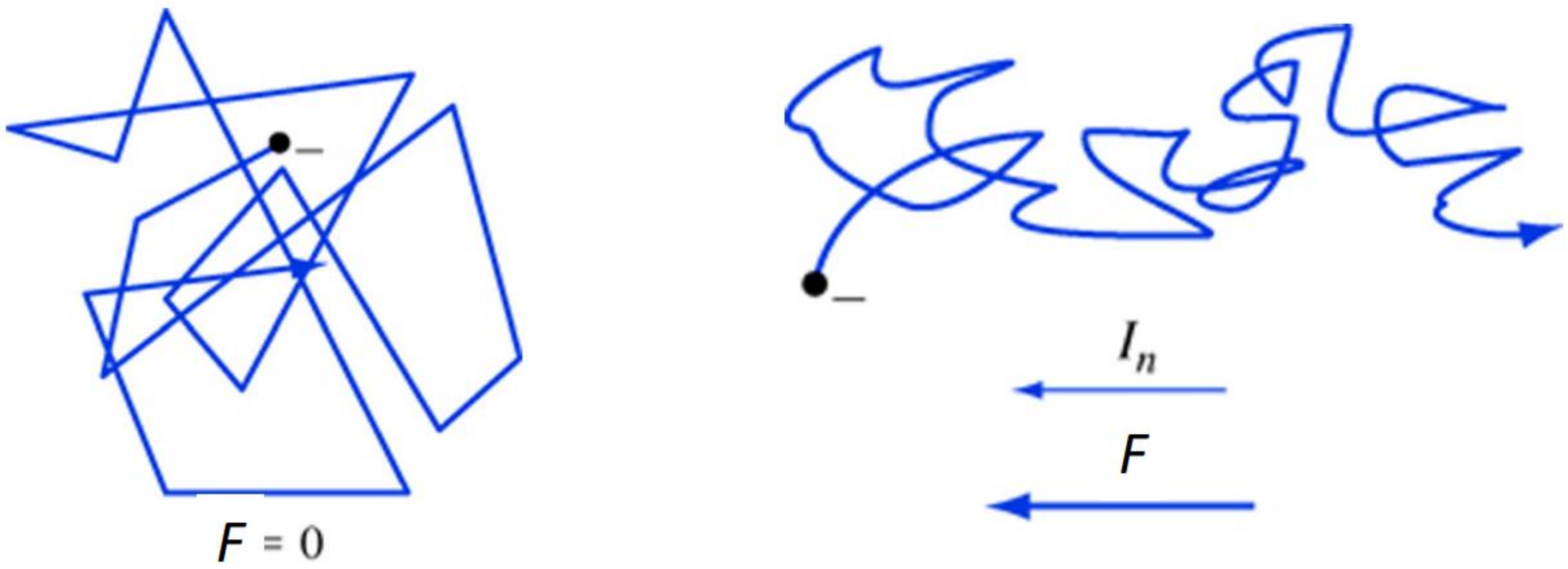
Mid2 RC part1

Drift current-electrons and holes are accelerated by the electric field

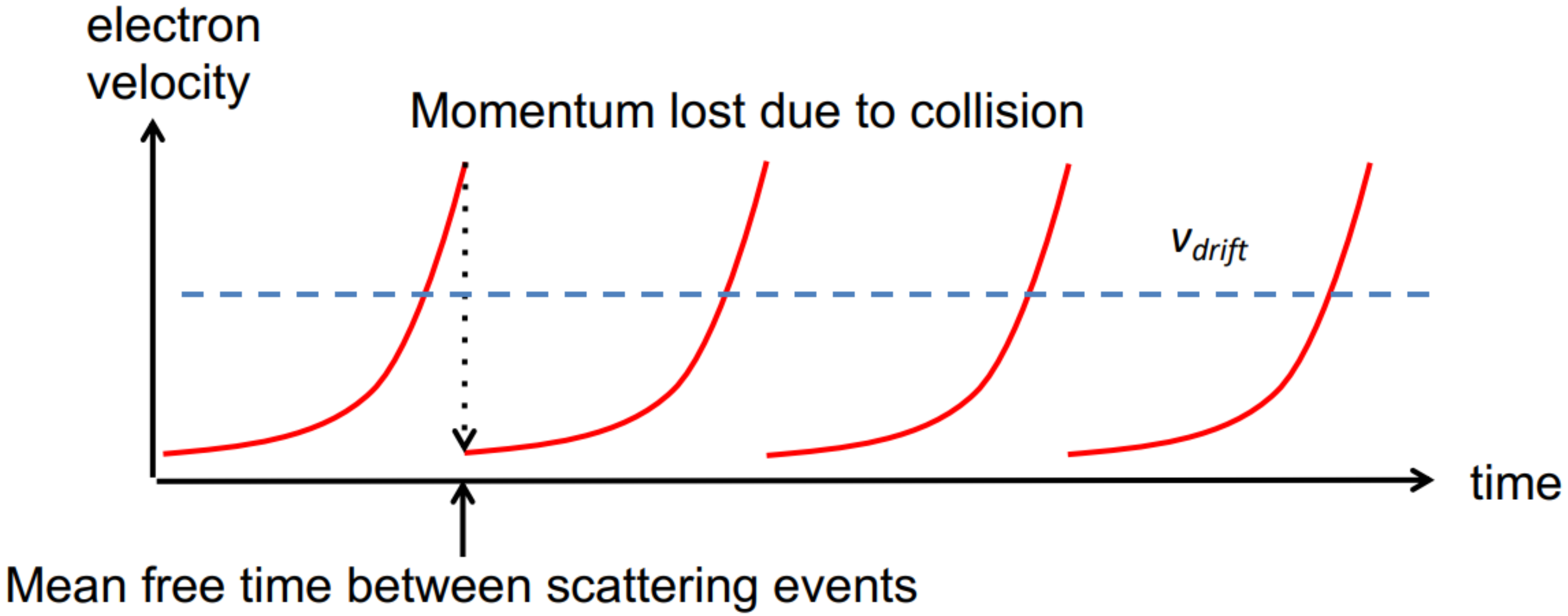
Electrons & holes respond to an electric field



- Carriers are in motion without an electric field (thermal energy)
- Carriers are accelerated by electric field in a particular direction
