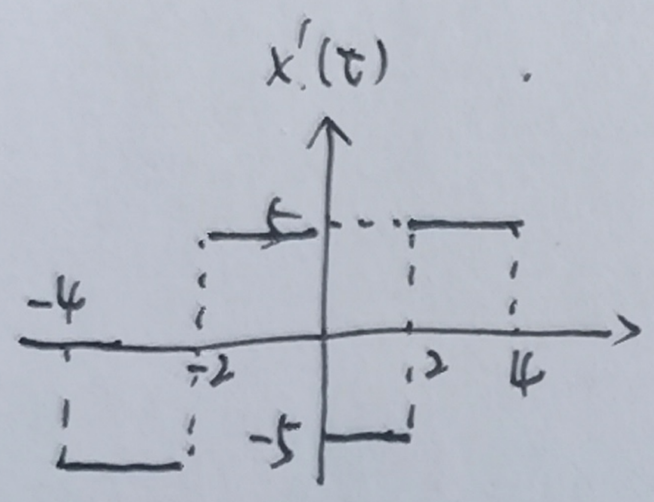
1.

From the graph :

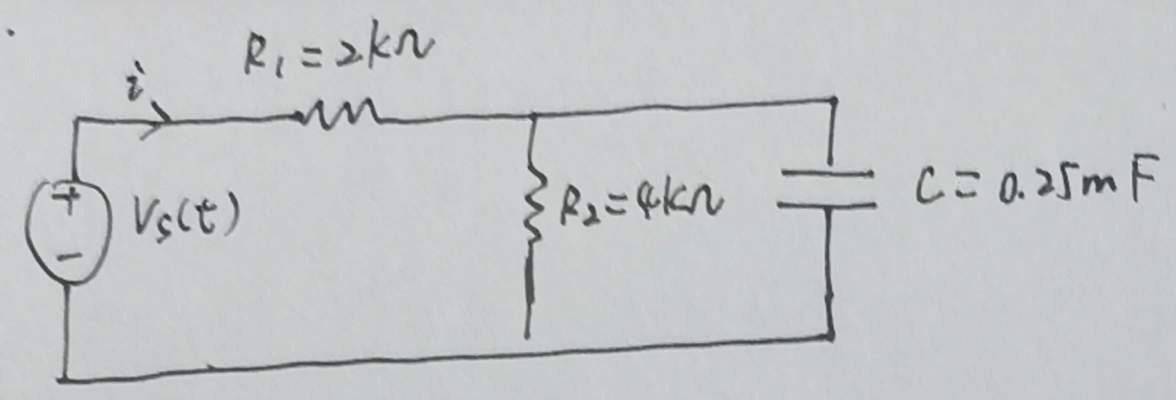


$$\frac{d}{dt}x(t) = 2Sa(w)e^{jw} - 2Sa(w)e^{j3w} - 2Sa(w)e^{-jw} + 2Sa(w)e^{-j3w}$$

$$\Rightarrow x(t) = \frac{1}{jw}\left(2Sa(w)e^{jw} - 2Sa(w)e^{j3w} - 2Sa(w)e^{-jw} + 2Sa(w)e^{-j3w}\right)$$

where $Sa(x) = \frac{\sin(x)}{x}$

2.



(1)

$$\Rightarrow i = \frac{V_s}{R_1 + \frac{\frac{R_2}{jwC}}{R_2 + \frac{1}{jwC}}} = \frac{V_s}{R_1 + \frac{R_2}{jwCR_2+1}}$$

$$\Rightarrow i_c = i \cdot \frac{R_2}{R_2 + \frac{1}{jwC}} = i \cdot \frac{jwCR_2}{jwCR_2+1}$$

$$= \frac{V_s}{R_1 + \frac{R_2}{jwCR_2+1}} \cdot \frac{jwCR_2}{jwCR_2+1}$$

$$\Rightarrow I_c(jw) = \frac{V_s \cdot jwCR_2}{(jwCR_2+1)R_1 + R_2} \quad ①$$

$$I_c(s) = \frac{\frac{10}{s} \cdot s \cdot 0.25 \times 10^{-3} \cdot 4 \times 10^3}{(s \cdot 0.25 \times 10^{-3} \times 4 \times 10^3) \times 2 \times 10^3 + 4 \times 10^3}$$

$$= 10^{-3} \cdot \frac{5}{s+2}$$

$$\Rightarrow I_c(jw) = 10^{-3} \cdot \frac{5}{jw+2} \Rightarrow 10^{-3} \times 5e^{-2t} u(t)$$

$$i_c(t) = 5e^{-2t} u(t) \ (mA)$$

(2)

