UM–SJTU JOINT INSTITUTE
SIGNALS AND SYSTEMS LABORATORY
(VE216)

Laboratory 3

Xu Duo, Zong Junzhe, Qiu Tian

April 21, 2023

Partners:

Name: Xu Duo	ID: 521370910024	Thursday 11
Name: Zong Junzhe	ID: 521370910068	Thursday 11
Name: Qiu Tian	ID: 521370910160	Thursday 11

Laboratory Date: April 6, 2023

Objectives

In the experiment, we would like to understand feedback control.

Experimental results

Open Loop Control——Plant

In the experiment, we assemble the plant circuit with two capacitors, two fixed-resistance resistor and an operational amplifier. The resistance of the resistor is $10k\Omega$, and the capacitance of the capacitor is $100\mu F$ and $0.22\mu F$. Note that the output signal is the output voltage of the op-amp.

(i) Impulse response of input signal with amplitude:1V, width:0.1s, frequency:1Hz

We set the function generator to impulse function with $A = 1V, width = 0.1s, f = 1Hz$ and observe the output signal.

According to the experiment, we get the input and output signals in the oscillator. The following picture should be what the result looks like. (Due to some technical issues, we lost our picture but TAs can verify that we got the correct result. This picture is from other group.)

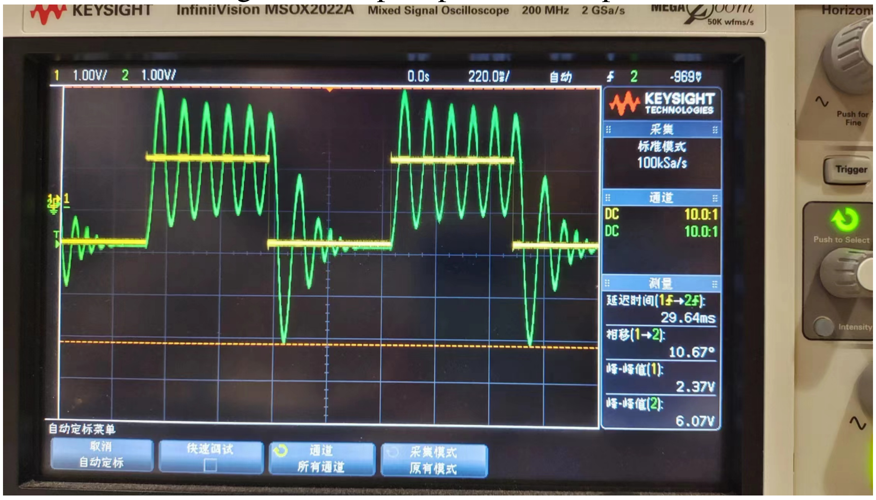
Figure 1: Impulse response of the plant

(ii) Step response of input signal with amplitude:1V, frequency:1Hz

We set the function generator to step function with $A = 1V, f = 1Hz$ and observe the output signal.

According to the experiment, we get the input and output signals in the oscillator as shown in the following picture.

Figure 2: Step response of the plant



Feedback Control circuit

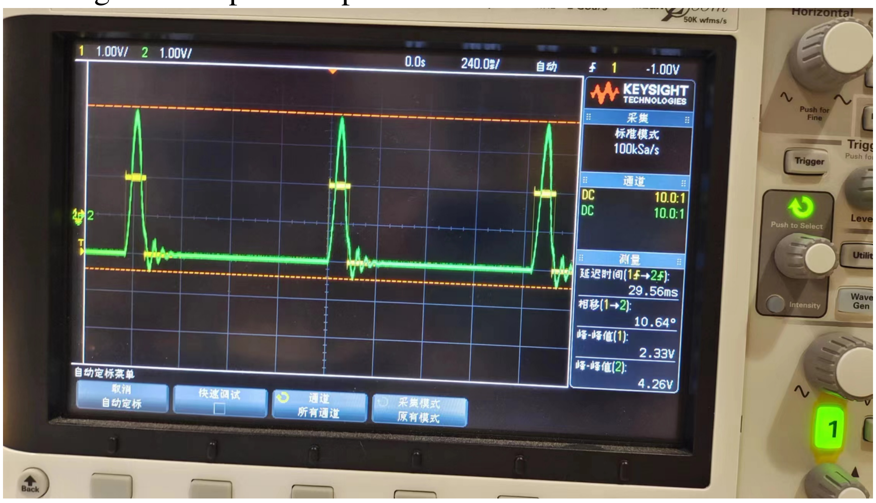
We add the feedback control circuit to the plant with $R_1 = R_3 = 150k\Omega, R_2 = 3k\Omega, C_3 = 0.47\mu F$.

