Bode 图绘制

⇒ 低频部分会过 $(1, 20\lg k)$ ⟶ 此处 $(1, 20\lg 25)$

① 化成标准形式：

如 $H(s) = \dfrac{25(0.1s+1)}{s^2(0.2s+1)}$

此时有比例环节(1个) $\boxed{k}$: 25

积分环节(2个) : $\dfrac{1}{s^2}$

导前环节(1个) : $0.1s+1$

惯性环节(1个) : $0.2s+1$

比例环节: $G(s)=k$

积分环节: $G(s)=\dfrac{1}{s}$

微分环节: $G(s)=s$

惯性环节: $G(s)=\dfrac{1}{Ts+1}$

导前环节: $G(s)=Ts+1$

确定了第一条线

Ⅱ型系统（有n个积分环节就是n型系统）

⇓

$V=2$ ⇒ 低频部分斜率为 -20 dB/dec · $V = -40$ dB/dec (此处)

② 求频率特性（把 s 变为 jw）

$H(jw) = \dfrac{25(j0.1w+1)}{(jw)^2(j0.2w+1)}$

$\begin{cases} \text{对导前环节} \quad j0.1w+1, \quad w_2 = \frac{1}{0.1} = 10 \ s^{-1} \\ \text{对惯性环节} \quad j0.2w+1, \quad w_1 = \frac{1}{0.2} = 5 \ s^{-1} \end{cases}$ （小的标1）

$\begin{cases} \text{对导前环节 } w_2 \text{ 开始斜率上升 } 20 \text{ dB/dec} \\ \text{对惯性环节 } w_2 \text{ 开始斜率下降 } 20 \text{ dB/dec} \end{cases}$

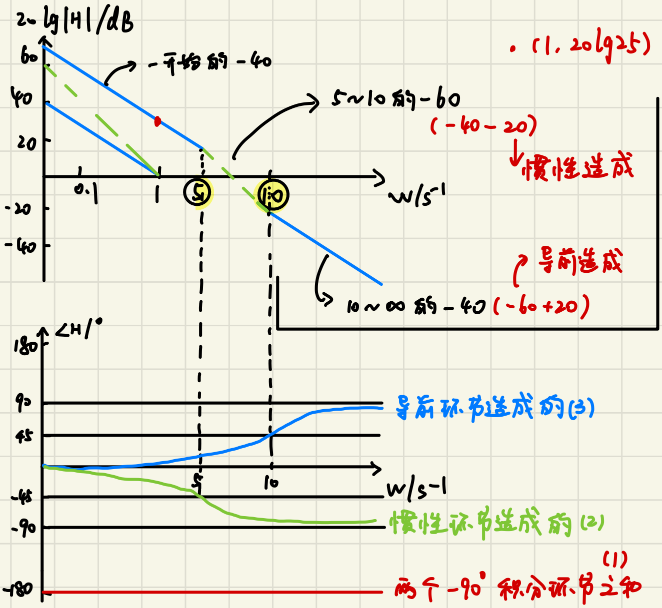
③ 画图

幅频图 ⟶

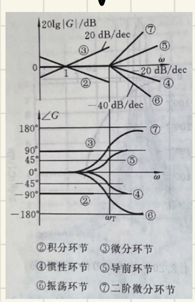
相频图 ⟶

↓

把(1)(2)(3)加起来即可得到结果

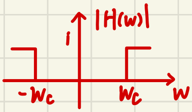


不用管比例环节

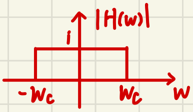


① Ideal Filters

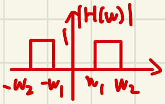
highpass filter



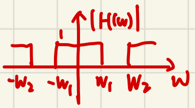
lowpass filter



bandpass filter



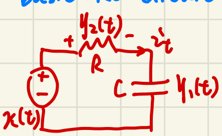
bandstop filter

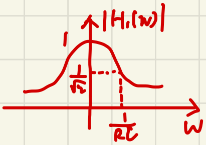


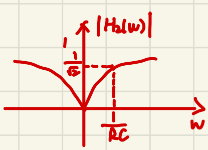
good ideal filter can't be implemented in real life since noncausal