- Carriers scatter from lattice ions & crystal imperfections
- Average velocity obtained between scattering = v_{drift}

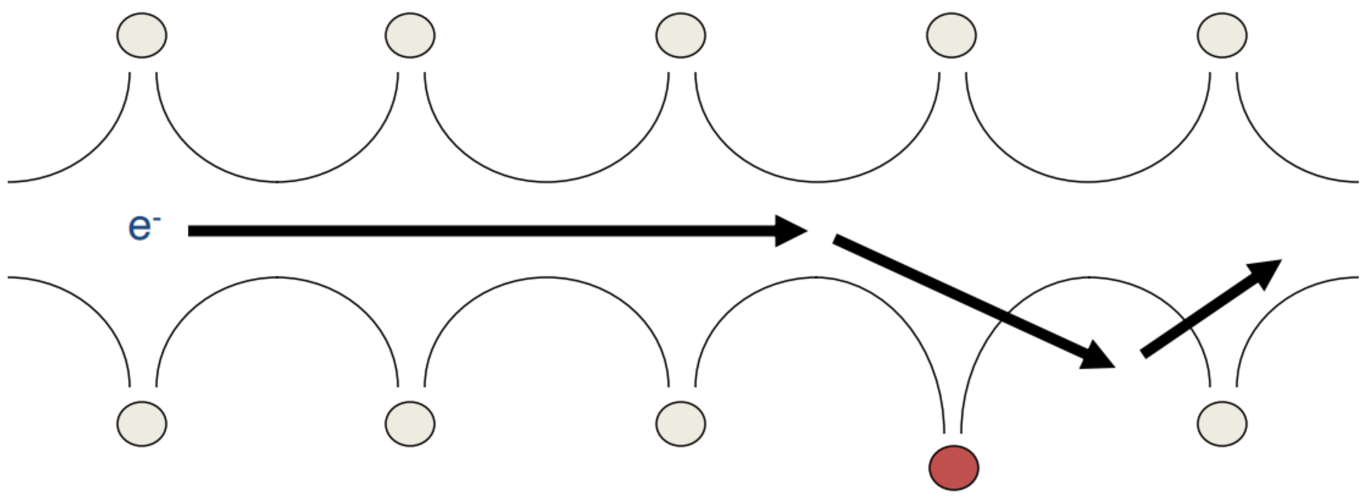
Drift current-drift velocity due to scattering

In semiconductors, a constant Coulomb force moves carriers at a *constant* velocity = **drift velocity**.



