1. (a) [10] Perform partial fraction expansion on a system's frequency response

$$H(j\omega) = \frac{2j\omega + 1}{-j\omega^3 - 5\omega^2 + 8j\omega + 4}.$$

Then use Matlab **residue** to verify your result. Be careful with the interpretation of Matlab outputs in the case of repeated pole(s).

(b) [10] Based on (a), find the unit impulse response $h(t)$ of this system and use Matlab to plot your answer as a function of t. Then use Matlab **impulse** to generate a same plot and verify your result.

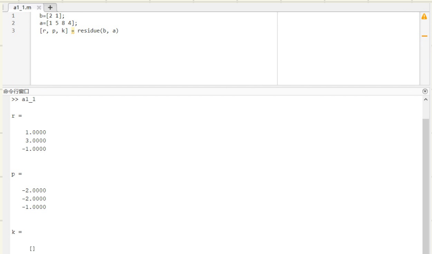
(a)

Letting $s = j\omega$, we have

$$H(s) = \frac{2s+1}{s^3+5s^2+8s+4}$$

$$= \frac{1}{s+2} + \frac{3}{(s+2)^2} - \frac{1}{s+1}$$

$\Rightarrow$



```
a1_1.m
1   b=[2 1];
2   a=[1 5 8 4];
3   [r, p, k] = residue(b, a)

命令行窗口
>> a1_1

r =

    1.0000
    3.0000
   -1.0000


p =

   -2.0000
   -2.0000
   -1.0000


k =

     []
```

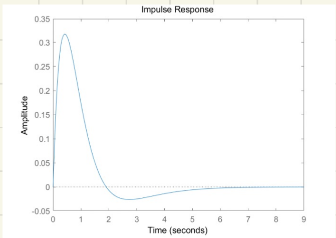
$\Rightarrow$ In the case of repeated holes, p is ordered in Ak. Therefore, it's exactly our result.

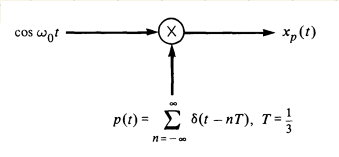
(b)

$$e^{-at}u(t) \leftrightarrow \frac{1}{j\omega + a},$$

$$te^{-at}u(t) \leftrightarrow \frac{1}{(j\omega+a)^2},$$

$$\Rightarrow h(t) = (3t+1)e^{-2t}u(t) - e^{-t}u(t)$$





(a) [10] Sketch $X_p(\omega)$ for $-9\pi \le \omega \le 9\pi$ for the following values of ω_0.

i. $\omega_0 = \pi$
ii. $\omega_0 = 2\pi$
iii. $\omega_0 = 3\pi$
iv. $\omega_0 = 5\pi$

(b) [4] For which of the preceding values of ω_0 is $x_p(t)$ identical? For which of the preceding values of ω_0 will NOT be able to recover the input sinusoidal signal after lowpass filtering $x_p(t)$?

(a) The spectrum of $p(t)$: $P(\omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega - \frac{2\pi k}{T}) = 6\pi\sum_{k=-\infty}^{\infty}\delta(\omega - 6\pi k)$

And the spectrum of $\cos(\omega_0 t)$ is $\pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)$

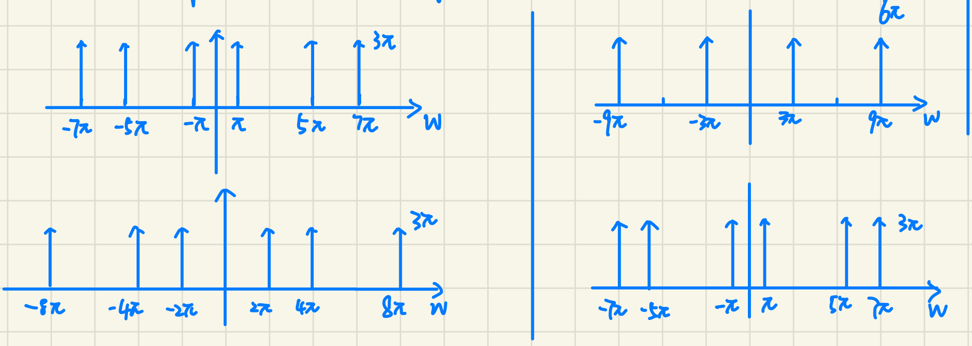
Hence, $X_p(\omega) = \frac{1}{2\pi}P(\omega) * [\pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)]$

and the four sketches are given below:

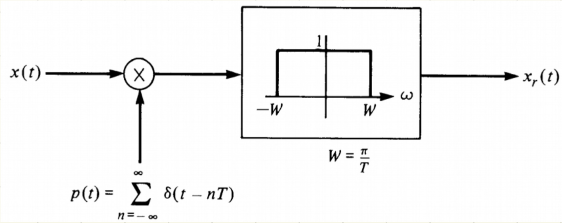
(b)

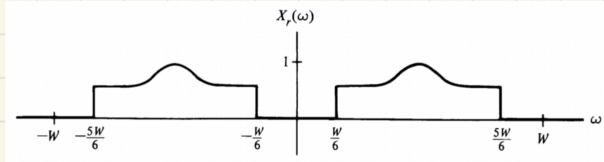
(i) and (iv) are identical.

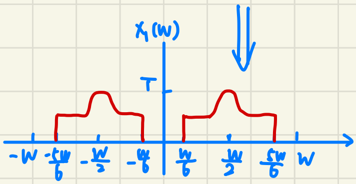
We will not be able to reconstruct (iv) from $x_p(t)$

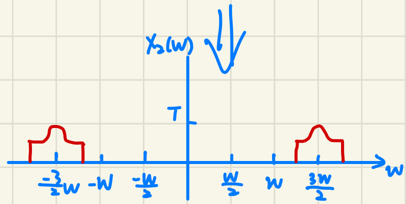


3. [12] Given the system in Figure 0503(a) and the Fourier transform of $x_r(t)$ in Figure 0503(b), sketch the Fourier transform of two different signals $x(t)$ that could have generated $x_r(t)$.









