Strings Interleaving

We'll cover the following

Problem Statement

Give three strings ‘m’, ‘n’, and ‘p’, write a method to find out if ‘p’ has been formed by interleaving ‘m’ and ‘n’. ‘p’ would be considered interleaving ‘m’ and ‘n’ if it contains all the letters from ‘m’ and ‘n’ and the order of letters is preserved too.

Example 1:

```
Input: m="abd", n="cef", p="abcdef"
Output: true
Explanation: 'p' contains all the letters from 'm' and 'n' and preserves their ord
er too.
```

Example 2:

```
Input: m="abd", n="cef", p="adcbef"
Output: false
Explanation: 'p' contains all the letters from 'm' and 'n' but does not preserve t
he order.
```

Example 3:

```
Input: m="abc", n="def", p="abdccf"
Output: false
Explanation: 'p' does not contain all the letters from 'm' and 'n'.
```

Example 4:

```
Input: m="abcdef", n="mnop", p="mnaobcdepf"
Output: true
Explanation: 'p' contains all the letters from 'm' and 'n' and preserves their ord
er too.
```

Basic Solution

The problem follows the Longest Common Subsequence (https://www.educative.io/collection/page/5668639101419520/5633779737559040/5657535201673216) (LCS) pattern and has some similarities with Subsequence Pattern Matching (https://www.educative.io/collection/page/5668639101419520/5633779737559040/5718922095493120/).

A basic brute-force solution could be to try matching 'm' and 'n' with 'p' one letter at a time. Let's assume `mIndex`, `nIndex`, and `pIndex` represent the current indexes of 'm', 'n', and 'p' strings respectively. Therefore, we have two options at any step:

1. If the letter at `mIndex` matches with the letter at `pIndex`, we can recursively match for the remaining lengths of 'm' and 'p'.
2. If the letter at `nIndex` matches with the letter at 'pIndex', we can recursively match for the remaining lengths of 'n' and 'p'.

Code

Here is the code:

Java | JS | Python3 | C++

```python
def find_SI(m, n,  p):
  return find_SI_recursive(m, n, p, 0, 0, 0)


def find_SI_recursive(m, n, p, mIndex, nIndex, pIndex):

  mLen, nLen, pLen = len(m), len(n), len(p)
  # if we have reached the end of the all the strings
  if mIndex == mLen and nIndex == nLen and pIndex == pLen:
    return True

  # if we have reached the end of 'p' but 'm' or 'n' still has some characters left
  if pIndex == pLen:
    return False

  b1, b2 = False, False
  if mIndex < mLen and m[mIndex] == p[pIndex]:
    b1 = find_SI_recursive(m, n, p, mIndex+1, nIndex, pIndex+1)

  if nIndex < nLen and n[nIndex] == p[pIndex]:
    b2 = find_SI_recursive(m, n, p, mIndex, nIndex+1, pIndex+1)

  return b1 or b2


def main():
  print(find_SI("abd", "cef", "abcdef"))
  print(find_SI("abd", "cef", "adcbef"))
```

```
  print(find_SI("abc", "def", "abdccf"))
  print(find_SI("abcdef", "mnop", "mnaobcdepf"))


main()
```

The time complexity of the above algorithm is exponential $O(2^{m+n})$, where 'm' and 'n' are the lengths of the two interleaving strings. The space complexity is $O(m+n)$, the value that is used to store the recursion stack.

Top-down Dynamic Programming with Memoization

This problem can have overlapping subproblems only when there are some common letters between 'm' and 'n' at the same index. Because whenever we hit such a scenario, we get an option to match with any one of them.

The three changing values in our recursive function are the three indexes `mIndex`, `nIndex`, and `pIndex`. Therefore, we can store the results of all the subproblems in a three-dimensional array. Alternately, we can use a hash-table whose key would be a string (mIndex + "|" + nIndex + "|" + pIndex).

