Homework 5

HW Notes:

- $\boxed{\text{Box}}$ your final answer. You will be graded on both the final answer and the steps leading to it. Correct intermediate steps will help earn partial credit.
- For full credit, ~~cross out~~ any incorrect intermediate steps.
- If you need to make any additional assumptions, state them clearly.
- Legible writing will help when it comes to partial credit.
- Simplify your result when possible.

Problems:

1. (a) [10] Perform partial fraction expansion on a system's frequency response

$$H(j\omega) = \frac{2j\omega + 1}{-j\omega^3 - 5\omega^2 + 8j\omega + 4}.$$

Then use Matlab `residue` to verify your result. Be careful with the interpretation of Matlab outputs in the case of repeated pole(s).

(b) [10] Based on (a), find the unit impulse response $h(t)$ of this system and use Matlab to plot your answer as a function of t. Then use Matlab `impulse` to generate a same plot and verify your result.

2. Consider the system in Figure 0502.

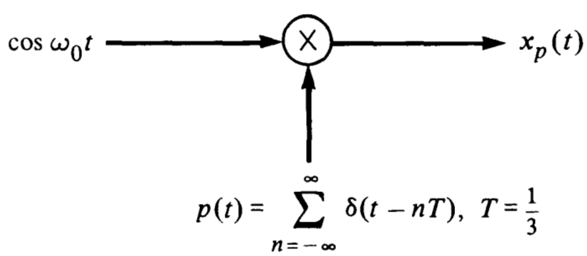


Figure 0502.

(a) [10] Sketch $X_p(\omega)$ for $-9\pi \le \omega \le 9\pi$ for the following values of ω_0.

i. $\omega_0 = \pi$
ii. $\omega_0 = 2\pi$
iii. $\omega_0 = 3\pi$
iv. $\omega_0 = 5\pi$

(b) [4] For which of the preceding values of ω_0 is $x_p(t)$ identical? For which of the preceding values of ω_0 will NOT be able to recover the input sinusoidal signal after lowpass filtering $x_p(t)$?

3. [12] Given the system in Figure 0503(a) and the Fourier transform of $x_r(t)$ in Figure 0503(b), sketch the Fourier transform of two different signals $x(t)$ that could have generated $x_r(t)$.

4. [16] Consider the system in Figure 0504.

If $X_1(\omega) = 0$ for $|\omega| > 2W$ and $X_2(\omega) = 0$ for $|\omega| > W$. For the following inputs $x(t)$, find the ranges for the cutoff frequency W_c in terms of T and W and find the maximum values of T and A, such that $x_r(t) = x(t)$.

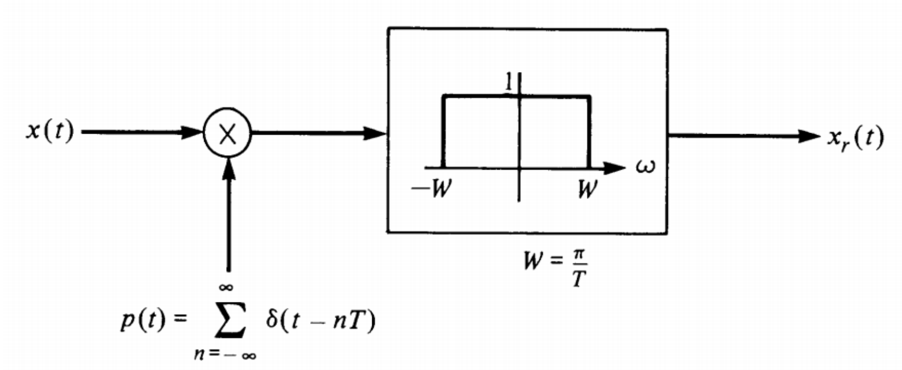


Figure 0503(a).

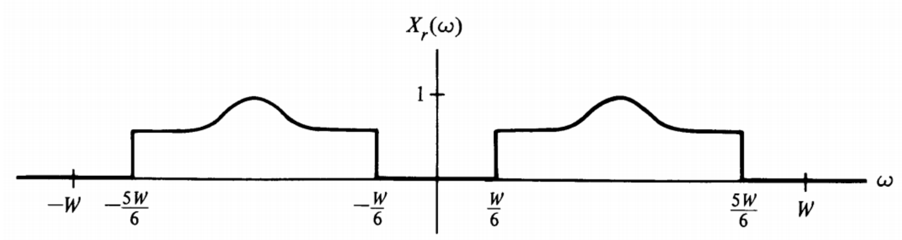


Figure 0503(b).

