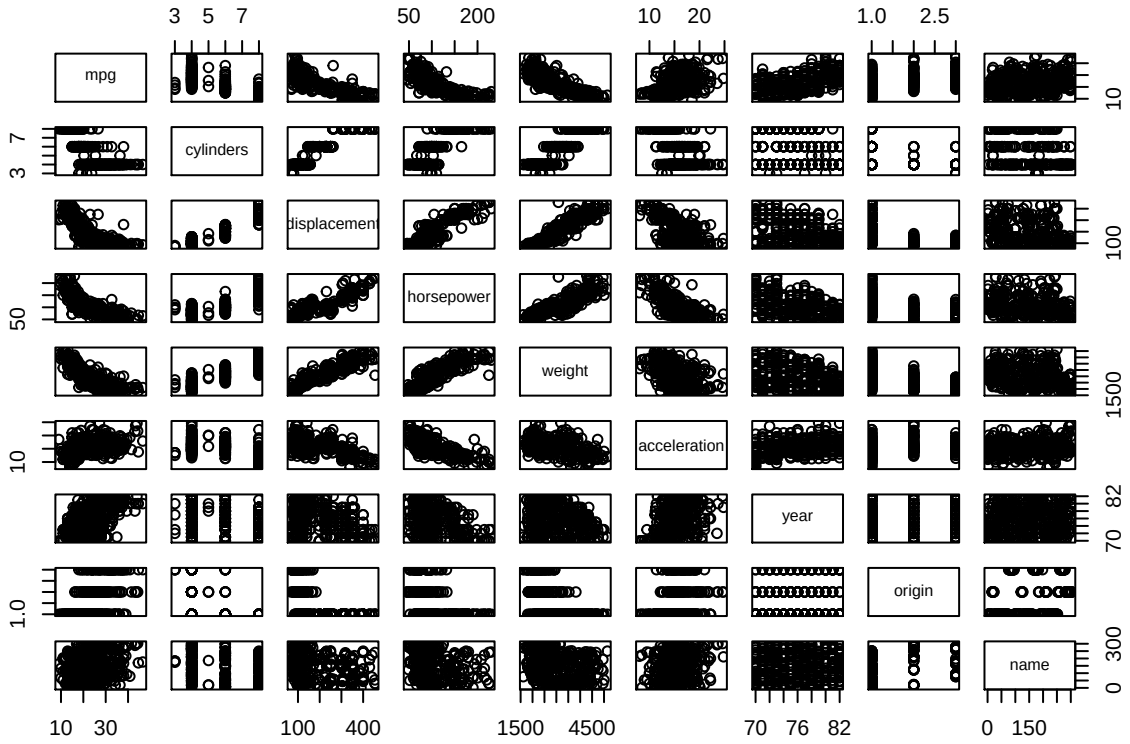
Problem6

a)

```
library(ISLR)
plot(Auto)
```



b)

```
cor(Auto[1:8])
```

```
##                     mpg  cylinders displacement horsepower     weight
## mpg           1.0000000 -0.7776175   -0.8051269 -0.7784268 -0.8322442
## cylinders    -0.7776175  1.0000000    0.9508233  0.8429834  0.8975273
## displacement -0.8051269  0.9508233    1.0000000  0.8972570  0.9329944
## horsepower   -0.7784268  0.8429834    0.8972570  1.0000000  0.8645377
## weight       -0.8322442  0.8975273    0.9329944  0.8645377  1.0000000
## acceleration  0.4233285 -0.5046834   -0.5438005 -0.6891955 -0.4168392
## year          0.5805410 -0.3456474   -0.3698552 -0.4163615 -0.3091199
## origin        0.5652088 -0.5689316   -0.6145351 -0.4551715 -0.5850054
##              acceleration       year     origin
## mpg             0.4233285  0.5805410  0.5652088
## cylinders      -0.5046834 -0.3456474 -0.5689316
## displacement   -0.5438005 -0.3698552 -0.6145351
## horsepower     -0.6891955 -0.4163615 -0.4551715
## weight         -0.4168392 -0.3091199 -0.5850054
## acceleration    1.0000000  0.2903161  0.2127458
## year            0.2903161  1.0000000  0.1815277
## origin          0.2127458  0.1815277  1.0000000
```

c)

```
lm.auto=lm(mpg~.-name,data=Auto)
summary(lm.auto)
```

```
##
## Call:
## lm(formula = mpg ~ . - name, data = Auto)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.5903 -2.1565 -0.1169  1.8690 13.0604
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -17.218435   4.644294  -3.707  0.00024 ***
## cylinders     -0.493376   0.323282  -1.526  0.12780
## displacement   0.019896   0.007515   2.647  0.00844 **
## horsepower    -0.016951   0.013787  -1.230  0.21963
## weight        -0.006474   0.000652  -9.929  < 2e-16 ***
## acceleration   0.080576   0.098845   0.815  0.41548
## year           0.750773   0.050973  14.729  < 2e-16 ***
## origin         1.426141   0.278136   5.127 4.67e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.328 on 384 degrees of freedom
## Multiple R-squared:  0.8215, Adjusted R-squared:  0.8182
## F-statistic: 252.4 on 7 and 384 DF,  p-value: < 2.2e-16
```

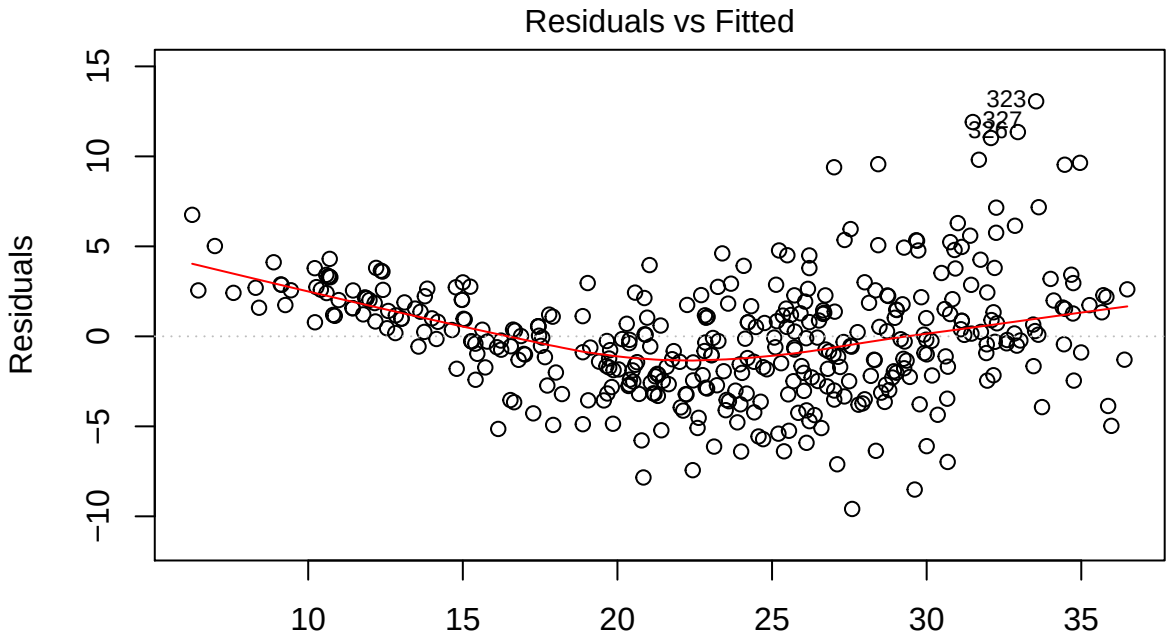
1.The whole model has a p value less than 2.2e-16, therefore there must be some predictors that have relationship with the response.

