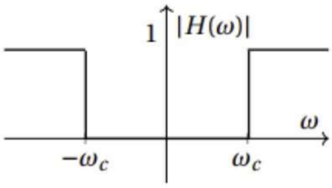
VE216 SU23 RC Chapter 6&7&8

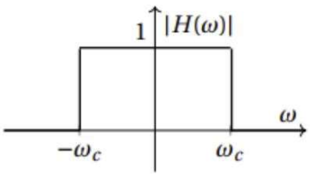
Chapter 6

Ideal Filters

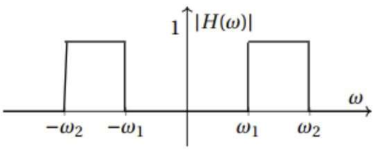
highpass filter



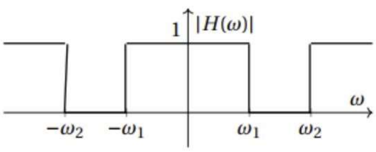
lowpass filter



bandpass filter



bandstop filter

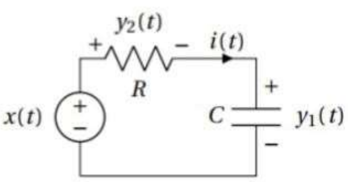


For those good looking ideal filter, they all cannot be implemented in real life as they are noncausal.
If we try to get the corresponding plot on time domain.
For example, we shall see one bandpass filter.

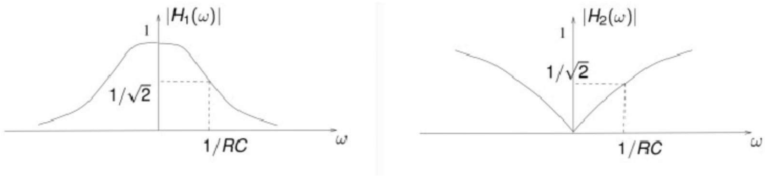
$$H(\omega) = \text{rect}\left(\frac{\omega}{2\omega_c}\right) \rightarrow h(t) = \frac{\omega_c}{\pi}\,\text{sinc}\left(\frac{\omega_c}{\pi}t\right)$$

Real Filter

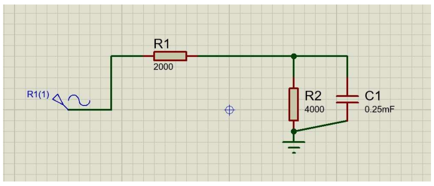
In real life, we will use some circuits as real filters.
For the following basic RC circuit,



the frequency response of $y_1(t)$ and $y_2(t)$ are



It is easy to tell that they are lowpass filter and highpass filter respectively.



Still remember this? What's the filter type of $|H(w)| = |\frac{I_C(w)}{V_{in}(w)}|$?

