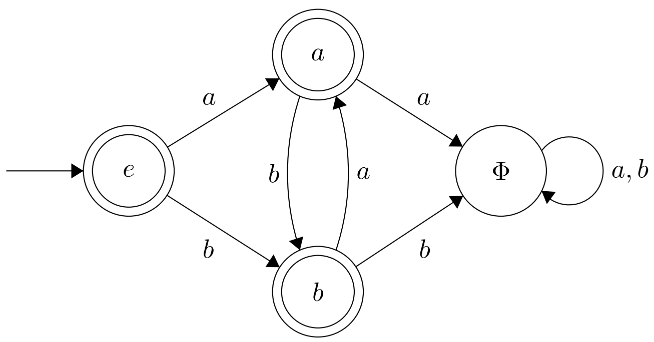
Foundations of Computation

Assignment 1

October 17, 2021

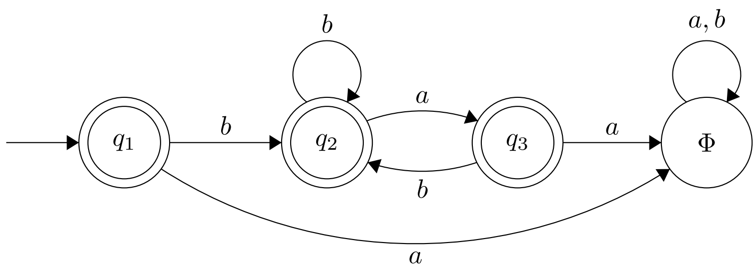
Q1: Construct deterministic finite automata which recognise each of the following languages. You should represent each automaton in the form of a diagram

1. $\{w \in \{a,b\}^* |\ w$ has neither aa nor bb as a substring $\}$

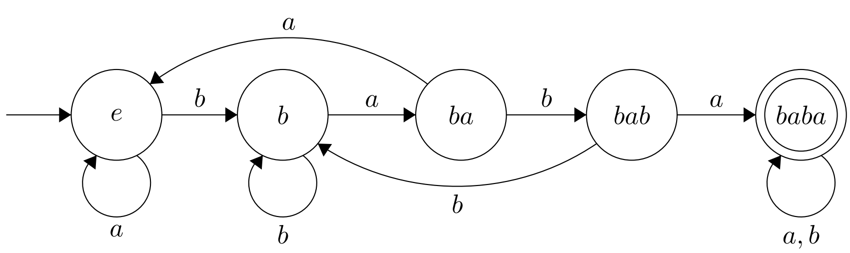


The states are labelled by the last letter. Here e means empty word.

2. $\{w \in \{a,b\}^* |$ each a in w is immediately preceded by at least one b $\}$

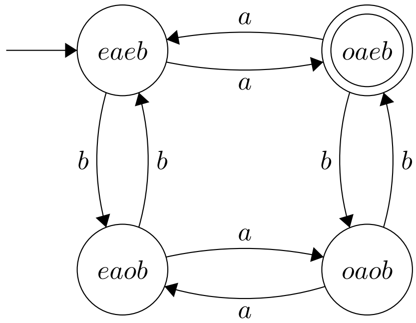


3. $\{w \in \{a,b\}^* |\ w$ has $baba$ as a substring $\}$



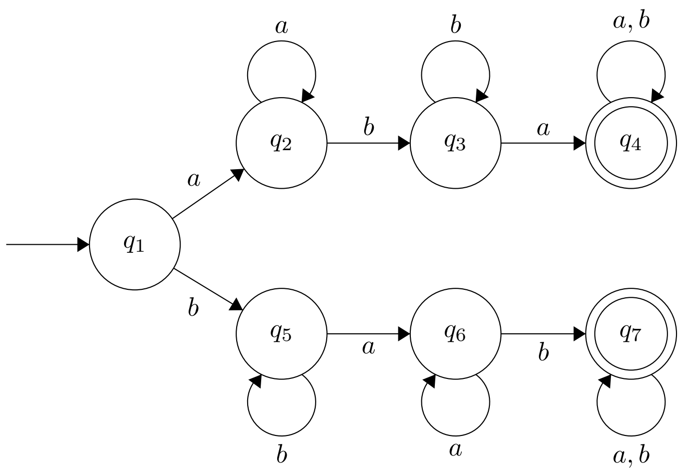
States are labelled by the parts found from the sought string $baba$