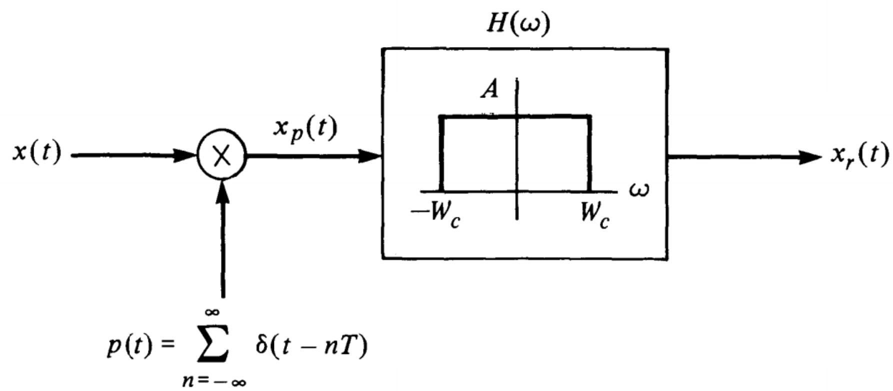


Figure 0504.

(a) $x(t) = x_1(t - \pi/2) + x_2(t)$

(b) $x(t) = x_1(t)x_2(t)$

(c) $x(t) = dx_2(t)/dt$

(d) $x(t) = x_2(t)\cos(2Wt)$

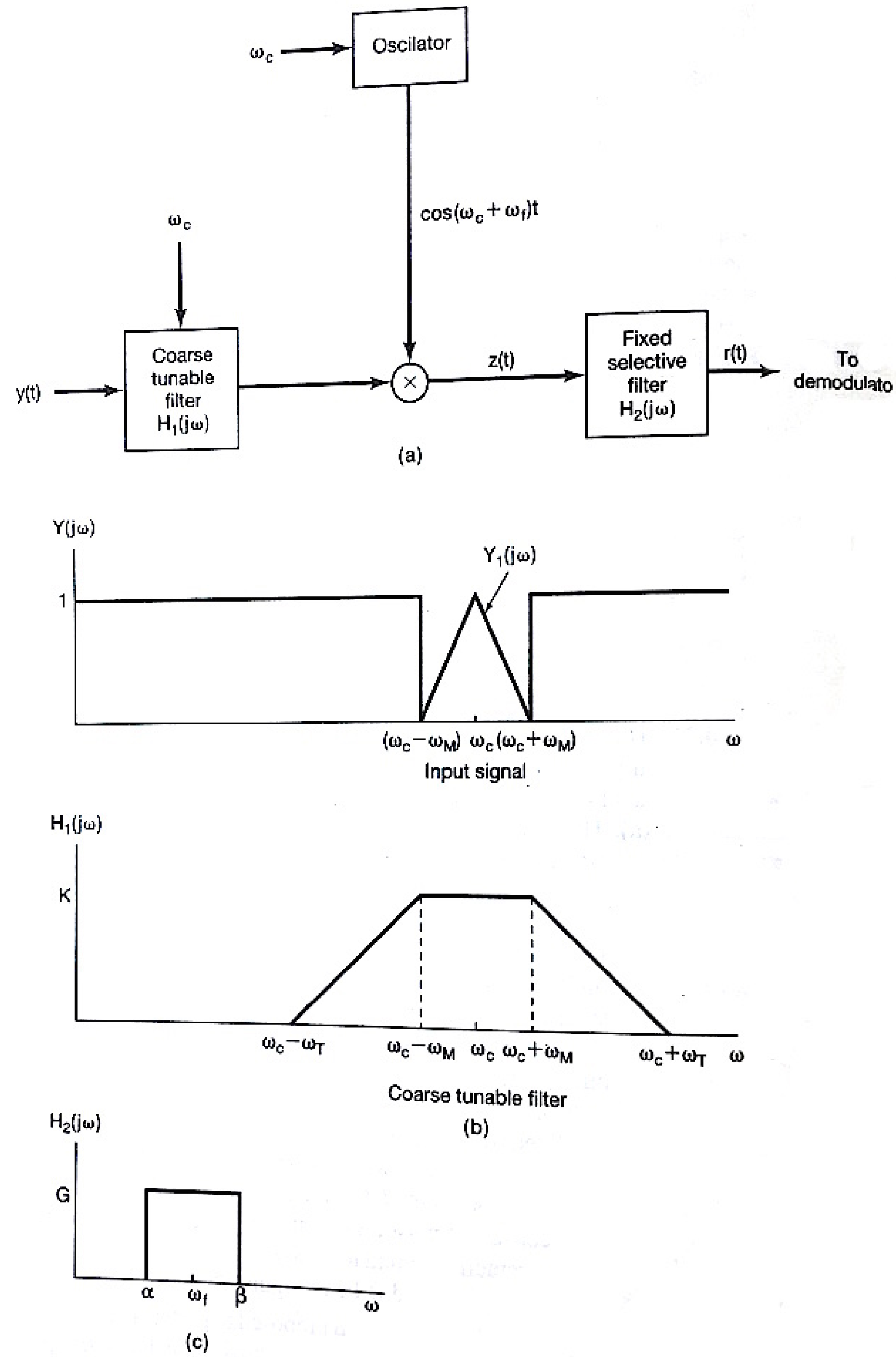
5. [20] A signal $x(t)$ with spectrum $X(\omega) = \left(1 - \left|\frac{\omega}{2\pi}\right|\right) \operatorname{rect}\left(\frac{\omega}{4\pi}\right)$ is filtered by a system with frequency response $H(\omega) = \operatorname{rect}\left(\frac{\omega}{2\pi}\right)$. The resulting signal is then multiplied by $\cos(8\pi t)$. Finally, that signal is passed through an integrator system, yielding a signal $y(t)$. Find and sketch $Y(\omega)$.