Code #

Here is the code:

Java | JS | Python3 | C++

```
def find_SI(m, n, p):
  return find_SI_recursive({}, m, n, p, 0, 0, 0)


def find_SI_recursive(dp, m, n, p, mIndex, nIndex, pIndex):
  mLen, nLen, pLen = len(m), len(n), len(p)
  # if we have reached the end of the all the strings

  if mIndex == mLen and nIndex == nLen and pIndex == pLen:
    return True

  # if we have reached the end of 'p' but 'm' or 'n' still has some characters left
  if pIndex == pLen:
    return False

  subProblemKey = str(mIndex) + "-" + str(nIndex) + "-" + str(pIndex)
  if subProblemKey not in dp:
    b1, b2 = False, False
    if mIndex < mLen and m[mIndex] == p[pIndex]:
      b1 = find_SI_recursive(dp, m, n, p, mIndex + 1, nIndex, pIndex + 1)

    if nIndex < nLen and n[nIndex] == p[pIndex]:
      b2 = find_SI_recursive(dp, m, n, p, mIndex, nIndex + 1, pIndex + 1)
```

```
    dp[subProblemKey] = b1 or b2

  return dp.get(subProblemKey)


def main():
  print(find_SI("abd", "cef", "abcdef"))
  print(find_SI("abd", "cef", "adcbef"))
  print(find_SI("abc", "def", "abdccf"))
  print(find_SI("abcdef", "mnop", "mnaobcdepf"))


main()
```

Bottom-up Dynamic Programming

Since we want to completely match ‘m’ and ‘n’ (the two interleaving strings) with ‘p’, we can use a two-dimensional array to store our results. The lengths of ‘m’ and ‘n’ will define the dimensions of the result array.

As mentioned above, we will be tracking separate indexes for ‘m’, ‘n’ and ‘p’, so we will have the following options for every value of `mIndex`, `nIndex`, and `pIndex`:

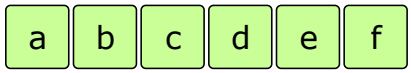
1. If the character `m[mIndex]` matches the character `p[pIndex]`, we will take the matching result up to `mIndex-1` and `nIndex`.
2. If the character `n[nIndex]` matches the character `p[pIndex]`, we will take the matching result up to `mIndex` and `nIndex-1`.

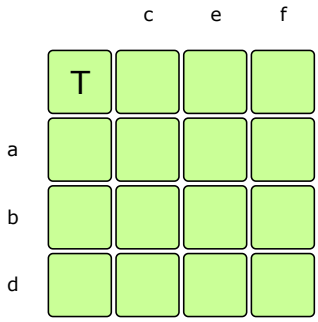
String ‘p’ will be interleaving strings ‘m’ and ‘n’ if any of the above two options is `true`. This is also required as there could be some common letters between ‘m’ and ‘n’.

So our recursive formula would look like:

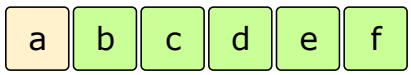
```
dp[mIndex][nIndex] = false
if m[mIndex] == p[pIndex]
  dp[mIndex][nIndex] = dp[mIndex-1][nIndex]
if n[nIndex] == p[pIndex]
 dp[mIndex][nIndex] |= dp[mIndex][nIndex-1]
```

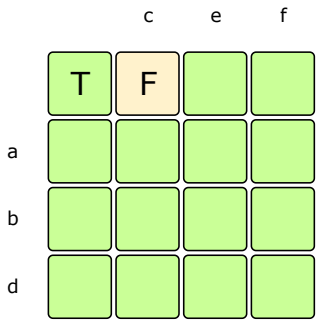
Let’s draw this visually:

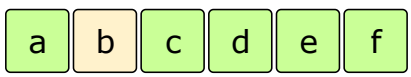


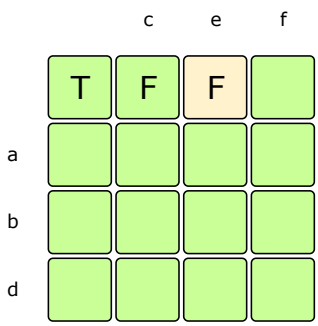


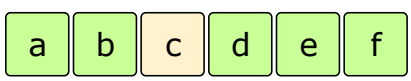
If 'm' and 'n' are empty, we can always find an empty string that will be interleaving them