4. $\{w \in \{a,b\}^* |\ w$ has an odd number of a's and an even number of b's $\}$

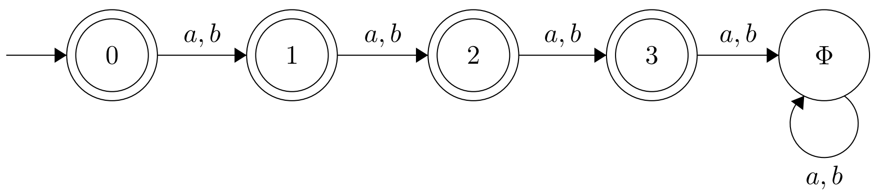


States are labelled by odd or even number's of a's and b's. e.g.for string that has odd a's and even b's, the state is denoted as $oaeb$ or oe.

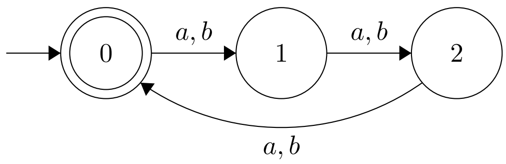
5. $\{w \in \{a,b\}^* |\ w$ has both ab and ba as substrings $\}$ Note: the string aba should be accepted.



6. $\{w \in \{a,b\}^* |\ 0 \leq |w| \leq 3\ \}$



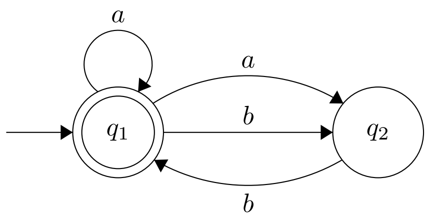
7. $\{w \in \{a,b\}^* |\ |w| = 3n, n = 0, 1, 2, ...\ \}$



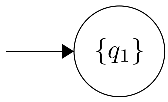
The states are labelled according to the remainder after dividing by 3.

Q2: Transform each of the following non-deterministic finite automata into equivalent deterministic finite automata. Represent the DFA in the form of a diagram, avoiding obsolete states which may appear in the process of transformation. Show your working.

1. The given NFA is the following:

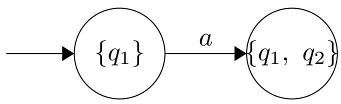


Step 0: the initial state of the new DFA is found by taking the initial state of the NFA, then applying any epsilon transition, and taking the union of these states. In this case, there is no epsilon transitions from the start state. So our initial state is $\{q_1\}$

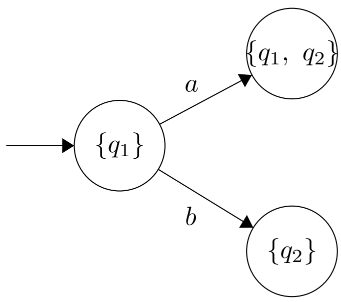


Step 1: $\{q_1\}$ only contains one state which is q_1. And q_1 has 2 transitions, which is a and b.

According to $E(\delta(q_1, a)) = \{q_1, q_2\}$, we will have the following:

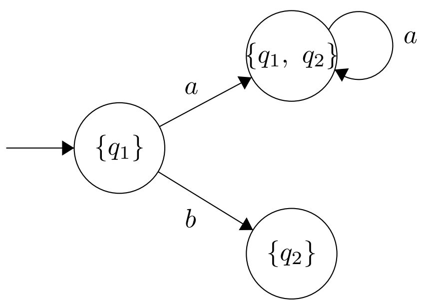


Based on $E(\delta(q_1, b)) = \{q_2\}$, we will have the following:

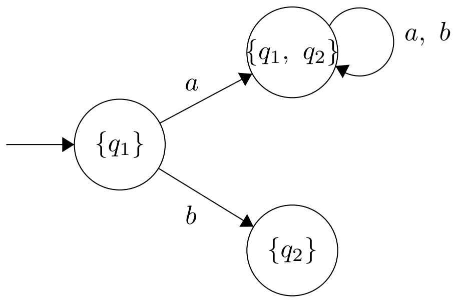


Step 2: All the states in the new DFA must have transitions for every letter. Now state $\{q_1\}$ has both transitions for a and b, we can move to the next state $\{q_1, q_2\}$.