6. [18] The accurate demultiplexing-demodulation of radio and television signals is generally performed using a system called the superheterodyne receiver, which is equivalent to a tunable filter. The basic system is shown as Figure 0506.

(a) [6] The input signal $y(t)$ consists of the superposition of many amplitude-modulated signals that have been multiplexed using frequency-division multiplexing, so that each signal occupies a different frequency channel. Let us consider one such channel that contains the amplitude-modulated signal $y_1(t) = x_1(t)\cos\omega_c t$, with spectrum $Y_1(\omega)$ within $[\omega_c - \omega_M, \omega_c + \omega_M]$ as depicted below. We want to demultiplex and demodulate $y_1(t)$ to recover the modulating signal $x_1(t)$, using the system in the figure. The coarse tunable filter has the spectrum $H_1(\omega)$. Determine the spectrum $Z(\omega)$ of the input signal to the fixed selective filter $H_2(j\omega)$. Sketch and label $Z(\omega)$ for $\omega > 0$.

(b) [6] The fixed frequency-seletive filter is a bandpass type centered around the fixed frequency ω_f. We would like the output of the filter with spectrum $H_2(\omega)$ to be $r(t) = x_1(t)\cos\omega_f t$. In terms of ω_c and ω_M, what constraint must ω_T satisfy to guarantee that an undistorted spectrum of $x_1(t)$ is center around $\omega = \omega_f$?

(c) [6] What must G, α, and β be in the figure so that $r(t) = x_1(t)\cos\omega_f t$.



ω_c
Oscilator
$\cos(\omega_c+\omega_f)t$
ω_c
y(t)
Coarse tunable filter $H_1(j\omega)$
z(t)
Fixed selective filter $H_2(j\omega)$
r(t)
To demodulato
(a)
$Y(j\omega)$
1
$Y_1(j\omega)$
$(\omega_c-\omega_M)$ ω_c $(\omega_c+\omega_M)$
ω
Input signal
$H_1(j\omega)$
K
$\omega_c-\omega_T$
$\omega_c-\omega_M$
ω_c
$\omega_c+\omega_M$
$\omega_c+\omega_T$
ω
Coarse tunable filter
(b)
$H_2(j\omega)$
G
α
ω_f
β
ω
(c)

Figure 0506.

Optional Problems:

1. Add a subsystem to Problem 2 in the above Problem part (with T undetermined) as shown in Figure Optional-0501(a), assume the final output is a single value Q. As ω changes, Q may change. Determine whether the following two plots 0501(b)(c) are possible for the variation of Q as a function of ω. State your reasoning.

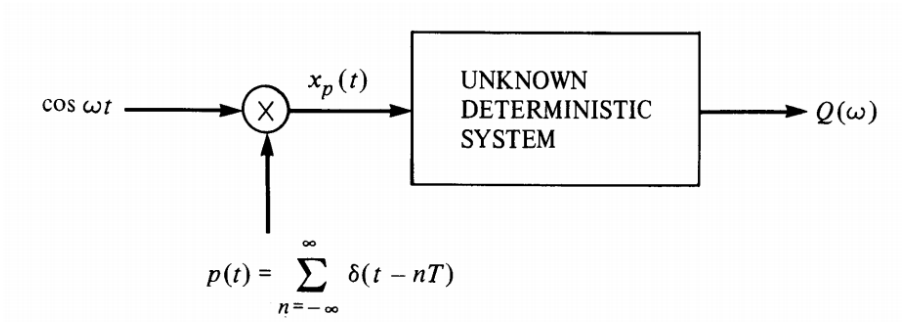


Figure Optional-0501(a).

