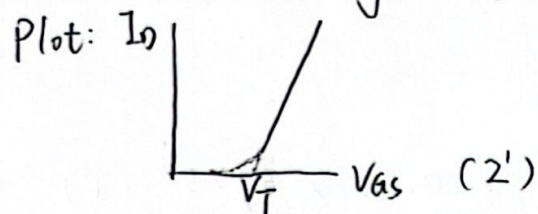
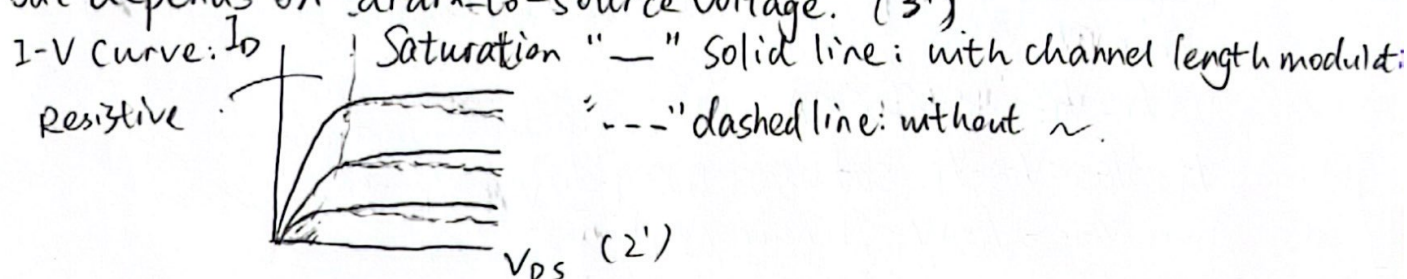


## Problem Set 8 Solution

1. (15 points) (a) When a reversed biased source-to-substrate voltage is applied, the depletion region of source-substrate region junction widens, so more gate voltage should be applied to invert the channel. (5')
- (b) Subthreshold conduction refers to the phenomenon where the drain current ( $I_D$ ) exists even when input voltage is less than or equal to the threshold voltage ( $V_{GS} \leq V_T$ ) (3')



- (c) Channel length modulation means when the MOSFET is biased in the saturation region, the effective channel length is not a constant but depends on drain-to-source voltage. (3')



2. (20 points) 5' + 5' + 5' + 5'

- (a)  $|V_{GS}| = 0.25V$ ,  $|V_{GS} - V_T| = 0.4V$ . Since  $|V_{DS}| < |V_{GS} - V_T|$ , in linear mode (2')

$$I_D = k_p' \cdot \frac{W}{L} (|V_{GS} - V_T| - \frac{1}{2} |V_{DS}|) |V_{DS}| = 0.1 \times 15 \times (0.4 - \frac{1}{2} \times 0.25) \times 0.25 = \boxed{0.103mA} \quad (3')$$

- (b)  $|V_{DS}| = 1V$ ,  $|V_{GS} - V_T| = 0.4V$ . Since  $|V_{DS}| > |V_{GS} - V_T|$  in saturation mode (2')

$$I_D = \frac{1}{2} k_p' \cdot \frac{W}{L} (|V_{GS} - V_T|)^2 = \frac{1}{2} \times 0.1 \times 15 \times 0.4^2 = \boxed{0.12mA} \quad (3')$$

- (c)  $|V_{DS}| = 1V$ ,  $|V_{GS} - V_T| = 0.8V$ . Since  $|V_{DS}| > |V_{GS} - V_T|$  in saturation mode (2')

$$I_D = \frac{1}{2} k_p' \cdot \frac{W}{L} (|V_{GS} - V_T|)^2 = \frac{1}{2} \times 0.1 \times 15 \times 0.8^2 = \boxed{0.48mA} \quad (3')$$

- (d)  $|V_{DS}| = 2V$ ,  $|V_{GS} - V_T| = 0.8V$ .  $|V_{DS}| > |V_{GS} - V_T|$  in sat mode (2')  $I_D = \boxed{0.48mA} \quad (3')$

3. (15 points) 5' + 5' + 5'

- (a) When  $V_{GS} = 0V$ ,  $V_{GS} < V_T$ ,  $I_D = 0$ .

