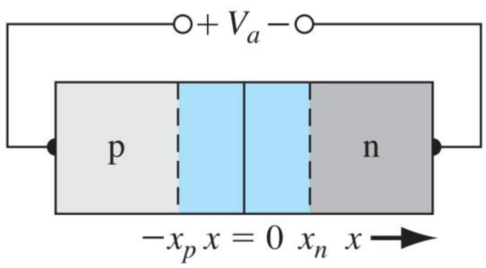
VE320 Intro to Semiconductor Devices
Summer 2024 — Problem Set 6
Due: 11:59pm 10th July

Note: In the following problems, assume T = 300K and the following parameters unless otherwise stated.
For silicon pn junctions: $D_n = 25\text{cm}^2/\text{s}$, $D_p = 10\text{cm}^2/\text{s}$, $\tau_{n0} = 5 \times 10^{-7}\text{s}$, $\tau_{p0} = 10^{-7}\text{s}$
For GaAs pn junctions: $D_n = 205\text{cm}^2/\text{s}$, $D_p = 9.8\text{cm}^2/\text{s}$, $\tau_{n0} = 5 \times 10^{-8}\text{s}$, $\tau_{p0} = 10^{-8}\text{s}$

10' 1) Explain the physical mechanism of the a) generation current and b) recombination current in depletion region of pn junction

10' 2) Consider an ideal pn junction diode at T = 300K operating in the forward-bias region.
a) Calculate the change in diode voltage that will cause a factor of 10 increase in current.
b) Repeat part a) for a factor of 100 increase in current.

30' 3) Consider an ideal silicon pn junction diode with the geometry shown in the figure.



The doping concentrations are $N_a = 5 \times 10^{16}\text{cm}^{-3}$ and $N_d = 1.5 \times 10^{16}\text{cm}^{-3}$, and the minority carrier lifetimes are $\tau_{n0} = 2 \times 10^{-7}\text{s}$, $\tau_{p0} = 8 \times 10^{-8}\text{s}$. The cross-sectional area is $A = 5 \times 10^{-4}\text{cm}^2$. Calculate:
a) the ideal reverse-saturation current due to holes
b) the ideal reverse-saturation current due to electrons
c) the hole concentration at $x = x_n$ for $V_a = 0.8V_{bi}$
d) the electron current at $x = x_n$ for $V_a = 0.8V_{bi}$
e) the electron current at $x = x_n + (1/2)L_p$ for $V_a = 0.8V_{bi}$

10' 4) Consider an ideal GaAs pn junction diode.
a) What must be the ratio of N_d/N_a so that 90 percent of the current in the depletion region is due to the flow of electrons?
b) Repeat part a) if 80 percent of the current in the depletion region is due to the flow of holes.

15' 5) The reverse-biased saturation current is a function of temperature.
a) Assuming that I_s varies with temperature only from the intrinsic carrier concentration, show that we can write $I_s = CT^3 \exp(-E_g/kT)$ where C is a constant and a function only of the diode parameters.
b) Determine the increase in I_s as the temperature increases from T = 300K to T = 400K for a
(i) germanium diode

(ii) silicon diode

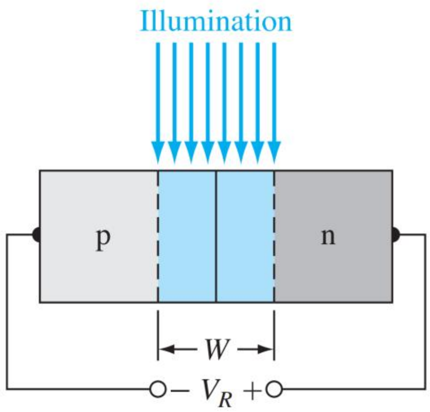
15′ 6) Consider a silicon pn junction diode with an applied reverse-biased voltage of $V_R = 5\text{V}$. The doping concentrations are $N_a = N_d = 4 \times 10^{16}\text{cm}^{-3}$ and the cross-sectional area is $A = 10^{-4}\text{cm}^2$. Assume minority carrier lifetimes of $\tau_0 = \tau_{n0} = \tau_{p0} = 10^{-7}\text{s}$. Calculate:

a) ideal reverse-saturation current

b) reverse-biased generation current

c) the ratio of the generation current to ideal saturation current

10′ 7) Consider a uniformly doped silicon pn junction at $T = 300\text{K}$ with impurity doping concentrations of $N_a = N_d = 5 \times 10^{15}\text{cm}^{-3}$ and minority carrier lifetimes of $\tau_0 = \tau_{n0} = \tau_{p0} = 10^{-7}\text{s}$. A reverse-biased voltage of $V_R = 10\text{V}$ is applied as shown in the figure. A light source is incident only on the space charge region, producing an excess carrier generation rate of $g' = 4 \times 10^{19}\text{cm}^{-3}\text{s}^{-1}$. Calculate the generation current density.



5'

1) a) generation current: under reverse bias, electrons and holes are generated within depletion region. Generated electrons and holes are swept out of the depletion region by the electric field, resulting in the generation current.

condition (forward / reverse bias): 1'

5'

b) recombination current: under forward bias, electrons and holes are injected across the space charge region and recombine within depletion region, reducing number of carriers. To make up the loss of carriers, additional carriers must be injected and the flow of additional carriers results in recombination current.

10'

2) $I = I_s\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$

$$\frac{I_1}{I_2} = \frac{\exp\left(\frac{eV_{a1}}{kT}\right) - 1}{\exp\left(\frac{eV_{a2}}{kT}\right) - 1} \approx \exp\left(\frac{e(V_{a1} - V_{a2})}{kT}\right)$$

$V_{a1} - V_{a2} = kT \ln\left(\frac{I_1}{I_2}\right)$ 5'

a) $V_{a1} - V_{a2} = 0.0259 \times \ln 10 = 0.0596\,V$ 5'

b) $V_{a1} - V_{a2} = 0.0259 \times \ln 100 = 0.119\,V$

5'

3) a) $I_{sp} = J_{sp} A = \frac{eD_p p_{n0}}{L_p} \cdot A = e\sqrt{\frac{D_p}{\tau_{p0}}} \cdot \frac{n_i^2}{N_d} \cdot A$ 2'

$= 1.6 \times 10^{-19} \times \sqrt{\frac{10}{8 \times 10^{-8}}} \times \frac{(1.5 \times 10^{10})^2}{1.5 \times 10^{16}} \times 5 \times 10^{-4}$

$= 1.342 \times 10^{-14}\,A$ 3'

5'

b) $I_{sn} = J_{sn} A = \frac{eD_n n_{p0}}{L_n} \cdot A = e\sqrt{\frac{D_n}{\tau_{n0}}} \cdot \frac{n_i^2}{N_a} \cdot A$ 2'

$= 1.6 \times 10^{-19} \times \sqrt{\frac{25}{2 \times 10^{-7}}} \cdot \frac{(1.5 \times 10^{10})^2}{5 \times 10^{16}} \times 5 \times 10^{-4}$

$= 4.025 \times 10^{-15}\,A$ 3'

5'
c) $V_{bi} = V_t \cdot \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$= 0.0259 \times \ln\left(\frac{5\times10^{16}\times1.5\times10^{16}}{(1.5\times10^{10})^2}\right)$

$= 0.747\,V$ 2'

$p_{no} = \frac{n_i^2}{N_d} = \frac{(1.5\times10^{10})^2}{1.5\times10^{16}} = 1.5\times10^4\ cm^{-3}$

$p_n(x_n) = p_{no}\exp\left(\frac{V_a}{V_t}\right)$ 1' $= 1.5\times10^4 \times \exp\left(\frac{0.8\times0.747}{0.0259}\right)$