② Real Filter

Basic RC circuit







③ Bandwidth Relationship

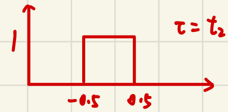
RMS bandwidth

$$W_{rms} = \sqrt{\frac{\int_{-\infty}^{\infty} w^2 |X(w)|^2 dw}{\int_{-\infty}^{\infty} |X(w)|^2 dw}}$$

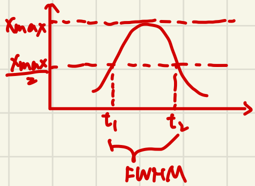
Time duration

Absolute time duration

$\tau = t_2 - t_1$



Full width at half maximum



Root mean squared time duration

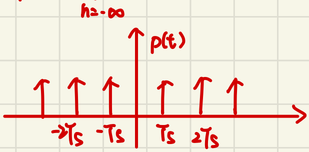
$$T_{rms} = \sqrt{\frac{\int_{-\infty}^{\infty} t^2 |x(t)|^2 dt}{\int_{-\infty}^{\infty} |x(t)|^2 dt}}$$

Time-Bandwidth product

$$W_{rms} \cdot T_{rms} \geq \frac{1}{2}$$

④ FT of impulse-train sampled signals

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$



Impulse-train sampling

$$x_s(t) = x(t)p(t) = x(t)\sum_{n=-\infty}^{\infty} \delta(t-nT_s)$$

$$= \sum_{-\infty}^{\infty} x(nT_s)\,\delta(t-nT_s)$$

FT of sampled signal

With FT of ideal sampling function:

$$p(t) \overset{F}{\longleftrightarrow} P(w) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{T_s}\delta(w - kw_s), \quad w_s = \frac{2\pi}{T_s}$$

So we can derive the shape of the sampled signal on the frequency domain

$$X_s(w) = \frac{1}{2\pi} X(w) * P(w) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X(w - kw_s)$$

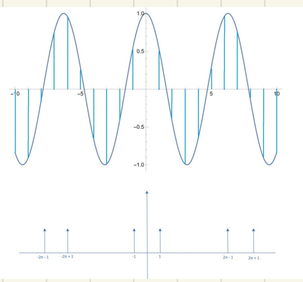
⑥ Sampling Theorem

If $x(t)$ is a band-limited signal with $X(w) = 0$ for $|w| > W_{max}$, then the sampling interval T_s should meet the need that $W_s > 2W_{max}$ to ensure that there is no overlape of $X(w-kw_s)$ ($-\infty < k < \infty$ on frequency domain)

$y = \cos t$,

$t_s = 1$,

$W_s = 2\pi$



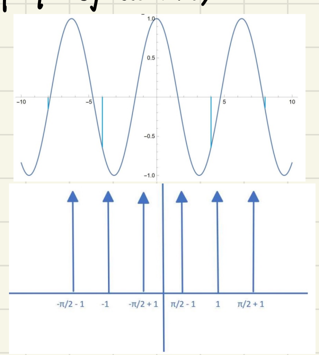
⑦ Aliasing

It will happen exactly when there are overlaps of $X(w-kw_s)$ ($-\infty < k < \infty$ on frequency domain)

$y = \cos(t)$

$t_s = 4$

$W_s = \frac{\pi}{2}$



⑧ Reconstruction via interpolation

Sinc interpolation (Ideal interpolation filter)

$$h(t) = \frac{\omega_c T_s}{\pi}\operatorname{sinc}\left(\frac{\omega_c t}{\pi}\right) \xleftrightarrow{F} H(\omega) = T_s \operatorname{rect}\left(\frac{\omega}{2\omega_c}\right)$$

$\omega_{max} < \omega_c < \omega_s - \omega_{max}$ and usually $\omega_c = \frac{\omega_s}{2}$

Linear interpolation (first-order holder filter)

$$h_1(t) = \operatorname{tri}\left(\frac{t}{T_s}\right) \xleftrightarrow{F} H_1(\omega) = T_s \operatorname{sinc}^2\left(\frac{\omega}{\omega_s}\right)$$

