Fibonacci numbers

We'll cover the following

Problem Statement

Write a function to calculate the nth Fibonacci number.

Fibonacci numbers are a series of numbers in which each number is the sum of the two preceding numbers. First few Fibonacci numbers are: 0, 1, 1, 2, 3, 5, 8, …

Mathematically we can define the Fibonacci numbers as:

```
    Fib(n) = Fib(n-1) + Fib(n-2), for n > 1

    Given that: Fib(0) = 0, and Fib(1) = 1
```

Basic Solution

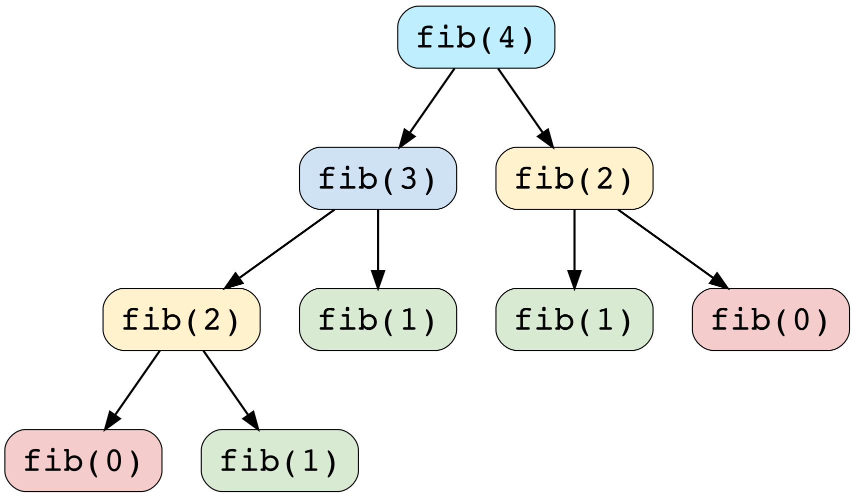
A basic solution could be to have a recursive implementation of the mathematical formula discussed above:

Java JS Python3 C++

```python
def calculateFibonacci(n):
  if n < 2:
    return n

  return calculateFibonacci(n - 1) + calculateFibonacci(n - 2)


def main():
  print("5th Fibonacci is ---> " + str(calculateFibonacci(5)))
  print("6th Fibonacci is ---> " + str(calculateFibonacci(6)))
  print("7th Fibonacci is ---> " + str(calculateFibonacci(7)))


main()
```

The time complexity of the above algorithm is exponential $O(2^n)$ as we are making two recursive calls in the same function. The space complexity is $O(n)$ which is used to store the recursion stack.

Let's visually draw the recursion for `CalculateFibonacci(4)` to see the overlapping subproblems:



Recursion tree for calculating Fibonacci numbers

We can clearly see the overlapping subproblem pattern: `fib(2)` has been called twice and `fib(1)` has been called thrice. We can optimize this using memoization to store the results for subproblems.

Top-down Dynamic Programming with Memoization

We can use an array to store the already solved subproblems. Here is the code:

Java JS Python3 C++

```python
def calculateFibonacci(n):
  memoize = [-1 for x in range(n+1)]
  return calculateFibonacciRecur(memoize, n)


def calculateFibonacciRecur(memoize, n):
  if n < 2:
    return n

  # if we have already solved this subproblem, simply return the result from the cache
  if memoize[n] >= 0:
    return memoize[n]

  memoize[n] = calculateFibonacciRecur(
    memoize, n - 1) + calculateFibonacciRecur(memoize, n - 2)
  return memoize[n]


def main():
  print("5th Fibonacci is ---> " + str(calculateFibonacci(5)))
  print("6th Fibonacci is ---> " + str(calculateFibonacci(6)))
  print("7th Fibonacci is ---> " + str(calculateFibonacci(7)))


main()

```

Bottom-up Dynamic Programming

Let’s try to populate our `dp[]` array from the above solution, working in a bottom-up fashion. Since every Fibonacci number is the sum of the previous two numbers, we can use this fact to populate our array.

Here is the code for the bottom-up dynamic programming approach:

Java | JS | Python3 | C++

```
def calculateFibonacci(n):
  if n < 2:
    return n
  dp = [0, 1]
  for i in range(2, n + 1):
    dp.append(dp[i - 1] + dp[i - 2])

  return dp[n]


def main():
  print("5th Fibonacci is ---> " + str(calculateFibonacci(5)))
  print("6th Fibonacci is ---> " + str(calculateFibonacci(6)))
  print("7th Fibonacci is ---> " + str(calculateFibonacci(7)))


main()

```

The above solution has time and space complexity of $O(n)$.

Memory optimization

We can optimize the space used in our previous solution. We don’t need to store all the Fibonacci numbers up to ‘n’, as we only need two previous numbers to calculate the next Fibonacci number. We can use this fact to further improve our solution:

Java | JS | Python3 | C++

```
def calculateFibonacci(n):
  if n < 2:
    return n
```

```python

  n1, n2, temp = 0, 1, 0
  for i in range(2, n + 1):
    temp = n1 + n2
    n1 = n2
    n2 = temp

  return n2


def main():
  print("5th Fibonacci is ---> " + str(calculateFibonacci(5)))
  print("6th Fibonacci is ---> " + str(calculateFibonacci(6)))
  print("7th Fibonacci is ---> " + str(calculateFibonacci(7)))


main()

```

The above solution has a time complexity of $O(n)$ but a constant space complexity $O(1)$.

← Back
Maximum Ribbon Cut

Next →
Staircase

✓ Mark as Completed

Report an Issue

Ask a Question (https://discuss.educative.io/tag/fibonacci-numbers__pattern-3-fibonacci-numbers__grokking-dynamic-programming-patterns-for-coding-interviews)