(i) Impulse response of input signal with amplitude:1V, width:0.1s, frequency:1Hz

We set the function generator to impulse function with $A = 1V, width = 0.1s, f = 1Hz$ and observe the output signal.

According to the experiment, we get the input and output signals in the oscillator as shown in the following picture.

Figure 3: Impulse response of the feedback control circuit

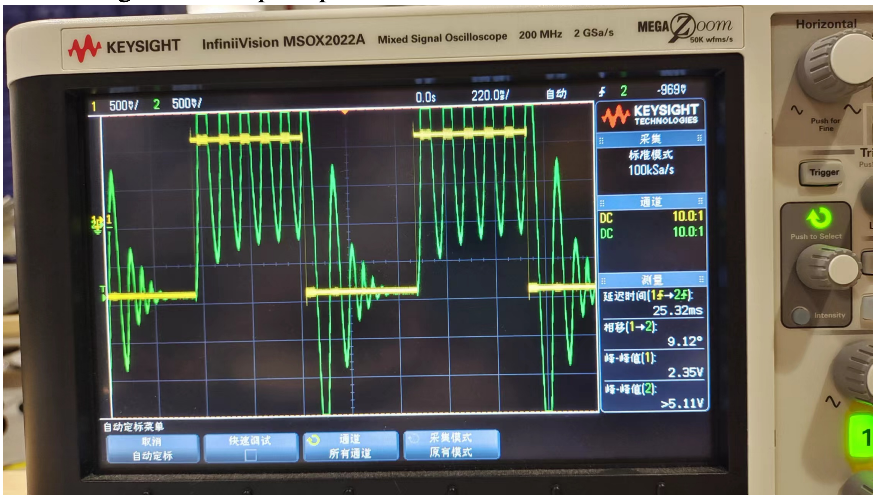


(ii) Step response of input signal with amplitude:1V, frequency:1Hz

We set the function generator to step function with $A = 1V, f = 1Hz$ and observe the output signal.

According to the experiment, we get the input and output signals in the oscillator as shown in the following picture.

Figure 4: Step response of the feedback control circuit



Error analysis and discussion

Part1: As shown in the pre-lab assignment, the transfer function of the plant is given by the following expression:

$$P(s) = \frac{a_1}{s^2 + a_2 s + a_3} \tag{1}$$

Where

$$a_1 = \frac{1}{R^2 C_1 C_2} \tag{2}$$

$$a_2 = \frac{2}{RC_1} \tag{3}$$

$$a_3 = \frac{1}{R^2 C_1 C_2} \tag{4}$$

By converting the values into SI units and plugging in the values, the numerical expression for the transfer function is as follow:

$$P(s) = \frac{455}{s^2 + 2s + 455} \tag{5}$$

Using Matlab, we get the following plot for the impulse response and the step response:

Figure 5: Impulse response of the plant (Matlab)

