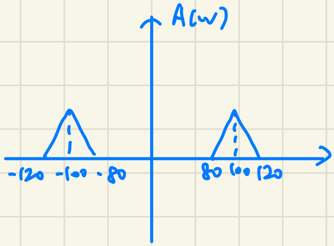
Q1:

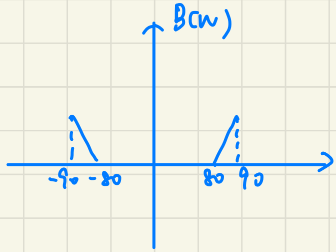
$$a(t) = x(t)\cdot G(t) \xrightarrow{F} \frac{F(w-w_0)+F(w+w_0)}{2} = \frac{X(w-100)(w+100)}{2}$$

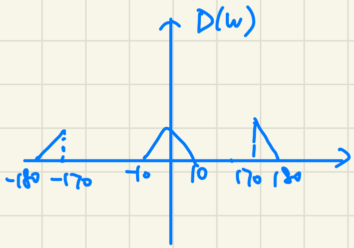
$$b(t) = A(w)\cdot H_1(w) = \frac{X(w+100)+X(w-100)}{2}\cdot \text{rect}\left(\frac{w}{180}\right)$$

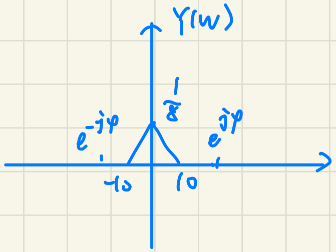
$$d(t) = b(t)\cdot G_2(t) \qquad D(w) = \frac{1}{2}\left(e^{j\varphi}B(w-90)+e^{j\varphi}B(w+90)\right)$$

$$y(t) = D(t)\cdot H_2(w)$$

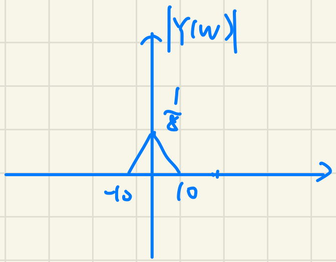


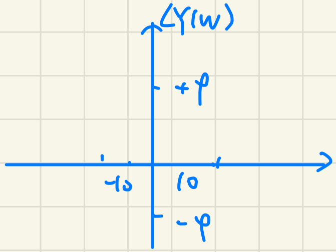






$$Y(w) = \frac{1}{8}e^{j\varphi}\,\text{tri}\,\frac{w}{10}\,\text{rect}\left(\frac{w-5}{10}\right) + \frac{1}{8}e^{-j\varphi}\,\text{tri}\left(\frac{w}{10}\right)\text{rect}\left(\frac{w+5}{10}\right)$$





Q2 :

① $Y(s) = 2W(s) - 4\frac{1}{s}W(s) - 6\frac{1}{s^2}W(s)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{Y(s)}{W(s)} \cdot \frac{W(s)}{X(s)}$$

$$= \frac{2-4s^{-1}-6s^{-2}}{1+2s^{-1}-9s^{-2}-18s^{-3}}$$

$$= \frac{2s(s+1)}{(s+2)(s+3)}$$

two poles $-2, -3$ negative $\Rightarrow$ BIBO stable

② : $H(s) = \frac{s+b}{s+2} + \frac{s+d}{s+3}$

$$(s+d)(s+2) + (s+3)(s+b) = 2s^2+2s$$

$$\Rightarrow \begin{cases} b=12 \\ d=-11 \end{cases}$$

$$2+\frac{4}{s+2} - \frac{12}{s+3} \Rightarrow 2\delta(t) + 4e^{-2t}u(t) - 12e^{-3t}u(t)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2s(s+1)}{(s+2)(s+3)}$$

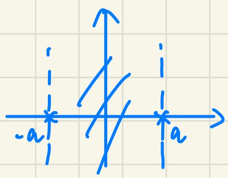
$$y'' + 5y' + 6y = 2x'' + 2x$$

Q3.

$a = 0 \quad x(t) = 1 \quad X(s) = \delta(s)$

$a > 0 \quad x(t) = -e^{at}u(t) + e^{at}u(-t)$

$X(s) = \frac{1}{s+a} - \frac{1}{s-a} = \frac{-2a}{(s+a)(s-a)} \quad (-a < Re(s) < a)$



Q4.

$$x(t) + \frac{d}{dt}\left(x(t) + w(t) \right) = y(t)$$

$$w(t) = \frac{d}{dt} y(t) \cdot \left(-\frac{1}{3}\right)$$

$$x(t) + \frac{d}{dt}\left(x(t) - \frac{1}{3}\frac{d}{dt} y(t) \right) = y(t)$$

$$x(t) + x'(t) - \frac{1}{3} y''(t) = y(t)$$

$$H(s) = \frac{s+1}{\frac{1}{3}s^2 + 1}$$

$\Rightarrow s = \pm\sqrt{3}\,j$ unstable for poles on imaginary axis