$V_s(t)$ now becomes $10e^{-2t} u(t)$

From ① we can still know that

$$V_c(jw) = I_c(jw) \cdot \frac{1}{jwC} \Rightarrow V_c(s) = \frac{10}{s+2} \cdot 4 \times 10^3 \cdot \frac{1}{s \cdot 2 \times 10^3 + 4 \times 10^3}$$

$$= \frac{20}{(s+2)^2} \Rightarrow V_c(t) = 20 \cdot e^{-2t} t\, u(t)$$

3.

$$H(w) = 3\,\mathrm{rect}\left(\frac{w-3\pi}{4\pi}\right) + 3\,\mathrm{rect}\left(\frac{w+3\pi}{4\pi}\right)$$

$$\left(\text{where } x(t) = \sum_{n=-\infty}^{\infty} (t-n)\,\mathrm{rect}\left(t-\tfrac{1}{2}-n\right)\right)$$

For $x_0(t) = t \cdot \mathrm{rect}(t-\frac{1}{2}) \longrightarrow T=1$

since $x_0(t) = t \cdot y_0(t) \Rightarrow Y_0(w) = 1 \cdot \mathrm{sinc}\left(1 \cdot \frac{w}{2\pi}\right) e^{-jw\frac{1}{2}}$

$$\Rightarrow X_0(w) = \frac{d}{dw} j \cdot Y_0(w) = j\,\frac{d\,\mathrm{sinc}\left(\frac{w}{2\pi}\right) \cdot e^{-\frac{1}{2}jw}}{dw}$$

$$= \left(\mathrm{sinc}\left(\frac{w}{2\pi}\right)\right)' \cdot e^{-\frac{1}{2}jw} \cdot j + \mathrm{sinc}\left(\frac{w}{2\pi}\right) \cdot \frac{1}{2} \cdot e^{-\frac{1}{2}jw}$$

$$= \left[\frac{1}{2}\,\mathrm{sinc}\left(\frac{w}{2\pi}\right) - j\,\mathrm{cosc}\left(\frac{w}{2\pi}\right)\right] \cdot e^{-\frac{1}{2}jw}$$

4.

(1) $y'(t)+7y(t)=x'(t)+4x(t)+\int_{-\infty}^{\infty} x(\tau)z(t-\tau)d\tau$

$(z(t)=4e^{-t}u(t)+\delta(t))$

$\Rightarrow jwY(jw)+7Y(jw)=jwX(jw)+4X(jw)+X(jw)\cdot\left[\frac{4}{jw+1}+1\right]$

$(jw+7)Y(jw)=\left(jw+4+\frac{jw+5}{jw+1}\right)X(jw)$

$\Rightarrow \frac{Y(jw)}{X(jw)}=\frac{(jw+4)(jw+1)+(jw+5)}{(jw+7)(jw+1)}$

$H(s)=\frac{s^2+5s+4+s+5}{s^2+8s+7}=\frac{s^2+6s+9}{s^2+8s+7}=1+\frac{A}{s+1}+\frac{B}{s+7}$

$\Rightarrow H(s)=1+\frac{2}{3}\frac{1}{s+1}-\frac{8}{3}\frac{1}{s+7}$

$\Rightarrow h(t)=8t+\frac{2}{3}e^{-t}u(t)-\frac{8}{3}e^{-7t}u(t)$

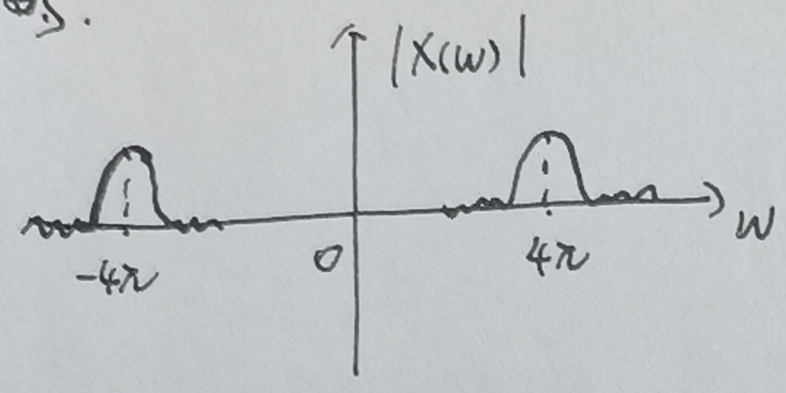
(2)