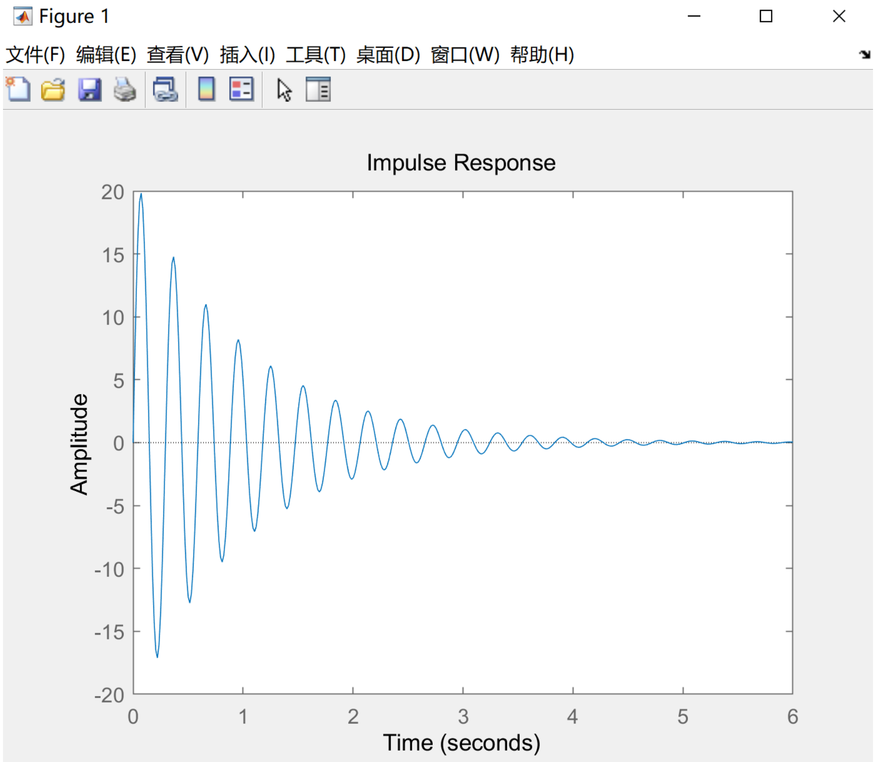
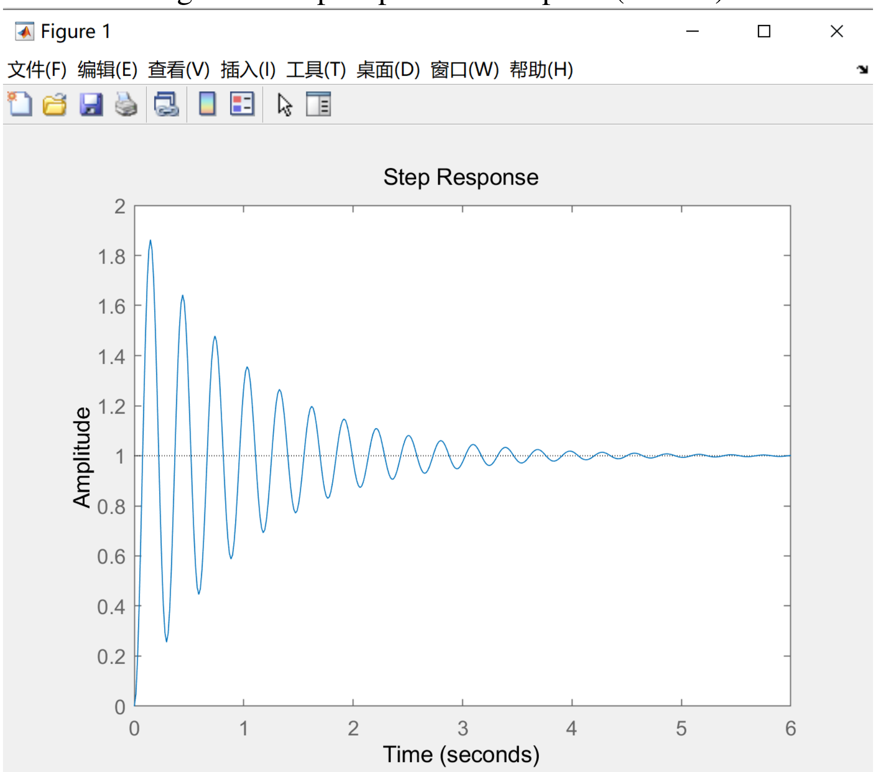


Figure 6: Step response of the plant (Matlab)



The signal's shape is generally consistent with the ideal shape. We can observe that after an impulse signal enters the system, the system's response has the similar shape as what is generated in Matlab. In particular, the amplitude of the wave becomes smaller and smaller as time goes by, which is also consistent with the theory. The same features also apply to the step response. Thus, we can conclude that the error of part1 is not significant. The approach of using the open loop control is very simple. The disadvantage is also obvious: It is not as stable as the feedback control and it comes with the problem of saturation.