Drift current-two scattering mechanisms

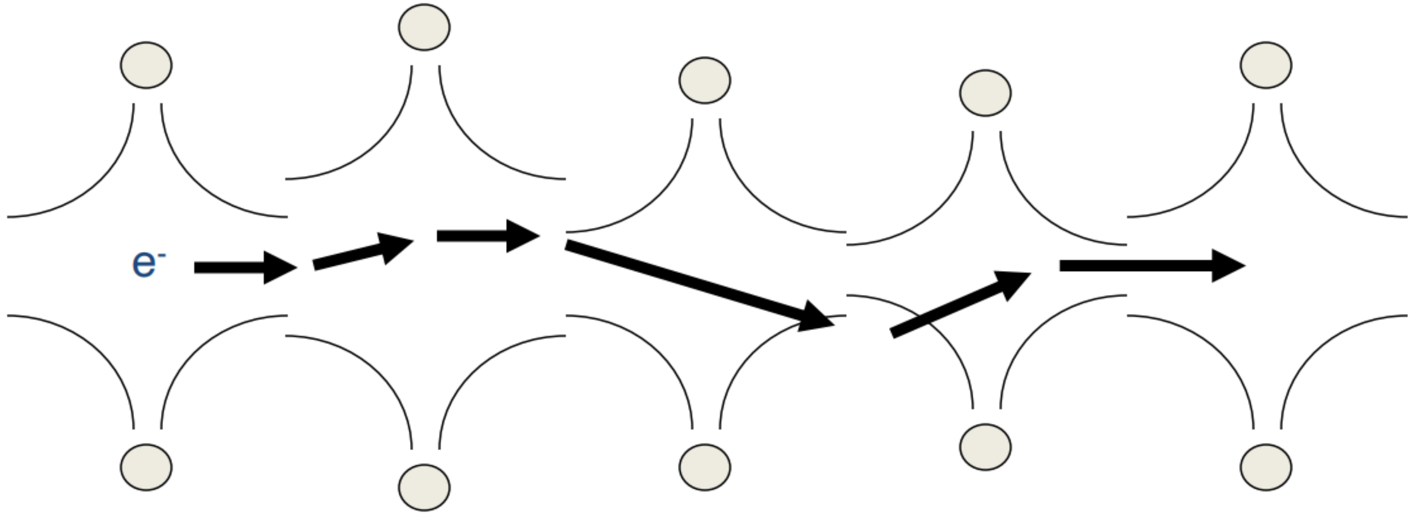
Mobility shows clear dependence on doping



Dopants are impurities in the crystal that cause local changes to the crystal potential seen by a moving electron

Drift current-two scattering mechanisms

Mobility also depends on temperature



Vibration of crystal atoms due to temperature causes a varying crystal potential as seen by a moving electron

