi_{Fn}| + |\phi_{Fp}| \\
&= \frac{kT}{e}\ln(\frac{N_a N_d}{n_i^2}) = V_t \ln(\frac{N_a N_d}{n_i^2})
\end{aligned}
$$

V_{bi}: built-in potential barrier; $V_t = kT/e$: thermal voltage

Charge Density

- Poisson's equation: $\dfrac{\mathrm{d}^2\phi(x)}{\mathrm{d}x^2} = \dfrac{-\rho(x)}{\epsilon_s} = -\dfrac{\mathrm{d}E(x)}{\mathrm{d}x}$, $\epsilon_s = \epsilon_0\epsilon_r$



- volume charge density:

$$\rho(x) = \begin{cases} -eN_a, & -x_p \le x \le 0 \\ +eN_d, & 0 \le x \le x_n \end{cases}$$

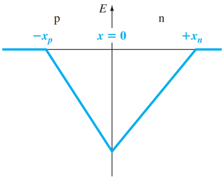
Electric Field

- electric field:

$$E(x) = \begin{cases} -\frac{eN_a}{\epsilon_s}(x + x_p), & -x_p \leq x \leq 0 \\ -\frac{eN_d}{\epsilon_s}(x_n - x), & 0 \leq x \leq x_n \end{cases}$$

- continuity at $x = 0$:

$$N_a x_p = N_d x_n$$



- maximum electric field intensity at $x = 0$:

$$E_{\max} = -\frac{eN_a x_p}{\epsilon_s} = -\frac{eN_d x_n}{\epsilon_s}$$

Electric Potential

- Set $\phi(x = -x_p) = 0$:

$$\phi(x) = \begin{cases} \frac{eN_a}{2\epsilon_s}(x + x_p)^2, & -x_p \leq x \leq 0 \\ \frac{eN_d}{\epsilon_s}(x_n \cdot x - \frac{x^2}{2}) + \frac{eN_d}{2\epsilon_s}x_p^2, & 0 \leq x \leq x_n \end{cases}$$

- when $x = x_n$, the potential is equal to the built-in potential barrier:

$$V_{bi} = |\phi(x = x_n| = \frac{e}{2\epsilon_s}(N_d x_n^2 + N_a x_p^2)$$

Space Charge Width

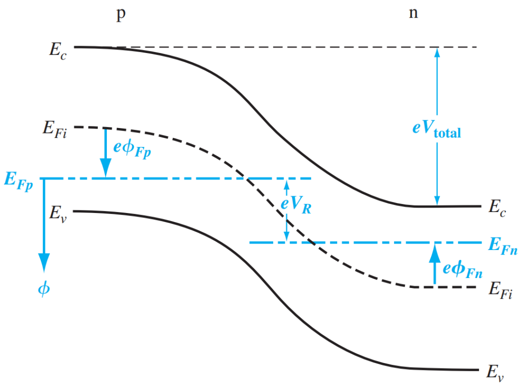
$$x_n = \sqrt{\frac{2\epsilon_s V_{bi}}{e}(\frac{N_a}{N_d})(\frac{1}{N_a + N_d})}$$

$$x_p = \sqrt{\frac{2\epsilon_s V_{bi}}{e}(\frac{N_d}{N_a})(\frac{1}{N_a + N_d})}$$

- total depletion region width W:

$$\begin{aligned} W &= x_n + x_p \\ &= \sqrt{\frac{2\epsilon_s V_{bi}}{e}(\frac{N_a + N_d}{N_a N_d})} \end{aligned}$$

Reverse Applied Bias



- total potential barrier V_{total}:

$$V_{\text{total}} = |\phi_{Fn}| + |\phi_{Fp}| + V_R = V_{bi} + V_R$$

Space Charge Width and Electric Field

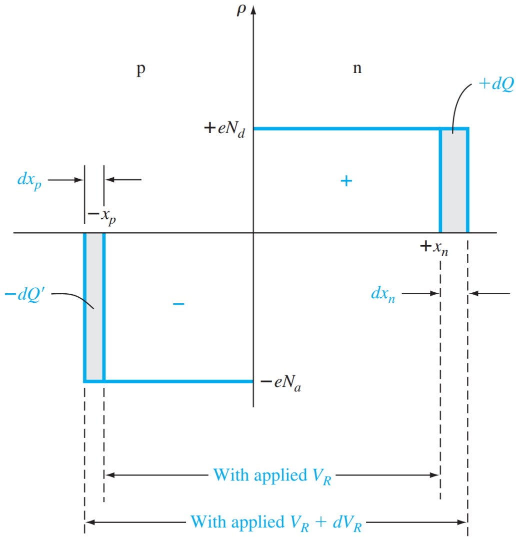
- In all of the previous equations, the built-in potential barrier can be replaced by the total potential barrier.
- total space charge width:

$$W = \sqrt{\frac{2\epsilon_s(V_{bi} + V_R)}{e}(\frac{N_a + N_d}{N_a N_d})}$$

- maximum electric field:

$$\begin{aligned} E_{\max} &= -\sqrt{\frac{2e(V_{bi} + V_R)}{\epsilon_s}(\frac{N_a N_d}{N_a + N_d})} \\ &= \frac{-2(V_{bi} + V_R)}{W} \end{aligned}$$

Junction Capacitance



Junction Capacitance

- seperated +/- charges in depletion region: capacitance behavior
- The junction capacitance is defined as

$$C' = \frac{\mathrm{d}Q'}{\mathrm{d}V_R}$$

where

$$\mathrm{d}Q' = eN_d\mathrm{d}x_n = eN_a\mathrm{d}x_p$$

$$C' = \sqrt{\frac{e\epsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)}} = \frac{\epsilon_s}{W}$$

- unit of C': F/cm^2

One-Sided Junctions

- If, for example, $N_a \gg N_d$, this junction is referred to as a p^+n junction.
- total space charge width reduces to

$$W \approx \sqrt{\frac{2\epsilon_s(V_{bi} + V_R)}{eN_d}}$$

- Consider expressions for x_n and x_p, for p^+n junction:

$$x_p \ll x_n$$

$$W \approx x_n$$

- junction capacitance for p^+n junction reduces to:

$$C' \approx \sqrt{\frac{e\epsilon_s N_d}{2(V_{bi} + V_R)}}$$

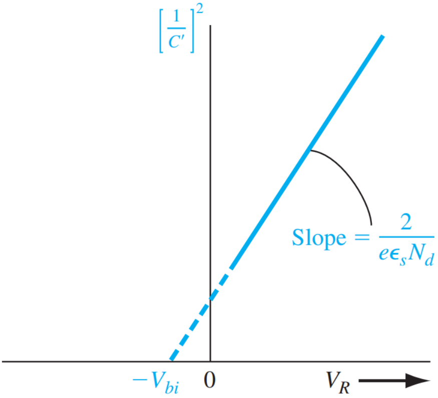
One-Sided Junctions

- The depletion layer capacitance of a one-sided junction is a function of the doping concentration in the low-doped region:

$$(\frac{1}{C'})^2 = \frac{2(V_{bi} + V_R)}{e\epsilon_s N_d}$$

- the inverse capacitance squared is a linear function of applied reverse-biased voltage: an experimental way to determine doping concentration

One-Sided Junctions



pn Junction Current

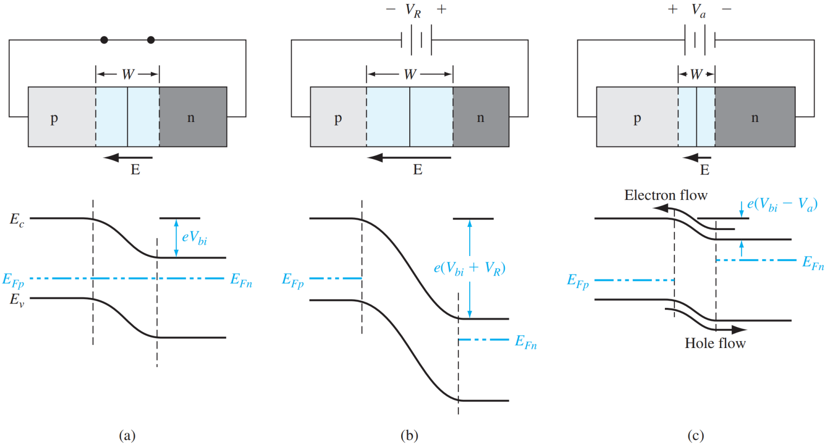


Figure 8.1 | A pn junction and its associated energy-band diagram for (a) zero bias, (b) reverse bias, and (c) forward bias.

Ideal Current-Voltage Relationship

The ideal current–voltage relationship of a pn junction is derived on the basis of four assumptions. (The last assumption has three parts, but each part deals with current.) They are:

1. The abrupt depletion layer approximation applies. The space charge regions have abrupt boundaries, and the semiconductor is neutral outside of the depletion region.