Part2: As shown in the pre-lab assignment, the following expression holds:

$$W(s) = K_p(X(s) - Y(s)) - K_D sY(s) \tag{6}$$

where W(s) is the input signal of the plant. Therefore,

$$\frac{Y(s)}{P(s)} = K_p(X(s) - Y(s)) - K_D sY(s) \tag{7}$$

Simplify the equation and the expression of the total transfer function of the PD controller is as follow:

$$H(s) = \frac{Y(s)}{X(s)} \quad = \frac{K_p}{\frac{1}{P(s)} + K_p + K_D s} \tag{8}$$

The constants K_p and K_D can be expressed in terms of R_2, R_3, and C_3.

$$K_p = \frac{R_3}{R_2} \tag{9}$$

$$K_D = R_3 C_3 \tag{10}$$

By converting the values into SI units and plugging in the values, the values of the constants is as follow:

$$K_p = \frac{R_3}{R_2} \quad = 50 \tag{11}$$

$$K_D = R_3 C_3 \quad = 0.0705 \tag{12}$$

Therefore, the transfer function of the whole system is

$$H(s) = \frac{K_p}{\frac{1}{P(s)} + K_p + K_D s} \quad = \frac{50}{\frac{s^2+2s+455}{455} + 50 + 0.0705s} \tag{13}$$

$$H(s) = \frac{22750}{s^2 + 34.078s + 23205} \tag{14}$$

Using Matlab, we get the following plot for the impulse response and the step response:

Figure 7: Impulse response of the feedback control system (Matlab)

