VE216 Recitation Class 1

Chapter 1

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Definition of Signals & Systems

- Signal is a function of time: x(t)
- System is a function of time AND the input x(t): F(t,x(t))

Examples

Amplifier applied at t=1 on a singer's voice which starts at t=0.

Time-Transformation

CHANGE OF VARIABLES!

- Folding/ Reflection/ Time-reversal $y(t) = x(-t)$
- Time-scaling $y(t) = x(at)$
- Time-shifting $y(t) = x(t - t_0)$

$$y(t) = x(at - b) = x(\frac{t - t_0}{w})$$

Example

$x(t) = t, t > 0$

- y(t)=x(-t)
- y(t)=x(2t)
- y(t)=x(t-2)

Amplitude-Transformation

- Amplitude-reversal $y(t) = -x(t)$
- Amplitude-scaling $y(t) = ax(t)$
- Amplitude-shifting $y(t) = x(t) + b$

Calculus

- Differentiator y=$\frac{d}{dt}$x(t)
- Integrator y=$\int_{-\infty}^{t} x(\tau)d\tau$

Two-Signal Operations

- Sum $y(t){=}x_1(t) + x_2(t)$
- Product $y(t){=}x_1(t) \cdot x_2(t)$

Signal Characteristics

- Period T: x(t+T) = x(t), $T > 0$,for any t
  - Fundamental period T_0: smallest period
  - Sum of two periodic signals is periodic $\Leftrightarrow$ $\frac{T_1}{T_2}$ is rational
- Even/Odd Symmetry: x(t)=x(-t)/ x(t)=-x(-t)
- Average value: $A = \lim_{x\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)dt$
- Energy: $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$
  - Energy signal: E $< \infty$
- Average power: $P = \lim_{x\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$
  - Power signal: E=∞ & $P < \infty$ & $P \neq 0$

Examples

- x(t)=sin(t)
- x(t)=u(t)$\cdot e^{-t+1}$

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Unit Step Function

- u(t)=$\begin{cases} 1, t > 0 \\ 0, t < 0 \end{cases}$

Caution

EDGE DOES NOT MATTER

Rect(angle) Function

- rect(t)= $\begin{cases} 1, -\frac{1}{2} < t < \frac{1}{2} \\ 0, \textit{otherwise} \end{cases}$
  - rect(t)=u(t+$\frac{1}{2}$)-u(t-$\frac{1}{2}$)
  - rect($\frac{t-t_0}{T}$): centered at t_0 with width T

Examples

- rect($\frac{t-1}{2}$)
- rect($\frac{t}{4} - \frac{3}{2}$)

Unit Impulse function

- $\delta(t)$: zero width & infinite height
  - Sampling: $x(t) \cdot \delta(t - t_0) = x(t_0) \cdot \delta(t - t_0)$
  - Convolution: $x(t) * \delta(t - t_0) = \int_{-\infty}^{\infty} x(t - \tau) \cdot \delta(\tau - t_0) d\tau = x(t - t_0)$
  - Shifting: $\int_{-\infty}^{\infty} x(t) \cdot \delta(t - t_0) dt = x(t_0)$
  - $\delta(t) = \frac{d}{dt} u(t)$, $u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$
  - Unit Area: $\int_{-\infty}^{\infty} \delta(t - t_0) dt = 1$, for any t_0
  - Scaling: $\delta(at + b) = \frac{1}{|a|} \delta(t + \frac{b}{a})$, for any $a \neq 0$
  - Symmetry: $\delta(t) = \delta(-t)$
  - Algebaic: $t \cdot \delta(t) = 0$

Examples

Evaluate the following

- $\int_{-\infty}^{\infty} sin(t) \cdot \delta(t - \pi) dt$
- $\frac{d}{dt} [e^{-3t} \cdot u(t)] |_{t=2}$

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Linearity

$$T[a_1x_1(t) + a_2x_2(t)] = a_1T[x_1(t)] + a_2T[x_2(t)]$$

Skill

1. $x_1(t) \to y_1(t)$ & $x_2(t) \to y_2(t)$
2. $a_1x_1(t) + a_2x_2(t) \to y(t)$
3. y(t) vs. $a_1y_1(t) + a_2y_2(t)$

Example

y(t)=$\frac{x(t+1)}{x(t-1)}$

Stability

Skill

1. Assume there exists M_x s.t. $|x(t)| <= M_x < \infty$
2. Substitute in y to see whether y is bounded

Example

y(t)=$\int_t^{t+T} x(\tau) d\tau$

Causality & Memory

- Causal:Depends only on present and past
- Memory: Depends only on present
  - Memoryless $\rightarrow$ Causal

Example

- $y = x(\cos(t + \frac{\pi}{4}))$

Time-Invariance

Skill

1. Find $y(t - t_0)$ by replacing every t in y with t-t_0
2. Find the output y_d when input is $x_d = x(t - t_0)$
3. $y(t - t_0)$ vs. y_d

Examples

- y(t)=(t-2)·x(t)
- y(t)=2·x(3t)
- y(t)=5·x(t-1)