Mathematical and numerical modeling of ice-ocean interface

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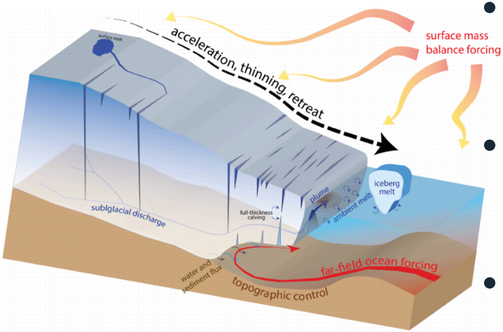
Introduction

- The Antarctic and Greenland ice sheets, glaciers are the important components of the global climate system.
- The global warming have caused the ocean to warm; as a result, its leads to mass loss in the global ice sheets, glaciers, and sea ice.
- The ice loss increased due to both increased surface melting and ice discharge to the ocean.

Ice-ocean interaction



Source: Catania et al. (2020)

- The mass balance of ice sheets, glaciers is determined by the interaction between ice and ocean.
- The region where that interaction occurs might include the base and front of the grounded ice.
- The basal ice melting is a significant component of mass loss from the Antarctic ice sheet and glaciers (Malyarenko et al., 2020).

Ice-ocean interaction

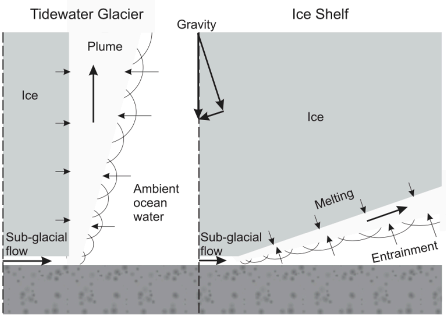
Credit: Henryk Sadura *Getty Images*

Source: (Scientific American) Why Are Glaciers Melting from the Bottom? It's Complicated

Ice-ocean interaction

- Poorly understood processes in ice-ocean interaction that could lead to rapid ice melting under future warming.
- Most melting estimates near the ice shelf grounding line have been based on observations of ice flux.
- Also, assumption of a steady-state, calculated melting or freezing and known surface melting.

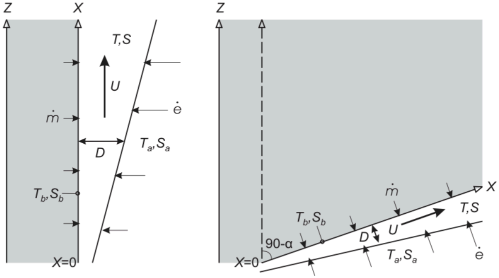
Plume dynamics



Source: Jenkins (2011)

- The meltwater from the ice sheets and glaciers generates buoyant plumes (Hewitt, 2020)
- The volume of plumes increases with height by the entrainment of fluid from the surroundings.
- The dynamics of these plumes are essential in regulating the heat transfer

Plume dynamics



Source: Jenkins (2011)

- Plume theory was initially developed by Morton et al. (1956)
- The model describes mass, momentum, heat, and salt conservation
- Prognostic variables: D, U, S, and T

Plume dynamics

- Wells and Worster (2008) also developed a detailed model for a plume rising along a vertical wall, suggesting two transitions.
- The first from a laminar state to a turbulent regime, and then to a second turbulent regime.
- Despite increasing applications on ocean forcing of glaciers, they have not been able to cover the region near the grounding line.

Plume dynamics

- These applications have failed to sample the ice-ocean boundary layer where the meltwater plume is expected to rise (Straneo et al., 2012).
- However, it is possible to better understand the dynamics at the ice-ocean boundary by studying the transformation of ocean waters by the glacier in conjunction with thermodynamic models of ice melt in saline waters (Holland and Jenkins, 1999).

Thermodynamic of ice–ocean interaction

- The aim of the ice-ocean interaction modeling is to obtain a melt rate, interface temperature, and salinity at the ice shelf base as realistic as possible.
- The determination of these characteristics are based on three physical constraints (Holland and Jenkins, 1999):
  - the interface must be at the freezing point,
  - the heat must be conserved at the interface during any phase change,
  - the slat must be conserved at the interface during any phase change.

Freezing point dependence

The relationship between the temperature and salinity at the ice-ocean interface, T_b and S_b is given by

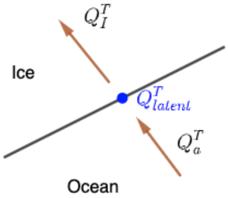
$$T_b = \lambda_1 S_b + \lambda_2 + \lambda_3 p_b \tag{1}$$

where p_b is the pressure at the interface and $\lambda_1, \lambda_2, \lambda_3$ are constants.

Conservation of heat

The heat conservation at the interface is

$$Q_i^T - Q_a^T = Q_{latent}^T.$$



The above equation becomes

$$\rho_i c_i \kappa_i^T \frac{\partial T_i}{\partial x}\bigg|_b - \rho_w c_a \kappa_a^T \frac{\partial T_a}{\partial x}\bigg|_b = \rho_a \dot{m} L_i. \tag{2}$$

where c_i, c_a are the specific heat capacities of ice and ocean; κ_i^T, κ_a^T are the thermal diffusivities of ice and ocean, ρ_i is the ice density, ρ_a is the density of the seawater.

