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Q1

(1) Detroit WJR

$S_{AM}(t) = [A_0 + \cos(2\pi f_m t)] \cos(2\pi f_c t)$

$= A_0 \cos(2\pi f_c t) + \frac{1}{2}\cos(2\pi(f_c + f_m)t)$

$+ \frac{1}{2}\cos(2\pi(f_c + f_A)t)$

S_{AM} have three frequency :

$f_c = 760\,kHz$

$f_c - f_m = 753\,kHz$

$f_c + f_m = 767\,kHz$

$\Rightarrow$ S_{AM} is in range of : $753\,kHz \sim 767\,kHz$

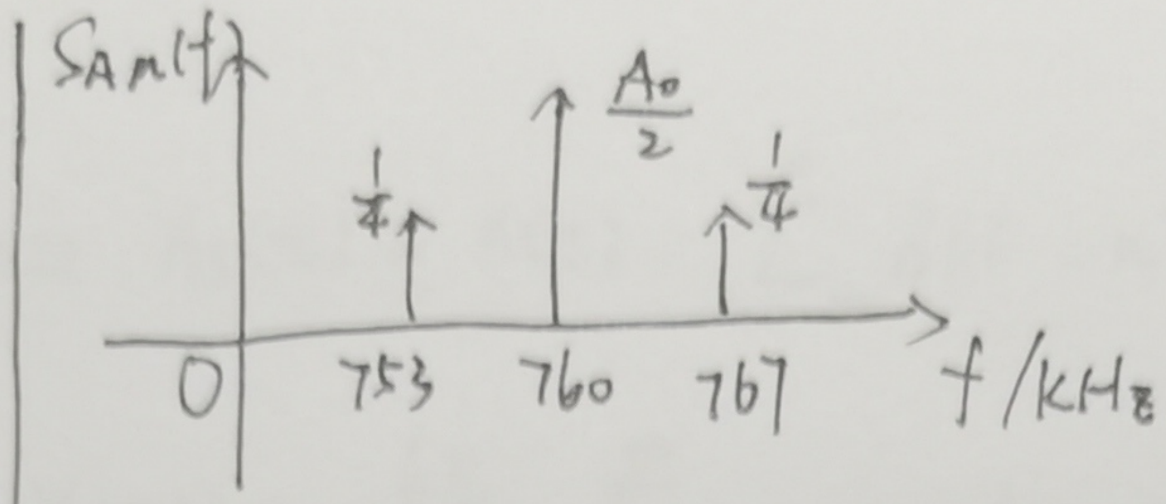
Detroit WJR

(2) Based on $S_{AM}(t)$ in (1)

$\Rightarrow S_{AM}(f) = \frac{A_0}{2}[\delta(f+760k) + \delta(f-760k)]$

$+ \frac{1}{4}[\delta(f+753k) + \delta(f-753k)]$

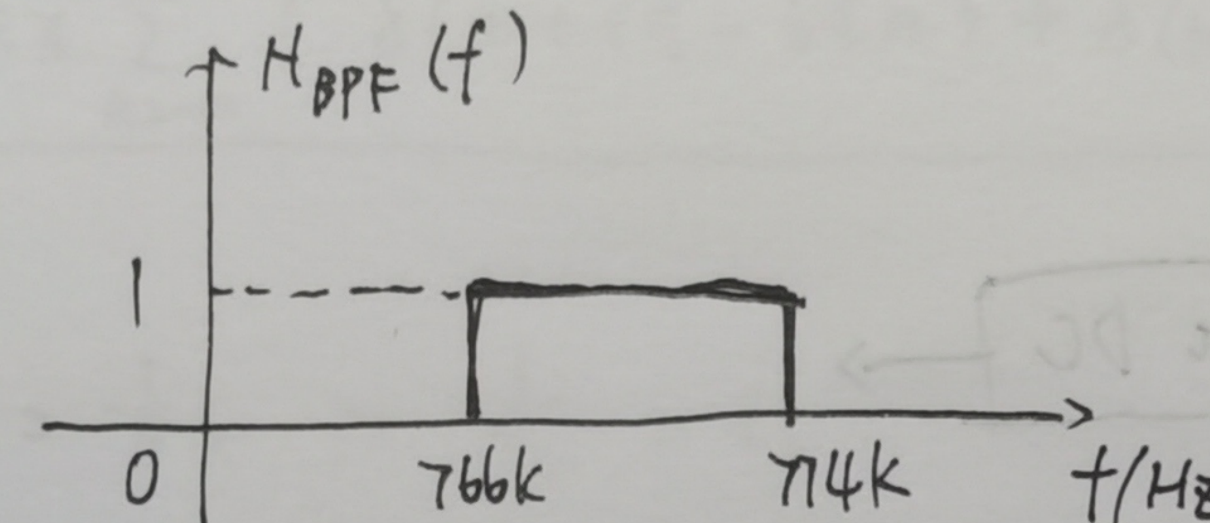
$+ \frac{1}{4}[\delta(f+767k) + \delta(f-767k)]$