Drift current-drift velocity and mobility

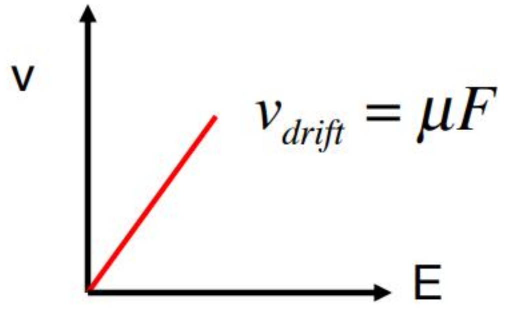
Drift current density

$\boxed{scatterings}$

$\boxed{v \sim E}$

$\boxed{v = \mu E}$

$\boxed{v_n = \mu_n E}$	$\boxed{v_p = \mu_p E}$
Electrons	Holes

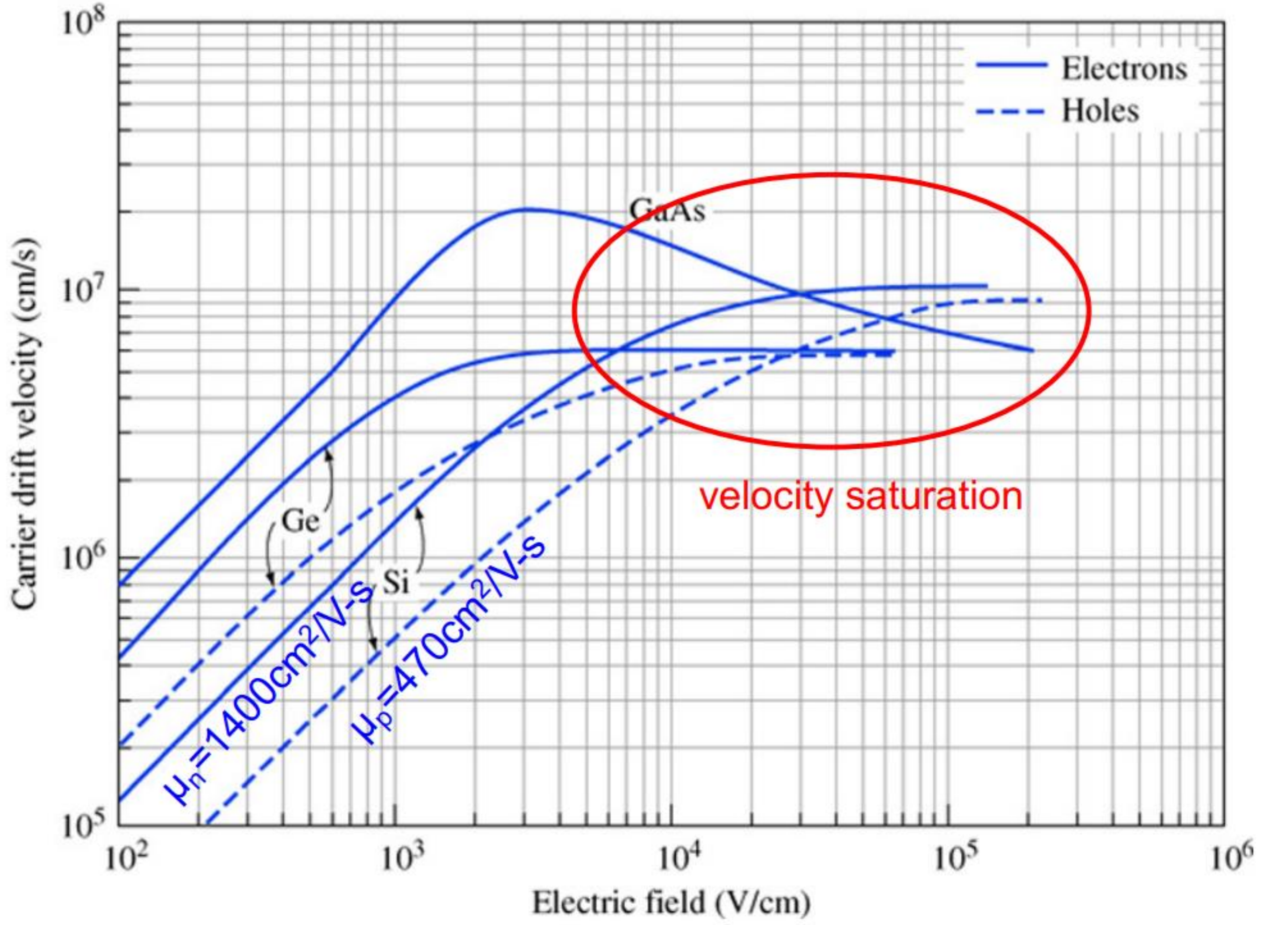


μ depends on mean time between scattering, τ, and m^*

$$\mu = \frac{q\tau}{m^*}$$

Drift current-drift velocity saturation

At high fields, mobility not constant, velocity "saturates"



Drift current-current density and conductivity

Drift current density

Hole drift current

$$J_{p|drf} = qp_0\mu_pE$$

Electron drift current

$$J_{n|drf} = qn_0\mu_nE$$

Both electrons and holes contribute to current:

$$J_{drf} = q(p_0\mu_p + n_0\mu_n)E$$

Table 5.1 | Typical mobility values at $T = 300$ K and low doping concentrations

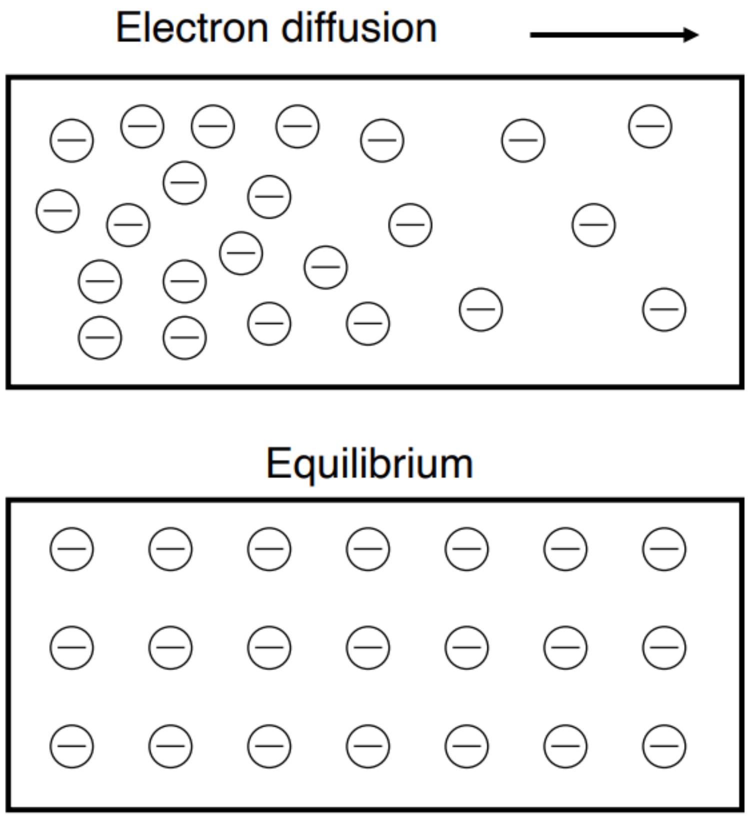
	μ_n (cm²/V-s)	μ_p (cm²/V-s)
Silicon	1350	480
Gallium arsenide	8500	400
Germanium	3900	1900