When  $V_{GS} = 0.6V$ ,  $V_{DS}(sat) = V_{GS} - V_T = \boxed{0.15V}$ ,  $I_D(sat) = \mu_n C_{ox} \frac{W}{2L} V_{DS,sat}^2$   
 $= 425 \times \frac{3.9 \times 8.85 \times 10^{-14}}{11 \times 10^{-7}} \times \frac{20}{2 \times 1.2} \times 0.15^2 = \boxed{0.025mA}$

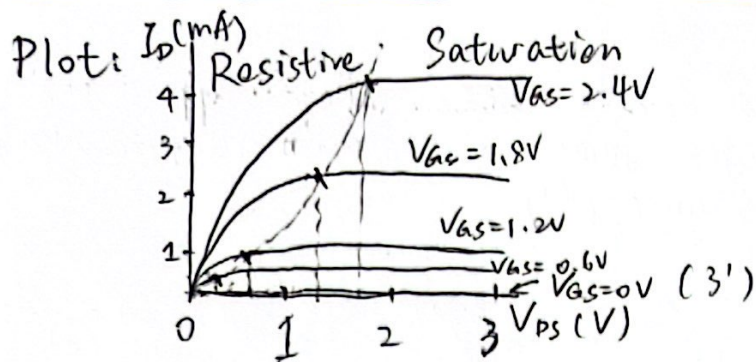
When  $V_{GS} = 1.2V$ ,  $V_{DS}(sat) = V_{GS} - V_T = \boxed{0.75V}$ ,  $I_D(sat) = \mu_n C_{ox} \frac{W}{2L} V_{DS,sat}^2 = \boxed{0.625mA}$

When  $V_{GS} = 1.8V$ ,  $V_{DS}(sat) = V_{GS} - V_T = \boxed{1.35V}$ ,  $I_D(sat) = \boxed{2.025mA}$

When  $V_{GS} = 2.4V$ ,  $V_{DS}(sat) = V_{GS} - V_T = \boxed{1.95V}$ ,  $I_D(sat) = \boxed{4.226mA} \quad (2')$

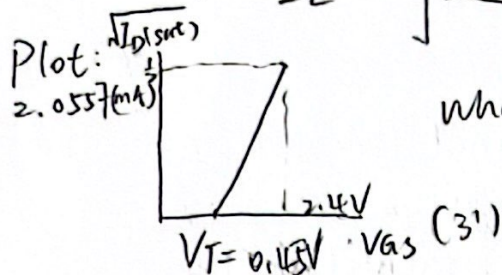






(b)  $\sqrt{I_D(sat)} = \sqrt{\frac{W\mu_n C_{ox}}{2L}} (V_{GS} - V_T)$

Slope =  $\sqrt{\frac{\mu_n C_{ox} W}{2L}} = \sqrt{\frac{425 \times \frac{3.9 \times 8.85 \times 10^{-14}}{11 \times 10^{-9}} \times 20}{2 \times 1.2}} = 0.0333 \text{ (F/V.S)}^{\frac{1}{2}} \text{ (1')}$



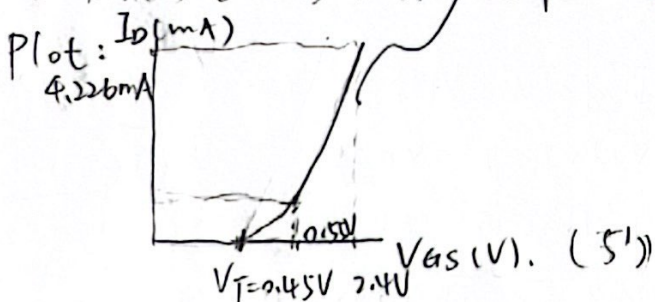
When  $V_{GS} = 2.4V$ ,  $\sqrt{I_D(sat)} \approx 2.0557 \text{ (mA)}^{\frac{1}{2}} \text{ (1')}$

(c) When  $0 < V_{GS} \leq V_T = 0.45V$ ,  $I_D = 0A$

When  $V_T < V_{GS} \leq V_{DS} + V_T$ , saturation region

and when  $V_{GS} \geq V_{DS} + V_T$ , linear region

$I_D = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)$ , slope =  $425 \times \frac{3.9 \times 8.85 \times 10^{-14}}{11 \times 10^{-9}} \times \frac{20}{1.2} = 2.223 \times 10^{-3} S$



4. (5 points)

$\phi_{fp} = \frac{kT}{q} \ln \frac{N_A}{n_i} = 0.0259 \times \ln \frac{10^{16}}{1.5 \times 10^{10}} \approx 0.347V \text{ (1')}$

$\Delta V_T = V_T' - V_T = \gamma \left[ \sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}} \right]$

$0.5 - V_T = 0.12 \times (\sqrt{0.347 \times 2 + 2.5} - \sqrt{0.347 \times 2}) \text{ (2')}$

$\Rightarrow V_T \approx \boxed{0.386V} \text{ (2')}$

5. (20 points) 5' + 5' + 5' + 5'

(a)  $\phi_{fp} = \frac{kT}{q} \ln \frac{N_A}{n_i} = 0.0259 \times \ln \frac{2 \times 10^{16}}{1.5 \times 10^{10}} \approx 0.365V$

