Ve216 Midterm Exam II, 07/04/2023, 10:00 - 11:40

!! KEEP THIS PAGE FACE-UP UNTIL YOU ARE TOLD TO BEGIN !!

- This is a closed book exam. You are permitted to use **two** A4 page of notes (both sides), all of which must be in your own handwriting. Submit your notes as a **pdf** file to "Canvas → Assignments → Midterm Exam II → "Content crib sheet" section by **11:50**.
- Electronic media with wireless capability are not allowed. You may use calculators without wireless capability.
- There are 5 problems worth a total of 100 points. The questions are not necessarily in order of increasing difficulty.
- This exam has 12 pages. Make sure your copy is complete.
- Problems where the number of points are followed by an exclamation point are basic skill problems and will be graded without partial credit.
- Box your final answer. You will be graded on both the final answer and the steps leading to it. Correct intermediate steps will help earn partial credit. For full credit, ~~cross out~~ any incorrect intermediate steps.
- If you need to make any additional assumptions, state them clearly.
- Legible writing will help when it comes to partial credits.
- Simplify your results when possible.
- **Any** writing after the time is up is an honor code violation. Write your name, ID, and sign the honor code pledge *before* starting the exam so that you can stop writing immediately when the time is up.
- Submit your **signed honor code pledge** as a **pdf** file to "Canvas → Assignments → Midterm Exam II → Honor Code Pledge Submission" section by **11:50**.
- Submit your **midter exam II solutions** as a **pdf** file to Canvas by **11:50**. **Name your pdf file as "Name (First Name Last Name) + student ID"**, *e.g.*, SanZhang+518370910999.pdf. **Submit your pdf file to "Canvas → Assignments → Midterm Exam II → Midterm Exam II Solutions" section**.
- In the unlikely event of a technical problem with Canvas, email your solutions to me (`yong.long@sjtu.edu.cn`) by the deadline, and then later upload when the system is working.
- Canvas records each of your submission times and the pdf file you uploaded. All of your submissions are on Canvas. **For students who have submitted multiple versions, please indicate which version you want TAs to grade, by writing in the comment session of your file submission.**
- **Scores for late exams will be reduced by 1 point per minute after the due time, up to the total points.**

Table of Fourier Series for Common Signals

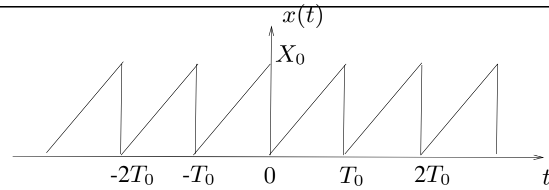
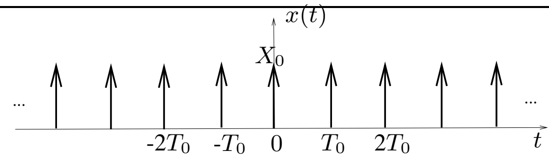
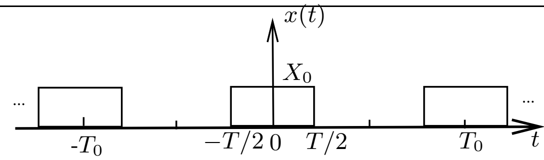
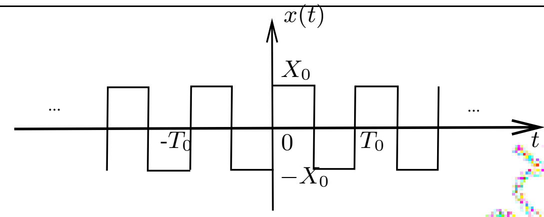
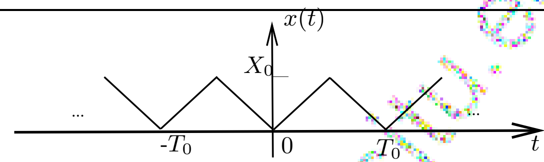
Name	Waveform	c_0	$c_k, k \neq 0$	Comments
Sawtooth		$\frac{X_0}{2}$	$j\frac{X_0}{2\pi k}$	
Impulse train		$\frac{X_0}{T_0}$	$\frac{X_0}{T_0}$	
Rectangular wave		$\frac{TX_0}{T_0}$	$\frac{TX_0}{T_0}\text{sinc}\left(\frac{Tk\omega_0}{2\pi}\right)$	$\frac{Tk\omega_0}{2\pi} = \frac{Tk}{T_0}$
Square wave		0	$-j\frac{2X_0}{\pi k}$	$c_k = 0$, k even
Triangular wave sine		$\frac{X_0}{2}$	$\frac{-2X_0}{(\pi k)^2}$	$c_k = 0$, k even

