1. [12] Is this system stable? Explain. (Note: the system is causal.)

$$2 \cdot 10^6 y(t) + 10^5 \frac{d}{dt} y(t) + 60 \frac{d^2}{dt^2} y(t) + \frac{d^3}{dt^3} y(t) = 8 \cdot 10^6 x(t) - 10^4 \frac{d}{dt} x(t)$$

$\Rightarrow$ Characteristic polynomial is:

$$s^3 + 60 s^2 + 10^5 s + 2 \cdot 10^6 = 0$$

Use Matlab to solve this equation:

ans =

1.0e+02 *

$-0.1992 + 3.1432i$

$-0.1992 - 3.1432i$

$-0.2016 + 0.0000i$

$\Rightarrow$ the poles are all in LHP

$\Rightarrow$ the system is stable

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2. [12] How many signals have a Laplace transform that may be experessed as

$$\frac{(s-1)}{(s+2)(s+3)(s^2+s+1)}$$

in its region of convergence?

We may find different signal with the given Laplace transform by choosing different regions of convergence, the poles of the given Laplace transform are:

$$S_0 = -2, \quad S_1 = -3, \quad S_2 = -\frac{1}{2} + \frac{\sqrt{3}}{2} j, \quad S_3 = -\frac{1}{2} - \frac{\sqrt{3}}{2} j$$

Based on the locations of these poles, we may choose the following regions of convergence:

$$Re\{s\} > \frac{1}{2}$$

$$-2 < Re\{s\} < -\frac{1}{2}$$

$$-3 < Re\{s\} < -2$$

$$Re\{s\} < -3$$

Therefore, we may find four different signals of the given Laplace transform.

3. [16] Consider an LTI system with input $x(t) = e^{-t}u(t)$ and impulse response $h(t) = e^{-2t}u(t)$.

(a) Determine the Laplace transform of $x(t)$ and $h(t)$.

(b) Using the convolution property, determine the Laplace transform $Y(s)$ of the output $y(t)$.

(c) From the Laplace transform of $y(t)$ as obtained in part(b), determine $y(t)$.

(d) Verify your result in part (c) by explicitly convolving $x(t)$ and $h(t)$.

(a)

$$x(t) = e^{-t}u(t) \xrightarrow{\mathcal{L}} X(s) = \frac{1}{s+1} \quad \text{ROC: real}\{s\} > -1$$

$$h(t) = e^{-2t}u(t) \xrightarrow{\mathcal{L}} H(s) = \frac{1}{s+2} \quad \text{ROC. real}\{s\} > -2$$

(b)

$$y(t) = x(t) * h(t) \xrightarrow{\mathcal{L}} Y(s) = X(s)H(s) = \frac{1}{(s+1)(s+2)}$$

(c)

$$Y(s) = \frac{1}{(s+1)(s+2)} = \frac{1}{s+1} - \frac{1}{s+2} \Rightarrow y(t) = e^{-t}u(t) - e^{-2t}u(t)$$

(d)

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} e^{-\tau}u(\tau)e^{-2(t-\tau)}u(t-\tau)\,d\tau$$

$$= \int_0^t e^{-\tau}e^{-2(t-\tau)}\,d\tau \quad (t>0) = e^{-t}u(t) - e^{-2t}u(t)$$

4. [12] The system function of a causal LTI system is

$$H(s) = \frac{s+1}{s^2+2s+2}$$

Determine and sketch the response $y(t)$ when the input is

$$e^{-|t|}, \quad -\infty < t < \infty$$

$$x(t) = e^{-|t|} = e^{-t}u(t) + e^{t}u(-t)$$

$$X(s) = \frac{1}{s+1} - \frac{1}{s-1} = \frac{-2}{(s+1)(s-1)}$$

$(-1 < R\{s\} < 1)$

Since we also have:

$$H(s) = \frac{s+1}{s^2+2s+2}$$

$\Rightarrow$ The poles of $H(s)$ are $-1 \pm j$.

and since $h(t)$ is causal, we have that the ROC of $H(s)$ is $R\{s\} > -1$

Rewrite it as:

$$Y(s) = -\frac{\frac{2}{5}}{s-1} + \frac{2}{5}\left[\frac{s+1}{(s+1)^2+1}\right] + \frac{4}{5}\left[\frac{1}{(s+1)^2+1}\right]$$