$V_{DS(sat)} = V_{GS} - V_T = 1 - 0.4 = 0.6V \text{ (2')}$



$$\Delta L = \sqrt{\frac{2\epsilon_s}{eN_A}} \left[ \sqrt{\phi_{fp} + V_{DS}} - \sqrt{\phi_{fp} + V_{DS(sat)}} \right] = \sqrt{\frac{2 \times 11.7 \times 8.85 \times 10^{-14}}{1.6 \times 10^{-19} \times 2 \times 10^{16}}} \times \left[ \sqrt{0.365 + 2} - \sqrt{0.365 + 0.6} \right]$$

$$\approx \boxed{1.413 \times 10^{-5} \text{ cm}} \quad (3')$$

$$(b) V_{DS(sat)} = V_{GS} - V_T = 1 - 0.4 = 0.6 \text{ V}. \quad \Delta L = 2.544 \times 10^{-5} \times (\sqrt{0.365 + 4} - \sqrt{0.365 + 0.6})$$

$$\approx \boxed{2.816 \times 10^{-5} \text{ cm}} \quad (3')$$

$$(c) V_{DS(sat)} = V_{GS} - V_T = 2 - 0.4 = 1.6 \text{ V}. \quad \Delta L = 2.544 \times 10^{-5} \times (\sqrt{0.365 + 2} - \sqrt{0.365 + 1.6}) \approx \boxed{3.462 \times 10^{-6} \text{ cm}} \quad (3')$$

$$(d) V_{DS(sat)} = 1.6 \text{ V}. \quad \Delta L = 2.544 \times 10^{-5} \times (\sqrt{0.365 + 4} - \sqrt{0.365 + 1.6}) \approx \boxed{1.749 \times 10^{-5} \text{ cm}} \quad (3')$$

6. (10 points)

$$\text{Since } V_{DS(sat)} = V_{GS} - V_T, \quad V_{GS} = V_{DS(sat)} + V_T = 5 \text{ V} + 1 \text{ V} = \boxed{6 \text{ V}} \quad (1')$$

$$\text{When } V_{DS} = 10 \text{ V} > V_{DS(sat)}, \text{ in saturation mode. } \boxed{g_D = \frac{dI_{DS}}{dV_{DS}} = 0} \quad (3')$$

$$g_m(sat) = \frac{dI_{DS}}{dV_{GS}} = \frac{W}{L} \mu_n C_{ox} (V_{GS} - V_T) = k_n (V_{GS} - V_T) \quad (1')$$

$$\text{Since } I_{D(sat)} = \frac{k_n}{2} (V_{GS} - V_T)^2 \Rightarrow 0.1 \text{ mA} = \frac{k_n}{2} \times (5 \text{ V})^2 \Rightarrow k_n = 8 \times 10^{-6} \text{ A/V}^2$$

$$g_m(sat) = 8 \times 10^{-6} \times 5 = \boxed{4 \times 10^{-5} \text{ S}} \quad (3')$$

$$\text{For } V_{DS} = 1 \text{ V. } \begin{array}{c} S \\ | \\ \boxed{k^+} \end{array} \begin{array}{c} D \\ | \\ \boxed{n^+} \end{array} \quad (1') \quad \text{For } V_{DS} = 5 \text{ V. } \begin{array}{c} S \\ | \\ \boxed{n^+} \end{array} \begin{array}{c} D \\ | \\ \boxed{n^+} \end{array} \quad (1')$$

$$\text{For } V_{DS} = 10 \text{ V. } \begin{array}{c} S \\ | \\ \boxed{n^+} \end{array} \begin{array}{c} D \\ | \\ \boxed{n^+} \end{array} \quad (1')$$

7. (15 points) 5' + 5' + 5'

$$(a) \frac{I_D'}{I_D} = \frac{I_S e^{V_{GS}'/nV_T}}{I_S e^{V_{GS}/nV_T}} = e^{(V_{GS}' - V_{GS})/nV_T} \quad V_{GS}' - V_{GS} = nV_T \ln \frac{I_D'}{I_D}$$

$$\text{for } n=1, \quad V_{GS}' - V_{GS} = 0.0259 \times \ln 10 \approx \boxed{0.0596 \text{ V}} \quad (5')$$

$$(b) \text{ for } n=1.5, \quad V_{GS}' - V_{GS} = 1.5 \times 0.0259 \times \ln 10 \approx \boxed{0.0895 \text{ V}} \quad (5')$$

$$(c) \text{ for } n=2, \quad V_{GS}' - V_{GS} = 2 \times 0.0259 \times \ln 10 \approx \boxed{0.125 \text{ V}} \quad (5')$$