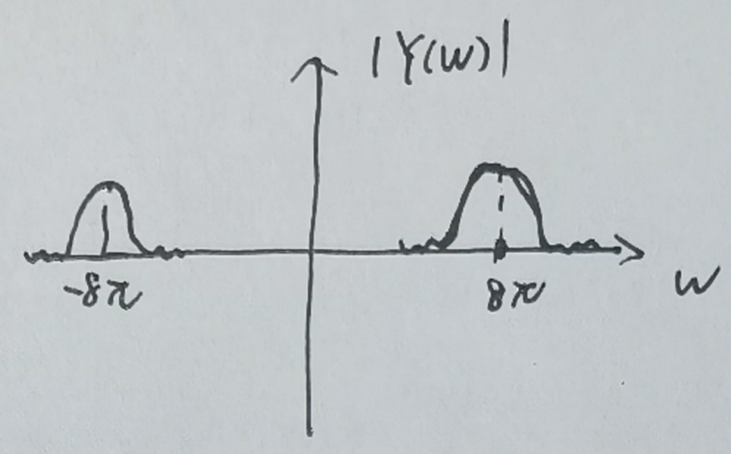
$H_{inverse}(s)=\frac{1}{H(s)}=\frac{(s+1)(s+7)}{(s+3)^2}=\frac{s^2+8s+7}{s^2+6s+9}=\frac{Y(s)}{X(s)}$

$\Rightarrow y''(t)+6y'(t)+9=x''(t)+8x'(t)+7x(t)$

where $\left(y'(t)=\frac{dy(t)}{dt}\right)$

⊕5.





$$f(t) = [u(t+1) - u(t)] \cdot (t+1) + [u(t) - u(t-1)] \cdot (-t+1)$$

$$s(t) = \cos(4\pi t)$$

$$\Rightarrow x(t) = \underbrace{\left([u(t+1) - u(t)] \cdot (1+t) + [u(t) - u(t-1)] \cdot (1-t) \right)} \cdot \cos(4\pi t)$$

since $x(t) = \cos(4\pi t) \cdot f(t)$

we just need to calculate $f(w)$ since $X(w) = \frac{1}{2}[F(w-4\pi) + F(w+4\pi)]$

$$\Rightarrow f(t) = u(t+1) \cdot (t+1) - u(t) \cdot 2t + (t-1) \cdot u(t-1)$$

$$F(w) = \cancel{u(t+1) + t \cdot u(t+1)} - \frac{2dj\left(\pi\delta(w) + \frac{1}{jw}\right)}{dw} + \frac{dj\left(\pi\delta(w) + \frac{1}{jw}\right) \cdot e^{-jw}}{dw}$$

$$\left(\pi\delta(w) + \frac{1}{jw}\right) \cdot e^{jw} + \frac{dj\left(\pi\delta(w) + \frac{1}{jw}\right) \cdot e^{jw}}{dw} - \left(\pi\delta(w) + \frac{1}{jw}\right) \cdot e^{-}$$

$$= \left(\pi\delta(w) + \frac{1}{jw}\right) \cdot \left(e^{jw} - e^{-jw}\right) + \frac{d\left(\pi\delta(w) + j\frac{1}{w}\right) j e^{jw}}{dw} + \frac{d\left(\pi\delta(w) + \frac{1}{jw}\right) j e^{-jw}}{dw}$$

$$- 2\frac{j\, d\left(\pi\delta(w) + j\frac{1}{w}\right)}{dw} = \left(\pi\delta(w) + \frac{1}{jw}\right)' \cdot \left(je^{jw} + je^{-jw} - 2j\right)$$

$$\Rightarrow X(w) = \left(\pi\delta(w-4\pi) + \frac{1}{j(w-4\pi)}\right)' \cdot \left(je^{j(w-4\pi)} + je^{-j(w-4\pi)} - 2j\right)$$

$$+ \left(\pi\delta(w+4\pi) + \frac{1}{j(w+4\pi)}\right)' \cdot \left(je^{j(w+4\pi)} + je^{-j(w+4\pi)} - 2j\right)$$

$$= \frac{j}{(w-4\pi)^2} \cdot \left(je^{j(w-4\pi)} + je^{-j(w-4\pi)} - 2j\right)$$

$$+ \frac{j}{(w+4\pi)^2} \cdot \left(je^{j(w+4\pi)} + je^{-j(w+4\pi)} - 2j\right)$$