Ve216 Midterm Exam 2, 03/31/2023, 10:00 - 11:40

!! KEEP THIS PAGE FACE-UP UNTIL YOU ARE TOLD TO BEGIN !!

- This is a closed book exam. You are permitted to use two A4 page of notes (both sides), all of which must be in your own handwriting.
- Electronic media with wireless capability are not allowed. You may use calculators without wireless capability.
- There are 5 problems worth a total of 100 points. The questions are not necessarily in order of increasing difficulty.
- This exam has 10 pages and 2 cover pages. Make sure your copy is complete.
- Problems where the number of points are followed by an exclamation point are basic skill problems and will be graded without partial credit.
- Box your final answer. You will be graded on both the final answer and the steps leading to it. Correct intermediate steps will help earn partial credit. For full credit, ~~cross out~~ any incorrect intermediate steps.
- If you need to make any additional assumptions, state them clearly.
- Legible writing will help when it comes to partial credits.
- Simplify your results when possible.
- **Any** writing after the time is up is an honor code violation. Write your name, ID, and sign the honor code pledge *before* starting the exam so that you can stop writing immediately when the time is up.

Table of Fourier Series for Common Signals

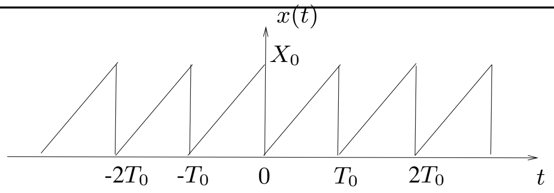
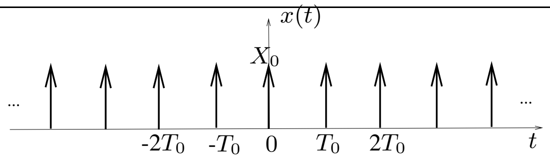
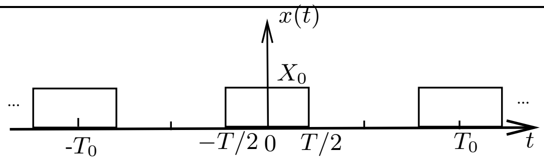
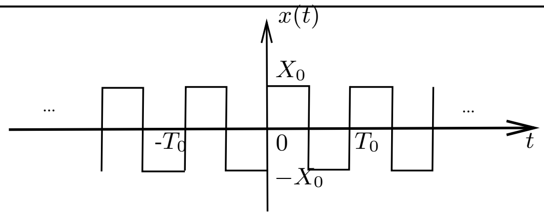
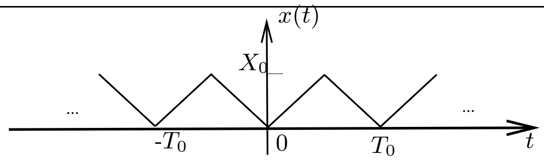
Name	Waveform	c_0	$c_k, k \neq 0$	Comments
Sawtooth		$\frac{X_0}{2}$	$j\frac{X_0}{2\pi k}$	
Impulse train		$\frac{X_0}{T_0}$	$\frac{X_0}{T_0}$	
Rectangular wave		$\frac{TX_0}{T_0}$	$\frac{TX_0}{T_0}\,\mathrm{sinc}\left(\frac{Tk\omega_0}{2\pi}\right)$	$\frac{Tk\omega_0}{2\pi} = \frac{Tk}{T_0}$
Square wave		0	$-j\frac{2X_0}{\pi k}$	$c_k = 0$, k even
Triangular wave sine		$\frac{X_0}{2}$	$\frac{-2X_0}{(\pi k)^2}$	$c_k = 0$, k even

Table of Fourier transform pairs

$f(t)$	$F(\omega)$
$\delta(t)$	1
1	$2\pi\,\delta(\omega) = \delta\left(\dfrac{\omega}{2\pi}\right)$
$u(t)$	$\pi\,\delta(\omega) + \dfrac{1}{j\omega}$
$\mathrm{sgn}(t)$	$\dfrac{2}{j\omega}$
$\mathrm{e}^{j\omega_0 t}$	$2\pi\,\delta(\omega - \omega_0)$
$\cos\omega_0 t$	$\pi\,\delta(\omega - \omega_0) + \pi\,\delta(\omega + \omega_0)$
$\sin\omega_0 t$	$\dfrac{\pi}{j}\,\delta(\omega - \omega_0) - \dfrac{\pi}{j}\,\delta(\omega + \omega_0)$
e^{-bt^2}	$\sqrt{\pi/b}\,\mathrm{e}^{-\omega^2/(4b)}$
$\displaystyle\sum_{n=-\infty}^{\infty} \delta(t - nT_0)$	$\displaystyle\sum_{k=-\infty}^{\infty} \omega_0\,\delta(\omega - k\omega_0)$

$f(t)$	$F(\omega)$
$\dfrac{1}{b^2 + t^2}$	$\dfrac{\pi}{b}\,\mathrm{e}^{-b\lvert\omega\rvert}$
$\mathrm{e}^{-b\lvert t\rvert}$	$\dfrac{2b}{b^2 + \omega^2}$
$\mathrm{rect}\left(\dfrac{t}{T}\right)$	$T\,\mathrm{sinc}\left(T\dfrac{\omega}{2\pi}\right)$
$\mathrm{tri}(t)$	$\mathrm{sinc}^2\left(\dfrac{\omega}{2\pi}\right)$
$\dfrac{\omega_0}{2\pi}\,\mathrm{sinc}\left(\dfrac{\omega_0}{2\pi}t\right)$	$\mathrm{rect}\left(\dfrac{\omega}{\omega_0}\right)$
$\mathrm{sinc}^2(t)$	$\mathrm{tri}\left(\dfrac{\omega}{2\pi}\right)$
$\mathrm{e}^{-at}\,u(t)$	$\dfrac{1}{j\omega + a}$
$\dfrac{t^{n-1}}{(n-1)!}\,\mathrm{e}^{-at}\,u(t)$	$\dfrac{1}{(j\omega + a)^n}$
$\dfrac{j}{\pi t}$	$\mathrm{sgn}(\omega)$

b is a real positive number throughout. a is a real or complex number throughout, with positive real part.