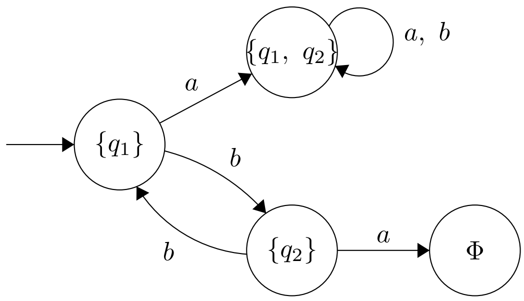
Based on $E(\delta(q_1, a)) = \{q_1, q_2\}$ and $E(\delta(q_2, a)) = \Phi$, we have:



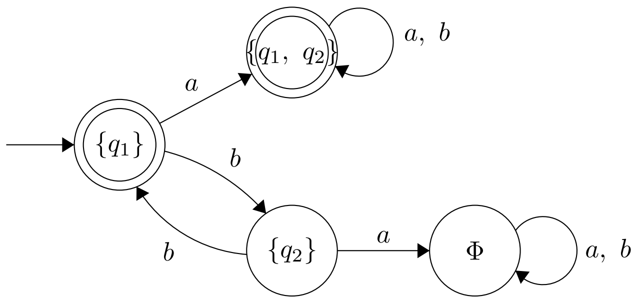
Based on $E(\delta(q_1, b)) = \{q_2\}$ and $E(\delta(q_2, b)) = \{q_1\}$, we have:



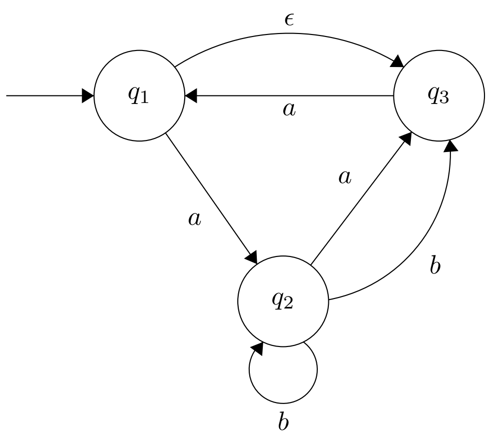
Step 3: Now we look at state $\{q_2\}$. We have $E(\delta(q_2, a)) = \Phi$ and $E(\delta(q_2, b)) = \{q_1\}$, therefore the diagram looks like:



Step 4: Check that all states in the new DFA have transitions for every letter. Then we mark the acceptable states for the new DFA, which should be all states that contains the original acceptable states from the NFA. In this case, the new accept states should be $\{q_1\}$ and $\{q_1, q_2\}$. The final new DFA should look like:

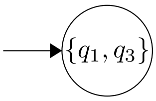


2. The given NFA is the following:

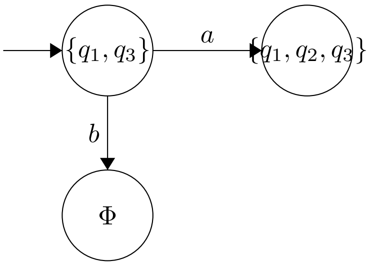


Step 0: The new set of states has to be the power set of the old set of states, so that's every subset of the original set of states. There are 8 possible states in this new DFA: empty set, $\{q_1\}$, $\{q_2\}$, $\{q_3\}$, $\{q_1, q_2\}$, $\{q_1, q_3\}$, $\{q_2, q_3\}$, $\{q_1, q_2, q_3\}$. The initial state of the new DFA

