1)

a) $\psi = 0$ for $|x| \geq \frac{L}{2}$ 2'

b) first: kinetic energy 2'

second: potential energy 2'

c) $\psi(x) = Ae^{jkx} + Be^{-jkx}$ 2'

or $A\sin(kx) + B\cos(kx)$

d) $\psi(-\frac{L}{2}) = 0$, $\psi(\frac{L}{2}) = 0$ 2'

e) 6'

$$\begin{cases} A\sin(k\cdot\frac{L}{2}) + B\cos(k\cdot\frac{L}{2}) = 0 \quad ① \\ A\sin(-k\cdot\frac{L}{2}) + B\cos(-k\cdot\frac{L}{2}) = 0 \Rightarrow -A\sin(k\cdot\frac{L}{2}) + B\cos(k\cdot\frac{L}{2}) = 0 \quad ② \end{cases}$$

① + ② , ① - ② $\Rightarrow \begin{cases} \sin(k\cdot\frac{L}{2}) = 0 \text{ or } A = 0 \\ \cos(k\cdot\frac{L}{2}) = 0 \text{ or } B = 0 \end{cases}$

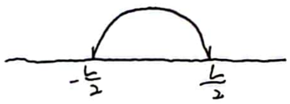
$$\begin{cases} \text{if } \sin(k\cdot\frac{L}{2}) = 0,\ B = 0 \Rightarrow k\cdot\frac{L}{2} = n\pi \Rightarrow k = \frac{2n\pi}{L} \Rightarrow \psi(x) = A\sin(\frac{2n\pi}{L}x) \quad n = 1, 2, 3\cdots \\ \text{if } \cos(k\cdot\frac{L}{2}) = 0,\ A = 0 \Rightarrow k\cdot\frac{L}{2} = (n+\frac{1}{2})\pi \Rightarrow k = \frac{(2n+1)\pi}{L} \Rightarrow \psi(x) = B\cos(\frac{(2n+1)\pi}{L}x) \quad n = 0, 1, 2, 3\cdots \end{cases}$$

$$E = E_p + E_k = E_k = \frac{p^2}{2m} = \frac{\hbar^2k^2}{2m} = \begin{cases} \frac{2\hbar^2n^2\pi^2}{mL^2} & \text{if } \psi(x) = A\sin(\frac{2n\pi}{L}x) \quad n = 1, 2, 3\cdots \\ \frac{\hbar^2(2n+1)^2\pi^2}{2mL^2} & \text{if } \psi(x) = B\cos(\frac{(2n+1)\pi}{L}x) \quad n = 0, 1, 2, 3\cdots \end{cases}$$

Normalize: $\int_{-\infty}^{\infty}\psi^*(x)\psi(x) = 0 \Rightarrow \int_{-\frac{L}{2}}^{\frac{L}{2}}\psi^2(x)\,dx = 0 \Rightarrow A = B = \sqrt{\frac{2}{L}}$

$$\text{So } \psi(x) = \begin{cases} \sqrt{\frac{2}{L}}\sin(\frac{2n\pi}{L}x) \quad n = 1, 2, 3\cdots \quad E_n = \frac{2\hbar^2n^2\pi^2}{mL^2} \quad \text{for } -\frac{L}{2} \leq x \leq \frac{L}{2} \\ \sqrt{\frac{2}{L}}\cos(\frac{(2n+1)\pi}{L}x) \quad n = 0, 1, 2, 3\cdots \quad E_n = \frac{\hbar^2(2n+1)^2\pi^2}{2mL^2} \quad \text{for } -\frac{L}{2} \leq x \leq \frac{L}{2} \end{cases}$$

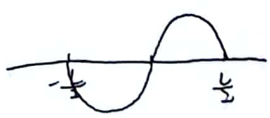
f) first: $\psi(x) = \sqrt{\frac{2}{L}}\cos\frac{\pi x}{L}$



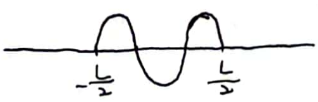
6'

second: $\psi(x) = \sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}$

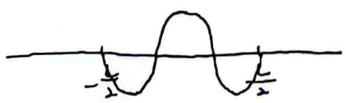
or



third: $\psi(x) = \sqrt{\frac{2}{L}}\cos\frac{3\pi x}{L}$



(this is also correct if you flip the sign of $\psi(x)$, even you multiply it by $|\alpha| = 1$, α is a complex number)