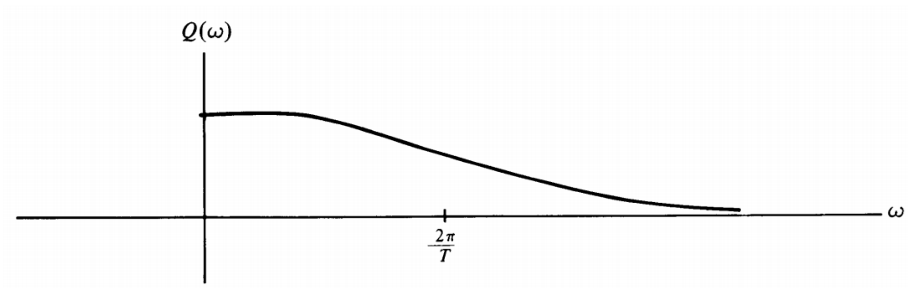


Figure Optional-0501(b).

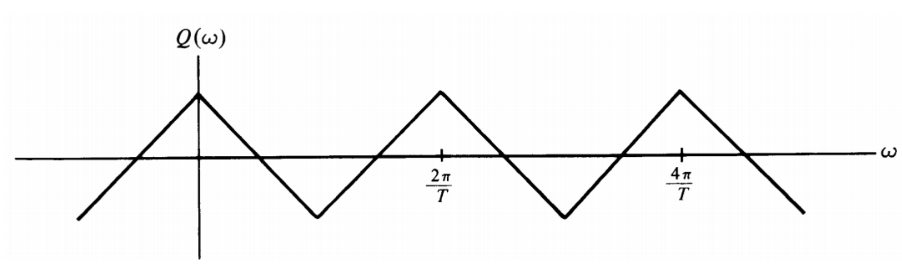


Figure Optional-0501(c).

2. Given the system in Figure Optional-0502(a) and the Fourier transforms in Figure Optional-0502(b), determine the maximum values for T and A in terms of W such that $y(t) = x(t)$ if $s(t)$ is the impulse train

$$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT).$$

State your reasoning.

3. We discussed the effect of a loss of synchronization in phase between the carrier signals in the modulator and demodulator in sinusoidal amplitude modulation. We showed that the output of the demodulation is

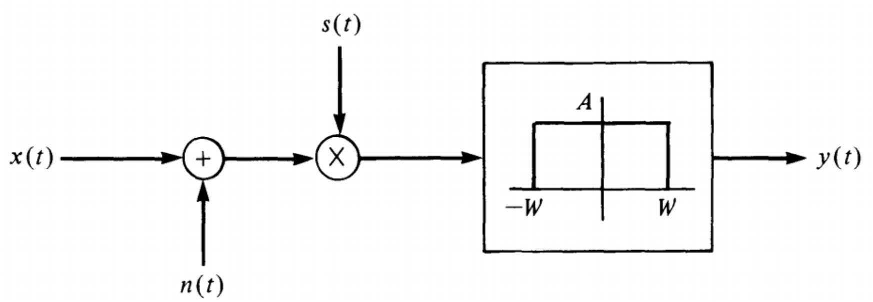


Figure Optional-0502(a).

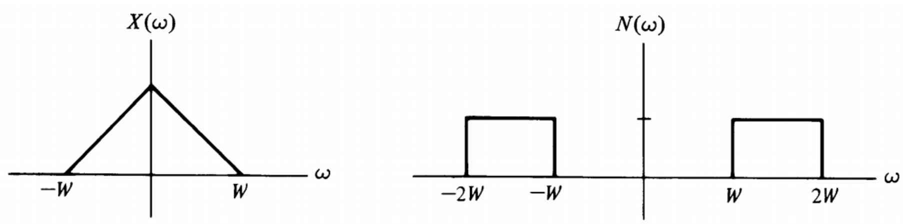


Figure Optional-0502(b).