We can get:

$$y(t) = \frac{2}{5}e^{t}u(-t) + \frac{2}{5}e^{-t}\cos(t)u(t) + \frac{4}{5}e^{-t}\sin(t)u(t)$$

5. [18] Draw a direct-form representation for the causal LTI systems with the following system fuctions:

(a)

$$H_1(s) = \frac{s+1}{s^2+5s+6}$$

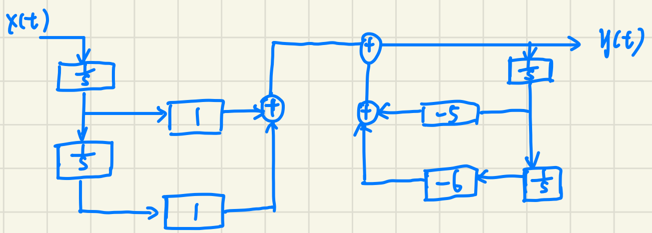
(b)

$$H_2(s) = \frac{s^2-5s+6}{s^2+7s+10}$$

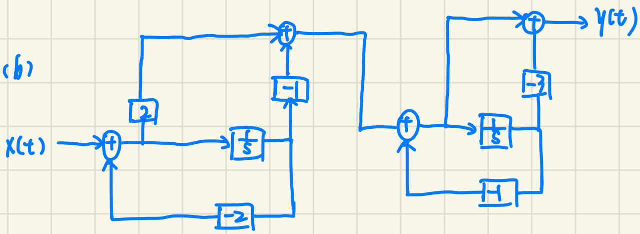
(c)

$$H_3(s) = \frac{s}{(s+2)^2}$$

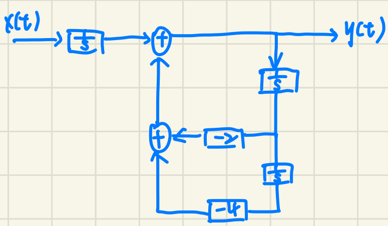
(a)



(b)



(c)



6. [10] A causal LTI system with impulse response h(t) has the following properties:1.When the input to the system is $x(t) = e^{2t}$ for all t, the output is $y(t) = \frac{1}{6}e^{2t}$ for all t. 2.The impulse respose h(t) satisfies the differential equation $\frac{dh(t)}{dt} + 2h(t) = (e^{-4t})u(t) + bu(t)$, where b is an unkhown constant.

Determine the system function H(s) of the system, consistent with information above. There should be no unkhowm constants in your answer, that is, the constant b should not appear in the answer.

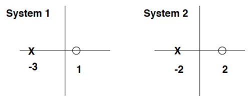
If $x(t) = e^{2t}$ produces $y(t) = \frac{1}{6}e^{2t}$, then $H(2) = \frac{1}{6}$. Also, by taking the Laplace transform of both sides of the given equation. we can get

$$H(s) = \frac{s + b(s+4)}{s(s+4)(s+2)}$$

Since $H(2) = \frac{1}{6}$, we may deduce that $b = 1$. Therefore

$$H(s) = \frac{2}{s(s+4)}$$

7. [10] A unit step signal is applied to a system consisting of two LTI system connected in parallel. The pole-zero plots of each of the system are shown below. Determine the output signal. Assume that each of the system has unit gain at DC.



Hint: first find the Laplace transform $Y(s)$ of the output signal using the convolution and linearity properties of the Laplace transform, Then take the inverse Laplace transform to get $y(t)$ using PFE. The "unit gain at DC" specifies $H_1(0)$ and $H_2(0)$, which you can use to determine the scaling factor.

$$H_1(s) = G_1 \frac{s-1}{s+3}, \quad H_2(s) = G_2 \frac{s-2}{s+2}$$

Since they all have unit DC gains. we can get $G_1 = -3$ and $G_2 = -1$

$$H_1(s) = -3\frac{s-1}{s+3}, \quad H_2(s) = -\frac{s-2}{s+3}$$

$$u(t) \xrightarrow{\mathcal{L}} \frac{1}{s}$$

$$Y(s) = \frac{-3(s-1)}{s(s+3)} + \frac{-(s-2)}{s(s+2)} = \frac{1}{s} + \frac{-4}{s+3} + \frac{1}{s} + \frac{-2}{s+2}$$

$$\Rightarrow y(t) = 2u(t) - 4e^{-3t}u(t) - 2e^{-2t}u(t)$$