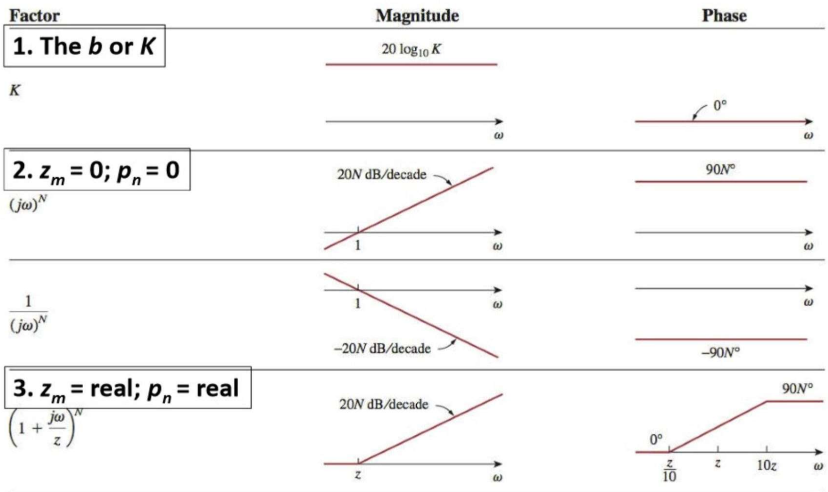
Bode plots

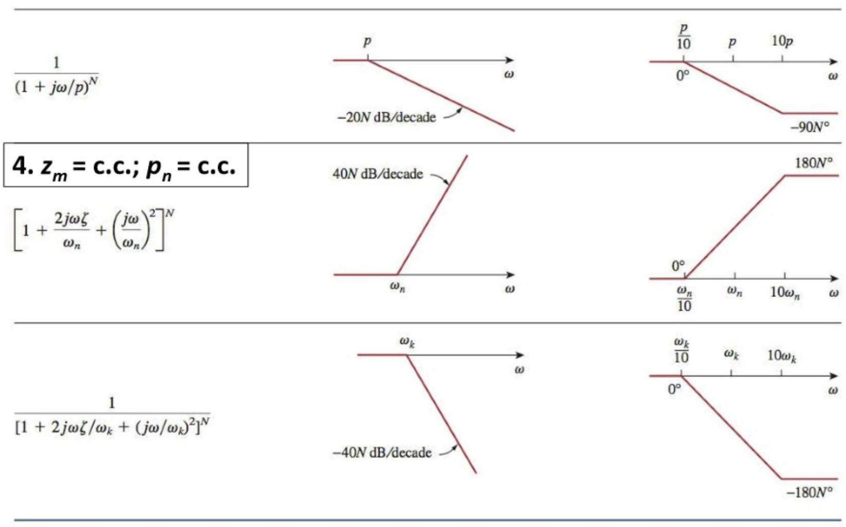
Plots of $20log_{10}|H(w)|$ and $\angle H(w)$ versus $log_{10}(w)$are referred as bode plots. It is used to faciliate the plotting.

For one signal on the frequency domain, it is composed of magnitude and phase
X(w) = $|X(w)|e^{i\angle X(w)}$

With Bode plotS, a filter
$Y(w) = X(w)H(w) = |X(w)|e^{i\angle X(w)} \cdot |H(w)|e^{i\angle H(w)} = |X(w)H(w)|e^{i(\angle X(w)+\angle H(w))}$
can be transformed to
$20log_{10}|Y(w)| = 20log_{10}|X(w)| + 20log_{10}|H(w)|$
$\angle Y(w) = \angle X(w) + \angle H(w)$

In this way, for a signal with form
$Y(w) = \frac{b_M(jw)^M + b_{M-1}(jw)^{M-1} + \cdots + b_0}{a_N(jw)^N + a_{N-1}(jw)^{N-1} + \cdots + a_0}$
can be transformed into
$\frac{b_M}{a_N} \cdot \frac{(jw+z_1)(jw+z_2)\cdots(jw+z_M)}{(jw+p_1)(jw+p_2)\cdots(jw+p_N)}$
and
$20log_{10}|Y(w)| = 20log_{10}|\frac{b_M}{a_N}| + 20log_{10}|jw+z_1| + \cdots + 20log_{10}|jw+z_M| + 20log_{10}|\frac{1}{jw+p_1}| + \cdots + 20log_{10}|\frac{1}{jw+p_N}|$
$\angle(Y(w)) = \angle(\frac{b_M}{a_N}) + \angle(jw+z_1) + \cdots + \angle(jw+z_M) + \angle(\frac{1}{jw+p_1}) + \cdots + \angle(\frac{1}{jw+p_N})$
And all those small elements have their approximated plot.





Bandwidth Relationship

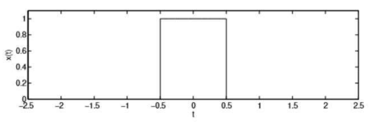
Root Mean Square(RMS) bandwidth

$$w_{rms} = \sqrt{\frac{\int_{-\infty}^{\infty} w^2|X(w)|^2 dw}{\int_{-\infty}^{\infty} |X(w)|^2 dw}}$$

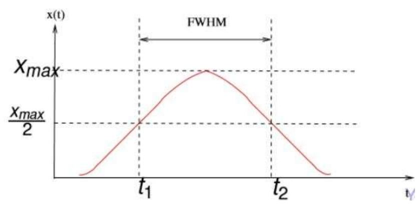
Time duration

Absolute time duration (time-limited)

$$\tau = t_2 - t_1.$$



Full-width at half maximum(FWHM)(non-time-limited)



Root mean-squared(RMS) time duration

$$\tau_{rms} = \sqrt{\frac{\int_{-\infty}^{\infty} t^2|x(t)|^2 dt}{\int_{-\infty}^{\infty} |x(t)|^2 dt}}$$