$= 1.57\times10^{14}\ cm^{-3}$ 2'

5'
d) $I_n(x_n) = I_n(-x_p) = I_{sn}\left[\exp\left(\frac{V_a}{V_t}\right)-1\right]$ 2'

$= 4.025\times10^{-15} \times \left[\exp\left(\frac{0.8\times0.747}{0.0259}\right)-1\right]$

$= 4.22\times10^{-5}\,A$ 3'

10'
e) $I_p(x_n) = I_{sp}\left[\exp\left(\frac{V_a}{V_t}\right)-1\right]$

$= 1.342\times10^{-14} \times \left[\exp\left(\frac{0.8\times0.747}{0.0259}\right)-1\right]$

$= 1.407\times10^{-4}\,A$ 3'

$I_{total} = I_n(x_n) + I_p(x_n) = 1.83\times10^{-4}\,A$ 2'

$I_p(x_n+\frac{1}{2}L_p) = I_{sp}\left[\exp\left(\frac{V_a}{V_t}\right)-1\right]\cdot\exp\left(-\frac{1}{2}\right) = 8.53\times10^{-5}\,A$ 3'

$I_n(x_n+\frac{1}{2}L_p) = I_{total} - I_p(x_n+\frac{1}{2}L_p)$

$= 9.77\times10^{-5}\,A$ 2'

4) 5'
a) $\frac{J_n}{J} = \frac{J_{sn}}{J_{sn}+J_{sp}} = 0.9$ 1' $\Rightarrow J_{sn} = 9J_{sp}$

$\sqrt{\frac{D_n}{\tau_{no}}}\cdot\frac{n_i^2}{N_a} = 9\sqrt{\frac{D_p}{\tau_{po}}}\cdot\frac{n_i^2}{N_d}$

$\frac{N_d}{N_a} = 9\sqrt{\frac{D_p}{\tau_{po}}\cdot\frac{\tau_{no}}{D_n}}$ 2' $= 9\times\sqrt{\frac{9.8}{10^{-8}}\times\frac{5\times10^{-8}}{205}} = 4.4001$ 2'

5'
b) $\frac{J_p}{J} = \frac{J_{sp}}{J_{sn}+J_{sp}} = 0.8$ 1' $\Rightarrow J_{sp} = 4J_{sn}$

$\sqrt{\frac{D_p}{\tau_{po}}}\cdot\frac{n_i^2}{N_d} = 4\sqrt{\frac{D_n}{\tau_{no}}}\cdot\frac{n_i^2}{N_a}$

$$\frac{N_d}{N_a} = \frac{1}{4}\sqrt{\frac{D_p}{\tau_{p0}}\cdot\frac{\tau_{n0}}{D_n}} \quad 2' \quad = \frac{1}{4}\times\sqrt{\frac{9.8}{10^{-8}}\times\frac{5\times10^{-8}}{205}} = 0.1222 \quad 2'$$

5) 5' a) $I_s = A\cdot J_s$

$$= Ae\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right] n_i^2$$

$$n_i^2 = N_cN_v \exp\left(-\frac{E_g}{kT}\right)$$

$$= 4\,\frac{(2\pi m_n^* k)^{3/2}}{h^3}\cdot\frac{(2\pi m_p^* k)^{3/2}}{h^3}\cdot T^3\cdot\exp\left(-\frac{E_g}{kT}\right)$$

$$C = 4eA\,\frac{(2\pi m_n^* k)^{3/2}\cdot(2\pi m_p^* k)^{3/2}}{h^6}\cdot\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right]$$

$$I_s = CT^3\exp\left(-\frac{E_g}{kT}\right)$$

10' b) $\frac{I_{s1}}{I_{s2}} = \left(\frac{T_1}{T_2}\right)^3\exp\left[-E_g\left(\frac{1}{kT_1}-\frac{1}{kT_2}\right)\right]$ 5'

i) germanium: $\frac{I_{s1}(T=400K)}{I_{s2}(T=300K)} = \left(\frac{4}{3}\right)^3\cdot\exp\left[-0.66\cdot\left(\frac{1}{0.0259\times\frac{4}{3}}-\frac{1}{0.0259}\right)\right] = 1.385\times10^3$