Table of Fourier transform pairs

$f(t)$	$F(\omega)$
$\delta(t)$	1
1	$2\pi\,\delta(\omega) = \delta\left(\frac{\omega}{2\pi}\right)$
$u(t)$	$\pi\,\delta(\omega) + \frac{1}{j\omega}$
$\operatorname{sgn}(t)$	$\frac{2}{j\omega}$
$\mathrm{e}^{j\omega_0 t}$	$2\pi\,\delta(\omega-\omega_0)$
$\cos\omega_0 t$	$\pi\,\delta(\omega-\omega_0) + \pi\,\delta(\omega+\omega_0)$
$\sin\omega_0 t$	$\frac{\pi}{j}\,\delta(\omega-\omega_0) - \frac{\pi}{j}\,\delta(\omega+\omega_0)$
e^{-bt^2}	$\sqrt{\pi/b}\,\mathrm{e}^{-\omega^2/(4b)}$
$\sum_{n=-\infty}^{\infty} \delta(t-nT_0)$	$\sum_{k=-\infty}^{\infty} \omega_0\,\delta(\omega-k\omega_0)$

$f(t)$	$F(\omega)$
$\frac{1}{b^2+t^2}$	$\frac{\pi}{b}\,\mathrm{e}^{-b\lvert\omega\rvert}$
$\mathrm{e}^{-b\lvert t\rvert}$	$\frac{2b}{b^2+\omega^2}$
$\operatorname{rect}\left(\frac{t}{T}\right)$	$T\operatorname{sinc}\left(T\frac{\omega}{2\pi}\right)$
$\operatorname{tri}(t)$	$\operatorname{sinc}^2\left(\frac{\omega}{2\pi}\right)$
$\frac{\omega_0}{2\pi}\operatorname{sinc}\left(\frac{\omega_0}{2\pi}t\right)$	$\operatorname{rect}\left(\frac{\omega}{\omega_0}\right)$
$\operatorname{sinc}^2(t)$	$\operatorname{tri}\left(\frac{\omega}{2\pi}\right)$
$\mathrm{e}^{-at}\,u(t)$	$\frac{1}{j\omega+a}$
$\frac{t^{n-1}}{(n-1)!}\,\mathrm{e}^{-at}\,u(t)$	$\frac{1}{(j\omega+a)^n}$
$\frac{j}{\pi t}$	$\operatorname{sgn}(\omega)$

b is a real positive number throughout. a is a real or complex number throughout, with positive real part.

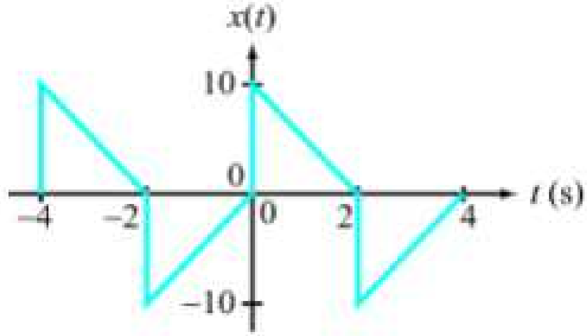
Properties of the Continuous-Time Fourier Transform