Conductivity depends on both carrier density and mobility

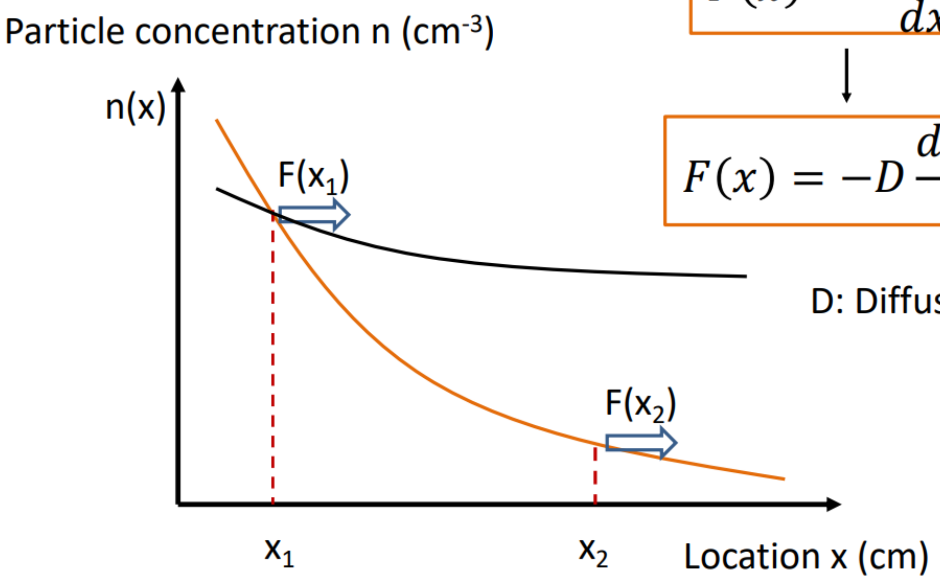
$$\sigma = qn\mu_n + qp\mu_p$$

Diffusion current

Carriers flow from high to low concentration



Diffusion current



$$\boxed{F(x) \sim \frac{dn(x)}{dx}}$$

$$\downarrow$$

$$\boxed{F(x) = -D\frac{dn(x)}{dx}}$$

D: Diffusivity

Diffusion current density

Electron diffusion current density: $J_{nx|dif} = -qF_n = qD_n\frac{dn}{dx}$

D_n is called the electron diffusion coefficient

Hole diffusion current density: $J_{px|dif} = qF_p = -qD_p\frac{dp}{dx}$

D_p is called the hole diffusion coefficient

Total current - drift + diffusion current

$$J = J_{drf} + J_{dif} = J_{n|drf} + J_{p|drf} + J_{n|dif} + J_{p|dif}$$

$$= qn\mu_n E_x + qp\mu_p E_x + qD_n \frac{dn}{dx} - qD_p \frac{dp}{dx}$$

$$\mathrm{J} = qn\mu_n E_x + qp\mu_p E_x + qD_n \nabla \mathrm{n} - qD_p \nabla p$$

How to link diffusion and drift

- Refer to the slide for detailed derivation
- Note the Einstein relationship has some restrictions. The Boltzmann approximation must can be applied to the semiconductor.

$$\boxed{D_n = \frac{\mu_n kT}{q}}$$

$$J_{n,drift} = qn(x)\mu_n|E| = qD_n\frac{dn(x)}{dx} = J_{n,diff}$$

$$qn(x)\mu_n\left(\frac{1}{q}\frac{kT}{n(x)}\frac{dn(x)}{dx}\right) = qD_n\frac{dn(x)}{dx}$$

$$\cancel{q}\cancel{n(x)}\mu_n\left(\frac{1}{q}\frac{kT}{\cancel{n(x)}}\cancel{\frac{dn(x)}{dx}}\right) = \cancel{q}D_n\cancel{\frac{dn(x)}{dx}}$$

Net recombination(generation) rate

- In one sentence, $np - n_i^2 = (n_0 + \Delta n)(p_0 + \Delta p) - n_i^2$ is the key parameter.
- If $np - n_i^2 = 0$, no net recombination(generation) rate. But generation and recombination still exist. The rate is 0 because they are balanced.