5'

ii) silicon: $\frac{I_{s1}(T=400K)}{I_{s2}(T=300K)} = \left(\frac{4}{3}\right)^3\cdot\exp\left[-1.12\cdot\left(\frac{1}{0.0259\times\frac{4}{3}}-\frac{1}{0.0259}\right)\right] = 1.17\times10^5$

6) 5' a) $I_s = J_sA = \left(\frac{eD_pp_{n0}}{L_p} + \frac{eD_nn_{p0}}{L_n}\right)A$

$$= en_i^2\left(\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right)\cdot A \quad 2'$$

$$= 1.6\times10^{-19}\times(1.5\times10^{10})^2\times\left(\frac{1}{4\times10^{16}}\sqrt{\frac{25}{10^{-7}}} + \frac{1}{4\times10^{16}}\sqrt{\frac{10}{10^{-7}}}\right)\times10^{-4}$$

$$= 2.323\times10^{-15}\,A \quad 3'$$

7' b) $J_{gen} = \int_0^W eG\,dx = \frac{en_iW}{2\tau_0}$

$$V_{bi} = V_t\ln\left(\frac{N_aN_d}{n_i^2}\right) = 0.0259\times\ln\left(\frac{4\times10^{16}\times4\times10^{16}}{(1.5\times10^{10})^2}\right)$$

$$= 0.7665\,V \quad 2'$$

$$W = \sqrt{\frac{2\varepsilon_s(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_aN_d}\right)}$$

$$= \sqrt{\frac{2\times11.7\times8.85\times10^{-14}\times(0.7665+5)}{1.6\times10^{-19}}\times\frac{4\times10^{16}+4\times10^{16}}{4\times10^{16}\times4\times10^{16}}}$$

$$= 6.109\times10^{-5}\,cm \quad 2'$$

$$I_{gen} = J_{gen} A$$

$$= \frac{1.6\times10^{-19} \times 1.5\times10^{10} \times 6.109\times10^{-5}}{2\times10^{-7}} \times 10^{-4}$$

$$= 7.3308\times10^{-11} A \quad 3'$$

3′ c) $\frac{I_{gen}}{I_s} = \frac{7.3308\times10^{-11}}{2.323\times10^{-15}} = 3.16\times10^4 \quad 3'$

10′

7) $J'_{gen} = \int_0^W eg'dx = eg'W$

$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = 0.0259\times \ln\left(\frac{5\times10^{15}\times5\times10^{15}}{(1.5\times10^{10})^2}\right) = 0.6587V \quad 2'$

$W = \sqrt{\frac{2\varepsilon_s(V_{bi}+V_R)}{e}\cdot\frac{N_a+N_d}{N_a N_d}} = \sqrt{\frac{2\times11.7\times8.85\times10^{-14}\times(0.6587+10)}{1.6\times10^{-19}}\times\frac{5\times10^{15}+5\times10^{15}}{5\times10^{15}\times5\times10^{15}}} = 2.349\times10^{-4}\, cm \quad 2'$

$J'_{gen} = 1.6\times10^{-19}\times4\times10^{19}\times2.349\times10^{-4} = 1.50\times10^{-3}\ A/cm^2 \quad 2'$

$J_{gen0} = \frac{en_iW}{2\tau_0} = \frac{1.6\times10^{-19}\times1.5\times10^{10}\times2.349\times10^{-4}}{2\times10^{-7}} = 2.82\times10^{-6} A/cm^2 \quad 2'$

$J_{gen} = J'_{gen} + J_{gen0} = 1.50\times10^{-3} + 2.82\times10^{-6} = 1.50\times10^{-3}\ A/cm^2 \quad 2'$