Table of Fourier Series for Common Signals

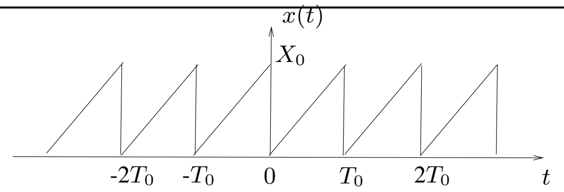
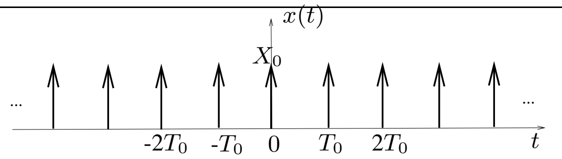
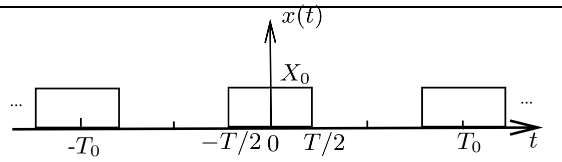
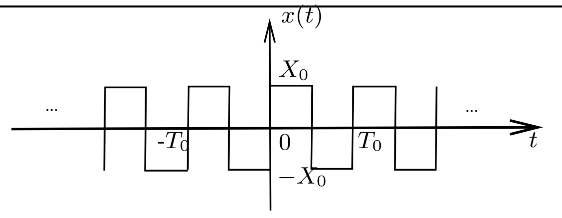
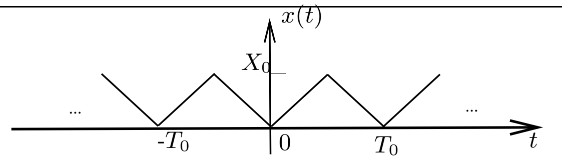
Name	Waveform	c_0	$c_k, k \neq 0$	Comments
Sawtooth		$\frac{X_0}{2}$	$j\frac{X_0}{2\pi k}$	
Impulse train		$\frac{X_0}{T_0}$	$\frac{X_0}{T_0}$	
Rectangular wave		$\frac{TX_0}{T_0}$	$\frac{TX_0}{T_0}\,\mathrm{sinc}\left(\frac{Tk\omega_0}{2\pi}\right)$	$\frac{Tk\omega_0}{2\pi} = \frac{Tk}{T_0}$
Square wave		0	$-j\frac{2X_0}{\pi k}$	$c_k = 0$, k even
Triangular wave sine		$\frac{X_0}{2}$	$\frac{-2X_0}{(\pi k)^2}$	$c_k = 0$, k even

Table of Fourier transform pairs

$f(t)$	$F(\omega)$
$\delta(t)$	1
1	$2\pi\,\delta(\omega) = \delta\left(\frac{\omega}{2\pi}\right)$
$u(t)$	$\pi\,\delta(\omega) + \frac{1}{j\omega}$
$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$
$\mathrm{e}^{j\omega_0 t}$	$2\pi\,\delta(\omega-\omega_0)$
$\cos\omega_0 t$	$\pi\,\delta(\omega-\omega_0) + \pi\,\delta(\omega+\omega_0)$
$\sin\omega_0 t$	$\frac{\pi}{j}\,\delta(\omega-\omega_0) - \frac{\pi}{j}\,\delta(\omega+\omega_0)$
e^{-bt^2}	$\sqrt{\pi/b}\,\mathrm{e}^{-\omega^2/(4b)}$
$\sum_{n=-\infty}^{\infty}\delta(t-nT_0)$	$\sum_{k=-\infty}^{\infty}\omega_0\,\delta(\omega-k\omega_0)$

$f(t)$	$F(\omega)$
$\frac{1}{b^2+t^2}$	$\frac{\pi}{b}\,\mathrm{e}^{-b\lvert\omega\rvert}$
$\mathrm{e}^{-b\lvert t\rvert}$	$\frac{2b}{b^2+\omega^2}$
$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}\left(T\frac{\omega}{2\pi}\right)$
$\mathrm{tri}(t)$	$\mathrm{sinc}^2\left(\frac{\omega}{2\pi}\right)$
$\frac{\omega_0}{2\pi}\,\mathrm{sinc}\left(\frac{\omega_0}{2\pi}t\right)$	$\mathrm{rect}\left(\frac{\omega}{\omega_0}\right)$
$\mathrm{sinc}^2(t)$	$\mathrm{tri}\left(\frac{\omega}{2\pi}\right)$
$\mathrm{e}^{-at}\,u(t)$	$\frac{1}{j\omega+a}$
$\frac{t^{n-1}}{(n-1)!}\,\mathrm{e}^{-at}\,u(t)$	$\frac{1}{(j\omega+a)^n}$
$\frac{j}{\pi t}$	$\mathrm{sgn}(\omega)$

b is a real positive number throughout. a is a real or complex number throughout, with positive real part.