Excess carrier generation and recombination

Net recombination rate

$$\frac{d\Delta p}{dt} = -(R_n - G_0) = -[\alpha_r(n_0 + \Delta n)(p_0 + \Delta p) - \alpha_r n_i^2]$$

$$= -\alpha_r \cdot \Delta p \cdot (p_0 + n_0) - \alpha_r \cdot (\Delta p)^2$$

$$if\ p_0 + n_0 \gg \Delta p \qquad \approx -\alpha_r \cdot \Delta p \cdot (p_0 + n_0)$$

(Small injection condition)

$$\Delta p(t) = \Delta p(0)\exp(-\frac{t}{\tau_{p0}}) \qquad \tau_{p0} = \frac{1}{\alpha_r(p_0 + n_0)}$$

Net recombination(generation) rate

- Note the small injection condition
- Note that $\Delta n, \Delta p$ can depend on both x and t

For n-type semiconductor, net recombination rate

$$R_n - G_0 = R_p - G_0 = \frac{\Delta p(t)}{\tau_{p0}}$$

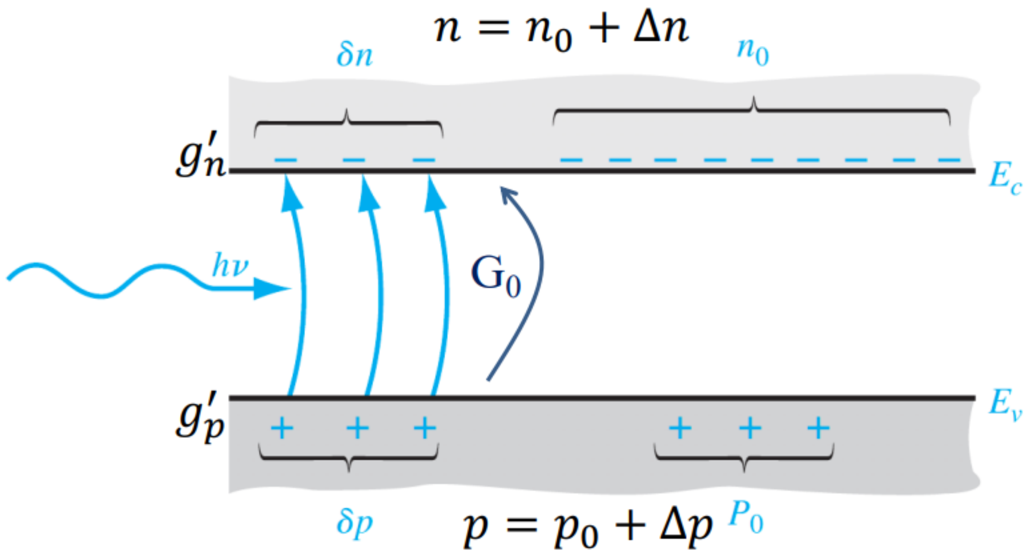
For p-type semiconductor, net recombination rate

$$R_n - G_0 = R_p - G_0 = \frac{\Delta n(t)}{\tau_{n0}}$$

External generation effect

- Note G_0 here is the intrinsic generation rate
- g' is the external generation rate

Excess carrier generation and recombination



g’ is not a function of n and p

$$g'_n = g'_p = g', \qquad \Delta n = \Delta p$$

Combination of those four effects give us the continuity equation

For an n-type semiconductor,

$$\boxed{\frac{d\Delta p}{dt} = D_p \frac{d^2 p}{dx^2} - \mu_p E \frac{dp}{dx} - p\mu_p \frac{dE}{dx} - R'_p + g'_p}$$

$$R'_p = \frac{\Delta p}{\tau_{p0}}$$

(minority carriers)

$$\boxed{\frac{d\Delta n}{dt} = D_n \frac{d^2 n}{dx^2} + \mu_n E \frac{dn}{dx} + n\mu_n \frac{dE}{dx} - R'_n + g'_n}$$

$$R'_n = R'_p = \frac{\Delta p}{\tau_{p0}}$$

(majority carriers)