2. The Maxwell–Boltzmann approximation applies to carrier statistics.

3. The concepts of low injection and complete ionization apply.

4a. The total current is a constant throughout the entire pn structure.

4b. The individual electron and hole currents are continuous functions through the pn structure.

4c. The individual electron and hole currents are constant throughout the depletion region.

Notation

Table 8.1 | Commonly used terms and notation for this chapter

Term	Meaning
N_a	Acceptor concentration in the p region of the pn junction
N_d	Donor concentration in the n region of the pn junction
$n_{n0} = N_d$	Thermal-equilibrium majority carrier electron concentration in the n region
$p_{p0} = N_a$	Thermal-equilibrium majority carrier hole concentration in the p region
$n_{p0} = n_i^2/N_a$	Thermal-equilibrium minority carrier electron concentration in the p region
$p_{n0} = n_i^2/N_d$	Thermal-equilibrium minority carrier hole concentration in the n region
n_p	Total minority carrier electron concentration in the p region
p_n	Total minority carrier hole concentration in the n region
$n_p(-x_p)$	Minority carrier electron concentration in the p region at the space charge edge
$p_n(x_n)$	Minority carrier hole concentration in the n region at the space charge edge
$\delta n_p = n_p - n_{p0}$	Excess minority carrier electron concentration in the p region
$\delta p_n = p_n - p_{n0}$	Excess minority carrier hole concentration in the n region

Boundary Conditions

- Assume complete ionization:

$$n_{n0} \approx N_d$$

- in the p region:

$$n_{p0} \approx \frac{n_i^2}{N_a}$$

$$n_{p0} = n_{n0}\exp\left(\frac{-eV_{bi}}{kT}\right)$$

This equation relates the minority carrier electron concentration on the p side of the junction to the majority carrier electron concentration on the n side of the junction in thermal equilibrium.

Boundary Conditions

- When a forward bias V_a is applied to the pn junction, the boundary conditions for total minority carrier concentrations are:

$$p_n(x_n) = p_{n0}\exp(\frac{eV_a}{kT})$$
$$n_p(-x_p) = n_{p0}\exp(\frac{eV_a}{kT})$$
$$p_n(x \to +\infty) = p_{n0}$$
$$n_p(x \to -\infty) = n_{p0}$$

Minority Carrier Distribution

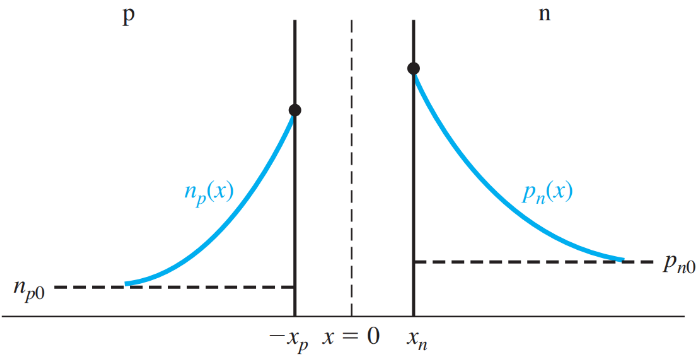
- Assume long pn junction.
- excess carrier concentrations for $x \geq x_n$:

$$\delta p_n(x) = p_n(x) - p_{n0} = p_{n0}[\exp(\frac{eV_a}{kT}) - 1]\exp(\frac{x_n - x}{L_p})$$

- for $x \leq -x_p$:

$$\delta n_p(x) = n_p(x) - n_{p0} = n_{p0}[\exp(\frac{eV_a}{kT}) - 1]\exp(\frac{x_p + x}{L_n})$$

Minority Carrier Distribution



Ideal pn Junction Current

- hole diffusion current density:

$$J_p(x_n) = \frac{eD_p p_{n0}}{L_p}[\exp(\frac{eV_a}{kT}) - 1]$$

- electron diffusion current density:

$$J_n(-x_p) = \frac{eD_n n_{p0}}{L_n}[\exp(\frac{eV_a}{kT}) - 1]$$

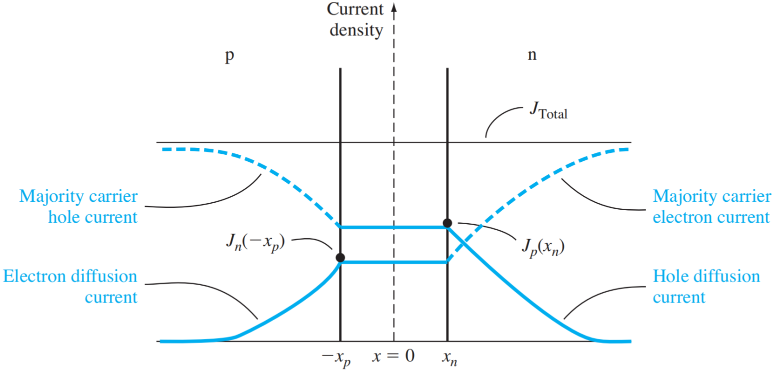
- for $x \geq x_n$:

$$J_p(x) = \frac{eD_p p_{n0}}{L_p}[\exp(\frac{eV_a}{kT}) - 1]\exp(\frac{x_n - x}{L_p})$$

- for $x \leq -x_p$:

$$J_n(x) = \frac{eD_n n_{p0}}{L_n}[\exp(\frac{eV_a}{kT}) - 1]\exp(\frac{x_p + x}{L_n})$$

Ideal pn Junction Current



Ideal pn Junction Current

- total current density:

$$J = J_p(x_n) + J_n(-x_p) = [\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}][\exp(\frac{eV_a}{kT}) - 1]$$

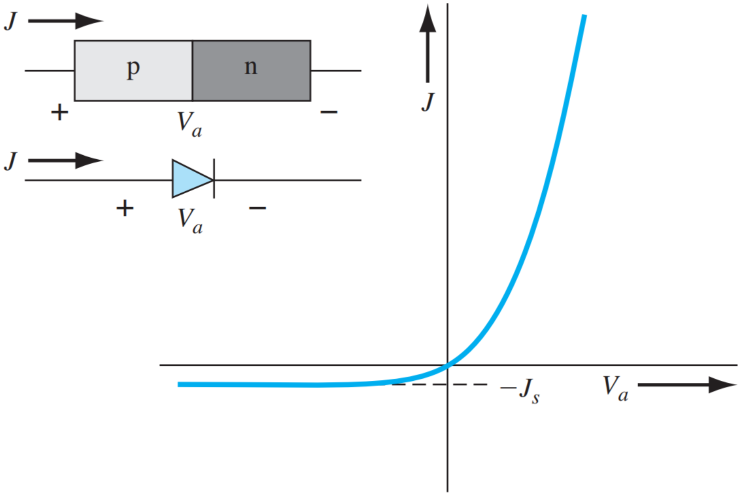
- define a parameter J_s as

$$J_s = \frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}$$

- so that the total current density can be written as:

$$J = J_s[\exp(\frac{eV_a}{kT}) - 1]$$

Ideal pn Junction Current



Reverse-Biased Generation Current

- mobile electrons and holes are swept out of the space charge width:

$$n \approx p \approx 0$$

- recombination rate:

$$R = \frac{-C_n C_p N_t n_i^2}{C_n n' + C_p p'}$$

- to simplify calculation, assume $E_t = E_i$, $\tau_{n0} = \tau_{p0} = \tau_0$:

$$R = \frac{-n_i}{2\tau_0} = -G$$

G: generation rate of electrons and holes in the depletion region

Reverse-Biased Generation Current

- generation current density:

$$J_{\text{gen}} = \int_0^W eG\,\mathrm{d}x$$

- If we assume that the generation rate is constant throughout the space charge region, then we obtain:

$$J_{\text{gen}} = \frac{en_i W}{2\tau_0}$$

- The total reverse-biased current density is the sum of the ideal reverse saturation current density and the generation current density:

$$J_R = J_s + J_{\text{gen}}$$

Forward-Biased Recombination Current

- The recombination current density may be calculated from

$$J_{\text{rec}} = \int_0^W eR\,\mathrm{d}x$$

- maximum recombination rate at the center of the space charge region:

$$R_{\text{max}} = \frac{n_i}{2\tau_0}[\exp(\frac{eV_a}{2kT}) - 1]$$

- recombination current density can be written as

$$J_{\text{rec}} = \frac{eWn_i}{2\tau_0}[\exp(\frac{eV_a}{2kT}) - 1]$$

Forward-Biased Recombination Current

- total forward-bias current:

$$I = I_s[\exp(\frac{eV_a}{nkT}) - 1]$$

- n: ideality factor
- diffusion dominates: $n \approx 1$
- recombination dominates: $n \approx 2$

Reference

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4. 2022Summer RC_Mid2_part2, Yucheng Huang