Ve 216: Introduction to Signals and Systems
Quiz

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Quiz 1

Quiz

Example (10!)

Consider the following CT system with input $x(t)$ and output $y(t)$

$$y(t) = x(\sin(t)) + \int_1^3 e^{-\tau^2} x(t-\tau)\, d\tau.$$

- Prove that it is linear or give a counter example. [2!]
- Prove that it is time-invariant or give a counter example. [2!]
- Determine whether it is causal or noncausal. [2!]
- Determine if it is a memoryless or memory system. [2!]
- Determine if it is BIBO stable or unstable? [2!]

Solution (1)

$$y(t) = y_1(t) + y_2(t)$$

$$y_1(t) = x(\sin(t)), \quad y_2(t) = \int_1^3 e^{-\tau^2} x(t-\tau)\, d\tau$$

$$\begin{aligned} y_2(t) &= \int_1^3 e^{-\tau^2} x(t-\tau)\, d\tau \\ &= \int_{-\infty}^{\infty} \mathrm{rect}\left(\frac{\tau-2}{2}\right) e^{-\tau^2} x(t-\tau)\, d\tau \\ &= \left(\mathrm{rect}\left(\frac{t-2}{2}\right) e^{-t^2}\right) * x(t) \end{aligned}$$

The second system is LTI.

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Solution (2)

This system is $\boxed{\textit{linear}}$. *Proof.*

$$y_1(t) = x_1(\sin(t)), \quad y_2(t) = x_2(\sin(t)),$$

$$x(\sin(t)) = a_1 x_1(\sin(t)) + a_2 x_2(\sin(t))$$

$$y(t) = x(\sin(t)) = a_1 x_1(\sin(t)) + a_2 x_2(\sin(t))$$

So

$$a_1 y_1(t) + a_2 y_2(t) = y(t)$$

Solution (3)

This system is $\boxed{\textit{time-varying}}$.

$$y(t-t_0) = x(\sin(t-t_0))$$

$$x_d(t) = x(t-t_0)$$

$$y_d(t) = x_d(\sin(t)) = x(\sin(t) - t_0)$$

So

$$y(t-t_0) \neq y_d(t)$$

A counter example

$$x(t) = \sin^{-1}(t) \Longrightarrow y(t) = \sin^{-1}(\sin(t)) = t$$

$$y(t-1) = t-1$$

$$x_d(t) = x(t-1) \Longrightarrow y_d(t) = \sin^{-1}(\sin(t) - 1) \neq y(t-1)$$

Solution (4)

The systen is $\boxed{\textit{non-causal}}$. *The first subsystem is non-causal.*

$$y(t) = x(\sin(t)) \Longrightarrow y(-\pi/2) = x(\sin(-\pi/2)) = x(-1)$$

So $y(-\pi/2)$ *depends on future input because* $-\pi/2 < -1$.

If there is any time instance such that $t < \sin(t)$, *this system is non-causal.*

Since $-1 \leq \sin(t) \leq 1$, *for any* $t < -1$, $\sin(t) > t$. *So this system is non-causal.*

Solution (4)

The second subsystem is causal.

$$y_2(t) = \left(\mathrm{rect}\left(\frac{t-2}{2}\right) e^{-t^2}\right) * x(t)$$

$$h(t) = \mathrm{rect}\left(\frac{t-2}{2}\right) e^{-t^2}$$

$$h(t) = 0, \quad for, \quad t < 0$$

Solution (4)

The second subsystem is causal.

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$$h(t) = \mathrm{rect}\left(\frac{t-2}{2}\right) e^{-t^2}$$

$$h(t) = 0, \quad for, \quad t < 0$$

Solution (5)

This system has $\boxed{\textit{memory}}$.
$\sin(t)$ *is not always equal to* t.

$$h(t) = \mathrm{rect}\left(\frac{t-2}{2}\right) e^{-t^2} \neq a\delta(t)$$

Solution (6)

This system is $\boxed{\textit{BIBO stable}}$.
Suppose $|x(t)| \leq M_x$,

$$|y_1(t)| = |x(\sin(t))| \leq M_x$$

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} \left| \text{rect}\left(\frac{t-2}{2}\right) e^{-t^2} \right| dt < \infty$$

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