1) Explain the physical meaning of the Fermi energy level.

The Fermi energy level, often referred to simply as the Fermi level, is the highest energy level that electrons occupy at absolute zero temperature. It represents the energy level at which the probability of finding an electron is 50%. In semiconductors and metals, the position of the Fermi level helps determine electrical properties, such as conductivity and carrier concentration.

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2) A silicon piece at T = 300K has $N_a = 7 \times 10^{14} cm^{-3}$ and $p_0 = 2 \times 10^5 cm^{-3}$

a) Is the material n type or p type?

b) What are the majority and minority carrier concentrations?

c) What must be the concentration of donor impurities?

a) $n_i = 1.5 \times 10^{10}$

$$\Rightarrow n_0 = \frac{n_i^2}{p_0} = \frac{(1.5 \times 10^{10})^2}{2 \times 10^5} = 1.125 \times 10^{15} \ cm^{-3}$$

since $n_0 >> p_0$, the material is n type. (Also we can see from c) that $N_d > N_a$)

b) majority carrier concentrations : n_0

minority carrier concentrations : p_0

c) Due to electric neutrality : $N_d + p_0 = N_a + n_0$

$\Rightarrow N_d + 2 \times 10^5 = 7 \times 10^{14} + 1.125 \times 10^{15}$ $\boxed{\text{Assume complete ionization}}$

Since $2 \times 10^5 << 7 \times 10^{14} < 1.125 \times 10^{15}$.

$N_d \approx 1.825 \times 10^{15} \ cm^{-3}$

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7)

a) What is meant by complete ionization?

b) What is meant by freeze-out?

(a) Complete Ionization

Complete ionization occurs when all dopant atoms in a semiconductor donate or accept electrons. This typically happens at higher temperatures where the thermal energy (>3kT> 3kT>3kT) is sufficient to ionize the dopants, ensuring that the majority of dopant atoms contribute free charge carriers (electrons or holes) to the conduction or valence band.

(b) Freeze-Out

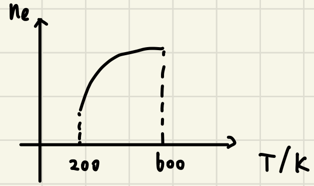
Freeze-out refers to the condition at low temperatures where the thermal energy is insufficient to ionize dopant atoms. As a result, most dopant atoms remain in their bound states, and the number of free charge carriers (electrons or holes) in the semiconductor is significantly reduced, approaching zero.

3) Silicon is doped at $N_d = 10^{15} cm^{-3}$ and $N_a = 0$.

a) Plot the concentration of electrons versus temperature over the range $200K \le T \le 600K$. (qualitatively)

b) For the device to operate properly, the intrinsic carriers must contribute no more than 5 percent to the total electron concentration. Calculate the maximum temperature it can work out.

(a)



(b)

$$\begin{cases} \left(n - \frac{N_d}{2}\right)^2 = \left(\frac{N_d}{2}\right)^2 + n_i^2 \\ n_i = 0.05\,n \end{cases}$$

$$\Rightarrow n = \frac{N_d}{1 - 0.05^2} = 1.0025 \times 10^{15}\ cm^{-3}$$

And $n_i^2 = N_v N_v e^{\frac{-E_g}{kT}}$

$$\Rightarrow T = -\frac{E_g}{k} \cdot \frac{1}{\ln \frac{n_i^2}{N_c N_v}} \approx 510\,K$$

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4) The magnitude of the product $g_c(E) f_F(E)$ in the conduction band is a function of energy. Assume the Boltzmann approximation is valid.

a) Determine the energy with respect to E_c at which the maximum occurs.

b) Repeat part a) for the magnitude of the product $g_v(E)[1 - f_F(E)]$ in the valence band.

a) $g_c(E) = \dfrac{\sqrt{2}\,(m_n^*)^{\frac{3}{2}} \sqrt{E - E_c}}{\pi^2 \hbar^3}$, $f_F(E) \approx e^{-\frac{E-E_F}{kT}}$ using Boltzmann approximation

Energy: $f: g_c(E) \cdot f_F(E) = c \cdot \sqrt{E - E_c} \cdot e^{-\frac{E-E_F}{kT}}$, where $c = \dfrac{\sqrt{2}\,(m_n^*)^{\frac{3}{2}}}{\pi^2 \hbar^3}$ is a constant

To find the maximum point: $\dfrac{\partial f}{\partial E} = \dfrac{c}{2} \dfrac{1}{\sqrt{E - E_c}} e^{-\frac{E-E_F}{kT}} - \dfrac{c}{kT}\sqrt{E - E_c}\, e^{-\frac{E-E_F}{kT}} = 0$

$$\Rightarrow \frac{kT}{2} = E - E_c \Rightarrow E = \frac{kT}{2} + E_c$$

b) Similarly: $\dfrac{\partial f}{\partial E} = -kT + 2(E_v - E) = 0 \Rightarrow E = E_v - \dfrac{kT}{2}$

5) For a particular semiconductor, $E_g = 1.50eV$, $m_p^* = 10m_n^*$, T = 300K, and $n_i = 1 \times 10^5 cm^{-3}$.