Time-Bandwidth product

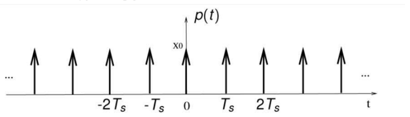
$$w_{rms}\tau_{rms} \geq \frac{1}{2}$$

Chapter7

FT of impulse-train sampled signals

Ideal sampling function

$$p(t) = \Sigma_{n=-\infty}^{\infty} \delta(t - nT_s)$$



Impulse-train sampling

$$x_s(t) = x(t)p(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t - nT_s) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s).$$

FT of sampled signal

With FT of ideal sampling function

$p(t) \overset{F}{\longleftrightarrow} P(w) = \Sigma_{k=-\infty}^{\infty}\frac{2\pi}{T_s}\delta(w - kw_s) \quad w_s = \frac{2\pi}{T_s}$

So we can derive the shape of the sampled signal on the frequency domain.

$$\begin{aligned} X_s(w) &= \frac{1}{2\pi}X(w) * P(w) = \frac{1}{2\pi}X(w) * \Sigma_{k=-\infty}^{\infty}\frac{2\pi}{T_s}\delta(t - kw_s) \\ &= \frac{1}{T_s}\Sigma_{k=-\infty}^{\infty}X(w - kw_s) \quad w_s = \frac{2\pi}{T_s} \end{aligned}$$

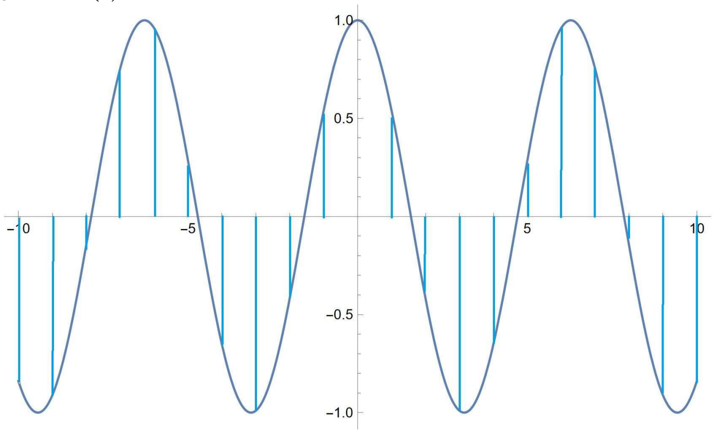
Sampling Theorem

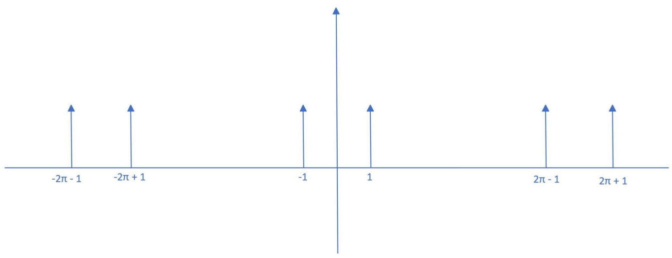
If $x(t)$ is a band-limited signal with $X(w) = 0$ for $|w| > w_{max}$, then the sampling interval T_s should meet the need that

$w_s > 2w_{max} \qquad w_s = \frac{2\pi}{T_s}$

to ensure that there is no overlape of $X(w - kw_s) \quad -\infty < k < \infty$ on frequency domain

