Question 1:

It can be derived from the MLR OLS formula by considering a special case. $p=1$

general MLR model: $y = \beta_0 + \beta_1 x_1 + \cdots \beta_p x_p + \varepsilon$

$$\hat{b}_p = \sum H_i (y_i - \bar{y}) / \sum H_i (x_i - \bar{x})$$

where H_i represents the leverage of the ith observation and defined as

$$H_i = x_i - \bar{x} / \sum_j (x_j - \bar{x})^2$$

If we consider the SLR case, there is only one independent variable

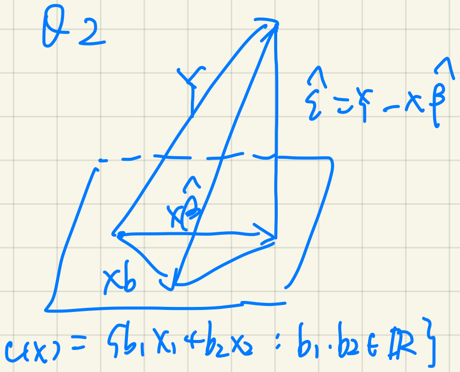
$$\hat{b} = (\sum (x_i - \bar{x})(y_i - \bar{y})) / \sum (x_i - \bar{x})^2$$

This is the OLS estimator for the slope parameter in SLR, denoted as $\hat{b}$.

The intercept parameter in SLR $\hat{a}$:

$$\hat{a} = \bar{y} - \hat{b}(\bar{x})$$

Q2



By projection, the residual vector $\hat{\varepsilon} = Y - X\hat{\beta}$ must be orthogonal to $C(X)$, or to $X_1 \ldots X_p$

This geometric implies that:

$$X_1^T (Y - X\hat{\beta}) = 0, \ \ldots \ X_p^T (Y - X\hat{\beta}) = 0$$

Q3:

$\hat{b} = Cov(X,Y)/Var(X)$

$\hat{Y} = \hat{b} \cdot X$

$\Rightarrow Cov(\hat{Y}, \hat{\varepsilon}) = Cov(\hat{b} \cdot X, \hat{\varepsilon}) = \hat{b} \cdot Cov(X, \hat{\varepsilon})$

since $E(\varepsilon | X) = 0$, $Cov(\varepsilon_i \cdot \varepsilon_j | X) = 0$ for $i \neq j$

$\Rightarrow Cov(X, \hat{\varepsilon}) = E(X, \hat{\varepsilon}) - E(X) E(\hat{\varepsilon}) = 0$

$\Rightarrow Cov(\hat{Y}, \hat{\varepsilon}) = \hat{b} \cdot Cov(X, \hat{\varepsilon}) = 0$

$\Rightarrow \hat{Y}$ is orthogonal to $\hat{\varepsilon}$

Q4

$E(\tilde{\mu}) = E[a_1 y_1 + a_2 y_2 \dots + a_n y_n] = a_1 E(y_1) + \dots a_n E(y_n) = \mu$

since $\sum_{i=1}^{n} a_n = 1 \Rightarrow a_1 - \frac{1}{n} \dots + a_n \cdot \frac{1}{n} = 0$

$\sum a_i = (a_i - y_j) / \sum n_j = y_j = 0$

$\Rightarrow \sum n_i = a_i y_i = a_i$

$\Rightarrow \tilde{\mu} = \frac{y_1}{n} + \frac{y_2}{n} \dots + \frac{y_n}{n}$