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Quiz4

(1)

$$h(t) = h_1(t) * h_2(t) \overset{F}{\longleftrightarrow} H(w) = H_1(w)\, H_2(w)$$

$$H_1(w) = \frac{1}{1+jw} \quad . \quad H_1(w) = \frac{1}{2+jw}$$

$$H(w) = \frac{1}{(jw+1)(jw+2)}$$

(2)

$$H(s) = \frac{1}{(s+1)(s+2)} = \frac{1}{s^2+3s+2}$$

or $$\frac{Y(w)}{X(w)} = H(w) = \frac{1}{(jw)^2+3jw+2}$$

Cross Multiplying, we have $\left((jw)^2+3jw+2\right) \cdot Y(w) = X(w)$

Using the time differentiation property:

$$\frac{d^2}{dt^2} y(t) + 3\frac{d}{dt} y(t) + 2y(t) = x(t)$$

(3)

$$Y(w) = H(w)\, X(w) = \frac{1}{(jw+5)(jw+2)(jw+1)} = \frac{A}{jw+1} + \frac{B}{jw+2} + \frac{C}{jw+5}$$

$$\Rightarrow \begin{cases} A+B+C=0 \\ 7A+6B+3C=0 \\ 10A+5B+2C=0 \end{cases} \Rightarrow \begin{cases} A = \frac{1}{4} \\ B = -\frac{1}{3} \\ C = \frac{1}{12} \end{cases}$$

$$\Rightarrow Y(w) = \frac{1}{4}\frac{1}{jw+1} - \frac{1}{3}\frac{1}{jw+2} + \frac{1}{12}\frac{1}{jw+5}$$

$$\Rightarrow y(t) = \frac{1}{4}e^{-t}u(t) - \frac{1}{3}e^{-2t}u(t) + \frac{1}{12}e^{-5t}u(t)$$

$$h(t) = \left\{ \frac{\pi}{5} \frac{\sin(5t)}{\pi t} \frac{\sin(15t)}{\pi t} \right\} 2\cos(25t)$$

$$= \frac{5}{\pi} \operatorname{sinc}\left(\frac{5t}{\pi}\right) \frac{15}{\pi} \operatorname{sinc}\left(\frac{15t}{\pi}\right) \frac{2\pi}{5} \cos(25t)$$

since

$$\frac{W_0}{2\pi} \operatorname{sinc}\left(\frac{W_0}{2\pi} t\right) \xleftrightarrow{F} \operatorname{rect}\left(\frac{w}{W_0}\right)$$

$$\Rightarrow \frac{5}{\pi} \operatorname{sinc}\left(\frac{5t}{\pi}\right) \xleftrightarrow{F} \operatorname{rect}\left(\frac{w}{10}\right)$$

$$\frac{15}{\pi} \operatorname{sinc}\left(\frac{15t}{\pi}\right) \xleftrightarrow{F} \operatorname{rect}\left(\frac{w}{30}\right)$$

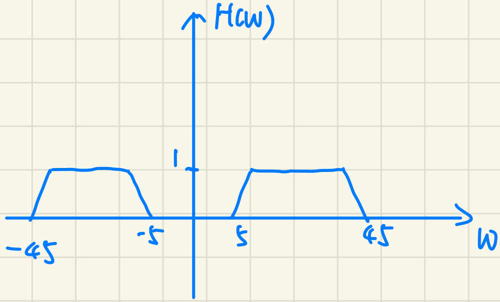
$$\frac{2\pi}{5} \cos(25t) \xleftrightarrow{F} \frac{2\pi^2}{5} \left(\delta(w-25) + \delta(w+25) \right)$$

Therefore. $H(w) = \frac{1}{4\pi^2} \frac{2\pi^2}{5} \operatorname{rect}\left(\frac{w}{10}\right) * \operatorname{rect}\left(\frac{w}{30}\right) \cdot \left(\delta(w-25) + \delta(w+25) \right)$

$$= \frac{1}{10} \left(u(w+5) - u(w-5) \right) * \left(u(w+15) - u(w-15) \right) * \left(\delta(w+25) + \delta(w-25) \right)$$

$$= \frac{1}{10} \left((t-5)u(t-5) - (t-35)u(t-35) - (t-15)u(t-15) + (t-45)u(t-45) \right)$$

$$+ \frac{1}{10} \left((t+45)u(t+45) - (t+15)u(t+15) - (t+35)u(t+35) + (t+5)u(t+5) \right)$$



$$T_0 = \frac{2\pi}{5} \Rightarrow w_0 = \frac{2\pi}{T_0} = 5$$

$$x(t) = \sum_{k=-\infty}^{\infty} \delta\left(t - n\frac{2\pi}{5}\right) \xleftrightarrow{F} X(w) = \sum_{k=-\infty}^{\infty} C_k\, 2\pi\, \delta(w - kw_0) = \sum_{k=-\infty}^{\infty} 5\,\delta(w-5k)$$

since $Y(w) = H(w)X(w)$, only the frequency components $(-45, -5)$ and $(5, 45)$ remains

$$Y(w) = \frac{5}{2}\delta(w-40) + \frac{5}{2}\delta(w-10) + \frac{5}{2}\delta(w+40) + \frac{5}{2}\delta(w+10)$$

$$+5\left(\delta(w-35) + \delta(w-30) + \delta(w-25) + \delta(w-20) + \delta(w-15)\right)$$

$$+5\left(\delta(w+35) + \delta(w+30) + \delta(w+25) + \delta(w+20) + \delta(w+15)\right)$$

since $\quad \cos(w_0 t) \xleftrightarrow{F} \pi\left(\delta(w-w_0) + \delta(w+w_0)\right)$

$$\Rightarrow y(t) = \frac{5}{2\pi}\left(\cos(40t) + \cos(10t)\right) + 5\pi\left(\cos(35t) + \cos(30t) + \cos(25t) + \cos(20t) + \cos(15t)\right)$$

(1)

$$c_k = \frac{1}{2}\frac{5}{\pi k}\sin\left(\frac{2\pi k}{5}\right)$$

$$= \frac{\sin\left(\frac{2\pi k}{5}\right)}{\frac{2\pi k}{5}} = \text{sinc}\left(\frac{2k}{5}\right)$$

For $k=0$, $c_0 = 1$

Since $\text{sinc}\left(\frac{2k}{5}\right) = 1$, when $k=0$

$$c_k = \text{sinc}\left(\frac{2k}{5}\right)$$

(2) From the code, clearly

$$w_0 = \frac{2\pi}{5} \Rightarrow T_0 = 5$$

From the FS table, the signal is a rectangular wave with

$$c_k = \frac{\tau X_0}{T_0}\,\text{sinc}\left(\frac{\tau k w_0}{2\pi}\right)$$

Compare with $c_k = \text{sinc}\left(\frac{2k}{5}\right)$, peak amplitude $X_0 = \frac{5}{2}$, fundamental period $T_0 = 5$, width $\tau = 2$