$y = cos(t) \quad t_s = 1 \quad w_s = 2\pi$

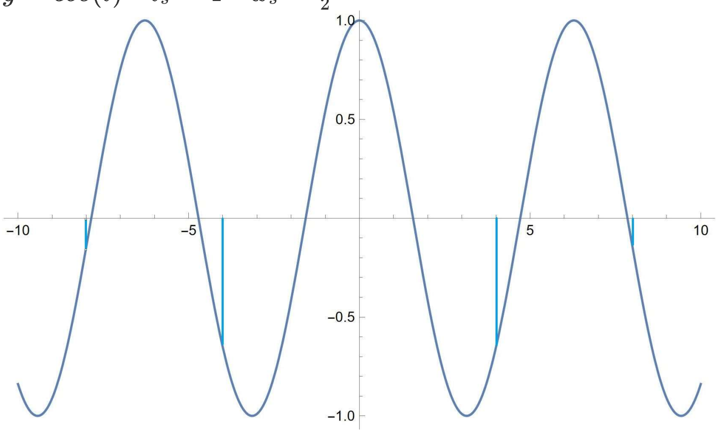


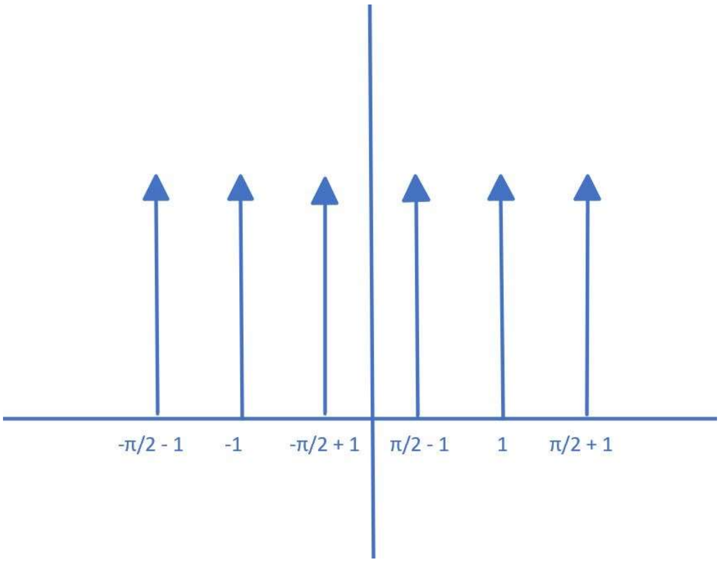


Aliasing

It will happen exactly when there are overlaps of $X(w - kw_s) \quad \infty < k < \infty$ on frequency domain.

$y = cos(t) \quad t_s = 4 \quad w_s = \frac{\pi}{2}$





Reconstruction via interpolation

Sinc interpolation (Ideal interpolation filter)

$$h(t) = \frac{\omega_c T_s}{\pi} \operatorname{sinc}\left(\frac{\omega_c t}{\pi}\right) \overset{\mathcal{F}}{\longleftrightarrow} H(\omega) = T_s \operatorname{rect}\left(\frac{\omega}{2\omega_c}\right)$$

$w_{max} < w_c < w_s - w_{max}$ and usually $w_c = \frac{w_s}{2}$

Linear interpolation (first-order hold filter)

$$h_1(t) = \operatorname{tri}(t/T_s) \overset{\mathcal{F}}{\longleftrightarrow} H_1(\omega) = T_s \operatorname{sinc}^2\left(\frac{\omega}{\omega_s}\right)$$

Nearest neighbor interpolation (zero-order hold filter)

$$h_2(t) = \operatorname{rect}\left(\frac{t}{T_s} - \frac{1}{2}\right) \overset{\mathcal{F}}{\longleftrightarrow} H_2(\omega) = T_s \operatorname{sinc}\left(\frac{\omega}{\omega_s}\right) e^{-j\omega T_s/2}$$

How to reconstruct

on time domain
$x_{recon}(t) = \Sigma_{n=-\infty}^{\infty} x_s[n] h_i(t - nT_s) \quad x[n] = x(nT_s)$
$\qquad\qquad = h_i(t) * x_s(t)$
and on frequency domain
$X_{recon}(w) = X_s(w) H_i(w)$

Chapter 8

Sinusoidal amplitude modulation

For a modulating signal $x(t)$, we modulate it with $c(t)$, called carrier signal. This method is called doublesideband, suppressed carrier, amplitude modulation or DSB/SC-AM.

$c(t)$ has the form of $cos(w_c t + \theta_c)$ and w_c is called the carrier frequency
$y(t) = x(t)c(t) = x(t)cos(w_c t + \theta_c)$
$\overset{F}{\longleftrightarrow}$
$Y(w) = \frac{1}{2}[e^{j\theta_c} X(w - w_c) + e^{-j\theta_c} X(w + w_c)]$
This faciliate the transmission of signal through antenna.

Synchronous Demodulation

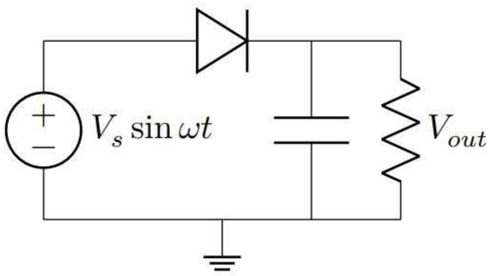
When we want to restore the signal above, apply one more $c(t)$ to $y(t)$
$w(t) = y(t)cos(w_c t + \theta_c)$
$W(w) = \frac{1}{4}[e^{2j\theta_c}X(w-2w_c) + 2X(w) + e^{-2j\theta_c}X(w+2w_c)]$
The $X(w)$ is right in the middle and a lowpass filter is enough to extract it.