$$g'_n = g'_p$$

We focus on minority carrier equation

- p-type: $D_n \frac{\mathrm{d}^2 n}{\mathrm{d}x^2} + \mu_n (E \frac{\mathrm{d}n}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x}) + g_n' - \frac{\delta n}{\tau_{n0}} = \frac{\mathrm{d}\delta n}{\mathrm{d}t}$
- n-type: $D_p \frac{\mathrm{d}^2 p}{\mathrm{d}x^2} - \mu_p (E \frac{\mathrm{d}p}{\mathrm{d}x} + p \frac{\mathrm{d}E}{\mathrm{d}x}) + g_p' - \frac{\delta p}{\tau_{p0}} = \frac{\mathrm{d}\delta p}{\mathrm{d}t}$
- For homogeneous semiconductor, $n(x) = n_0 + \delta n(x)$, the equation can be simplified as

$$D_n \frac{\mathrm{d}^2 \delta n}{\mathrm{d}x^2} + \mu_n (E \frac{\mathrm{d}\delta n}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x}) + g_n' - \frac{\delta n}{\tau_{n0}} = \frac{\mathrm{d}\delta n}{\mathrm{d}t}$$
$$D_p \frac{\mathrm{d}^2 \delta p}{\mathrm{d}x^2} - \mu_p (E \frac{\mathrm{d}\delta p}{\mathrm{d}x} + p \frac{\mathrm{d}E}{\mathrm{d}x}) + g_p' - \frac{\delta p}{\tau_{p0}} = \frac{\mathrm{d}\delta p}{\mathrm{d}t}$$

At most cases, you can simplify the continuity equations by some assumptions

Table 6.2 |

Specification	Effect
Steady state	$\frac{\partial(\delta n)}{\partial t} = 0, \quad \frac{\partial(\delta p)}{\partial t} = 0$
Uniform distribution of excess carriers (uniform generation rate) + no boundary confinement	$D_n \frac{\partial^2(\delta n)}{\partial x^2} = 0, \quad D_p \frac{\partial^2(\delta n)}{\partial x^2} = 0$
Zero electric field	$\mathrm{E}\frac{\partial(\delta n)}{\partial x} = 0, \quad \mathrm{E}\frac{\partial(\delta p)}{\partial x} = 0$
No excess carrier generation	$g' = 0$
No excess carrier recombination (infinite lifetime)	$\frac{\delta n}{\tau_{n0}} = 0, \quad \frac{\delta p}{\tau_{p0}} = 0$

We focus on minority carrier equation

- p-type: $D_n \frac{\mathrm{d}^2 n}{\mathrm{d}x^2} + \mu_n (E \frac{\mathrm{d}n}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x}) + g_n' - \frac{\delta n}{\tau_{n0}} = \frac{\mathrm{d}\delta n}{\mathrm{d}t}$
- n-type: $D_p \frac{\mathrm{d}^2 p}{\mathrm{d}x^2} - \mu_p (E \frac{\mathrm{d}p}{\mathrm{d}x} + p \frac{\mathrm{d}E}{\mathrm{d}x}) + g_p' - \frac{\delta p}{\tau_{p0}} = \frac{\mathrm{d}\delta p}{\mathrm{d}t}$
- For homogeneous semiconductor, $n(x) = n_0 + \delta n(x)$, the equation can be simplified as

$$D_n \frac{\mathrm{d}^2 \delta n}{\mathrm{d}x^2} + \mu_n (E \frac{\mathrm{d}\delta n}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x}) + g_n' - \frac{\delta n}{\tau_{n0}} = \frac{\mathrm{d}\delta n}{\mathrm{d}t}$$

$$D_p \frac{\mathrm{d}^2 \delta p}{\mathrm{d}x^2} - \mu_p (E \frac{\mathrm{d}\delta p}{\mathrm{d}x} + p \frac{\mathrm{d}E}{\mathrm{d}x}) + g_p' - \frac{\delta p}{\tau_{p0}} = \frac{\mathrm{d}\delta p}{\mathrm{d}t}$$