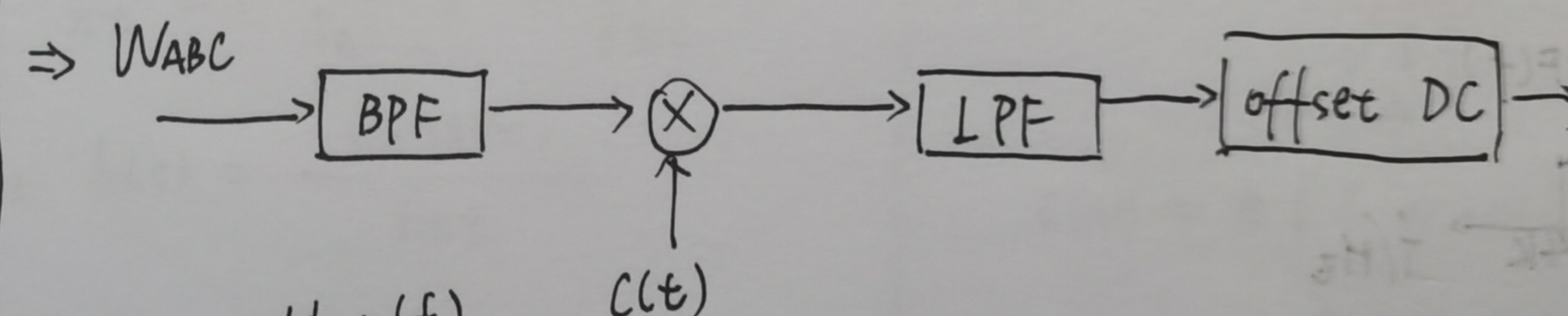


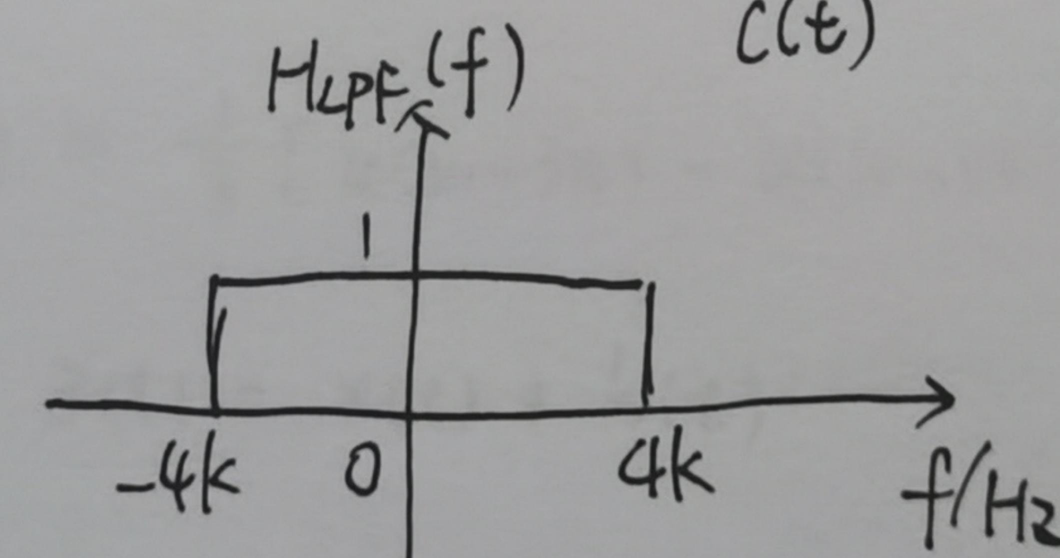
(3)