A more clever approach is to switch the coordinate from $0 \to L$ to $-\frac{L}{2} \to \frac{L}{2}$, the case of $0 \to L$ is discussed in the class!

in this case $\psi(x) = \sqrt{\frac{2}{L}}\sin[\frac{n\pi}{L}(x+\frac{L}{2})]$ $n = 1, 2, 3\cdots$

$$E = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

2)

a) I: $-\frac{\hbar^2}{2m}\frac{\partial^2 \psi_1(x)}{\partial x^2} + V_0 \psi_1(x) = E\psi_1(x) \Rightarrow \psi_1(x) = A_1 e^{k_1 x}$ 2' $\quad k_1 = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$ $\quad e^{-k_1 x}$ is excluded since it reaches infinity when $x \to -\infty$

II: $-\frac{\hbar^2}{2m}\frac{\partial^2 \psi_2(x)}{\partial x^2} = E\psi_2(x) \Rightarrow \psi_2(x) = A_2 \sin(k_2 x) + B_2 \cos(k_2 x)$ 2' $\quad k_2 = \sqrt{\frac{2mE}{\hbar^2}}$

III: $V = \infty \Rightarrow \psi_3(x) = 0$ 2'

b) $x=0$: $\begin{cases} \psi_1(x) = \psi_2(x) \Rightarrow A_1 = B_2 & ① \quad 2' \\ \psi_1'(x) = \psi_2'(x) \Rightarrow k_1 A_1 = k_2 A_2 & ② \quad 2' \end{cases}$

$x=a$: $\psi_2(x) = \psi_3(x) = 0 \Rightarrow A_2 \sin(k_2 a) + B_2 \cos(k_2 a) = 0$ ③ 2'

c) $B_2 = A_1 = \frac{k_2}{k_1} A_2$ from ① and ②

From ③: $\cancel{A_2} \sin(k_2 a) + \frac{k_2}{k_1} \cancel{A_2} \cos(k_2 a) = 0$ $\quad A_2$ can't be 0

$$\sin\left(\sqrt{\frac{2mE}{\hbar^2}}a\right) + \sqrt{\frac{E}{V_0-E}}\cos\left(\sqrt{\frac{2mE}{\hbar^2}}a\right) = 0$$

E should be quantized 6'

4)

a) $1/a^3$ 3'

b) $1/(\sqrt{2}a^2)$ 4'

c) [1,1,1] 3'

7)

a) indirect 2'

b) $E_g \sim 0.4eV$ 2'

c) hole: $m_2 > m_1$ 2'

electron: $m_3 > m_4$ 2'

d) $p = \hbar k = -\frac{\pi}{4a}\hbar$ 3'

$E_K = -0.4eV$ if $m_p^* < 0$ but generally we consider $m_p^* > 0$ so $E_K = 0.4eV$ 3'

e) $E_K = \frac{p^2}{2m_p^*} = \frac{\pi^2\hbar^2}{32a^2 m_p^*} = 0.4eV$

$m_p^* = \frac{\pi^2\hbar^2}{32a^2 \cdot 0.4(eV)}$ 3' or $5.35\times10^{-50}\frac{1}{a^2}$

$V = \frac{p}{m_p^*} = -\frac{8a\cdot 0.4(eV)}{\pi\hbar}$ 3' or $-1.55\times10^{15} a$

3: photon-electric effect, double-slit interference for single electron or photon, hydrogen spectrum, stability of atom, Compton scattering, randomness of radioactivity··· List whatever you find on the Internet will be OK.

5: band gap: Insulator>semiconductor>metal

At 0K: No electron can receive thermal energy to be excited from the valence band to the conduction band in semiconductor. In metal, electrons don't need to be excited to participate in the conduction process.

6: Doping is we add impurities intentionally to semiconductor like silicon. It enables us to control semiconductors' properties like conductivity so that we can engineer these materials based on our need. P-type dopants tend to attract electrons from semiconductor while N-type dopants tend to give electrons. P-type: B, Al, Ga···. N-type: N, P, As···.