# Longest Alternating Subsequence

**We'll cover the following**

## Problem Statement #

Given a number sequence, find the length of its Longest Alternating Subsequence (LAS). A subsequence is considered alternating if its elements are in alternating order.

A three element sequence (a1, a2, a3) will be an alternating sequence if its elements hold one of the following conditions:

```
{a1 > a2 < a3 } or { a1 < a2 > a3}.
```

**Example 1:**

```
Input: {1,2,3,4}
Output: 2
Explanation: There are many LAS: {1,2}, {3,4}, {1,3}, {1,4}
```

**Example 2:**

```
Input: {3,2,1,4}
Output: 3
Explanation: The LAS are {3,2,4} and {2,1,4}.
```

**Example 3:**

```
Input: {1,3,2,4}
Output: 4
Explanation: The LAS is {1,3,2,4}.
```

## Basic Solution #

A basic brute-force solution could be to try finding the LAS starting from every number in both ascending and descending order. So for every index 'i' in the given array, we will have three options:

1. If the element at 'i' is bigger than the last element we considered, we include the element at 'i' and recursively process the remaining array to find the next element in descending order.
2. If the element at 'i' is smaller than the last element we considered, we include the element at 'i' and recursively process the remaining array to find the next element in ascending order.
3. In addition to the above two cases, we can always skip the element 'i' to recurse for the remaining array. This will ensure that we try all subsequences.

LAS would be the maximum of the above three subsequences.

Code #

Here is the code:

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```python
def find_LAS_length(nums):
  # we have to start with two recursive calls, one where we will consider that the first e
  # bigger than the second element and one where the first element is smaller than the sec
  return max(find_LAS_length_recursive(nums, -1, 0, True), find_LAS_length_recursive(nums,


def find_LAS_length_recursive(nums,  previousIndex,  currentIndex,  isAsc):
  if currentIndex == len(nums):
    return 0

  c1 = 0
  # if ascending, the next element should be bigger
  if isAsc:
    if previousIndex == -1 or nums[previousIndex] < nums[currentIndex]:
      c1 = 1 + find_LAS_length_recursive(nums, currentIndex, currentIndex + 1, not isAsc)
  else:  # if descending, the next element should be smaller
    if previousIndex == -1 or nums[previousIndex] > nums[currentIndex]:
      c1 = 1 + find_LAS_length_recursive(nums, currentIndex, currentIndex + 1, not isAsc)
  # skip the current element
  c2 = find_LAS_length_recursive(
    nums, previousIndex, currentIndex + 1, isAsc)
  return max(c1, c2)


def main():
  print(find_LAS_length([1, 2, 3, 4]))
  print(find_LAS_length([3, 2, 1, 4]))
  print(find_LAS_length([1, 3, 2, 4]))


main()
```

The time complexity of the above algorithm is exponential $O(2^n)$, where 'n' is the lengths of the input array. The space complexity is $O(n)$ which is used to store the recursion stack.

## Top-down Dynamic Programming with Memoization #

To overcome the overlapping subproblems, we can use an array to store the already solved subproblems.

The three changing values for our recursive function are the current and the previous indexes and the `isAsc` flag. Therefore, we can store the results of all subproblems in a three-dimensional array, where the third dimension will be of size two, to store the boolean flag `isAsc`. (Another alternative could be to use a hash-table whose key would be a string (currentIndex + "|" + previousIndex + "|" + isAsc)).

### Code #

Here is the code:

Java JS Python3 C++

```python
def find_LAS_length(nums):
  n = len(nums)
  dp = [[[-1 for _ in range(2)] for _ in range(n)] for _ in range(n)]
  return max(find_LAS_length_recursive(dp, nums, -1, 0, True),
             find_LAS_length_recursive(dp, nums, -1, 0, False))


def find_LAS_length_recursive(dp, nums, previousIndex, currentIndex,  isAsc):

  if currentIndex == len(nums):
    return 0

  if dp[previousIndex + 1][currentIndex][1 if isAsc else 0] == -1:
    c1 = 0
    # if ascending, the next element should be bigger
    if isAsc:
      if previousIndex == -1 or nums[previousIndex] < nums[currentIndex]:
        c1 = 1 + find_LAS_length_recursive(dp, nums, currentIndex, currentIndex + 1, not i
    else:  # if descending, the next element should be smaller
      if previousIndex == -1 or nums[previousIndex] > nums[currentIndex]:
        c1 = 1 + find_LAS_length_recursive(dp, nums, currentIndex, currentIndex + 1, not i

    # skip the current element
    c2 = find_LAS_length_recursive(
      dp, nums, previousIndex, currentIndex + 1, isAsc)
    dp[previousIndex + 1][currentIndex][1 if isAsc else 0] = max(c1, c2)

  return dp[previousIndex + 1][currentIndex][1 if isAsc else 0]


def main():
  print(find_LAS_length([1, 2, 3, 4]))
  print(find_LAS_length([3, 2, 1, 4]))
  print(find_LAS_length([1, 3, 2, 4]))


main()
```

## Bottom-up Dynamic Programming #

The above algorithm tells us three things:

1. We need to find an ascending and descending subsequence at every index.
2. While finding the next element in the ascending order, if the number at the current index is bigger than the number at the previous index, we increment the count for a LAS up to the current index. But if there is a bigger LAS without including the number at the current index, we take that.
3. Similarly for the descending order, if the number at the current index is smaller than the number at the previous index, we increment the count for a LAS up to the current index. But if there is a bigger LAS without including the number at the current index, we take that.

To find the largest LAS, we need to find all of the LAS for a number at index 'i' from all the previous numbers (i.e. number till index 'i-1').

We can use two arrays to store the length of LAS, one for ascending order and one for descending order. (Actually, we will use a two-dimensional array, where the second dimension will be of size two).

If 'i' represents the currentIndex and 'j' represents the previousIndex, our recursive formula would look like:

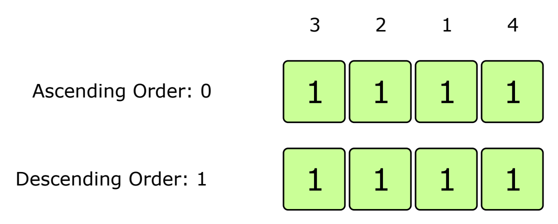
- If `nums[i]` is bigger than `nums[j]` then we will consider the LAS ending at 'j' where the last two elements were in descending order =>

