VE216 Recitation Class 2

Chapter 1&2

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Transformations

- Folding/ Reflection/ Time-reversal $y(t) = x(-t)$
- Time-scaling $y(t) = x(at)$
- Time-shifting $y(t) = x(t - t_0)$
- Amplitude-reversal $y(t) = -x(t)$
- Amplitude-scaling $y(t) = ax(t)$
- Amplitude-shifting $y(t) = x(t) + b$

$$y(t) = x(at - b) = x(\tfrac{t-t_0}{w})$$

Signal Characteristics

- Period T: x(t+T) = x(t), $T > 0$,for any t
  - Fundamental period T_0: smallest period
  - Sum of two periodic signals is periodic $\Leftrightarrow$ $\frac{T_1}{T_2}$ is rational
- Average value: $A = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)dt$
- Energy: $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$
  - Energy signal: E $< \infty$
- Average power: $P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$
  - Power signal: E$=\infty$ & $P < \infty$ & $P \neq 0$

Singularity Functions

- u(t)=$\begin{cases} 1, t > 0 \\ 0, t < 0 \end{cases}$
  - u(t-t_0) turns on at t=t_0
- rect(t)=$\begin{cases} 1, -\frac{1}{2} < t < \frac{1}{2} \\ 0, otherwise \end{cases}$
  - rect($\frac{t-t_0}{T}$): centered at t_0 with width T
- $\delta(t)$: zero width & infinite height
  - Sampling: $x(t) \cdot \delta(t-t_0) = x(t_0) \cdot \delta(t-t_0)$
  - Convolution: $x(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} x(t-\tau) \cdot \delta(\tau - t_0) d\tau = x(t-t_0)$
  - Shifting: $\int_{-\infty}^{\infty} x(t) \cdot \delta(t-t_0) dt = x(t_0)$
  - $\delta(t) = \frac{d}{dt} u(t)$, $u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$
  - Unit Area: $\int_{-\infty}^{\infty} \delta(t-t_0) dt = 1$, for any t_0
  - Scaling: $\delta(at+b) = \frac{1}{|a|} \delta(t + \frac{b}{a})$, for any $a \neq 0$
  - Algebaic: $t \cdot \delta(t) = 0$

System Characteriscs

- Linearity: $T[a_1x_1(t) + a_2x_2(t)] = a_1T[x_1(t)] + a_2T[x_2(t)]$
  1. $x_1(t) \rightarrow y_1(t)$ & $x_2(t) \rightarrow y_2(t)$
  2. $a_1x_1(t) + a_2x_2(t) \rightarrow y(t)$
  3. y(t) vs. $a_1y_1(t) + a_2y_2(t)$
- Stability
  1. Assume there exists M_x s.t. $|x(t)| <= M_x < \infty$
  2. Substitute in y to see whether y is bounded
- Causal:Depends only on present and past
- Memoryless: Depends only on present
  - Memoryless $\rightarrow$ Causal
- Time-Invariance
  1. Find $y(t - t_0)$ by replacing every t in y with t-t_0
  2. Find the output y_d when input is $x_d = x(t - t_0)$
  3. $y(t - t_0)$ vs. y_d

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Impulse Response

- The output of the system when the input is delta function.
- $y(t)=x(t)*h(t) \leftrightarrow$ LTI system
- An LTI system is completely characterized by h(t).

LTI System Properties & h(t)

- Causality $\leftrightarrow$ h(t)=0 when t$<$ 0
- Static/Memoryless $\leftrightarrow$ h(t)=a$\cdot\delta$(t)
- BIBO Stable $\leftrightarrow$ $\int_{-\infty}^{\infty}|h(t)|dt < \infty$
- Intertibility $\leftrightarrow$ h(t)$*h_i(t)$=δ(t)

Step Response

- The output of the system when input is step function u(t)
- $h(t) = \frac{d}{dt} s(t)$

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Convolution Properties

- Commutative: $h(t) * x(t) = x(t) * h(t)$
- Associative: $[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)]$
- Distributive: $x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$
- Delay: $x(t) * \delta(t - t_0) = x(t - t_0)$
- Integration: $[\frac{d}{dt}x(t)] * h(t) = x(t) * [\frac{d}{dt}h(t)]$
- $y(at) = (h * x)(at) \neq h(at) * x(at)$

How to Proceed

Discuss Piecewise!

Example

$rect(\frac{t-2}{3}) * rect(\frac{t+1}{4})$