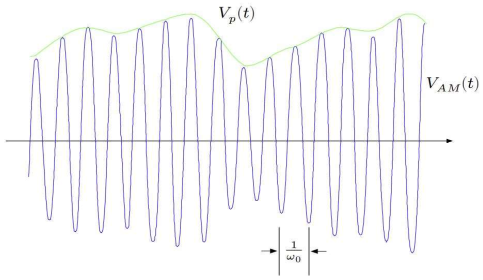
Asynchronous Demodulation

Transmitting some of the modulation signal is called double sideband, with carrier, amplitude modulation or DSB/WC-AM.

The modulated signal is
$y(t) = (A + x(t))cos(w_c t)$
and on the frequency domain
$Y(w) = A\pi[\delta(w-w_c) + \delta(w+w_c)] + \frac{1}{2}[X(w-w_c) + X(w+w_c)]$

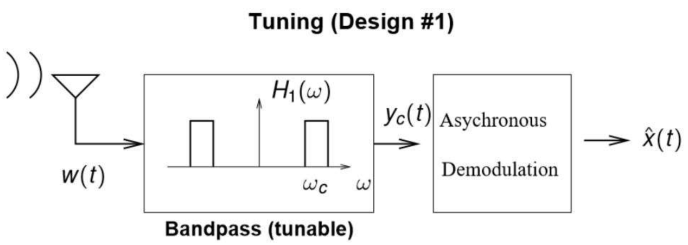
To demodulate this signal, an envelope tractor is used to extract the original signal $x(t)$

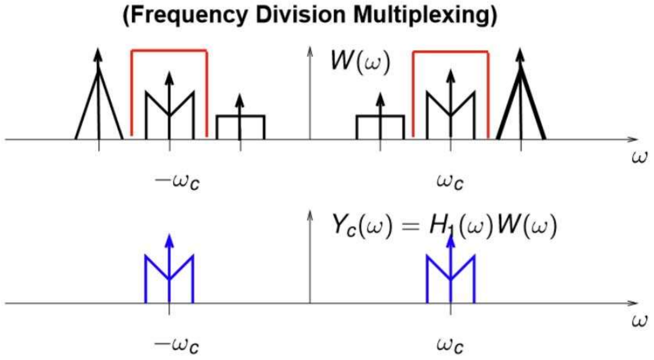




Frequency-division multiplexing

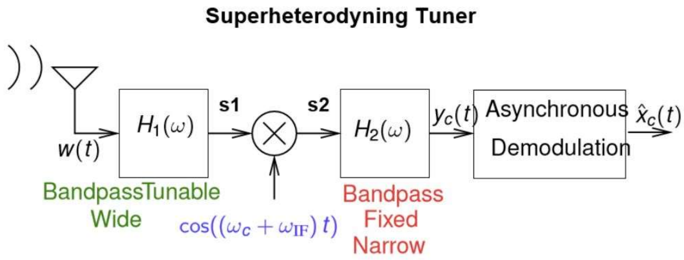
Tunable bandpass filter

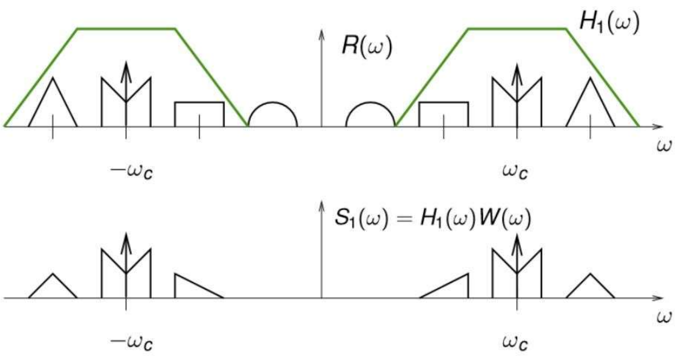




Feature: one narrow and tunable filter.

Superheterodyning receiver





Feature: one wide and tunable filter, shift the resulting signal to target frequency and filter it again with a narrow and fixed filter.

Reference

[1] ve216.chap6.study.pdf
[2] ve216.chap7.study.pdf
[3] ve216.chap8.study.pdf