Properties of the Continuous-Time Fourier Transform

	Time	Fourier
Synthesis, Analysis	$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega$	$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$
Eigenfunction	$h(t) * e^{j\omega_0 t} = H(\omega_0)e^{j\omega_0 t}$	$H(\omega)2\pi\delta(\omega-\omega_0)$ $= H(\omega_0)2\pi\delta(\omega-\omega_0)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Time transformation	$f(at+b),\ a \neq 0$	$\frac{1}{\lvert a\rvert} e^{j\omega b/a} F(\omega/a)$
Time shift	$f(t-\tau)$	$F(\omega)e^{-j\omega\tau}$
Time reversal	$f(-t)$	$F(-\omega)$
Time-scaling	$f(at),\ a \neq 0$	$\dfrac{1}{\lvert a\rvert} F\left(\dfrac{\omega}{a}\right)$
Convolution	$f_1(t) * f_2(t)$	$F_1(\omega) \cdot F_2(\omega)$
Time-domain Multiplication	$f_1(t) \cdot f_2(t)$	$\dfrac{1}{2\pi} F_1(\omega) * F_2(\omega)$
Frequency shift	$f(t)e^{j\omega_0 t}$	$F(\omega-\omega_0)$
Modulation (cosine)	$f(t)\cos(\omega_0 t)$	$\dfrac{F(\omega-\omega_0) + F(\omega+\omega_0)}{2}$
Time. Differentiation	$\dfrac{d^n}{dt^n} f(t)$	$(j\omega)^n F(\omega)$
Freq. Differentiation	$(-jt)^n f(t)$	$\dfrac{d^n}{d\omega^n} F(\omega)$
Integration	$\int_{-\infty}^{t} f(\tau)\,d\tau = f(t) * u(t)$	$\dfrac{1}{j\omega} F(\omega) + \pi F(0)\delta(\omega)$
Conjugation	$f^*(t)$	$F^*(-\omega)$
Symmetry properties	$f(t)$ real	$F(\omega) = F^*(-\omega)$
	$f(t) = f^*(-t)$	$F(\omega)$ real
Duality	$F^*(t)$	$2\pi f^*(\omega)$
Duality	$F(t)$	$2\pi f(-\omega)$
Relation to Laplace	$F(\omega) = F(s)\rvert_{s=j\omega}$, if ROC includes $j\omega$ axis	
Parseval's Theorem	$\int_{-\infty}^{\infty} f_1(t) f_2^*(t)\,dt = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} F_1(\omega)F_2^*(\omega)\,d\omega$	
Parseval/Rayleigh Theorem	$E = \int_{-\infty}^{\infty} \lvert f(t)\rvert^2\,dt = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} \lvert F(\omega)\rvert^2\,d\omega$	
DC Value	$\int_{-\infty}^{\infty} f(t)\,dt = F(0)$	

A function that satisfies $f(t) = f^*(-t)$ is said to have **Hermitian symmetry**.

1. (20 points)
Consider the following cascade of LTI systems:

$$x(t) \rightarrow \boxed{h_1(t)} \rightarrow \boxed{h_2(t)} \rightarrow y(t),$$

where $h_1(t) = e^{-t}u(t)$ and $h_2(t) = e^{-2t}u(t)$.

- [5 points] Find the frequency response of the overall system.

- [5 points] Find the linear constant coefficient differential equation that describes this system.

- [10 points] If the input to the system is $x(t) = e^{-5t}u(t)$, find the corresponding output $y(t)$.

2. (30 points)
Consider an LTI system with impulse response

$$h(t) = \left\{ \frac{\pi}{5} \frac{\sin(5t)}{\pi t} \frac{\sin(15t)}{\pi t} \right\} 2\cos(25t)$$

- [15 points] Determine and sketch the frequency response, $H(\omega)$, which is the Fourier Transform of $h(t)$.

- [15 points] Determine the output $y(t)$ for the input $x(t)$ is an impulse train:

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\frac{2\pi}{5}).$$

(The final answer needs to be expressed as a sum of sine waves or cosine waves.)

3. (20 points)
An engineer types the following MATLAB commands to plot a signal.

```
t = linspace(0, 15, 1001);
x = ones(size(t));
for k = 1:50
    x = x + 5/(pi*k)*sin(2*pi*k/5) * cos(2*pi*k*t/5);
end
plot(t, x), xlabel('t'), ylabel('x(t)'), title('Wave Exam 2')
set(gca,'XTick',0:1:15);
print('wave_exam2', '-deps')
```

- [10 points] Find Fourier Seris coefficients c_k of the singal the engineer wants to plot.

- [10 points] Carefully sketch the signal that is produced in the engineer's MATLAB plot.

end