Nearest neighbor interpolation (0 order hold filter)

$$h_2(t) = \operatorname{rect}\left(\frac{t}{T_s} - \frac{1}{2}\right) \xleftrightarrow{F} H_2(\omega) = T_s \operatorname{sinc}\left(\frac{\omega}{\omega_s}\right) e^{-j\omega T_s/2}$$

How to reconstruct

$$x_{recon}(t) = \sum_{n=-\infty}^{\infty} x_s[n]\, h_i(t - nT_s) \qquad x(n) = x(nT_s)$$

$$= h_i(t) * x_s(t) \quad \text{on time domain}$$

$$X_{recon}(\omega) = X_s(\omega) H_i(\omega) \quad \text{frequency domain}$$

⑨ Sinusoidal amplitude modulation

$$y(t) = x(t)c(t) = x(t)\cos(\omega_c t + \theta_c)$$

$$\xleftrightarrow{F} Y(\omega) = \frac{1}{2}\left[e^{j\theta_c} X(\omega - \omega_c) + e^{-j\theta_c} X(\omega + \omega_c)\right]$$

⑩ Synchronous Demodulation

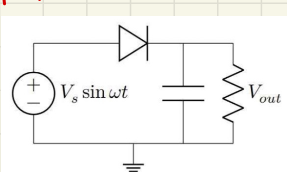
$$w(t) = y(t)\cos(\omega_c t + \theta_c)$$

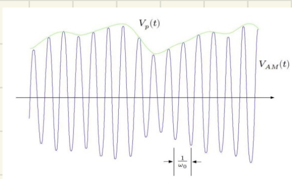
$$W(\omega) = \frac{1}{4}\left[e^{2j\theta_c} X(\omega - 2\omega_c) + 2X(\omega) + e^{-2j\theta_c} X(\omega + 2\omega_c)\right]$$

⑪ Asynchronous Demodulation

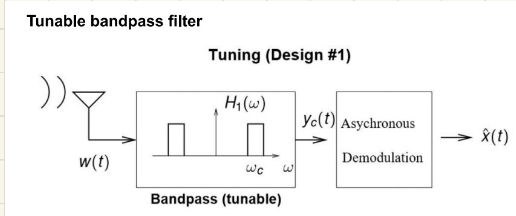
$$y(t) = (A + x(t))\cos(\omega_c t)$$

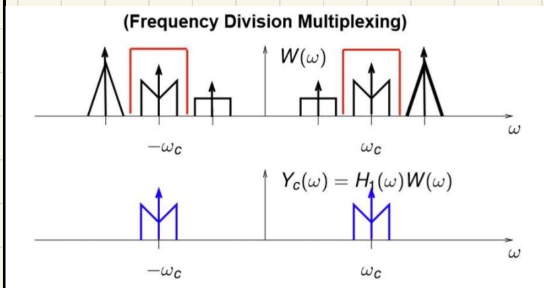
$$Y(\omega) = A\pi\left[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)\right] + \frac{1}{2}\left[X(\omega - \omega_c) + X(\omega + \omega_c)\right]$$

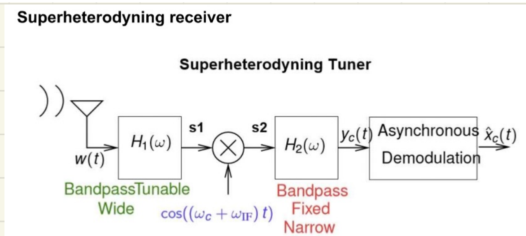


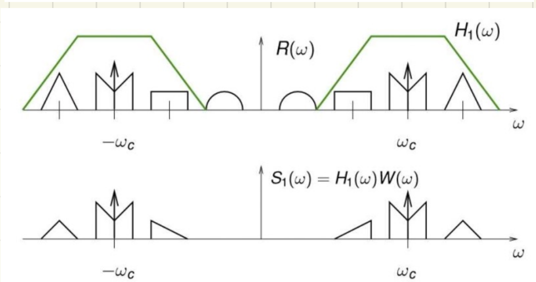


⑫ Frequency-division multiplexing









⑬ Laplace Transforms

$$e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t} e^{j\omega t}$$

Bilateral Laplace Transform

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}$$

$$x(t) = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} X(s)\, e^{st}$$

$$x(t) \xleftrightarrow{L} X(s)$$