Quasi-Fermi level

Fermi energy level considering excess carriers:

$$n_0 + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

$$p_0 + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

Quasi-Fermi level-band diagram

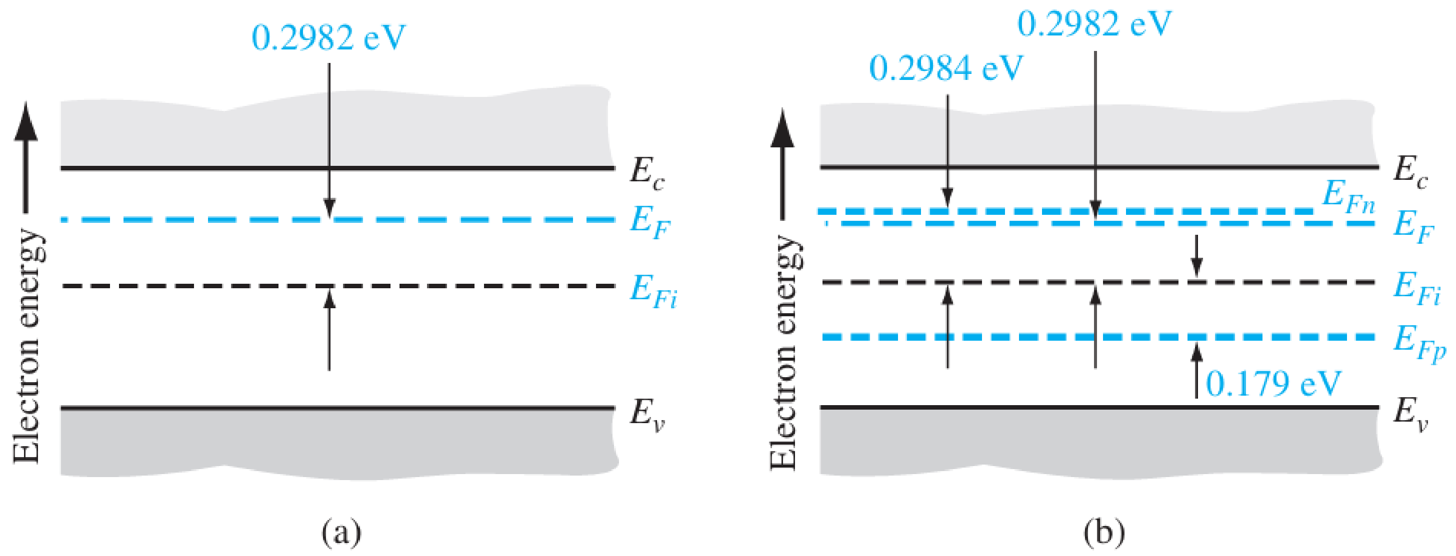


Figure 6.15 | (a) Thermal-equilibrium energy-band diagram for $N_d = 10^{15}$ cm^{-3} and $n_i = 10^{10}$ cm^{-3}. (b) Quasi-Fermi levels for electrons and holes if 10^{13} cm^{-3} excess carriers are present.

SRH recombination

$$R_n = R_p = \frac{C_n C_p N_t (np - n_i^2)}{C_n(n + n') + C_p(p + p')}$$

$$= \frac{np - n_i^2}{\tau_{p0}(n + n') + \tau_{n0}(p + p')}$$

where

$$n' = N_c \exp[-\frac{E_c - E_t}{kT}], \;\; p' = N_v \exp[-\frac{E_t - E_v}{kT}]$$

$$\tau_{n0} = \frac{1}{C_n N_t}, \;\; \tau_{p0} = \frac{1}{C_p N_t}$$

Surface effect- In practice, you can view it as a boundary condition for solving continuity equation.

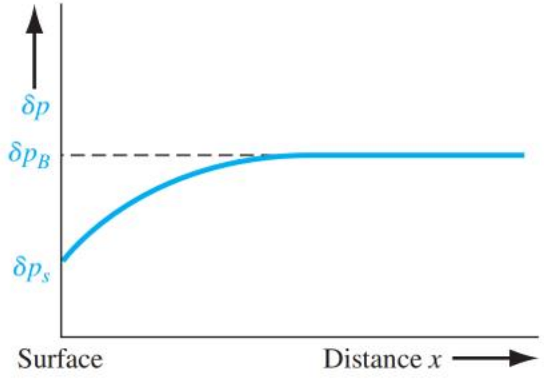


Figure 6.18 | Steady-state excess hole concentration versus distance from a semiconductor surface.

$$-D_p[\hat{n} \cdot \frac{\mathrm{d}(\delta p)}{\mathrm{d}x}]|_{\mathrm{surf}} = s\delta p|_{\mathrm{surf}}$$

s: surface recombination velocity

Combine continuity equation and surface effect

Problem example #5

A n-type semiconductor wafer is uniformly doped and uniformly illuminated by light. There is no electric field. The illumination generation rate is g and the minority carrier lifetime is τ. The semiconductor is long enough on the right side. On the left side edge, the surface recombination velocity is s. Find how does the concentration of the excess minority carriers change along x coordinate at steady state. Small injection condition is always maintained.