attenuated by the cosine of the phase difference, and in particular, when the modulator and demodulator have a phase difference of $\pi/2$, the demodulator output is zero. As we demonstrate in this problem, it is also important to have frequency synchronization between the modulator and demodulator.
Consider the amplitude modulation and demodulation systems with $\theta_c = 0$ and with a change in the frequency of the modulator carrier so that

$$w(t) = y(t)\cos\omega_d t$$

where

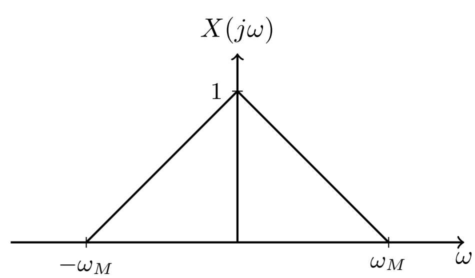
$$y(t) = x(t)\cos\omega_c t$$

Let us denote the difference in frequency between the modulator and demodulator as $\Delta\omega$ (i.e., $\omega_d - \omega_c = \Delta\omega$). Also assume that $x(t)$ is band limited with $X(j\omega) = 0$ for $|\omega| \geq \omega_M$, and assume that the cufoff frequency ω_{co} of the lowpass filter in the demodulator satisfies the inequality

$$\omega_M + \Delta\omega < \omega_{co} < 2\omega_c + \Delta\omega - \omega_M$$

(a) [5] Show that the output of the lowpass filter in the demodulator is proportional to $x(t)\cos(\Delta\omega t)$.

(b) [5] If the spectrum of $x(t)$ is that shown in figure below, sketch the spectrum of the output of the demodulator.



4. (This problem is related to how AM radios work.) The signal $x(t) = \text{sinc}(t/\pi)$ is modulated to form $y(t) = \cos(\omega_0 t)x(t)$, and then demodulated by forming $z(t) = cos(\omega_0 t)y(t)$. Find and sketch $Z(\omega)$, assuming ω_0 is large.

5. Suppose we have the system in Figure Optional-0505(a) and Optional-0505(b), in which $x(t)$ is sampled with an impulse train. Sketch $x_p(t)$, $y(t)$ and $w(t)$. State your reasoning.

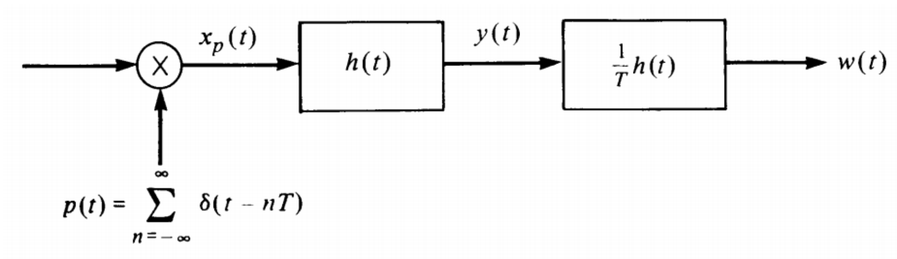


Figure Optional-0505(a).

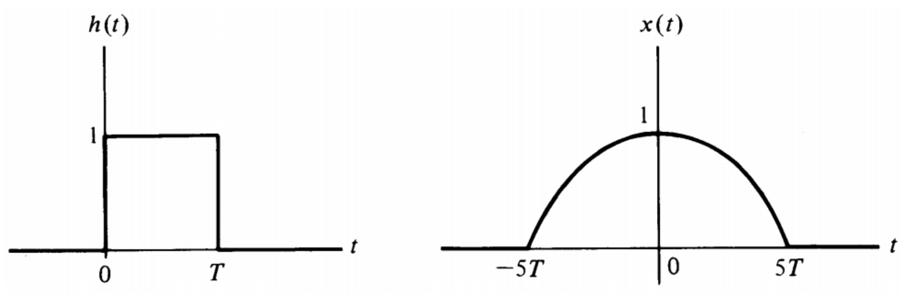


Figure Optional-0505(b).

6. Asynchronous modulation-demodulation requires the injection of the carrier signal so that the modulated signal is of the form

$$y(t) = [A + x(t)]\cos(\omega_c t + \theta_c)$$

where $A + x(t) > 0$ for all t. The presence of the carrier means that more transmitter power is required, representing an inefficiency.

(a) [4] Let $x(t) = \cos\omega_M t$ with $\omega_M < \omega_c$ and $A + x(t) > 0$. For a periodic signal $y(t)$ with period T, the average power over time is defined as $P_y = (1/T)\int_T y^2(t)dt$. Determine P_y for $y(t)$ defined above. Express your answer as a function of the modulation index m, defined as the maximum absolute value of $x(t)$ divided by A.

(b) [4] The efficiency of transmission of an amplitude-modulated signal is defined to be the ratio of the power in the sidebands (i.e., power of modulated signal that does not come from DC input) of the signal to the total power in the signal. With $x(t) = \cos\omega_M t$, and with $\omega_M < \omega_c$ and $A + x(t) > 0$, determine and sketch the efficiency of the modulated signal as a function of the modulation index m.