	Time	Fourier
Synthesis, Analysis	$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega$	$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$
Eigenfunction	$h(t) * e^{j\omega_0 t} = H(\omega_0)e^{j\omega_0 t}$	$H(\omega)2\pi\delta(\omega-\omega_0)$ $= H(\omega_0)2\pi\delta(\omega-\omega_0)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Time transformation	$f(at+b),\ a \neq 0$	$\frac{1}{\lvert a\rvert} e^{j\omega b/a} F(\omega/a)$
Time shift	$f(t-\tau)$	$F(\omega)e^{-j\omega\tau}$
Time reversal	$f(-t)$	$F(-\omega)$
Time-scaling	$f(at),\ a \neq 0$	$\frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)$
Convolution	$f_1(t) * f_2(t)$	$F_1(\omega)\cdot F_2(\omega)$
Time-domain Multiplication	$f_1(t)\cdot f_2(t)$	$\frac{1}{2\pi}F_1(\omega) * F_2(\omega)$
Frequency shift	$f(t)e^{j\omega_0 t}$	$F(\omega-\omega_0)$
Modulation (cosine)	$f(t)\cos(\omega_0 t)$	$\frac{F(\omega-\omega_0)+F(\omega+\omega_0)}{2}$
Time. Differentiation	$\frac{d^n}{dt^n}f(t)$	$(j\omega)^n F(\omega)$
Freq. Differentiation	$(-jt)^n f(t)$	$\frac{d^n}{d\omega^n}F(\omega)$
Integration	$\int_{-\infty}^{t} f(\tau)\,d\tau = f(t) * u(t)$	$\frac{1}{j\omega}F(\omega) + \pi F(0)\delta(\omega)$
Conjugation	$f^*(t)$	$F^*(-\omega)$
Symmetry properties	$f(t)$ real	$F(\omega) = F^*(-\omega)$
	$f(t) = f^*(-t)$	$F(\omega)$ real
Duality	$F^*(t)$	$2\pi f^*(\omega)$
Duality	$F(t)$	$2\pi f(-\omega)$
Relation to Laplace	$F(\omega) = F(s)\rvert_{s=j\omega}$, if ROC includes $j\omega$ axis	
Parseval's Theorem	$\int_{-\infty}^{\infty} f_1(t)f_2^*(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(\omega)F_2^*(\omega)\,d\omega$	
Parseval/Rayleigh Theorem	$E = \int_{-\infty}^{\infty} \lvert f(t)\rvert^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert F(\omega)\rvert^2\,d\omega$	
DC Value	$\int_{-\infty}^{\infty} f(t)\,dt = F(0)$	

A function that satisfies $f(t) = f^*(-t)$ is said to have **Hermitian symmetry**.

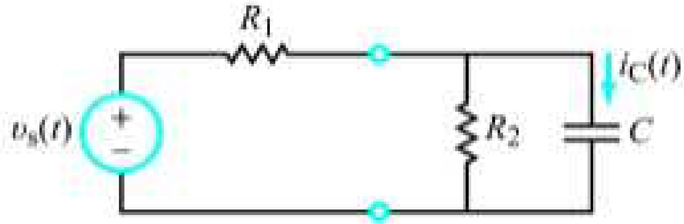
1. (25 points)

The waveform signal $x(t)$ is illustrated below. Using the differentiation property to obtain the real-valued Fourier-series representation for this waveform signal.



2. (30 points)

The RC circuit shown below is excited by a voltage source $v_S(t)$. The element values are $R_1 = 2k\Omega, R_2 = 4k\Omega$, and $C = 0.25mF$.



- [15 points] Apply Fourier analysis to determine $i_C(t)$ if $v_S(t) = 10u(t)$.

- [15 points] Determine the voltage across the capacitor $v_C(t)$ for $v_S(t) = 10e^{-2t}u(t)$ V.

3. (15 points)
The signal $x(t) = \sum_{n=-\infty}^{\infty} (t-n)\,\mathrm{rect}(t-1/2-n)$ is passed through a filter with frequency response

$$H(\omega) = 3\,\mathrm{rect}\left(\frac{\omega-3\pi}{4\pi}\right) + 3\,\mathrm{rect}\left(\frac{\omega+3\pi}{4\pi}\right).$$

Determine the output signal $y(t)$. Express your results in a real form.