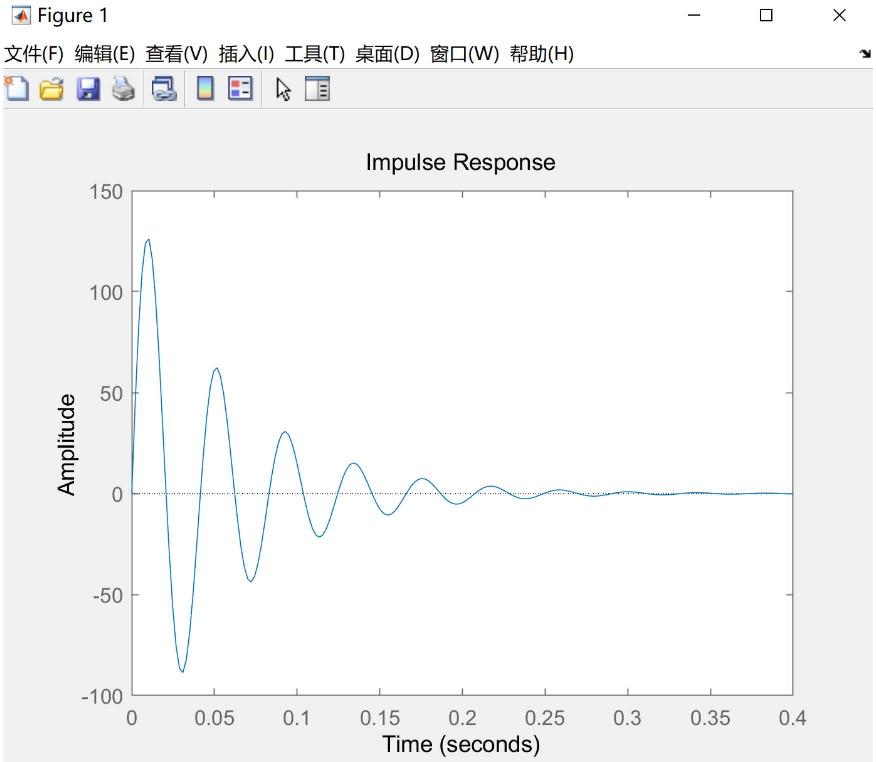
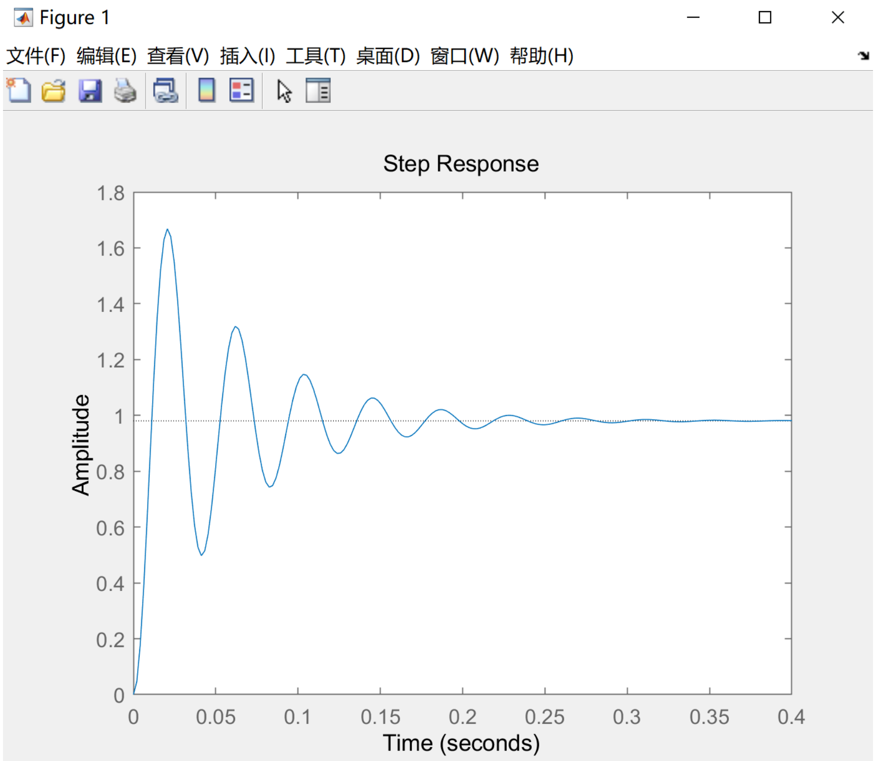


Figure 8: Step response of the feedback control system (Matlab)



The output signal's shape also shares lots of similarities with the theoretical shape. Take the step response for example. If we take the step response of the feedback control circuit and compare it with the step response of the plant, we can find out that the output signal of the step response of the feedback control circuit changes more drastically, or "converges" quicker. This is because usually, using the feedback control has the advantage of stabilizing the circuits. In our laboratory, since the plant is already stable (with both poles situated in the LHP, real parts equal to -1), adding the features of feedback control becomes less significant, but it could be observe if a closer look is taken. (The poles of the feedback control system is both in the LHP as well, with real parts equal to -17, hence more stable.) The approach of using the feedback control is more complicated, but it also comes with the advantage of making the system more stable and avoids the problem of saturation.

Conclusion

In this experiment, we have measure responses of our plant and feedback control system to different input signals. The experimental results are compared with the theoretical calculations. The results are in general similar and the errors are not too large. Possible reasons for the error could be due to the fact that the input signal is a train of impulse and step functions, so the response may be influenced by previous signals. We have discovered the advantage of the open loop circuit is that it is simple to build, and by using the feedback control we can not only avoid the problem of saturation, but also the system is more stable. We believe this experiment is of high practical importance. For real life applications, the feedback control is very important, in a way that our input usually needs to be based on the outputs and responses of the environment.

References

[1]Yong, Long. *Ve216 Signals and Systems Lab 3 Manual*. UM-SJTU Joint Institute, 2023.

[2]Kim, Winick. *VE 216 Spring 2023 Lab 3: Feedback Control (Part I: Intro Pre-lab Assignment)*. University of Michigan, 2008.