$$x(t) = e^{st}$$

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, e^{st-s\tau}\, d\tau$$

$$= e^{st} \int_{-\infty}^{\infty} h(\tau) e^{-s\tau}\, d\tau$$

$$= x(t) \cdot H(s)$$

When we take s to be purely imaginary, $s = jw$

$$H(s)\big|_{s=jw} = H(jw) = H(w)$$

$$= \int_{-\infty}^{\infty} h(t)\, e^{-jwt}\, dt$$

$$= |H(w)|\, e^{j\angle H(w)}$$

Synthesis : $x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jkw_0 t}$

Analysis : $C_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jkw_0 t}\, dt$

$$C_0 = \frac{1}{T_0} \int_{T_0} x(t)\, dt$$

Result : $y(t) = \sum_{-\infty}^{\infty} H(kw_0)\, C_k e^{jkw_0 t}$

combined trigonometric

$$y(t) = C_0 + 2 \sum_{k=1}^{\infty} |C_k| \cos(kw_0 t + \angle C_k)$$

trigonometric

$$y(t) = C_0 + 2 \sum_{k=1}^{\infty} Re(C_k) \cdot \cos(kw_0 t) - I_m(C_k) \cdot \sin(kw_0 t)$$