4. (18 points)
The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}y(t) + 7y(t) = \frac{\mathrm{d}}{\mathrm{d}t}x(t) + 4x(t) + \int_{-\infty}^{\infty} x(\tau)z(t-\tau)\,d\tau,$$

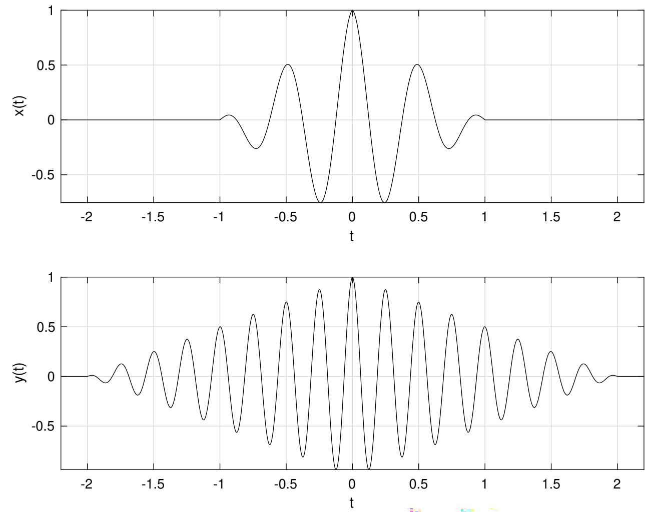
where $z(t) = 4e^{-t}u(t) + \delta(t)$.

- [10 points] Find the impulse response of the system.

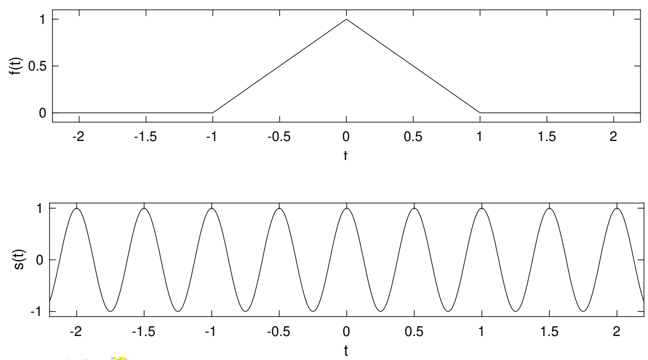
- [8 points] The inverse of this system is also LTI and described by a differential equation. Find the differential equation describing the inverse system.

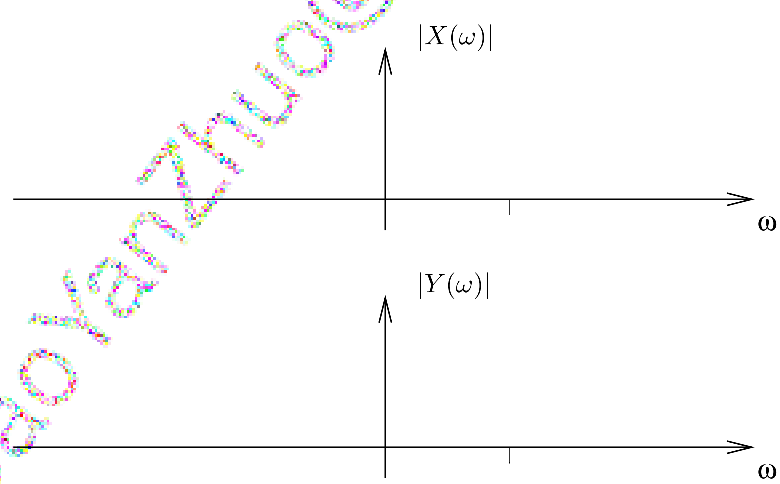
5. (12 points)

Consider the signals $x(t)$ and $y(t)$ shown to the right. Find the Fourier transform $X(\omega)$, $Y(\omega)$ of $x(t)$ and $y(t)$. Using the axes below, sketch the magnitude spectrum of $X(\omega)$ and $Y(\omega)$.
The *relative* characteristics of your two plots are particularly important.



Hint: $x(t)$ is the product of the two signals shown to the right.





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