4. [16] Consider the system in Figure 0504.

If $X_1(\omega) = 0$ for $|\omega| > 2W$ and $X_2(\omega) = 0$ for $|\omega| > W$. For the following inputs $x(t)$, find the ranges for the cutoff frequency W_c in terms of T and W and find the maximum values of T and A, such that $x_r(t) = x(t)$.

(a) $x(t) = x_1(t - \pi/2) + x_2(t)$
(b) $x(t) = x_1(t)x_2(t)$
(c) $x(t) = dx_2(t)/dt$
(d) $x(t) = x_2(t)\cos(2Wt)$

(a) Time shift doesn't affect bandwidth.

$X(w) = 0$ for $|w| > 2W$. Hence. $T_{max} = \frac{\pi}{2W}$, $A = T$.

$2W < W_c < \frac{2\pi}{T} - 2W$

(b) $X(w) = 0$ for $|w| > 3W$. Hence, $T_{max} = \frac{\pi}{3W}$, $A = T$. $3W < W_c < \frac{2\pi}{T} - 3W$

(c) $X(w) = 0$ for $|w| > W$. Hence, $T_{max} = \frac{\pi}{W}$, $A = T$, $W < W_c < \frac{2\pi}{T} - W$

(d) $X(w) = \frac{1}{2}(X(w-2W) + X(w+2W))$. $X(w) = 0$ for $|w| > 3W$.

Hence, $T_{max} = \frac{\pi}{3W}$. $A = T$, $3W < W_c < \frac{2\pi}{T} - 3W$

5. [20] A signal $x(t)$ with spectrum $X(\omega) = \left(1 - \left|\frac{\omega}{2\pi}\right|\right) \mathrm{rect}\left(\frac{\omega}{4\pi}\right)$ is filtered by a system with frequency response $H(\omega) = \mathrm{rect}\left(\frac{\omega}{2\pi}\right)$. The resulting signal is then multiplied by $\cos(8\pi t)$. Finally, that signal is passed through an integrator system, yielding a signal $y(t)$. Find and sketch $Y(\omega)$.

After filtering. $X_1(w) = X(w)H(w) = (1 - |\frac{w}{2\pi}|)\,\mathrm{rect}(\frac{w}{4\pi})\,\mathrm{rect}(\frac{w}{2\pi}) = (1 - |\frac{w}{2\pi}|)\,\mathrm{rect}(\frac{w}{2\pi})$

After modulation. $X_2(w) = \frac{1}{2}(X_1(w+8\pi) + X_1(w-8\pi))$

Passing through an intergrator.

$$Y(w) = X_2(w) \cdot \frac{1}{jw} + \pi X_2(0) \cdot \delta(w) = \frac{1}{jw} X_2(w) = \frac{1}{2jw}(X_1(w+8\pi) + X_1(w-8\pi))$$

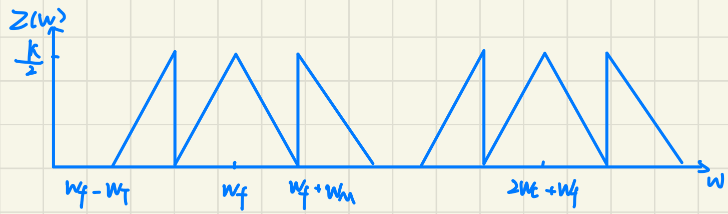
6. [18] The accurate demultiplexing-demodulation of radio and television signals is generally performed using a system called the superheterodyne receiver, which is equivalent to a tunable filter. The basic system is shown as Figure 0506.

(a) [6] The input signal $y(t)$ consists of the superposition of many amplitude-modulated signals that have been multiplexed using frequency-division multiplexing, so that each signal occupies a different frequency channel. Let us consider one such channel that contains the amplitude-modulated signal $y_1(t) = x_1(t)\cos\omega_c t$, with spectrum $Y_1(\omega)$ within $[\omega_c - \omega_M, \omega_c + \omega_M]$ as depicted below. We want to demultiplex and demodulate $y_1(t)$ to recover the modulating signal $x_1(t)$, using the system in the figure. The coarse tunable filter has the spectrum $H_1(\omega)$. Determine the spectrum $Z(\omega)$ of the input signal to the fixed selective filter $H_2(j\omega)$. Sketch and label $Z(\omega)$ for $\omega > 0$.

(b) [6] The fixed frequency-seletive filter is a bandpass type centered around the fixed frequency ω_f. We would like the output of the filter with spectrum $H_2(\omega)$ to be $r(t) = x_1(t)\cos\omega_f t$. In terms of ω_c and ω_M, what constraint must ω_T satisfy to guarantee that an undistorted spectrum of $x_1(t)$ is center around $\omega = \omega_f$?

(c) [6] What must G, α, and β be in the figure so that $r(t) = x_1(t)\cos\omega_f t$.

(a) The spectrum $Z(w)$



(b) From the figure above, we know that to avoid aliasing:

$$w_f + w_M \le 2w_c + w_f - w_T \Rightarrow w_T \le 2w_c - w_M$$

and in case $w_f - w_T$ is negative, we also have to make sure:

$$-w_f + w_T \le w_f - w_M \Rightarrow w_T \le 2w_f - w_M$$

(c)

$$G = \frac{2}{k}, \quad \alpha = w_f - w_M, \quad \beta = w_f + w_M$$