Properties of the Continuous-Time Fourier Transform

	Time	Fourier
Synthesis, Analysis	$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega$	$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$
Eigenfunction	$h(t) * e^{j\omega_0 t} = H(\omega_0)e^{j\omega_0 t}$	$H(\omega)2\pi\delta(\omega-\omega_0)$ $= H(\omega_0)2\pi\delta(\omega-\omega_0)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Time transformation	$f(at+b),\ a \neq 0$	$\frac{1}{\lvert a\rvert}e^{j\omega b/a}F(\omega/a)$
Time shift	$f(t-\tau)$	$F(\omega)e^{-j\omega\tau}$
Time reversal	$f(-t)$	$F(-\omega)$
Time-scaling	$f(at),\ a \neq 0$	$\frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)$
Convolution	$f_1(t) * f_2(t)$	$F_1(\omega)\cdot F_2(\omega)$
Time-domain Multiplication	$f_1(t)\cdot f_2(t)$	$\frac{1}{2\pi}F_1(\omega) * F_2(\omega)$
Frequency shift	$f(t)e^{j\omega_0 t}$	$F(\omega-\omega_0)$
Modulation (cosine)	$f(t)\cos(\omega_0 t)$	$\frac{F(\omega-\omega_0)+F(\omega+\omega_0)}{2}$
Time. Differentiation	$\frac{d^n}{dt^n}f(t)$	$(j\omega)^n F(\omega)$
Freq. Differentiation	$(-jt)^n f(t)$	$\frac{d^n}{d\omega^n}F(\omega)$
Integration	$\int_{-\infty}^{t} f(\tau)\,d\tau = f(t) * u(t)$	$\frac{1}{j\omega}F(\omega) + \pi F(0)\delta(\omega)$
Conjugation	$f^*(t)$	$F^*(-\omega)$
Symmetry properties	$f(t)$ real	$F(\omega) = F^*(-\omega)$
	$f(t) = f^*(-t)$	$F(\omega)$ real
Duality	$F^*(t)$	$2\pi f^*(\omega)$
Duality	$F(t)$	$2\pi f(-\omega)$
Relation to Laplace	$F(\omega) = F(s)\rvert_{s=j\omega}$, if ROC includes $j\omega$ axis	
Parseval's Theorem	$\int_{-\infty}^{\infty} f_1(t)f_2^*(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(\omega)F_2^*(\omega)\,d\omega$	
Parseval/Rayleigh Theorem	$E = \int_{-\infty}^{\infty} \lvert f(t)\rvert^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert F(\omega)\rvert^2\,d\omega$	
DC Value	$\int_{-\infty}^{\infty} f(t)\,dt = F(0)$	

A function that satisfies $f(t) = f^*(-t)$ is said to have **Hermitian symmetry**.

Table of Laplace transform pairs

$f(t)$	$F(s)$	ROC
$\delta(t)$	1	$\forall s$
$u(t)$	$\frac{1}{s}$	$\text{real}\{s\} > 0$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$	$\text{real}\{s\} > 0$
$e^{-at}u(t)$	$\frac{1}{s+a}$	$\text{real}\{s\} > \text{real}\{-a\}$
$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\text{real}\{s\} < \text{real}\{-a\}$
$t^n e^{-at}u(t)$	$\frac{n!}{(s+a)^{n+1}}$	$\text{real}\{s\} > \text{real}\{-a\}$

$f(t)$	$F(s)$	ROC
$\sin(\omega_0 t)\, u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\text{real}\{s\} > 0$
$\cos(\omega_0 t)\, u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\text{real}\{s\} > 0$
$e^{-at}\cos(\omega_0 t)\, u(t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$	$\text{real}\{s\} > \text{real}\{-a\}$
$e^{-at}\sin(\omega_0 t)\, u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\text{real}\{s\} > \text{real}\{-a\}$
$u_n(t) = \frac{d^n}{dt^n}\delta(t)$	s^n	$\forall s$
$u_{-n}(t) = \underbrace{u(t) * \ldots * u(t)}_{\text{n times}}$	$\frac{1}{s^n}$	$\text{real}\{s\} > 0$

Properties of the Laplace Transform

	Time	Laplace	ROC (of result)
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$	contains $\mathrm{ROC}_1 \cap \mathrm{ROC}_2$
Time shift	$f(t-\tau)$	$e^{-s\tau} F(s)$	same
Time-scaling	$f(at),\ a \neq 0$	$\frac{1}{\lvert a\rvert} F\left(\frac{s}{a}\right)$	aROC
Time reversal	$f(-t)$	$F(-s)$	$-$ROC
Convolution	$f_1(t) * f_2(t)$	$F_1(s) \cdot F_2(s)$	contains $\mathrm{ROC}_1 \cap \mathrm{ROC}_2$
Frequency shift	$f(t)e^{j\omega_0 t}$	$F(s - j\omega_0)$	same
Frequency shift	$f(t)e^{s_0 t}$	$F(s - s_0)$	ROC $+$ real$\{s_0\}$
Time Differentiation	$\frac{d^n}{dt^n} f(t)$	$s^n F(\omega)$	contains ROC
s-domain Differentiation	$(-t)^n f(t)$	$\frac{d^n}{ds^n} F(\omega)$	same
Integration	$\int_{-\infty}^{t} f(\tau)\, d\tau = f(t) * u(t)$	$\frac{1}{s} F(s)$	contains ROC $\cap$ $\{$real$\{s\} > 0\}$
DC Value	$\int_{-\infty}^{\infty} f(t)\, dt = F(0)$		must contain $s = 0$