WABC $\longrightarrow$ 770 kHz , $f_m \rightarrow 4\,kHz$

$C(t) = \cos(2\pi \times 770k\,t)$







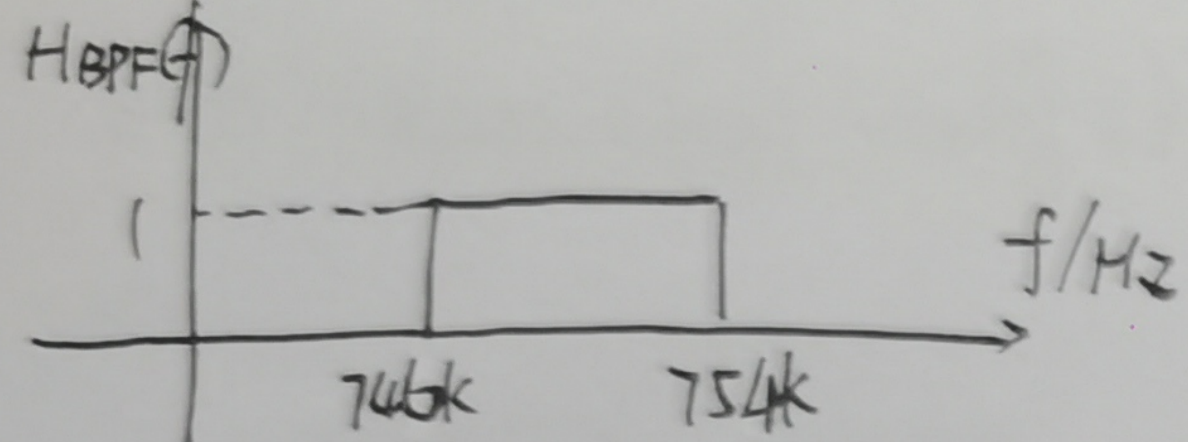
$\Rightarrow$ interference signal frequency : 3k Hz

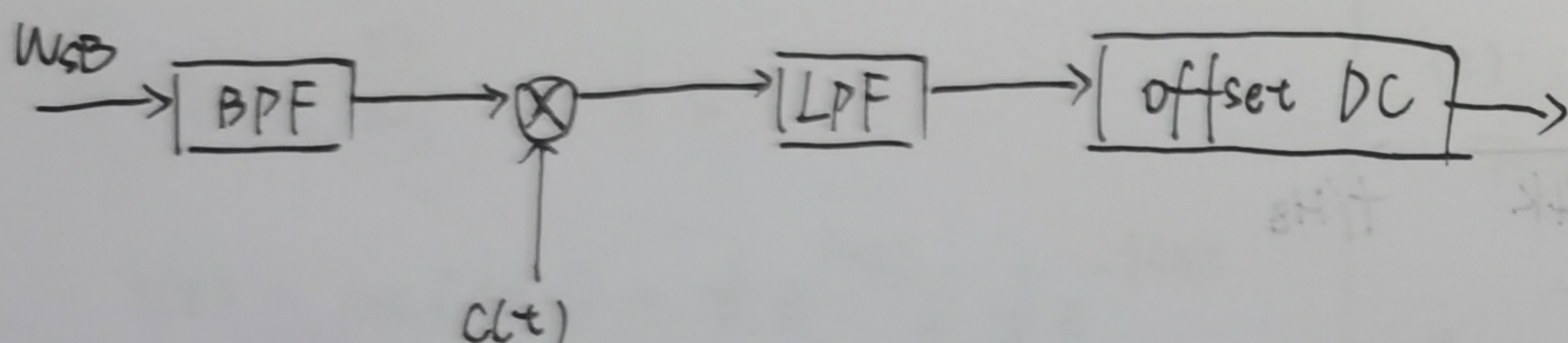
(4)

WSB $\longrightarrow$ 750 kHz . $f_m \longrightarrow$ 4kHz

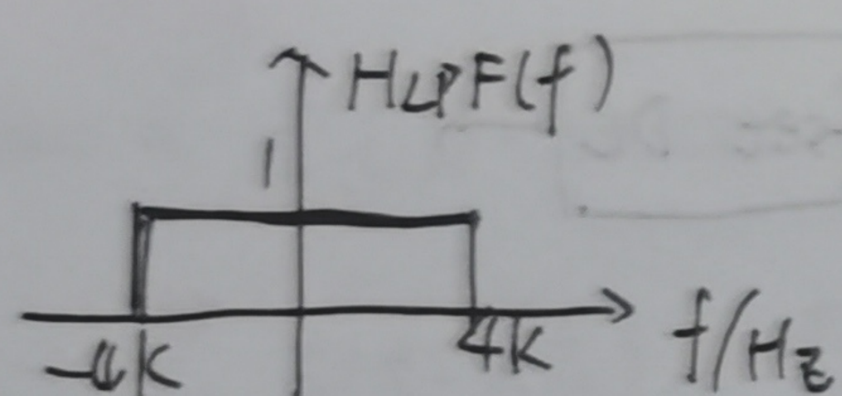
Similarly to (3)

$c(t) = \cos(2\pi \times 750k t)$





$\Rightarrow$



$\Rightarrow$ interference signal frequency : 4kHz

Q 2

$x(t) = \cos(4\pi t)$

$x[n] = x(nT_s)$, $T_s > 0$

$x_s(t) = \sum_{n=-\infty}^{\infty} x[n]\,\delta(t-nT_s)$

$z(t) = x_s(t) * \left[\frac{T_s}{\pi t}\sin\left(\frac{\pi}{T_s}t\right)\right]$

(1)

since $w_s = 6\pi$

$\Rightarrow T_s = \frac{2\pi}{w_s} = \frac{1}{3}\,s$

$x(t) = \cos(4\pi t) = \frac{1}{2}e^{j4\pi t} + \frac{1}{2}e^{-j4\pi t}$

$X(w) = \pi\left(\delta(w+4\pi) + \delta(w-4\pi)\right)$

Also, $\sum_{n=-\infty}^{\infty}\delta(t-nT_s) = \sum_{n=-\infty}^{\infty} a_n e^{j\frac{2\pi}{T_s}nt}$

where $a_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t)e^{-j6\pi nt}\,dt = 3$

$\Rightarrow \sum_{n=-\infty}^{\infty}\delta(t-nT_s) = \sum_{n=-\infty}^{\infty} 3e^{j6\pi nt}$

$\updownarrow$

$6\pi\sum_{n=-\infty}^{\infty}\delta(w-6\pi n)$

since $X_s(t) = X(t)\cdot\sum_{n=-\infty}^{\infty}\delta(t-nT_s)$

$\Rightarrow X_s(w) = \frac{6\pi}{2\pi}\sum_{n=-\infty}^{\infty}\delta(w-6\pi n) * X(w)$

$= 3\sum_{n=-\infty}^{\infty} X(w-6\pi n)$

$= 3\pi\sum_{n=-\infty}^{\infty}\left[\delta(w+4\pi-6\pi n) + \delta(w-4\pi-6\pi n)\right]$

(2)

$T_s = \frac{1}{3}s \Rightarrow \frac{1}{T_s} = 3s$

$\Rightarrow \frac{T_s}{\pi t}\sin\frac{\pi t}{T_s} = \frac{\sin(3\pi t)}{3\pi t}$

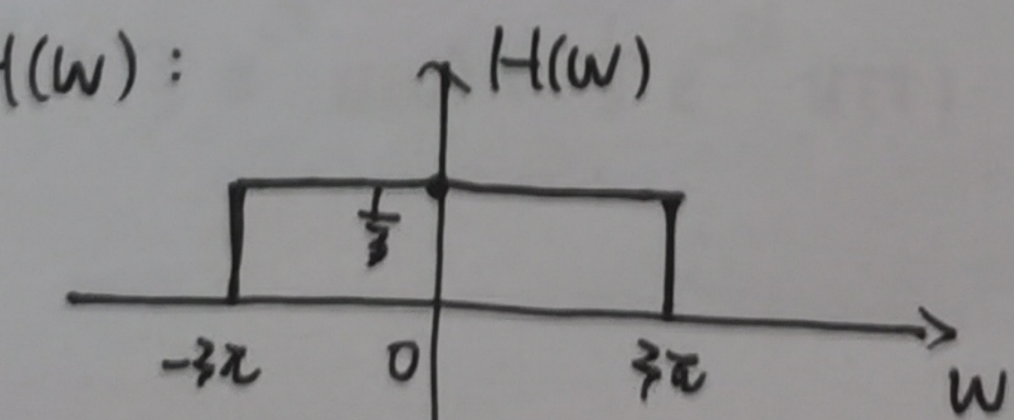
Let $h(t) = \frac{\sin(3\pi t)}{3\pi t}$

$\Rightarrow H(w) = \frac{1}{3}\left[u(w+3\pi) - u(w-3\pi)\right]$

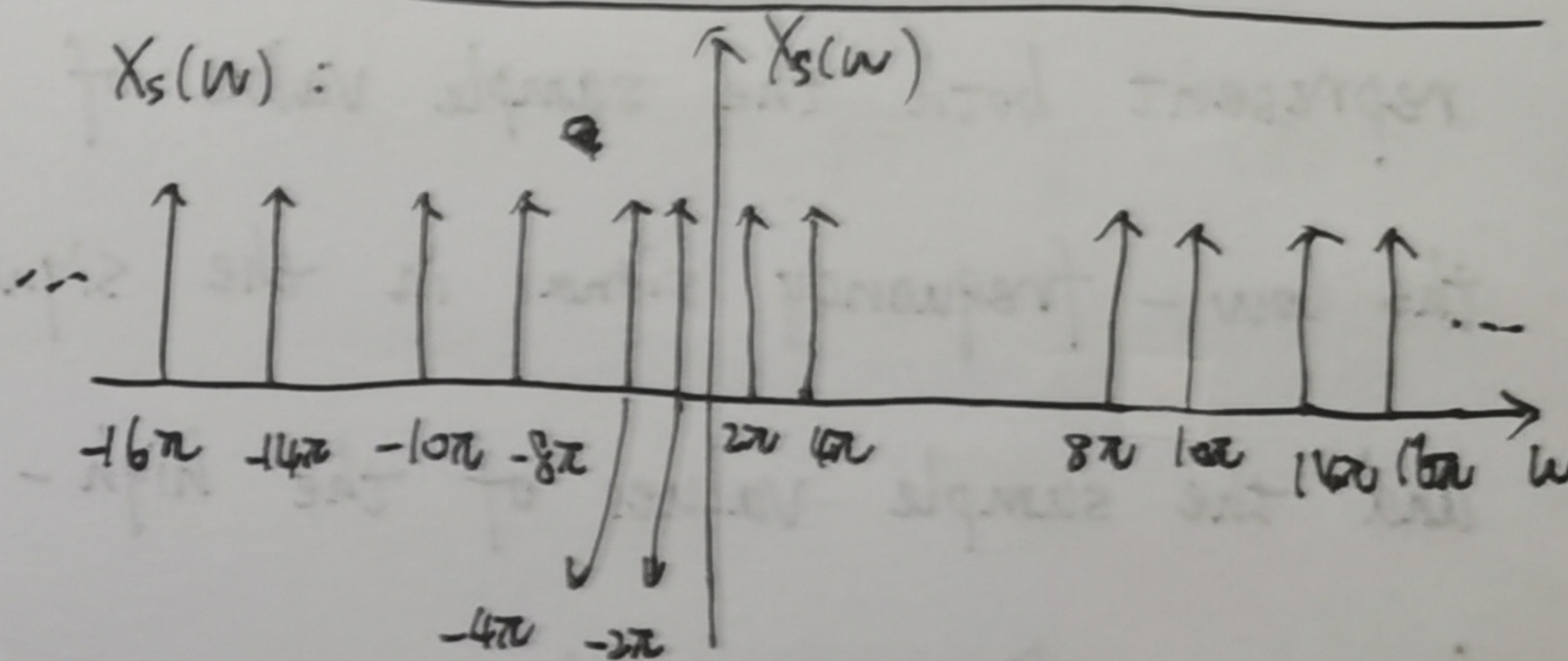
Since $z(t) = x(t) * h(t)$

$\Rightarrow Z(w) = X_s(w)H(w)$

H(w):



$X_s(w)$:



$Z(w) = \pi\left[\delta(w+2\pi) + \delta(w-2\pi)\right]$

$\Rightarrow z(t) = F^{-1}[Z(w)] = \cos(2\pi t)$

(3)

Distortions:

A phenomenon when high and low frequency components are confused due to changes in the sampled signal spectrum. When sampling, the frequency is not high enough. The sampled points represent both the sample value of the low-frequency signal in the signal and the sample values of the high-frequency. During signal reconstruction, the high-frequency signal is replaced by the low-frequency signal, and the two waveforms completely overlap, forming serious distortion.

Q3

① $V_{in} = V_R + V_C$

$V_R = i_t \cdot R$

$V_C = \int i \, dt \cdot \frac{1}{C}$

$\Rightarrow V_{in} = iR + \frac{1}{C}\int i dt$

Taking the Laplace transform of both sides :

$V_{in}(s) = I(s)\, R + \frac{1}{C}\,\frac{I(s)}{s}$

$\Rightarrow I(s) = V_{in}(s) \,/\, [R + \frac{1}{sC}]$

since $V_C = I(s)/sC$ and

$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{sRC+1}$

$\Rightarrow h(t) = L^{-1}\{H(s)\}$

$= \frac{1}{RC}\, e^{\frac{-1}{RC}} \quad u(t)$

②

since $\frac{1}{RC} = 2$

$\Rightarrow H(s) = \frac{2}{s+2}$

$Y(s) = LT[\,y(t)]$

$= \frac{\frac{2}{3}}{s+1} + \frac{\frac{2}{3}}{s+2} - \frac{\frac{2}{3}}{s+3}$

$= \frac{\frac{4}{3}s+2}{(s+1)(s+2)} - \frac{\frac{2}{3}}{s+3}$

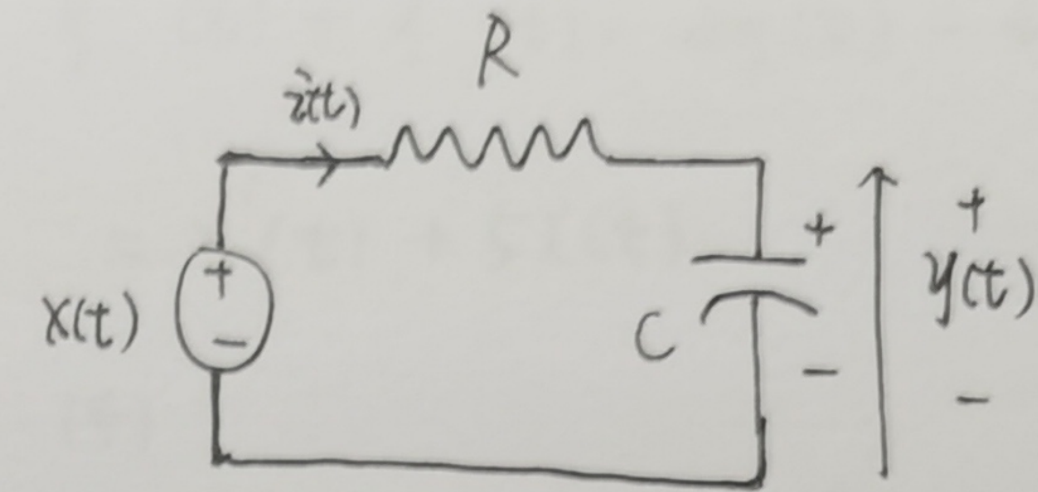
$= \frac{\frac{2}{3}(s+6s+7)}{(s+1)(s+2)(s+3)}$

$\Rightarrow X(s) = \frac{Y(s)}{H(s)}$

$= \frac{\frac{1}{3}(s^2+6s+7)}{(s+1)(s+3)}$

$= \frac{1}{3} + \frac{\frac{1}{3}}{s+1} + \frac{\frac{1}{3}}{s+3}$

$\Rightarrow X(t) = \frac{1}{3}\delta(t) + \frac{1}{3}e^{-t}u(t) + \frac{1}{3}e^{-3t}u(t)$



$Q = CV$

$\frac{dQ}{dt} = C\frac{dv}{dt} \Rightarrow i = C\frac{dy(t)}{dt}$

$\Rightarrow y(t) = \frac{1}{C}\int i dt$

Q4

(1)

$x_1(t) \overset{L}{\longleftrightarrow} X_1(s)$

$x(t) \longleftrightarrow X(s)$

take LT for equation's two sides:

$X_1(s)(s^2+2s-8) = X(s)(s+3)$

$\Rightarrow H_1(s) = \frac{s+3}{s^2+2s-8}$

(2)

$H_2(s) = \frac{1+9\frac{1}{s}+20\frac{1}{s^2}}{1+6\frac{1}{s}+11\frac{1}{s^2}+6\frac{1}{s^3}}$

$\Leftrightarrow \frac{s^2+9s+20}{s^3+6s^2+11s+6}$

$= \frac{s^2+9s+20}{(s+1)(s+2)(s+3)}$

$= \frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{s+3}$

$\Rightarrow A = \frac{s^2+9s+20}{(s+2)(s+3)}\Big|_{s=-1} = 6$

Similarly, $B = -6$ and $C = 1$

$\Rightarrow H_2(s) = \frac{6}{s+1} - \frac{6}{s+2} + \frac{1}{s+3}$

since the system is causal:

ROC: $Re\{s\} > -1$

$\Rightarrow h_2(t) = L^{-1}[H_2(s)]$

$= (6e^{-t} - 6e^{-2t} + e^{-3t})\,u(t)$

(ROC includes $j\omega$ axis. therefore the system $h_2(t)$ is stable)

(3)

$H(s) = \frac{Y(s)}{X(s)} = H_1(s)\,H_2(s)$

$= \frac{s+3}{s^2+2s-8}\;\frac{s^2+9s+20}{s^3+6s^2+11s+6}$

$= \frac{s+3}{(s+4)(s-2)}\;\frac{(s+4)(s+5)}{(s+1)(s+2)(s+3)}$

$= \frac{s+5}{s^3+s^2-4s-4}$

⇒ differential equation

$y'''(t)+y''(t)-4y'(t)-4y(t)$

$= x'(t)+5x(t)$

(4)

$X(s) = \frac{1}{s+2}$

$Y(s) = X(s)\,H(s)$

$= \frac{s+5}{(s+1)(s+2)^2(s-2)}$

⇒ poles: $-1, -2, -2, 2$

ROC: $Re\{s\} > 2$