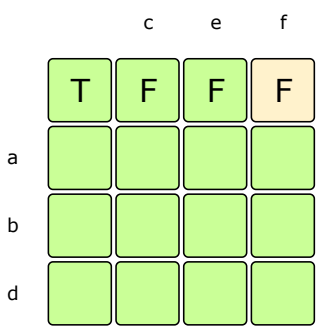




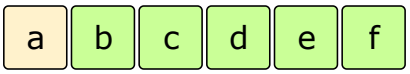


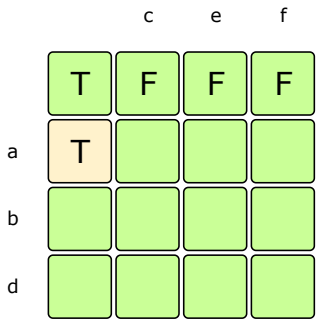






Interleaving string





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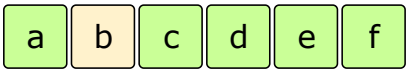

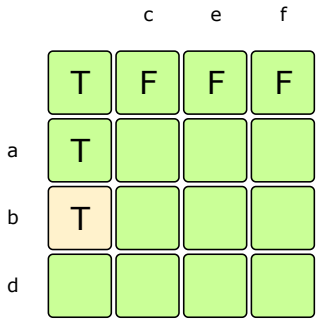

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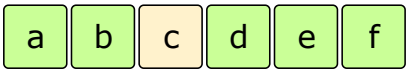

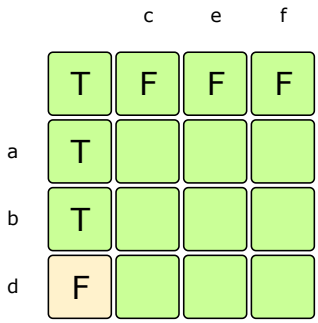

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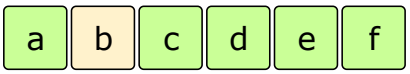

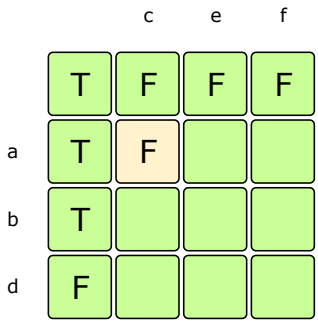

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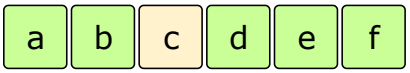

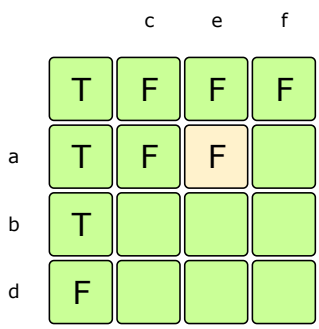

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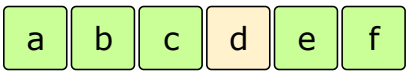

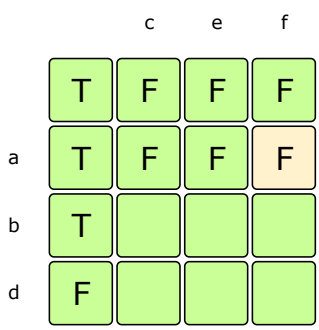

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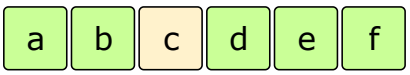

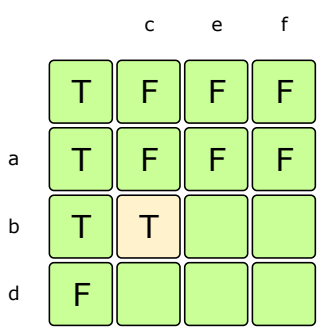

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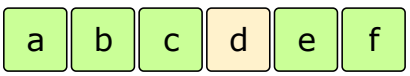

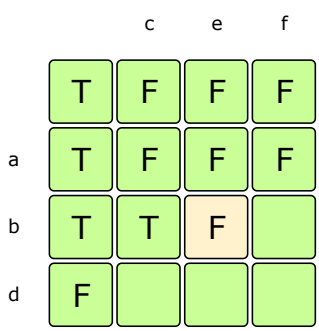

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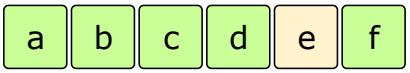
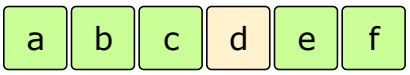


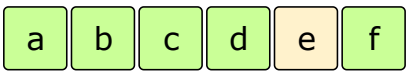
		c	e	f
	T	F	F	F
a	T	F	F	F
b	T	T	F	F
d	F			

Interleaving string



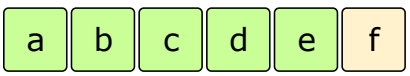
		c	e	f
	T	F	F	F
a	T	F	F	F
b	T	T	F	F
d	F	T		

Interleaving string



		c	e	f
	T	F	F	F
a	T	F	F	F
b	T	T	F	F
d	F	T	T	

Interleaving string



		c	e	f
	T	F	F	F
a	T	F	F	F
b	T	T	F	F
d	F	T	T	T

Code #

Here is the code for our bottom-up dynamic programming approach:

Java JS Python3 C++

```python
def find_SI(m, n, p):
  mLen, nLen, pLen = len(m), len(n), len(p)
  # dp[mIndex][nIndex] will be storing the result of string interleaving
  # up to p[0..mIndex+nIndex-1]
  dp = [[False for _ in range(nLen+1)] for _ in range(mLen+1)]

  # make sure if lengths of the strings add up
  if mLen + nLen != pLen:
    return False

  for mIndex in range(mLen+1):
    for nIndex in range(nLen+1):
      # if 'm' and 'n' are empty, then 'p' must have been empty too.
      if mIndex == 0 and nIndex == 0:
        dp[mIndex][nIndex] = True
      # if 'm' is empty, we need to check the interleaving with 'n' only
      elif mIndex == 0 and n[nIndex - 1] == p[mIndex + nIndex - 1]:
        dp[mIndex][nIndex] = dp[mIndex][nIndex - 1]
      # if 'n' is empty, we need to check the interleaving with 'm' only
      elif nIndex == 0 and m[mIndex - 1] == p[mIndex + nIndex - 1]:
        dp[mIndex][nIndex] = dp[mIndex - 1][nIndex]
      else:
        # if the letter of 'm' and 'p' match, we take whatever is matched till mIndex-1
        if mIndex > 0 and m[mIndex - 1] == p[mIndex + nIndex - 1]:
          dp[mIndex][nIndex] = dp[mIndex - 1][nIndex]
        # if the letter of 'n' and 'p' match, we take whatever is matched till nIndex-1 to
        # note the '|=', this is required when we have common letters
        if nIndex > 0 and n[nIndex - 1] == p[mIndex + nIndex - 1]:
          dp[mIndex][nIndex] |= dp[mIndex][nIndex - 1]

  return dp[mLen][nLen]


def main():
  print(find_SI("abd", "cef", "abcdef"))
  print(find_SI("abd", "cef", "adcbef"))
  print(find_SI("abc", "def", "abdccf"))
  print(find_SI("abcdef", "mnop", "mnaobcdepf"))


main()
```

The time and space complexity of the above algorithm is $O(m * n)$, where 'm' and 'n' are the lengths of the two interleaving strings.

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