Conservation of salinity

The salt conservation at the interface is given by

$$Q_i^S - Q_a^S = Q_{brine}^S.$$

Considering both Q_i^S and S_i negligible (Oerter et al., 1992; Gayen et al., 2016), the salt conservation at the interface becomes

$$\rho_a \kappa_a^s \frac{\partial S_a}{\partial x}\bigg|_b = \rho_a \dot{m} S_b. \tag{3}$$

where κ_a^S is the salinity diffusivity of the seawater.
The equations (2) and (3) represent the boundary conditions at the ice-ocean interface.

Three-equation formulation

The physical constraints

- $T_b = \lambda_1 S_b + \lambda_2 + \lambda_3 p_b$
- $\rho_i c_i \kappa_i^T \dfrac{\partial T_i}{\partial x}\bigg|_b - \rho_w c_a \kappa_a^T \dfrac{\partial T_a}{\partial x}\bigg|_b = \rho_a \dot{m} L_i$
- $\rho_a \kappa_a^s \dfrac{\partial S_a}{\partial x}\bigg|_b = \rho_a \dot{m} S_b$

The three-equation formulation is

- $T_b = \lambda_1 S_b + \lambda_2 + \lambda_3 p_b$
- $c_i \dot{m}\left(T_i - T_b\right) + c_a \gamma_T (T_a - T_b) = \dot{m} L_i$
- $\gamma_S (S_a - S_b) = \dot{m} S_b$

where γ_T and γ_S are temperature and salinity exchange velocity respectively.

Two-equation formulation

Although the three-equation formulation is commonly used in recent studies (eg. Gayen et al. (2016); Gwyther et al. (2020)); Jenkins et al. (2010) and Jenkins (2011) used the two equation formulation define by

$$T_f = \lambda_1 S + \lambda_2 + \lambda_3 p_b$$
$$c_i \dot{m} \left(T_i - T_f\right) + c_a c_d^{1/2} \gamma_{TS}(T - T_f) = \dot{m} L_i$$

where T_f is the freezing temperature of the plume, and $c_d^{1/2}\gamma_{TS}$ is called Stanton number.

Vertical ice face

- The melting rate depends not only on the thermal and salt diffusivities but also on the structure of the boundary layer.
- Kerr and McConnochie (2015) and Gayen et al. (2016) found that the melting rate is independent of the depth and plume velocity, and

$$\dot{m} = const \cdot (T_a - T_f)^{1.34},$$

  which is consistent with the theory prediction.
- For a vertical interface (McConnochie and Kerr, 2017), the transition to a shear regime occurs at a plume speed of 3-5 cm/s.

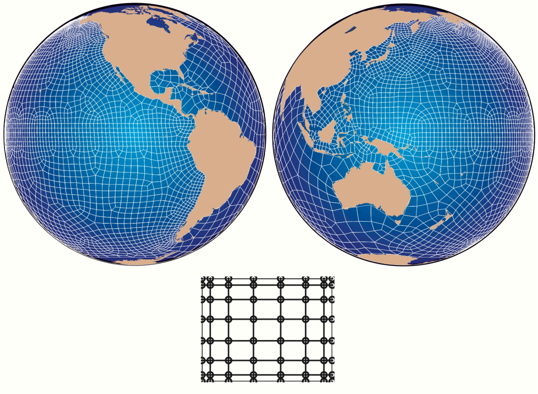
Numerical Models

- Numerical models that include the thermodynamic interaction between the ocean and the ice sheet are the best option for studying current and future Antarctic contributions to sea level.
- Among these models we have ROMS, MITgcm, FESOM, COCO, FVCOM, MPAS-O.
- The best current option is ice shelf-ocean models that neglect ice dynamics and assume steady-state ice geometry (Gwyther et al., 2020).

Numerical Models

- Given the non-homogeneity of ice-ocean interface, some of the current numerical models lack high-order algorithms.
- The spectral element method (SEM) approach has been used to address the problems of multi-scale simulations in complex geometrical region.

Ice-ocean interaction



Source: Iskandarani et al. (2002)

Limitations

- The three-equation formulation in some cases does not accurately resolve the melt rate (e.g. the observations of double diffusive, Kimura et al. (2015)).
- The wall assumption inherent in the three-equation formulation does not hold for flow at low rates (Davis and Nicholls, 2019) .
- The numerical models have not been able to cover the region near the grounding line in the region such as in the large vertical tidewater glaciers of Greenland, extends 5-10 km from the terminus.

Limitations

- The current models have not been able to resolve well the topography of the ice face.
- Plume models assume a vertical ice front, but real ice fronts have a more complicated geometry due to nonuniform submarine melting.

Future work possibilities

- Improve model resolution to well capture the plume at the interface.
- Prescribe different boundary conditions for the governing equations (e.g., Neumann or Robin).
- Numerical ice-ocean interaction models based on the spectral element method or the discontinuous Galerkin (DG) method.

Artifact

- We solve 1D-diffusion problem using the continuous Galerkin (CG) method CG-method.pdf.
- The method is implemented with three different boundary conditions, Dirichlet, Neumann, and Robin Artifact.

Thank you!

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