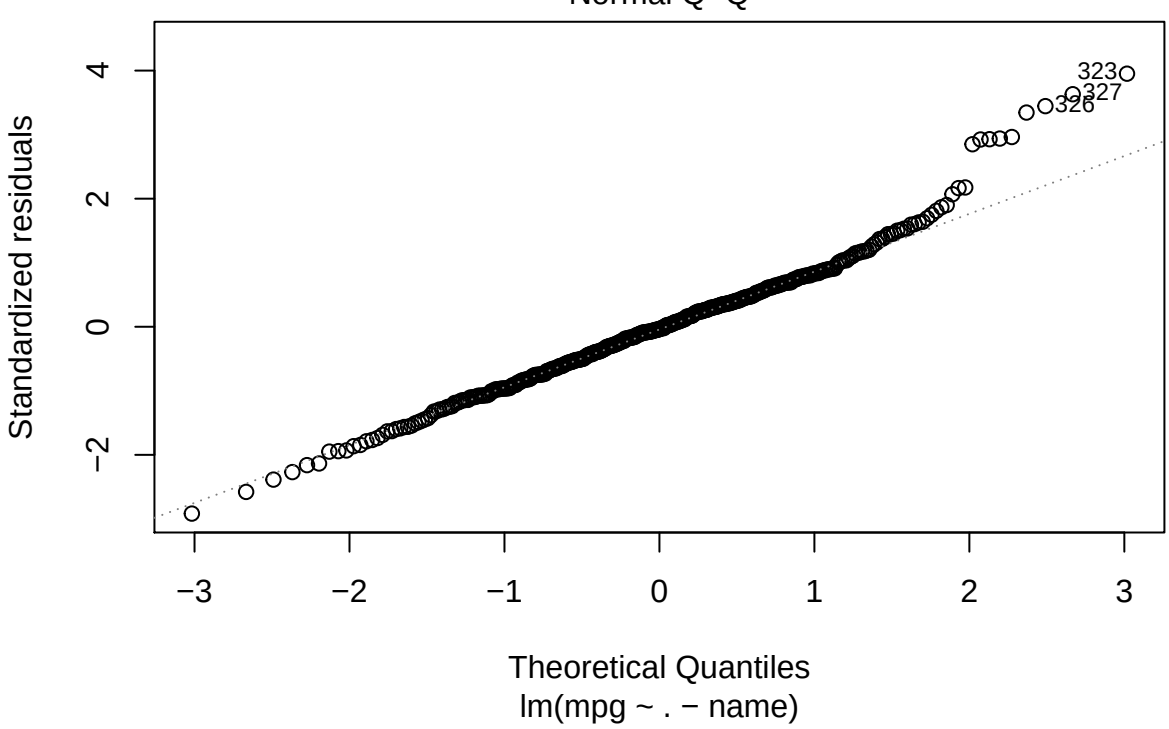
2.displacement,weight, year, origin have statistically significant relationship with the response.

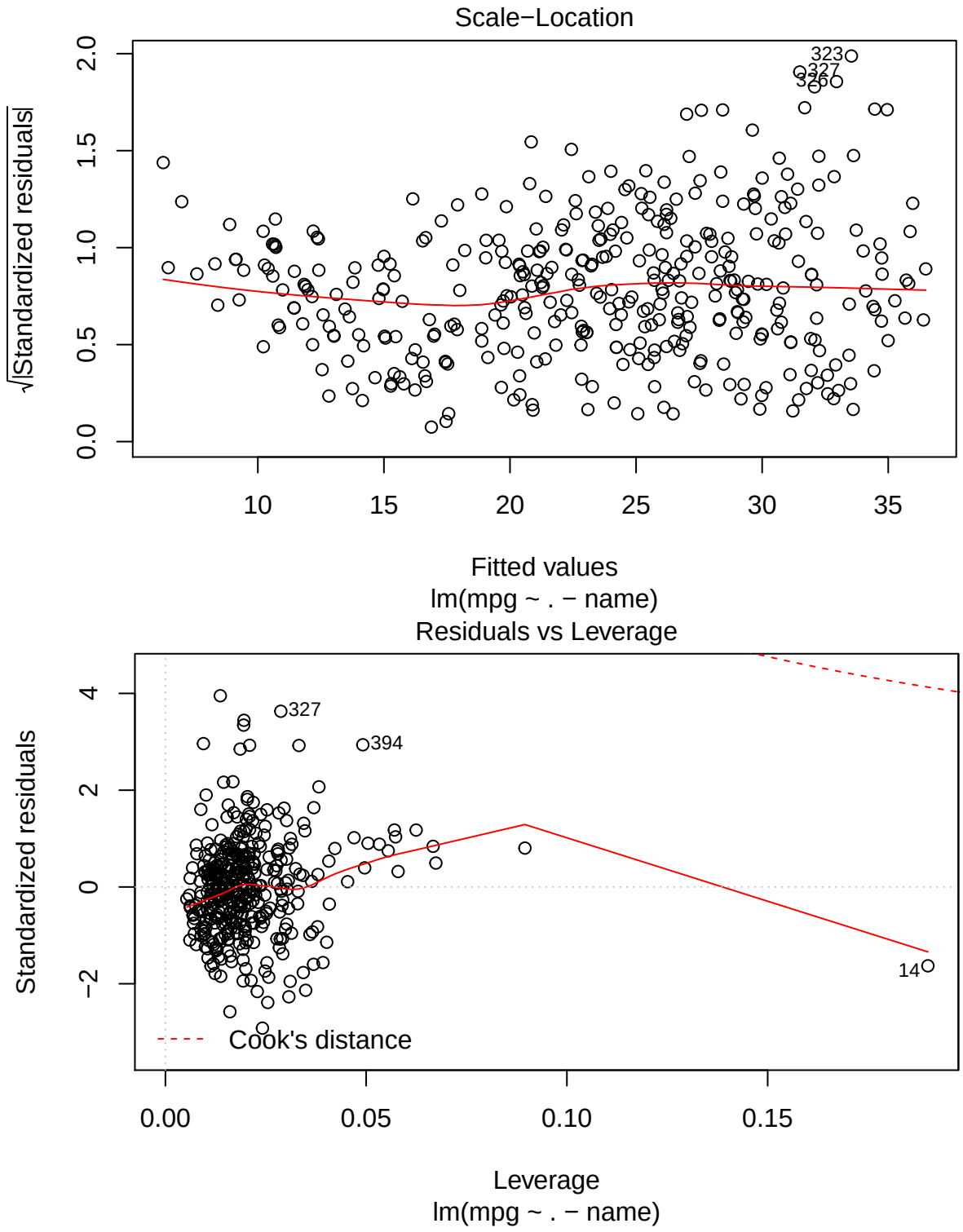
3.The coefficient of year suggests that if the average effect of year goes up 1 is that mpg will go up by 0.75.

d)

```
plot(lm.auto)
```


Residuals vs Fitted
Residuals
Fitted values
lm(mpg ~ . − name)
323
327
326
15
10
5
0
−5
−10
10
15
20
25
30
35


Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(mpg ~ . − name)
323
327
326
4
2
0
−2
−3
−2
−1
0
1
2
3



1.point 323, 326, 327 are unusually large outliers

2.point 14 has unusually large leverage

3.Accoring to the normal q-q plot, the residule is not nicely normal-distributed, it is somewhat right-skewed.

Problem 7

a)

```
set.seed(1)
x1=runif(100)
x2=0.5*x1+rnorm(100)/10
y=2+2*x1+0.3*x2+rnorm(100)
```

The linear model is:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$$

The coefficients are:

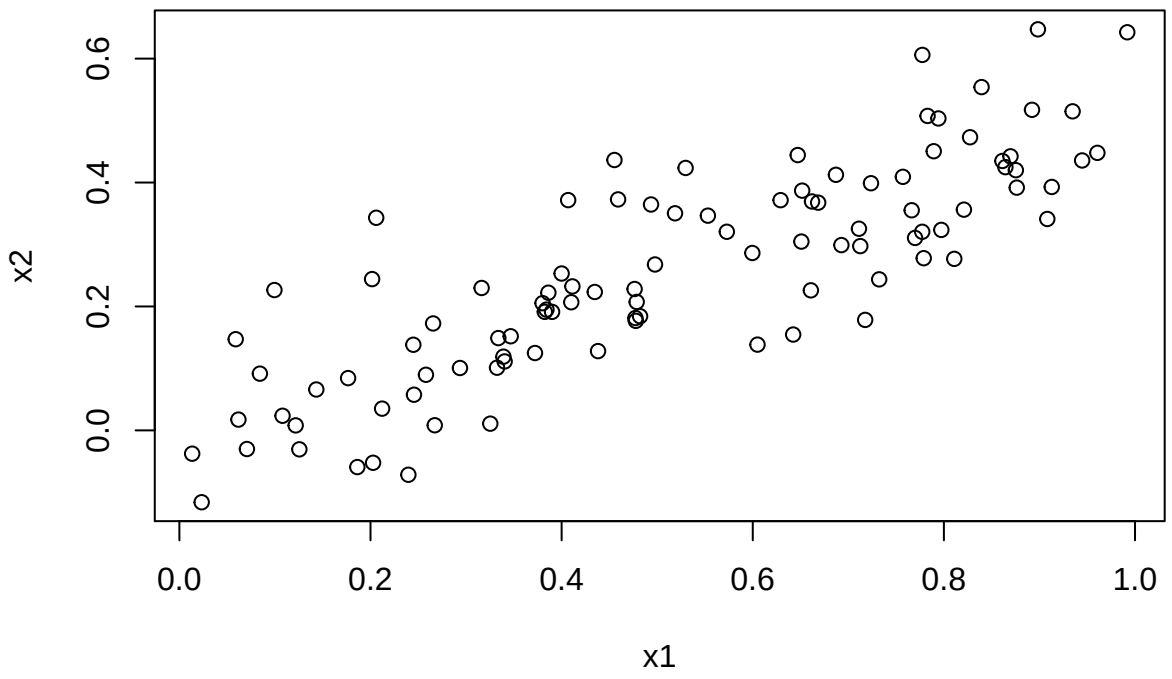
$$\beta_0 = 2, \beta_1 = 2, \beta_2 = 0.3$$

b)

```
cor(x1,x2)
```

```
## [1] 0.8351212
```

```
plot(x1,x2)
```



c)

```
lm.out1<-lm(y~x1+x2)
summary(lm.out1)
```

```
##
## Call:
## lm(formula = y ~ x1 + x2)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.8311 -0.7273 -0.0537  0.6338  2.3359
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.1305     0.2319   9.188 7.61e-15 ***
## x1            1.4396     0.7212   1.996   0.0487 *
## x2            1.0097     1.1337   0.891   0.3754
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.056 on 97 degrees of freedom
## Multiple R-squared:  0.2088, Adjusted R-squared:  0.1925
## F-statistic:  12.8 on 2 and 97 DF,  p-value: 1.164e-05
```

The estimators are:

$$\hat{\beta_0} = 2.1305, \hat{\beta_1} = 1.4396, \hat{\beta_2} = 1.0097$$

The β_0 is almost accurate but β_1 and β_2 are not. We can reject the null hypothesis $H_0 : \beta_1 = 0$ but we cannot reject the null hypothesis $H_0 : \beta_2 = 0$

d)

```
lm.out2<-lm(y~x1)
summary(lm.out2)
```

```
##
## Call:
## lm(formula = y ~ x1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.89495 -0.66874 -0.07785  0.59221  2.45560
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.1124     0.2307   9.155 8.27e-15 ***
## x1            1.9759     0.3963   4.986 2.66e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.055 on 98 degrees of freedom
## Multiple R-squared:  0.2024, Adjusted R-squared:  0.1942
## F-statistic: 24.86 on 1 and 98 DF,  p-value: 2.661e-06
```

Yes, we can reject the null hypothesis $H_0 : \beta_1 = 0$

e)

```
lm.out3<-lm(y~x2)
summary(lm.out3)
```

```
##
## Call:
## lm(formula = y ~ x2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.62687 -0.75156 -0.03598  0.72383  2.44890
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.3899     0.1949   12.26  < 2e-16 ***
## x2            2.8996     0.6330    4.58 1.37e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.072 on 98 degrees of freedom
## Multiple R-squared:  0.1763, Adjusted R-squared:  0.1679
## F-statistic: 20.98 on 1 and 98 DF,  p-value: 1.366e-05
```

Yes, we can reject the null hypothesis $H_0 : \beta_2 = 0$

f) No, they don't. This is because in c), the fact that we can not reject $H_0 : \beta_2 = 0$ is in the presence one x1. What it means is that in the presence of x1, x2 provides no statistically significant additional information about y. While d) and e) say that x1 or x2 alone provide statistically information about y.

The reason why this is happening is that x1 and x2 are highly correlated. We have collinearity. Collinearity reduces the accuracy of the estimates of the regression coefficients.

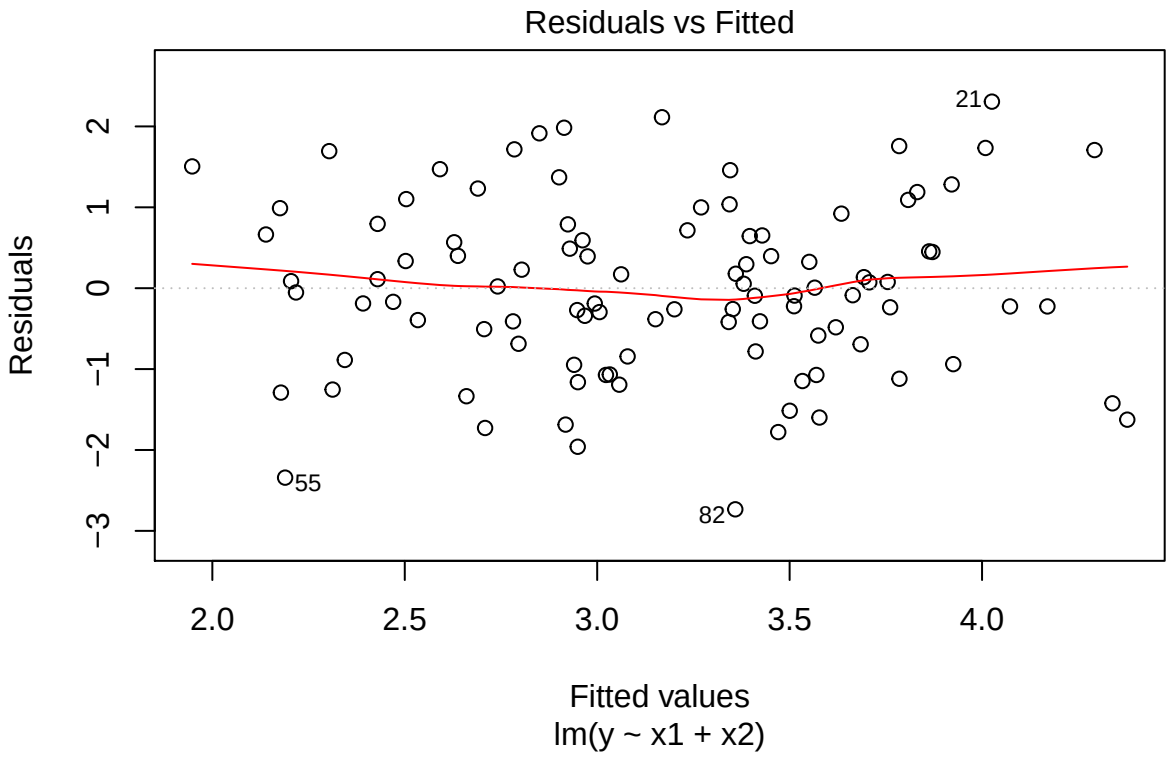
g)

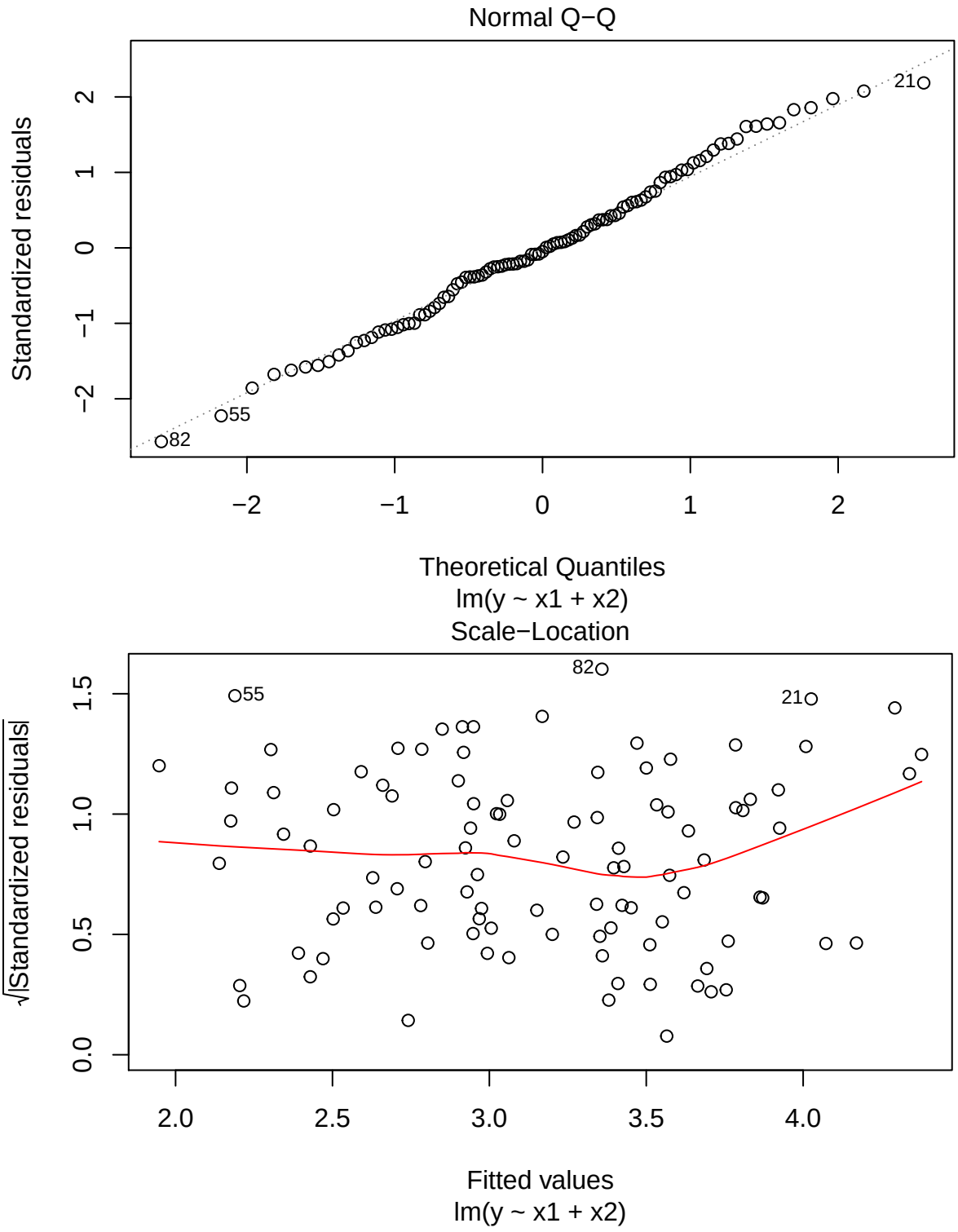
```
x1=c(x1,0.1)
x2=c(x2,0.8)
y=c(y,6)
lm.out1<-lm(y~x1+x2)
summary(lm.out1)
```

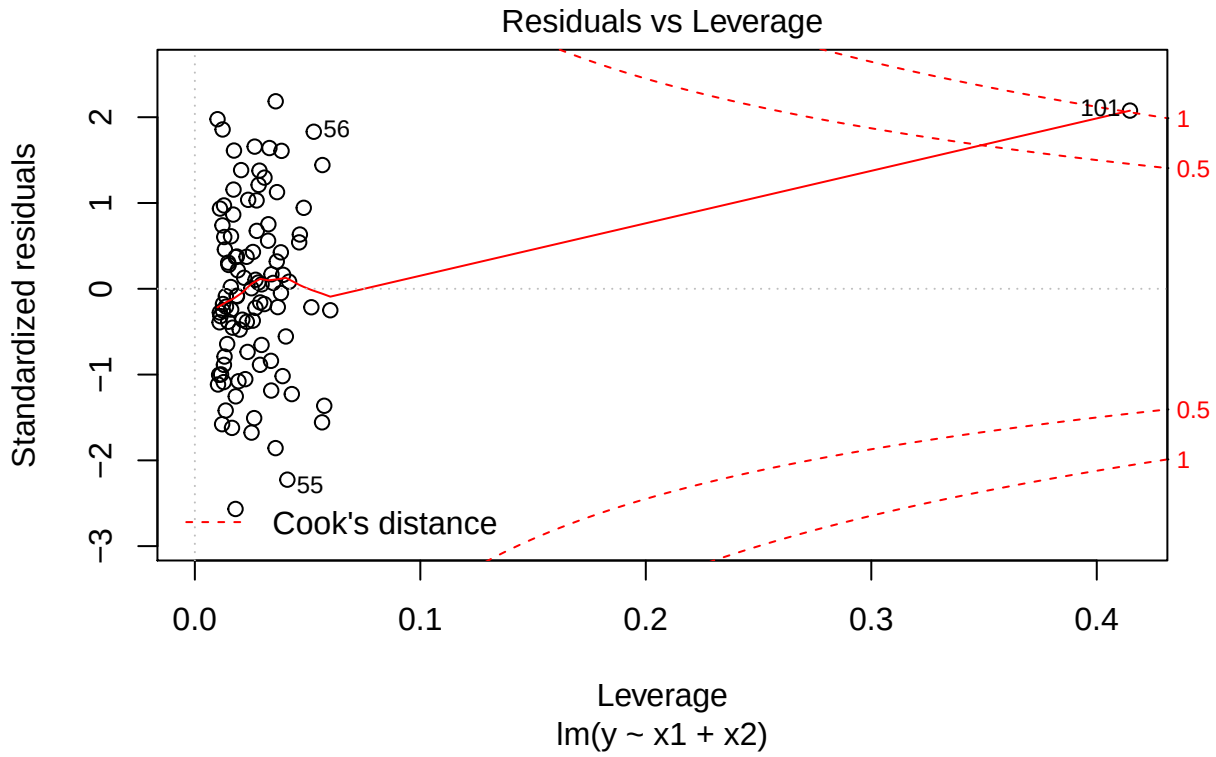
```
##
## Call:
## lm(formula = y ~ x1 + x2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.73348 -0.69318 -0.05263  0.66385  2.30619
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.2267     0.2314   9.624 7.91e-16 ***
## x1            0.5394     0.5922   0.911  0.36458
```

```
## x2            2.5146     0.8977   2.801  0.00614 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.075 on 98 degrees of freedom
## Multiple R-squared:  0.2188, Adjusted R-squared:  0.2029
## F-statistic: 13.72 on 2 and 98 DF,  p-value: 5.564e-06
```

```
plot(lm.out1)
```



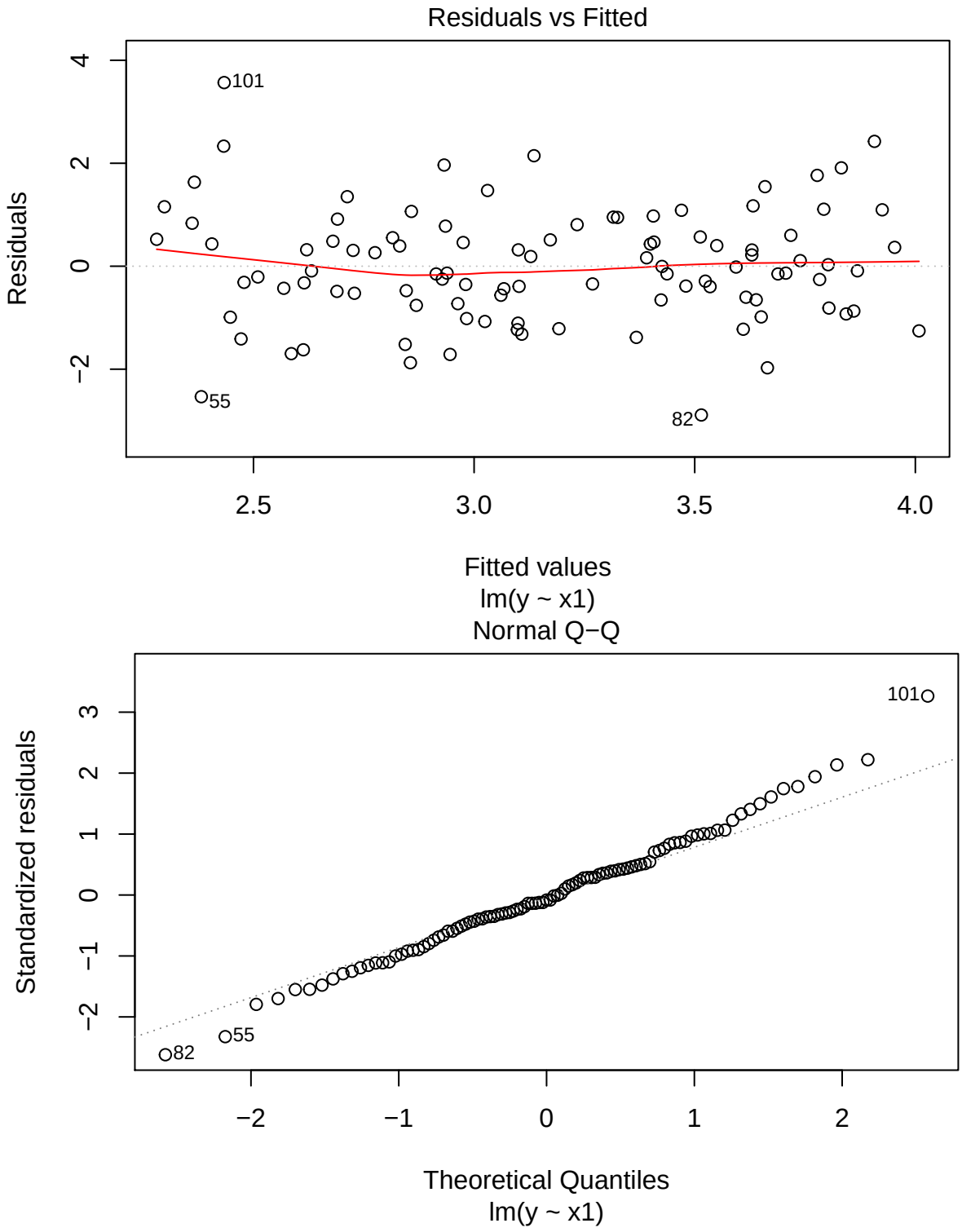

Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(y ~ x1 + x2)
Scale−Location
√|Standardized residuals|
Fitted values
lm(y ~ x1 + x2)
82
55
21

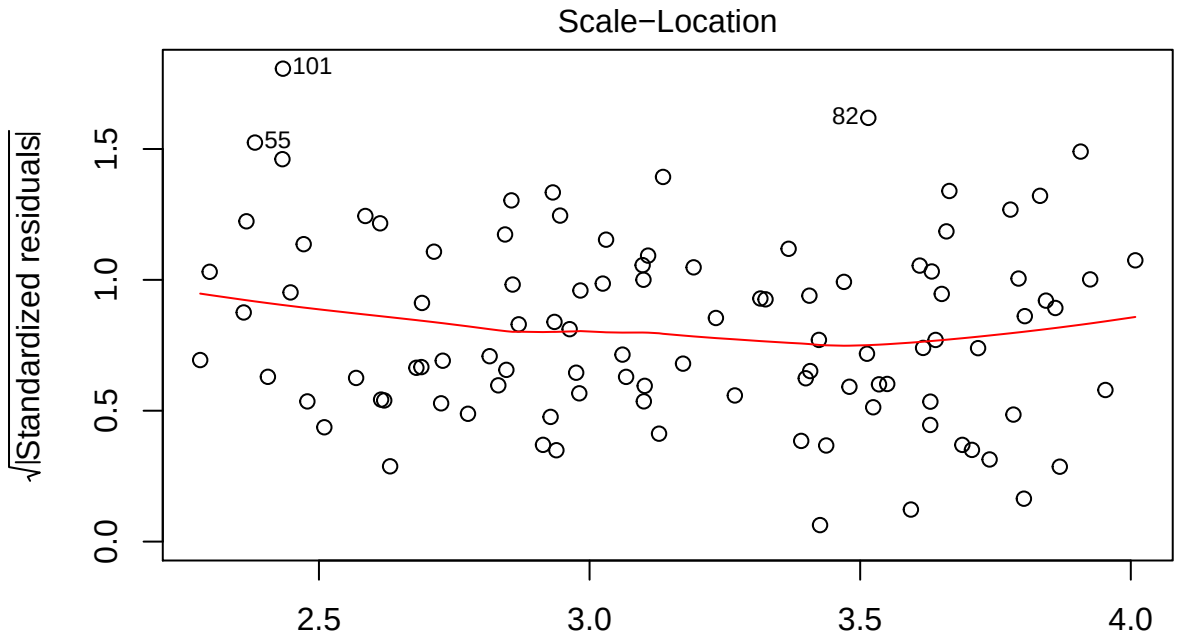


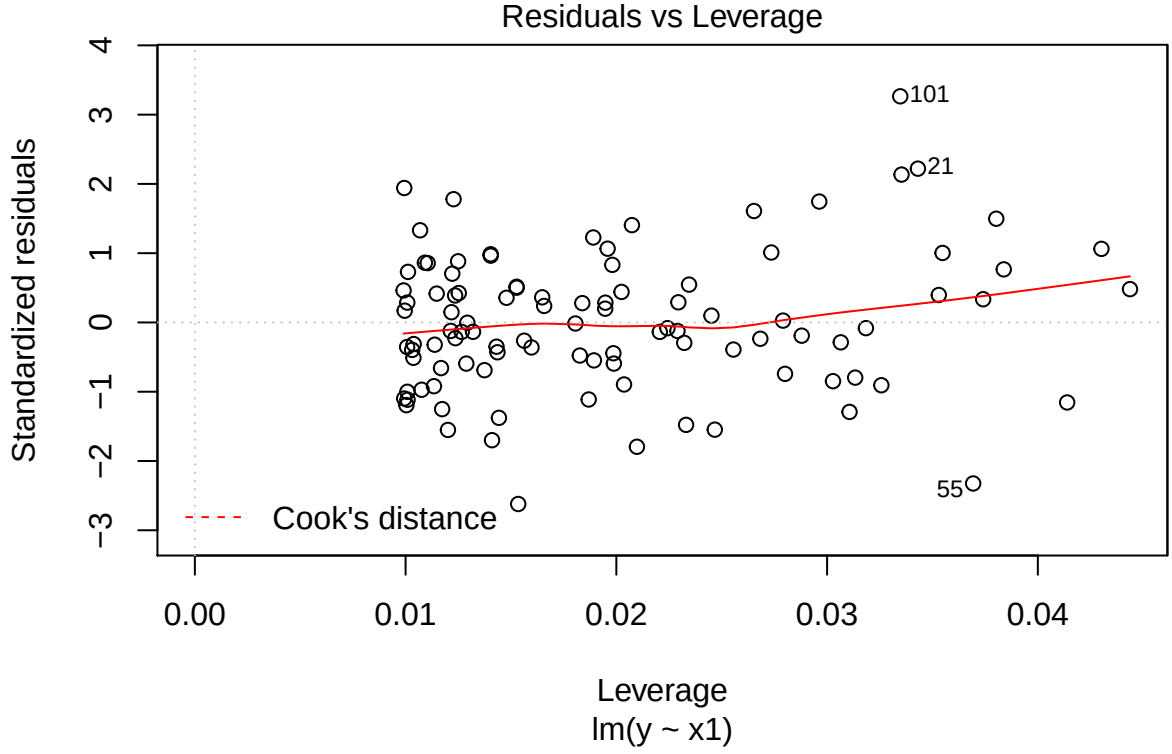

```
lm.out2<-lm(y~x1)
summary(lm.out2)
```

```
##
## Call:
## lm(formula = y ~ x1)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.8897 -0.6556 -0.0909  0.5682  3.5665
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.2569     0.2390   9.445 1.78e-15 ***
## x1            1.7657     0.4124   4.282 4.29e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.111 on 99 degrees of freedom
## Multiple R-squared:  0.1562, Adjusted R-squared:  0.1477
## F-statistic: 18.33 on 1 and 99 DF,  p-value: 4.295e-05
```

```
plot(lm.out2)
```


Residuals vs Fitted
Residuals
Fitted values
lm(y ~ x1)
101
55
82
Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(y ~ x1)



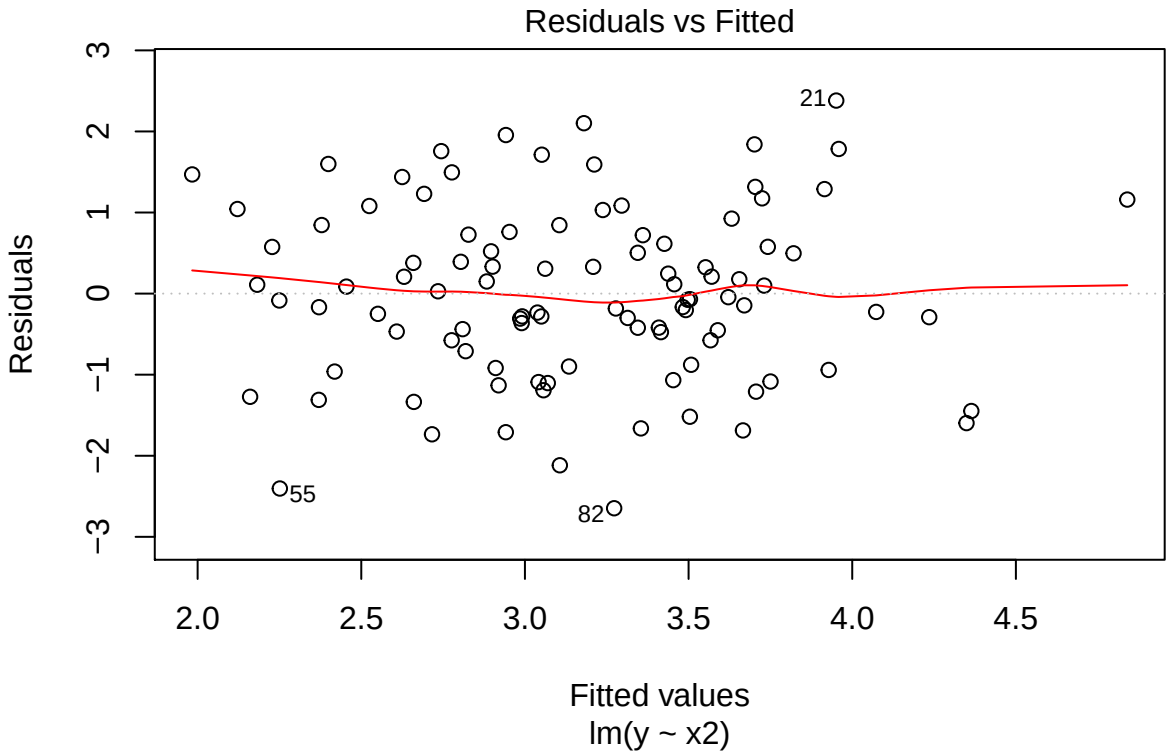


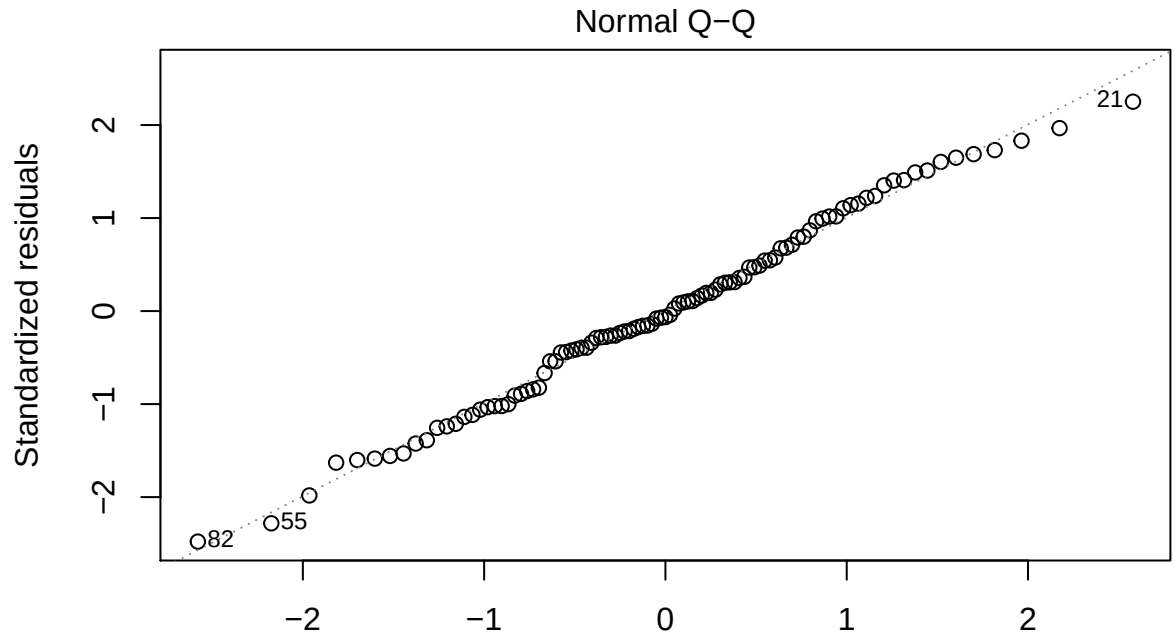

```
lm.out3<-lm(y~x2)
summary(lm.out3)
```

