Quiz 1

2023-05-23

Question 1

Derive the OLS estimator of SLR based on the Multiple Linear Regression OLS formula.

Question 2

Prove the following statement:

For any $b \in \mathbb{R}^p$, we have the following decomposition

$$||Y - Xb||^2 = ||Y - X\hat{\beta}||^2 + ||X(\hat{\beta} - b)||^2$$

where implies that $||Y - Xb||^2 \geq ||Y - X\hat{\beta}||^2$ with equality holding **if and only if** $b = \hat{\beta}$, where $\hat{\beta}$ is the OLS estimator of MLR.

Question 3

Under Gauss-Markov model, prove $\hat{Y}$ and $\hat{\epsilon}$ are orthogonal

Question 4

Assume that y_i has mean μ and σ^2 with $y_i, (i = 1, \ldots, n)$ being uncorrelated. A linear estimator of the mean μ is $\hat{\mu} = \sum_{i=1}^{n} a_i y_i$, which is not unique. Find the best linear unbiased estimator for μ.