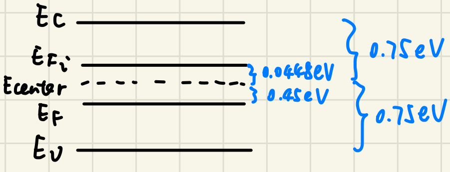
a) Determine the position of the intrinsic Fermi energy level with respect to the center of the bandgap.

b) Impurity atoms are added so that the Fermi energy level is 0.45eV below the center of the bandgap.

i) Are acceptor or donor atoms added?

ii) What is the concentration of impurity atoms added?

a) $$E_{Fi} - \frac{E_c + E_v}{2} = \frac{3}{4} kT \ln \frac{m_p^*}{m_n^*} = \frac{3}{4} \times 8.62 \times 10^{-5} \times 300 \cdot \ln 10 = 0.0448\,eV$$

b) 

(i) since $E_{Fi} > E_F \Rightarrow$ acceptor atoms are added

(ii) $$n_i^2 = n_o p_o = N_c \cdot e^{\frac{E_F - E_c}{kT}} \cdot N_v \cdot e^{\frac{E_v - E_F}{kT}} = N_c \cdot N_v \cdot e^{\frac{E_v - E_c}{kT}} = N_c \cdot N_v \cdot e^{\frac{-1.5}{8.62 \times 10^{-5} \cdot 300}}$$

Also, $$\frac{N_c}{N_v} = \frac{\frac{2(2\pi m_n^* kT)^{\frac{3}{2}}}{h^3}}{\frac{2(2\pi m_p^* kT)^{\frac{3}{2}}}{h^3}} = \left(\frac{m_n^*}{m_p^*}\right)^{\frac{3}{2}} = \left(\frac{1}{10}\right)^{\frac{3}{2}}$$

$$\Rightarrow \begin{cases} (10^5)^2 = N_c N_v \cdot e^{\frac{-1.5}{8.62 \times 10^{-5} \cdot 300}} \\ N_c = \left(\frac{1}{10}\right)^{\frac{3}{2}} N_v \end{cases} \Rightarrow (10^5)^2 = \left(\frac{1}{10}\right)^{\frac{3}{2}} N_v^2 \cdot e^{\frac{-1.5}{8.62 \times 10^{-5} \cdot 300}}$$

Therefore, $$N_v = \left(10^{11.5} \cdot e^{\frac{1.5}{8.62 \times 10^{-5} \cdot 300}}\right)^{\frac{1}{2}} = 2.22 \times 10^{18}\ cm^{-3}$$

According to $E_F = E_v + kT \ln \frac{N_v}{N_a}$, we have $(0.75 - 0.45) = 8.62 \times 10^{-5} \cdot 300 \ln \frac{2.22 \times 10^{18}}{N_a}$

$$\Rightarrow \frac{100}{8.62} = \ln \frac{2.22 \times 10^{18}}{N_a} \Rightarrow N_a = \frac{2.22 \times 10^{18}}{e^{\frac{100}{8.62}}} = 2.03 \times 10^{13}\ cm^{-3}$$

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6) A particular semiconductor material is doped at $N_d = 2 \times 10^{14} cm^{-3}$, and $N_a = 1.2 \times 10^{14} cm^{-3}$. The thermal equilibrium electron concentration is found to be $n_0 = 1.1 \times 10^{14} cm^{-3}$. Assuming complete ionization, determine:

a) the intrinsic carrier concentration

b) the thermal equilibrium hole concentration

b) $$N_d + p_o = N_a + n_o \Rightarrow p_o = 1.2 \times 10^{14} + 1.1 \times 10^{14} - 2 \times 10^{14} = 0.3 \times 10^{14}\ cm^{-3}$$

a) $$n_i = \sqrt{n_o p_o} = \sqrt{1.1 \times 10^{14} \times 0.3 \times 10^{14}} = \sqrt{33} \times 10^{13}\ cm^{-3} = 5.744 \times 10^{13}\ cm^{-3}$$