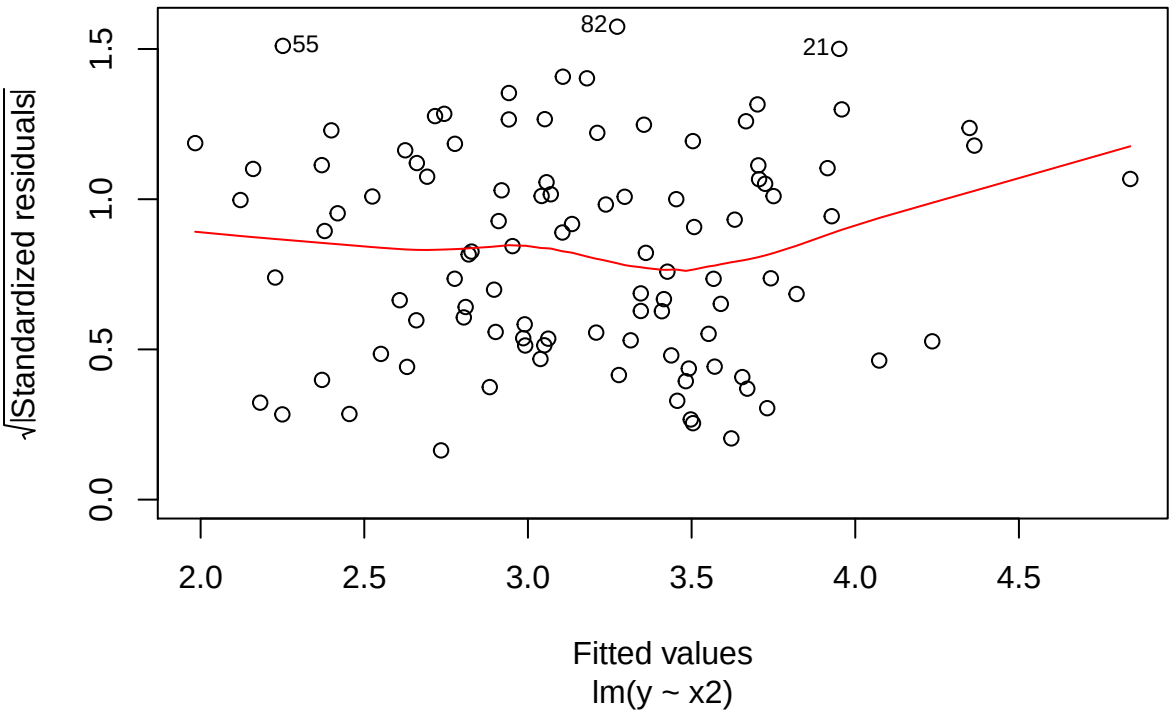
```
##
## Call:
```

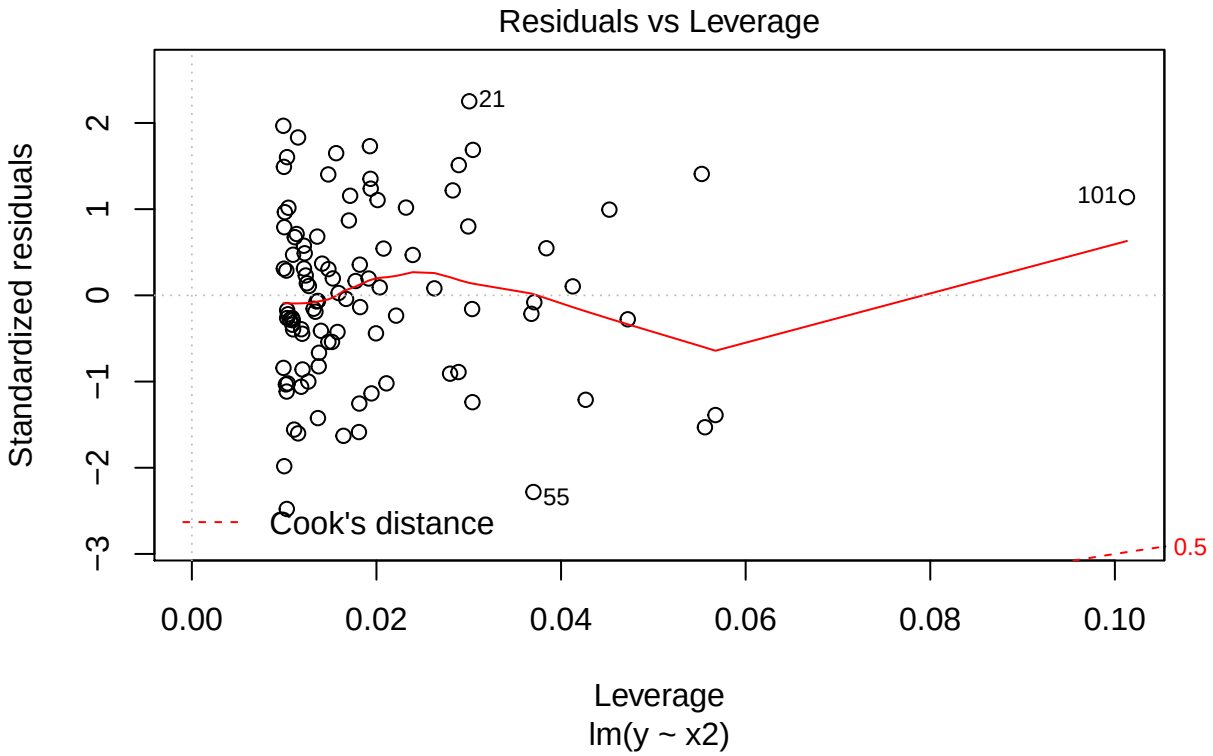
```
## lm(formula = y ~ x2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.64729 -0.71021 -0.06899  0.72699  2.38074
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.3451     0.1912  12.264  < 2e-16 ***
## x2            3.1190     0.6040   5.164 1.25e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.074 on 99 degrees of freedom
## Multiple R-squared:  0.2122, Adjusted R-squared:  0.2042
## F-statistic: 26.66 on 1 and 99 DF,  p-value: 1.253e-06
```

```
plot(lm.out3)
```




Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(y ~ x2)
82
55
21
−2
−1
0
1
2


Scale−Location
√|Standardized residuals|
Fitted values
lm(y ~ x2)
55
82
21
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5



This point is a high-leverage point in the model with both x1 and x2; it is both an outlier and a high-leverage point in the model with only x1; it is a high-leverage point in the model with only x2.

Problem 8

a)

```
library(MASS)
attach(Boston)
name=names(Boston)
single_coef=rep(0,13)
for(i in 2:14)
{
  print(paste('result for ',name[i],sep=''))
  lm.fit=lm(crim~Boston[,i],data=Boston)
  print(summary(lm.fit))
  single_coef[i-1]=lm.fit$coefficients[2]
}
```

zn, indus, nox, rm, age, dis, rad, tax, ptratio, black, lstat, medv have statistically significant association with crim.

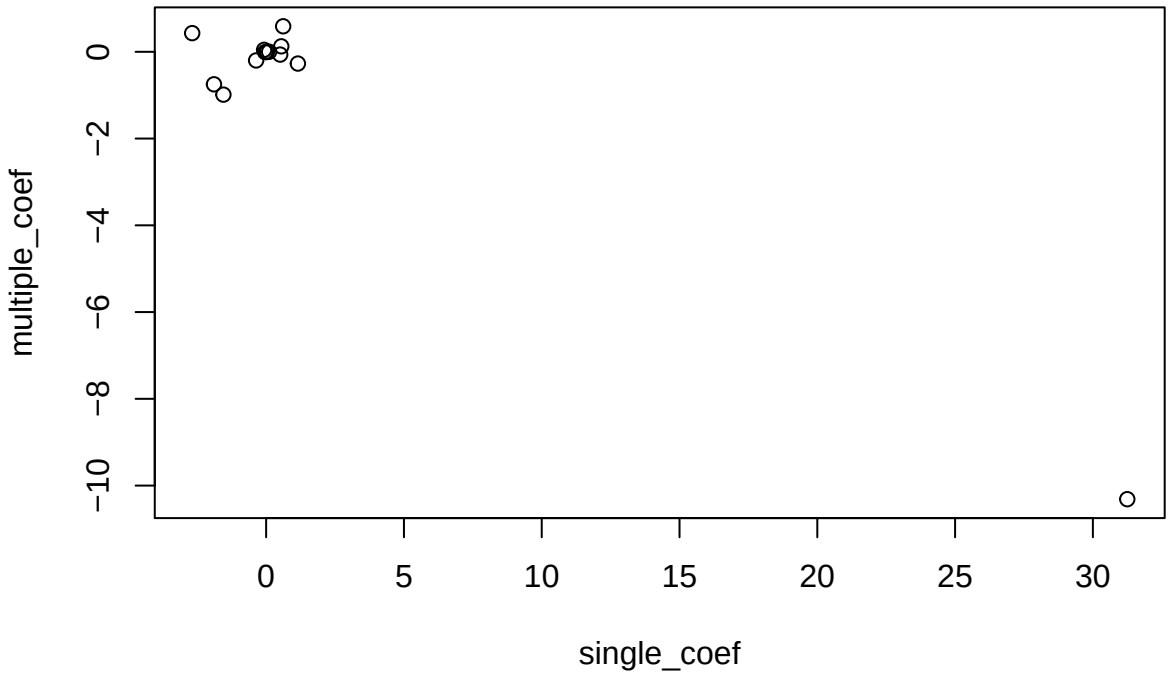
b)

```
lm.fit=lm(crim~.,data=Boston)
summary(lm.fit)
multiple_coef=lm.fit$coefficients[2:14]
```

For zn, dis, rad,black and medv, we can reject the null hypothesis $H_0 : \beta_j = 0$

c) Some of the predictors that are previously significant in a) are no-longer significant in b)

```
plot(x=single_coef,y=multiple_coef)
```



d)

```
for(i in c(2:14))
{
  print(paste('result for ',name[i],sep=''))
  lm.fit=lm(crim~poly(Boston[,i],3,raw=T),data=Boston)
  print(summary(lm.fit))
}
```

For indus, nox, age, dis, ptratio, medv, they have non-linear association with the response crim.