```
    if num[i] > num[j] => dp[i][0] = 1 + dp[j][1], if there is no bigger LAS for
'i'
```

- If `nums[i]` is smaller than `nums[j]` then we will consider the LAS ending at 'j' where the last two elements were in ascending order =>

```
    if num[i] < num[j] => dp[i][1] = 1 + dp[j][0], if there is no bigger LAS for
'i'
```

Let's draw this visually for {3,2,1,4}. Start with a subsequence of length '1', as every number can be a LAS of length '1':



Every single element can be considered as LAS of length 1

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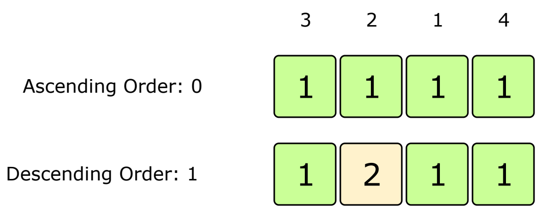

i=1, j=0 => since nums[i] < nums[j], so dp[i][1] = max(dp[i][1], 1 + dp[j][0])

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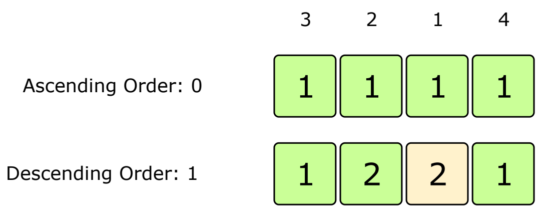

i=2, j=0 => since nums[i] < nums[j], so dp[i][1] = max(dp[i][1], 1 + dp[j][0])

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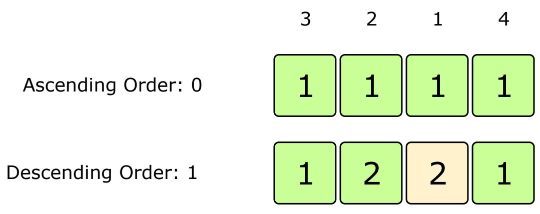

i=2, j=1 => since nums[i] < nums[j], so dp[i][0] = max(dp[i][0], 1 + dp[j][1])

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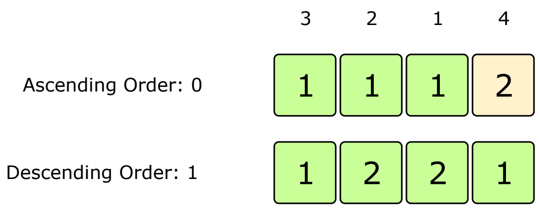

i=3, j=0 => since nums[i] > nums[j], so dp[i][0] = max(dp[i][0], 1 + dp[j][1])

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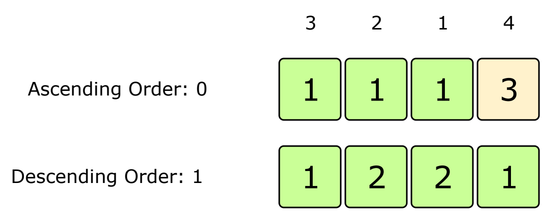

i=3, j=1-2 => since nums[i] > nums[j], so dp[i][0] = max(dp[i][0], 1 + dp[j][1])

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Here is the code for our bottom-up dynamic programming approach:

Java JS Python3 C++

```python
def find_LAS_length(nums):
  n = len(nums)
  if n == 0:
    return 0
  # dp[i][0] = stores the LAS ending at 'i' such that the last two elements are in ascendi
  # dp[i][1] = stores the LAS ending at 'i' such that the last two elements are in descend
  dp = [[0 for _ in range(2)] for _ in range(n)]
  maxLength = 1
  for i in range(n):
    # every single element can be considered as LAS of length 1
    dp[i][0] = dp[i][1] = 1
    for j in range(i):
      if nums[i] > nums[j]:
        # if nums[i] is BIGGER than nums[j] then we will consider the LAS ending at 'j' wh
        # last two elements were in DESCENDING order
        dp[i][0] = max(dp[i][0], 1 + dp[j][1])
        maxLength = max(maxLength, dp[i][0])
      elif nums[i] != nums[j]:  # if the numbers are equal don't do anything
        # if nums[i] is SMALLER than nums[j] then we will consider the LAS ending at
        # 'j' where the last two elements were in ASCENDING order
        dp[i][1] = max(dp[i][1], 1 + dp[j][0])
        maxLength = max(maxLength, dp[i][1])
  return maxLength


def main():
  print(find_LAS_length([1, 2, 3, 4]))
  print(find_LAS_length([3, 2, 1, 4]))
  print(find_LAS_length([1, 3, 2, 4]))


main()
```

The time complexity of the above algorithm is $O(n^2)$ and the space complexity is $O(n)$.

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