Example (5)

A signal $x(t)$ with spectrum $X(\omega) = (1 - 4|\omega|)\,\mathrm{rect}(2\omega)$ is modulated by the following modified impulse train: $p(t) = \sum_{n=-\infty}^{\infty} 2\delta(t-5n) - \delta(t-5n-1) - \delta(t-5n+1)$. Determine and sketch the magnitude spectrum of the resulting signal.

$$g(t) = 2\delta(t) - \delta(t-1) - \delta(t+1)$$

with $G(t) = 2 - e^{-jw} - e^{jw} = 2(1-\cos w)$

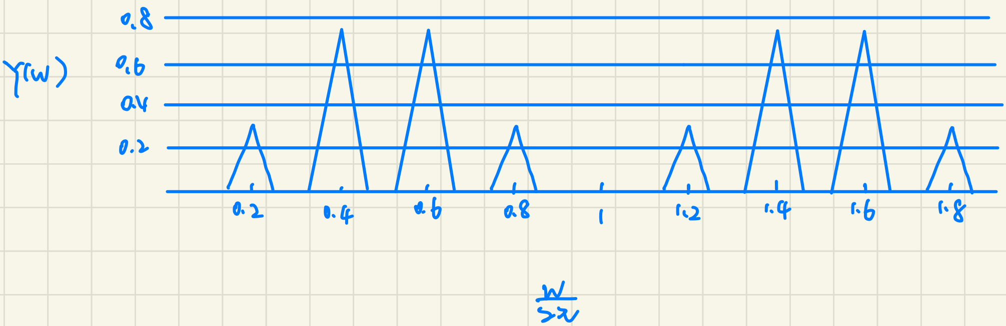
$$\Rightarrow P(w) = \sum_{k=-\infty}^{\infty} C_k\, 2\pi\, \delta(w - kw_0) = \sum_{k=-\infty}^{\infty} \frac{1}{T_0} G(w)\big|_{w=kw_0}\, 2\pi\, \delta(w-kw_0)$$

$$= \sum_{k=-\infty}^{\infty} w_0\, 2[1-\cos(kw_0)]\, \delta(w-kw_0) \quad \text{where } w_0 = \frac{2\pi}{5}$$

$$\Rightarrow Y(w) = \frac{1}{2\pi} X(w) * P(w)$$

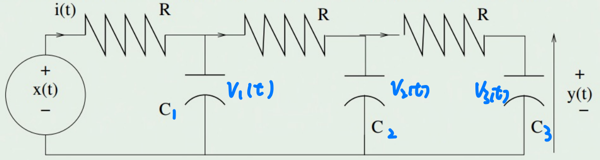
$$= \frac{1}{2\pi} X(w) * \sum_{k=-\infty}^{\infty} \frac{2\pi}{5}\, 2\,[1-\cos(k w_0)]\,\delta(w-kw_0)$$

$$= \sum_{k=-\infty}^{\infty} \frac{2}{5}[1-\cos(kw_0)]\,(1-4|w-kw_0|)\,\mathrm{rect}(2(w-kw_0))$$



Example (5)

Three of the RC circuits discussed in class are connected in series. Find the frequency response $H(\omega)$ for this circuit.



$$\Rightarrow \frac{Y(w)}{V_2(w)} = \frac{\frac{1}{jwC}}{\frac{1}{jwC}+R} = \frac{1}{1+jwRC}$$

$$Z_2(w) = \left[jwC + \left[\frac{1}{jwC}+R\right]^{-1}\right]^{-1}$$

$$\Rightarrow \frac{V_2(w)}{V_1(w)} = \frac{Z_2(w)}{Z_2(w)+R}$$

$$Z_1(w) = \left[jwC + [R+Z_2(w)]^{-1}\right]^{-1}$$

$$\Rightarrow \frac{V_1(w)}{X(w)} = \frac{Z_1(w)}{Z_1(w)+R} = \frac{1}{1+RZ_1(w)^{-1}}$$

$$= \frac{1}{1+jwRC+\frac{R}{R+Z_2(w)}}$$

$$\Rightarrow H(w) = \frac{Y(W)}{X(W)} = \frac{Y(w)}{V_2(w)} \cdot \frac{V_2(w)}{V_1(w)} \cdot \frac{V_1(w)}{X(w)}$$

$$= \frac{1}{(jwRC)^3 + 5(jwRC)^2 + 6jwRC + 1}$$

and

$$Z_2(w) = \left[jwC + \left[\frac{1}{jwC}+R\right]^{-1}\right]^{-1}$$

Example (5)

A 2 Hz cosinusoidal signal of 4 volt peak-to-peak amplitude is applied to a system described by the following differential equation

$$3y(t) + 2\frac{\mathrm{d}}{\mathrm{d}t}y(t) = 6x(t) - 4\frac{\mathrm{d}}{\mathrm{d}t}x(t).$$

Determine the output signal. (selected from Exam 2 in Summer 2014)

$x(t) = 2\cos w_0 t$ with $w_0 = 4\pi$

$$H(w) = \frac{6-4jw}{3+2jw}$$

$H(w) = H^*(-w) \Rightarrow h(t)$ is real

$$H(w_0) = \frac{6-j16\pi}{3+j8\pi}$$

So $|H(w_0)| = 2$. and $\angle H(w_0) = \angle \frac{6-j16\pi}{3+j8\pi}$

Thus $y(t) = 4\cos\left(4\pi t + \angle \frac{6-j16\pi}{3+j8\pi}\right)$