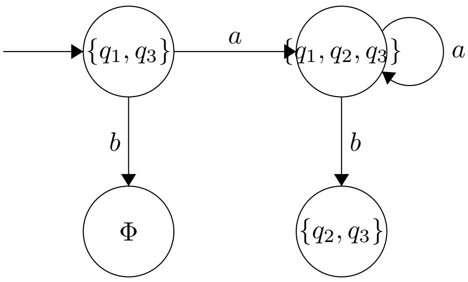
is found by taking the initial state of the NFA and then apply any epsilon transitions and taking the union of those states, so the initial state should be $\{q_1, q_3\}$



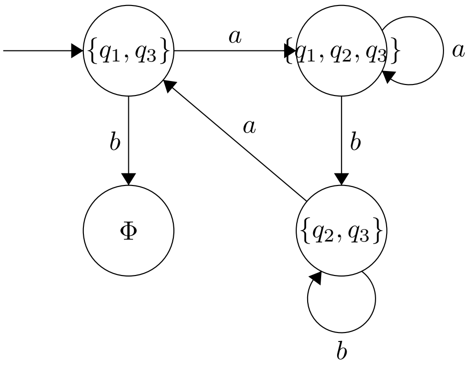
Step 1: So the $\{q_1, q_3\}$ contains 2 states which are $\{q_1\}$ and $\{q_3\}$. And $\{q_1\}$ has 2 transitions, one is an a, which takes it to $\{q_2\}$, and another one which is ϵ, which takes it to $\{q_3\}$. And $\{q_3\}$ has only 1 transition, which is an a, which takes it to $\{q_1\}$. More formally, $E(\delta(q_1, a)) = \{q_2\}$ and $E(\delta(q_1, \epsilon)) = \{q_3\}$ and $E(\delta(q_3, a)) = \{q_1\}$. The next question is what to do when there is not a transition for a particular letter. And we do need to have transitions for b from this first state. By looking at this diagram, we can see the original transition function must have looked like this: $E(\delta(q_1, b)) = \Phi$ and $E(\delta(q_3, b)) = \Phi$. So now the initial state is finished because we have 2 arrows coming out of it.



Step 1: Now let's look at state $\{q_1, q_2, q_3\}$. This state contains 3 states from the original NFA, so we have to look at their transitions. First let's look at transition for a. Let's consider $E(\delta(q_1, a))$, $E(\delta(q_2, a))$, $E(\delta(q_3, a))$ where q_1, q_2, q_3 are in this set $\{q_1, q_2, q_3\}$. So if we look at where we can get to by taking a transition from q_1, we can get to q_2, but we can also get to q_1 because we can take epsilon transition to q_3 and then an a transition to q_1, so that actually results in this set $\{q_1, q_2\}$. More formally, we have $E(\delta(q_1, a)) = \{q_1, q_2\}$. Following similar process for the other states, we have $E(\delta(q_2, a)) = \{q_3\}$ and $E(\delta(q_3, a)) = \{q_1, q_3\}$. Next let's look at transition for b. Similarly, we have $E(\delta(q_1, b)) = \Phi$, $E(\delta(q_2, b)) = \{q_2, q_3\}$ and $E(\delta(q_3, b)) = \Phi$. Now we are done with the state $\{q_1, q_2, q_3\}$.



Step 2: Now let's look at the transition from $\{q_2, q_3\}$. Now we have to consider the transitions that are possible from q_2 and q_3. Going through each letter, let's look at 'a' first. We have $E(\delta(q_2, a)) = \{q_3\}$ and $E(\delta(q_3, a)) = \{q_1, q_3\}$. Then let's look at b, we have $E(\delta(q_2, b)) = \{q_2, q_3\}$ and $E(\delta(q_3, b)) = \Phi$



Step 3: Now all the states in the new DFA must have transitions for every letter. It seems reasonable that we would just have all transitions from empty states going back to empty states. We don't have to worry about any of these other subsets because we can't reach any of them using transitions that are valid in the original NFA. The only thing left to do is to mark the accept states. And q_3 is the only acceptable state in the original NFA, so any state containing q_3 is now an accept state. The final DFA should be: