Subset Sum

We'll cover the following

Problem Statement

Given a set of positive numbers, determine if there exists a subset whose sum is equal to a given number 'S'.

Example 1:

```
Input: {1, 2, 3, 7}, S=6
Output: True
The given set has a subset whose sum is '6': {1, 2, 3}
```

Example 2:

```
Input: {1, 2, 7, 1, 5}, S=10
Output: True
The given set has a subset whose sum is '10': {1, 2, 7}
```

Example 3:

```
Input: {1, 3, 4, 8}, S=6
Output: False
The given set does not have any subset whose sum is equal to '6'.
```

Basic Solution

This problem follows the **0/1 Knapsack pattern** and is quite similar to Equal Subset Sum Partition (https://www.educative.io/collection/page/5668639101419520/5633779737559040/575275462662

5536). A basic brute-force solution could be to try all subsets of the given numbers to see if any set has a sum equal to ‘S’.

So our brute-force algorithm will look like:

```
for each number 'i'
  create a new set which INCLUDES number 'i' if it does not exceed 'S', and recursively
     process the remaining numbers
  create a new set WITHOUT number 'i', and recursively process the remaining numbers
return true if any of the above two sets has a sum equal to 'S', otherwise return false
```

Since this problem is quite similar to Equal Subset Sum Partition (https://www.educative.io/collection/page/5668639101419520/5633779737559040/5752754626625536), let’s jump directly to the bottom-up dynamic programming solution.

Bottom-up Dynamic Programming

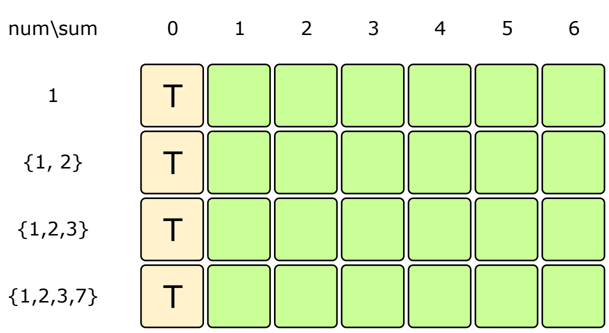
We’ll try to find if we can make all possible sums with every subset to populate the array `dp[TotalNumbers][S+1]`.

For every possible sum ‘s’ (where 0 <= s <= S), we have two options:

1. Exclude the number. In this case, we will see if we can get the sum ‘s’ from the subset excluding this number => `dp[index-1][s]`
2. Include the number if its value is not more than ‘s’. In this case, we will see if we can find a subset to get the remaining sum => `dp[index-1][s-num[index]]`

If either of the above two scenarios returns true, we can find a subset with a sum equal to ‘s’.

Let’s draw this visually, with the example input {1, 2, 3, 7}, and start with our base case of size zero:



'0' sum can always be found through an empty set

1 of 10

num\sum	0	1	2	3	4	5	6
1	T	T	F	F	F	F	F
{1, 2}	T						
{1,2,3}	T						
{1,2,3,7}	T						

With only one number, we can form a subset only when the required sum is equal to that number

2 of 10

num\sum	0	1	2	3	4	5	6
1	T	T	F	F	F	F	F
{1, 2}	T	T					
{1,2,3}	T						
{1,2,3,7}	T						

sum: 1, index:1=> (dp[index-1][sum] , as the 'sum' is less than the number at index '1' (i.e., 1 < 2)

3 of 10

num\sum	0	1	2	3	4	5	6
1	T	T	F	F	F	F	F
{1, 2}	T	T	T				
{1,2,3}	T						
{1,2,3,7}	T						

sum: 2, index:1=> (dp[index-1][sum] || dp[index-1][sum-2])

4 of 10

num\sum	0	1	2	3	4	5	6
1	T	T	F	F	F	F	F
{1, 2}	T	T	T	T			
{1,2,3}	T						
{1,2,3,7}	T						

sum: 3, index:1=> (dp[index-1][sum] || dp[index-1][sum-2])

5 of 10

num\sum	0	1	2	3	4	5	6
1	T	T	F	F	F	F	F
{1, 2}	T	T	T	T	F	F	F
{1,2,3}	T						
{1,2,3,7}	T						

sum: 4-6 index:1=> (dp[index-1][sum] || dp[index-1][sum-2])

6 of 10

num\sum	0	1	2	3	4	5	6
1	T	T	F	F	F	F	F
{1, 2}	T	T	T	T	F	F	F
{1,2,3}	T	T	T	T			
{1,2,3,7}	T						

sum: 1,2,3, index:2=> (dp[index-1][sum] || dp[index-1][sum-3])

7 of 10

num\sum	0	1	2	3	4	5	6
1	T	T	F	F	F	F	F
{1, 2}	T	T	T	T	F	F	F
{1,2,3}	T	T	T	T	T		
{1,2,3,7}	T						

sum: 4, index:2=> (dp[index-1][sum] || dp[index-1][sum-3])

8 of 10

num\sum	0	1	2	3	4	5	6
1	T	T	F	F	F	F	F
{1, 2}	T	T	T	T	F	F	F
{1,2,3}	T	T	T	T	T	T	T
{1,2,3,7}	T						

sum: 5, 6, index:2=> (dp[index-1][sum] || dp[index-1][sum-3])

9 of 10

num\sum	0	1	2	3	4	5	6
1	T	T	F	F	F	F	F
{1, 2}	T	T	T	T	F	F	F
{1,2,3}	T	T	T	T	T	T	T
{1,2,3,7}	T	T	T	T	T	T	T

sum: 1-6, index:3=> (dp[index-1][sum] || dp[index-1][sum-7])

10 of 10

Code

Here is the code for our bottom-up dynamic programming approach:

Java | JS | Python3 | C++

```
def can_partition(num, sum):
  n = len(num)
  dp = [[False for x in range(sum+1)] for y in range(n)]
```

```
  dp = [[False for x in range(sum+1)] for y in range(n)]

  # populate the sum = 0 columns, as we can always form '0' sum with an empty set
  for i in range(0, n):
    dp[i][0] = True

  # with only one number, we can form a subset only when the required sum is
  # equal to its value
  for s in range(1, sum+1):
    dp[0][s] = True if num[0] == s else False

  # process all subsets for all sums
  for i in range(1, n):
    for s in range(1, sum+1):
      # if we can get the sum 's' without the number at index 'i'
      if dp[i - 1][s]:
        dp[i][s] = dp[i - 1][s]
      elif s >= num[i]:
        # else include the number and see if we can find a subset to get the remaining sum
        dp[i][s] = dp[i - 1][s - num[i]]

  # the bottom-right corner will have our answer.
  return dp[n - 1][sum]


def main():
  print("Can partition: " + str(can_partition([1, 2, 3, 7], 6)))
  print("Can partition: " + str(can_partition([1, 2, 7, 1, 5], 10)))
  print("Can partition: " + str(can_partition([1, 3, 4, 8], 6)))


main()
```

The above solution has time and space complexity of $O(N * S)$, where 'N' represents total numbers and 'S' is the required sum.

Challenge

Can we further improve our bottom-up DP solution? Can you find an algorithm that has $O(S)$ space complexity?

Show Hint

Try it yourself

Java | JS | Python3 | C++

```
def can_partition(num, sum):
   #TODO: Write - Your - Code
  return False
```

Solution

```
    dp = [False for x in range(sum+1)]

    # handle sum=0, as we can always have '0' sum with an empty set
    dp[0] = True

    # with only one number, we can have a subset only when the required sum is equal
    for s in range(1, sum+1):
        dp[s] = num[0] == s

    # process all subsets for all sums
    for i in range(1, n):
        for s in range(sum, -1, -1):
            # if dp[s]==true, this means we can get the sum 's' without num[i], hence
            # the next number else we can include num[i] and see if we can find a sub
            # remaining sum
            if not dp[s] and s >= num[i]:
                dp[s] = dp[s - num[i]]

    return dp[sum]


def main():
    print("Can partition: " + str(can_partition([1, 2, 3, 7], 6)))
    print("Can partition: " + str(can_partition([1, 2, 7, 1, 5], 10)))
    print("Can partition: " + str(can_partition([1, 3, 4, 8], 6)))


main()
```